

# First extraction of Transversity PDF from a global analysis of lepton-hadron scattering and hadronic collisions data



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in collaboration with A. Bacchetta (Univ. Pavia)

based on  
P.R.L. **120** (2018) 192001 , arXiv:1802.05212  
**plus updates**

# a phase transition

quark polarization

nucleon polarization	U	L	T
U	$f_1$		$h_1^\perp$
L		$g_{1L}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}$	$h_1 \ h_{1T}^\perp$

1Dim (polarized) PDFs

Explorations

Parton model

Phase 1

3Dim TMDs

first global fit  
of  $f_1(x, k_\perp)$

*Bacchetta et al.,  
JHEP 1706 (17) 081*

Global fits

QCD analysis  
+ data

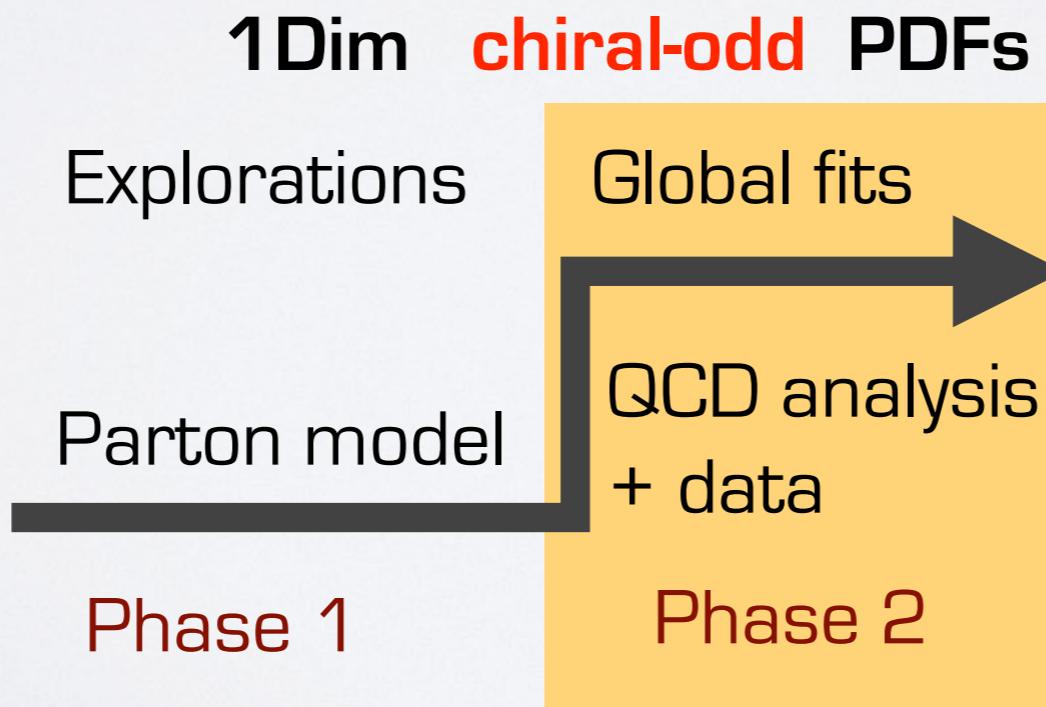
Phase 2

# a phase transition

quark polarization

nucleon polarization	U	L	T
U	$f_1$		$h_{1^\perp}$
L		$g_{1L}$	$h_{1L^\perp}$
T	$f_{1T^\perp}$	$g_{1T}$	$h_1$ $h_{1T^\perp}$

first global fit  
(= lepton-hadron scatt.  
and hadron collisions)  
of PDF  $h_1$



# transversity: a chiral-odd PDF

$$f_1 = \bullet$$

$$g_1 = \bullet \rightarrow - \leftarrow$$

transversity PDF  $h_1(x)$

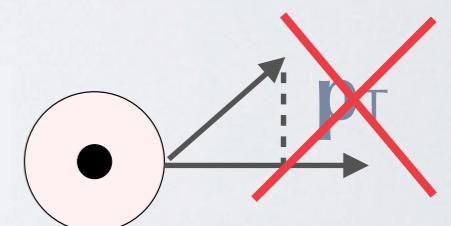
$$h_1 = \uparrow - \downarrow$$

it is a **chiral-odd** function (flips quark helicity)

- needs a chiral-odd partner
- suppressed in inclusive DIS

which channel addresses it at leading twist as a **PDF** ?

di-hadron semi-inclusive production

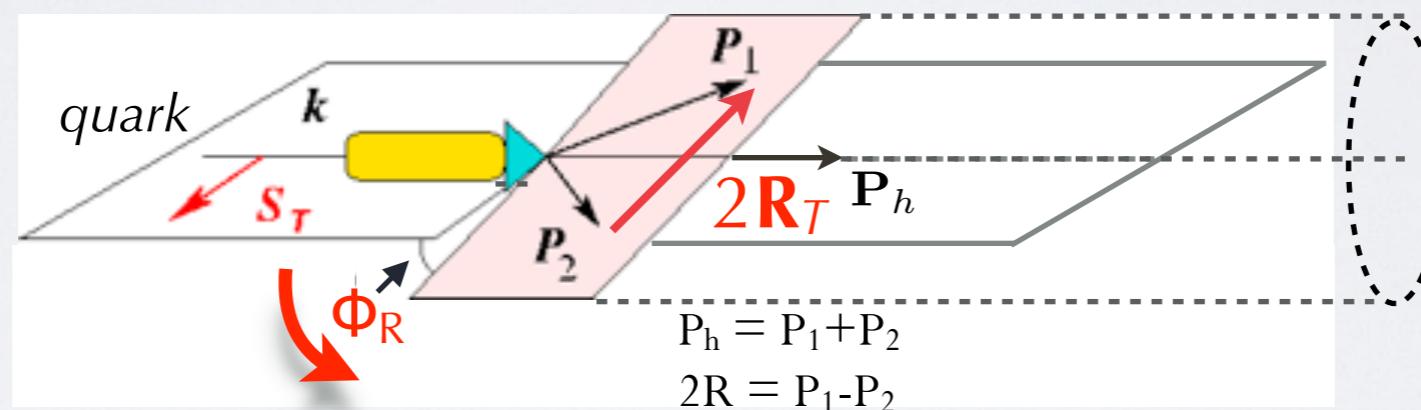


# 2-hadron-inclusive production

Collins, Heppelman, Ladinsky,  
N.P. **B420** (94)

$$R_T \ll Q \quad H_1^{\triangleleft}$$

↑  
 $M_h$



correlation  $s_T$  and  $R_T \rightarrow$  **azimuthal asymmetry**

invariant mass

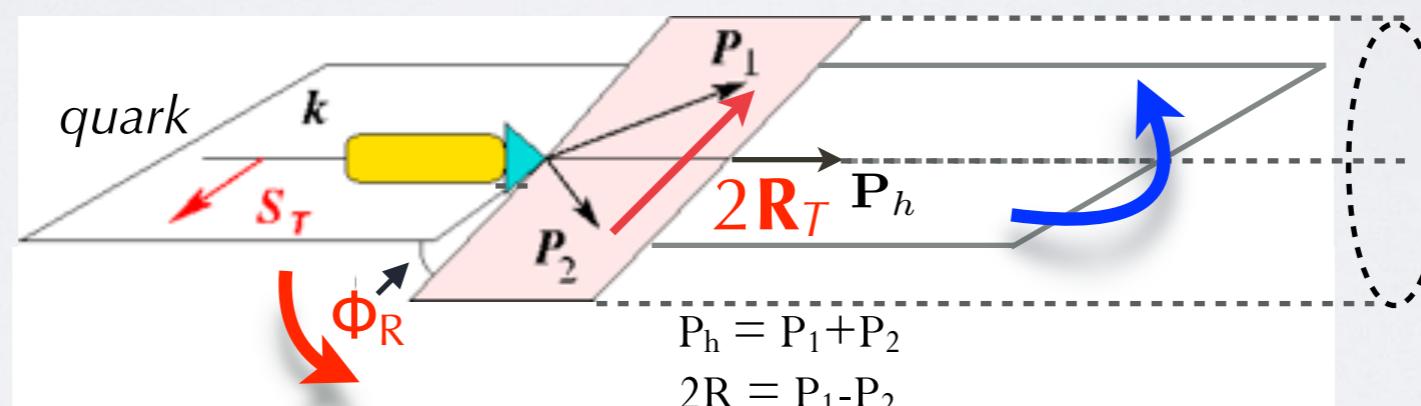
# 2-hadron-inclusive production

framework  
collinear  
factorization

Collins, Heppelman, Ladinsky,  
N.P. **B420** (94)

$$R_T \ll Q \quad H_1^{\triangleleft}$$

↔

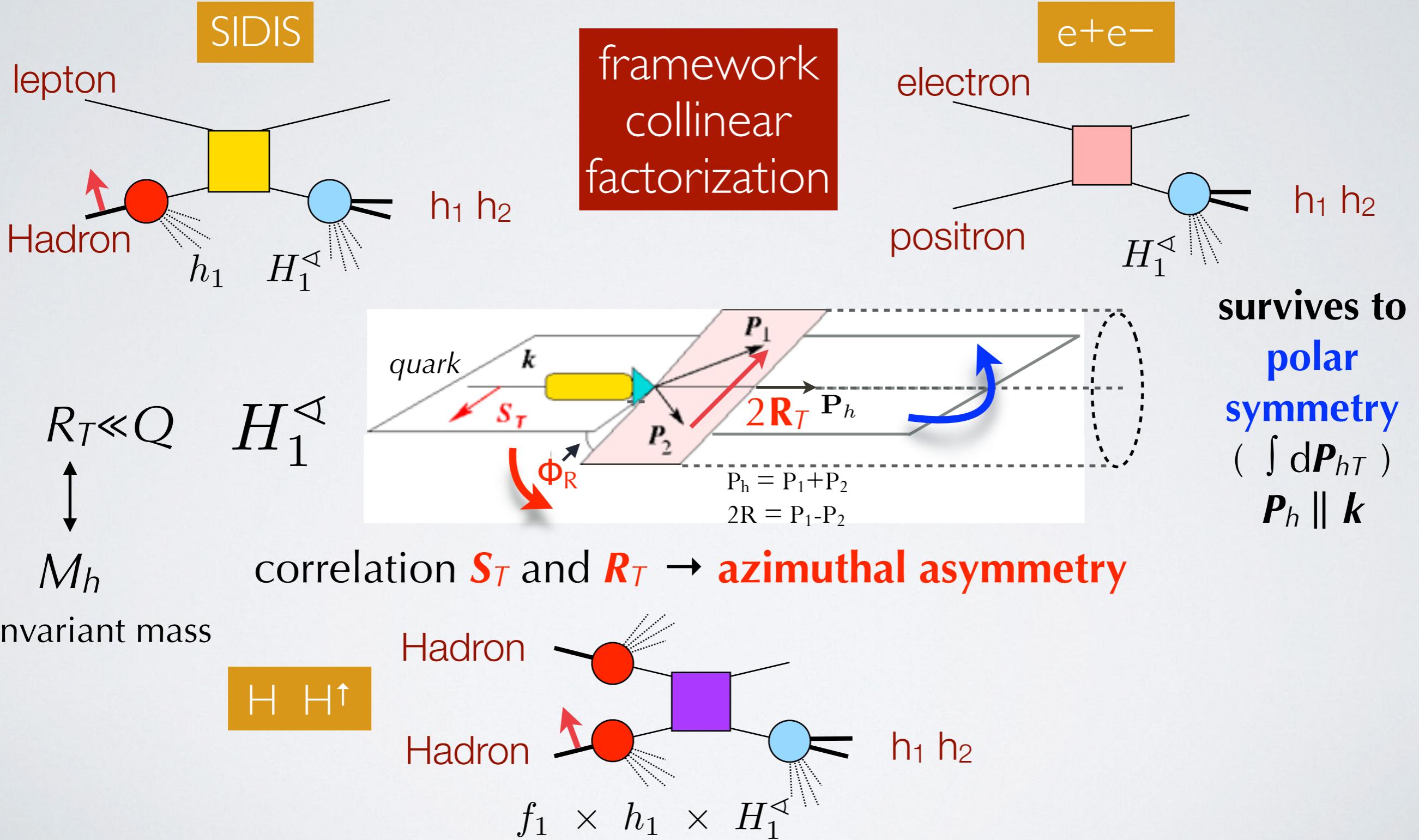


survives to  
polar  
symmetry  
 $(\int d\mathbf{P}_{hT})$   
 $\mathbf{P}_h \parallel \mathbf{k}$

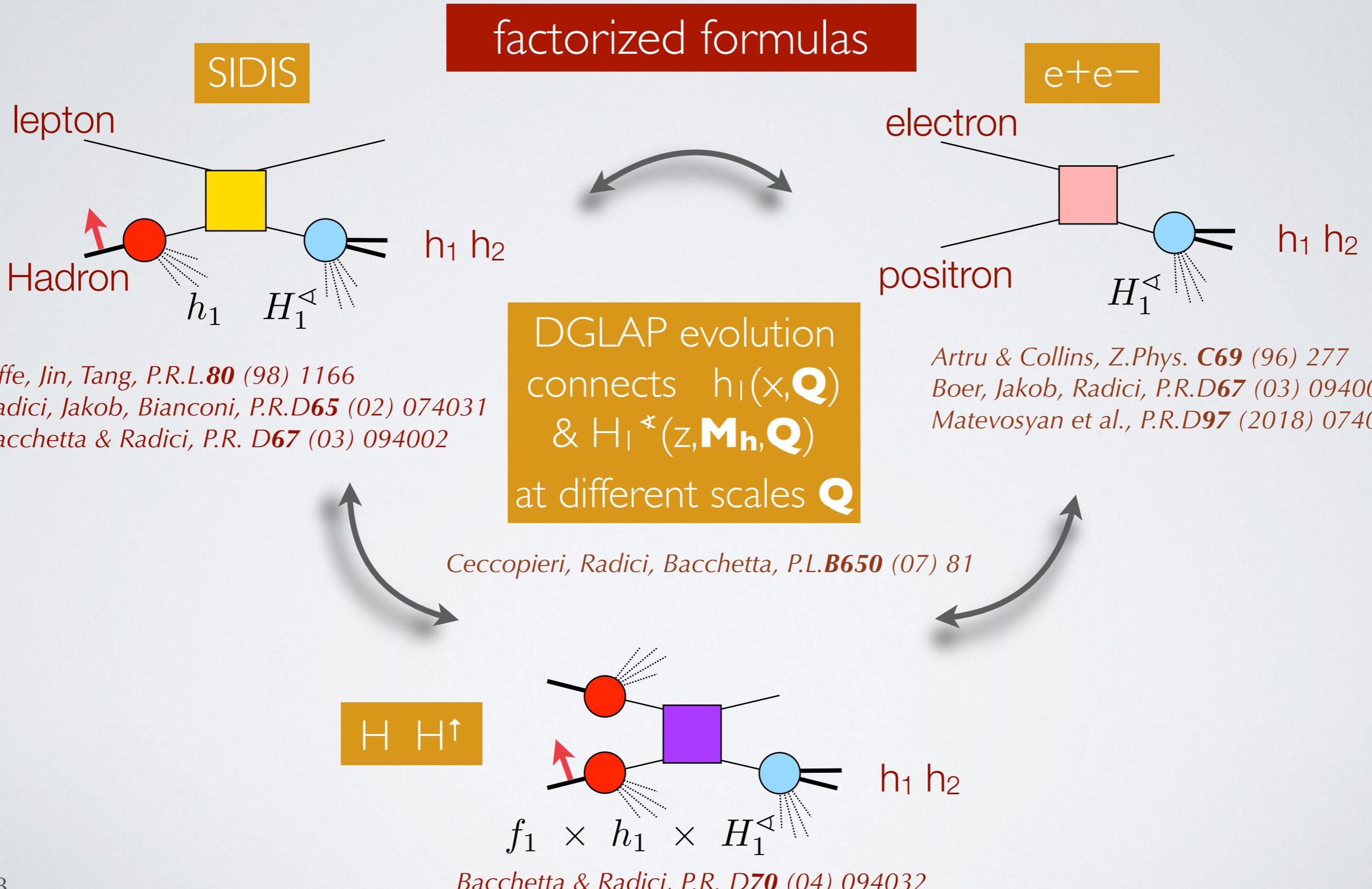
correlation  $S_T$  and  $R_T \rightarrow$  azimuthal asymmetry

invariant mass

# 2-hadron-inclusive production



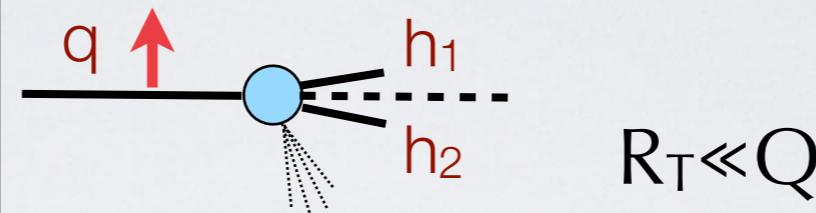
# 2-hadron-inclusive production



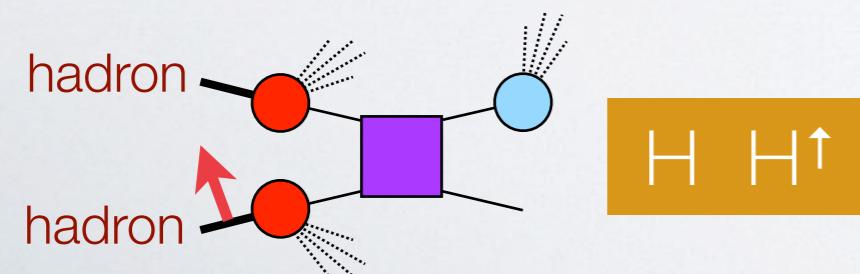
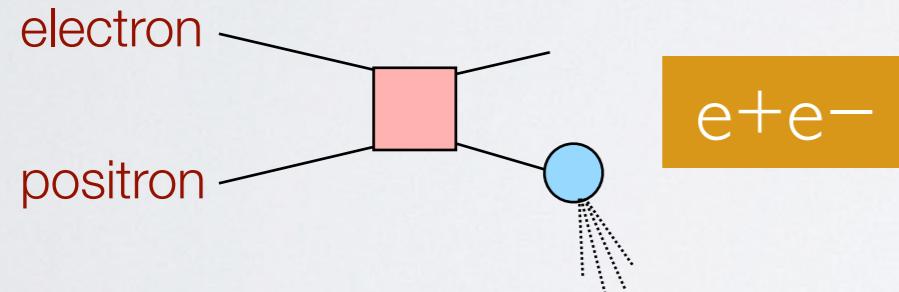
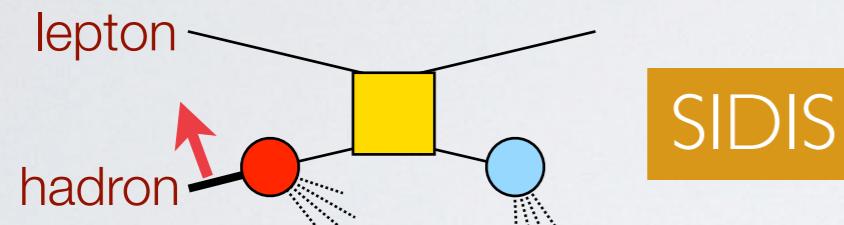
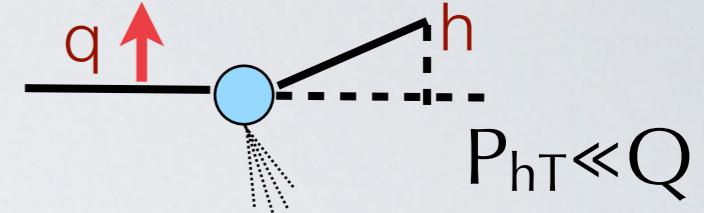
# advantage of 2-hadron-inclusive mechanism

## factorized formulas

Dihadron fragmentation  
collinear framework

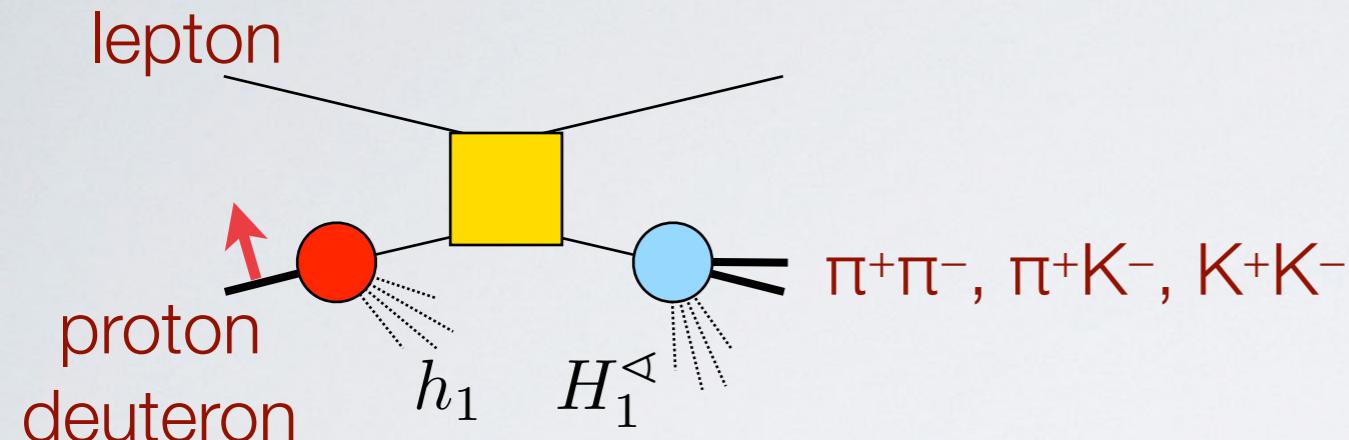


Collins effect  
TMD framework



# exp. data for 2-hadron-inclusive production

SIDIS  $\ell^- H^\uparrow \rightarrow \ell^+ (h_1 h_2) X$

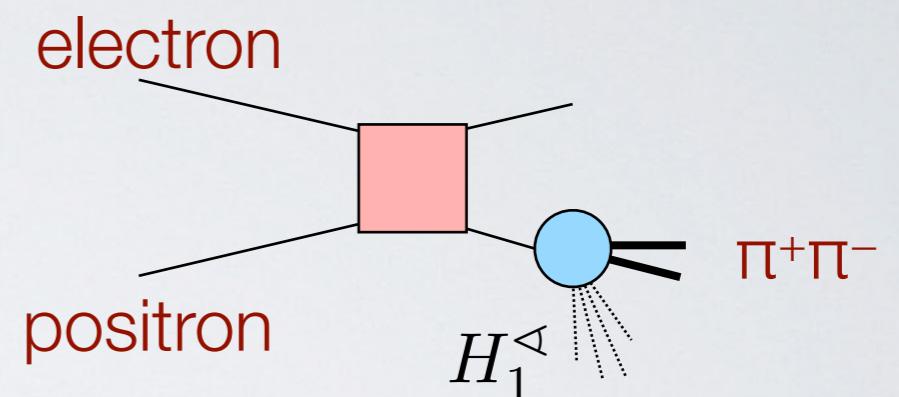


Airapetian et al.,  
*JHEP* **0806** (08) 017



Adolph et al., *P.L.* **B713** (12); *P.L.* **B736** (14)  
Braun et al., *E.P.J. Web Conf.* **85** (15) 02018

$e^+e^- \rightarrow (h_1 h_2) X$



Vossen et al., *P.R.L.* **107** (11) 072004

$D_1$  Seidl et al., *P.R.* **D96** (17) 032005

$H^- H^\uparrow \rightarrow (h_1 h_2) X$

proton

proton

$f_1 \times h_1 \times H_1^<$



run 2006 ( $s=200$ )

Adamczyk et al. (STAR),  
*P.R.L.* **115** (2015) 242501

run 2011 ( $s=500$ )

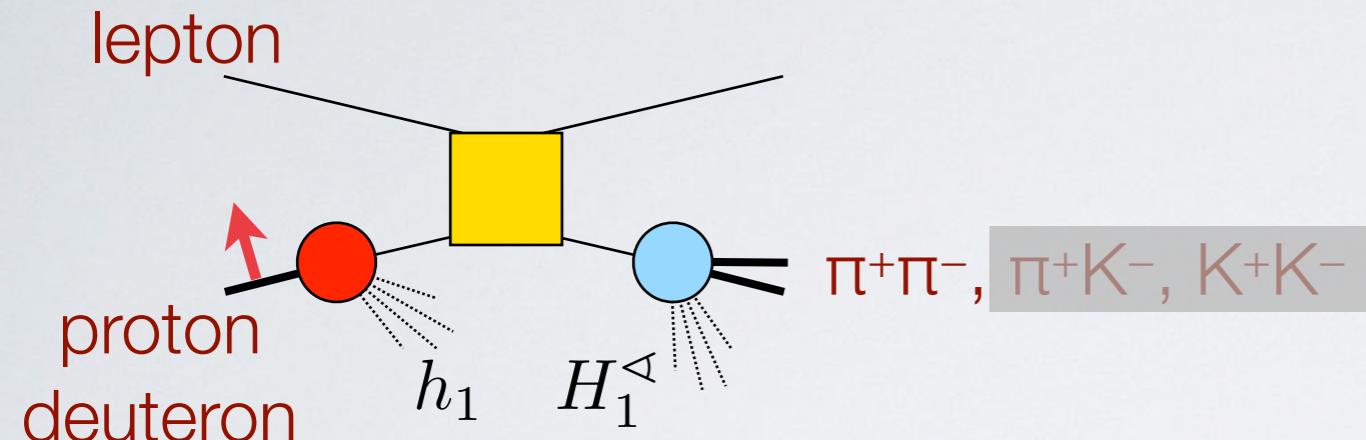
Adamczyk et al. (STAR),  
*P.L.* **B780** (18) 332

$\pi^+\pi^-$

$A_{UT}(\eta, M_h, P_T)$

# exp. data for 2-hadron-inclusive production

SIDIS  $\ell^- H^\uparrow \rightarrow \ell' (h_1 h_2) X$

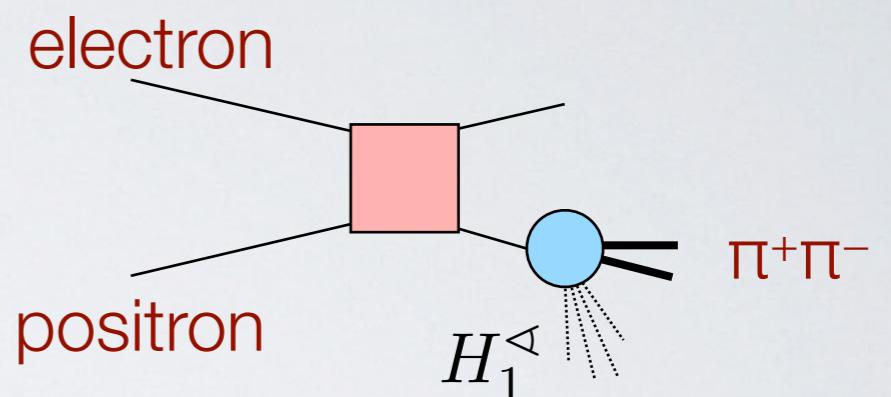


Airapetian et al.,  
JHEP **0806** (08) 017



Adolph et al., P.L. **B713** (12); P.L. **B736** (14)  
Braun et al., E.P.J. Web Conf. **85** (15) 02018

