Tests for Boer-Mulders, Sivers and transversity distributions in SIDIS

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Plan of the talk:

1. Test of the BM and Sivers relation
2. Test of factorization via BM and Sivers asymmetries
3. always difference asymmeries
4. the tests involve only measurable quanities, no TMDs

The difference asymmetries

$$A^{h^+} = \frac{\Delta\sigma^{h^+}}{\sigma^{h^+}},\; A^{h^-} = \frac{\Delta\sigma^{h^-}}{\sigma^{h^-}},\; \Rightarrow\; A^{h^+-h^-} = \frac{\Delta\sigma^{h^+-h^-}}{\sigma^{h^+-h^-}}$$

$$A^{h^+-h^-} = \frac{1}{1-r}\left(A^{h^+} - r\,A^{h^-}\right), \qquad r = \frac{\sigma^{h^-}}{\sigma^{h^+}}$$

needed: the same A^{h^+} and A^{h^-} + multiplicities σ^{h^-} and σ^{h^-}

$\Rightarrow$ it's not a new measurement!

Why difference asymmetries?

The goal: $f_q =?$

The usual asymmetries:

$$A^{h^+} \simeq \sum_q f_q \cdot D_q^{h^+}$$
$$A^{h^-} \simeq \sum_q f_q \cdot D_q^{h^-}$$

$$q = u_V,\, d_V, \bar{u},\, \bar{d},\, s, \bar{s}$$

too many unknowns:

1. we cannot avoid assumptions on $\bar{q}$!

2. or assume relations b/n PDFs

The difference asymmetries:

E.Ch. & E. Leader

- $A^{h-\bar{h}}$ determine **only** valence quark densities:

$$\Delta u_V = \Delta u - \Delta\bar{u}, \quad \Delta d_V = \Delta d - \Delta\bar{d}$$

but without any assumptions on $\bar{q}$, used general syms. of str. ints. only:

$$C - inv: \qquad D^h_{\bar{q}} = D^{\bar{h}}_q, \qquad D^h_G = D^{\bar{h}}_G$$

$$SU(2) - inv. \qquad D^{\pi^+}_u = D^{\pi^-}_d, \qquad D^{\pi^+}_s = D^{\pi^-}_s$$

true in LO and NLO – based only on symms.

Measured in the collinear picture: $\Delta q = ?$

- polarized SIDIS: $\overrightarrow{l} + \overrightarrow{N} \to l' + h^{\pm} + X$ – only $z_h \simeq E_h$ & $x_{_B}$ measured
  - A_{1N}^h determine Δq & $\Delta\bar{q}$:

$$A_{1N}^h = \frac{\Delta\sigma^h}{\sigma^h} \simeq e_q^2 \left(\Delta q D_q^h + \Delta\bar{q} D_{\bar{q}}^h\right)$$

but assumptions on $\Delta\bar{q}$: $\Delta\bar{u} = \Delta\bar{d} = \Delta\bar{s}$, $\quad \frac{\Delta\bar{u}}{\bar{u}} = \frac{\Delta\bar{d}}{\bar{d}} = \frac{\Delta\bar{s}}{\bar{s}}$

- 2008 COMPASS: measured the diff. asymms:

$$A_{1d}^{h^+-h^-} = \frac{\Delta\sigma^{h^+-h^-}}{\sigma^{h^+-h^-}} \simeq \frac{\Delta u_V + \Delta d_V}{u_V + d_V}, \qquad q_V = q - \bar{q}$$

showed:

1) $\Delta\bar{u} = \Delta\bar{d} = \Delta s = \Delta\bar{s}$ disfavoured at 2 σ

2) $\Delta s \neq 0$, $\Delta\bar{u} = -\Delta\bar{d}$??

Now we ask:

What happens in the non-collinear picture?

the direction of h is also measured: z_h, P_T and ϕ_h

The parton model

- the parton model formulated the collinear picture:
  - q in a relativ. nucleon are almost free
  - the q-momenta are parallel to proton momenta:

$$\overrightarrow{p}_q = x_{_B}\overrightarrow{P}, \qquad x_{_B} = \frac{Q^2}{2(Pq)}$$

collinear picture: 3 distribution functions describe the nucleon:

$$\begin{array}{ll} q\ in\ p: & q(x) = f_q(x) = \overrightarrow{q} + \overleftarrow{q} \\ \overrightarrow{q}\ in\ \overrightarrow{p}: & \Delta q(x) = g(x) = \overrightarrow{q} - \overleftarrow{q} \Rightarrow helicity \\ q^{\uparrow}\ in\ p^{\uparrow}: & \Delta_T q(x) = h(x) = q^{\uparrow} - q^{\downarrow} \Rightarrow transversity \end{array}$$

But what about transverse momenta of q:

$$\overrightarrow{p}_q = x_B \overrightarrow{P} + \overrightarrow{k}_\perp$$

the collinear picture is only an approximations!

