

GPD phenomenology: towards the extraction of nucleon tomography and energy-momentum form factors

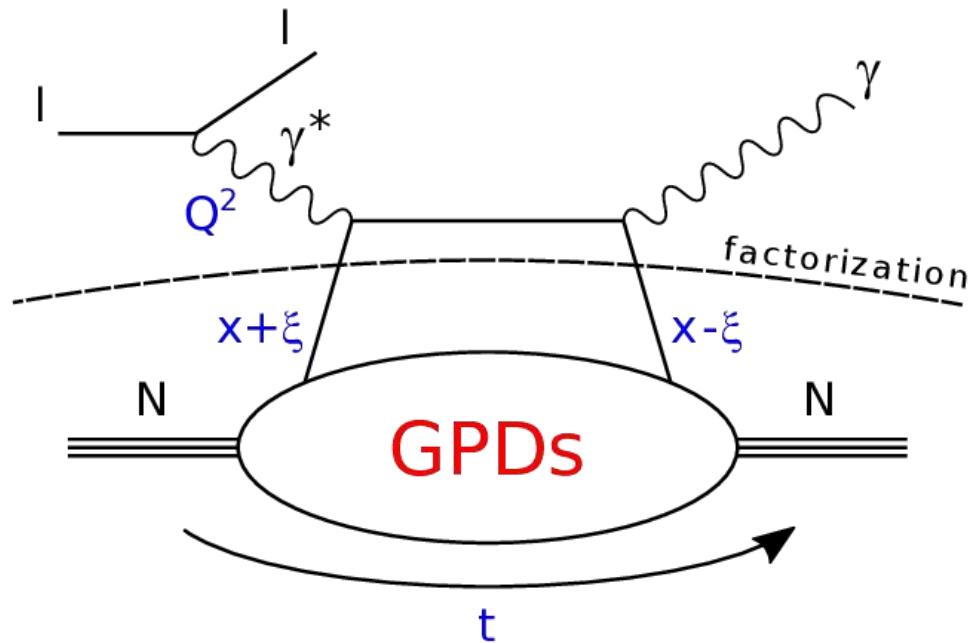
Paweł Sznajder
National Centre for Nuclear Research, Warsaw



COMPASS Analysis Meeting Seminar
CERN, November 14, 2018

- Introduction
- PARTONS project
- Global analysis of DVCS
- Summary

Deeply Virtual Compton Scattering (DVCS)

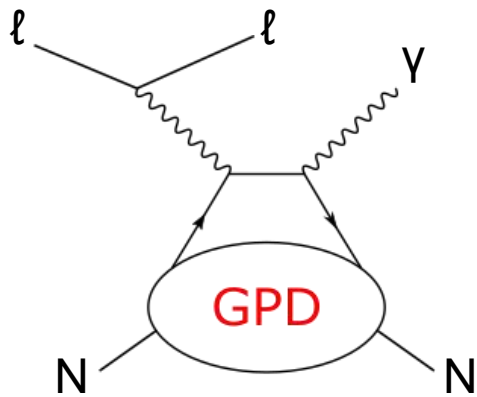


factorization for $|t|/Q^2 \ll 1$

Chiral-even GPDs:
(helicity of parton conserved)

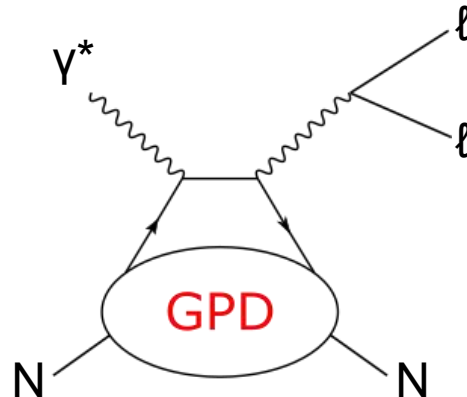
$H^{q,g}(x, \xi, t)$	$E^{q,g}(x, \xi, t)$	for sum over parton helicities
$\tilde{H}^{q,g}(x, \xi, t)$	$\tilde{E}^{q,g}(x, \xi, t)$	for difference over parton helicities
nucleon helicity conserved	nucleon helicity changed	

GPDs accessible in various production channels and observables
→ **experimental filters**



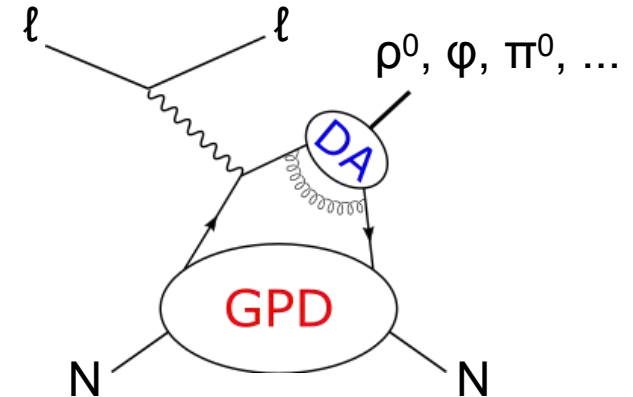
DVCS

*Deeply Virtual Compton
Scattering*



TCS

*Timelike Compton
Scattering*



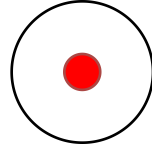
HEMP

*Hard Exclusive Meson
Production*

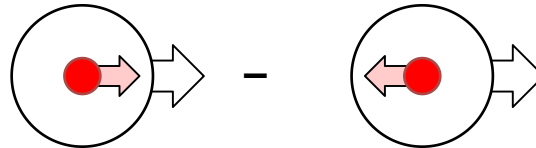
more production channels sensitive to GPDs exist!

■ Reduction to PDFs:

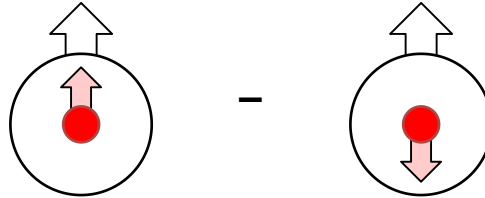
$$H^q(x, 0, 0) \equiv q(x)$$



$$\tilde{H}^q(x, 0, 0) \equiv \Delta q(x)$$



$$H_T^q(x, 0, 0) \equiv h_1(x)$$



no corresponding relations exist for other GPDs

■ Reduction to Elastic Form Factors (EFFs):

$$\int_{-1}^1 dx H^q(x, \xi, t) \equiv F_1^q(t)$$

$$\int_{-1}^1 dx E^q(x, \xi, t) \equiv F_2^q(t)$$

$$\int_{-1}^1 dx \tilde{H}^q(x, \xi, t) \equiv g_A^q(t)$$

$$\int_{-1}^1 dx \tilde{E}^q(x, \xi, t) \equiv g_P^q(t)$$

Polynomiality - non-trivial consequence of Lorentz invariance:

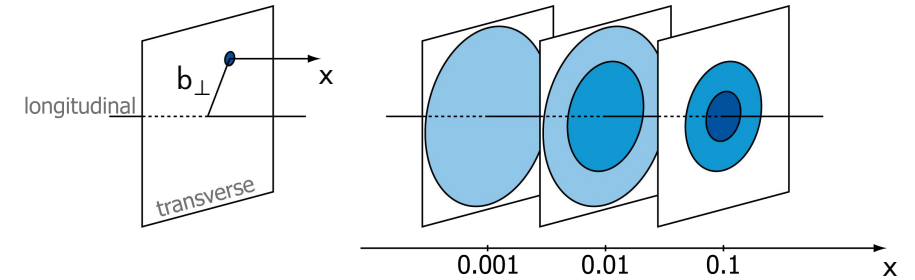
$$\int_{-1}^1 dx \, x^n H^q(x, \xi, t) = h_0^{q,n}(t) + \xi^2 h_2^{q,n}(t) + \dots + \text{mod}(n, 2) \xi^{n+1} h_{n+1}^{q,n}(t)$$

$$\int_{-1}^1 dx \, x^n \tilde{H}^q(x, \xi, t) = \tilde{h}_0^{q,n}(t) + \xi^2 \tilde{h}_2^{q,n}(t) + \dots + \text{mod}(n+1, 2) \xi^n \tilde{h}_n^{q,n}(t)$$

strong constraint on GPD parameterizations

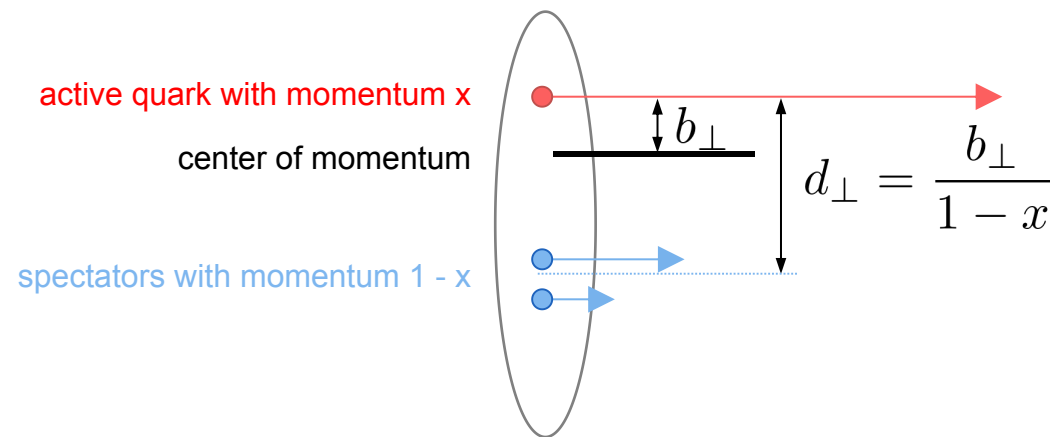
- Nucleon tomography

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta}{4\pi^2} e^{-i\mathbf{b}_\perp \cdot \Delta} H^q(x, 0, t = -\Delta^2)$$



- Study of long. polarization with GPD $\tilde{H}$
- Study of distortion in transv. polarized nucleon with GPD E

- Impact parameter $\mathbf{b}_\perp$ defined w.r.t. center of momentum, such as $\sum x \mathbf{b}_\perp = 0$

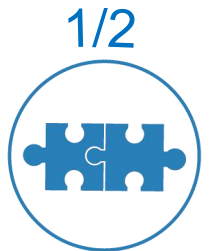


Energy momentum tensor in terms of form factors:

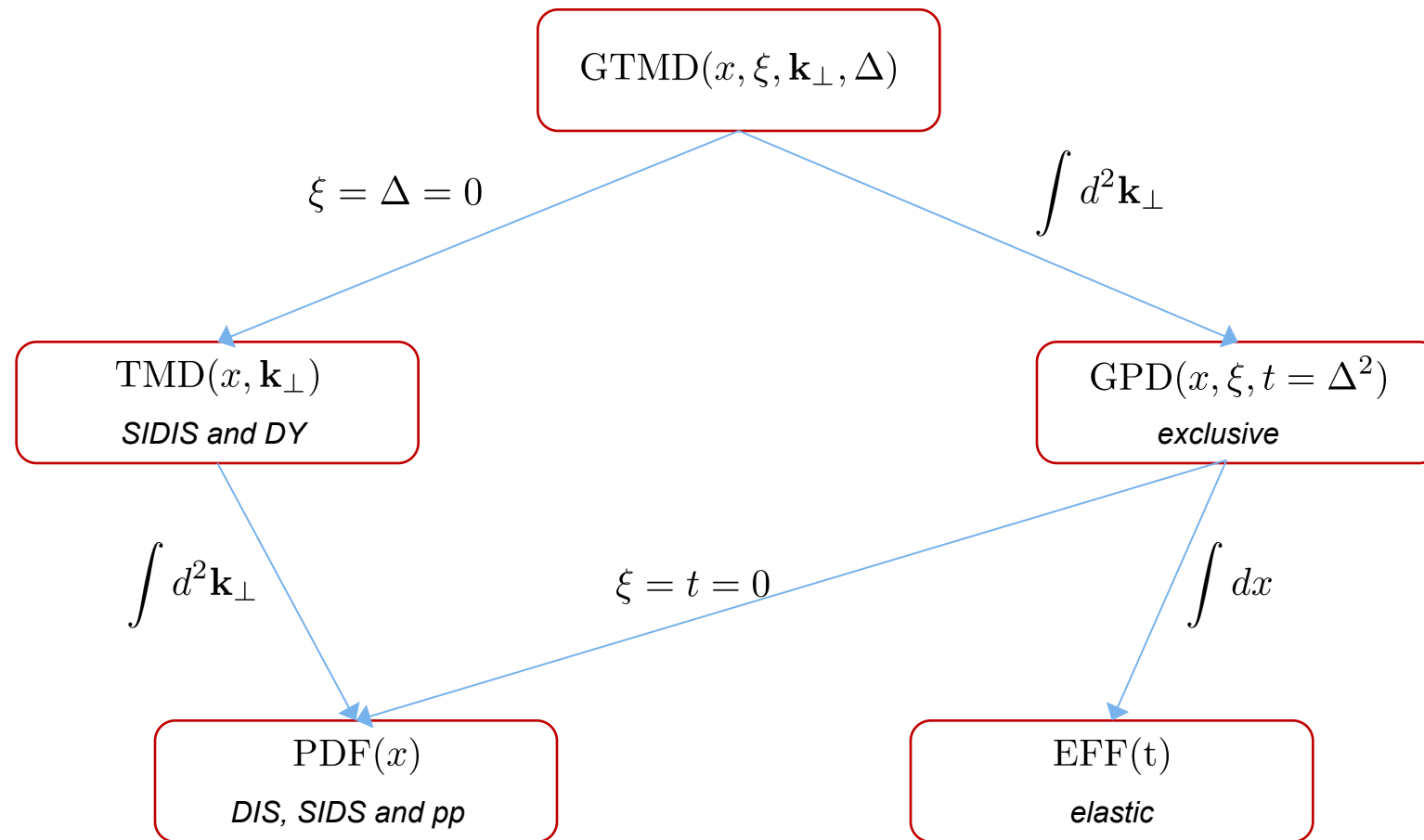
$$\langle p', s' | \hat{T}^{\mu\nu} | p, s \rangle = \bar{u}(p', s') \left[\frac{P^\mu P^\nu}{M} A(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu} \Delta^2}{M} C(t) + M \eta^{\mu\nu} \bar{C}(t) + \right. \\ \left. \frac{P^\mu i \sigma^{\nu\lambda} \Delta_\lambda}{4M} [A(t) + B(t) + D(t)] + \frac{P^\nu i \sigma^{\mu\lambda} \Delta_\lambda}{4M} [A(t) + B(t) - D(t)] \right] u(p, s)$$

Access to total angular momentum and forces acting on quarks

$$A^q(0) + B^q(0) = \int_{-1}^1 x [H^q(x, \xi, 0) + E^q(x, \xi, 0)] = 2J^q$$



Ji's sum rule



H. Moutarde, P. S., J. Wagner "*Border and skewness functions from a leading order fit to DVCS data*"
Eur. Phys. J. C78 (2018), 890

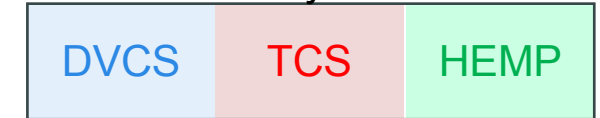
Goal: global extraction of Compton Form Factors (CFFs) from DVCS data using LO/LT formalism