$e^+e^- \rightarrow (h_1 h_2) X$



Vossen et al., P.R.L. **107** (11) 072004

$D_1$  Seidl et al., P.R. **D96** (17) 032005

from Montecarlo

$H^- H^\uparrow \rightarrow (h_1 h_2) X$

proton

proton

$f_1 \times h_1 \times H_1^<$



run 2006

Adamczyk et al. (STAR),  
P.R.L. **115** (2015) 242501

run 2011

Adamczyk et al. (STAR),  
P.L. **B780** (18) 332

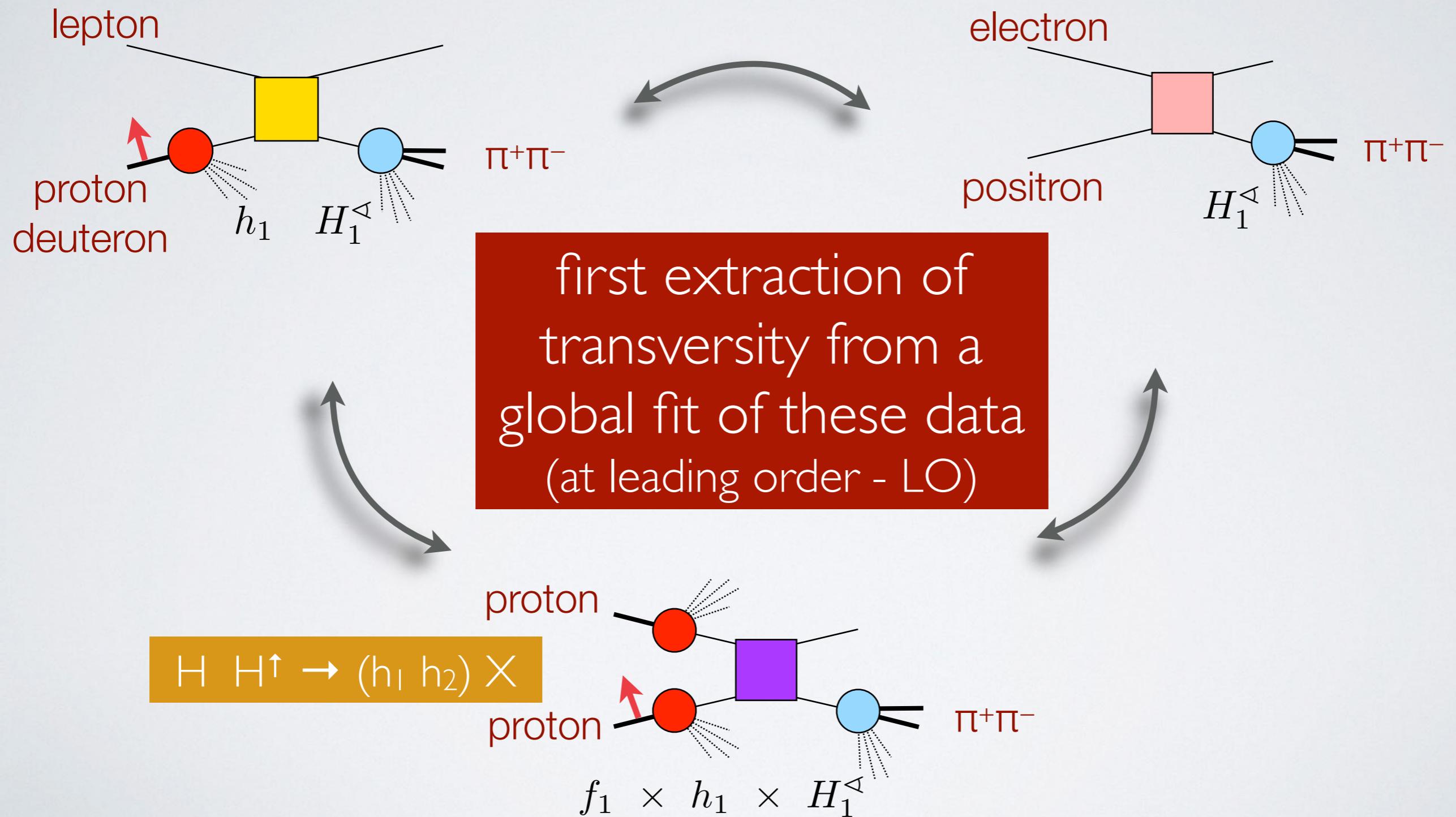
$\pi^+\pi^-$

$A_{UT}(\eta, M_h, P_T)$

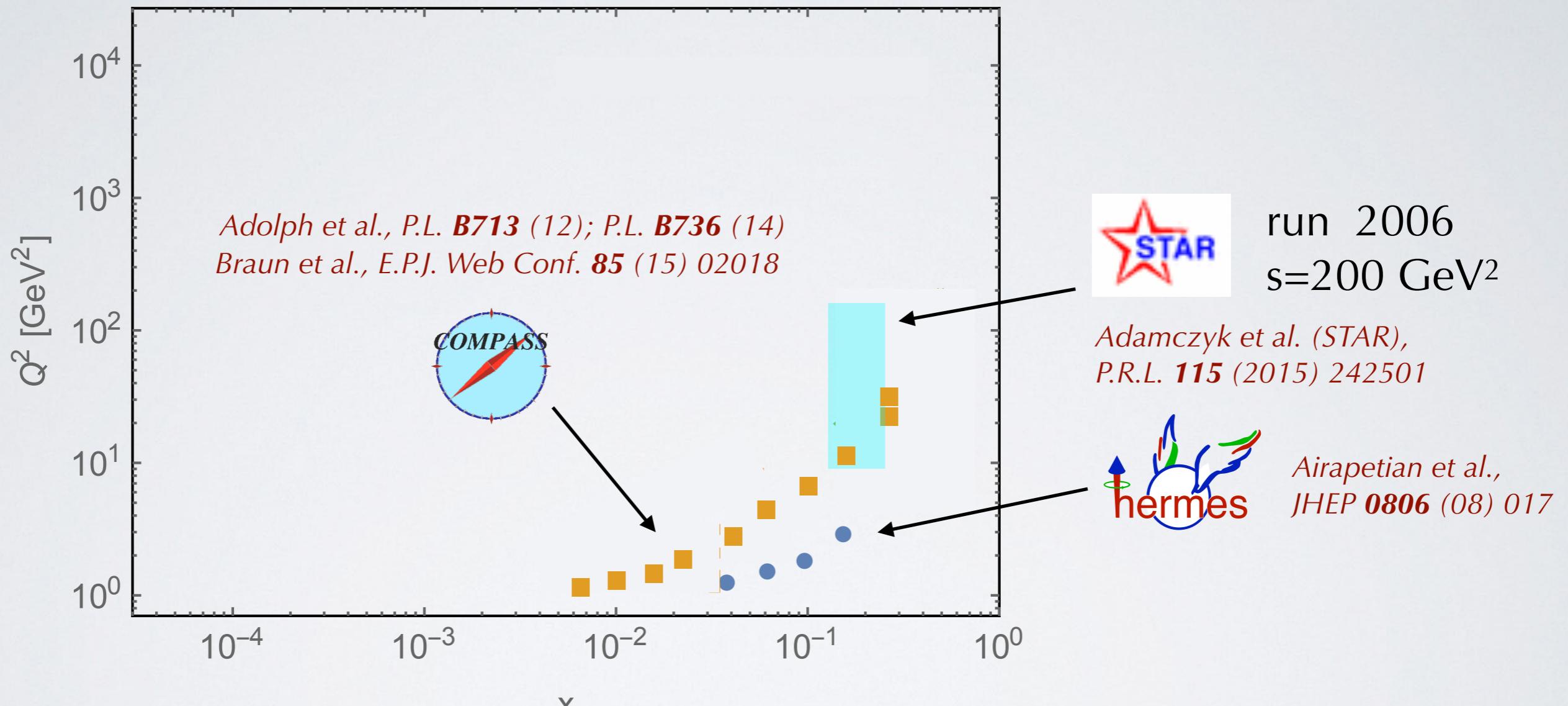
# take-away message

SIDIS  $\ell^- H^\uparrow \rightarrow \ell' (h_1 h_2) X$

$e^+e^- \rightarrow (h_1 h_2) X$

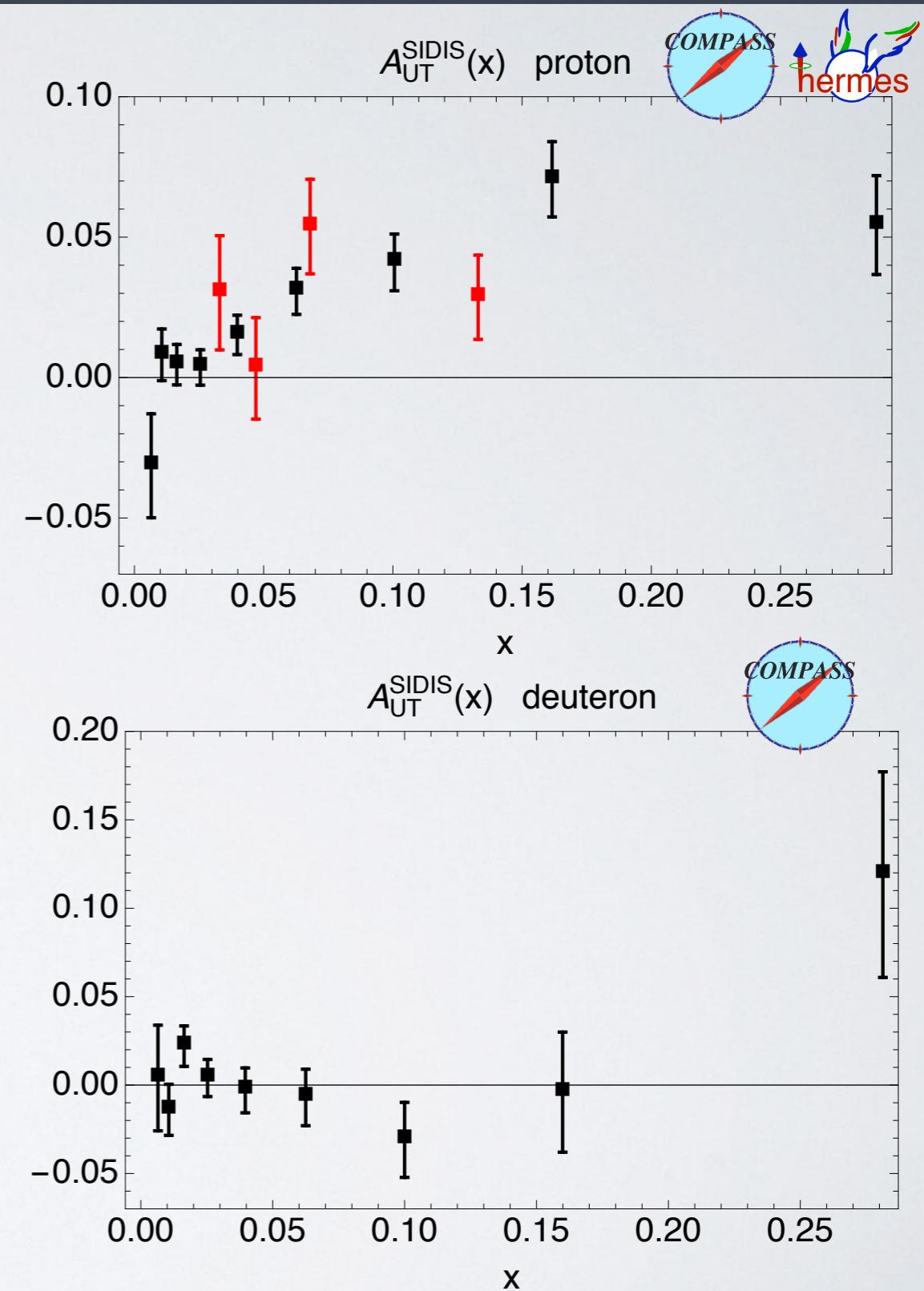


# the kinematics

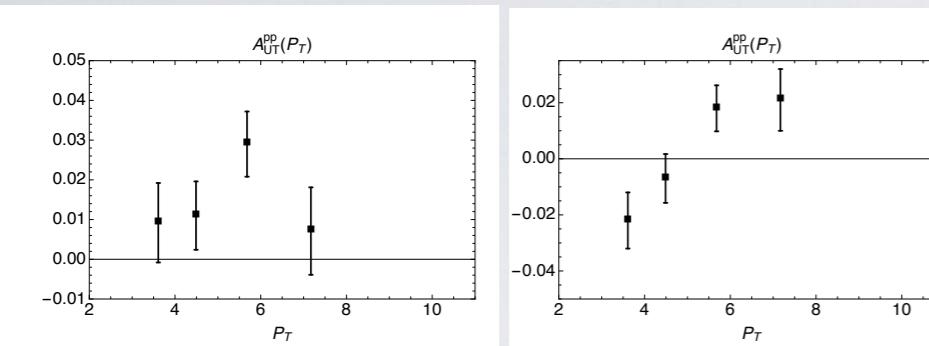


explore only valence quarks

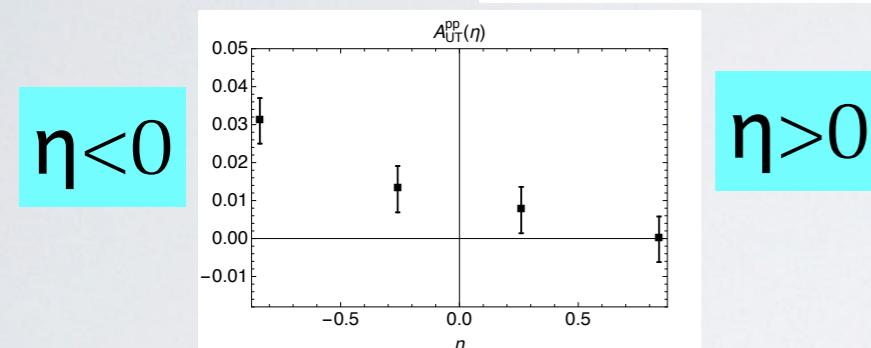
# the data set in more detail



# the data set in more detail

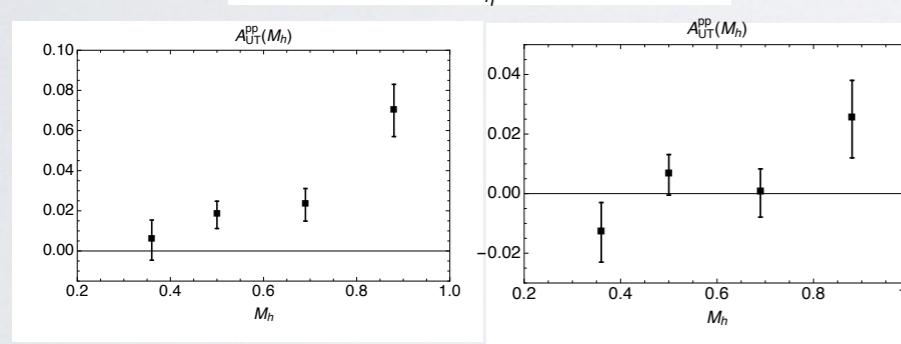


$A_{UT}^{pp}(P_T)$



$\eta > 0$

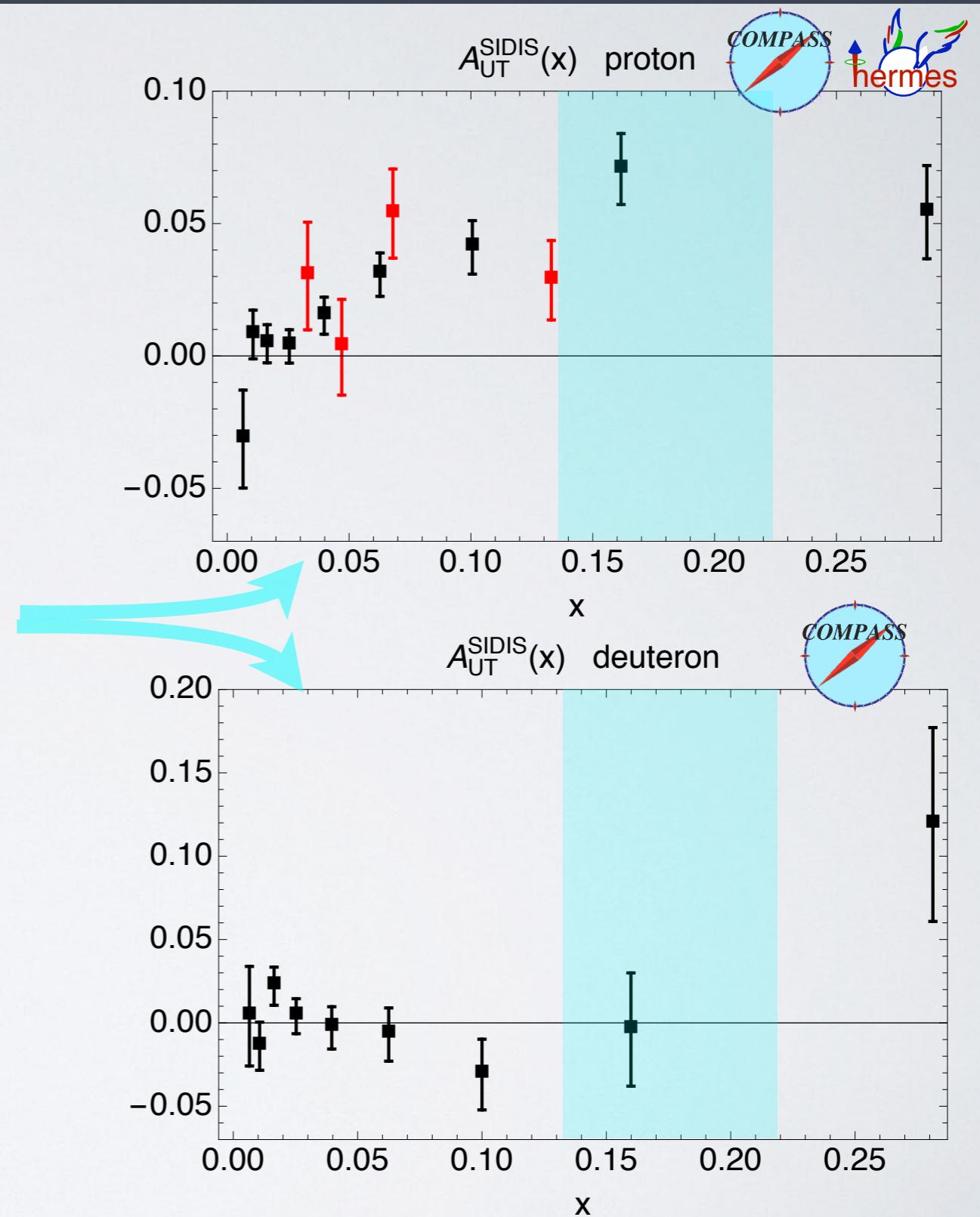
$A_{UT}^{pp}(\eta)$



$A_{UT}^{pp}(M_h)$



run 2006  $s=200 \text{ GeV}^2$   
(effective coverage in  $x$  [ ])



# choice of functional form

different funct. form whose Mellin transform can be computed analytically  
but keep main feature: comply with Soffer Bound at any x and scale  $Q^2$

$$h_1^{q_v}(x; Q_0^2) = F^{q_v}(x) \left[ \text{SB}^q(x) + \overline{\text{SB}}^{\bar{q}}(x) \right]$$



Soffer Bound

$$2|h_1^q(x, Q^2)| \leq 2 \text{ SB}^q(x, Q^2) = |f_1^q(x, Q^2) + g_1^q(x, Q^2)|$$

MSTW08 DSSV

# choice of functional form

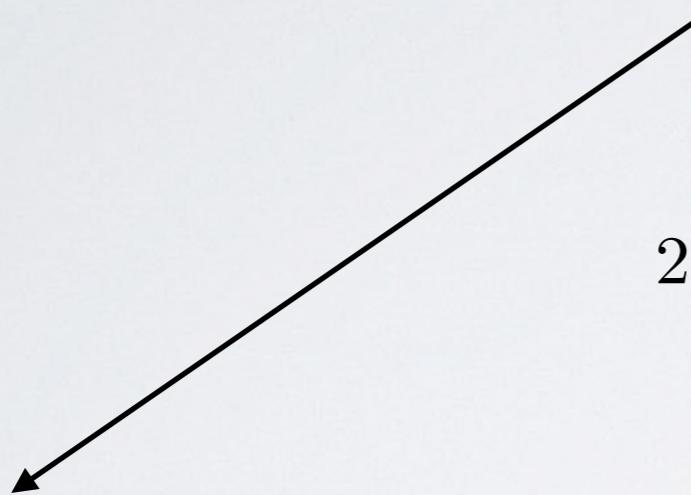
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$$h_1^{q_v}(x; Q_0^2) = F^{q_v}(x) \left[ \text{SB}^q(x) + \overline{\text{SB}}^{\bar{q}}(x) \right]$$

↓  
**Soffer Bound**

$$2|h_1^q(x, Q^2)| \leq 2 \text{ SB}^q(x, Q^2) = |f_1^q(x, Q^2) + g_1^q(x, Q^2)|$$

MSTW08      DSSV



$$F^{q_v}(x) = \frac{N_{q_v}}{\max_x [|F^{q_v}(x)|]} x^{A_{q_v}} [1 + B_{q_v} \text{Ceb}_1(x) + C_{q_v} \text{Ceb}_2(x) + D_{q_v} \text{Ceb}_3(x)]$$

Ceb<sub>n</sub>(x) Cebyshev polynomial  
10 fitting parameters

constrain parameters

$$|N_{q_v}| \leq 1 \Rightarrow |F^{q_v}(x)| \leq 1 \quad \text{Soffer Bound ok at any } Q^2$$

# choice of functional form

$$h_1^{q_v}(x; Q_0^2) = F^{q_v}(x) \left[ \text{SB}^q(x) + \overline{\text{SB}}^{\bar{q}}(x) \right]$$

$$F^{q_v}(x) = \frac{N_{q_v}}{\max_x [|F^{q_v}(x)|]} x^{A_{q_v}} [1 + B_{q_v} \text{Ceb}_1(x) + C_{q_v} \text{Ceb}_2(x) + D_{q_v} \text{Ceb}_3(x)]$$

if  $\lim_{x \rightarrow 0} x \text{SB}^q(x) \propto x^{a_q}$  then  $h_1^q(x) \underset{x \rightarrow 0}{\approx} x^{A_q + a_q - 1}$

constrain parameters

tensor charge  $\delta q(Q^2) = \int_{x_{\min}}^1 dx h_1^{q-\bar{q}}(x, Q^2)$

low-x behavior is important outside data range

- 1<sup>st</sup> option: finite tensor charge  $\rightarrow A_q + a_q > \frac{1}{3}$  grants also error  $O(1\%)$  for MSTW08  $x_{\min}=10^{-6}$

- 2<sup>nd</sup> option: finite violation of Burkhardt-Cottingham sum rule

*Accardi and Bacchetta,  
P.L. **B773** (17) 632*

$$\int_0^1 dx g_2(x) \propto \int_0^1 dx \frac{h_1(x)}{x} \rightarrow A_q + a_q > 1$$

# theoretical uncertainties

## unpolarized Di-hadron Fragmentation Function $D_1$

- **quark**  $D_{1q}$  is **well** constrained by  $e^+e^- \rightarrow (\pi^+\pi^-) X$  (Montecarlo)
- **gluon**  $D_{1g}$  is **not** constrained by  $e^+e^- \rightarrow (\pi^+\pi^-) X$  (currently, LO analysis)
- **no data** available yet for  $p p \rightarrow (\pi^+\pi^-) X$

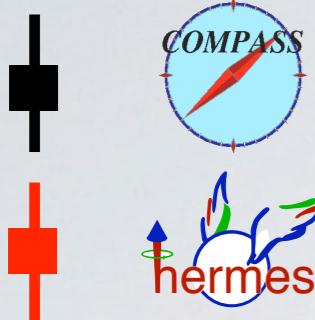
we don't know anything about the gluon  $D_{1g}$

our choice: set  $D_{1g}(Q_0) = \begin{cases} 0 \\ D_{1u}(Q_0) / 4 \\ D_{1u}(Q_0) \end{cases}$

deteriorates our  $e^+e^-$  fit as  $\chi^2/\text{dof} = \begin{cases} 1.69 & 1.28 \\ 1.81 & 1.37 \\ 2.96 & 2.01 \end{cases}$

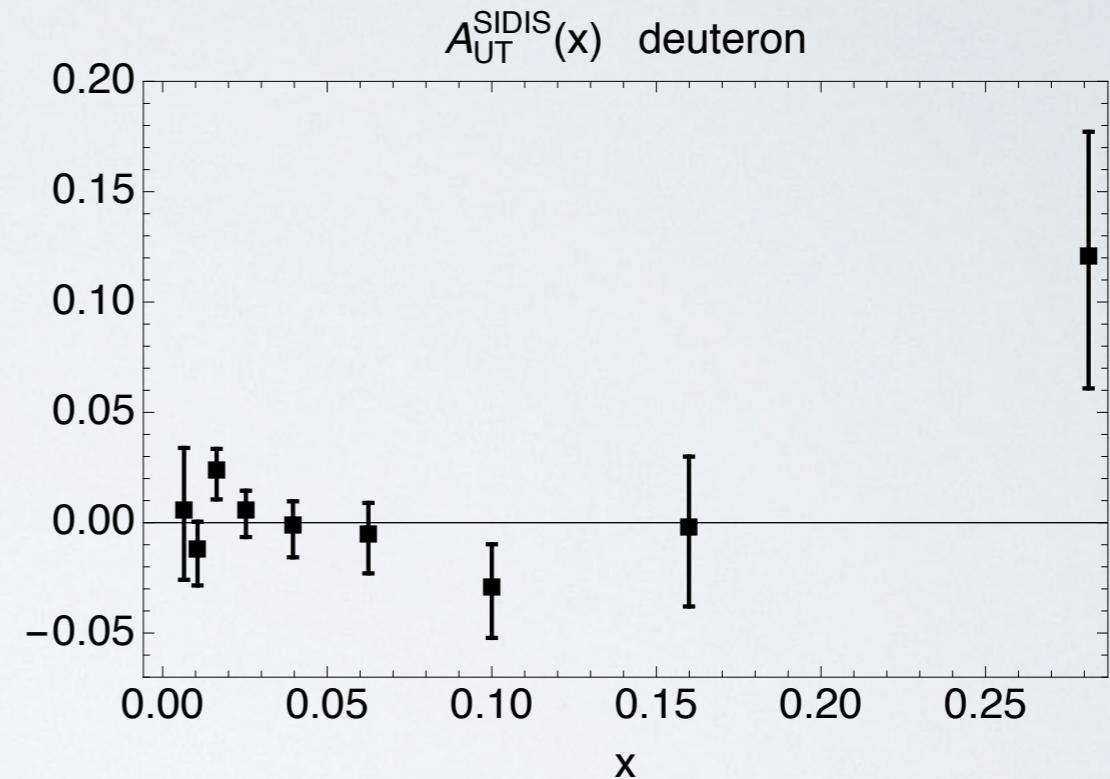
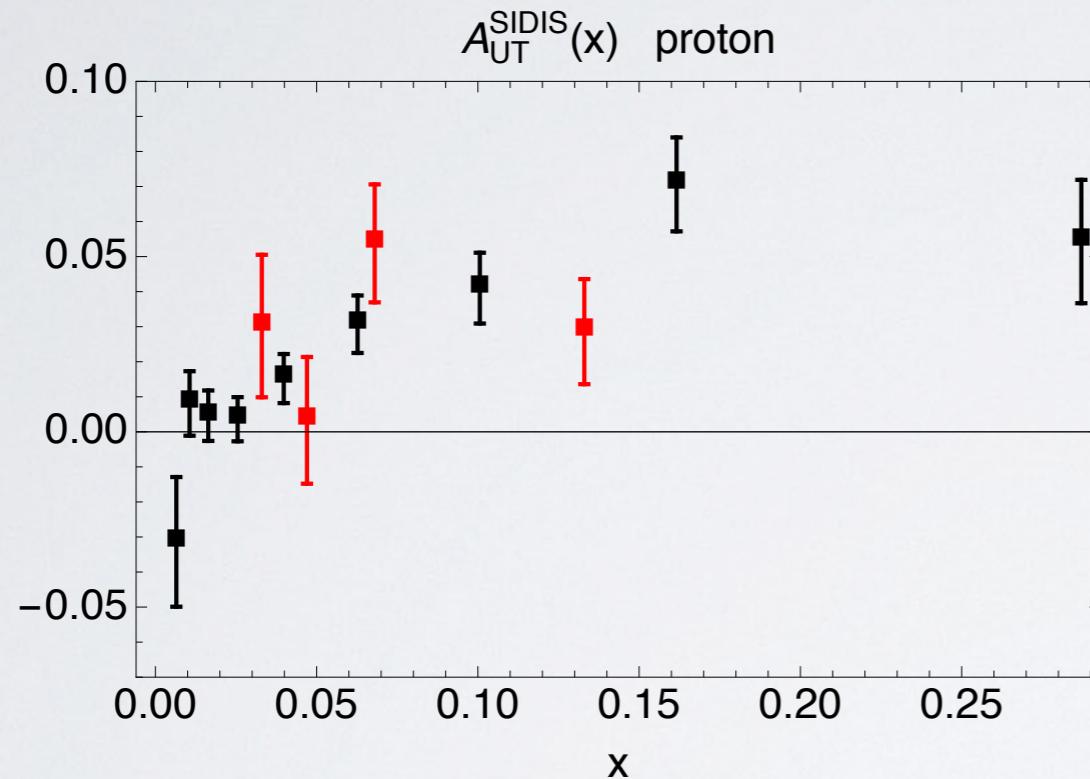
background     $\rho$     channels

# statistical uncertainty: the bootstrap method



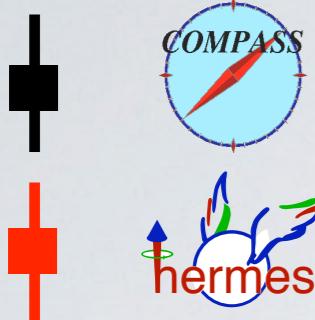
Braun et al., E.P.J. Web Conf. **85** (15) 02018

Airapetian et al., JHEP **0806** (08) 017



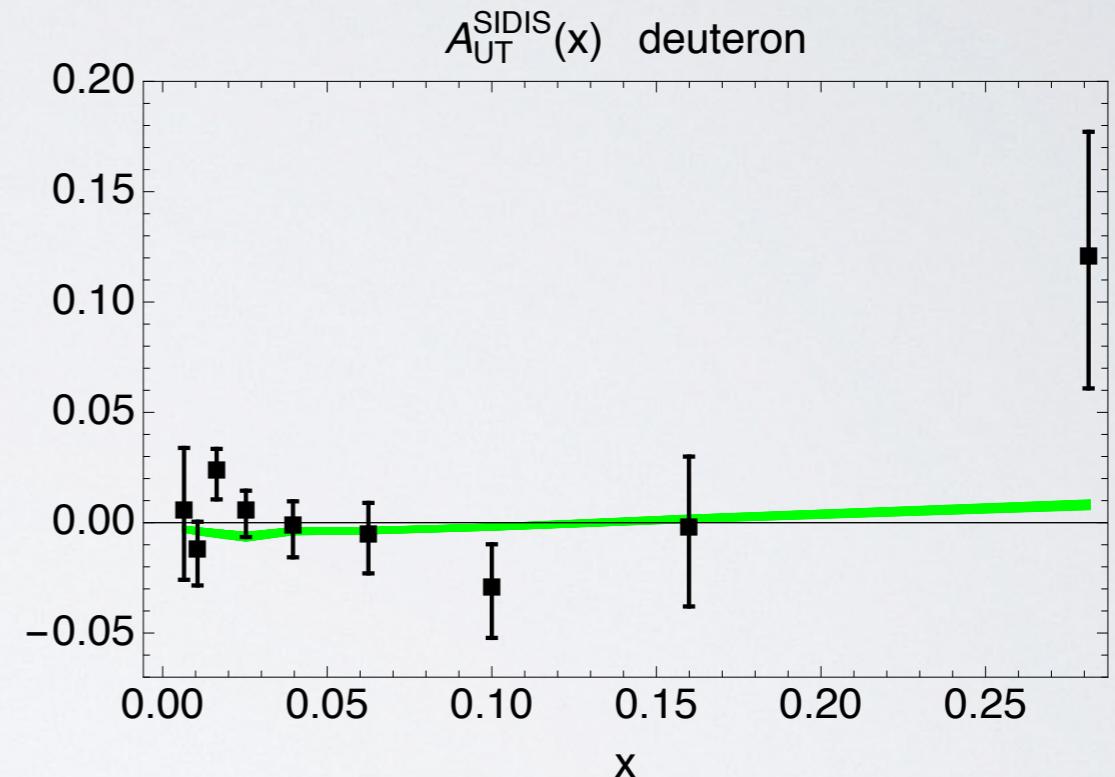
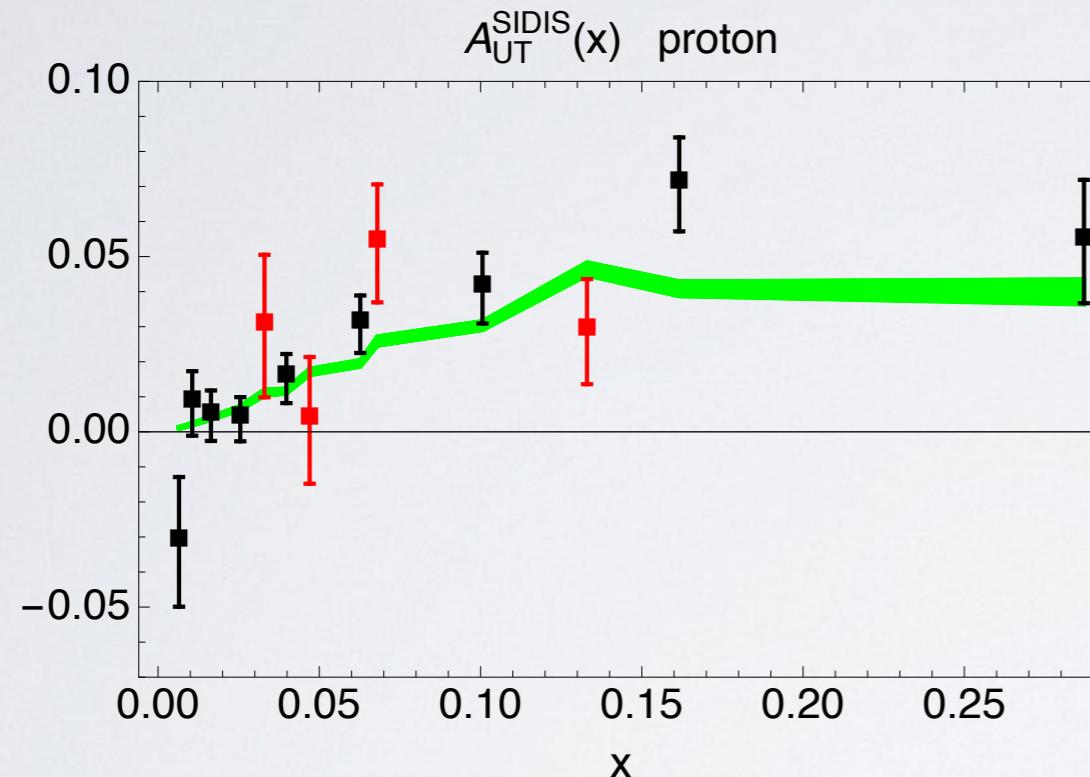
shift each exp. data point by Gaussian noise within exp. variance  
→ create a replica of all exp. data points and fit them