The non-collinear picture - the TMD distributions

What changes at $\vec{k}_\perp \neq 0$?

1. **PDF** $\Rightarrow$ **TMD-PDF** = transverse momentum dependent:

$$q(x) \to q(x, k_\perp)$$
$$\Delta q(x) \to \Delta q(x, k_\perp)$$
$$\Delta_T q(x) \to \Delta_T q(x, k_\perp)$$

$(x, \vec{k}_\perp)$ = the long. & transv. moment of q with respect to the proton momentum

2. + 5 more TMD distributions $\Delta^N f_q^J(x, k_\perp) \neq 0$ only at $\vec{k}_\perp \neq 0$:

these TMD's are completely new objects

The TMD distributions with $\vec{k}_\perp \neq 0$:

- **The Sivers function = unpolarized q in transv. pol. protons $p^\uparrow$:**

$$\text{only if } \vec{k}_\perp \neq 0 : p^\uparrow = (q^\uparrow + \underbrace{q}_{Sivers}) \quad \Rightarrow \quad \underbrace{\Delta^N f_{q/p^\uparrow}(x_{_B}, k_\perp)}_{Sivers} (\vec{S}_T \cdot \vec{P} \times \vec{k}_\perp)$$

- **The Boer-Mulders (BM) distributions = $q^\uparrow$ in unpol. proton:**

$$\text{only if } \vec{k}_\perp \neq 0 : p = (q + \underbrace{q^\uparrow}_{BM}) \quad \Rightarrow \quad \underbrace{\Delta f_{q^\uparrow, s_q/p}(x_{_B}, k_\perp)}_{BM} (\vec{s}_q \cdot \vec{P} \times \vec{k}_\perp)$$

- **BM and Siv. $\simeq (\vec{s} \cdot \vec{P} \times \vec{k}_\perp)$ = T-odd:**

$$\mathrm{T-odd}: \qquad t \to -t : \quad (\vec{s}_q \cdot \vec{P} \times \vec{k}_\perp) \to -(\vec{s}_q \cdot \vec{P} \times \vec{k}_\perp)$$

$$\mathrm{T-inv}: \qquad (t \to -t) + (|i> \leftrightarrow <f|)$$

Are they $\neq 0$? How big are they?

Fragmentation Functions (FF)

D_q^h = the probability for q to fragment into h:

$$D_q^h(z): \qquad q(p_q) \to h(P_h)$$

collinear picture: hadron mom. parallel to quark momenta:

$$\vec{P}_h = z_h\, \vec{p}_q, \qquad z_h = \frac{(PP^h)}{(Pq)} \simeq E^h$$

but in general $\vec{P}_h$ and $\vec{p}_q$ are not collinear:

$$\vec{P}_h = z_h\, \vec{p}_q + \vec{p}_\perp \;\Rightarrow\; D_q^h(z) \to D_q^h(z, p_\perp)$$

$(z, \vec{p}_\perp)$ = the energy & transv. moment of h with respect to the momentum $\vec{p}_q$ of the fragm. q

- TMD Collins function = $q^\uparrow \to h$:

$$\text{only if } \vec{p}_\perp \neq 0: \qquad (q + \underbrace{q^\uparrow}_{Collins}) \to h \;\Rightarrow\; \underbrace{\Delta D_{q^\uparrow}^h (z, p_\perp)}_{Collins}(\vec{s}_q \cdot \vec{p}_q \times \vec{p}_\perp)$$

How to measure TMDs

We consider semi-inclusive DIS (SIDIS):

$$\begin{aligned} l + N &\to l' + h + X \\ l + N^{\uparrow} &\to l' + h + X, \qquad h = \pi^{\pm}, K^{\pm}, h^{\pm} \end{aligned}$$

