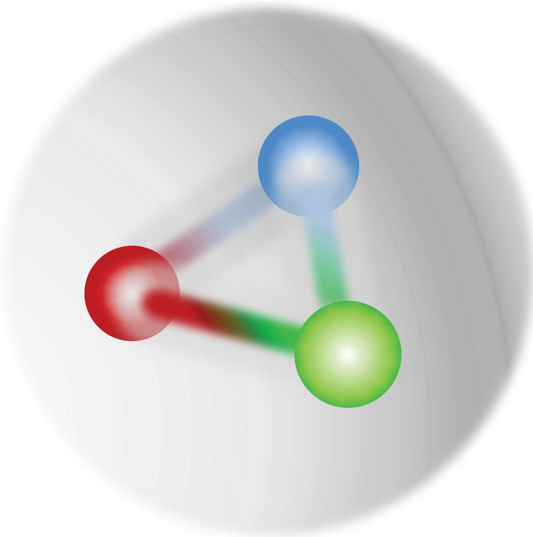
Quarkonium polarization from high to low p_T

Pietro Faccioli, LIP, Lisbon
In collaboration with C. Lourenço, M. Araújo and J. Seixas

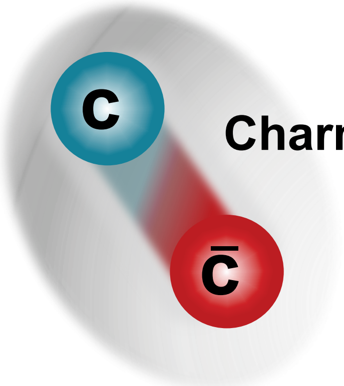
COMPASS Seminar
May 18th, 2018

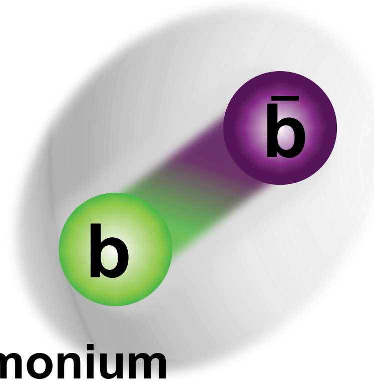
1. **Quarkonium production: LHC data vs NRQCD**
2. **Beyond the polarization puzzle: new insights from recent data**
3. **What about the $p_T \to 0$ limit?**
4. **Low-p_T polarization: puzzles and opportunities**
5. **"Offline" appendix: polarization basics, invariants and the Lam-Tung relation**



Almost all the visible matter is made of *hadrons*
The *dynamics* of hadron formation is not well understood
How do quarks combine into a bound state?

The ideal case study is Quarkonium,
bound state of a heavy quark and its antiquark