Analysis done within **PARTONS** project

- **PARTONS** - platform to study GPDs
- Come with number of available physics developments implemented
- Addition of new developments as easy as possible
- To support effort of GPD community
- Can be used by both theorists and experimentalists
- More info in: [Eur. Phys. J. C78 \(2018\) 6, 478](#)

<http://partons.cea.fr>

Observable Layer



Process Layer



CCF Layer



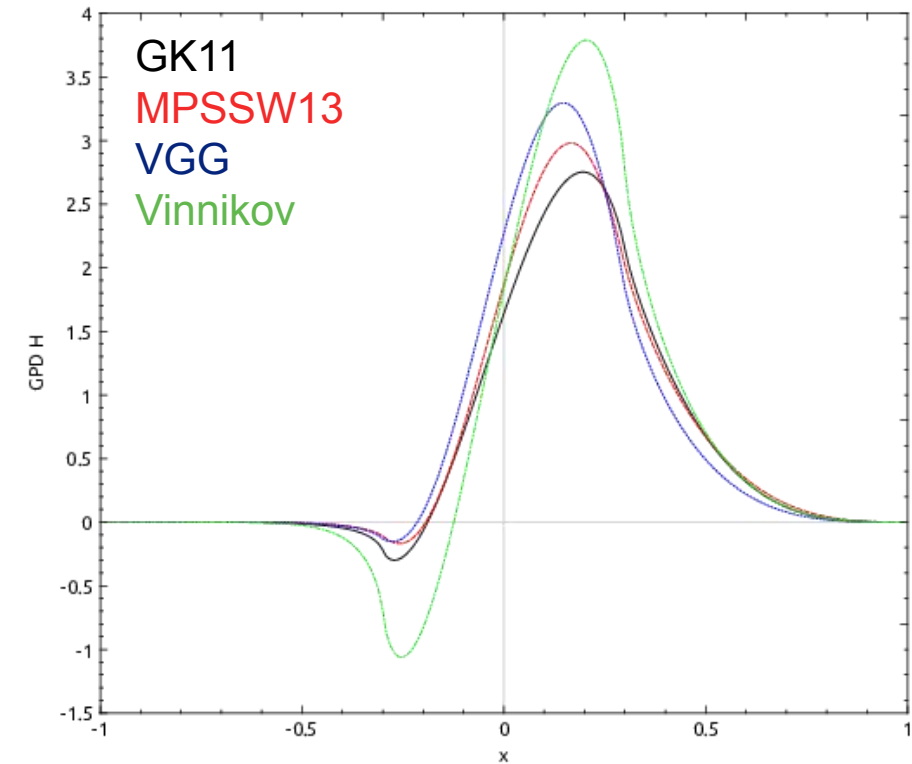
GPD Layer



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<http://partons.cea.fr>

$H^u @ x = 0.2, t = -0.1 \text{ GeV}^2, \mu_F^2 = \mu_R^2 = 2 \text{ GeV}^2$



■ imaginary part $Im\mathcal{G}(\xi, t) = \pi G^{(+)}(\xi, \xi, t) = \pi \sum_q e_q^2 G^{q(+)}(\xi, \xi, t)$

$$G^{q(+)}(x, \xi, t) = G^q(x, \xi, t) \mp G^q(-x, \xi, t)$$

$$G^{q(+)}(\xi, \xi, t) = G^{q_{\text{val}}}(\xi, \xi, t) + 2G^{q_{\text{sea}}}(\xi, \xi, t)$$

"-" for $G \in \{H, E\}$
 "+" for $G \in \{\tilde{H}, \tilde{E}\}$

■ real part

$$Re\mathcal{G}(\xi, t) = \text{P.V.} \int_0^1 G^{(+)}(x, \xi, t) \left(\frac{1}{\xi - x} \mp \frac{1}{\xi + x} \right) dx$$

$$Re\mathcal{G}(\xi, t) = \text{P.V.} \int_0^1 G^{(+)}(x, x, t) \left(\frac{1}{\xi - x} \mp \frac{1}{\xi + x} \right) dx + C_G(t)$$

$$C_H(t) = -C_E(t) \quad C_{\tilde{H}}(t) = C_{\tilde{E}}(t) = 0$$

Relation between subtraction constant and D-term:

$$C_G^q(t) = 2 \int_{-1}^1 \frac{D^q(z, t)}{1 - z} dz \equiv 4D^q(t)$$

where

$$z = \frac{x}{\xi}$$

Decomposition into Gegenbauer polynomials:

$$D^q(z, t) = (1 - z^2) \sum_{i=0}^{\infty} d_i^q(t) C_{2i+1}^{3/2}(z)$$

Connection to EMT FF:

$$D^q(t) = \sum_{\substack{i=1 \\ \text{odd}}}^{\infty} d_i^q(t)$$

$$d_1^q(t) = 5C^q(t)$$

Comparing CFFs evaluated with two methods

$$C_G^q(t) = \int_0^1 \left(G^{q(+)}(x, \xi, t) - G^{q(+)}(x, x, t) \right) \left(\frac{1}{\xi - x} - \frac{1}{\xi + x} \right) dx$$

for $\xi = 0$

$$C_G^q(t) = 2 \int_0^1 \left(G^{q(+)}(x, x, t) - G^{q(+)}(x, 0, t) \right) \frac{1}{x} dx$$

divergent integral!

but

$$C_{G,j}^q(t) = 2 \int_0^1 \left(G^{q(+)}(x, x, t) - G^{q(+)}(x, 0, t) \right) x^j dx$$

well defined for odd positive j

Subtraction constant as analytic continuation of Mellin moments to $j = -1$

$$C_G^q(t) = C_{G,-1}^q(t) = 2 \int_{(0)}^1 \left(G^{q(+)}(x, x, t) - G^{q(+)}(x, 0, t) \right) \frac{1}{x} dx$$

Analytic regularization prescription

$$\int_{(0)}^1 \frac{f(x)}{x^{a+1}} = \int_0^1 \frac{f(x) - f(0) - xf'(0) - \dots}{x^{a+1}} + f(0) \int_{(0)}^1 \frac{dx}{x^{a+1}} + f'(0) \int_{(0)}^1 \frac{dx}{x^a} + \dots =$$
$$\int_0^1 \frac{f(x) - f(0) - xf'(0) - \dots}{x^{a+1}} - \frac{f(0)}{a} - \frac{f'(0)}{a-1} + \dots$$

applicable if $f(x)$ analytic and not-vanishing at $x = 0$

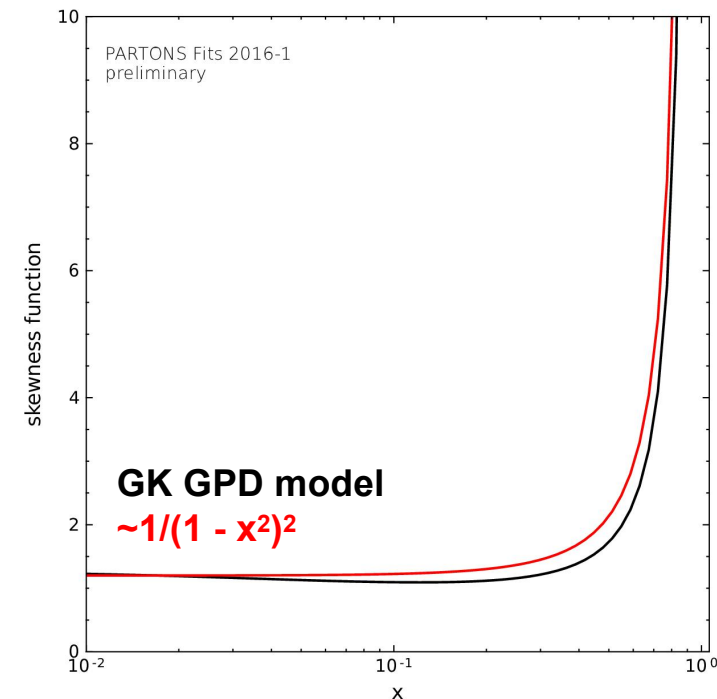
$$G^q(x, 0, t) = \text{pdf}_G^q(x) \exp(f_G^q(x)t)$$

$$f_G^q(x) = A_G^q \log(1/x) + B_G^q(1-x)^2 + C_G^q(1-x)x$$

- modify "classical" $\log(1/x)$ term by $B_G^q(1-x)^2$ in low- x and by $C_G^q(1-x)x$ in high- x regions
- polynomials found in analysis of EFF data $\rightarrow$ good description of data
- allows to use the analytic regularization prescription
- finite proton size at $x \rightarrow 1$

$$G^q(x, x, t) = G^q(x, 0, t) g_G^q(x, x, t)$$

$$g_G^q(x, x, t) = \frac{a_G^q}{(1 - x^2)^2} (1 + t(1 - x)(b_G^q + c_G^q \log(1 + x)))$$



- at $x \rightarrow 0$ constant skewness effect
- at $x \rightarrow 1$ reproduce power behavior predicted for GPDs in Phys. Rev. D69, 051501 (2004)
- t-dependence similar to DD-models with $(1-x)$ to avoid any t-dep. at $x = 1$

■ for GPD E

$$E^{q_{\text{val}}}(x, 0, t) = e^{q_{\text{val}}}(x) \exp(f_E^{q_{\text{val}}}(x)t)$$

$$e^{q_{\text{val}}}(x) = \kappa_q N_{q_{\text{val}}} x^{-\alpha_{q_{\text{val}}}} (1-x)^{\beta_{q_{\text{val}}}} (1 + \gamma_{q_{\text{val}}} \sqrt{x})$$

from Eur. Phys.J. C73 (2013) 4, 2397

$$E^{q_{\text{val}}}(x, x, t) = E^{q_{\text{val}}}(x, 0, t) g_E^{q_{\text{val}}}(x, t)$$

$$g_E^{q_{\text{val}}}(x, t) = \frac{a_E^{q_{\text{val}}}}{(1-x^2)^3}$$

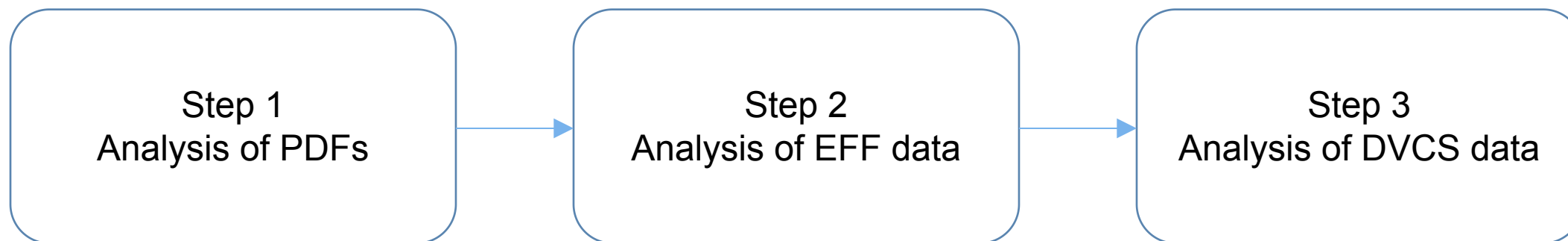
from Phys. Rev. D69, 051501 (2004)

■ for GPD $\tilde{E}$

$$\tilde{\mathcal{E}}(\xi, t) = N_{\tilde{E}} \tilde{\mathcal{E}}_{\text{GK}}(\xi, t)$$

CFF from GK GPD model

Steps of analysis:



Effectively we combine (semi-)inclusive, pp, elastic and exclusive data in a single analysis

$$H^q(x, 0, 0) \equiv q(x)$$

$$\tilde{H}^q(x, 0, 0) \equiv \Delta q(x)$$

Ansatz:

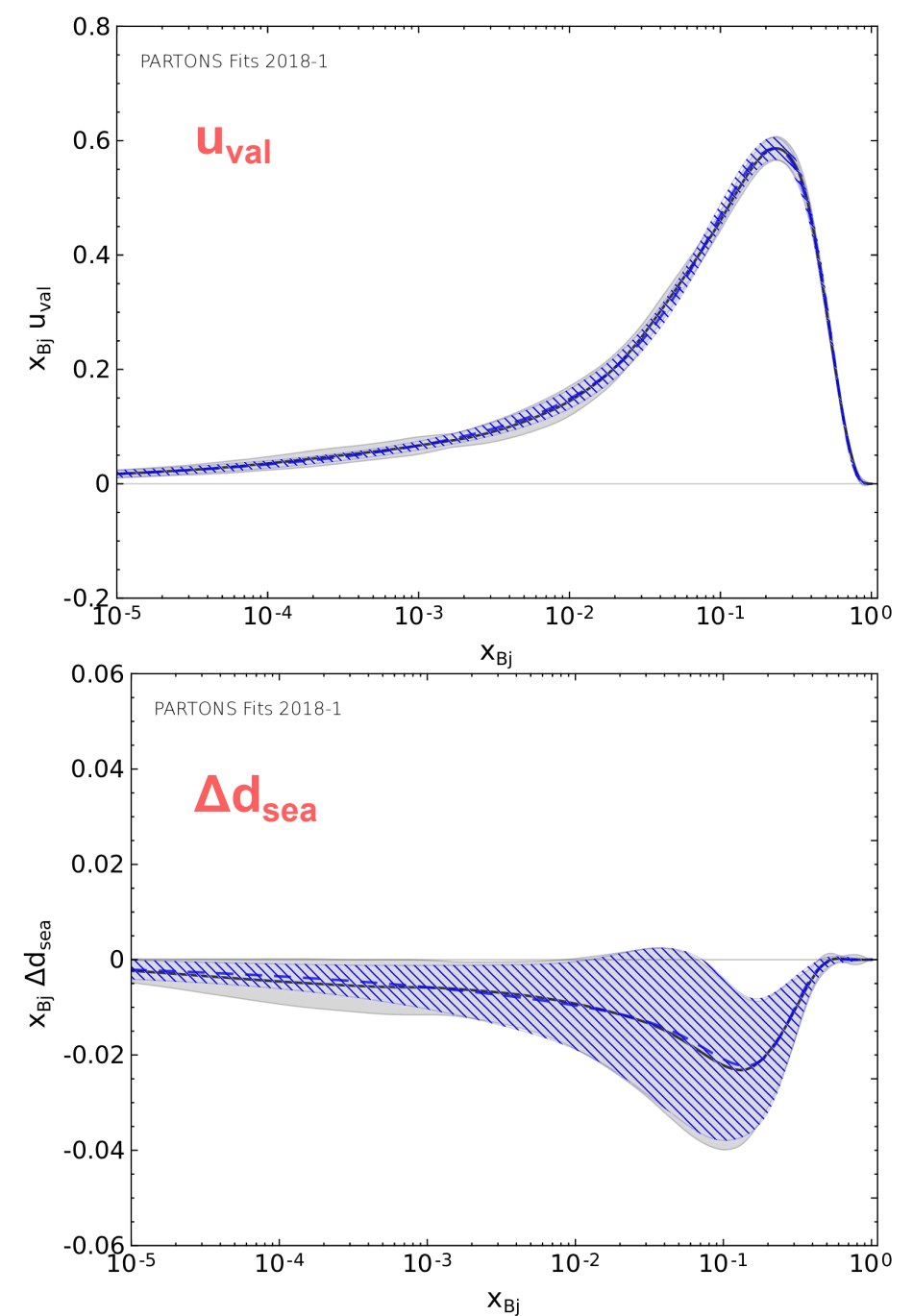
$$\text{pdf}_G(x, Q^2) = x^{-g(\delta_p, \delta_q, Q^2)} (1-x)^\alpha \sum_{i=0}^4 g(p_i, q_i, Q^2) x^i$$

$$g(p, q, Q^2) = p + q \log \frac{Q^2}{Q_0^2}$$

13 parameters:

$$\delta_p, \delta_q, \alpha, p_i, q_i \quad \text{where} \quad i = 0, 1, \dots, 4$$

constrained by NNPDF3.0 and NNPDFpol11 sets (per each flavor and each PDF replica)