In the collinear picture: $d\sigma^h = d\sigma^h(x_{_B}, z_h)$

In the non-collinear picture: $d\sigma^h = d\sigma^h(x_{_B}, z_h, \phi_h)$

TMDs induce azimuthal depends. on ϕ_h:

$$\begin{aligned} d\sigma &= d\sigma_0 \left\{ 1 + A^h_{BM} \cos 2\phi_h + ... \right. \\ &\left. + S_T \left[A^h_{Siv} \sin(\phi_s - \phi_h) + A^h_{Coll} \sin(\phi_s + \phi_h) + .. \right] \right\} \end{aligned}$$

- no azimuthal dependence in the collinear picture

Question:

- How to determine the new TMD's: $\Delta^N f_q^J$ =?

$$A_J(x_{_B}, z_h, Q^2, P_T) \simeq \sum_q \Delta^N f_q^J \otimes \Delta^N D_q^h, \qquad q = u_V,\, d_V,\, s,\, \bar{s}, \ldots$$

we need to simplify: different assumptions:

- assumptions about the sea-quark $\Delta f_{\bar{q}}$:

$$\Delta f_{\bar{u}} = \Delta f_{\bar{d}} = \Delta f_{\bar{s}},\ or\ \Delta f_{\bar{q}} = 0,\ etc.$$

- relations:

$$\Delta f_{_{BM}} = \lambda_q \, \Delta f_{Siv}, \ldots$$

and simple - the standard, parametrizations:

- $\Delta^N f_q(x_{_B}, k_\perp, Q^2) \simeq \Delta f_q(x_{_B}, Q^2) \cdot h(k_\perp) \Rightarrow x_{_B}$ and $k_\perp$ factorize
- $h(k_\perp)$ is flavour independent
- $h(k_\perp)$ is Gaussian $\simeq e^{-k_\perp^2/<k_\perp^2>}$
- $\Delta f_q(x_{_B}, Q^2) \propto \mathcal{N}_q(x_{_B})\, q(x_{_B}, Q^2) \;\Rightarrow\; Q^2$ is in the collin. PDFs

$$\Delta^N f_J^q(x_{_B}, k_\perp, Q^2) \simeq \mathcal{N}_q(x_{_B}) \; \underbrace{q(x_{_B}, Q^2)}_{DGLAP\,\text{evol.}} \; \underbrace{e^{-k_\perp^2/<k_\perp^2>_J}}_{\text{no}\, Q^2, x_B, q}$$

The problem:

different assumptions lead to different TMD's

We ask:

- Can we avoid the assumptions on $\bar{q}$ etc?
- Can we check $\Delta f_{_{BM}} = \lambda_q \, \Delta f_{Siv}$
- ...
- Can we check the standard parametrization?
  - factorization?
  - the Gaussian-form?
  - $h(k_\perp)$ is flv. independent?
  - ...

We ask: How can can difference asymmetries help?

We show (as expected):

- $d\sigma^{h-\bar{h}}$ keep the same ϕ_h-dependence – the same azim. A_J:
- but – only u_V & d_V enter A_J – no assumps. on $\bar{q}$
- on $d = p + n$ → only $Q_V = u_V + d_V$

also we obtain (!! new !!)

tests for the params. of Δf_q through relations b/n the asymms:

- relations b/n BM and Collins asymmetries
- relations b/n BM and Sivers asymmetries

Here we present: these relations in SIDIS on $d = p + n$

Test of assumption on BM function

assumption: BM is proportional to Siv. function:

$$\bullet\, \Delta f^{BM}_{q\Uparrow/p}(x_{_B}, k_\perp) = \lambda_q\, \Delta f^{Siv}_{q/p^\uparrow}(x_{_B}, k_\perp), \qquad \lambda_q = const$$

we show:

1. on $d = p + n$, $A_{_{BM}}(x_{_B})$ is proportional to $A_{Siv}(x_{_B})$:

$$A^{h-\bar{h}}_{BM}(x_{_B}) = \mathcal{C}^h(x_{_B})\, A^{h-\bar{h}}_{Siv}(x_{_B}), \qquad h = \pi^+, K^+, h^+$$

$\mathcal{C}^h$ = indep. of Δf^{BM} and Δf^{Siv}, but deps. on FFs.

$$recall: \quad d\sigma = d\sigma_0 \left\{1 + A^h_{BM}\,\cos 2\phi_h + .. + S_T\left[A^h_{Siv}\,\sin(\phi_s - \phi_h) + ...\right]\right\}$$