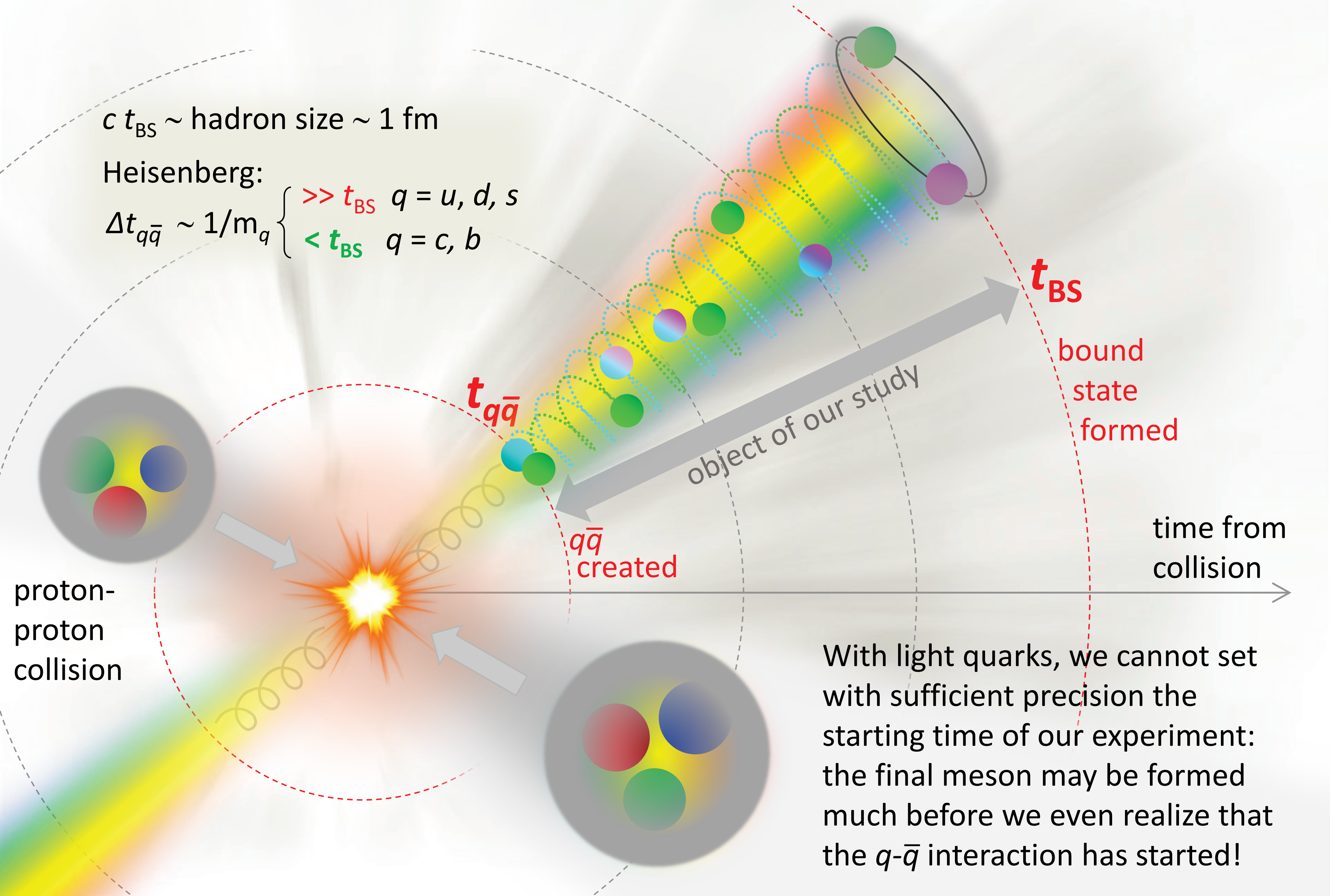


Studying quarkonia we can *see* the bound-state formation

$c\, t_{\mathrm{BS}} \sim$ hadron size $\sim$ 1 fm

Heisenberg:

$$\Delta t_{q\bar{q}} \sim 1/m_q \begin{cases} \gg t_{\mathrm{BS}} & q = u, d, s \\ < t_{\mathrm{BS}} & q = c, b \end{cases}$$



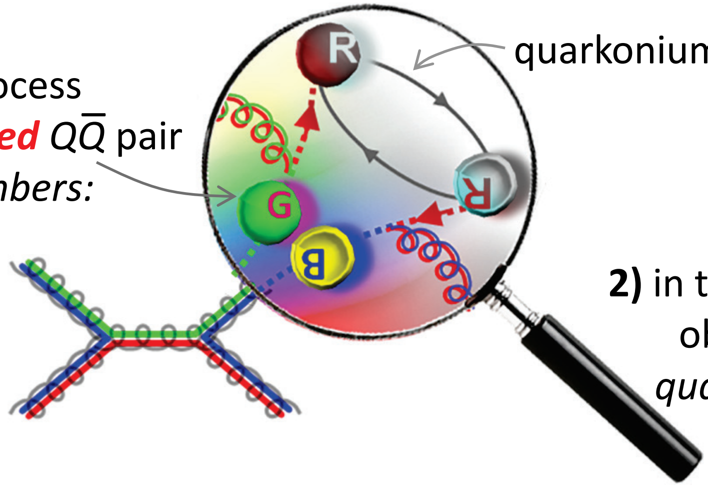
With light quarks, we cannot set with sufficient precision the starting time of our experiment: the final meson may be formed much before we even realize that the q-$\bar{q}$ interaction has started!

Theory of quarkonium production

- Nonrelativistic QCD (NRQCD) is the most complete approach
- In the "factorization" hypothesis, cornerstone of NRQCD, a variety of production mechanisms is in principle foreseen for each quarkonium state

1) short-distance partonic process produces *neutral* or ***coloured*** $Q\bar{Q}$ pair *of any* $^{2S+1}L_J$ *quantum numbers:*

$^1S_0$ $^1S_0$ $^3S_1$ $^3P_0$ $^3P_2$ $^1D_2$ $^1P_1$ $^3S_1$ $^3P_1$ $^3P_2$ $^3D_3$ $^1P_1$ $^3D_2$ $^3D_1$ $^3P_1$



quarkonium ($\mathcal{Q}$)

η_c , η_b [$^1S_0$]

ψ, Υ [$^3S_1$] χ_{c0} , χ_{b0} [$^3P_0$]

χ_{c1} , χ_{b1} [$^3P_1$] χ_{c2} , χ_{b2} [$^3P_2$]

2) in the **long-distance** evolution to the observed (neutral) bound state *quantum numbers change* to final

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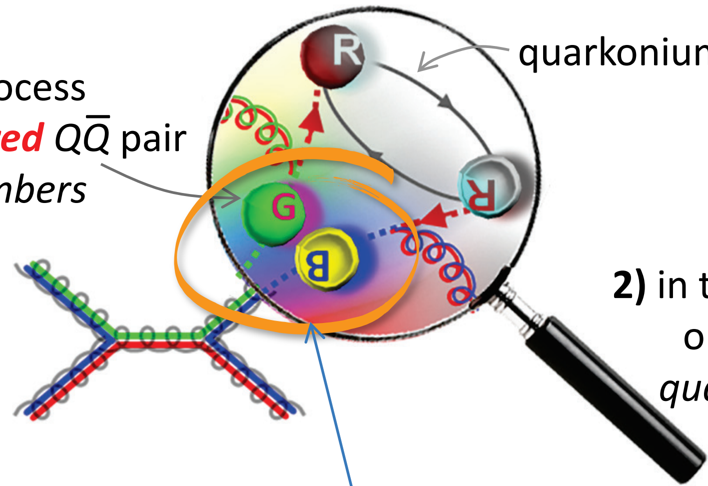

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2) in the **long-distance** evolution to the observed (neutral) bound state *quantum numbers change* to final

What is produced in the hard scattering
(and determines kinematics and polarization)
is a *pre-resonance* $Q\bar{Q}$ state
with its own quantum properties