# statistical uncertainty: the bootstrap method



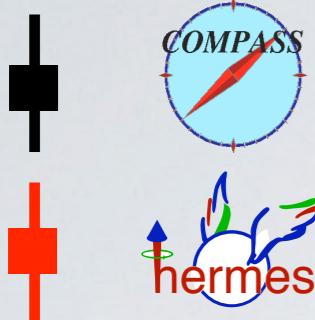
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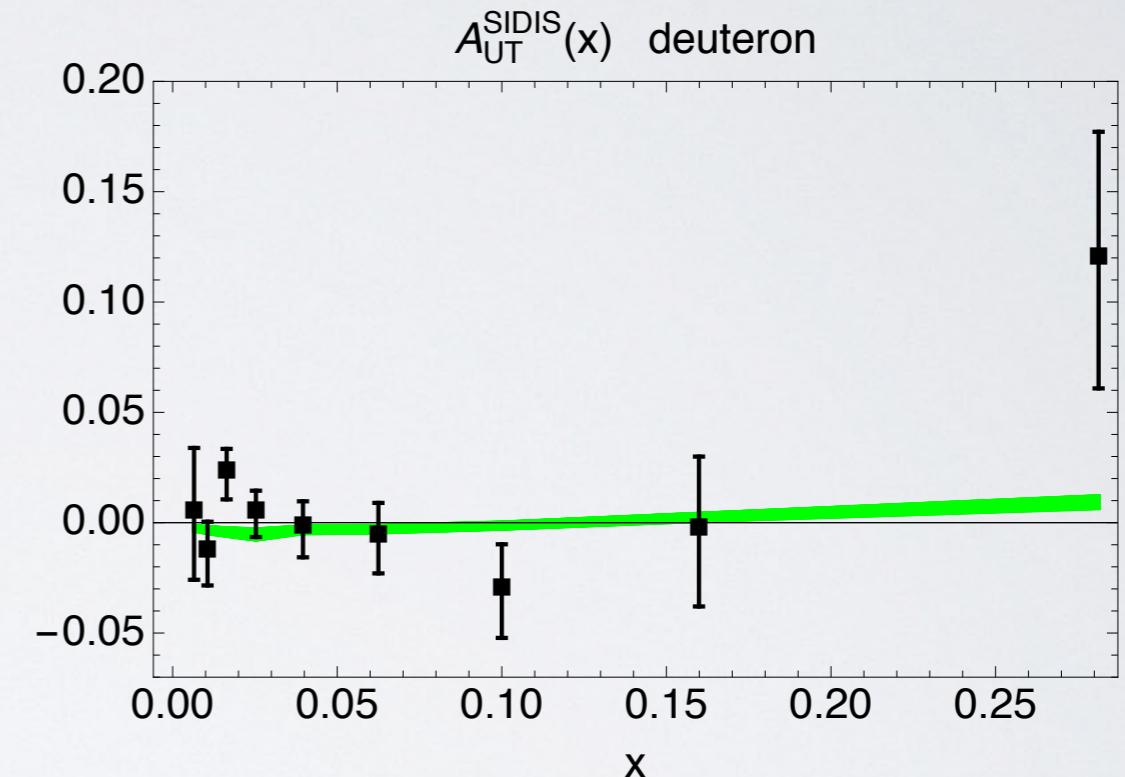
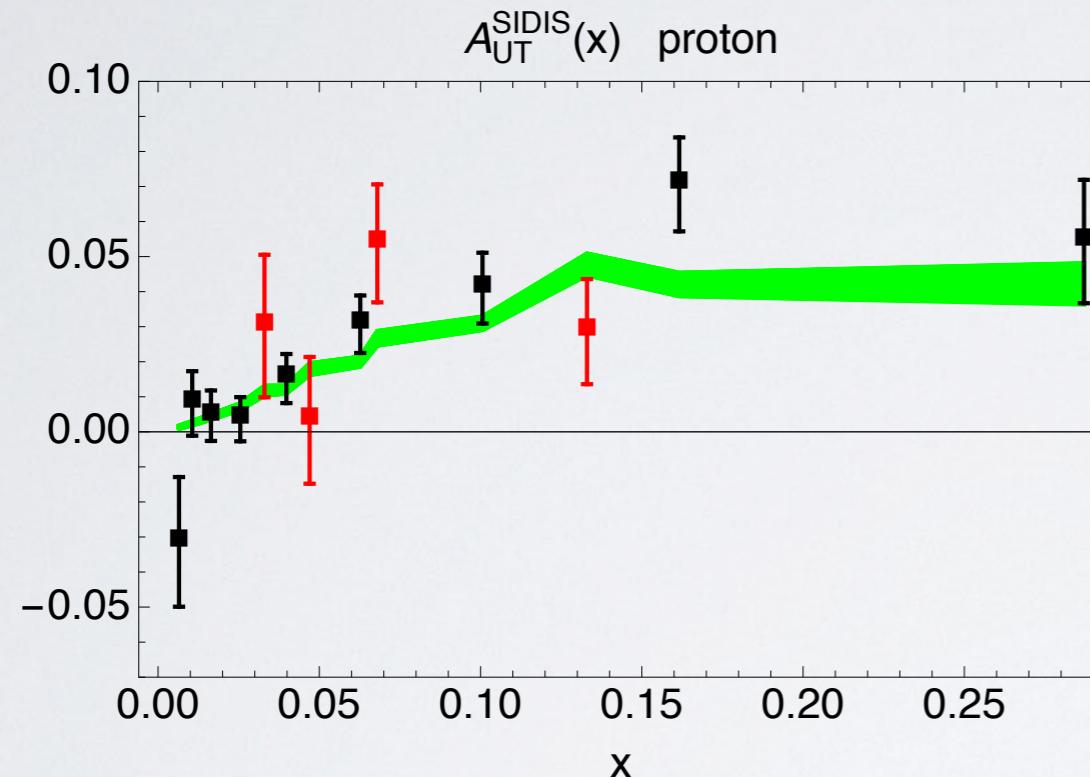
50 replicas

# statistical uncertainty: the bootstrap method



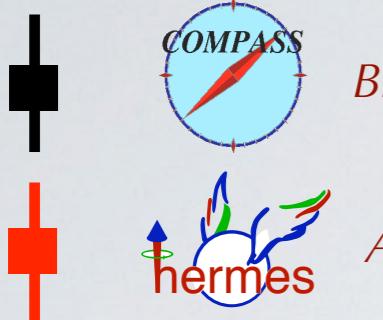
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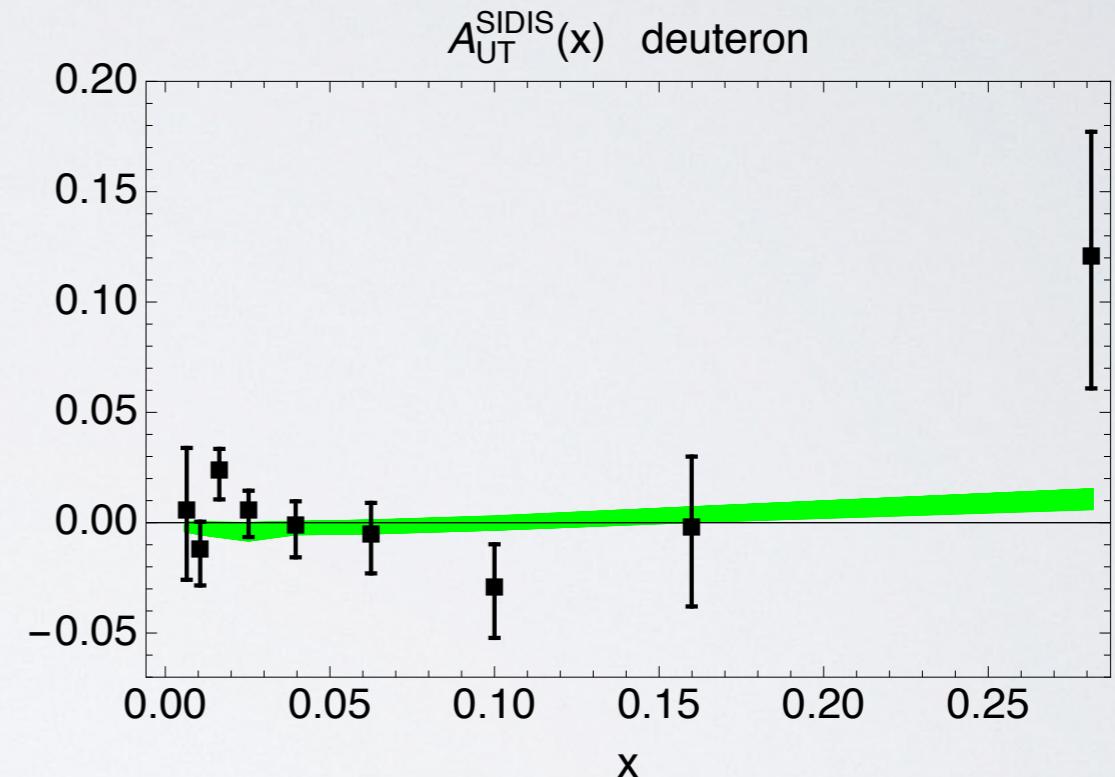
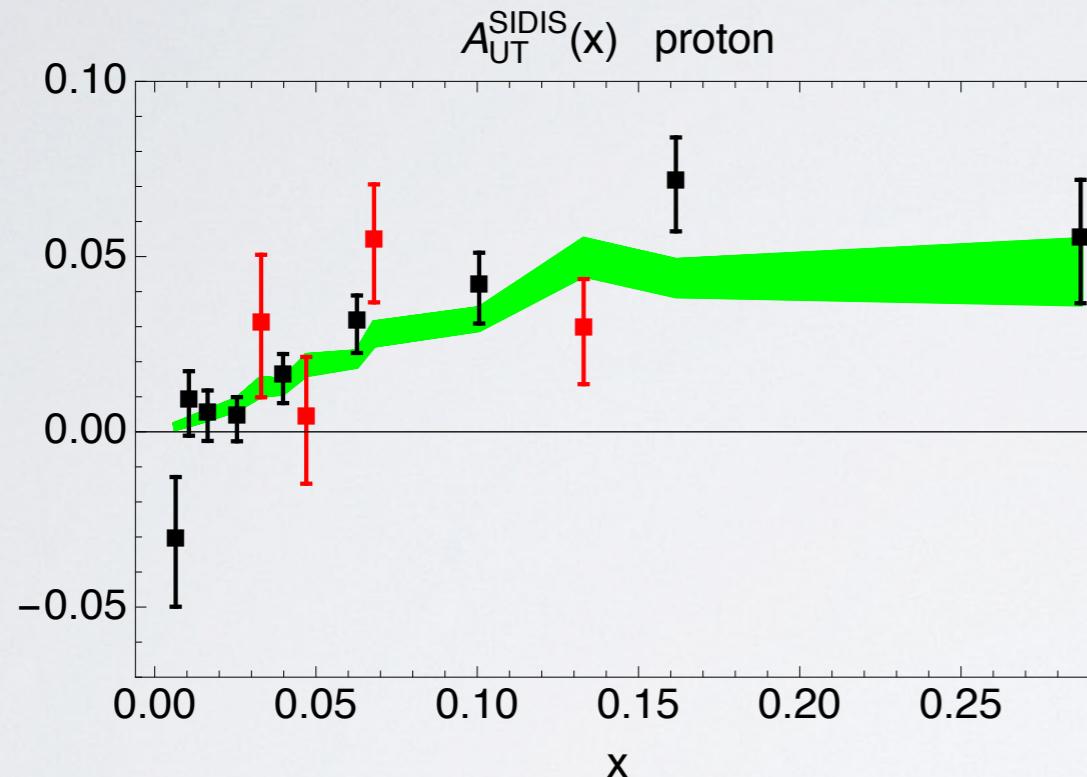
100 replicas

# statistical uncertainty: the bootstrap method



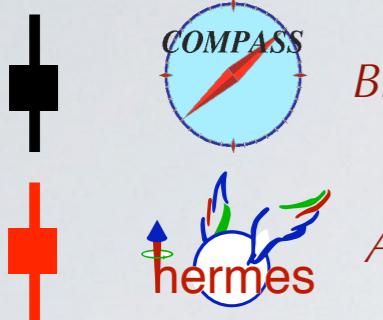
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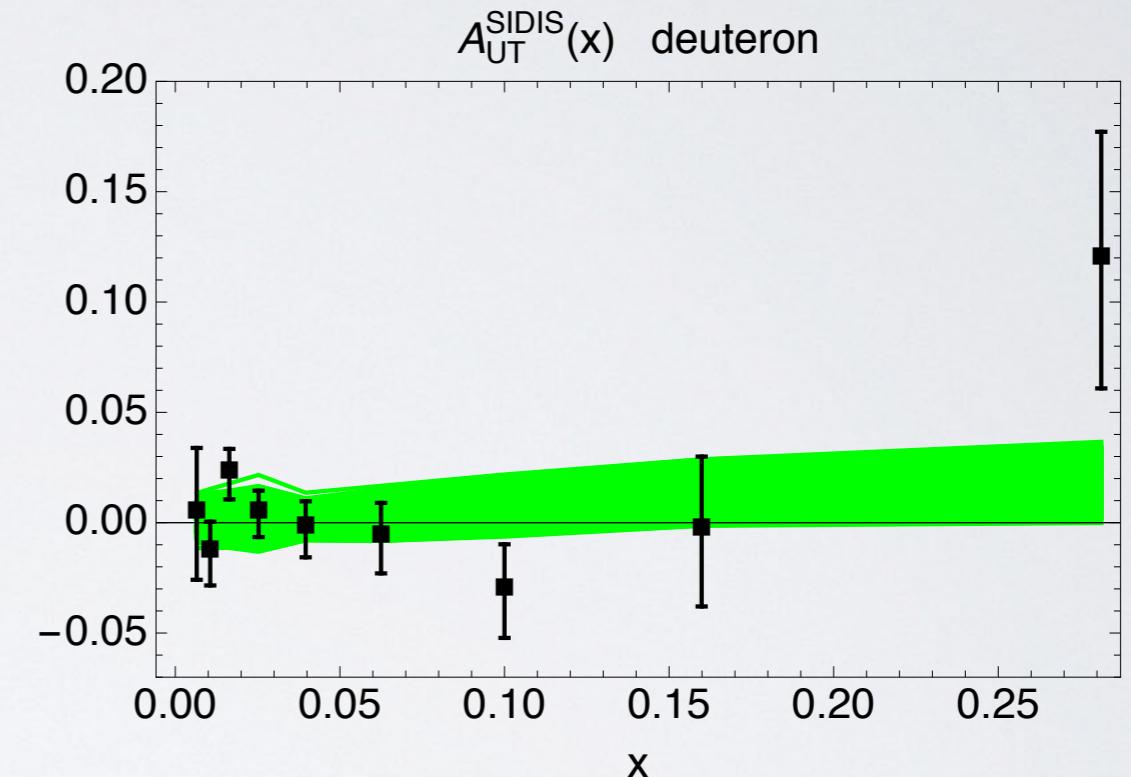
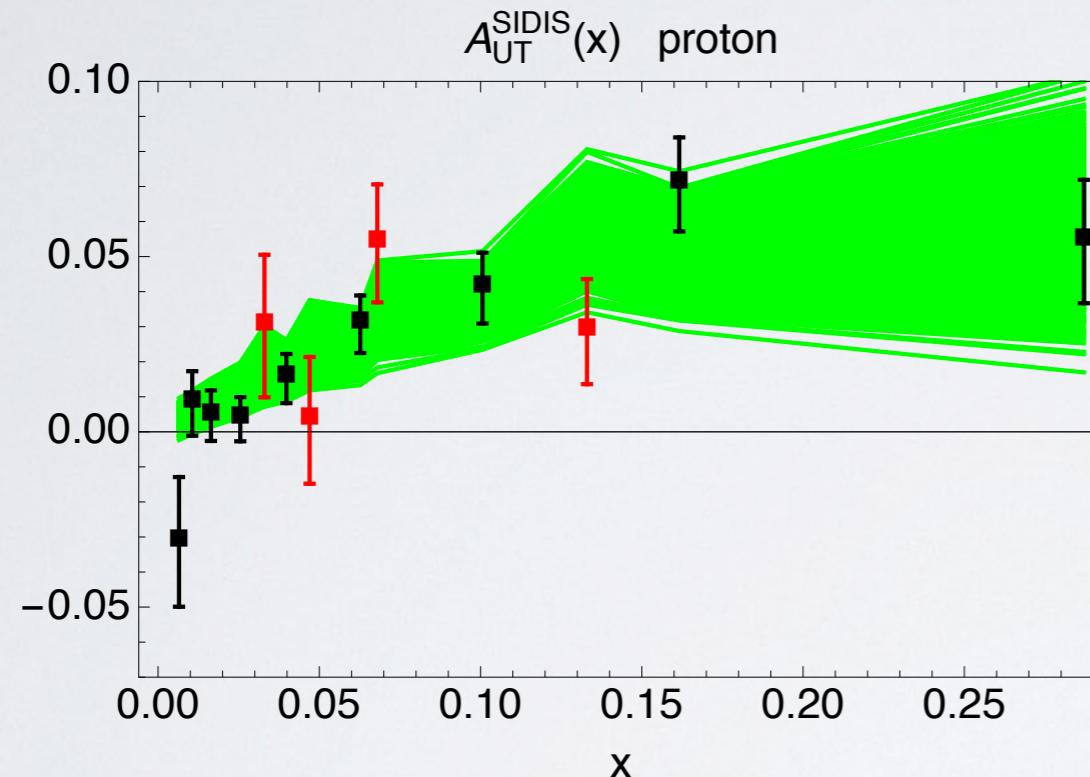
200 replicas

# statistical uncertainty: the bootstrap method



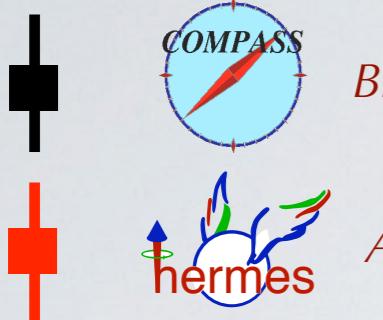
Braun et al., E.P.J. Web Conf. **85** (15) 02018

Airapetian et al., JHEP **0806** (08) 017



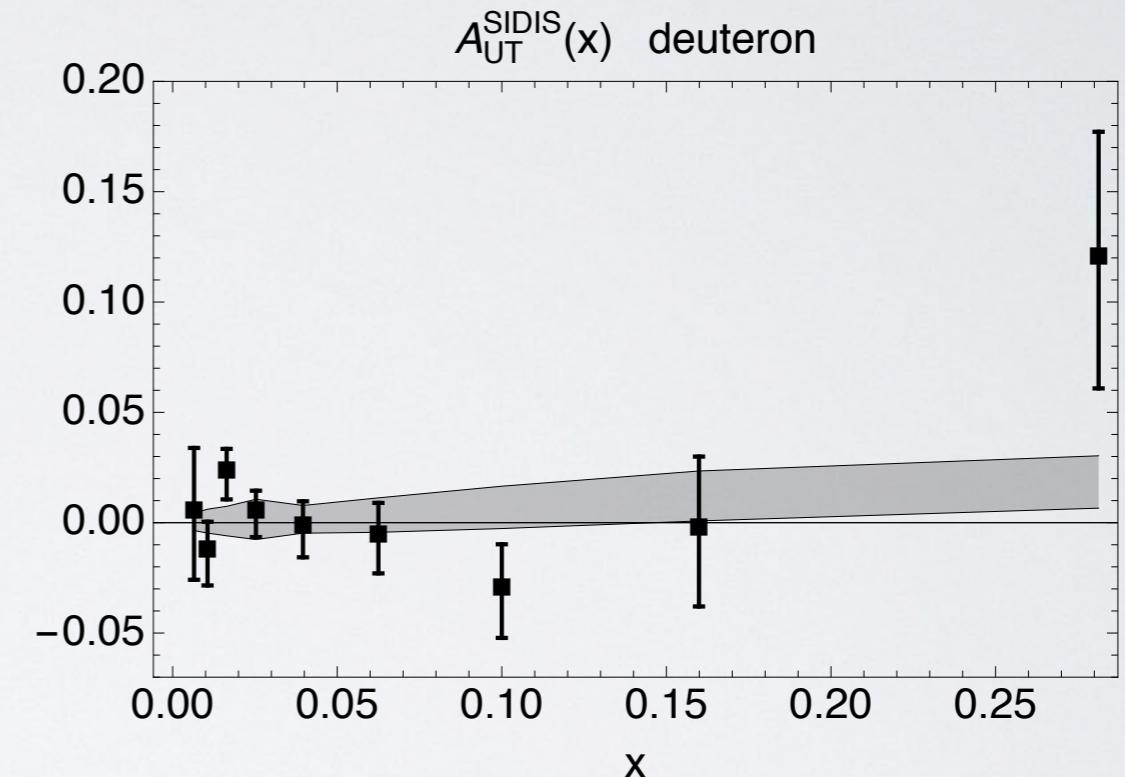
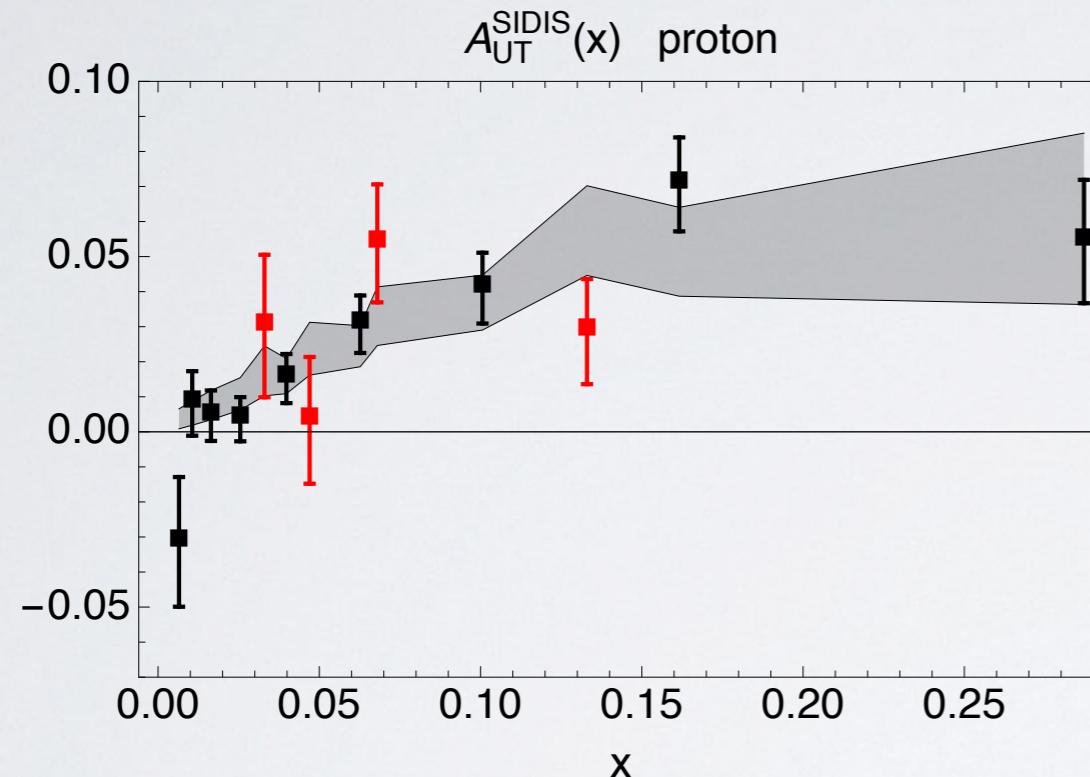
all 600 replicas

# statistical uncertainty: the bootstrap method



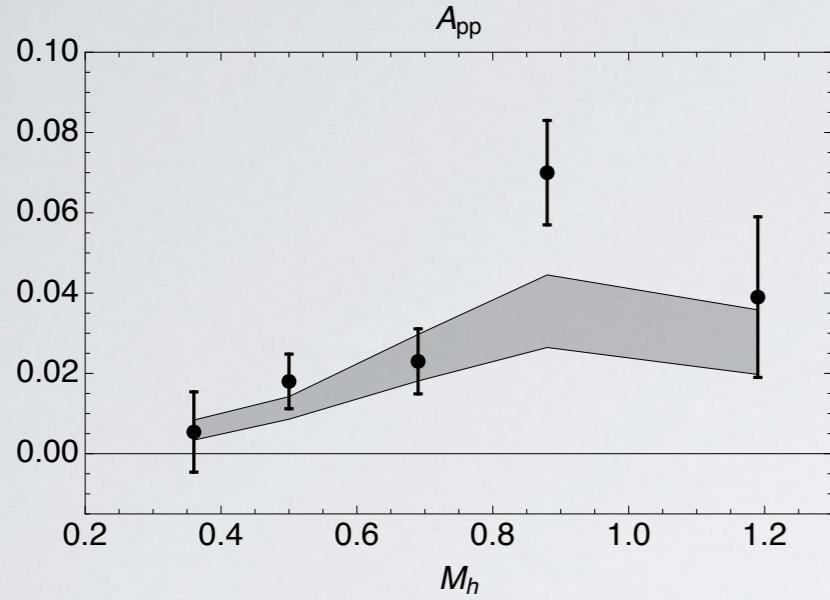
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Airapetian et al., JHEP **0806** (08) 017

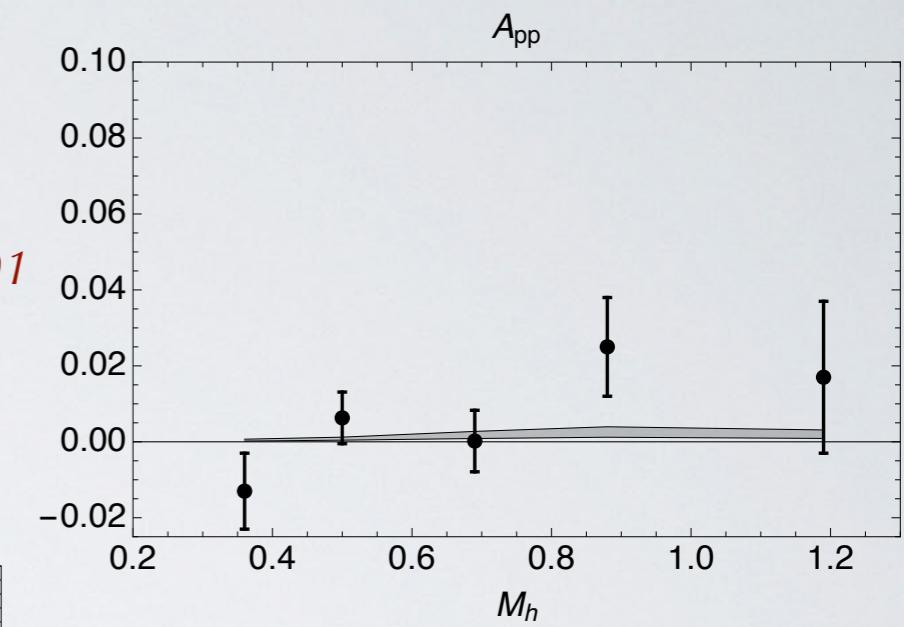


90% replicas

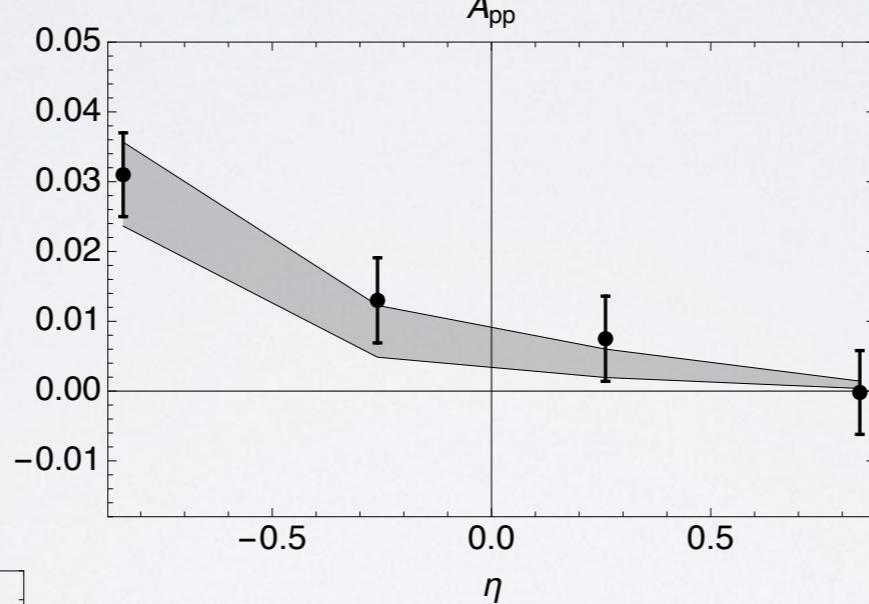
# fit STAR asymmetry



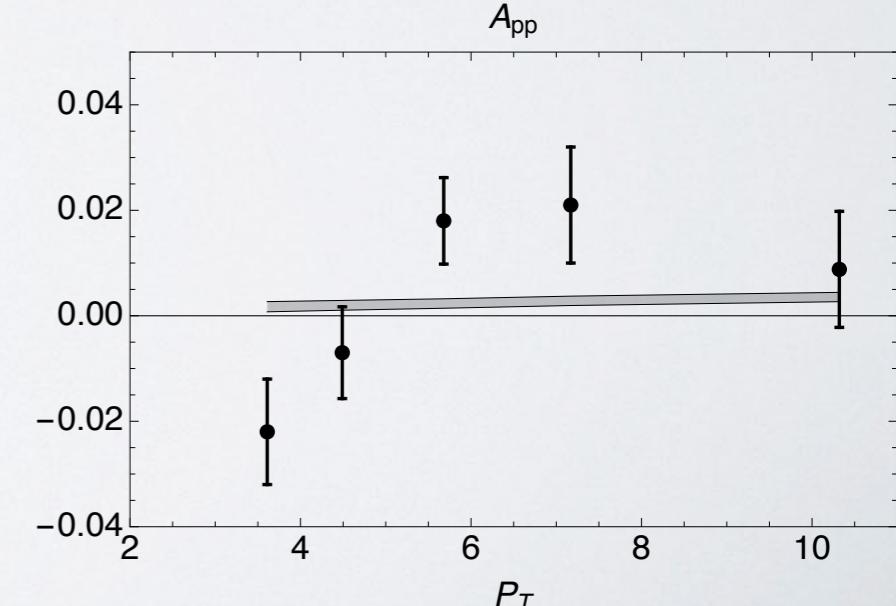
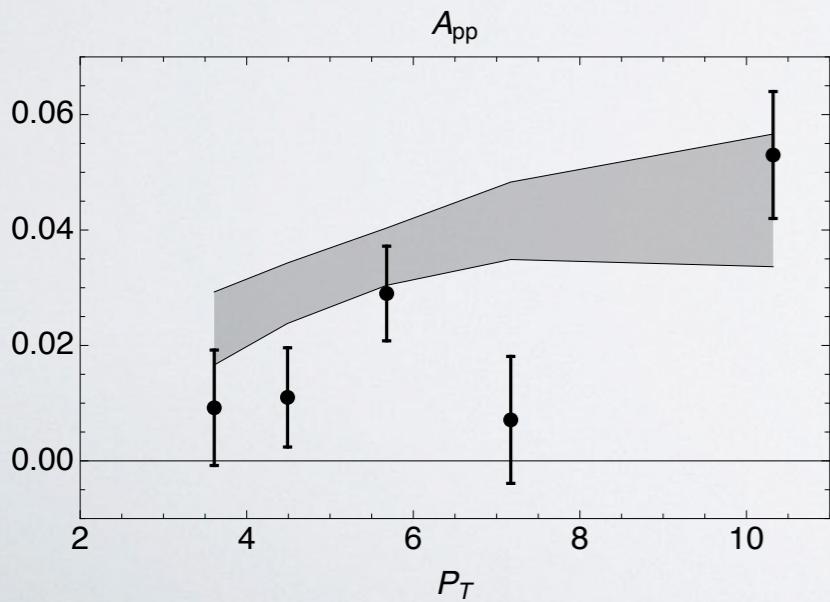
Adamczyk et al. (STAR),  
P.R.L. 115 (2015) 242501



$\eta > 0$



90% uncertainty band



# $\chi^2$ of the fit

**46** data points, **10** parameters  
global  $\chi^2/\text{dof} = 2.08 \pm 0.09$

$\approx 38\%$

$\approx 62\%$

**SIDIS**

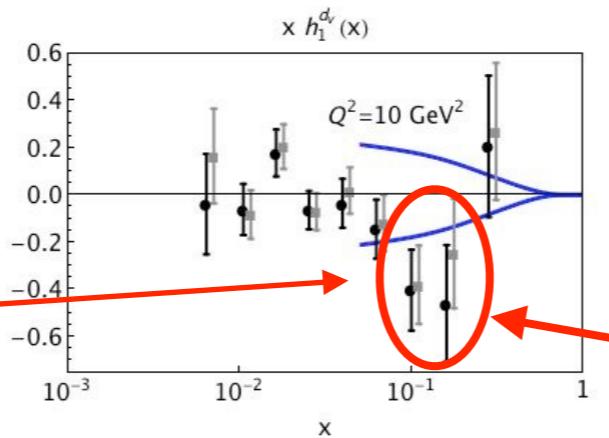
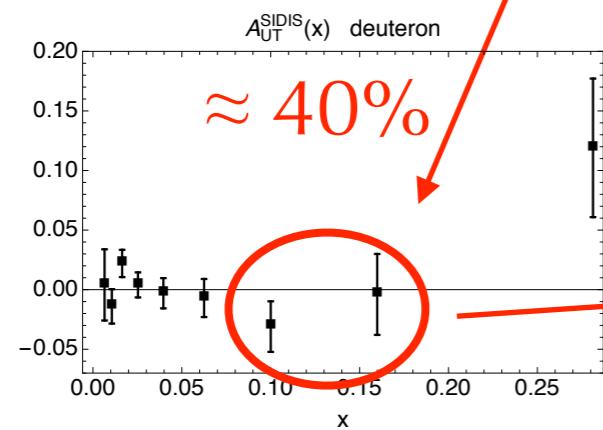