2. if Q^2-evol. can be neglected - very simple:

$$\begin{aligned} A_{BM}^{h^+-h^-}(x_{_B}) &= C^h \phi(x_{_B})\, A_{Siv}^{h^+-h^-}(x_{_B}) \\ A_{BM}^{\pi^+-\pi^-}(x_{_B}) &= C^\pi \phi(x_{_B})\, A_{Siv}^{\pi^+-\pi^-}(x_{_B}) \\ A_{BM}^{K^+-K^-}(x_{_B}) &= C^K \phi(x_{_B})\, A_{Siv}^{K^+-K^-}(x_{_B}) \end{aligned}$$

C = constant

$\phi(x_{_B})$ = the same $\forall\, h$, completely known function of kinem. vars:

$$\phi(x_{_B}) = \frac{\int dQ^2\,(1-y)/Q^4}{\int dQ^2\,(1+(1-y)^2)/Q^4}, \qquad y = \frac{Q^2}{sx}$$

- test of the assumption on theo. TMDs only thru meas. asymms.

Why is this possible?

- **on $d = p + n$ only $Q_V = (u_V + d_V)$ enters:**

$$BM: \sum \Delta f_{q^\uparrow/p} \otimes \Delta_T D^h_{q^\uparrow} \quad \rightarrow \quad \Delta f_{Q_V^\uparrow/p} \otimes \Delta_T D^h_{u_V^\uparrow}$$

$$Siv.: \sum \Delta f_{q/p^\uparrow} \otimes D^h_q \quad \rightarrow \quad \Delta f_{Q_V/p^\uparrow} \otimes D^h_{u_V}$$

$$\Delta f_{Q_V/p}(x_B, k_\perp) \equiv \Delta f_{(u_V+d_V)/p}(x_B, k_\perp)$$

- **PDF & FFs factorize - cancelations possible**

Test with COMPASS data

E.Ch., E.Leader, M.Stoilov

COMPASS data, 2007, 2014, $E_\mu = 160$ GeV:

$$\mu + d \to \mu + h^\pm + X, \qquad \mu + d^\uparrow \to \mu + h^\pm + X$$

$$A_{Siv}^{h^\pm}(x,Q^2) \simeq \frac{\int d\phi_h d\phi_s dz(\sigma_\uparrow^h - \sigma_\downarrow^h)\,\sin(\phi_h - \phi_S)}{\int d\phi_h d\phi_s dz(\sigma_\uparrow^h + \sigma_\downarrow^h)}, \qquad A_{BM}^{h^\pm}(x,Q^2) \simeq \frac{\int d\phi_h dz \sigma^h\,\cos(2\phi_h)}{\int d\phi_h dz \sigma^h}$$

$$0,003 < x < 0,13, \qquad 1 < Q^2 < 16.7\, GeV^2$$

In 3 steps:

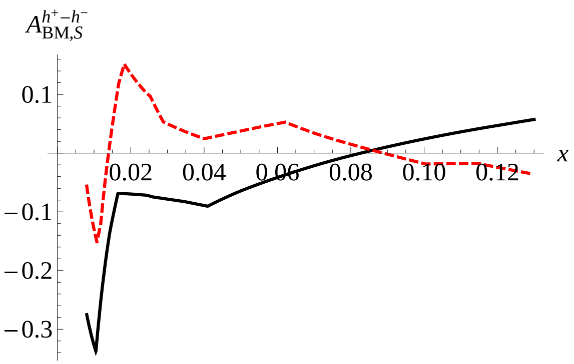
1. **form the diff. asymmetry:**

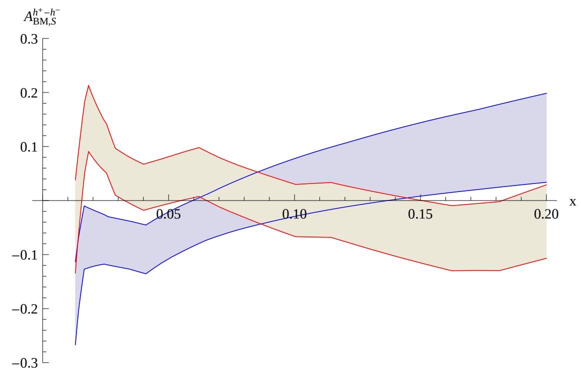
$$A^{h^+ - h^-} = \frac{1}{1-r}\left(A^{h^+} - r A^{h^-}\right), \qquad r = \frac{\sigma^{h^+}}{\sigma^{h^-}}$$

2. **choose the interval Q^2:** $Q_V(x,Q^2) \simeq Q_V(x)$, $\qquad D_{u_V}(z,Q^2) \simeq D_{u_V}(z)$
3. C=?