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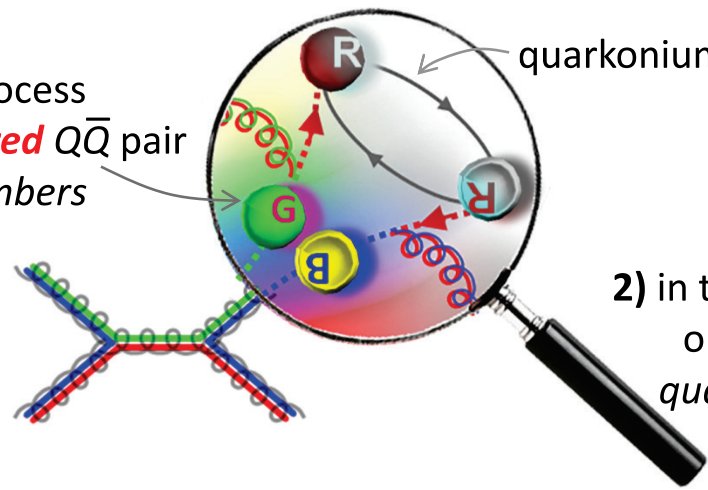

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2) in the **long-distance** evolution to the observed (neutral) bound state *quantum numbers change* to final

1) *short-distance coefficients* (**SDCs**): p_T-dependent partonic cross sections

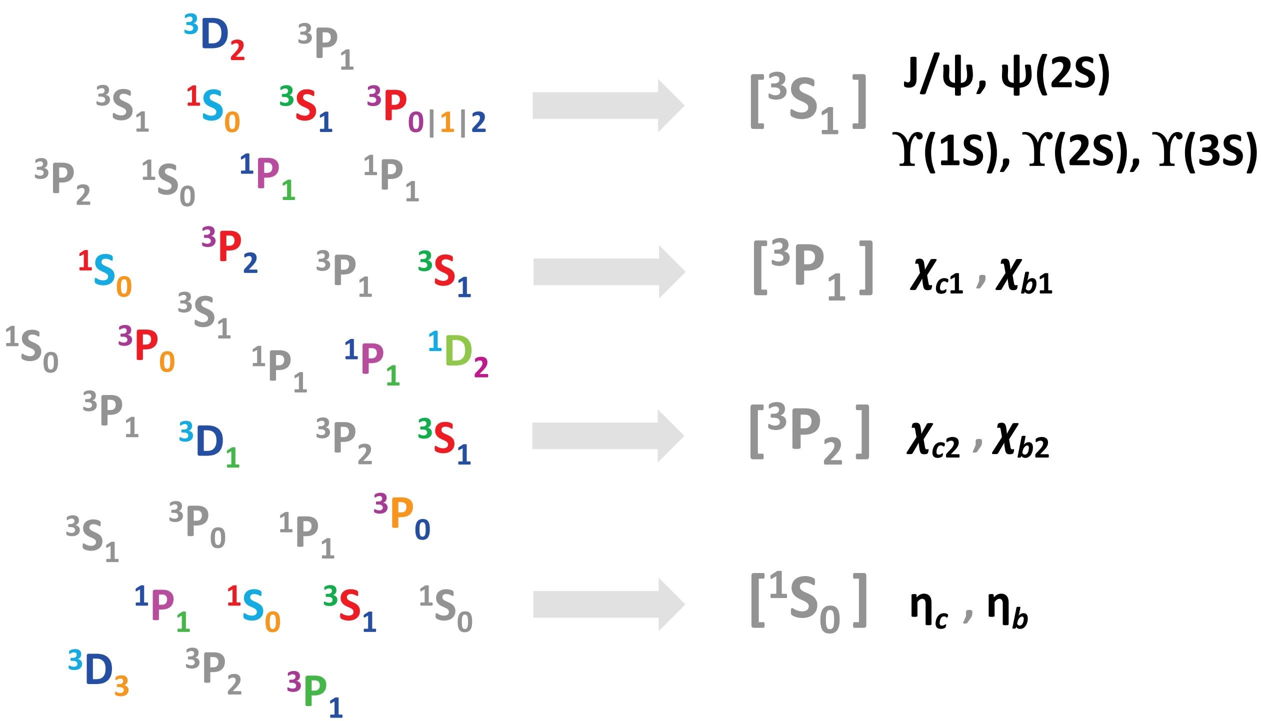
$$\sigma(A + B \to \mathcal{Q} + X) = \sum_{S,\,L,\,C} \mathcal{S}\{A + B \to (Q\bar{Q})_C[^{2S+1}L_J] + X\} \cdot \mathcal{L}\{(Q\bar{Q})_C[^{2S+1}L_J] \to \mathcal{Q}\}$$

$Q\bar{Q}$ **angular momentum** and **colour** configuration

2) *long-distance matrix elements* (**LDMEs**): constant, **fitted from data**

NRQCD hierarchies

Approximations (***heavy-quark limit***) and calculations induce hierarchies and links between pre-resonance contributions