Free parameters for valance quarks and GPDs H and E constrained by EFF data

$$\int_{-1}^1 dx H^q(x, \xi, t) \equiv F_1^q(t)$$

$$\int_{-1}^1 dx E^q(x, \xi, t) \equiv F_2^q(t)$$

From Dirac and Pauli partonic FFs to Sachs nucleon FFs

$$F_i^p = e_u F_i^u + e_d F_i^d \quad i = 1, 2$$

$$F_i^n = e_u F_i^d + e_d F_i^u$$

$$G_M^i = F_1^i + F_2^i \quad i = p, n$$

$$G_E^i = F_1^i + \frac{t}{4m^2} F_2^i$$

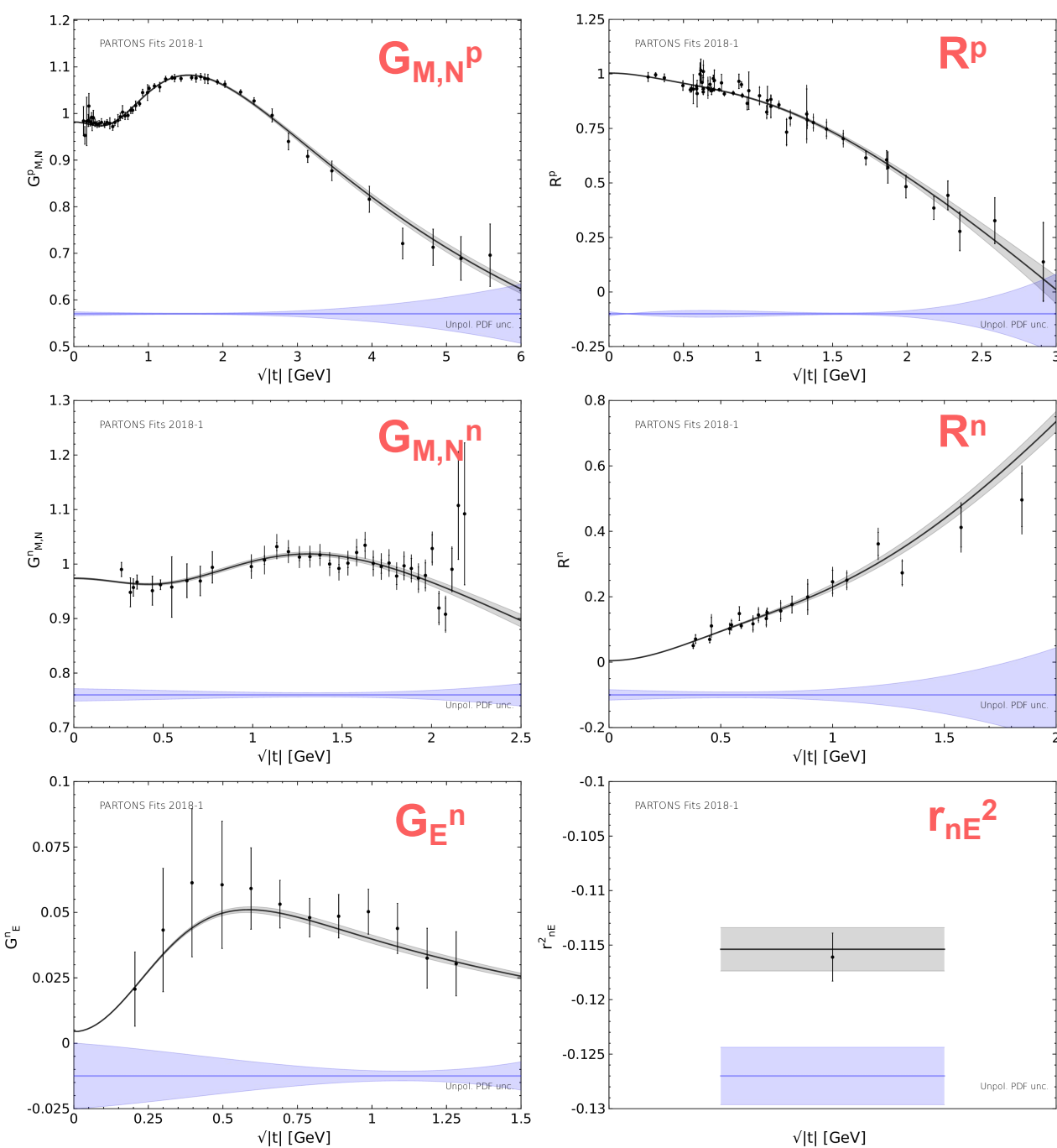
Observables

$$G_{M,N}^i(t) = \frac{G_M^i(t)}{\mu_i G_D(t)} \quad i = p, n$$

$$R^i(t) = \frac{\mu_i G_E^i(t)}{G_M^i(t)}$$

$$r_{nE}^2 = 6 \left. \frac{dG_E^n(t)}{dt} \right|_{t=0}$$

for the selection of observables and experimental data we follow *Eur. Phys.J. C73 (2013) 4, 2397*



Performance:

$$\chi^2/\text{ndf} = 129.6/(178 - 9) \approx 0.77$$

Replication of experimental data to estimate corresponding uncertainties:

$$v_i \pm \Delta_i^{\text{tot}} \xrightarrow{\text{replica } j} \text{rnd}_j(v_i, \Delta_i^{\text{tot}}) \pm \Delta_i^{\text{tot}}$$

$$\Delta_i^{\text{tot}} = \sqrt{(\Delta_i^{\text{stat}})^2 + (\Delta_i^{\text{sys}})^2}$$

Fitted values:

Parameter	Mean	Data unc.	Unpol. PDF unc.
$A_{H/E}^{u_{\text{val}}}$	0.99	0.01	0.08
$B_H^{u_{\text{val}}}$	-0.50	0.02	0.14
$A_{H/E}^{d_{\text{val}}}$	0.70	0.02	0.08
$B_H^{d_{\text{val}}}$	0.47	0.07	0.24
α	0.69	0.01	0.03
$B_E^{u_{\text{val}}}$	-0.69	0.04	0.18
$C_E^{u_{\text{val}}}$	-0.92	0.04	0.09
$B_E^{d_{\text{val}}}$	-0.54	0.06	0.20
$C_E^{d_{\text{val}}}$	-0.73	0.06	0.22

All DVCS proton data used in the fit, except:

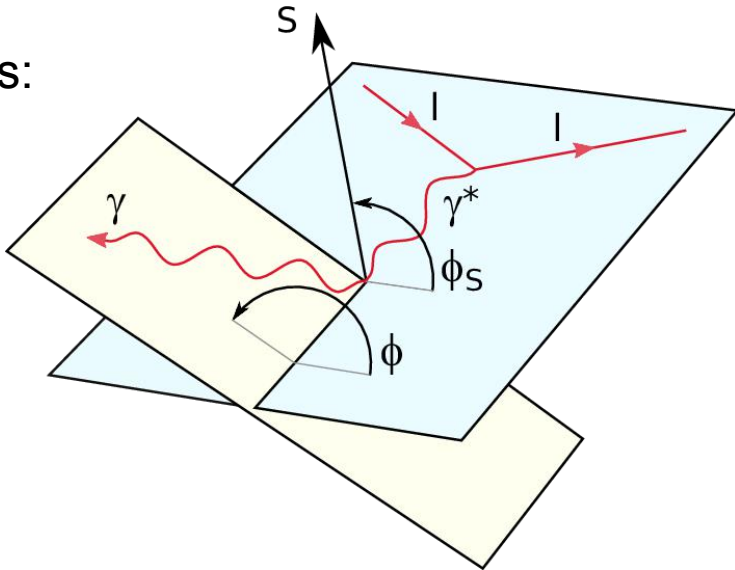
- HERA data
- Hall A cross sections

Kinematic cuts:

$$Q^2 > 1.5 \text{ GeV}^2$$

$$-t/Q^2 < 0.25$$

Angles:



No.	Collab.	Year	Observable		Kinematic dependence	No. of points used / all
1	HERMES	2001	A_{LU}^+		ϕ	10 / 10
2		2006	$A_C^{\cos i\phi}$	$i = 1$	t	4 / 4
3		2008	$A_C^{\cos i\phi}$	$i = 0, 1$	x_{Bj}	18 / 24
			$A_{UT,DVCS}^{\sin(\phi-\phi_S) \cos i\phi}$	$i = 0$		
			$A_{UT,I}^{\sin(\phi-\phi_S) \cos i\phi}$	$i = 0, 1$		
			$A_{UT,I}^{\cos(\phi-\phi_S) \sin i\phi}$	$i = 1$		
4		2009	$A_{LU,I}^{\sin i\phi}$	$i = 1, 2$	x_{Bj}	35 / 42
			$A_{LU,DVCS}^{\sin i\phi}$	$i = 1$		
			$A_C^{\cos i\phi}$	$i = 0, 1, 2, 3$		
5		2010	$A_{UL}^{+, \sin i\phi}$	$i = 1, 2, 3$	x_{Bj}	18 / 24
			$A_{LL}^{+, \cos i\phi}$	$i = 0, 1, 2$		
6		2011	$A_{LT,DVCS}^{\cos(\phi-\phi_S) \cos i\phi}$	$i = 0, 1$	x_{Bj}	24 / 32
			$A_{LT,DVCS}^{\sin(\phi-\phi_S) \sin i\phi}$	$i = 1$		
			$A_{LT,I}^{\cos(\phi-\phi_S) \cos i\phi}$	$i = 0, 1, 2$		
			$A_{LT,I}^{\sin(\phi-\phi_S) \sin i\phi}$	$i = 1, 2$		
7		2012	$A_{LU,I}^{\sin i\phi}$	$i = 1, 2$	x_{Bj}	35 / 42
			$A_{LU,DVCS}^{\sin i\phi}$	$i = 1$		
			$A_C^{\cos i\phi}$	$i = 0, 1, 2, 3$		
8		2001	$A_{LU}^{-, \sin i\phi}$	$i = 1, 2$	—	0 / 2
9	CLAS	2006	$A_{UL}^{-, \sin i\phi}$	$i = 1, 2$	—	2 / 2
10		2008	A_{LU}^-		ϕ	283 / 737
11		2009	A_{LU}^-		ϕ	22 / 33
12		2015	$A_{LU}^-, A_{UL}^-, A_{LL}^-$		ϕ	311 / 497
13		2015	$d^4\sigma_{UU}^-$		ϕ	1333 / 1933
14		2015	$\Delta d^4\sigma_{LU}^-$		ϕ	228 / 228
15	Hall A	2017	$\Delta d^4\sigma_{LU}^-$		ϕ	276 / 358
16	COMPASS	2018	b		—	1 / 1
SUM:						2600 / 3970

Performance:

$$\chi^2/\text{ndf} = 2346.3/(2600 - 13) \approx 0.91$$

No.	Collab.	Year	χ^2	n	χ^2/n
1	HERMES	2001	9.8	10	0.98
2		2006	2.9	4	0.72
3		2008	24.2	18	1.35
4		2009	40.1	35	1.15
5		2010	40.3	18	2.24
6		2011	14.5	24	0.60
7		2012	25.4	35	0.73
8	CLAS	2001	—	0	—
9		2006	0.9	2	0.47
10		2008	371.1	283	1.31
11		2009	36.4	22	1.66
12		2015	351.4	311	1.13
13	Hall A	2015	937.9	1333	0.70
14		2015	220.2	228	0.97
15		2017	258.8	276	0.94
16	COMPASS	2018	10.7	1	10.67

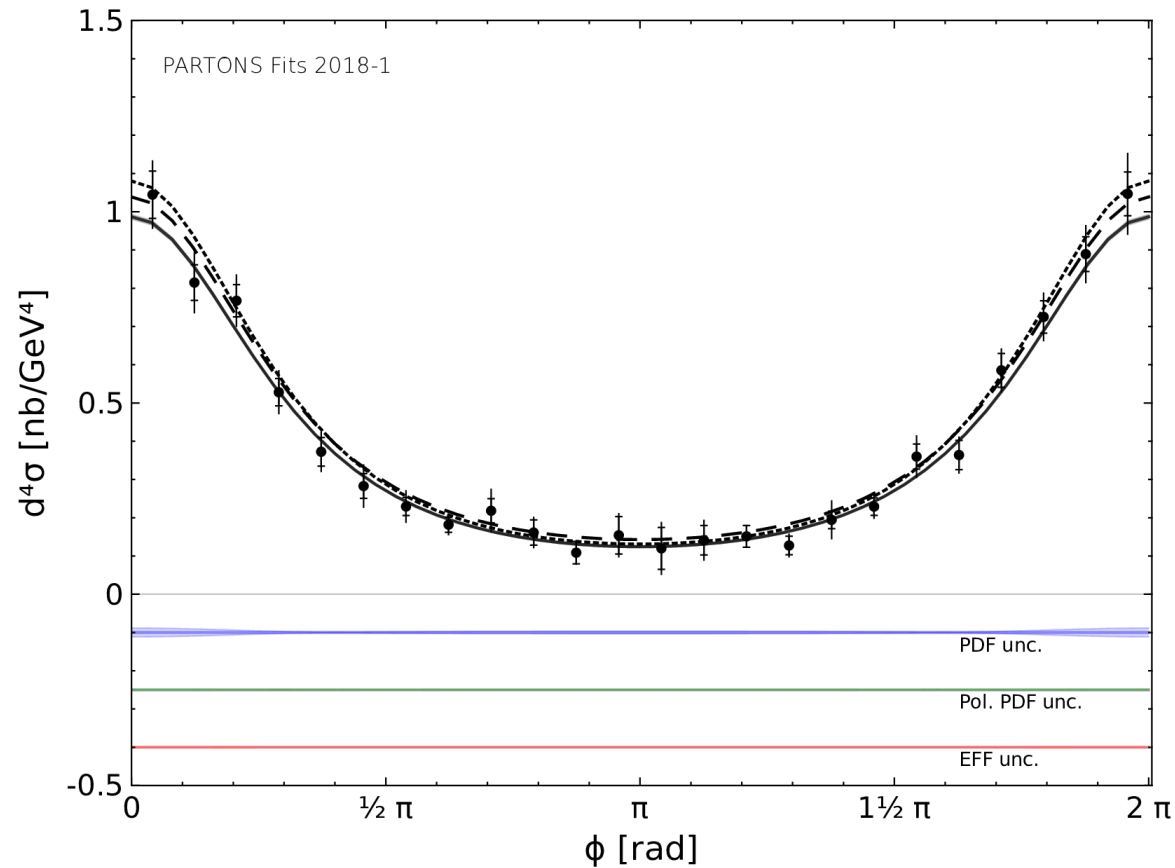
Fitted values:

Parameter	Mean	Data unc.	Unpol. PDF unc.	Pol. PDF unc.	EFF unc.
$a_H^{q_{\text{val}}}$	0.81	0.04	0.17	0.02	< 0.01
$a_H^{q_{\text{sea}}}$	0.99	0.01	0.02	< 0.01	< 0.01
$a_{\widetilde{H}}^q$	1.03	0.04	0.30	0.24	0.01
$N_{\widetilde{E}}$	-0.46	0.10	0.09	0.06	0.01
$A_H^{q_{\text{sea}}}$	2.56	0.23	0.30	0.09	0.03
$B_H^{q_{\text{sea}}}$	-5		at the limit		
$C_H^{q_{\text{sea}}}$	34	27	49	14	3
$A_{\widetilde{H}}^{u_{\text{val}}}$	0.77	0.12	0.30	0.23	0.07
$B_{\widetilde{H}}^{u_{\text{val}}}$	-0.02	0.26	0.75	0.24	0.15
$C_{\widetilde{H}}^{u_{\text{val}}}$	-0.92	0.07	0.44	0.24	0.04
$A_{\widetilde{H}}^{d_{\text{val}}}$	0.64	0.24	0.30	0.28	0.05
$B_{\widetilde{H}}^{d_{\text{val}}}$	-1.19	0.45	0.91	0.98	0.22
$C_{\widetilde{H}}^{d_{\text{val}}}$	-0.55	0.24	0.26	0.27	0.10

Replication of experimental data to estimate corresponding uncertainties:

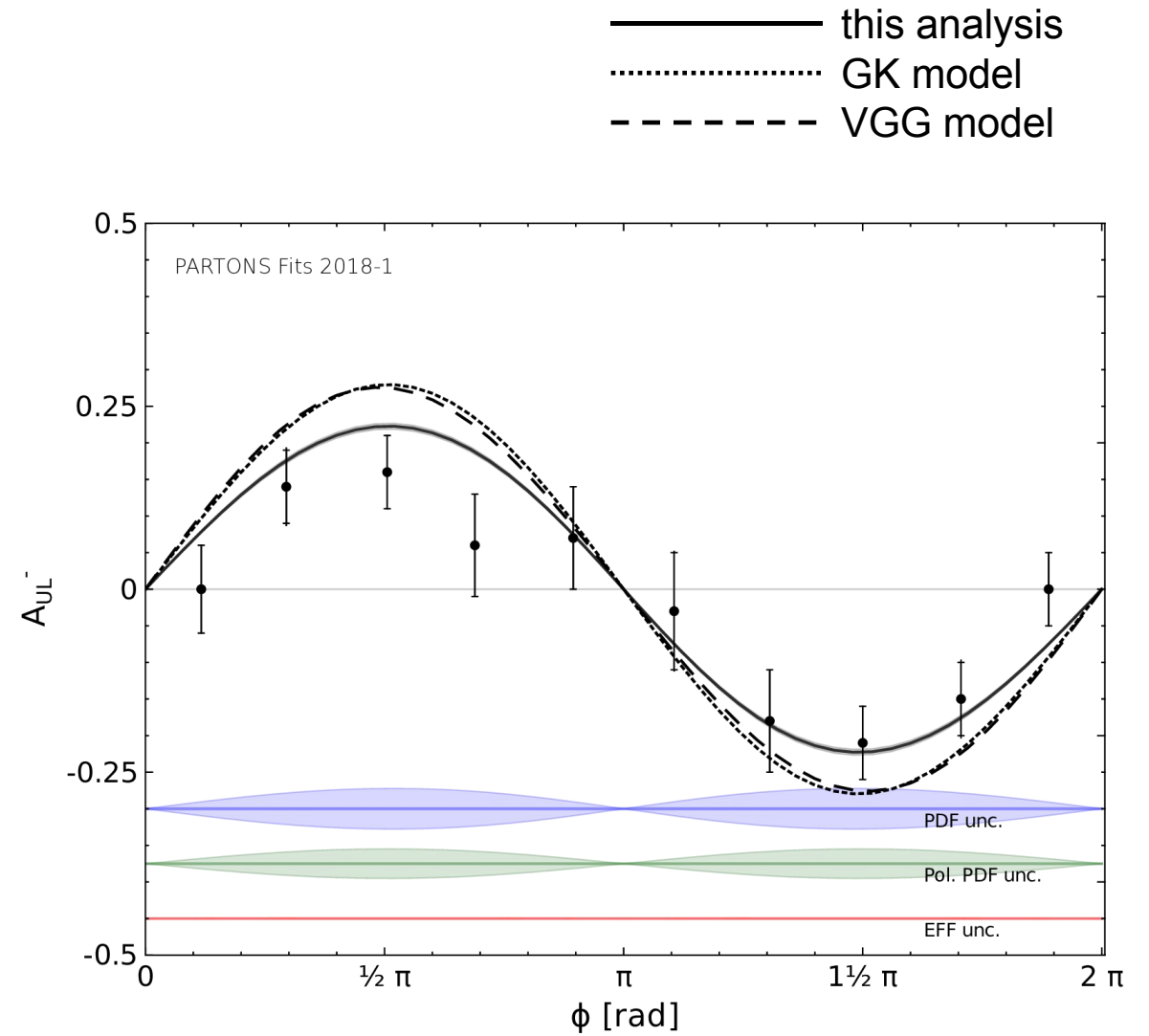
$$v_i \pm \Delta_i^{\text{tot}} \xrightarrow{\text{replica } j} (\text{rnd}_j(v_i, \Delta_i^{\text{tot}}) \pm \Delta_i^{\text{tot}}) \times \text{rnd}_j(1, \Delta_i^{\text{norm}}) \quad \Delta_i^{\text{tot}} = \sqrt{(\Delta_i^{\text{stat}})^2 + (\Delta_i^{\text{sys}})^2}$$

CLAS data:



Phys. Rev. Lett. 115(21), 212003 (2015)

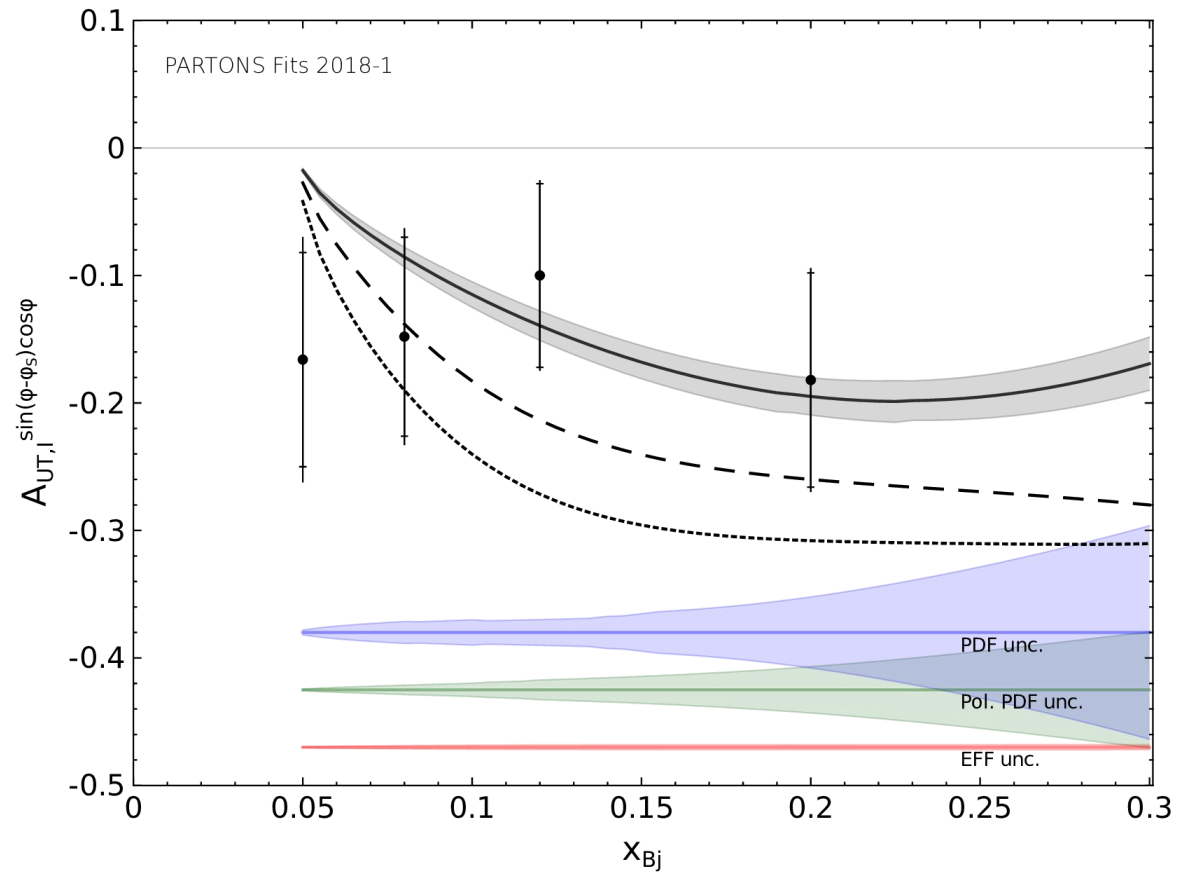
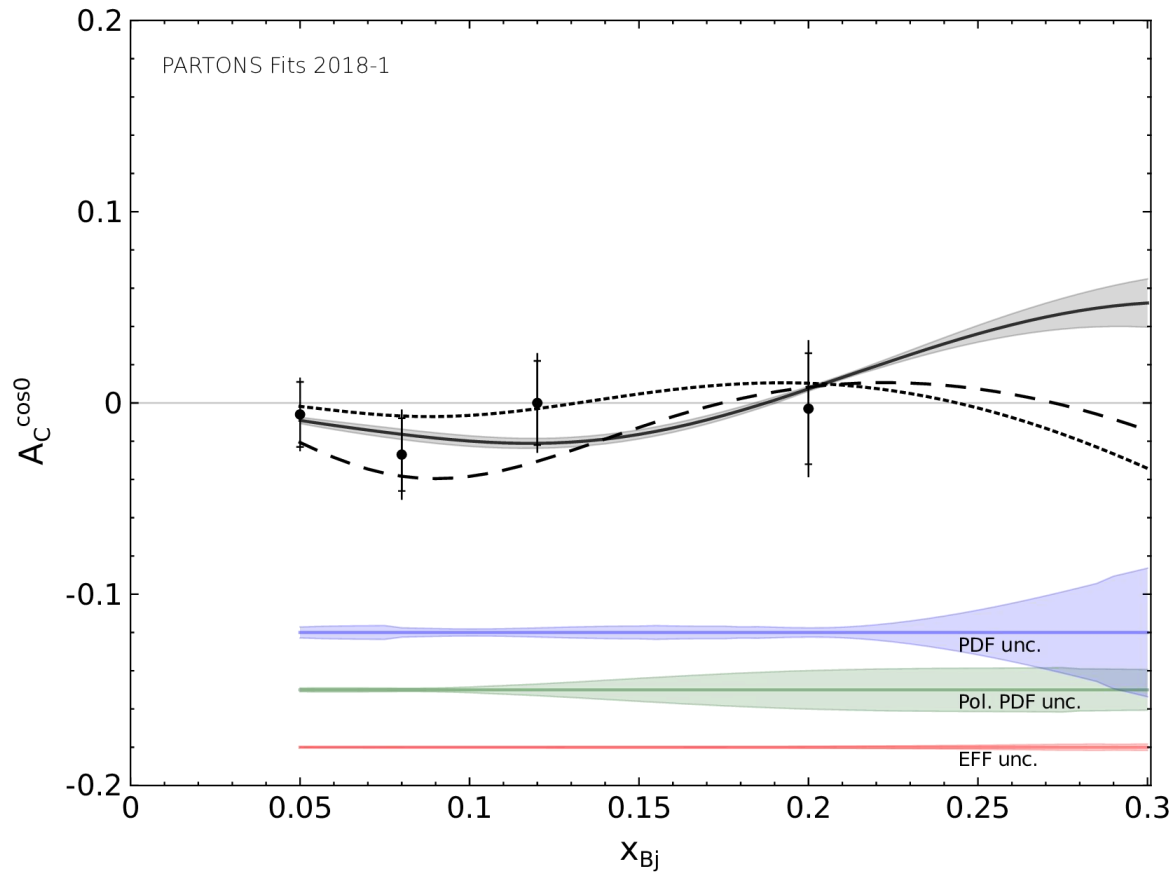
$x_{Bj} = 0.244$, $t = -0.15$ GeV², $Q^2 = 1.79$ GeV²



Phys. Rev. D91(5), 052014 (2015)