1. form the difference asymmetries with COMPASS data

$A_{BM}^{h^+-h^-}$= full line, and $A_{Siv}^{h^+-h^-}$ = dashed

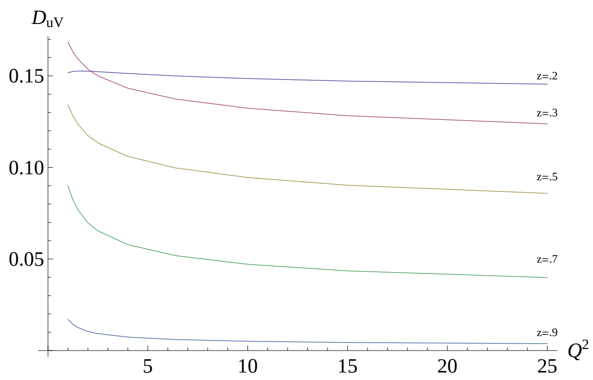




at $x > 0.02$: $A_{BM}^{h^+-h^-}$ and $A_{Siv}^{h^+-h^-}$ with opposite signs

2. choose the interval in Q^2

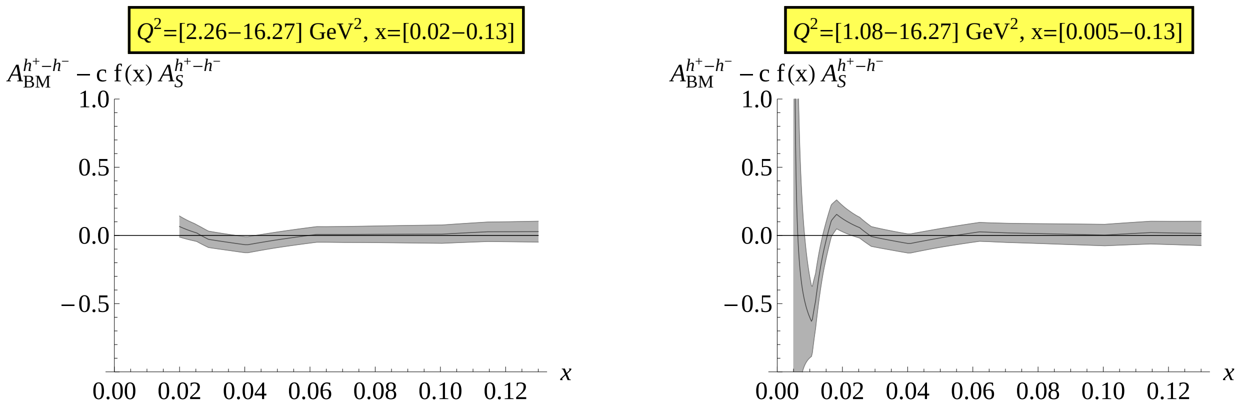
$D_{uV}(Q^2)$



choose 2 intervals: $Q^2 \in [2.26 - 16.27] GeV^2$ **and** $Q^2 \in [1.08 - 16.27] GeV^2$

$$A_{BM}(x) - C^h \phi(x) A_{Siv}(x)$$

3. C=fitting parameter



$Q^2 \in [2.26 - 16.27] GeV^2 \rightarrow C = -1.60$ and $Q^2 \in [1.08 - 16.27] GeV^2 \rightarrow C = -2.26$

we obtain:

the relation b/n BM and Siv is OK for $x > 0.02 \rightarrow C = -1.60$

Relation b/n Collins and BM asymms.