NRQCD hierarchies

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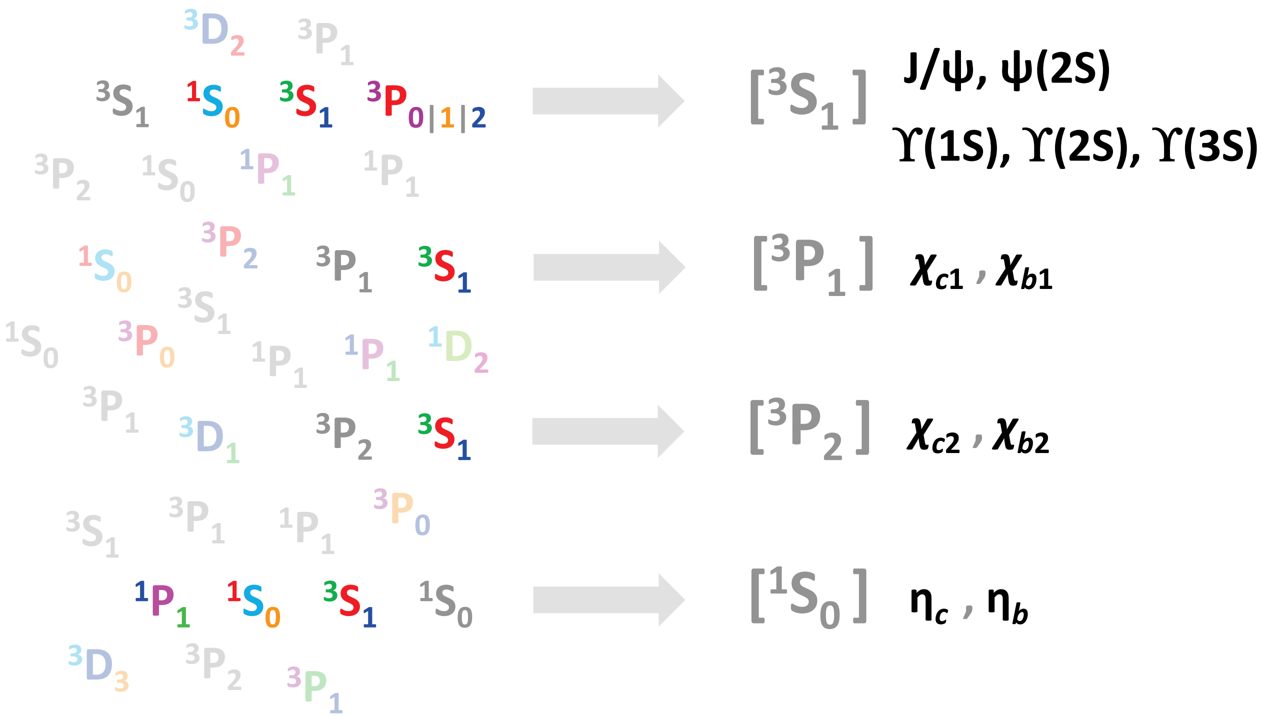
1) Small quark velocities v in the bound state → "***v*-scaling**" rules for LDMEs



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${}^3S_1$ ${}^1S_0$ ${}^3S_1$ ${}^3P_{0\|1\|2}$	→	$[{}^3S_1]$	J/ψ, ψ(2S) ϒ(1S), ϒ(2S), ϒ(3S)
${}^3P_1$ ${}^3S_1$	→	$[{}^3P_1]$	χ_{c1} , χ_{b1}
${}^3P_2$ ${}^3S_1$	→	$[{}^3P_2]$	χ_{c2} , χ_{b2}
${}^1P_1$ ${}^1S_0$ ${}^3S_1$ ${}^1S_0$	→	$[{}^1S_0]$	η_c , η_b

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2) **Perturbative calculations** $\rightarrow$ some SDCs are negligible:

$^3S_1$ $^1S_0$ $^3S_1$ $^3P_{0|1|2}$ $\Rightarrow$ $[^3S_1]$ **J/ψ, ψ(2S)** **Υ(1S), Υ(2S), Υ(3S)**

$^3P_1$ $^3S_1$ $\Rightarrow$ $[^3P_1]$ χ_{c1} , χ_{b1}

$^3P_2$ $^3S_1$ $\Rightarrow$ $[^3P_2]$ χ_{c2} , χ_{b2}

$^1P_1$ $^1S_0$ $^3S_1$ $^1S_0$ $\Rightarrow$ $[^1S_0]$ η_c , η_b

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$^3P_2$ $^3S_1$	$\Rightarrow$	$[^3P_2]$	χ_{c2} , χ_{b2}
$^3S_1$ $^1S_0$	$\Rightarrow$	$[^1S_0]$	η_c , η_b

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$^3P_1 \quad ^3S_1 \Rightarrow [^3P_1]$ χ_{c1}, χ_{b1}