$x_{Bj} = 0.257$, $t = -0.23$ GeV², $Q^2 = 2.02$ GeV²

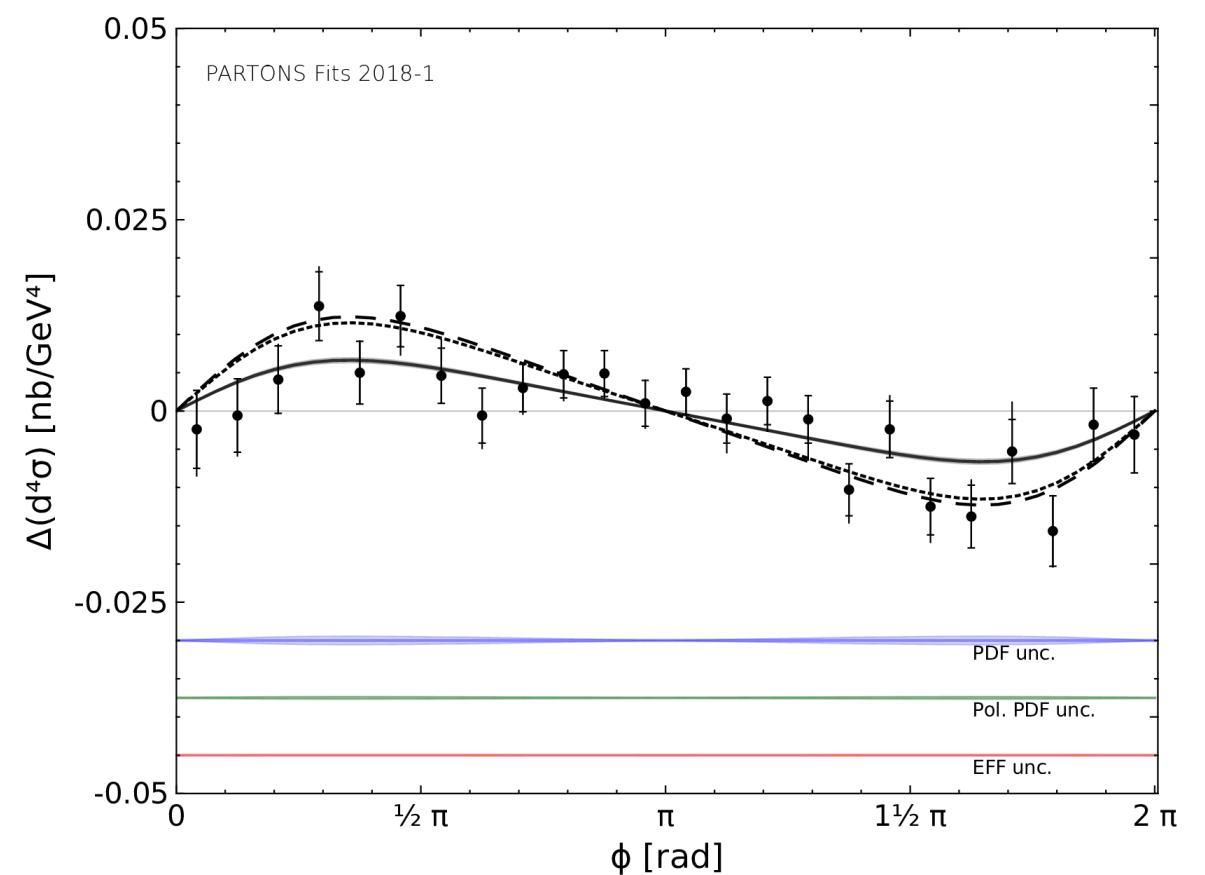
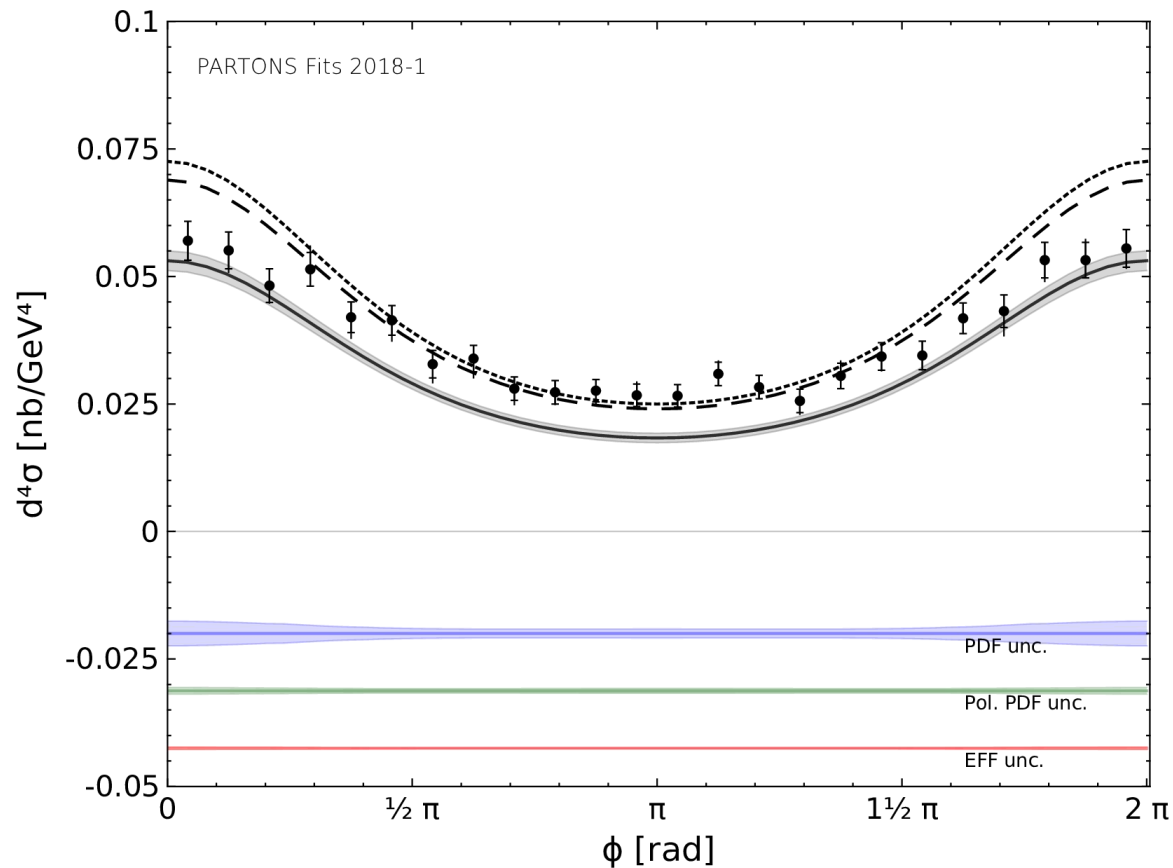
HERMES data:



JHEP 06, 066 (2008)

$t = -0.12 \text{ GeV}^2$, $Q^2 = 2.5 \text{ GeV}^2$

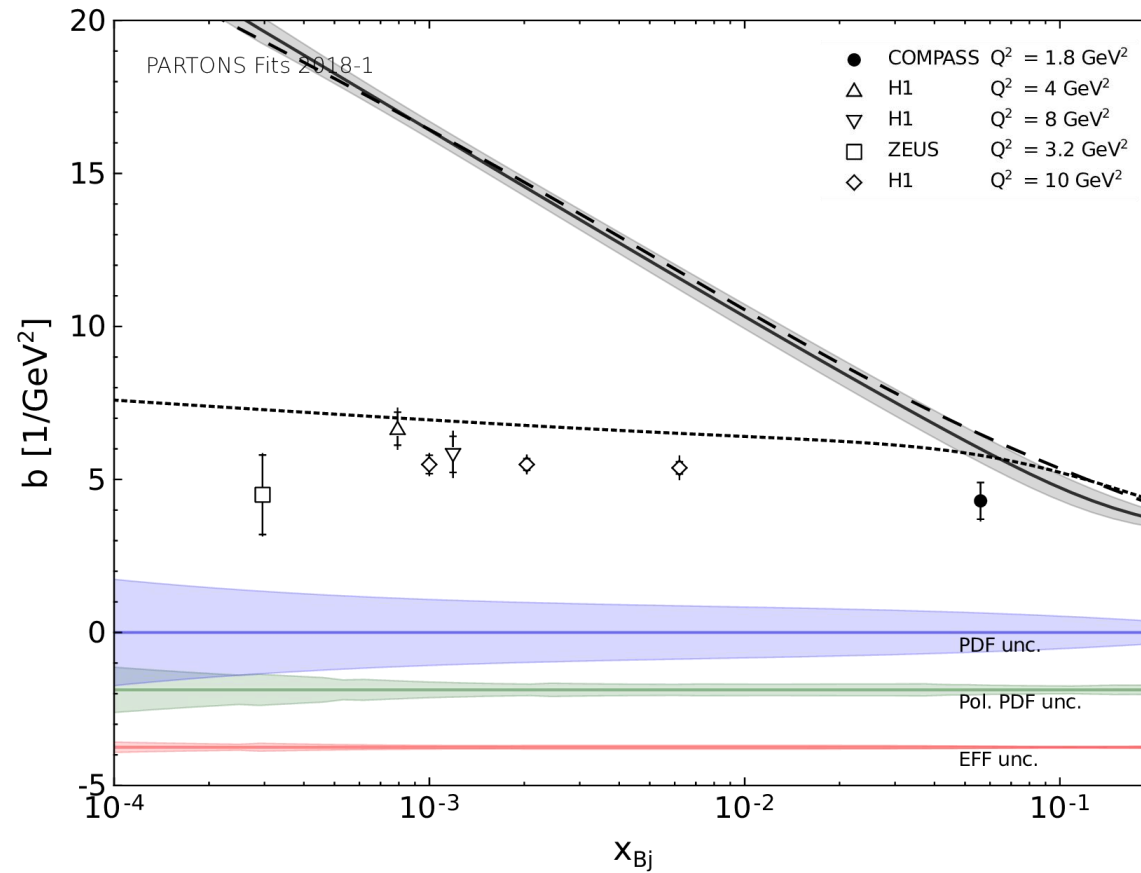
Hall A data:



Phys. Rev. C92(5), 055202 (2015)

$x_{Bj} = 0.392$, $t = -0.233 \text{ GeV}^2$, $Q^2 = 2.054 \text{ GeV}^2$

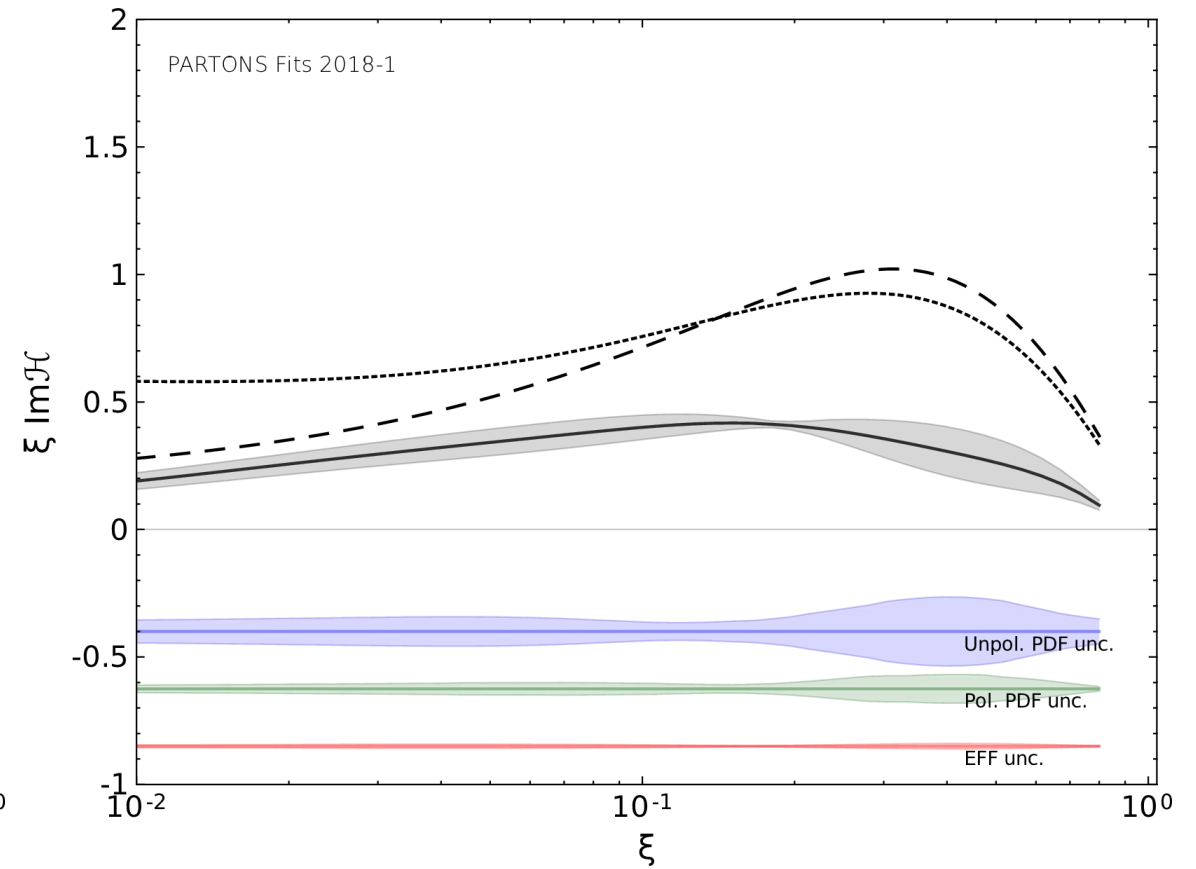
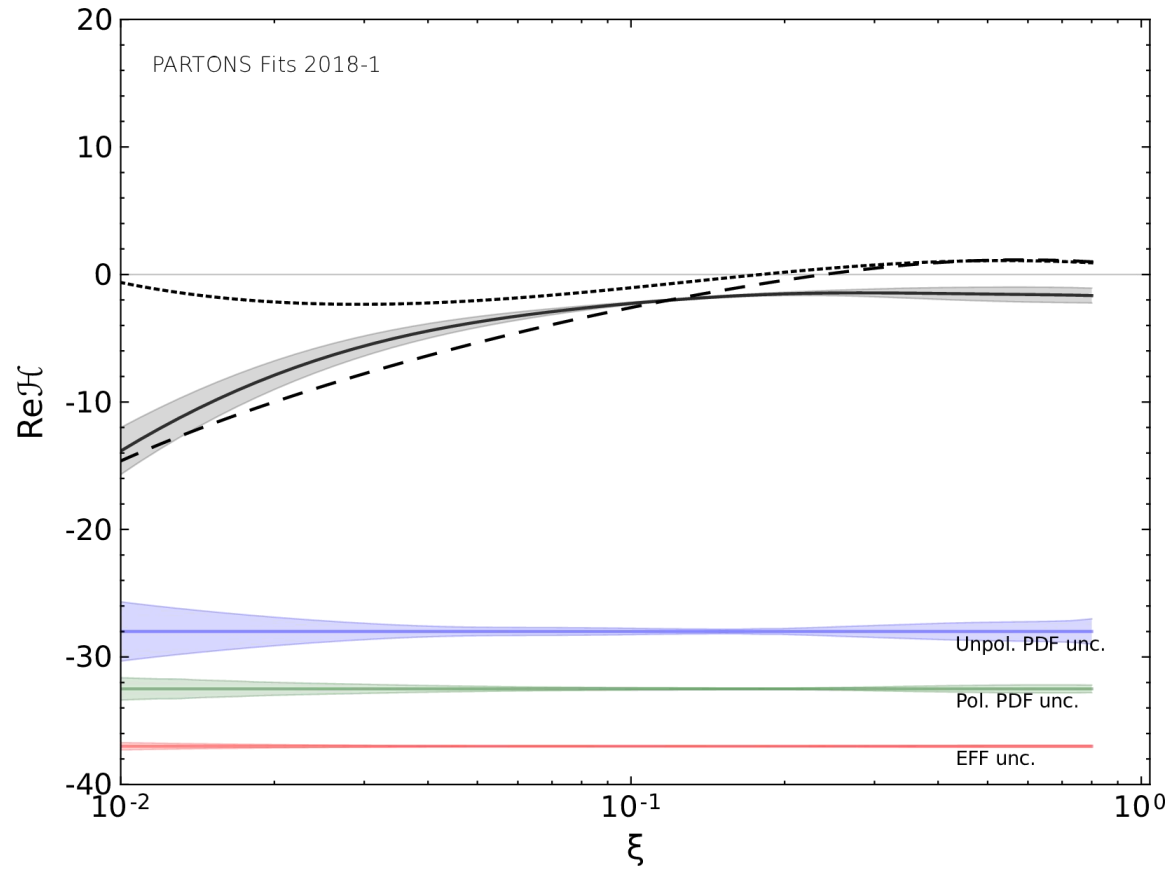
COMPASS and HERA:



arXiv: hep-ex/1802.02739

$Q^2 = 1.8 \text{ GeV}^2$

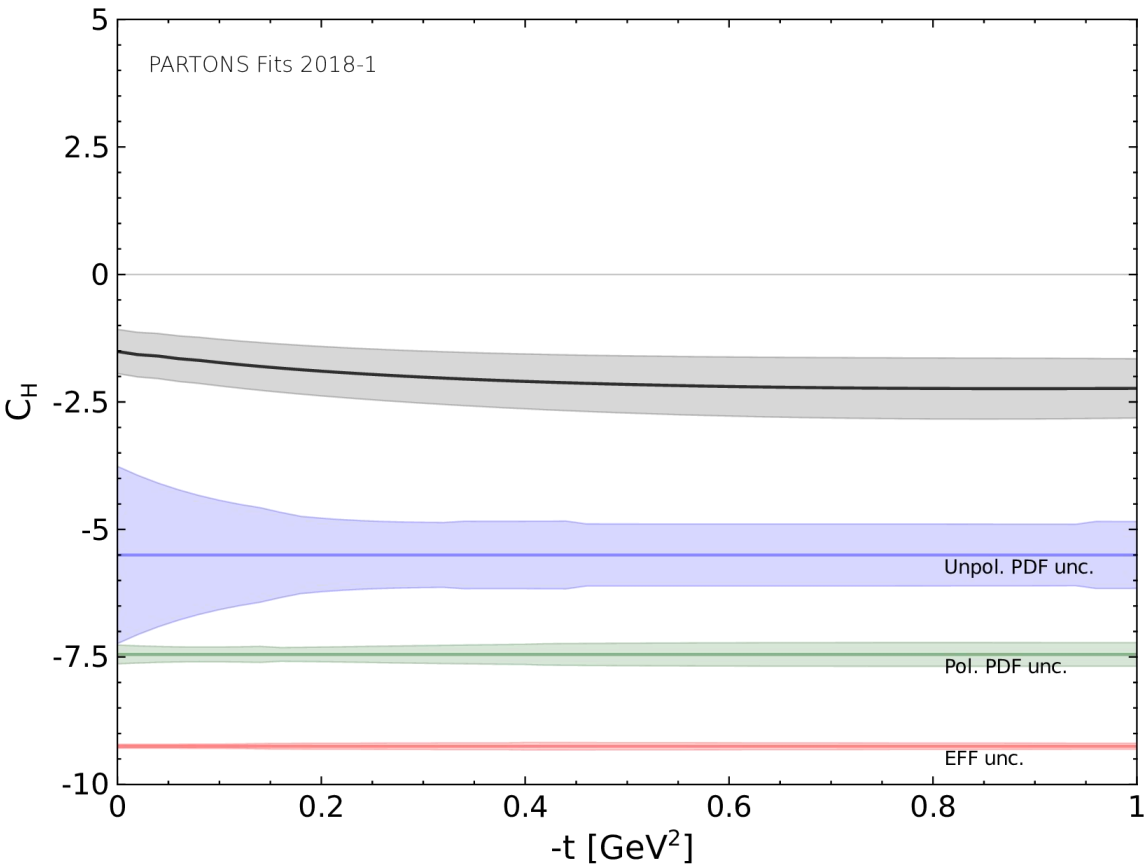
Compton Form Factors:



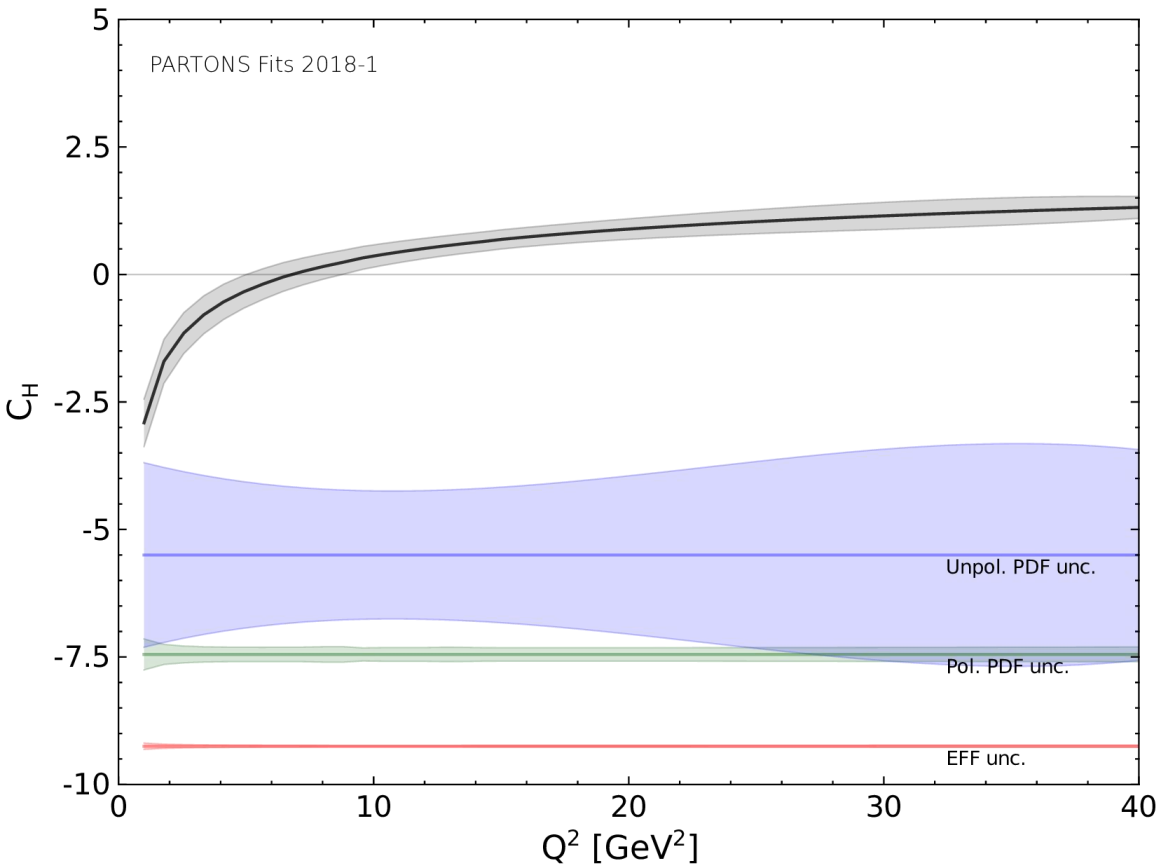
$$t = -0.3 \text{ GeV}^2, Q^2 = 2 \text{ GeV}^2$$

for other CFFs see the backup slides

Subtraction constant:

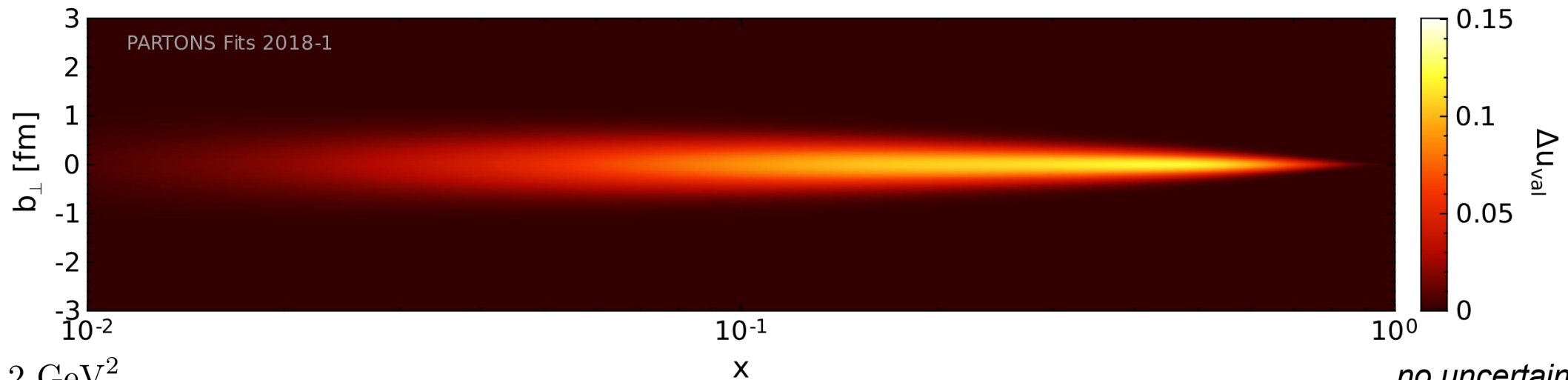
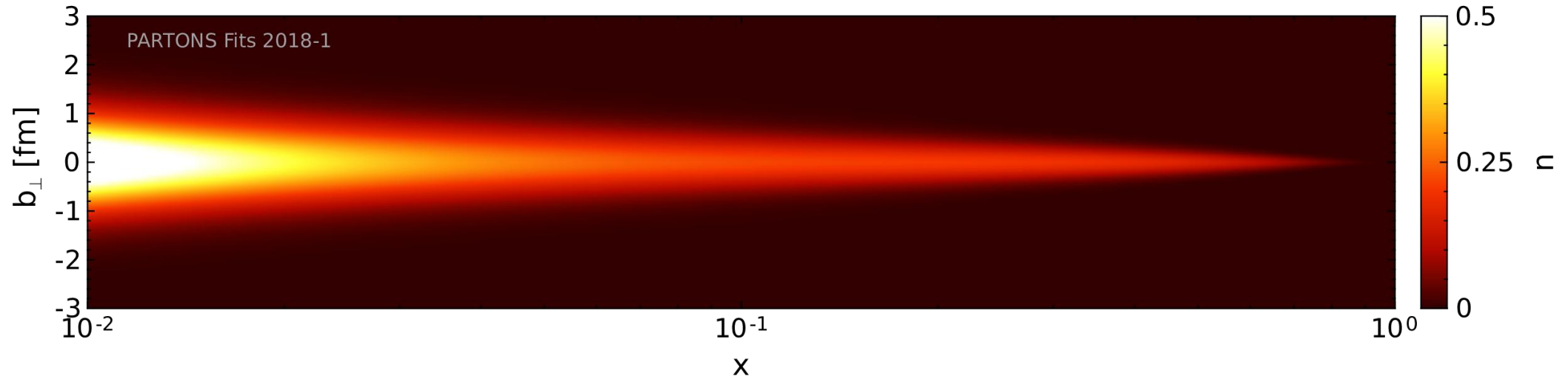


$Q^2 = 2 \text{ GeV}^2$



$t = 0$

Nucleon tomography:



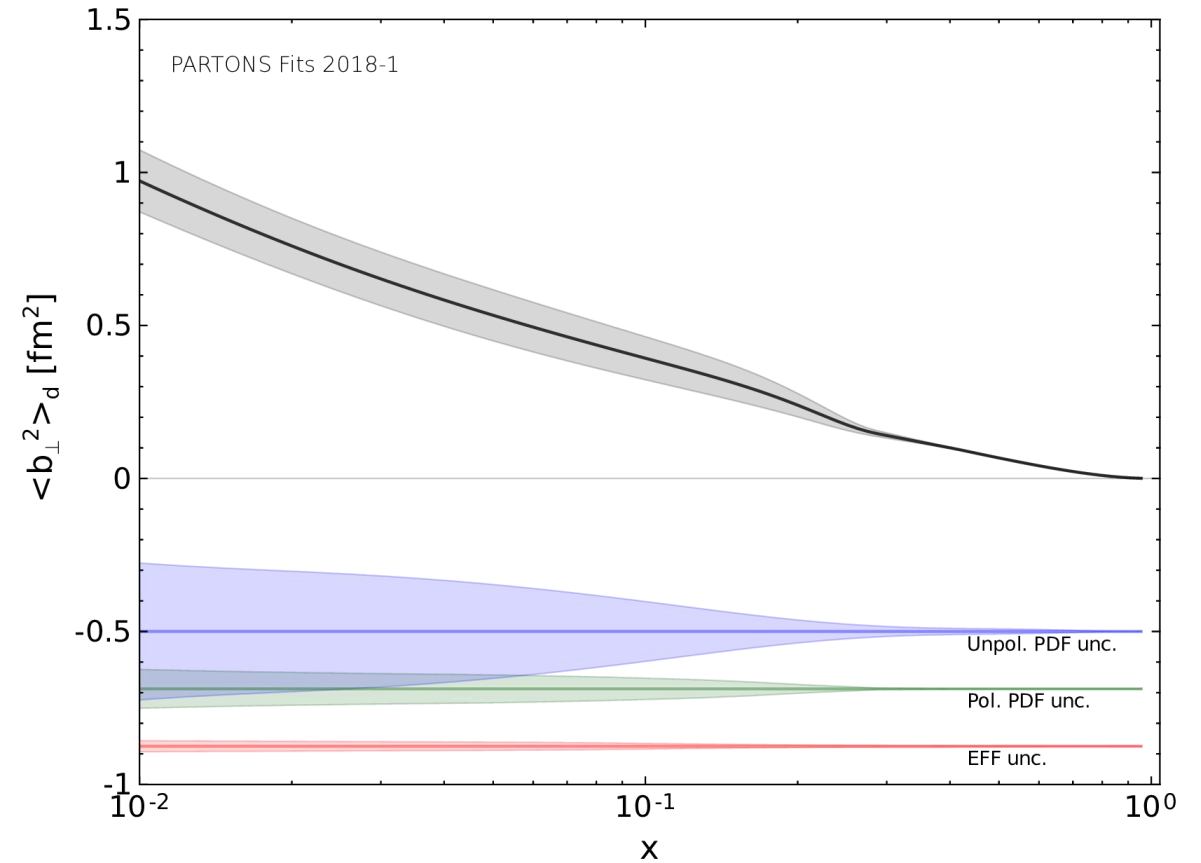
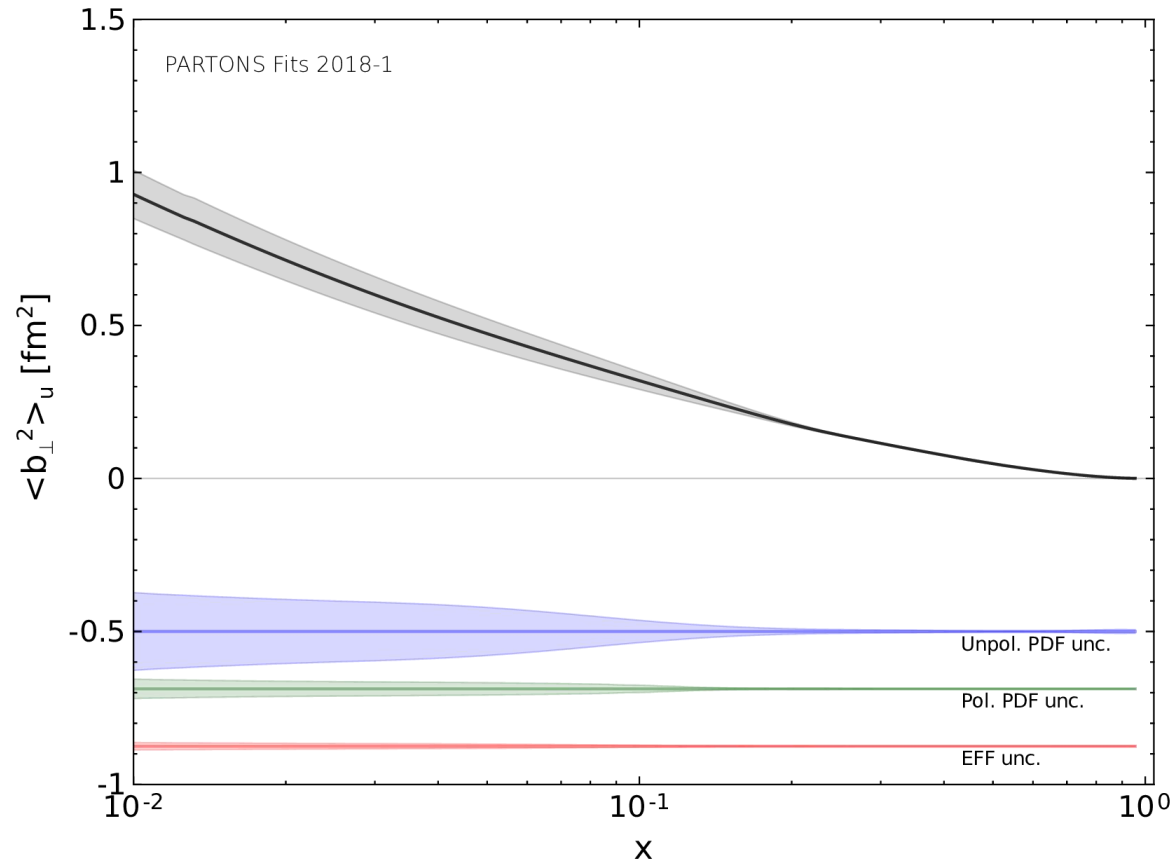
$$Q^2 = 2 \text{ GeV}^2$$

no uncertainties!