$\approx 24\%$



$\approx 76\%$

$\approx 60\%$   
rest



**STAR**

$\rightarrow P_T$  bins  $\approx 70\%$

$\rightarrow M_h$  bins  $\approx 28\%$

$\rightarrow \eta$  bins  $\approx 2\%$

# results

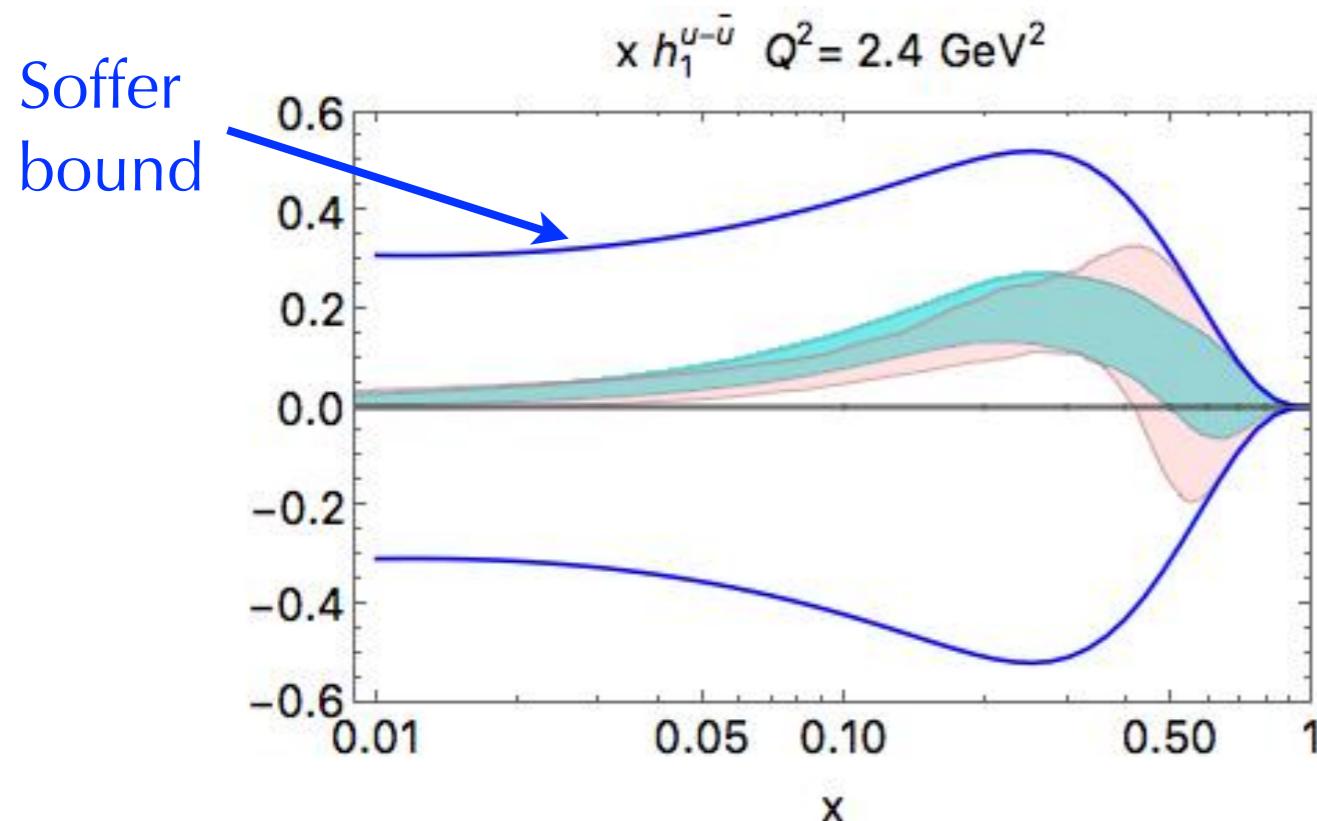
$$h_1^q(x) \stackrel{x \rightarrow 0}{\approx} x^{A_q + a_q - 1}$$

- 1<sup>st</sup> option: finite tensor charge

$$\rightarrow A_q + a_q > \frac{1}{3}$$

grants also error  $O(1\%)$  in calculation of tensor charge for MSTW08  $x_{\min}=10^{-6}$

# comparison with previous fit



*Radici & Bacchetta,  
P.R.L. **120** (18) 192001*

global fit

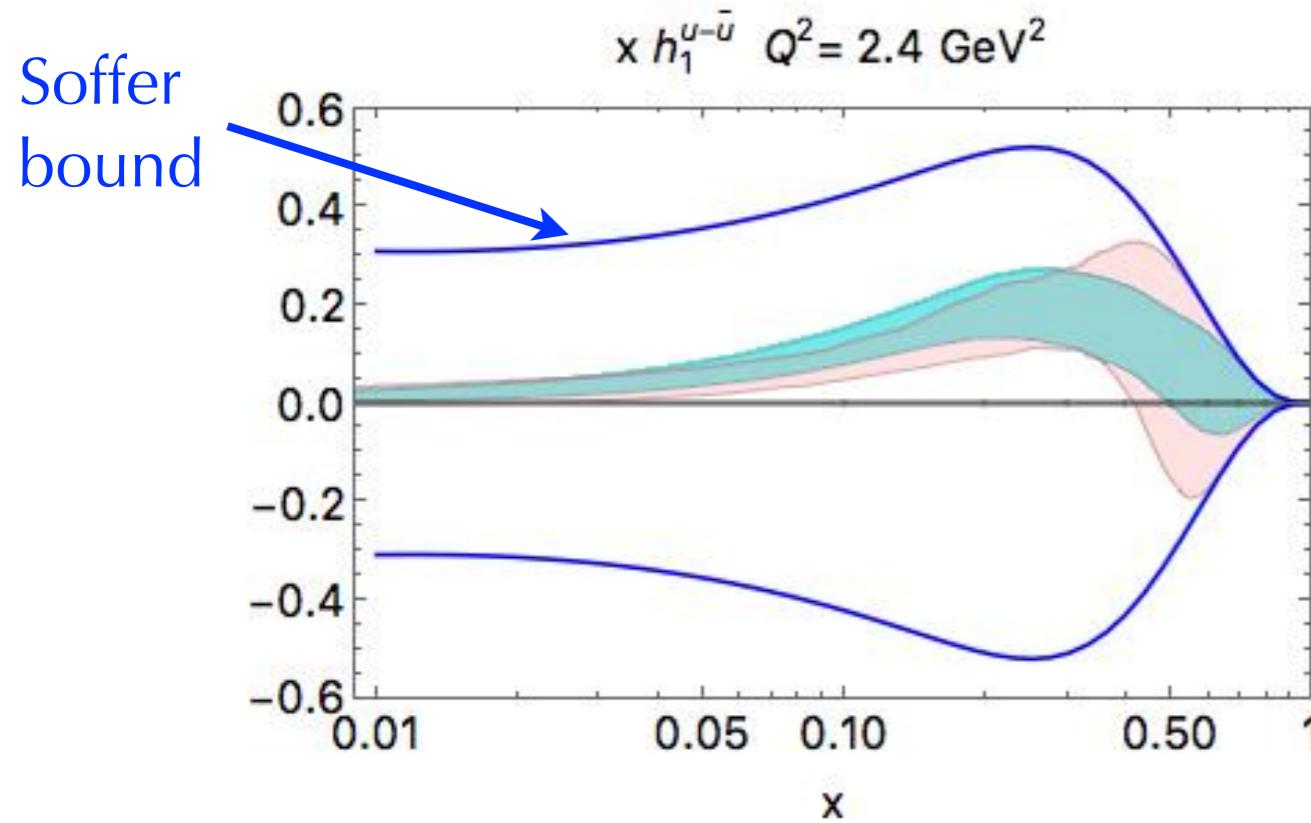
up

higher  
precision

old fit (only SIDIS data)

*Radici et al.,  
JHEP **1505** (15) 123*

# comparison with previous fit



*Radici & Bacchetta,  
P.R.L. **120** (18) 192001*

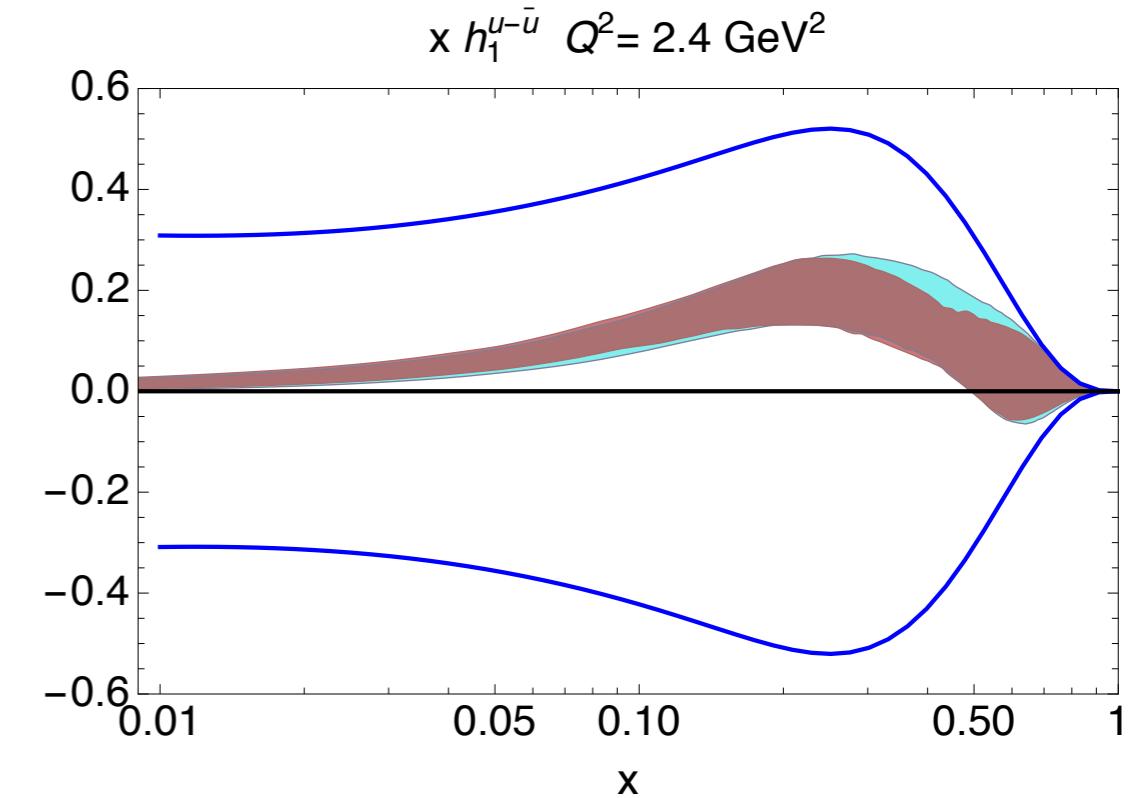
up

higher precision

up

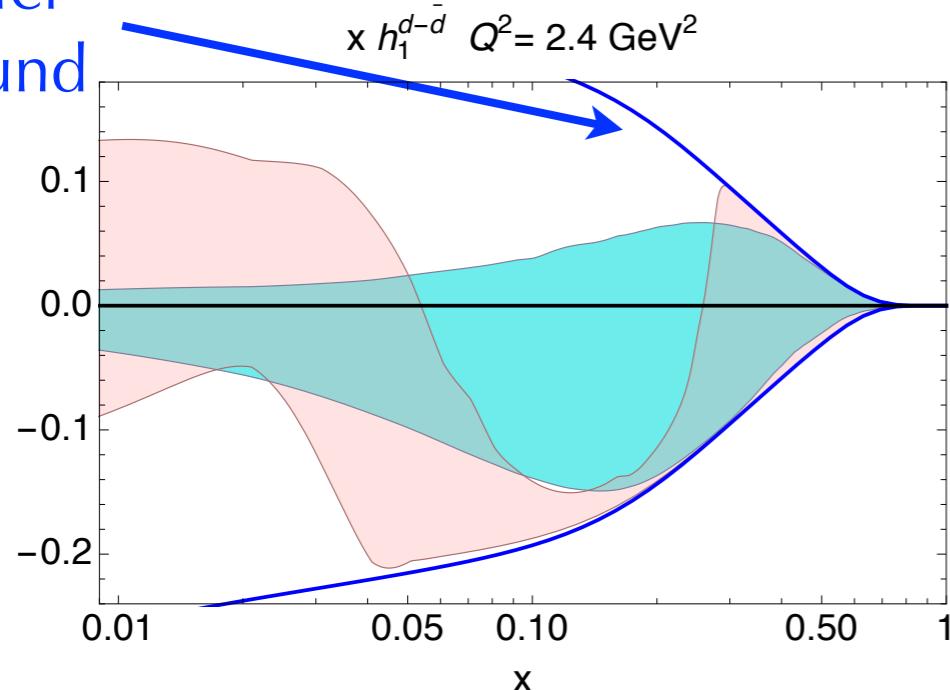
insensitive to  
uncertainty on  
gluon  $D_1$

$$D_{1g}(Q_0) = \begin{cases} 0 \\ D_{1u}/4 \\ D_{1u} \end{cases}$$



# comparison with previous fit

Soffer  
bound



down

sensitive to  
uncertainty on  
gluon  $D_1$

Radici & Bacchetta,  
P.R.L. 120 (18) 192001

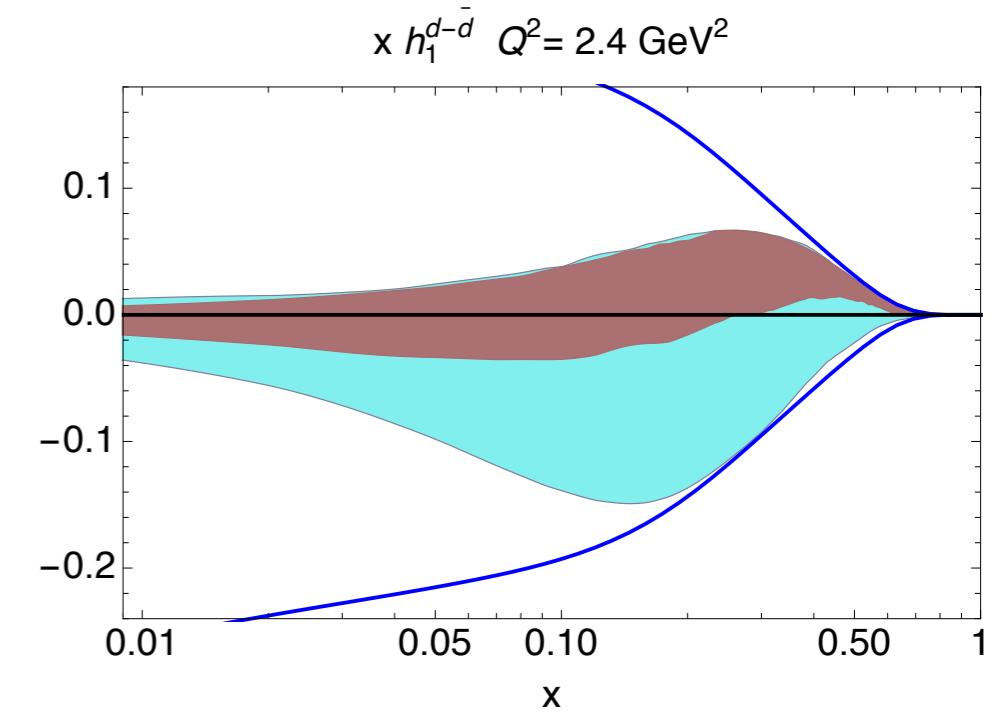
global fit

old fit

Radici et al.,  
JHEP 1505 (15) 123

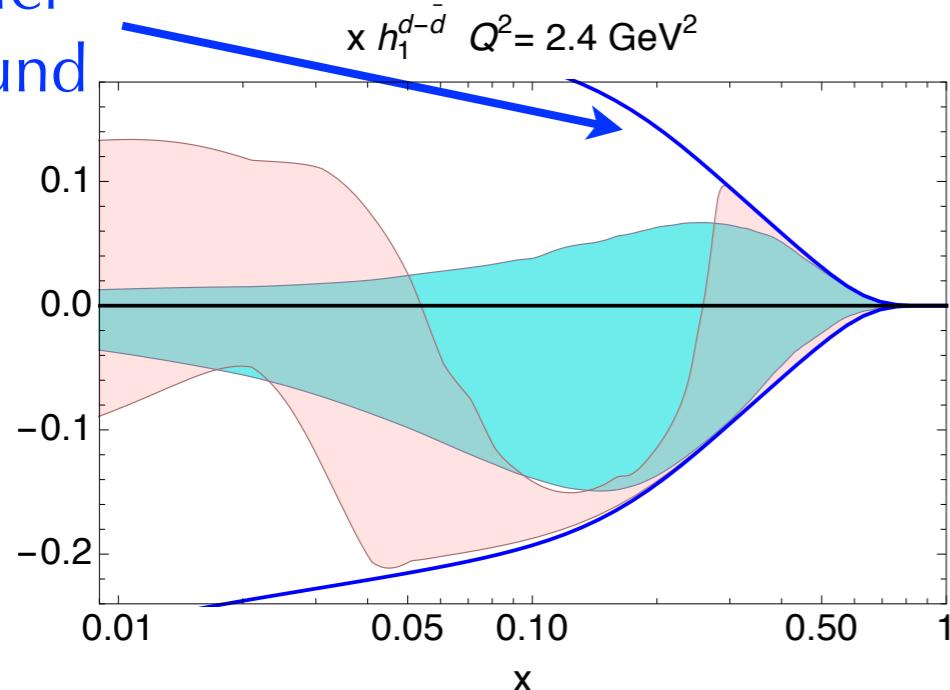
down

$$D_{1g}(Q_0) = 0$$
$$D_{1g}(Q_0) = \begin{cases} 0 \\ D_{1u}/4 \\ D_{1u} \end{cases}$$



# comparison with previous fit

Soffer  
bound



*Radici & Bacchetta,  
P.R.L. **120** (18) 192001*

global fit

old fit

*Radici et al.,  
JHEP **1505** (15) 123*

down

need NLO analysis

+

dihadron multiplicities in pp

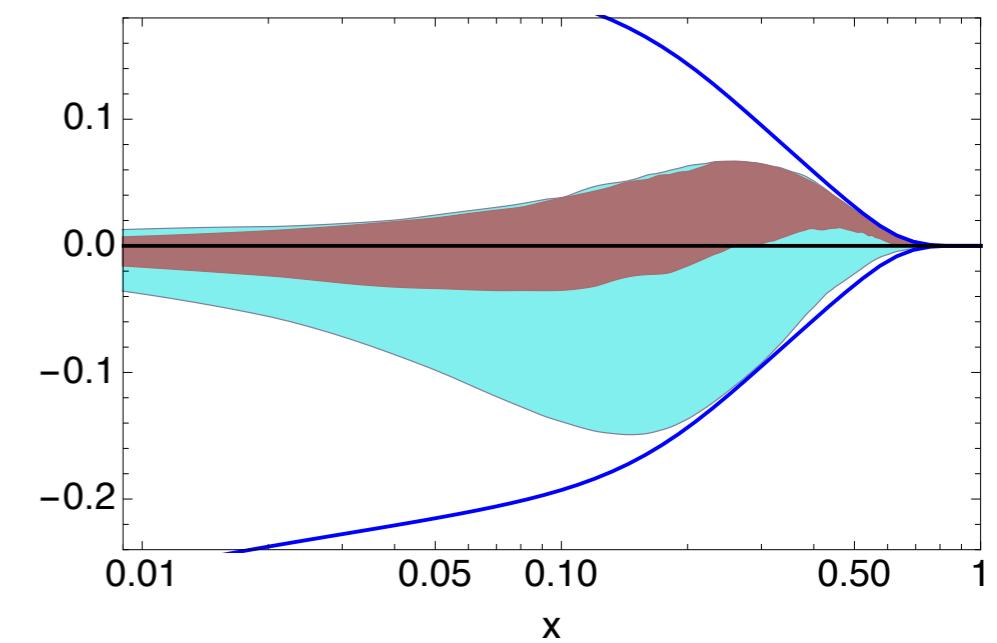
down

sensitive to  
uncertainty on  
gluon  $D_1$

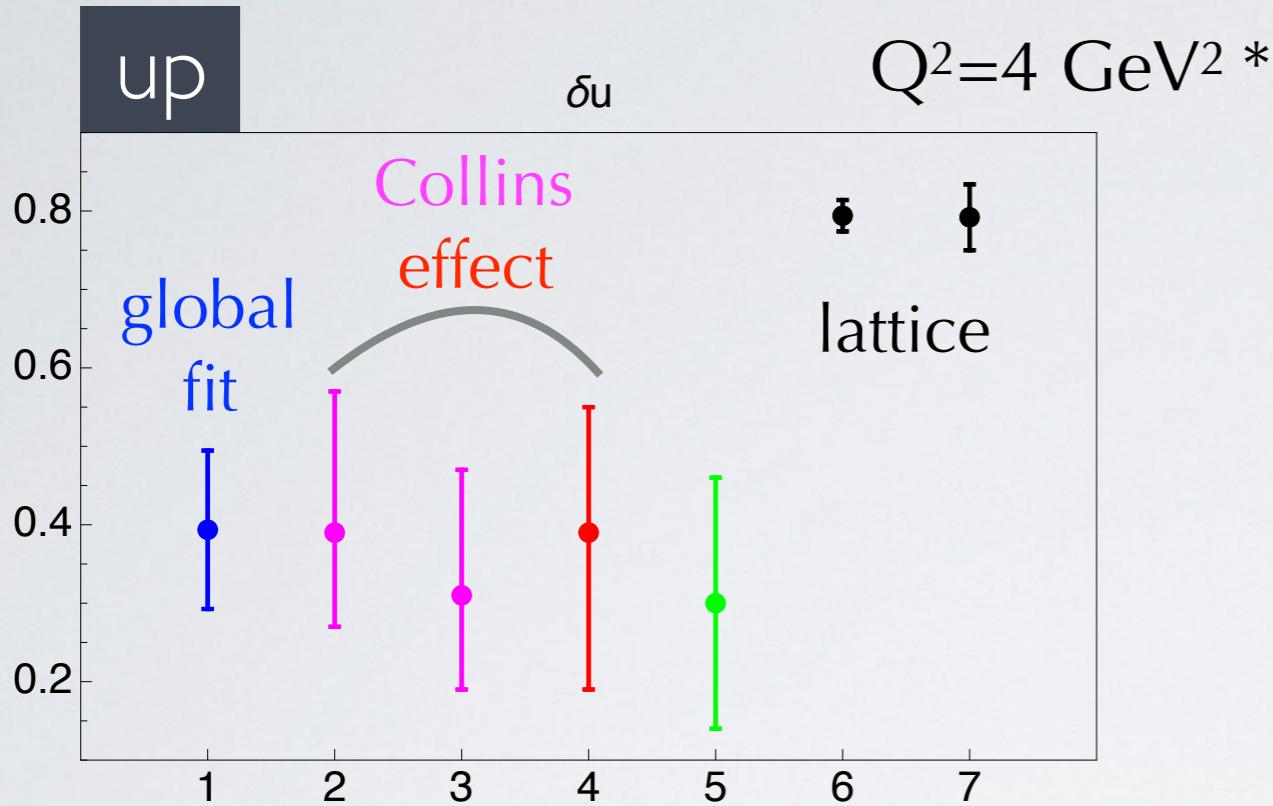
$$D_{1g}(Q_0) = 0$$

$$D_{1g}(Q_0) = \begin{cases} 0 \\ D_{1u}/4 \\ D_{1u} \end{cases}$$

$x h_1^{d-d} Q^2 = 2.4 \text{ GeV}^2$



**tensor charge**  $\delta q(Q^2) = \int dx h_1 q\bar{q} (x, Q^2)$



- 1- global fit** *Radici & Bacchetta, P.R.L.120 (18) 192001*

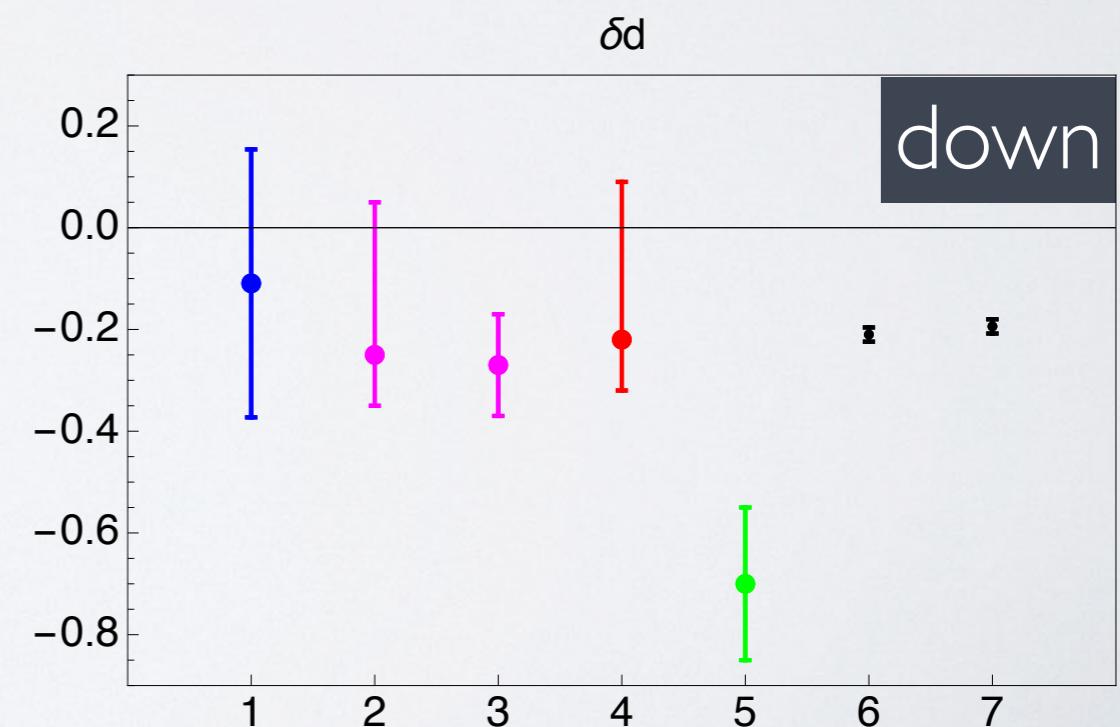
**2,3- Torino** *Anselmino et al., P.R. D87 (13) 094019* \*  $Q^2=1$

**4- TMD fit** *Kang et al., P.R. D93 (16) 014009* \*  $Q^2=10$

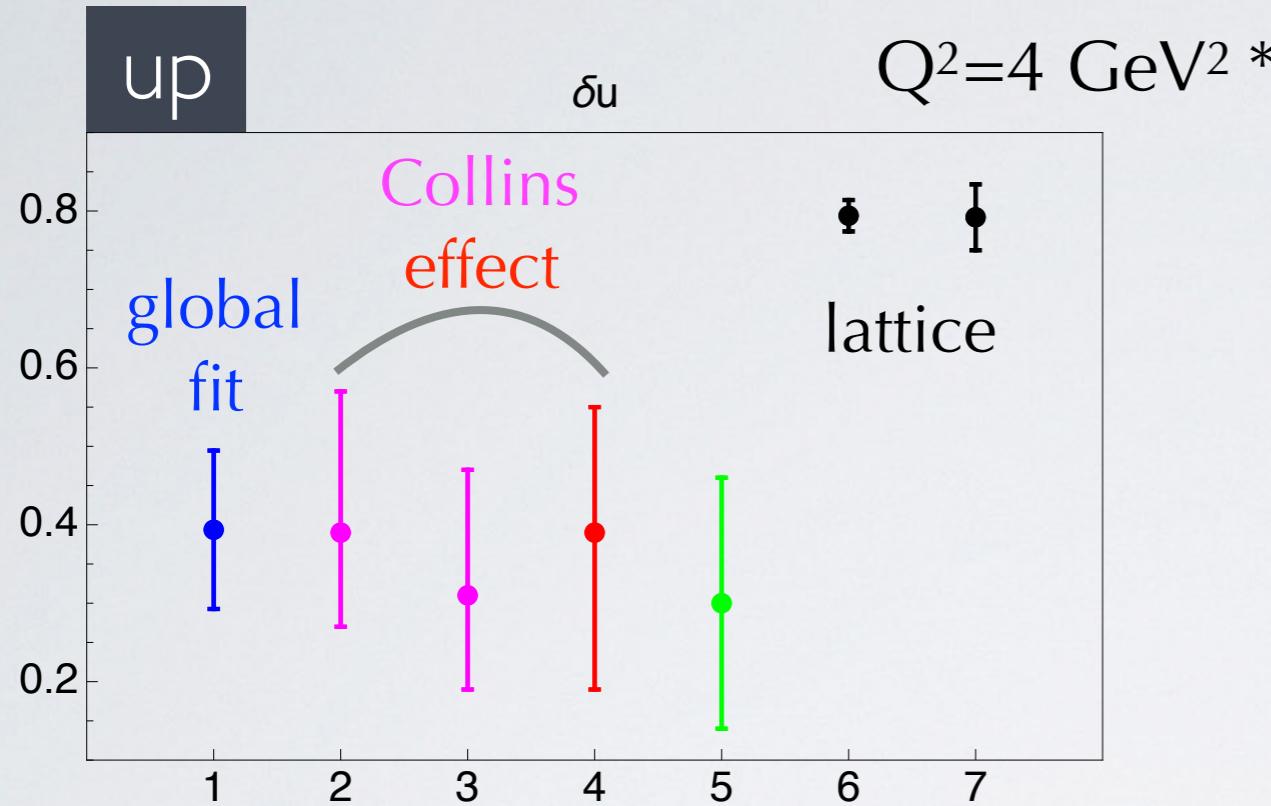
**5- JAM fit** *Lin et al., P.R.L.120 (18) 152502* {Collins effect + lattice  $g_T = \delta u - \delta d$ } \*  $Q_0^2=2$

**6- ETMC17** *Alexandrou et al., P.R. D95 (17) 114514;  
E P.R. D96 (17) 099906*

**7- PNDME16** *Bhattacharya et al., P.R. D94 (16) 054508*



$$\text{tensor charge } \delta q(Q^2) = \int dx h_1 q\bar{q} (x, Q^2)$$



incompatibility for up  
compatible for down  
but with large errors  
(except JAM)

**1- global fit** *Radici & Bacchetta, P.R.L. 120 (18) 192001*

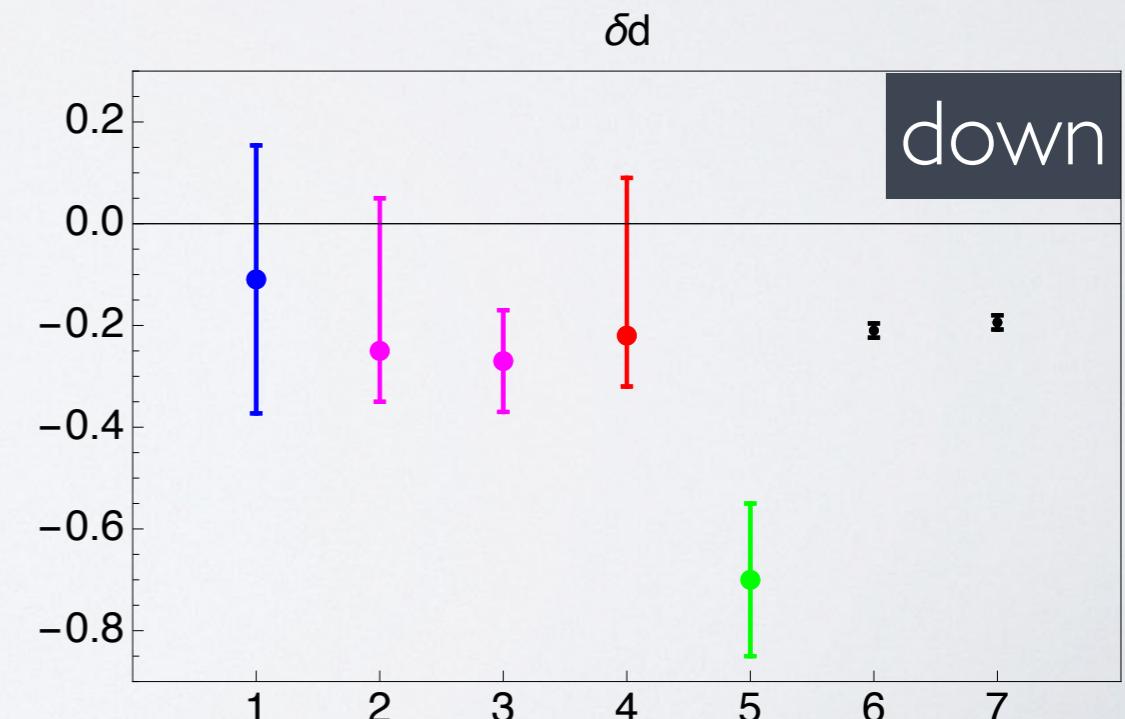
**2,3- Torino** *Anselmino et al., P.R.D 87 (13) 094019* \*  $Q^2=1$