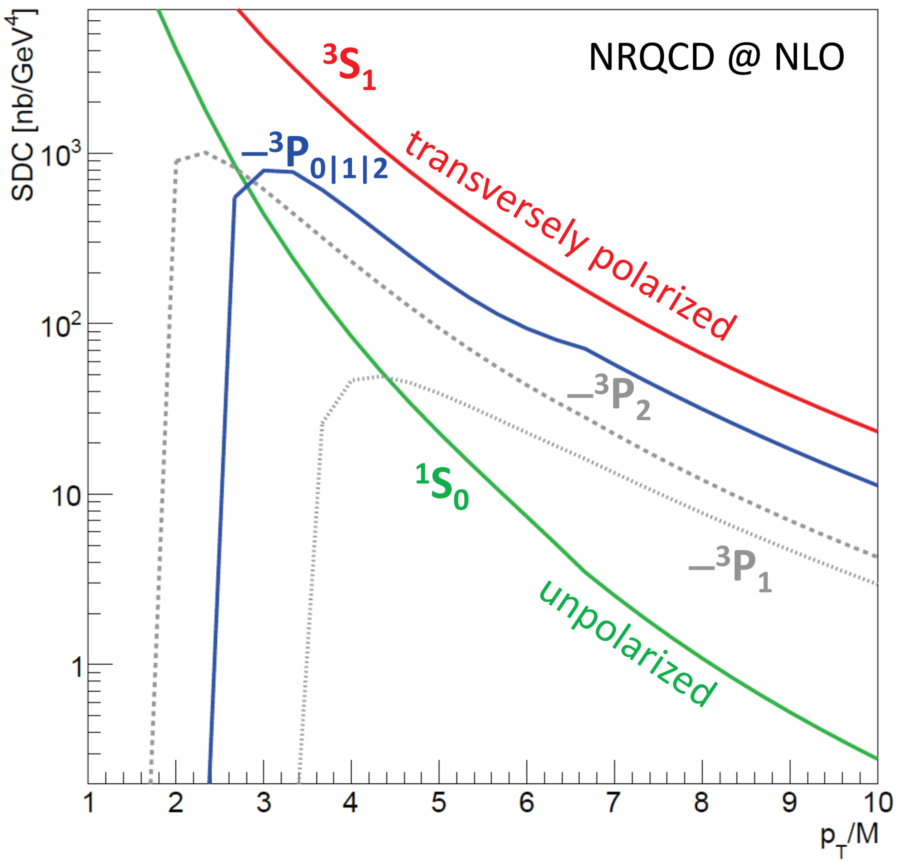
$^3P_2 \quad ^3S_1 \Rightarrow [^3P_2]$ χ_{c2}, χ_{b2}

$^3S_1 \quad ^1S_0 \Rightarrow [^1S_0]$ η_c, η_b

3) **Heavy-quark spin symmetry** $\rightarrow$ relations between LDMEs of different states

$$\frac{^3S_1 \rightarrow \chi_{c2}}{^3S_1 \rightarrow \chi_{c1}} = \frac{^3S_1 \rightarrow \chi_{b2}}{^3S_1 \rightarrow \chi_{b1}} = \frac{5}{3}\,, \qquad \begin{matrix} ^3S_1 \rightarrow \eta_c = {^1S_0} \rightarrow J/\psi \\ ^3S_1 \rightarrow \eta_b = {^1S_0} \rightarrow \Upsilon \end{matrix}\,, \text{ etc.}$$

The dominant short-distance cross section contributions



$^1S_0$ $^3S_1$ $^3P_{0|1|2}$ → J/ψ, ψ(2S), ϒ(1S), ϒ(2S), ϒ(3S)

$^3P_1$ $^3S_1$ → χ_{c1} , χ_{b1}

$^3P_2$ $^3S_1$ → χ_{c2} , χ_{b2}

negative P-wave contributions,
with large unphysical polarizations,
require proper cancellations
to recover physical result

Mixture of different pre-resonance contributions, with rather **diversified** kinematics and characteristic polarizations

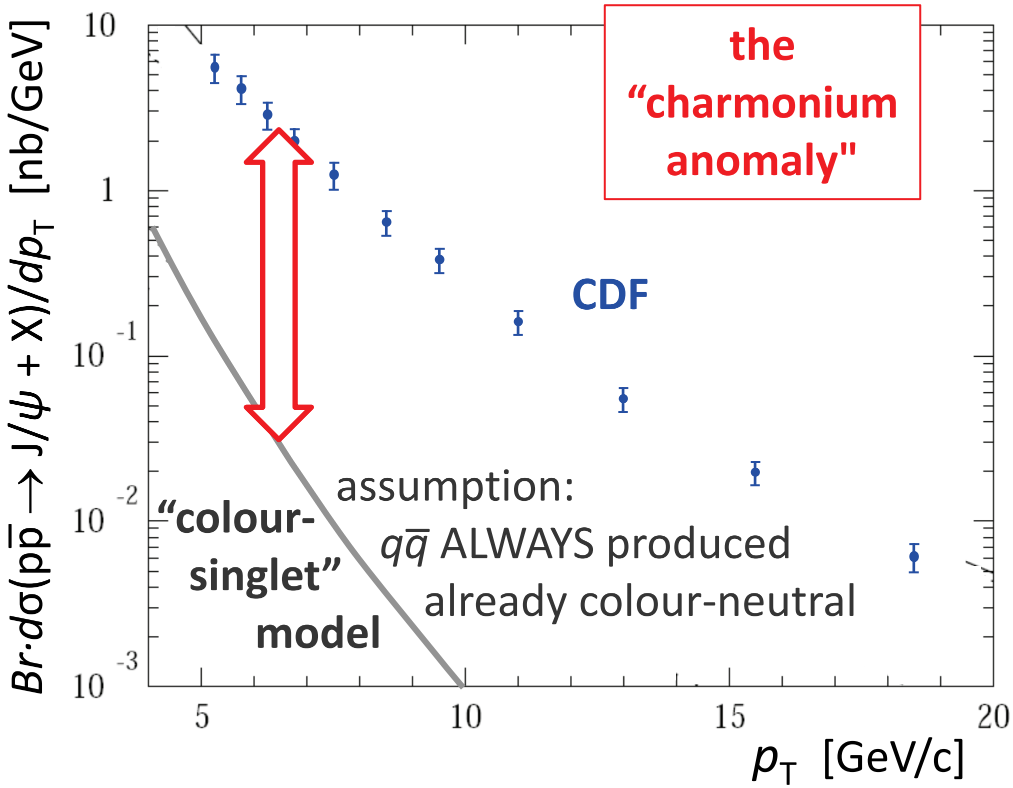
→ by fitting the experimental p_T distributions it is possible to determine the coefficients of all terms (LDMEs) and consequently predict the polarizations

...a delicate procedure!