Compton Form Factors:

$$\langle b_{\perp}^2 \rangle_q(x) = \frac{\int d^2 \mathbf{b}_{\perp} \mathbf{b}_{\perp}^2 q(x, \mathbf{b}_{\perp})}{\int d^2 \mathbf{b}_{\perp} q(x, \mathbf{b}_{\perp})}$$

— this analysis
 GK model
 - - - - VGG model

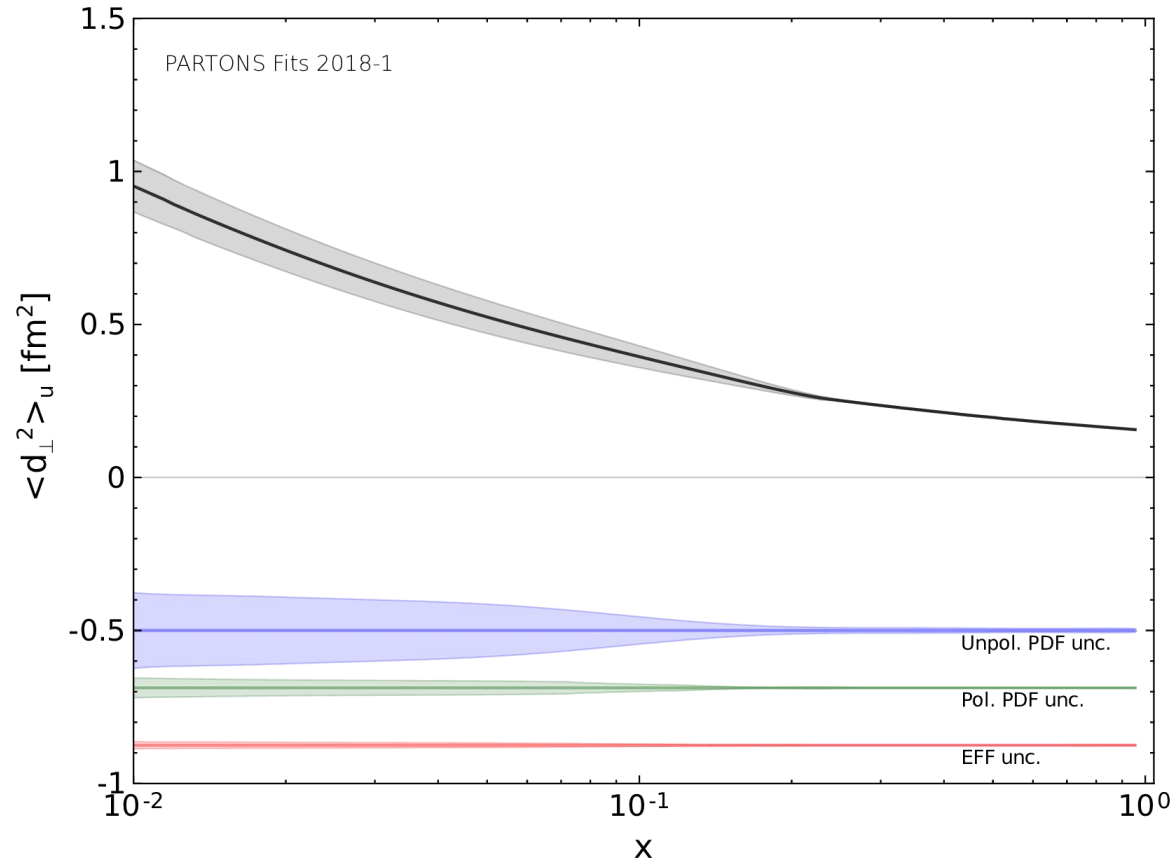
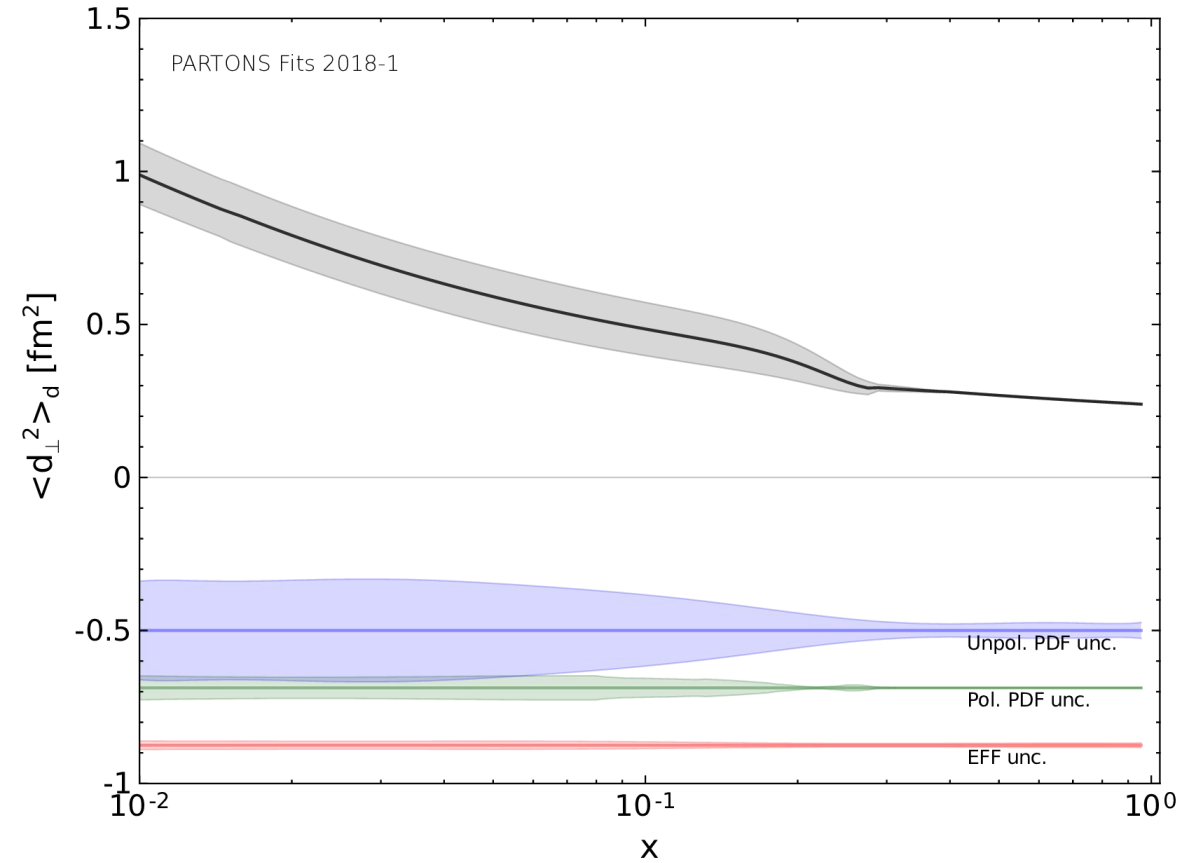


$Q^2 = 2 \text{ GeV}^2$

Compton Form Factors:

$$\langle d_{\perp}^2 \rangle_q(x) = \frac{\langle b_{\perp}^2 \rangle_q(x)}{(1-x)^2}$$

— this analysis
 GK model
 - - - - VGG model


 $Q^2 = 2 \text{ GeV}^2$


Fits to DVCS data

- New parameterizations of border and skewness function proposed
 - basic properties of GPD as building blocks
 - small number of parameters
 - encoded access to nucleon tomography and subtraction constant
- Successful to fit EFF and DVCS data
 - replica method for a careful propagation of uncertainties

What next?

- Neural network parameterization of CFFs
- Include other channels and more observables
- Include new and already existing theory developments
- Make consistent analysis of all those ingredients → **PARTONS**

Inequalities:

$$|\Delta q(x, \mathbf{b}_\perp)| \leq q(x, \mathbf{b}_\perp)$$

$$\frac{\mathbf{b}_\perp^2}{m^2} \left(\frac{\partial}{\partial \mathbf{b}_\perp^2} e(x, \mathbf{b}_\perp) \right)^2 \leq (q(x, \mathbf{b}_\perp) + \Delta q(x, \mathbf{b}_\perp)) \times (q(x, \mathbf{b}_\perp) - \Delta q(x, \mathbf{b}_\perp))$$

to avoid violation of the positivity in the impact parameter space

"trouble" with analytic regularization

$$\int_{(0)}^1 \frac{f(x)}{x^{a+1}} = \int_0^1 \frac{f(x) - f(0) - x f'(0) - \dots}{x^{a+1}} - \frac{f(0)}{a} - \frac{f'(0)}{a-1} + \dots$$

where in our case

$$a = \delta + A_G^q t$$

$$q(x) \sim x^{-\delta}$$

$$f(x) = \frac{G^q(x, x, t) - G^q(x, 0, t)}{x^{-a}} = \frac{G^q(x, 0, t) (g_G^q(x, t) - 1)}{x^{-a}}$$

compensating terms singular for $t \equiv t_0^\infty = -\delta/A_G^q$ and $t \equiv t_1^\infty = (1-\delta)/A_G^q$ unless $f(0) = 0$ at t_0^∞ and $f'(0) = 0$ at t_1^∞ , condition provided by:

$$b_G^q = \frac{A_G^q (a_G^q - 1)}{a_G^q \delta} \quad c_G^q = \frac{(a_G^q - 1)}{p_0 (\delta - 1) a_G^q \delta} (p_0 (2B_G^q - C_G^q) (\delta - 1) + A_G^q p_0 (\delta - 1 - \alpha) + A_G^q p_1)$$

where δ, α, p_0, p_1 are PDF parameterization parameters

Layered structure:

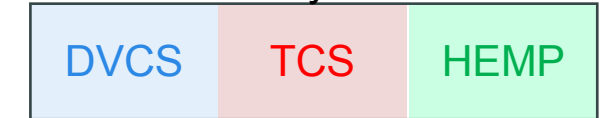
- one layer = collection of objects designed for common purpose
- one module = one physical development
- operations on modules provided by Services, e.g. for GPD Layer

```

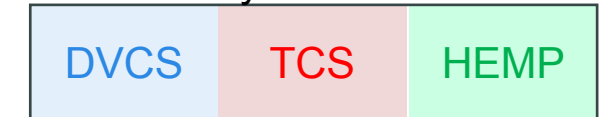
GPDResult computeGPDModel
    (const GPDKinematic& gpdKinematic, GPDModule* pGPDModule) const;
GPDResult computeGPDModelRestrictedByGPDType
    (const GPDKinematic& gpdKinematic, GPDModule* pGPDModule,
     GPDType::Type gpdType) const;
GPDResult computeGPDModelWithEvolution
    (const GPDKinematic& gpdKinematic, GPDModule* pGPDModule,
     GPEvolutionModule* pEvolQCDModule) const;
...
    
```

- what can be automated is automated
- features improving calculation speed
e.g. CFF Layer Service stores the last calculated values