**4- TMD fit** *Kang et al., P.R.D 93 (16) 014009* \*  $Q^2=10$

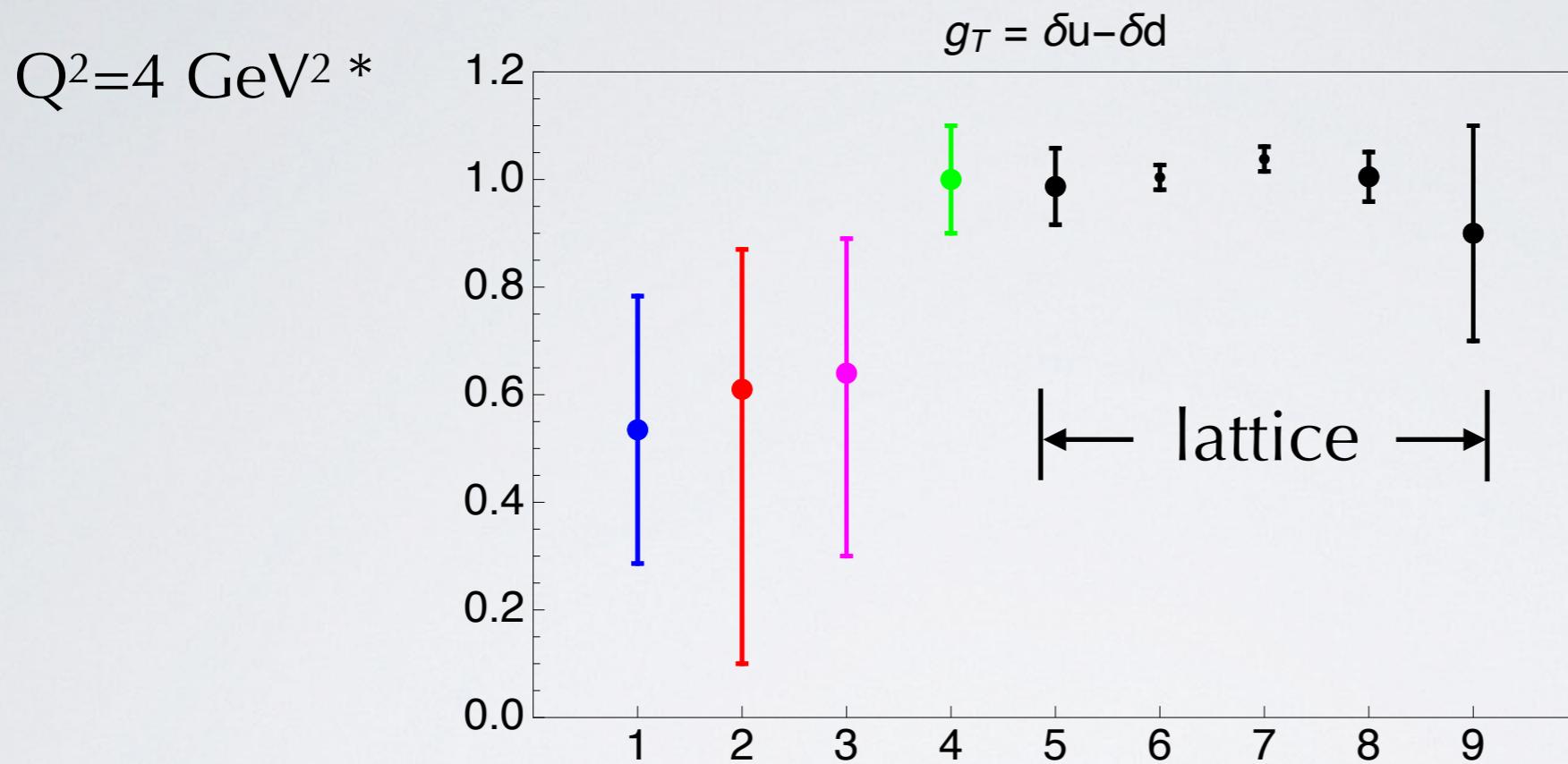
**5- JAM fit** *Lin et al., P.R.L. 120 (18) 152502* {Collins effect + lattice  $g_T = \delta u - \delta d$  \*  $Q_0^2=2$

**6- ETMC17** *Alexandrou et al., P.R.D 95 (17) 114514; E P.R.D 96 (17) 099906*

**7- PNNDME16** *Bhattacharya et al., P.R.D 94 (16) 054508*



# isovector tensor charge $g_T = \delta u - \delta d$



Radici & Bacchetta,  
P.R.L. 120 (18) 192001

Kang et al., P.R. D93 (16) 014009

Anselmino et al., P.R. D87 (13) 094019

Lin et al., P.R.L. 120 (18) 152502

1) **global fit '17**

2) **"TMD fit" \*  $Q^2=10$**

3) **Torino fit \*  $Q^2=1$**

4) **JAM fit '17 \*  $Q_0^2=2$**

5) PNDME '16

6) ETMC '17

7) LHPC '12

8) RQCD '14

9) RBC-UKQCD

Bhattacharya et al., P.R. D94 (16) 054508

Alexandrou et al., P.R. D95 (17) 114514;  
E P.R. D96 (17) 099906

Green et al., P.R. D86 (12)

Bali et al., P.R. D91 (15)

Aoki et al., P.R. D82 (10)

# “transverse-spin puzzle” ?

there seems to be no simultaneous compatibility  
about  $\delta u$ ,  $\delta d$ ,  $g_T = \delta u - \delta d$   
between lattice and  
phenomenological extractions  
of transversity

so far, shown results from published PRL paper

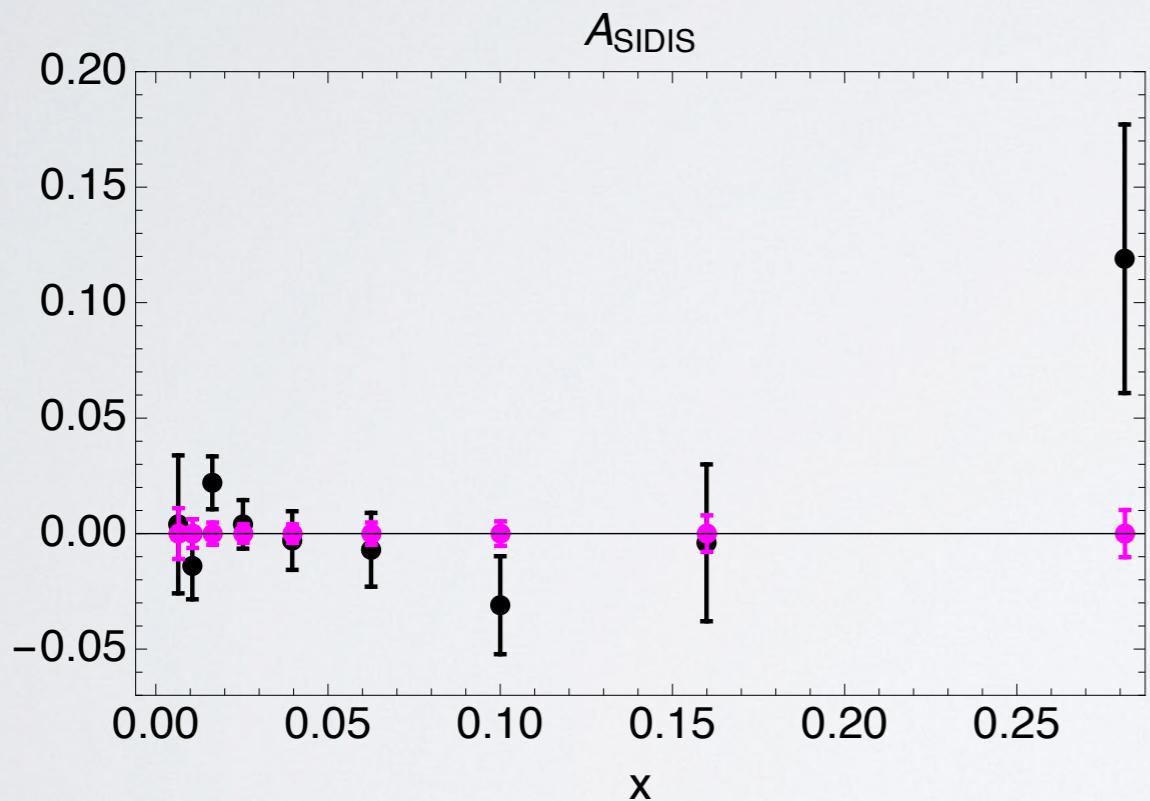
# add Compass deuteron pseudodata



*Adolph et al., P.L. B713 (12)*



*private communication*



1) recall: deuteron  $A_{\text{SIDIS}} \sim h_1^{u_v} + h_1^{d_v}$

proton  $A_{\text{SIDIS}} \sim 4h_1^{u_v} - h_1^{d_v}$

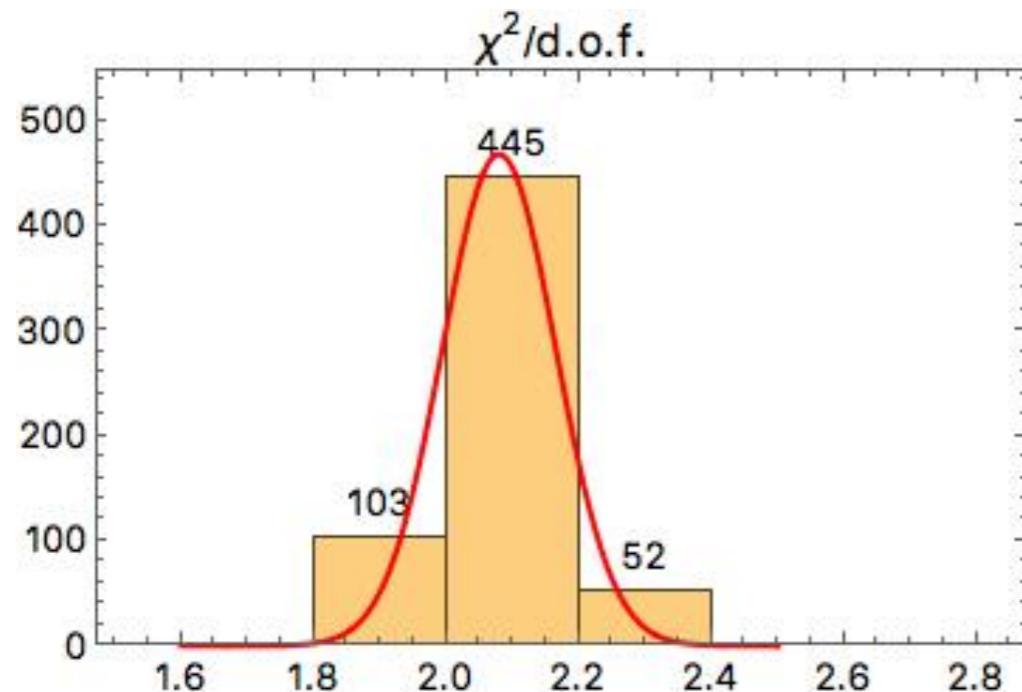
2) pseudodata with central value = 0

tried with values of old run 2004,  
but too strong tension with other data  
 $\rightarrow \chi^2/\text{dof} \gtrsim 3-4$

future deuteron measurement  
will have strong selective impact  
on replicas

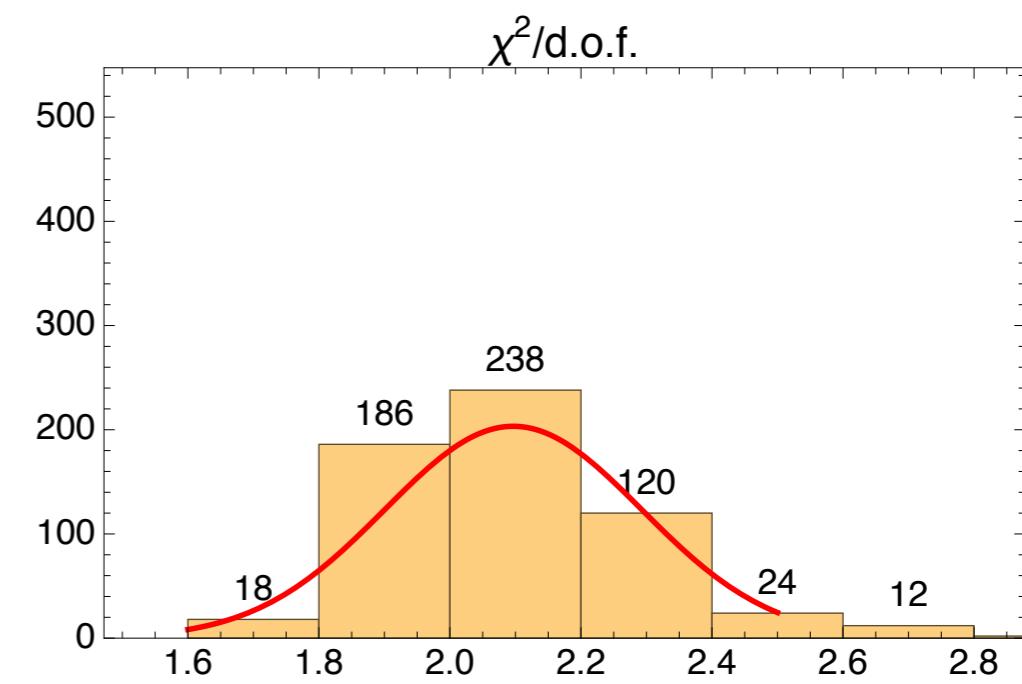
# $\chi^2$ of the fit

global fit



$$\chi^2/\text{dof} = 2.08 \pm 0.09$$

+ pseudodata

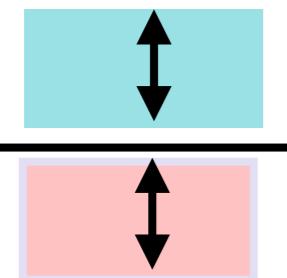
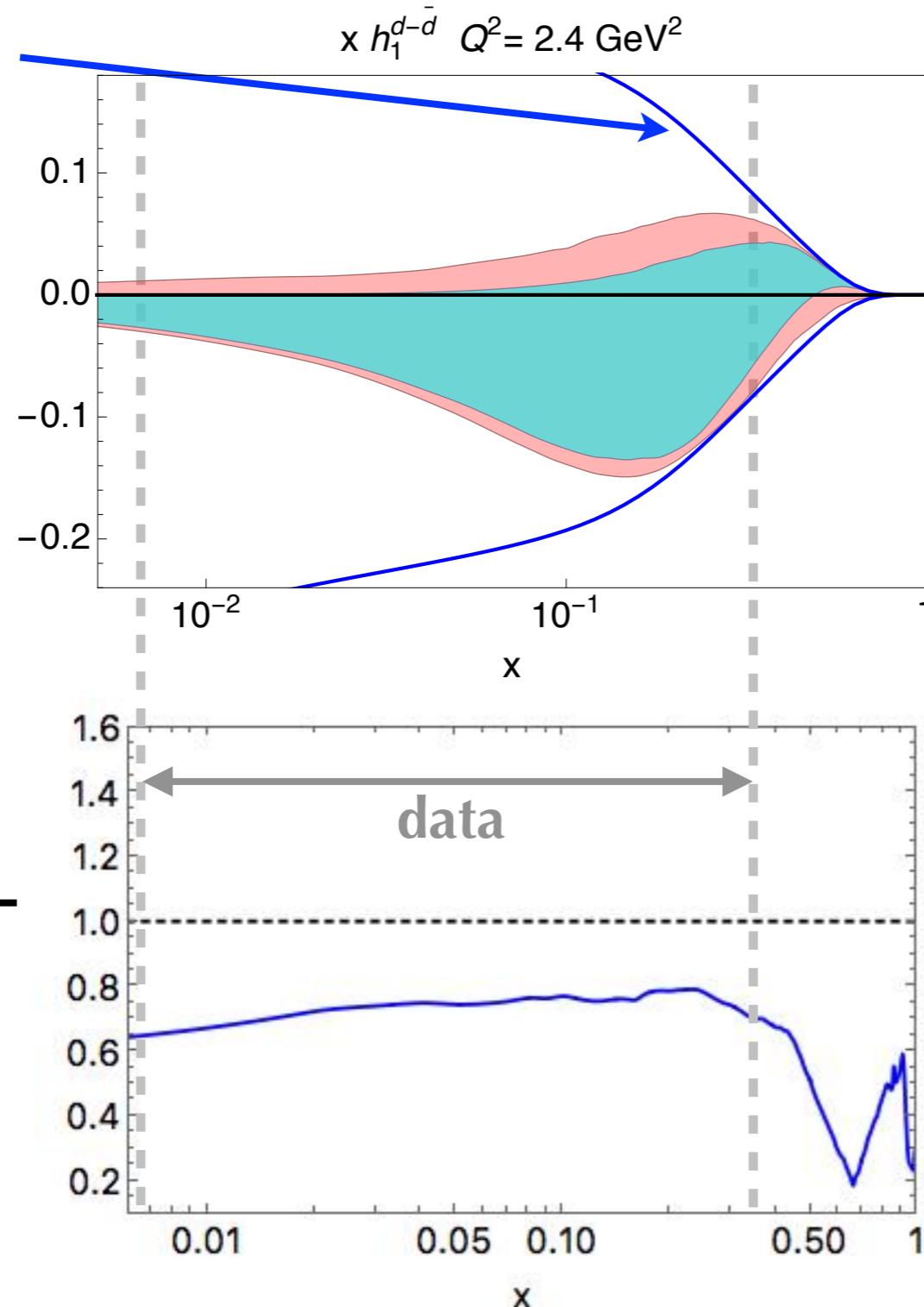


$$\chi^2/\text{dof} = 2.10 \pm 0.20$$

# pseudodata impact on down

Soffer  
bound

$$D_{1g}(Q_0) = \begin{cases} 0 \\ D_1^u/4 \\ D_1^u \end{cases}$$



ratio of  
widths

global fit + pseudodata

global fit

*Radici & Bacchetta,  
PRL 120 (18) 192001*

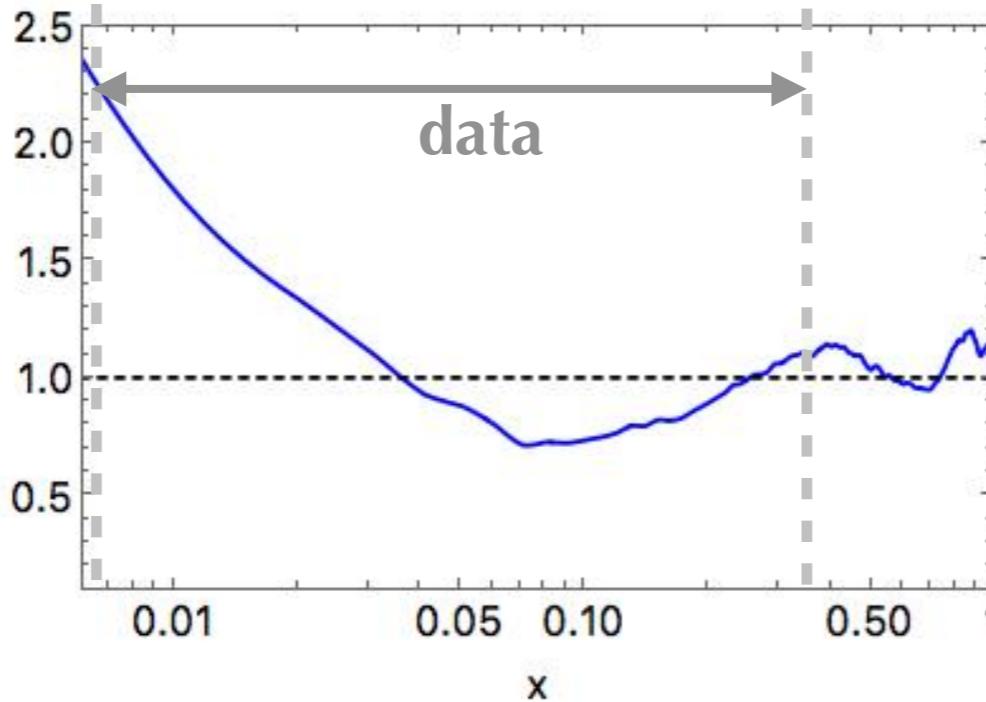
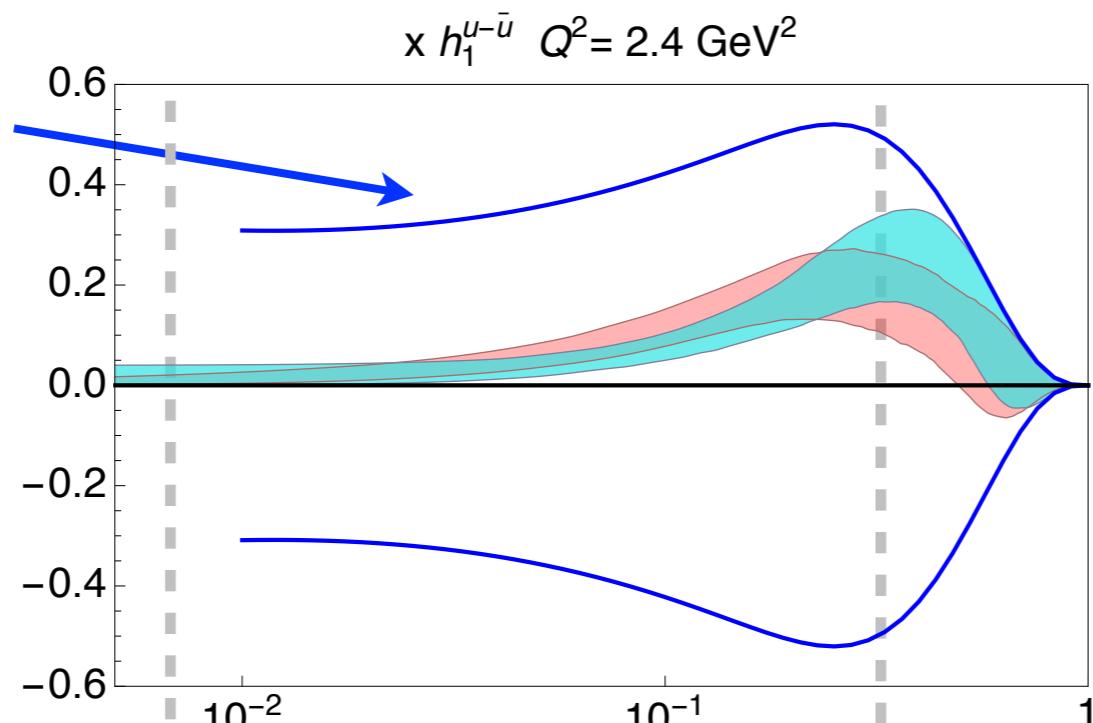
down

< 20% >  
increase in  
precision

# pseudodata impact on up

Soffer  
bound

$$D_{1g}(Q_0) = \begin{cases} 0 \\ D_1^u/4 \\ D_1^u \end{cases}$$



ratio of  
widths

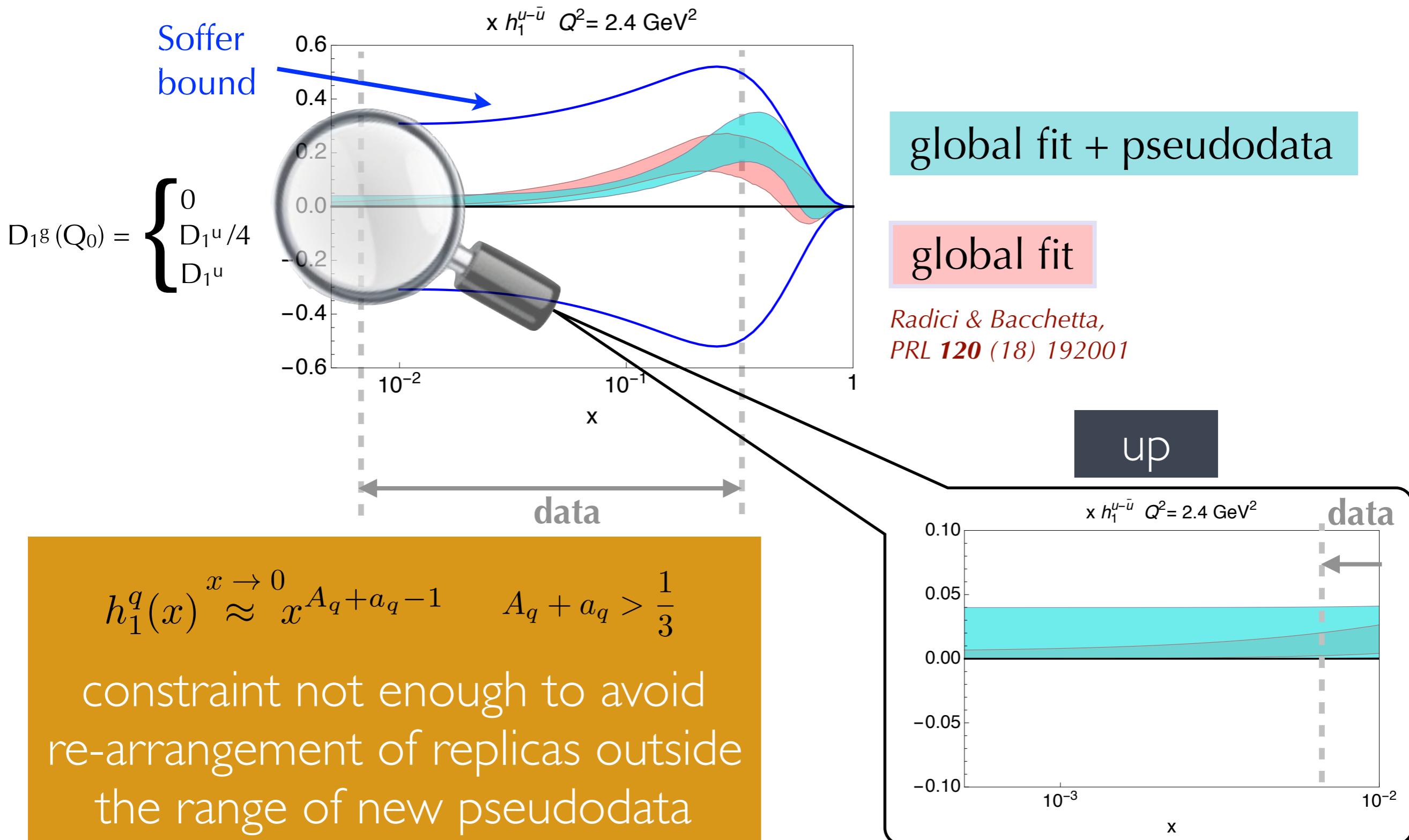
global fit + pseudodata

global fit

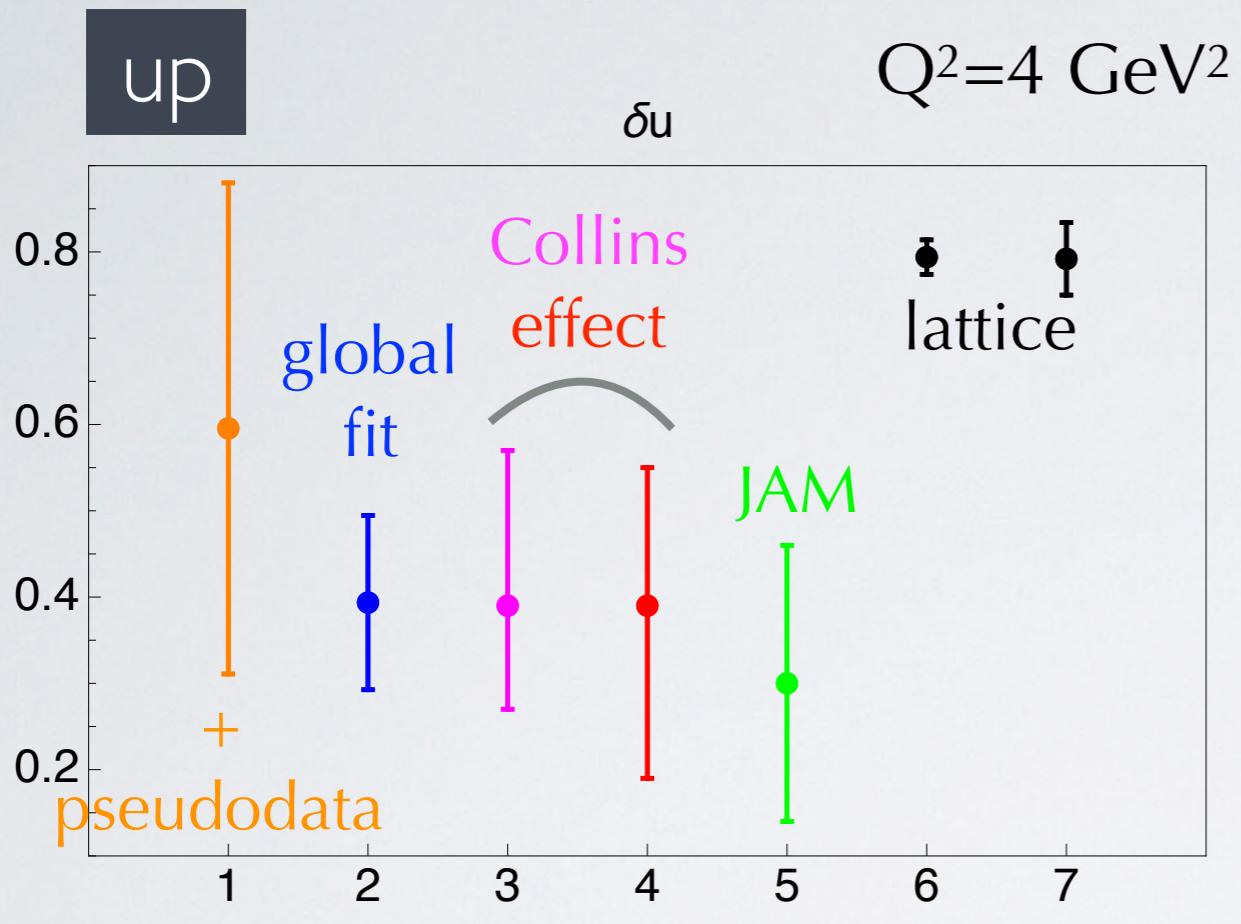
Radici & Bacchetta,  
*PRL* **120** (18) 192001