Curves from
H.-S. Shao et. al., PRL 108, 242004; PRL 112, 182003; Comput. Phys. Commun. 198, 238

A journey through anomalies...

In 1995, the **CDF** experiment at Fermilab observed J/ψ and ψ(2S) production yields ~50 times larger than predicted by the “colour-singlet model”



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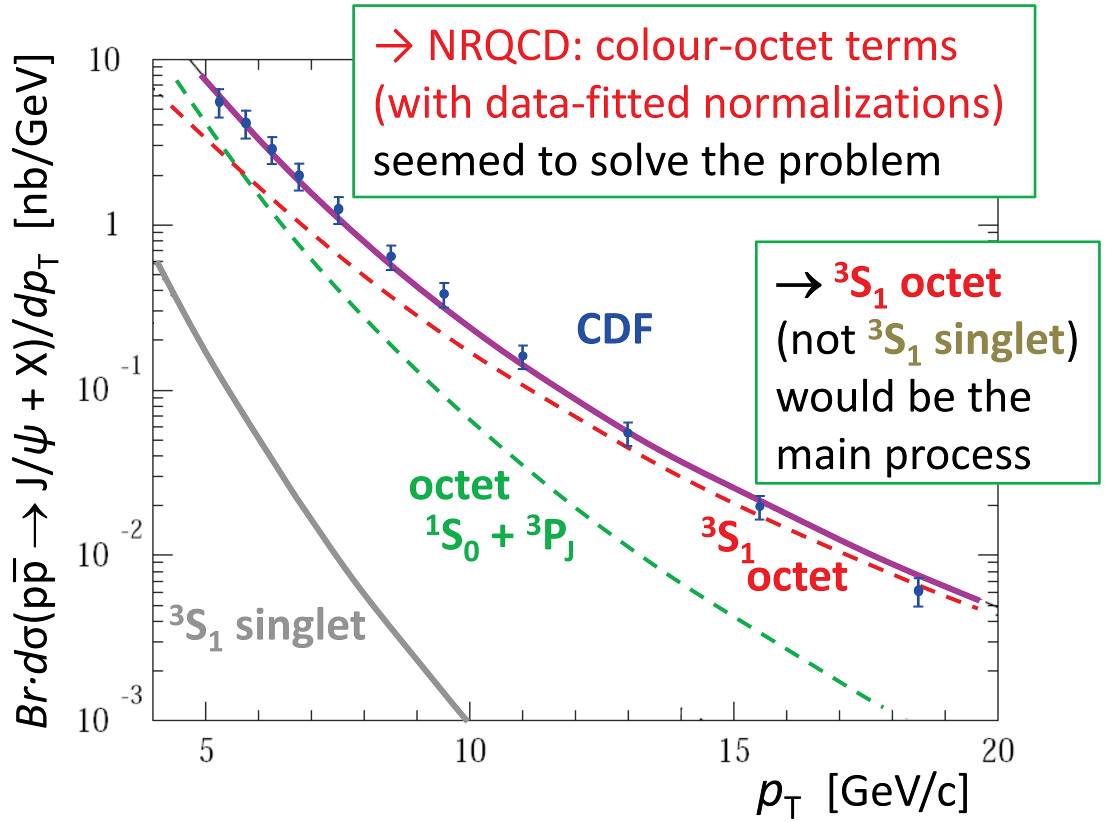
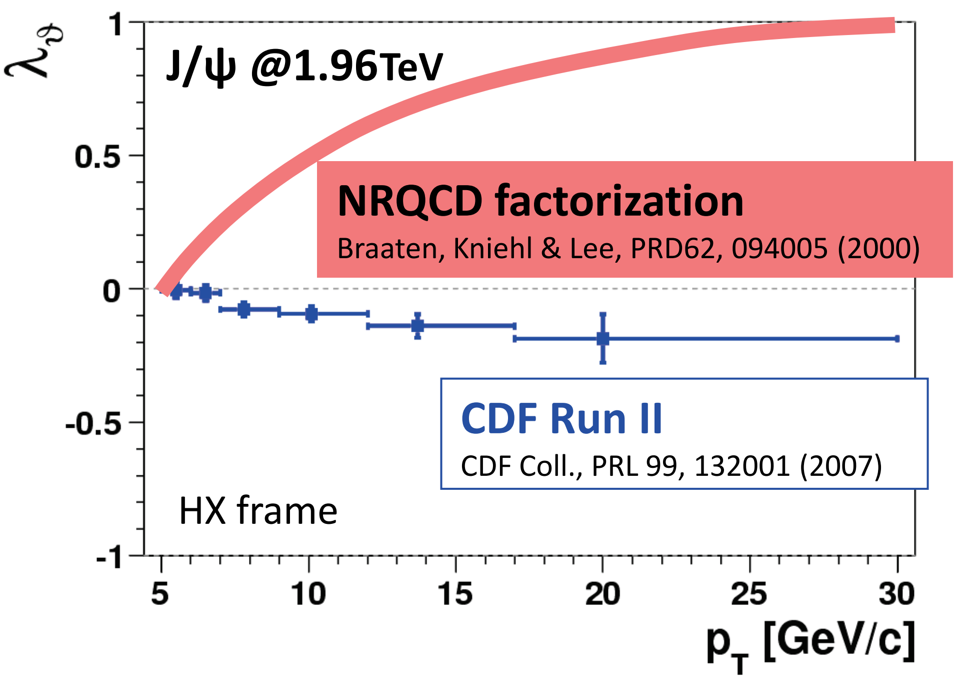