Common for Collins and BM asymms. – the same $\Delta^N D_{h^+/u_V\uparrow}$:

$$Coll: \quad A^{h-\bar{h}}_{Coll}(x_{_B}, z_h) \propto h_{1Q_V}(x_{_B}, Q^2)\, \Delta^N D_{h^+/u_V\uparrow}$$

$$BM: \quad A^{h-\bar{h}}_{_{BM}}(x_{_B}, z_h) \propto \Delta^N f_{Q_V\uparrow/p}(x_{_B}, Q^2)\, \Delta^N D_{h^+/u_V\uparrow}$$

- we can elliminte $\Delta^N D_{h^+/u_V\uparrow}$: $A^{h-\bar{h}}_{_{BM}}(z_h) \Leftrightarrow A^{h-\bar{h}}_{Coll}(z_h)$

in Q^2-interval, where Q^2-evol. can be neglected: – very simple:

$\Downarrow$

Relation b/n Collins and BM asymms. for d & p:

If standard params. holds $A^{h-\bar{h}}_{BM}(z_h)$ & $A^{h-\bar{h}}_{Coll}(z_h)$ are related:

$$A^{h-\bar{h}}_{BM,d}(z_h) = \mathcal{B}^h_1\,\Phi(z_h)\,A^{h-\bar{h}}_{Coll,d}(z_h), \qquad h = \pi^{\pm},\, K^{\pm}$$

$$A^{h-\bar{h}}_{BM,p}(z_h) = \mathcal{B}^h_2\,\Phi(z_h)\,A^{h-\bar{h}}_{Coll,p}(z_h)$$

$$A^{h-\bar{h}}_{BM,p}(z_h) = \mathcal{B}^h_3\,\Phi(z_h)\,A^{h-\bar{h}}_{Coll,d}(z_h)$$

- $\Phi(z_h)$ = the same, fixed by Gaussian $\Rightarrow \langle k_\perp^2\rangle$, $\langle k_\perp^2\rangle_s$, $\langle p_\perp^2\rangle_c$ = known:

$$\Phi(z_h) = \frac{z_h\sqrt{\langle p_\perp^2\rangle_C + z_h^2\langle k_\perp^2\rangle}}{\langle p_\perp^2\rangle_C + z_h^2\langle k_\perp^2\rangle_S}, \qquad \mathcal{B}^h = const$$

- Tests for the QCD picture and paramtrs. of TMDs without requiring knowledge about the TMDs!

Relation for Sivers asymms. on $d = p + n$:

when Q^2-evol. can be neglected:

$$1.\ A_{Siv}^{h-\bar{h}}(z_h, \bar{Q}^2) = B(\bar{Q}^2)\, \frac{z_h}{\langle p_\perp^2\rangle + z_h \langle k_\perp^2\rangle_S}, \qquad h = \pi,\ K,\ h$$

- a fixed z_h-dependence, the same for all h
- B =const, the same for all $h \Rightarrow k_\perp, p_\perp = h$ & q -indep.

$$2.\ A_{Siv}^{h-\bar{h}}(x_{_B}, \bar{Q}^2) = C^h(\bar{Q}^2)\, \mathcal{N}_{Siv}^{Q_V}(x_{_B}), \qquad h = \pi,\ K,\ h$$

- the same $\mathcal{N}_{Siv}^{Q_V}(x_{_B})$
- $A_{Siv}^{\pi^+-\pi^-}(x_{_B}) = C_1\, A_{Siv}^{K^+-K^-}(x_{_B}) = C_2\, A_{Siv}^{h^+-h^-}(x_{_B})$

All started with "The spin crisis of the proton"

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L$$

$\Delta\Sigma =$ spin carried by the quarks:

$$\Delta\Sigma = (\Delta u + \Delta\bar{u}) + (\Delta d + \Delta\bar{d}) + (\Delta s + \Delta\bar{s})$$

ΔG = spin carried by the gluons
L = quark and gluon orbital momenta

1988, EMC at CERN – measured $\Delta\Sigma$:

$$\Delta\Sigma = 0.12 \pm 0.17$$

"spin crisis": $\Delta\Sigma_{TH} \simeq 1$

$$2009: \qquad \Delta\Sigma = 0.33 \pm 0.03 \pm 0.05, \quad |\Delta G| < 0.3$$

The problem still remains:

Who carries the spin of the proton?: $\Delta q_V =?$, $\Delta\bar{q} =?$, the role of Δs-quarks

Summary

TMDs are new and unexpected objects that may help!

exp. measurements of TMDs in nucleon have just started:
SIDIS: COMPASS at CERN, HERMESS at DESY, JLab; etc.

the goal: **to determine the TMDs correctly**

$A^{h-\bar{h}}$ **are important $\Rightarrow$ we obtain:**

- **general and simple relations that test the stand. param.**
- **with no assumptions**
- **if they don't hold $\Rightarrow$ s.thing is wrong with the stand. param.**

Thanks!