up

# pseudodata impact on up

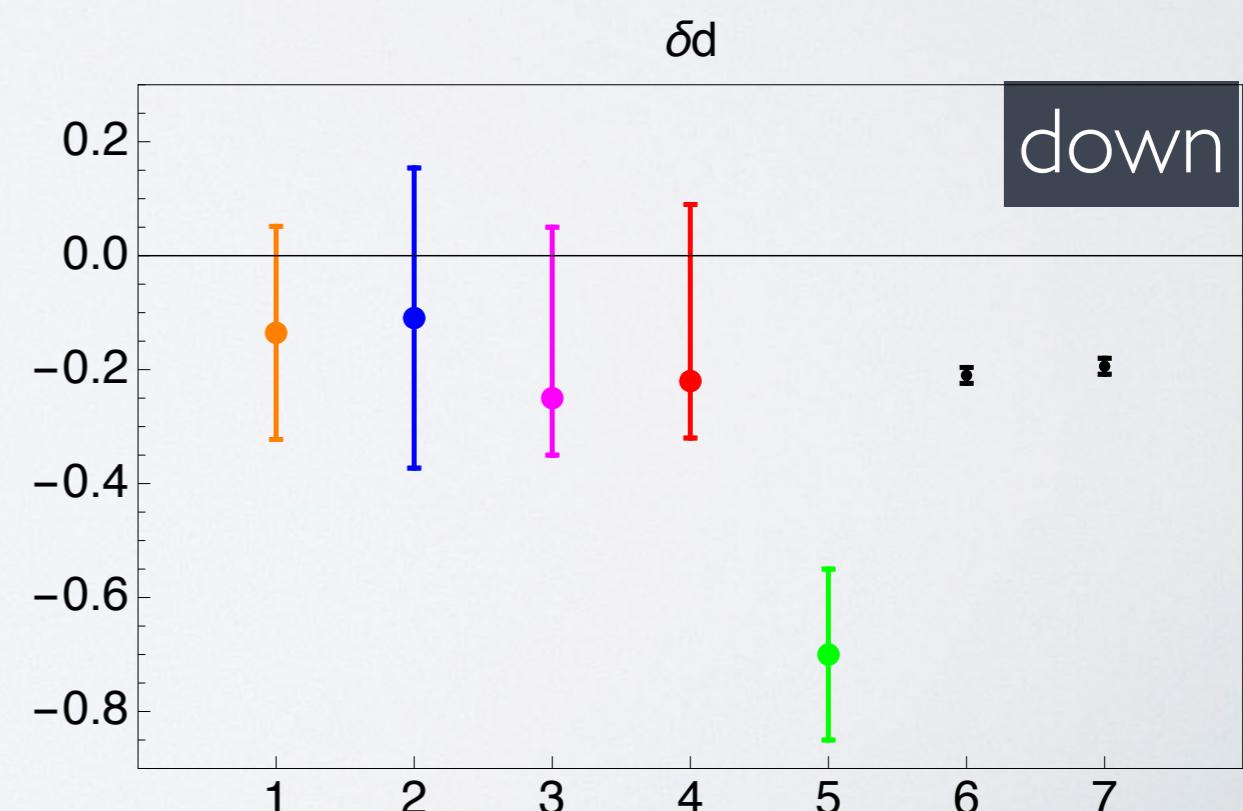


# pseudodata impact on tensor charge



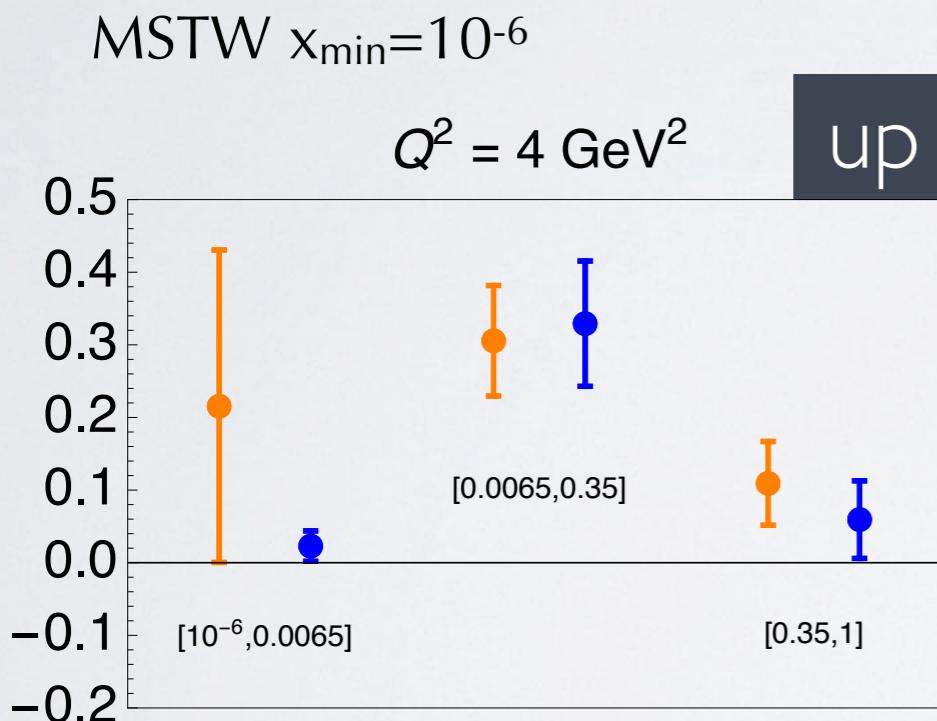
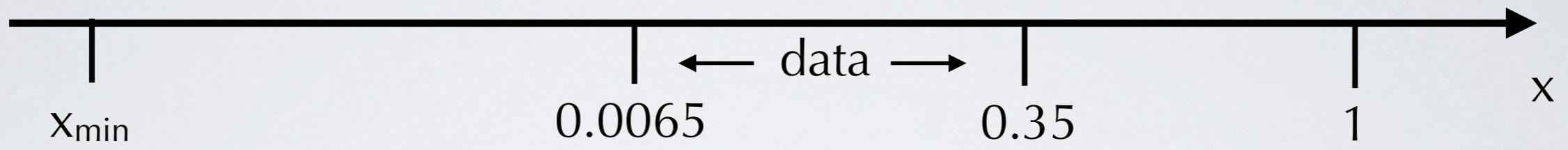
better precision on down  
larger uncertainty on up  
("reversed role" of flavors..)

full compatibility  
with lattice ??

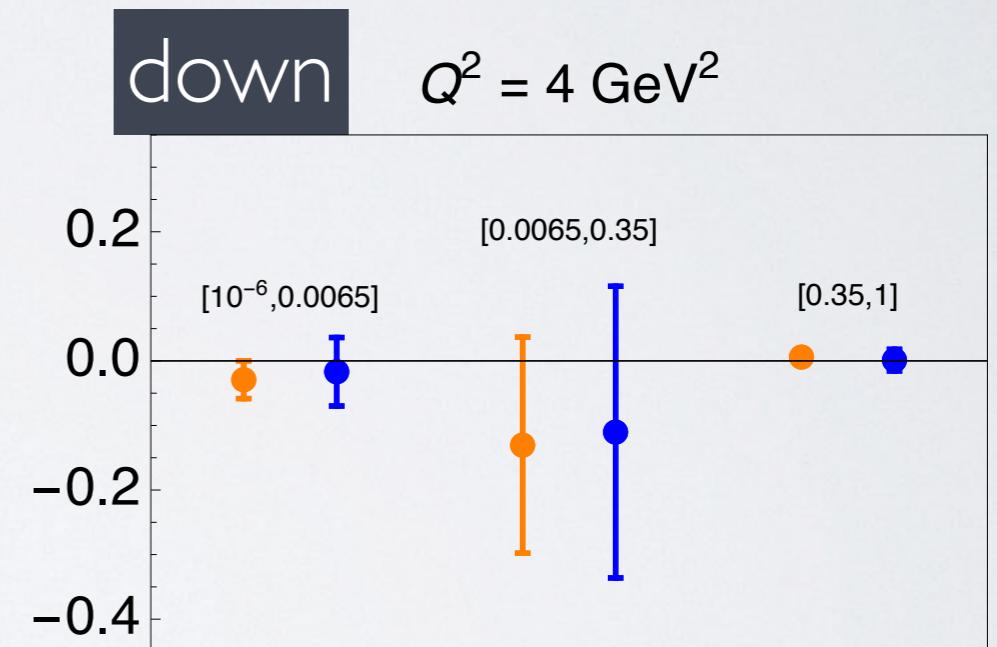


# impact of extrapolation outside data

$$\delta q(Q^2) = \int_{x_{\min}}^1 dx h_1^{q-\bar{q}}(x, Q^2)$$

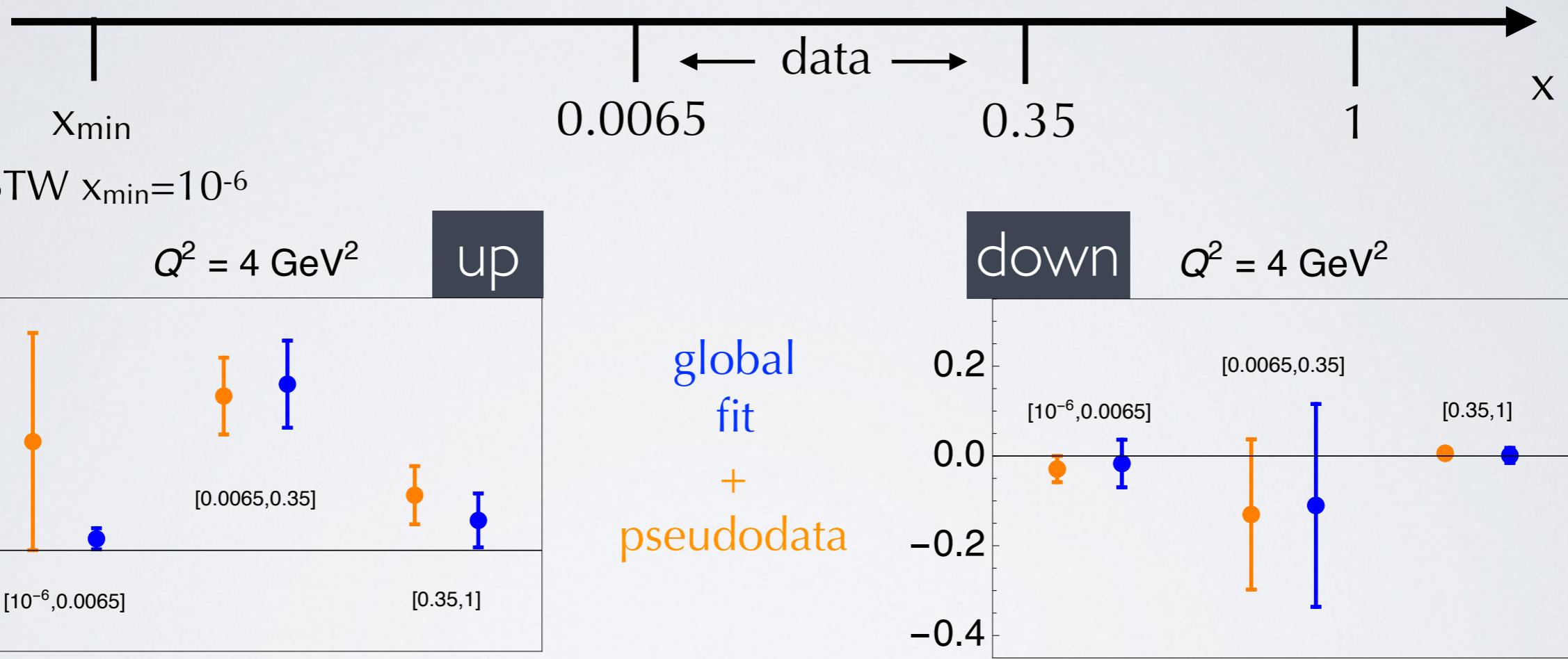


global  
fit  
+  
pseudodata



# impact of extrapolation outside data

$$\delta q(Q^2) = \int_{x_{\min}}^1 dx h_1^{q-\bar{q}}(x, Q^2)$$



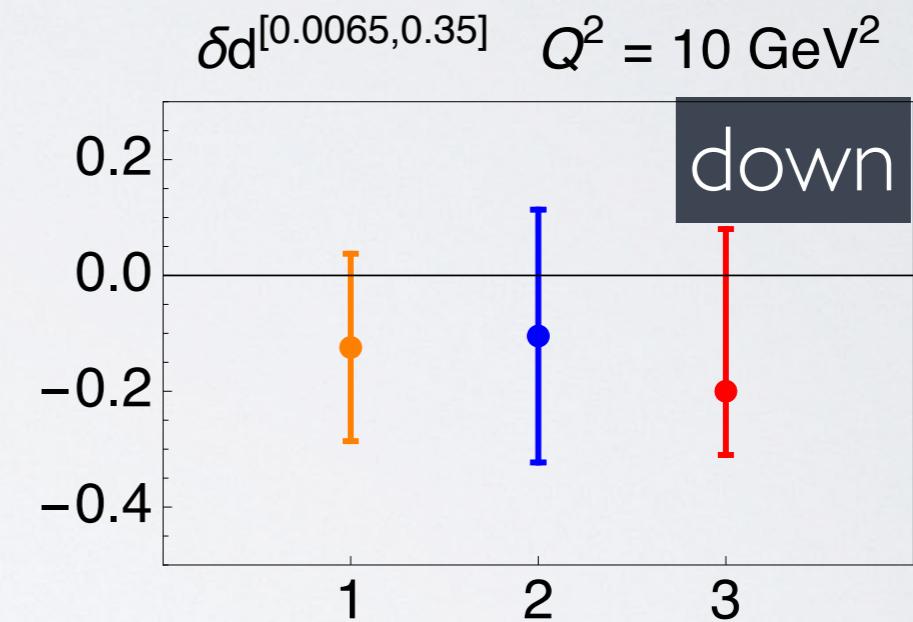
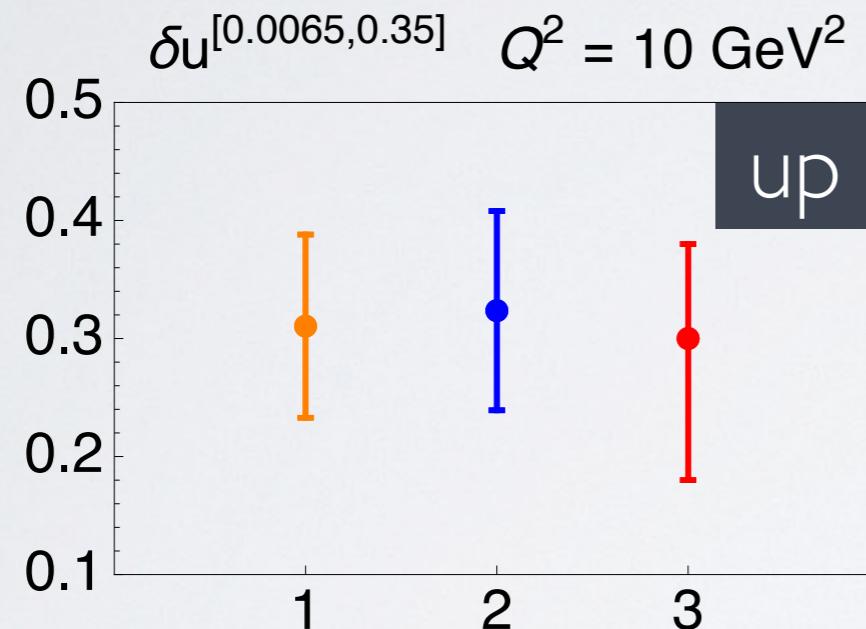
impact of pseudodata

for down: better precision everywhere

for up: large uncertainties in extrapolation at low  $x$

tensor charge  $\delta q(Q^2) = \int dx h_1 q\bar{q} (x, Q^2)$

truncated  
 $\delta q^{[0.0065, 0.35]} \quad Q^2 = 10$

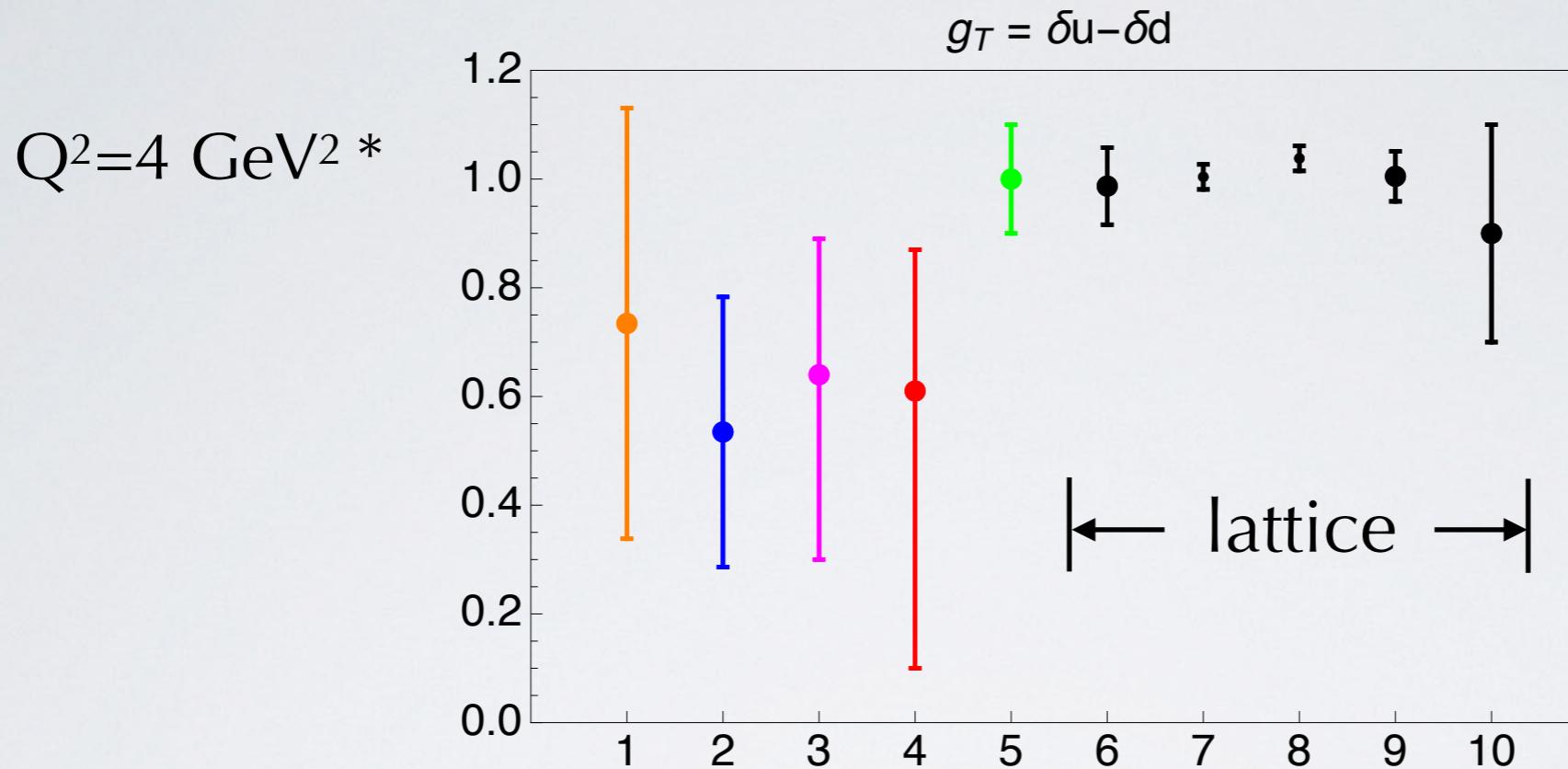


+  
pseudodata

global fit  
*Radici & Bacchetta,  
P.R.L. **120** (18) 192001*

TMD fit  
*Kang et al.,  
P.R. D93 (16) 014009*

# pseudodata impact on isovector tensor charge



apparent simultaneous compatibility  
because of large uncertainties coming from  
extrapolation outside the x-range of data (mainly at low x)

# results

$$h_1^q(x) \xrightarrow{x \rightarrow 0} x^{A_q + a_q - 1}$$

- 2<sup>nd</sup> option: finite violation of Burkhardt-Cottingham sum rule

$$\longrightarrow \quad A_q + a_q > 1$$

# impact of low-x constraint

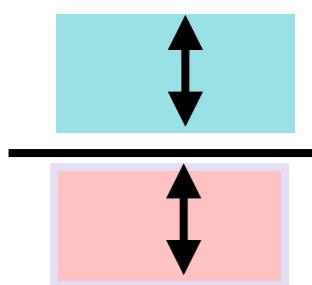
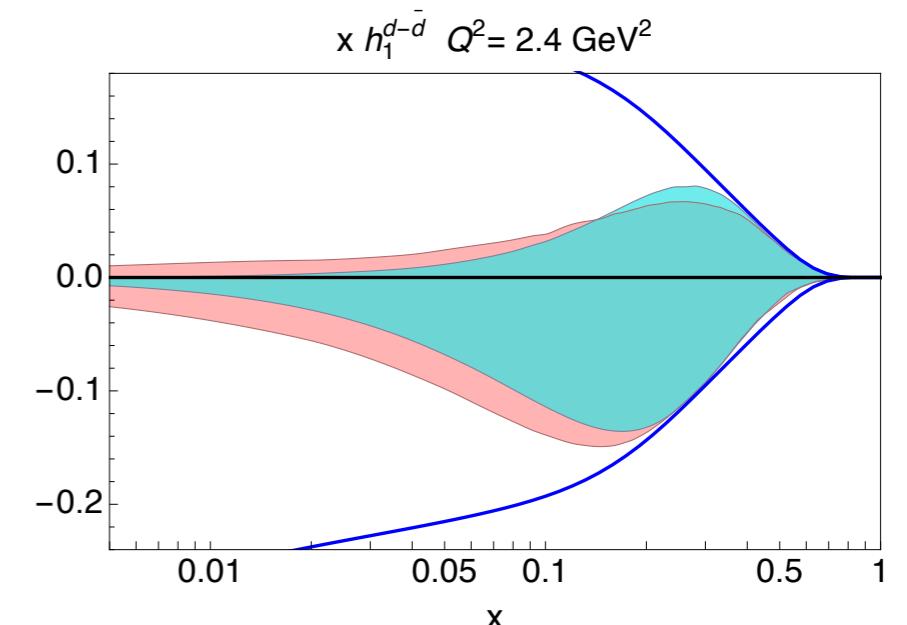
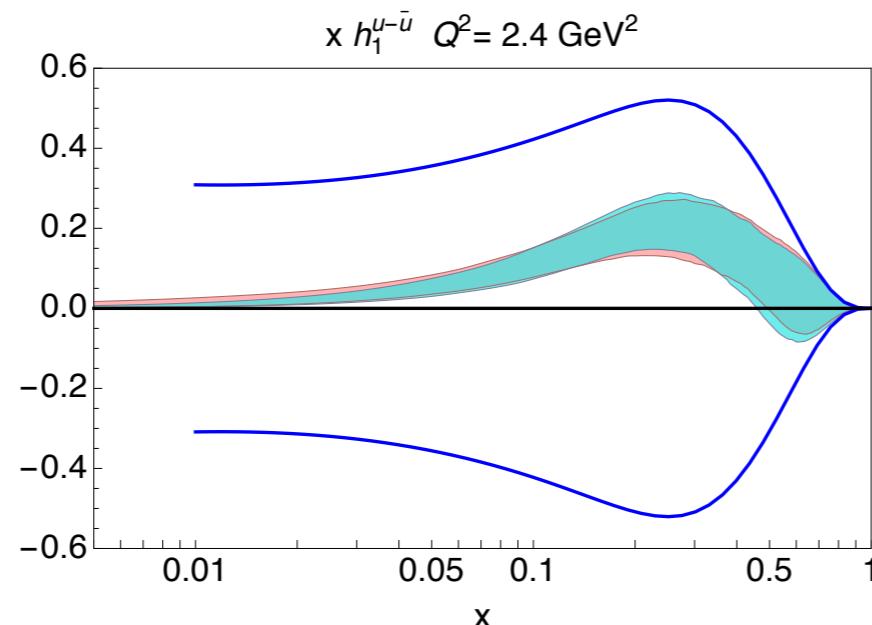
global fit 2<sup>nd</sup> option

global fit 1<sup>st</sup> option

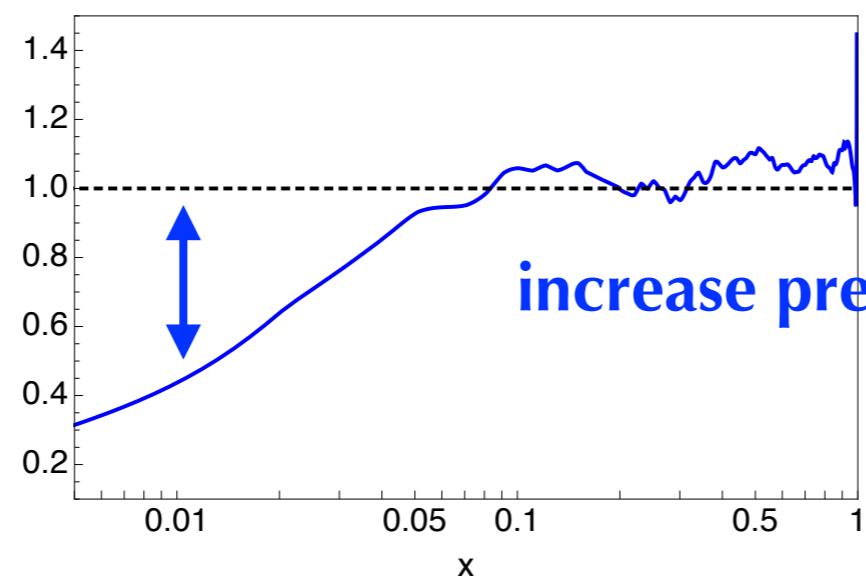
up

down

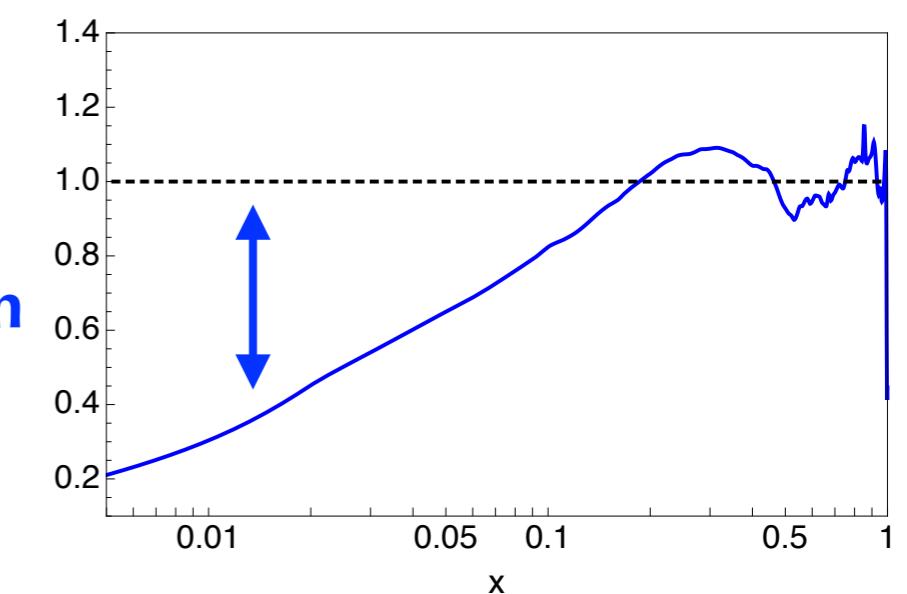
$$D_{1g}(Q_0) = \begin{cases} 0 \\ D_{1u}/4 \\ D_{1u} \end{cases}$$



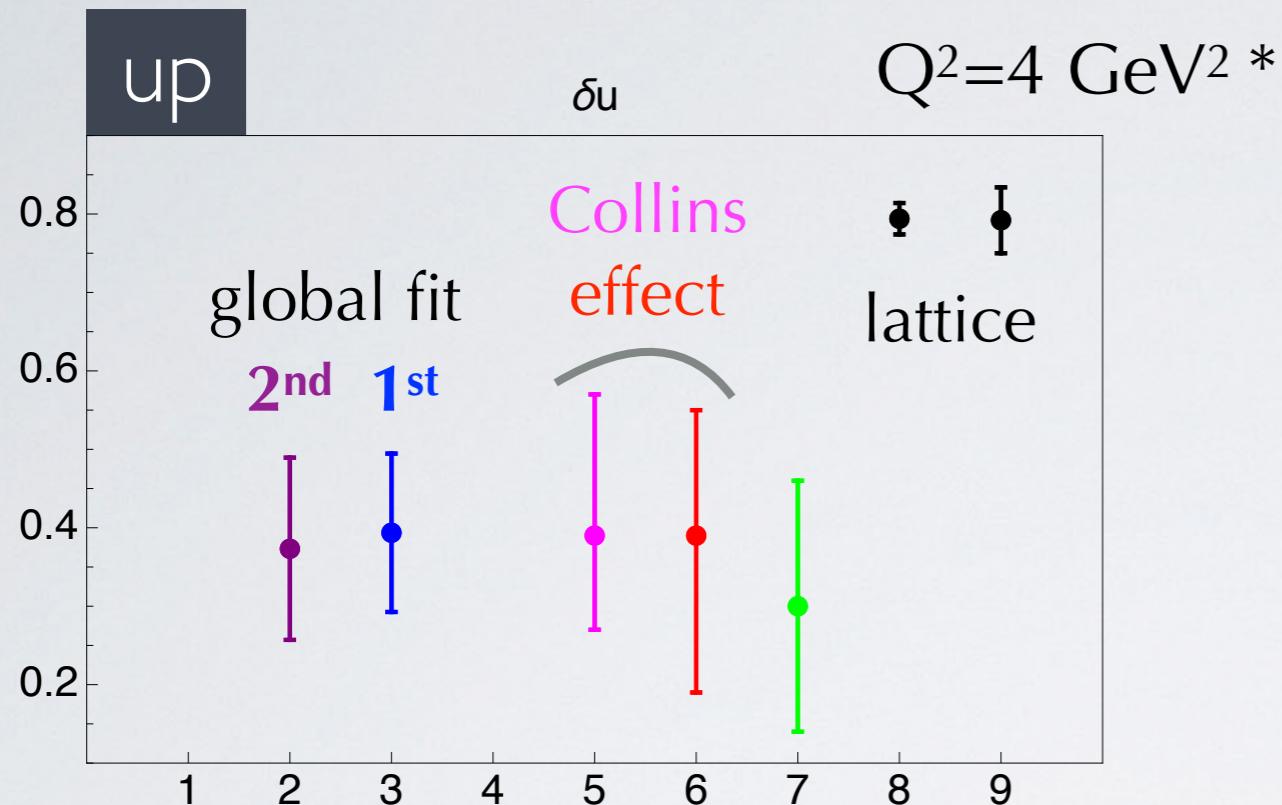
ratio of widths



increase precision



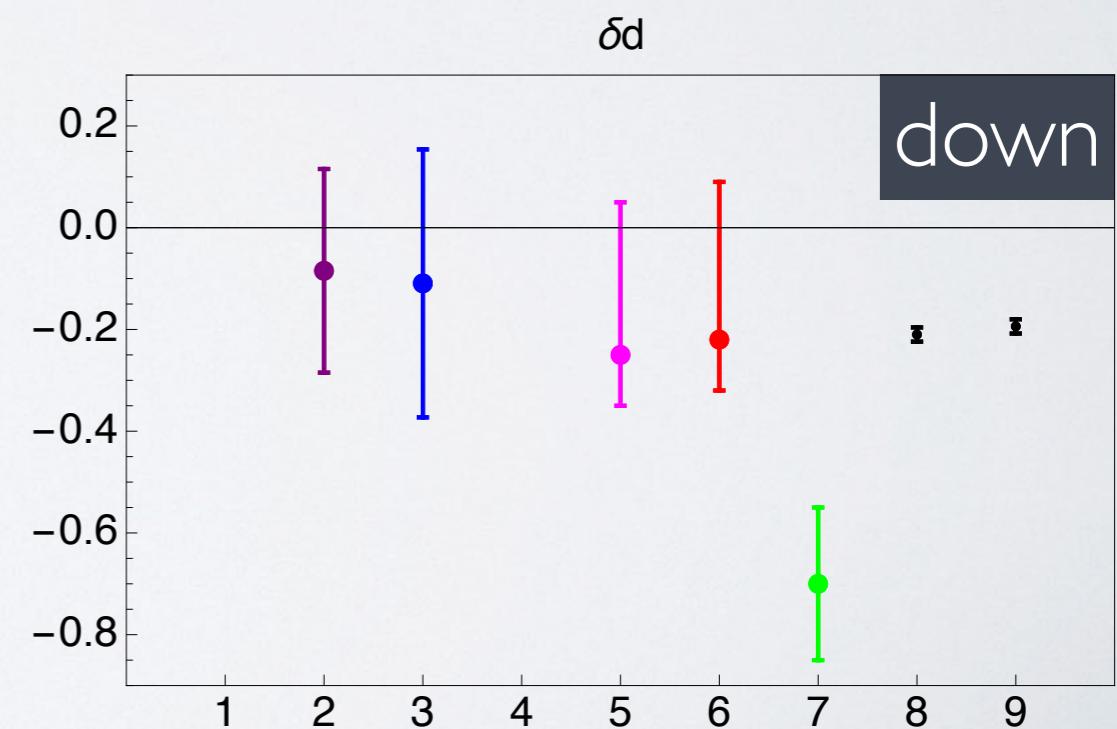
# impact of low-x constraint



better down  
up still incompatible  
(similarly for isovector  $g_T$ )  
general scenario confirmed