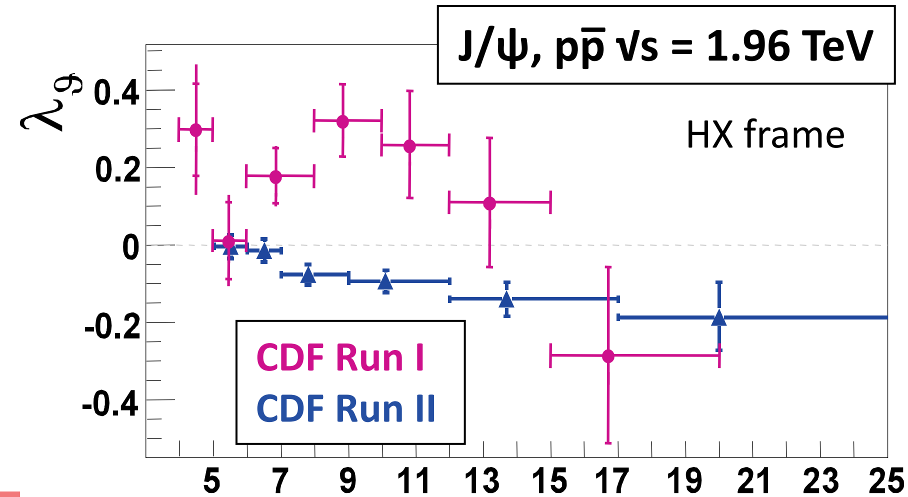
…puzzles

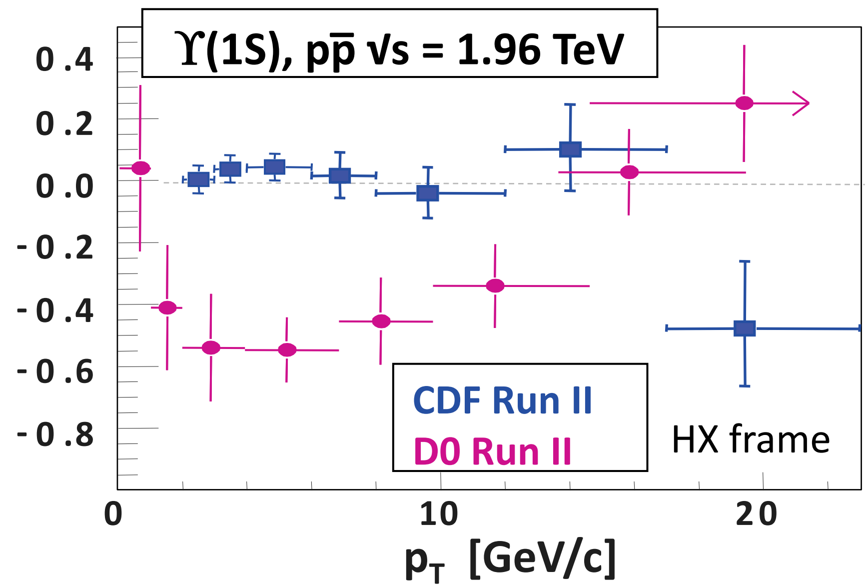
THEORY:
the fits of J/ψ p_T distributions seemingly indicated a strong dominance of the transversely polarized $^3S_1$-octet term
→ prediction: $\lambda_\theta(HX) \sim +1$ at high pT
CDF measurements contradicted it



…and inconsistencies

EXPERIMENT: first polarization measurements, from Tevatron, mutually excluded each other…





The experimental situation has been clarified…

Improved measurement techniques have been adopted by all LHC experiments

PRL 105, 061601 (2010) PHYSICAL REVIEW LETTERS week ending 6 AUGUST 2010

Rotation-Invariant Relations in Vector Meson Decays into Fermion Pairs

Pietro Faccioli,[1] Carlos Lourenço,[2] and João Seixas[1,3]

[1]*LIP, 1000-149 Lisbon, Portugal*

Eur. Phys. J. C (2010) 69: 657–673

Cited > 100 times

Special Article - Tools for Experiment and Theory

Towards the experimental clarification of quarkonium polarization

Pietro Faccioli[1], Carlos Lourenço[2,a], João Seixas[1,3], Hermine K. Wöhri[1]

[1]Laboratório de Instrumentação e Física Experimental de Partículas (LIP), Lisbon, Portugal
[2]CERN, Geneva, Switzerland
[3]Physics Department, Instituto Superior Técnico (IST), Lisbon, Portugal

RAPID COMMUNICATIONS

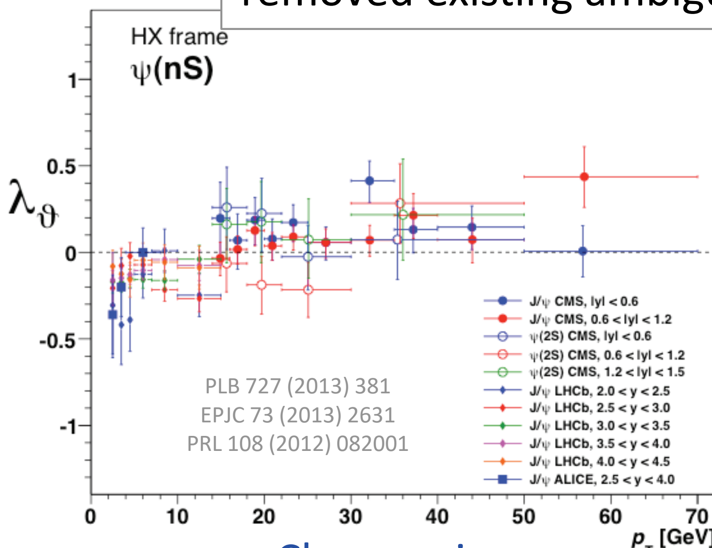
PHYSICAL REVIEW D **81,** 111502(R) (2010)