Observable Layer



Process Layer



CFF Layer



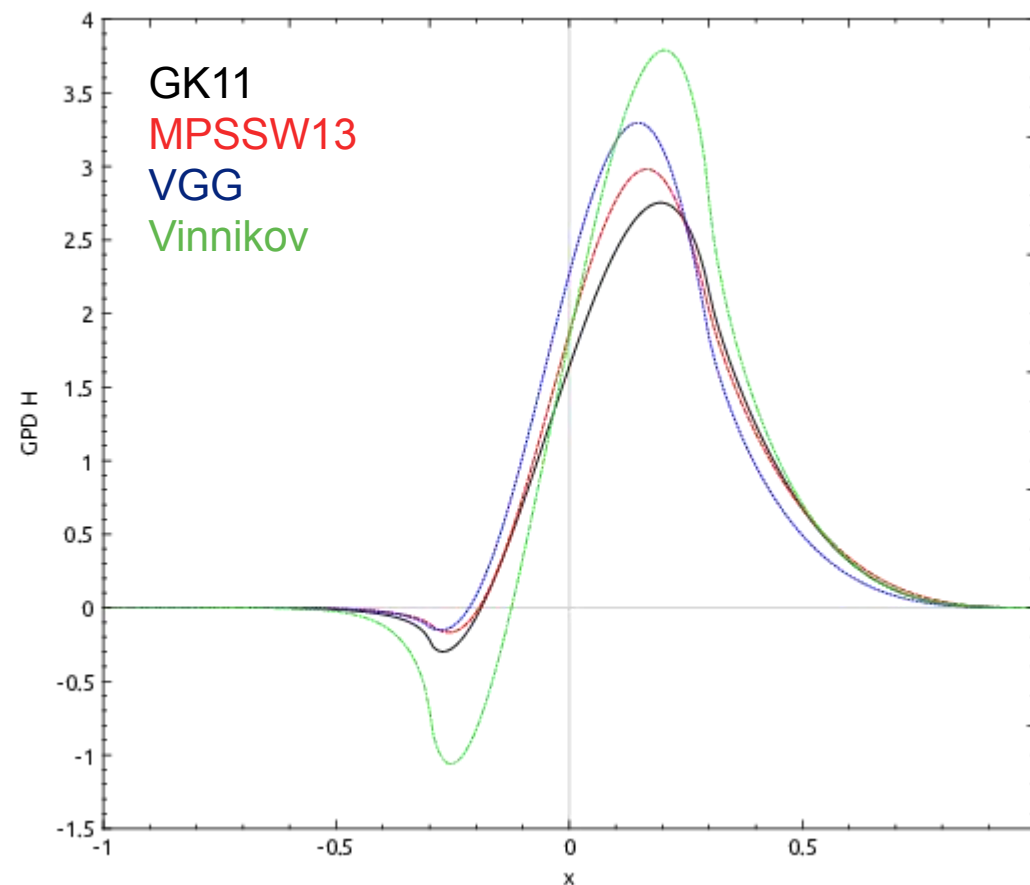
GPD Layer

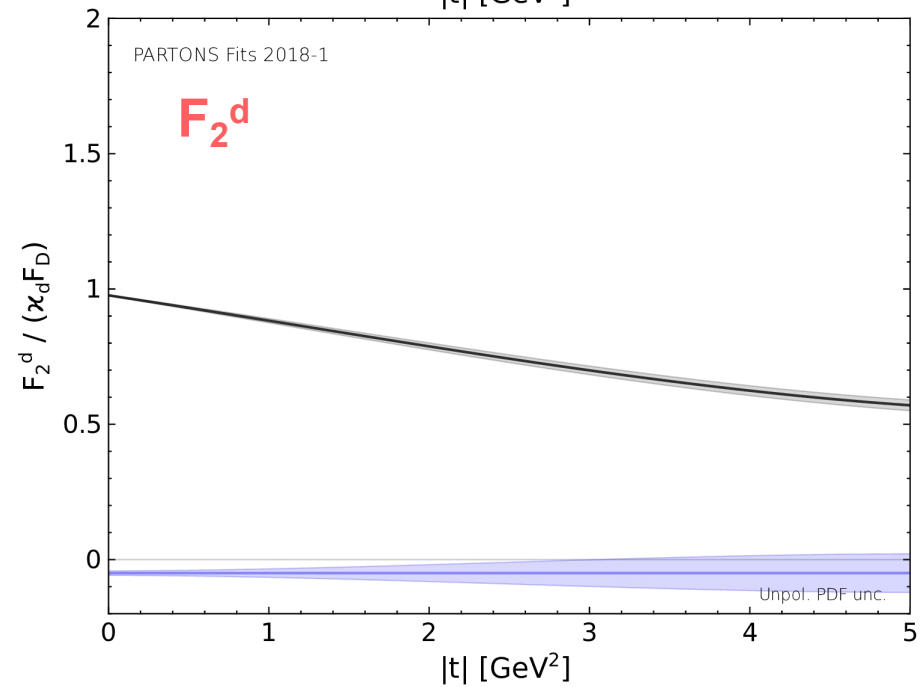
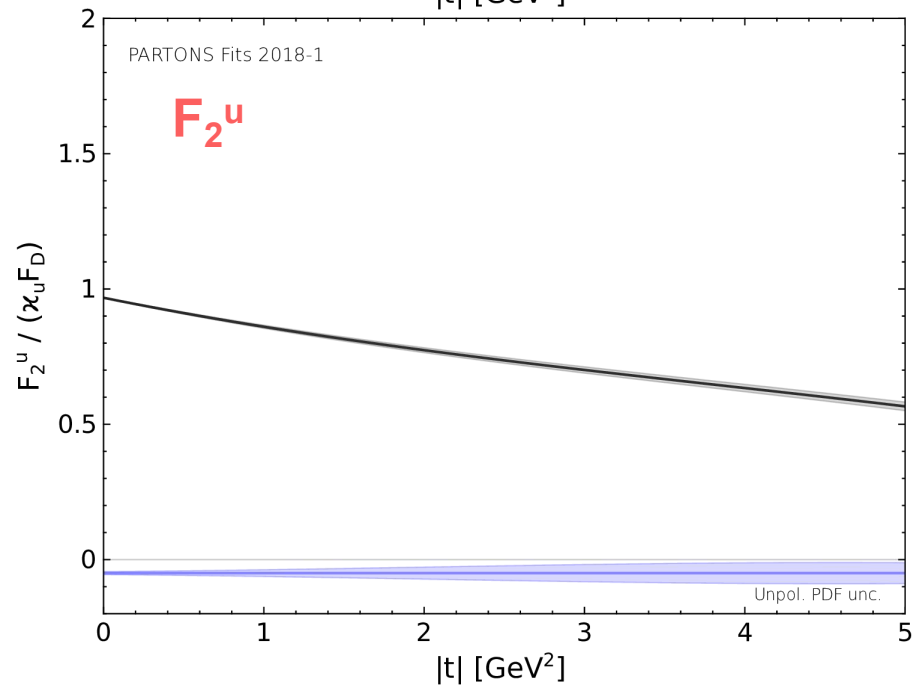
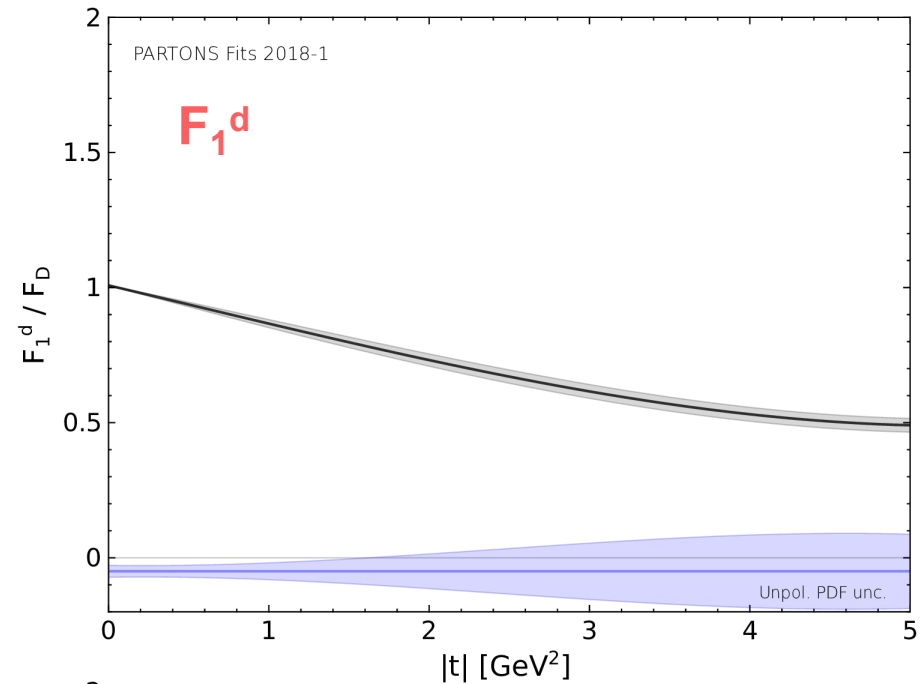
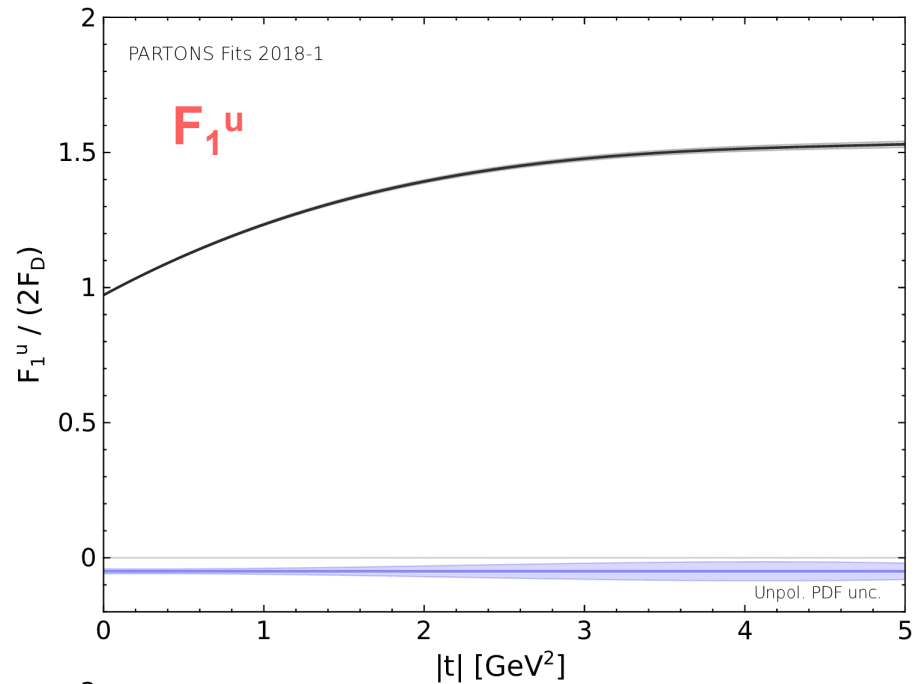


Existing modules:

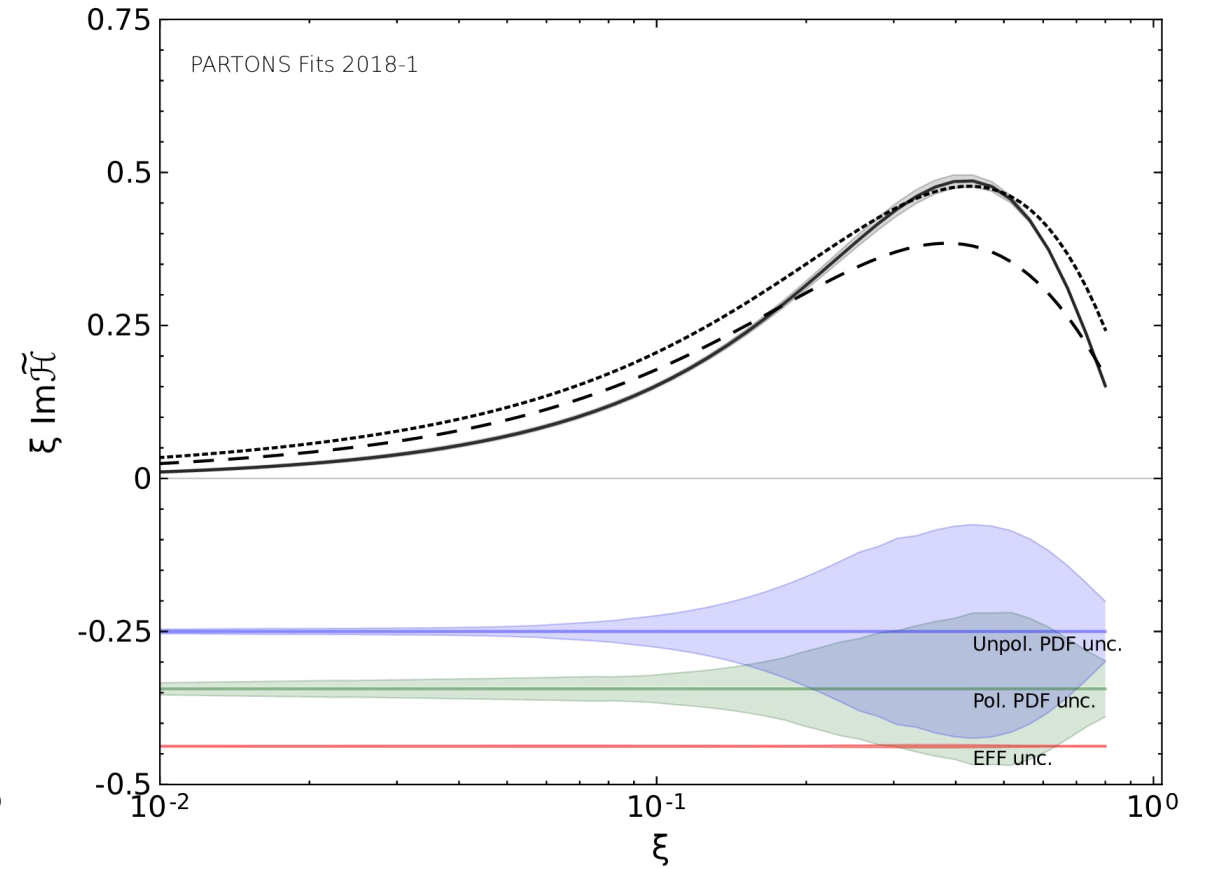
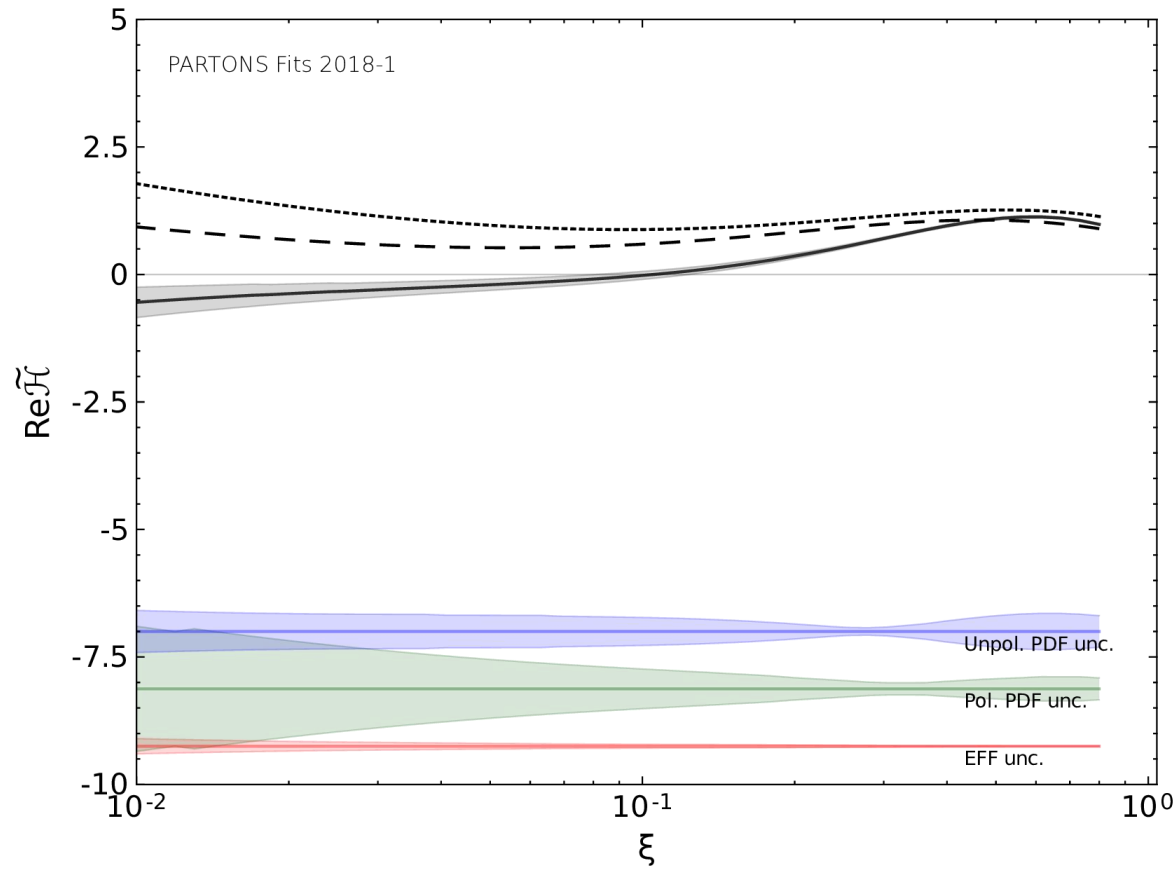
- GPD: GK11, VGG, Vinnikov, MPSSW13, MMS13
- Evolution: Vinnikov code
- CFF (DVCS only): LO, NLO (gluons and light or light + heavy quarks)
- Cross Section (DVCS only): VGG, BMJ, GV
- Running coupling: 4-loop PDG expression, constant value

$H^u @ x = 0.2, t = -0.1 \text{ GeV}^2, \mu_F^2 = \mu_R^2 = 2 \text{ GeV}^2$





Compton Form Factors:



$$t = -0.3 \text{ GeV}^2, Q^2 = 2 \text{ GeV}^2$$