- 2- global fit 2<sup>nd</sup> option (finite violation of BC sum rule)  
3- global fit 1<sup>st</sup> option (finite tensor charge)

*Radici & Bacchetta, P.R.L. 120 (18) 192001*



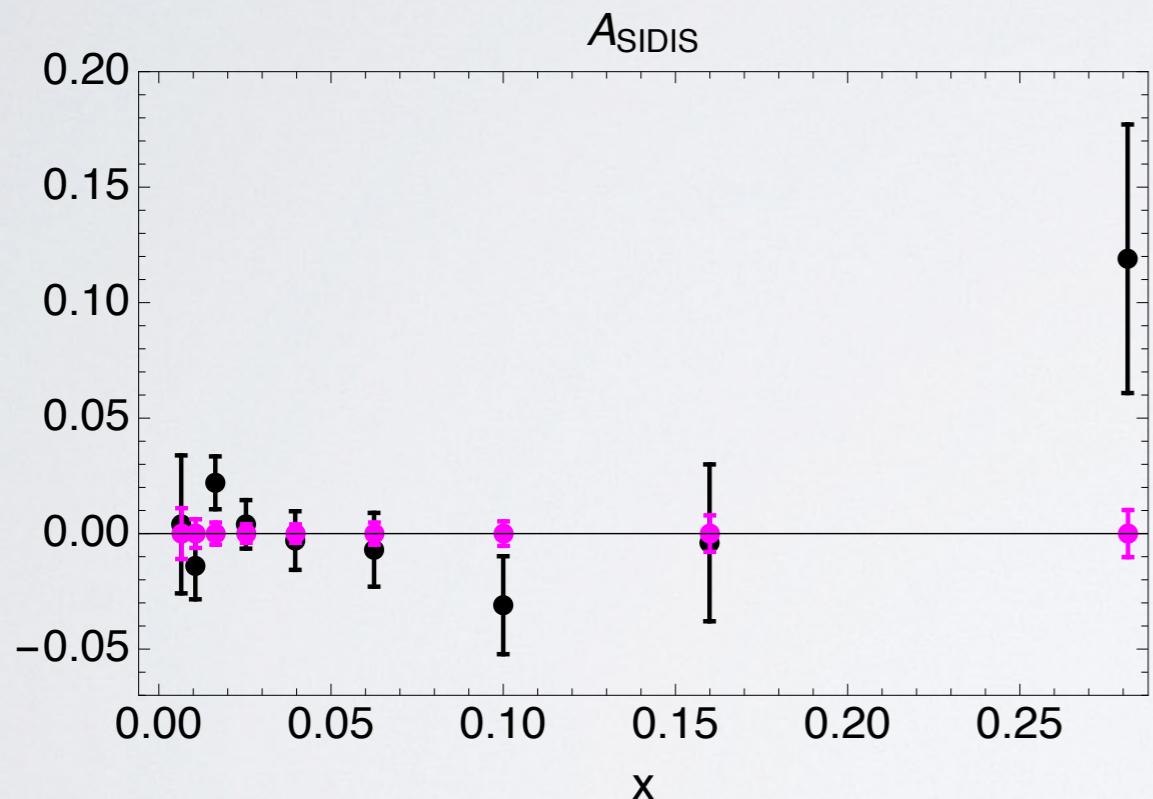
# add Compass deuteron pseudodata



*Adolph et al., P.L. B713 (12)*



*private communication*



adding again  
Compass deuteron pseudodata

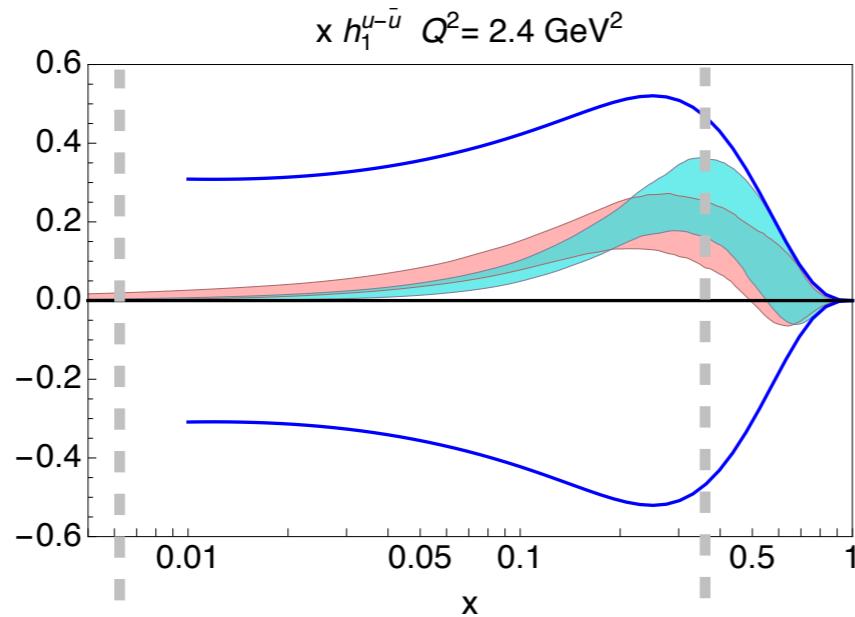
# impact of pseudodata

global fit + pseudodata

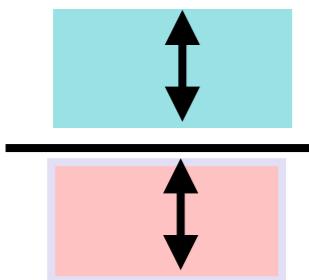
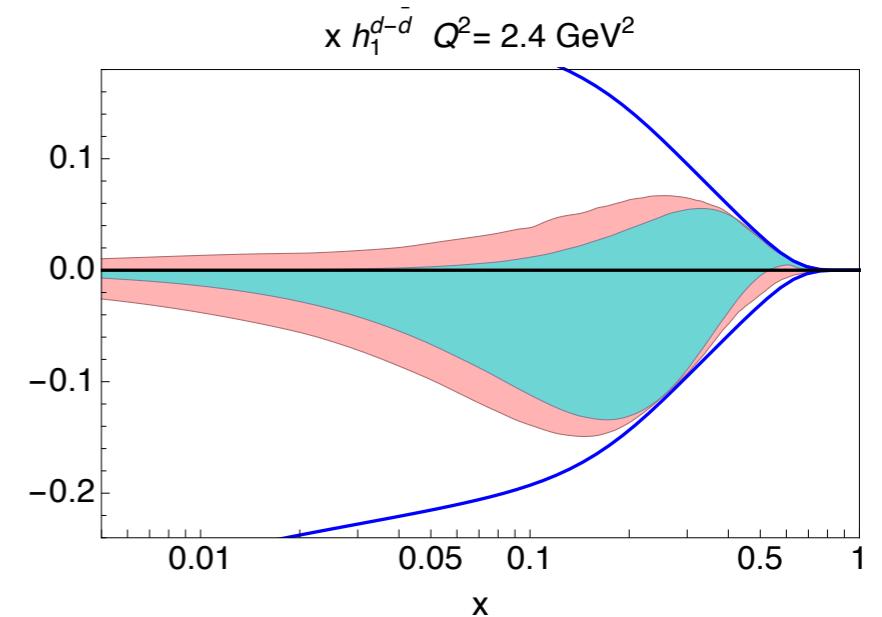
global fit

$$D_{1g}(Q_0) = \begin{cases} 0 \\ D_{1^u}/4 \\ D_{1^u} \end{cases}$$

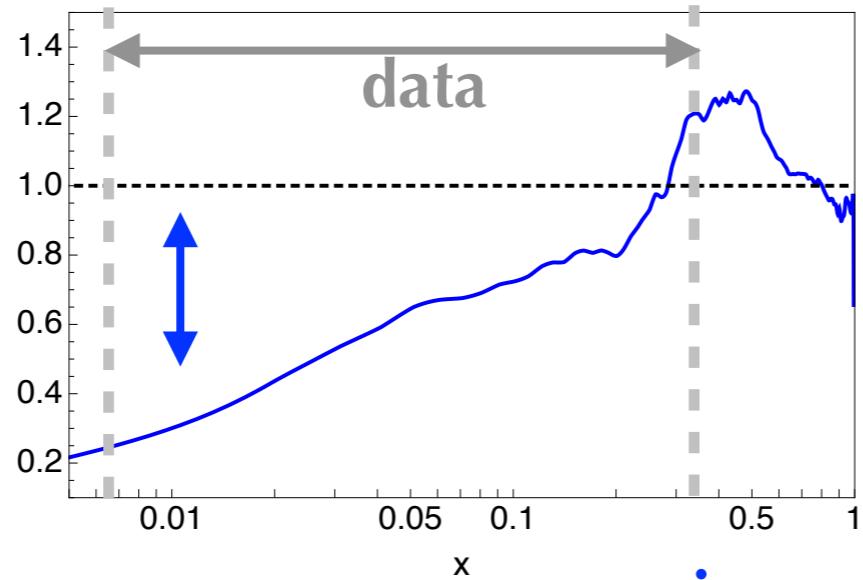
up



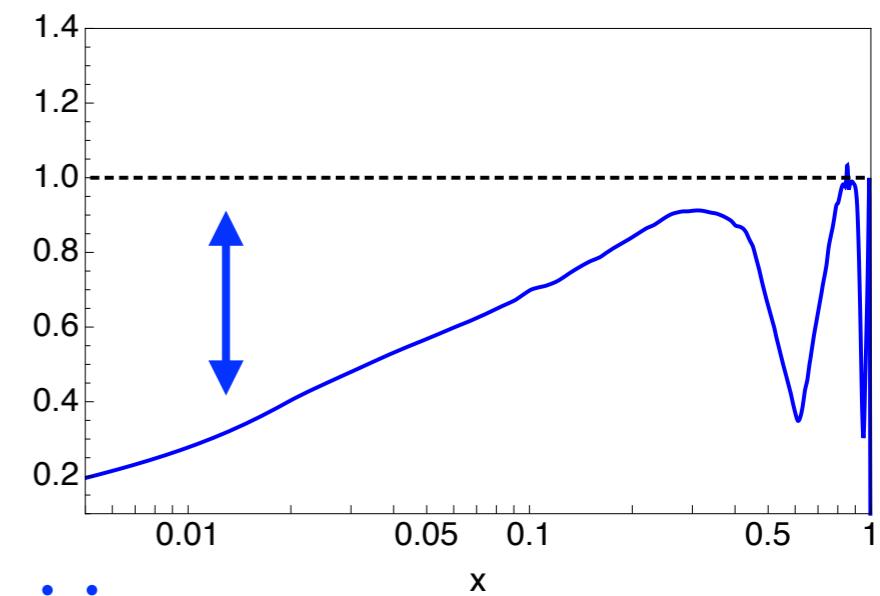
down



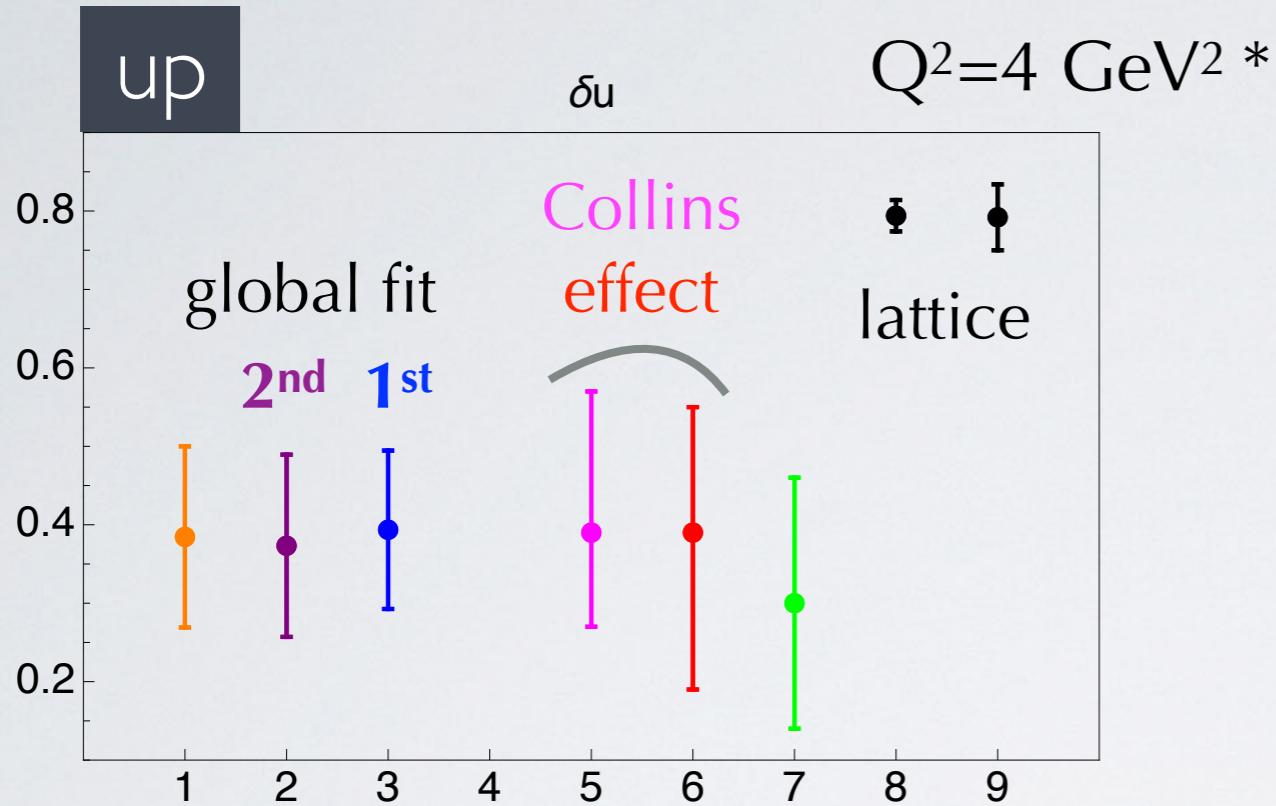
ratio of  
widths



increase precision

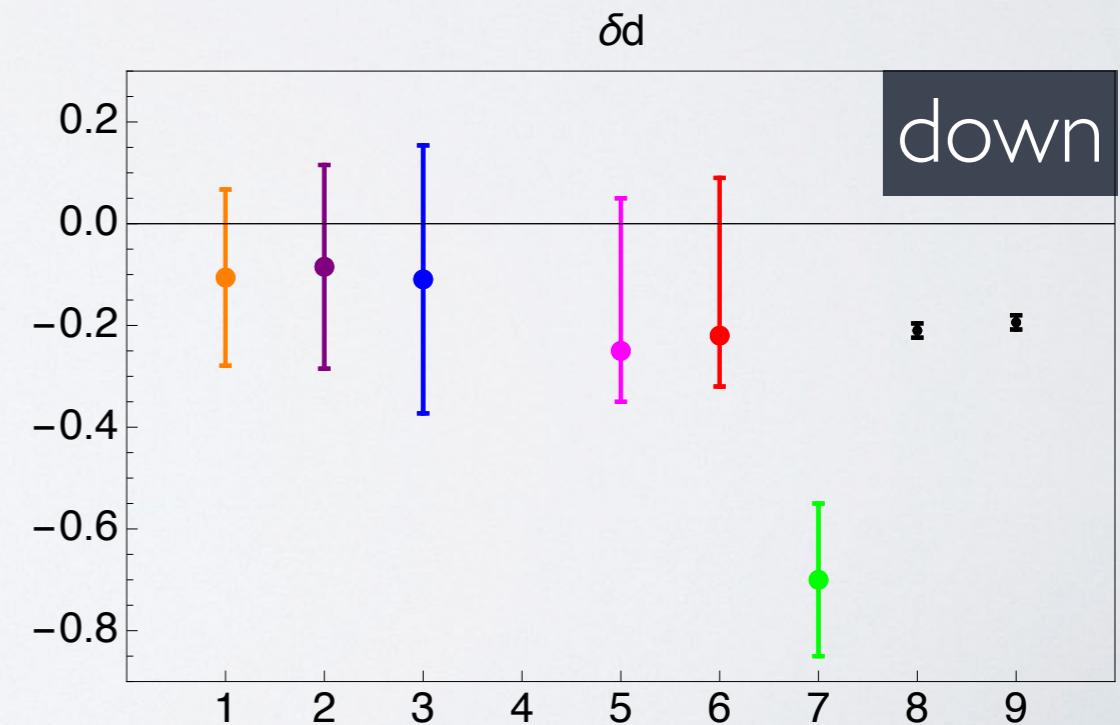


# impact of pseudodata



- 1- global fit 2<sup>nd</sup> option + pseudodata
- 2- global fit 2<sup>nd</sup> option (finite violation of BC sum rule)
- 3- global fit 1<sup>st</sup> option (finite tensor charge)

Radici & Bacchetta, P.R.L. 120 (18) 192001



# summarizing

color code of  
global fit

global fit 1<sup>st</sup> option (finite tensor charge)

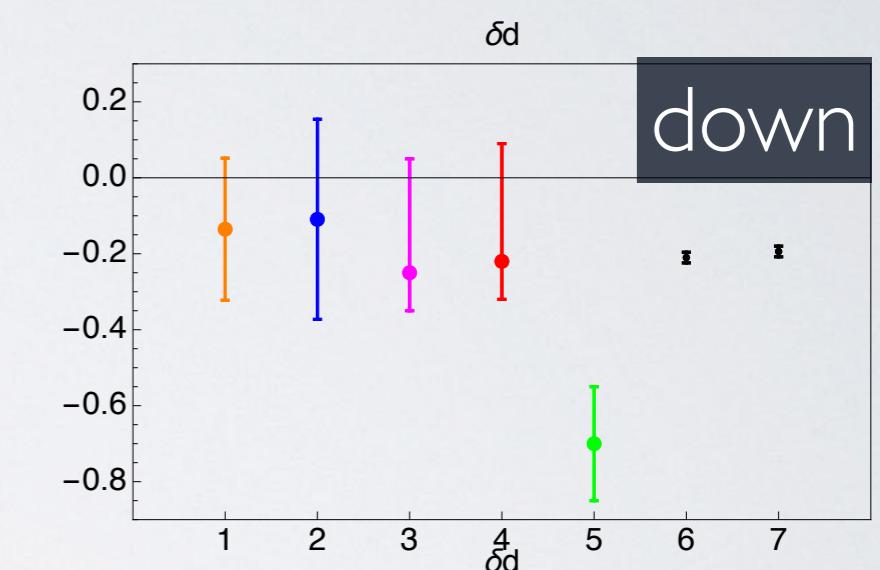
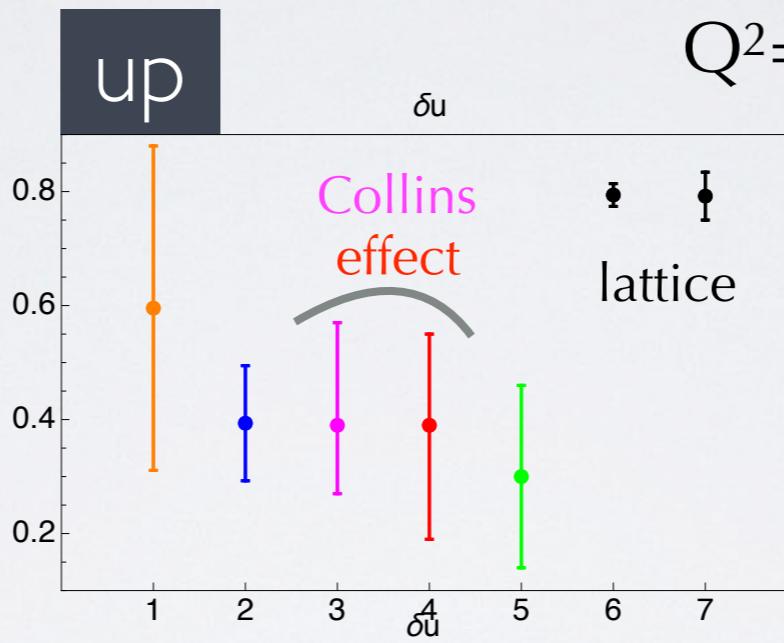
*Radici & Bacchetta, P.R.L. 120 (18) 192001*

global fit 2<sup>nd</sup> option (finite violation of BC sum rule)

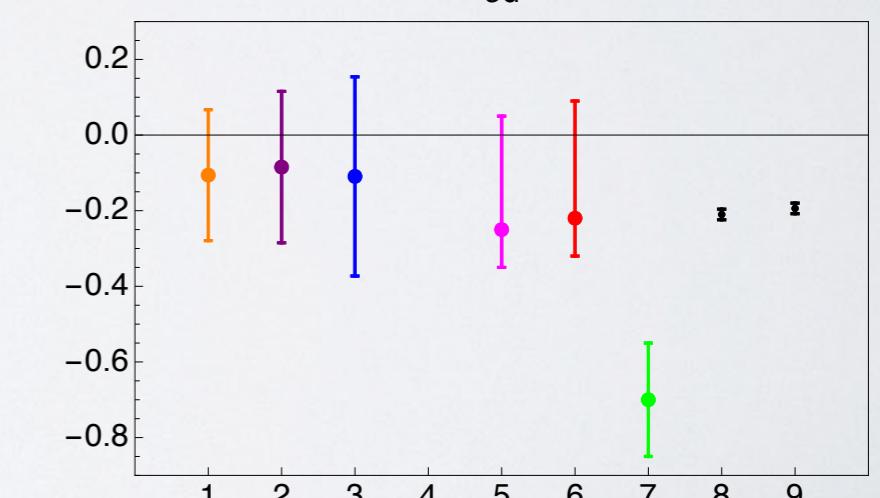
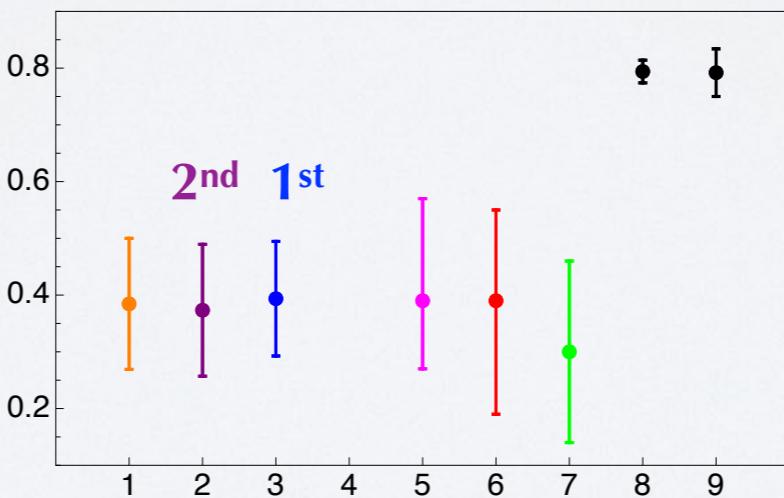
adding  
pseudodata



1<sup>st</sup> option + pseudodata



2<sup>nd</sup> option + pseudodata



# summarizing

color code of  
global fit

**global fit 1<sup>st</sup> option (finite tensor charge)**

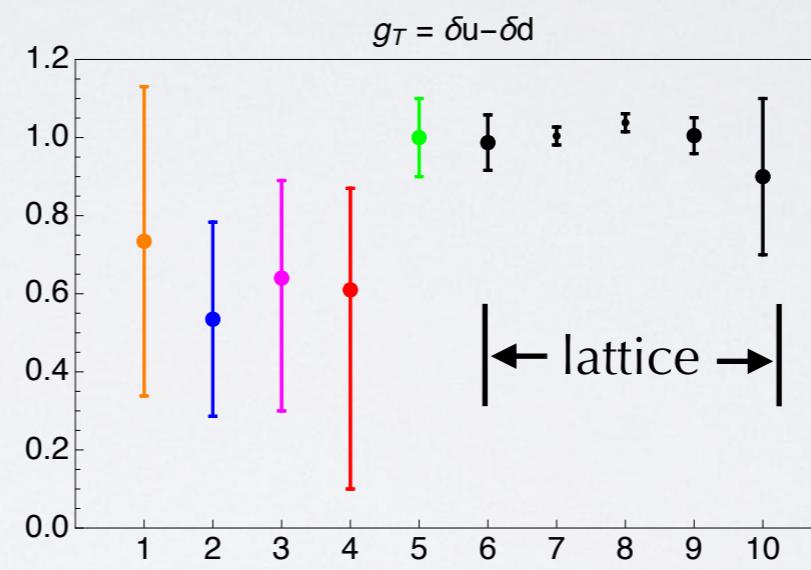
*Radici & Bacchetta, P.R.L. 120 (18) 192001*

**global fit 2<sup>nd</sup> option (finite violation of BC sum rule)**

adding  
pseudodata

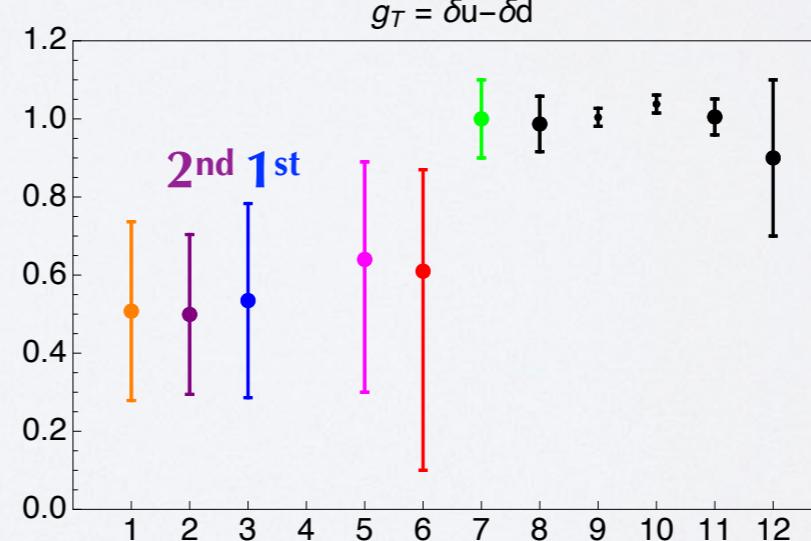


**1<sup>st</sup> option + pseudodata**

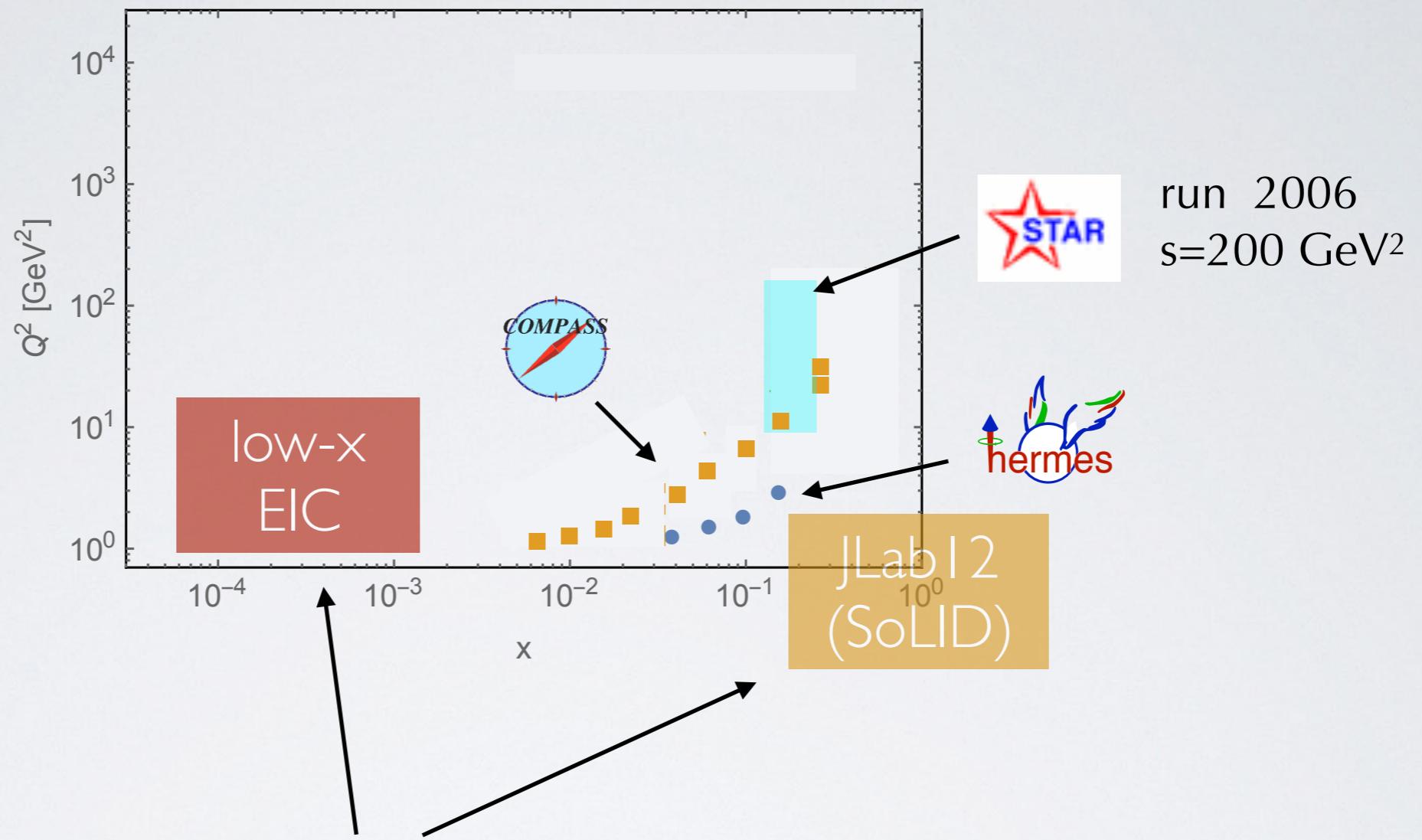


isovector

**2<sup>nd</sup> option + pseudodata**



# more constraints on extrapolation



- of course, need more data
- theoretical constraints from low-x behavior in dipole picture  
(generalize work on helicity)  $\Delta q^S(x, Q^2) \approx \left(\frac{1}{x}\right)^{\alpha_h}$   $\alpha_h = \frac{4}{\sqrt{3}} \sqrt{\frac{\alpha_s N_c}{2\pi}}$  by  
*Kovchegov et al., P.L. B772 (17) 136*  
polarized BFKL: from  $S(\text{inglet})$  to  $NS(\text{inglet})$  to  $\delta q(x, Q^2)$

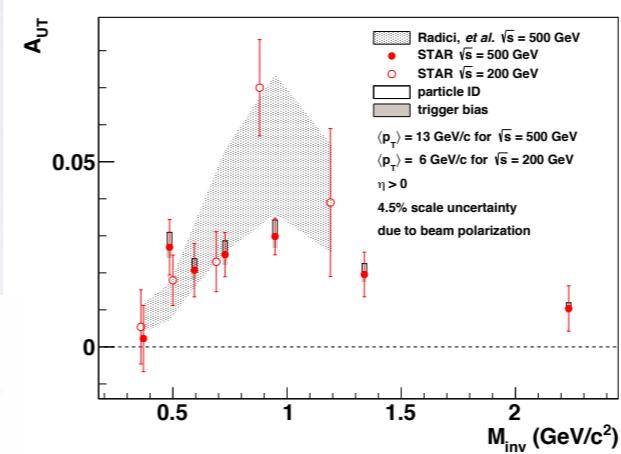
# To do list

- refit di-hadron fragmentation functions using new data:  
 $e^+e^- \rightarrow (\pi\pi) X$  constrains  $D_{1q}$   
 (currently only by Montecarlo)

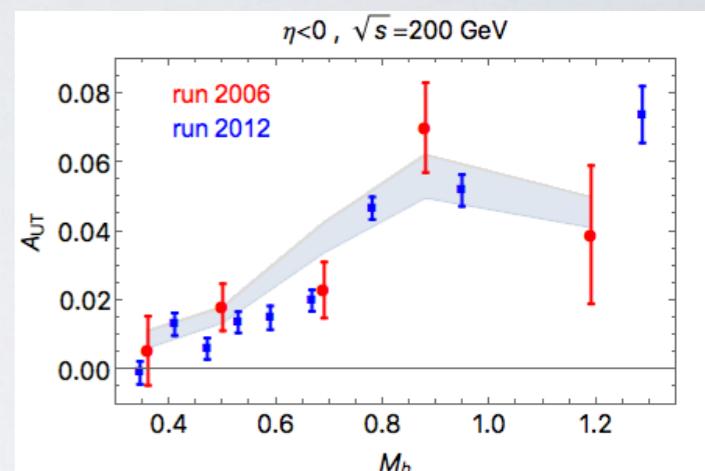


*Seidl et al.,  
P.R. D96 (17) 032005*

- use also other (multi-dimensional) data from STAR run 2011 ( $s=500$ ) and (later) run 2012 ( $s=200$ )



*Adamczyk et al. (STAR), P.L. B780 (18) 332*



*Radici et al., P.R. D94 (16) 034012*

- use COMPASS data on  $\pi K$  and  $KK$  channels, and from  $\Lambda^\uparrow$  fragmentation: constrain strange contribution ?
- need data on  $p+p \rightarrow (\pi\pi) X$  constrains gluon  $D_{1g}$
- explore other channels, like inclusive DIS via Jet fragm. funct.'s

# $e^+e^-$ cross section for $(\pi\pi)$ in same hemisphere

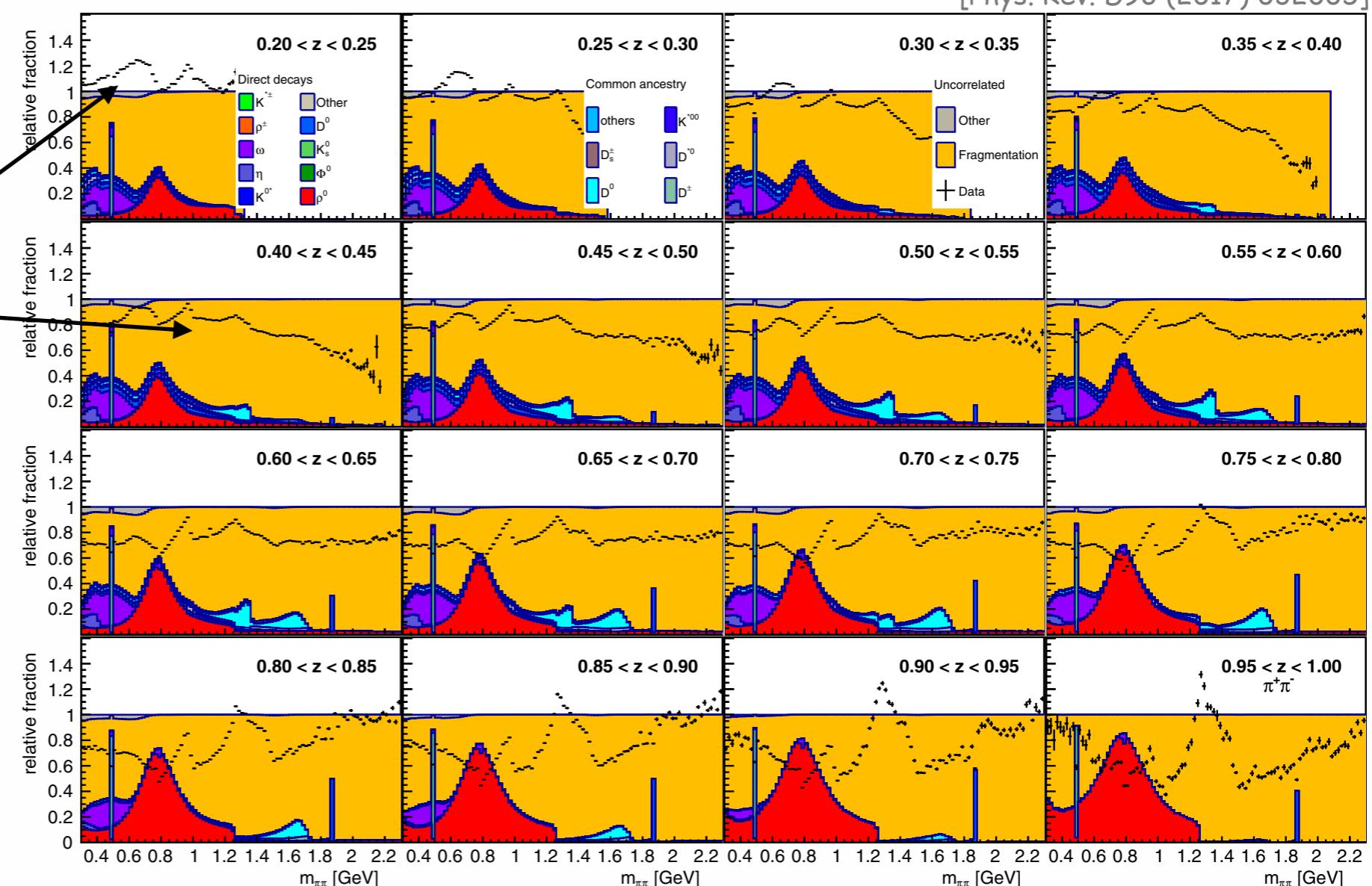


## same-hemisphere data: $M_{h1h2}$ dependence

unlike-sign  
pion pairs

data  $\neq$  MC

$T > 0.8$   
 $z_{1,2} > 0.1$



- decomposition based on PYTHIA simulation
- clear differences in invariant-mass dependence between MC and data

# Conclusions

- first global fit of di-hadron inclusive data leading to extraction of transversity as a PDF in collinear framework
- inclusion of STAR p-p<sup>↑</sup> data increases precision of up channel; large uncertainty on down due to unconstrained gluon unpolarized di-hadron fragmentation function
- no apparent simultaneous compatibility with lattice for tensor charge of up, down, and isovector
- adding Compass pseudodata for deuteron confirms the scenario, but can be potentially very selective when inserting real central values
- need data spanning larger x range; meantime, look for other theoretical constraints on extrapolation (mostly, at low x)

**THANK YOU**

# Back-up

# the SIDIS Single-Spin Asymmetry

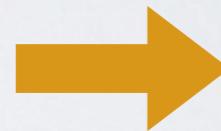
$$A_{UT}^{\sin(\phi_R + \phi_S)} = -\frac{B(y)}{A(y)} \frac{|\mathbf{R}|}{M_h} \sin \theta \frac{\sum_q e_q^2 h_1^q(x) H_{1,sp}^{< q}(z, M_h^2)}{\sum_q e_q^2 f_1^q(x) D_1^q(z, M_h^2)}$$

x-dep. of SSA given by PDFs only



$$\begin{aligned} n_q^\uparrow &= \int dz \int dM_h^2 \frac{|\mathbf{R}|}{M_h} H_{1,sp}^{< q}(z, M_h^2) \\ n_q &= \int dz \int dM_h^2 D_1^q(z, M_h^2) \end{aligned}$$

separate valence u and d using symmetry of DiFFs



$$\begin{aligned} n_q &= n_{\bar{q}} \\ n_q^\uparrow &= -n_{\bar{q}}^\uparrow \\ n_u^\uparrow &= -n_d^\uparrow \end{aligned}$$

proton

$$\begin{aligned} xh_1^p(x) &\equiv xh_1^{u_v}(x) - \frac{1}{4} xh_1^{d_v}(x) \\ &= -\frac{A(y)}{B(y)} \frac{[A_{UT}^{\sin(\phi_R + \phi_S)}]_p}{e_u^2 n_u^\uparrow} \frac{9}{4} \sum_{q=u,d,s} e_q^2 x f_1^{q+\bar{q}}(x) n_q \end{aligned}$$

deuteron

$$\begin{aligned} xh_1^D(x) &\equiv xh_1^{u_v}(x) + xh_1^{d_v}(x) \\ &= -\frac{A(y)}{B(y)} \frac{[A_{UT}^{\sin(\phi_R + \phi_S)}]_D}{e_u^2 n_u^\uparrow} 3 \sum_{q=u,d,s} [e_q^2 n_q + e_{\tilde{q}}^2 n_{\tilde{q}}] x f_1^{q+\bar{q}}(x) \end{aligned}$$

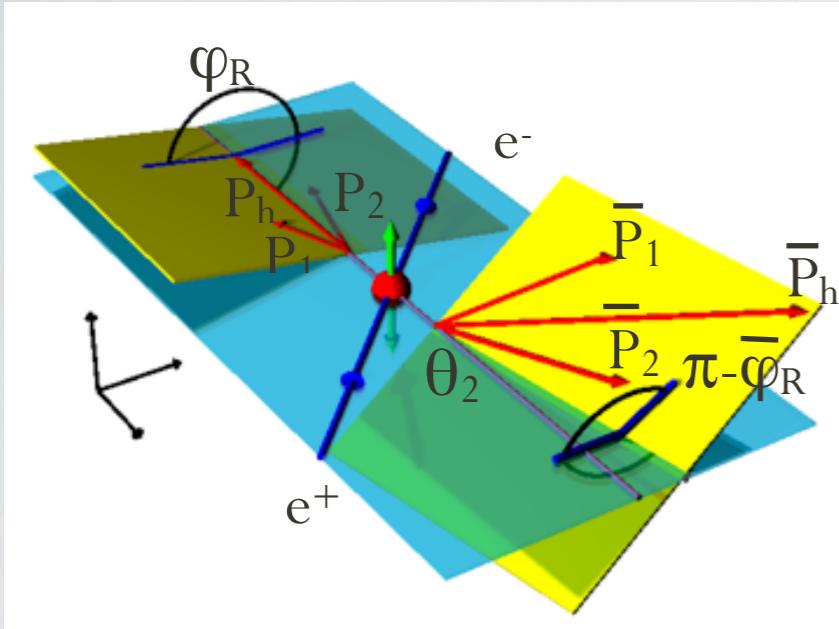
$$\tilde{q} = d, u, s$$

point-by-point  
extraction

Bacchetta, Courtoy, Radici,  
P.R.L. **107** (11) 012001

Martin, Bradamante, Barone,  
P.R. **D91** (15) 014034

# extraction of DiFF from $e^+e^-$



*Artru & Collins, Z.Ph. C69 (96) 277*

$$A^{\cos(\phi_R + \bar{\phi}_R)} = \frac{\sin^2 \theta_2}{1 + \cos^2 \theta_2} \frac{|\mathbf{R}_T|}{M_h} \frac{|\overline{\mathbf{R}}_T|}{\overline{M}_h}$$

$$\times \frac{\sum_q e_q^2 H_{1,sp}^{\leftarrow q}(z, M_h^2) \overline{H}_{1,sp}^{\leftarrow \bar{q}}(\bar{z}, \overline{M}_h^2)}{\sum_q e_q^2 D_1^q(z, M_h^2) \overline{D}_1^{\bar{q}}(\bar{z}, \overline{M}_h^2)}$$

*Boer, Jakob, Radici,  
P.R.D67 (03) 094003  
Matevosyan et al.,  
P.R.D97 (2018) 074019*

integrate on one hemisphere

$$A^{\cos(\phi_R + \bar{\phi}_R)} = \frac{\sin^2 \theta_2}{1 + \cos^2 \theta_2} \frac{|\mathbf{R}|}{M_h} \sin \theta \langle \sin \bar{\theta} \rangle \frac{\sum_q e_q^2 H_{1,sp}^{\leftarrow q}(z, M_h^2) n_q^\uparrow}{\sum_q e_q^2 D_1^q(z, M_h^2) n_{\bar{q}}}$$

$$n_q^\uparrow = \int dz \int dM_h^2 \frac{|\mathbf{R}|}{M_h} H_{1,sp}^{\leftarrow q}(z, M_h^2)$$

$$n_q = \int dz \int dM_h^2 D_1^q(z, M_h^2)$$



Belle data for  
 $A^{\cos(\Phi_R + \bar{\Phi}_R)}$

*Vossen et al., P.R.L. 107 (11) 072004*



symmetry  
of DiFFs

$$n_q = n_{\bar{q}}$$

$$n_q^\uparrow = -n_{\bar{q}}^\uparrow$$

$$n_u^\uparrow = -n_d^\uparrow$$



first extraction  
of DiFFs

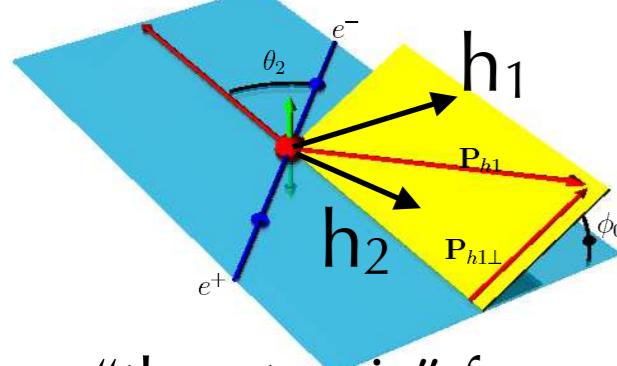
(determines the sign of  $A^{\cos(\Phi_R + \bar{\Phi}_R)}$ )

*Courtoy et al.,  
P.R.D85 (12) 114023  
Radici et al.,  
JHEP 1505 (15) 123*

# $e^+e^-$ cross section for $(\pi\pi)$ in same hemisphere

$e^+e^- \rightarrow (h_1 h_2) X$

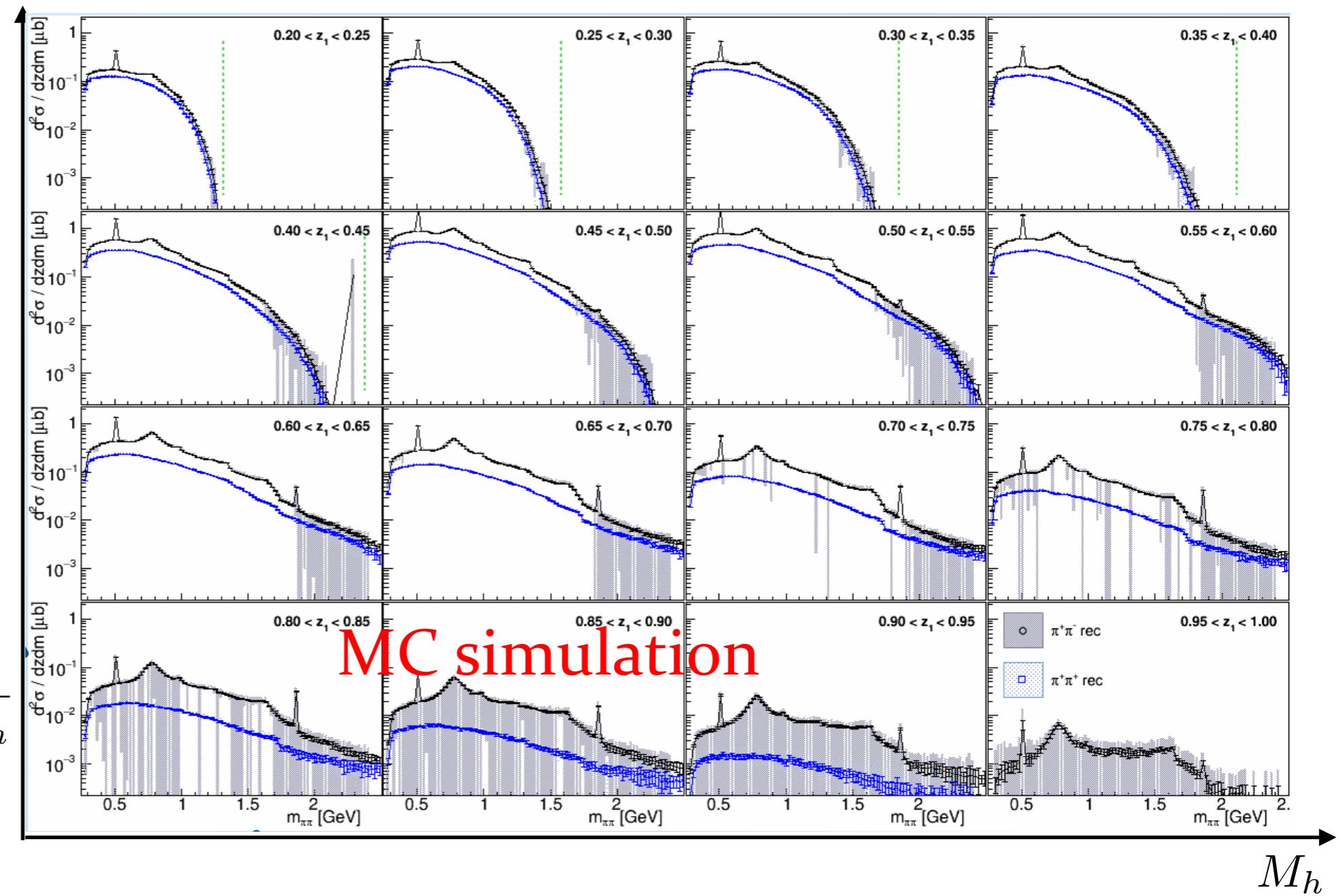
thrust



"thrust-axis" frame

- $\pi^+\pi^-$
- $\pi^+\pi^+$

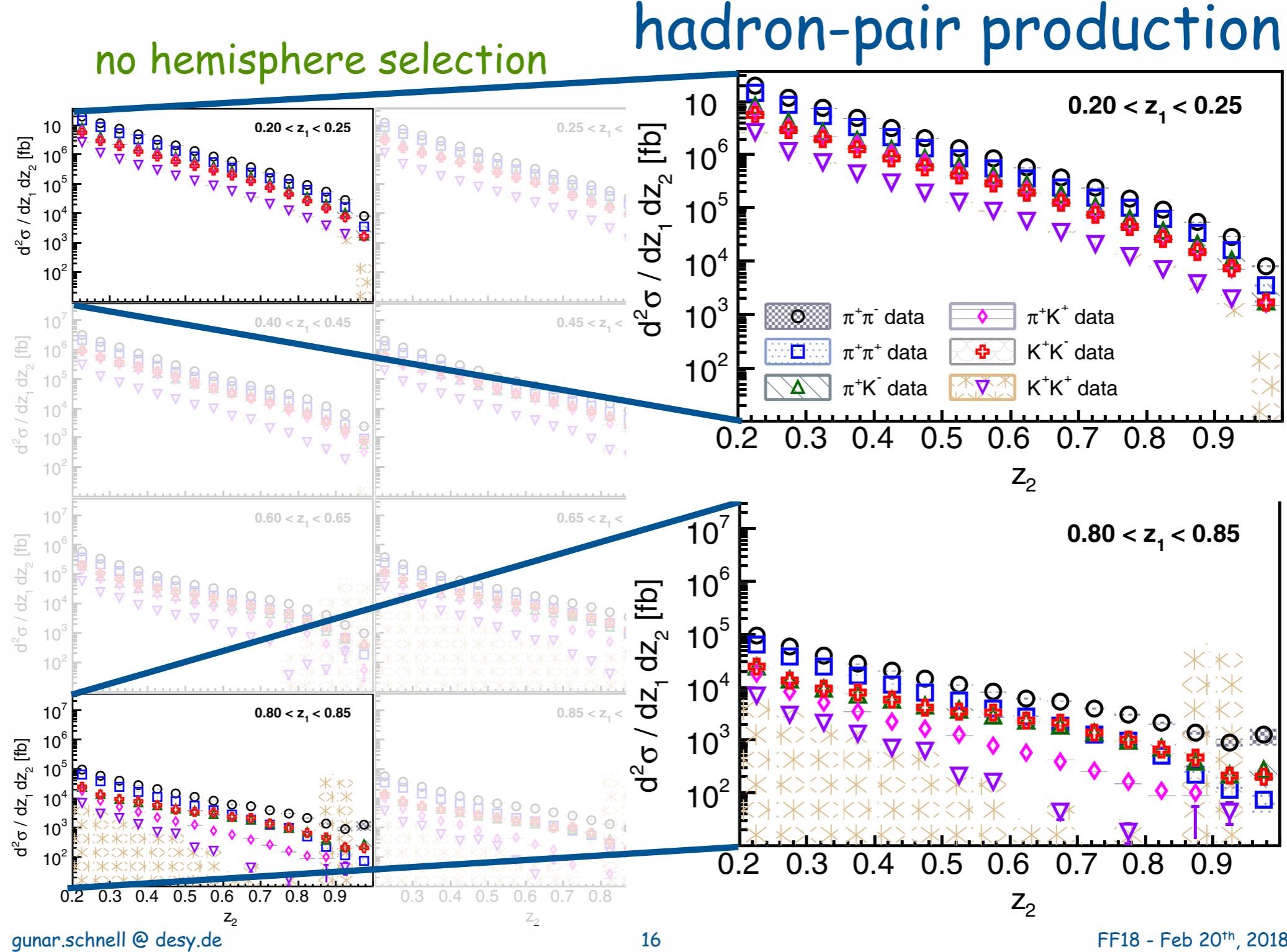
$$\frac{d\sigma^0}{dz dM_h}$$



R. Seidl, talk at SPIN2016

upcoming Belle data for  $(z, M_h)$  binning of  
unpolarized di-hadron  $e^+e^-$  cross section

# e<sup>+</sup>e<sup>-</sup> cross section for (hh) from all hemispheres



gunar.schnell @ desy.de

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FF18 - Feb 20<sup>th</sup>, 2018

D<sub>I</sub> (z<sub>2</sub>) @low z<sub>I</sub> ≠ D<sub>I</sub> (z<sub>2</sub>) @high z<sub>I</sub>

# $e^+e^-$ cross section for (hh) in different hemispheres



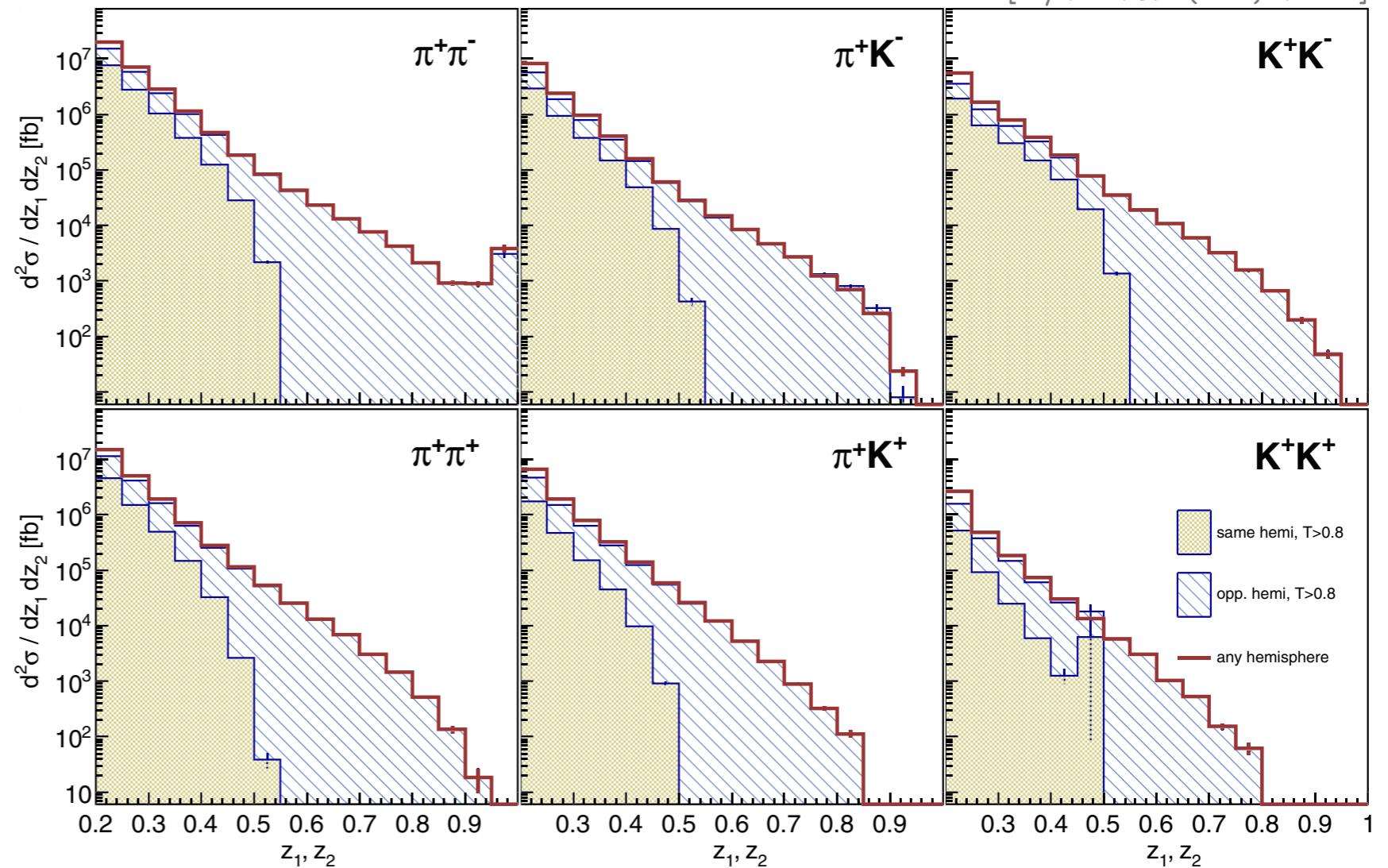
## hadron-pairs: topology comparison

- any hemisphere vs. opposite- & same-hemisphere pairs
- same-hemisphere pairs with kinematic limit at  $z_1=z_2=0.5$

[Phys. Rev. D92 (2015) 092007]

opposite hemisphere  
 $0 < z_1=z_2 < 1$

same hemisphere  
 $0 < z_1+z_2 < 1$   
 $0 < z_1=z_2 < 0.5$



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$$D_{\text{IFF}}(z_1, z_2) \neq D_{\text{I}}(z_1) D_{\text{I}}(z_2)$$

# hadronic collisions in Mellin space

$d\sigma(\eta, M_h, P_T)$  typical cross section for  $a+b^\uparrow \rightarrow c^\uparrow + d$  process

$$\frac{d\sigma_{UT}}{d\eta} \propto \int d|\mathbf{P}_T| dM_h \sum_{a,b,c,d} \int \frac{dx_a dx_b}{8\pi^2 \bar{z}} f_1^a(x_a) h_1^b(x_b) \frac{d\hat{\sigma}_{ab^\uparrow \rightarrow c^\uparrow d}}{d\hat{t}} H_1^{\leftarrow c}(\bar{z}, M_h)$$

to be computed thousands times... usual trick: use **Mellin anti-transform**

$$h_1(x, Q^2) = \int_{C_N} dN \ x^{-N} \ h_1^N(Q^2) \quad N \in \mathbb{C}$$

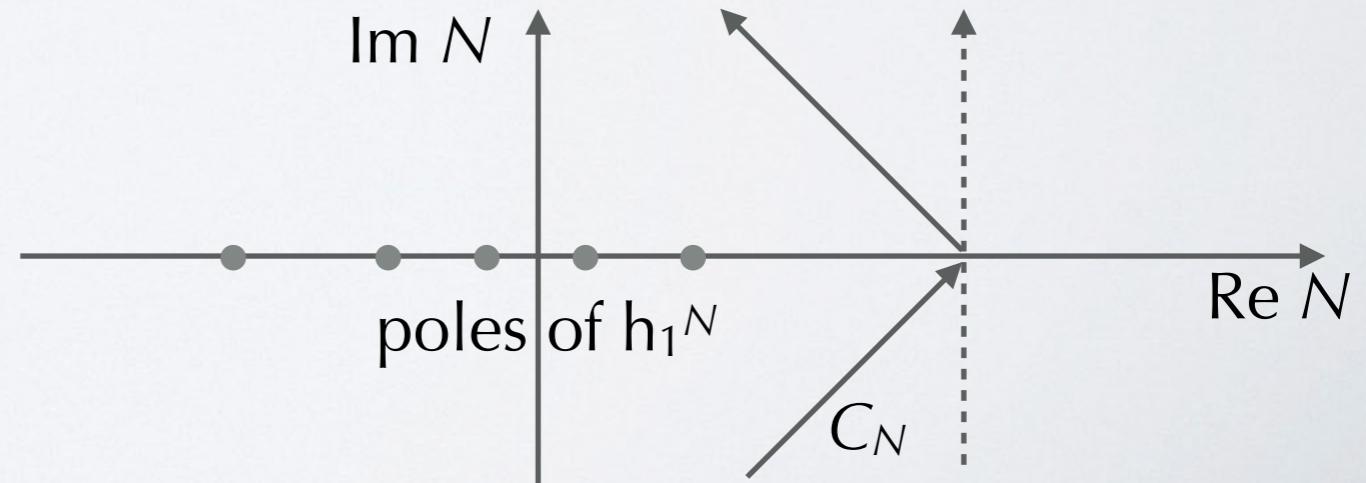
Stratmann & Vogelsang,  
P.R. D64 (01) 114007

$$\frac{d\sigma_{UT}}{d\eta} \propto \sum_b \int_{C_N} dN \int d|\mathbf{P}_T| h_{1b}^N(P_T^2) \int dM_h \sum_{a,c,d} \int \frac{dx_a dx_b}{8\pi^2 \bar{z}} f_1^a(x_a) x_b^{-N} \frac{d\hat{\sigma}_{ab^\uparrow \rightarrow c^\uparrow d}}{d\hat{t}} H_1^{\leftarrow c}(\bar{z}, M_h)$$

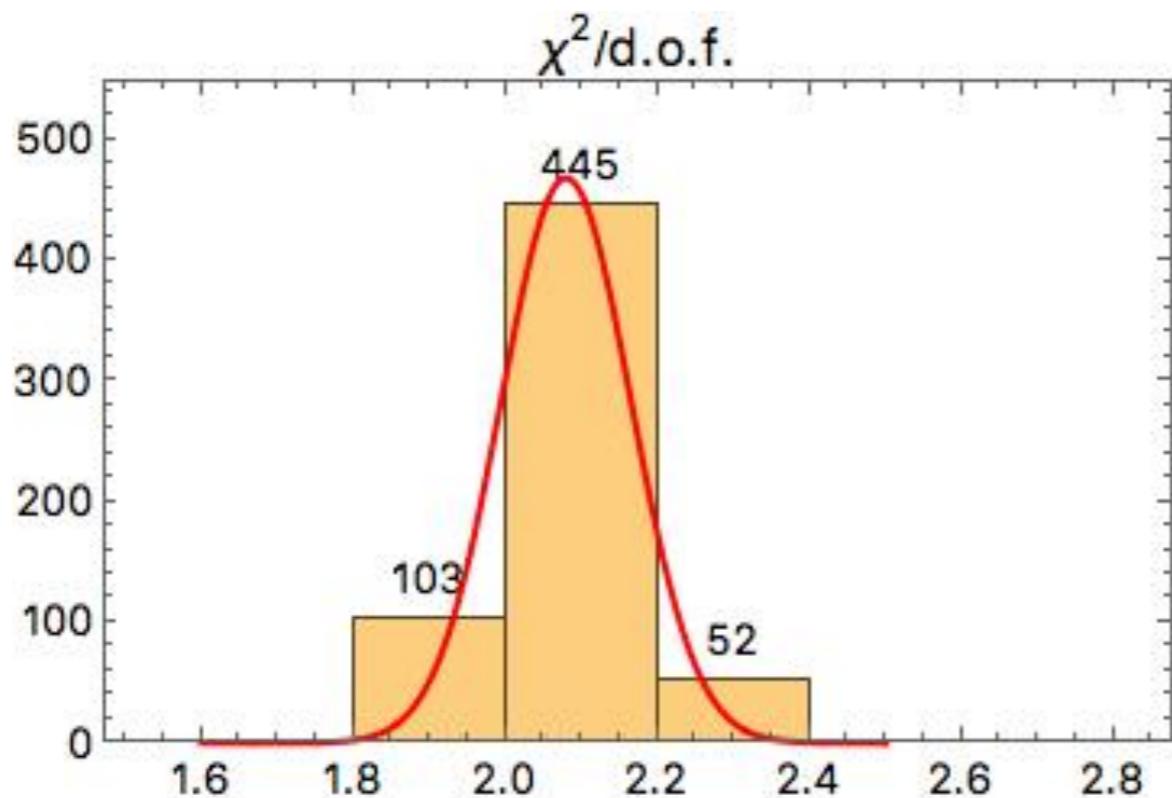
$F_b(N, \eta, |\mathbf{P}_T|, M_h)$

pre-compute  $F_b$  only one time  
on contour  $C_N$

this **speeds up** convergence  
and facilitates  $\int dN$ , provided  
that  **$h_1^N$  is known analytically**



# $\chi^2$ of the fit



$$\chi^2/\text{dof} = 2.08 \pm 0.09$$