New approach to quarkonium polarization studies

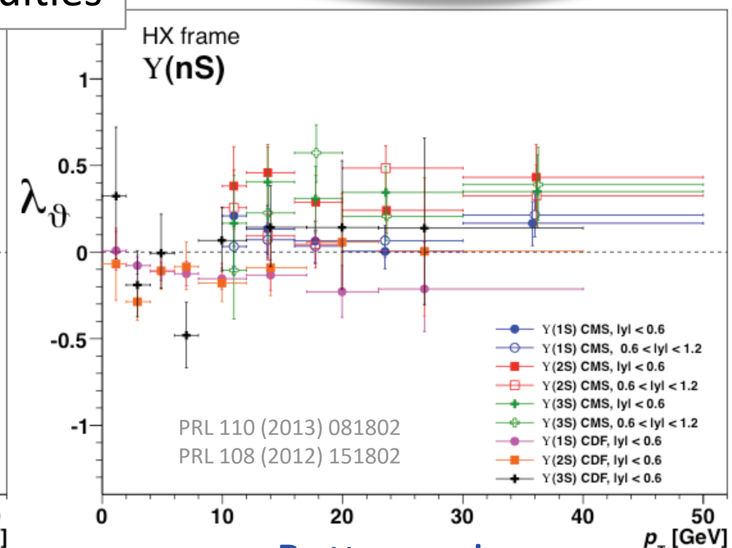
Pietro Faccioli,[1] Carlos Lourenço,[2] and João Seixas[1,3]

[1]*LIP, 1000-149 Lisbon, Portugal*
[2]*CERN, 1211 Geneva, Switzerland*
[3]*Physics Department, IST, 1049-001 Lisbon, Portugal*
(Received 15 April 2010; published 11 June 2010)

The new measurements removed existing ambiguities

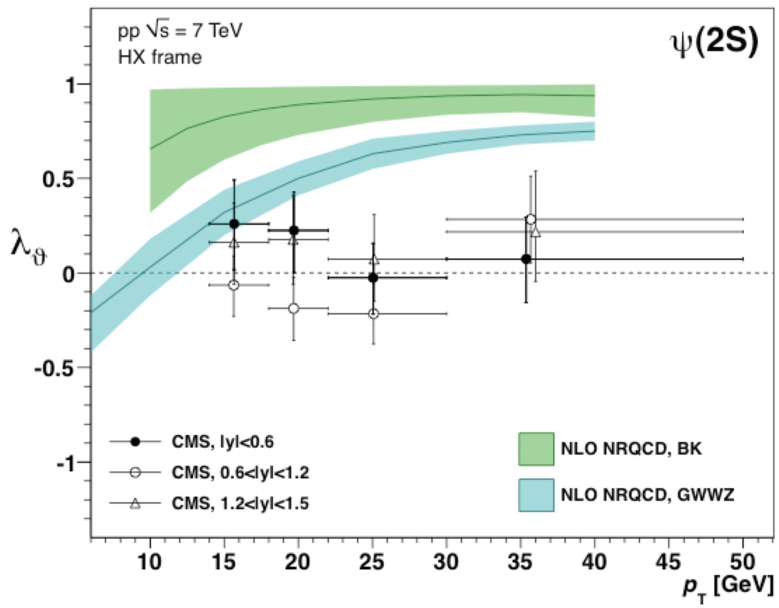


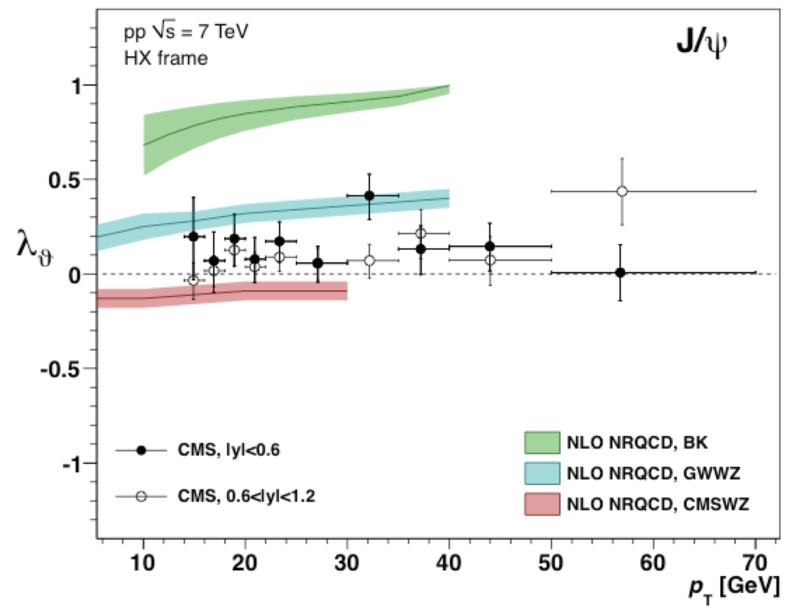
Charmonia



Bottomonia

… and the theory puzzle consolidated!



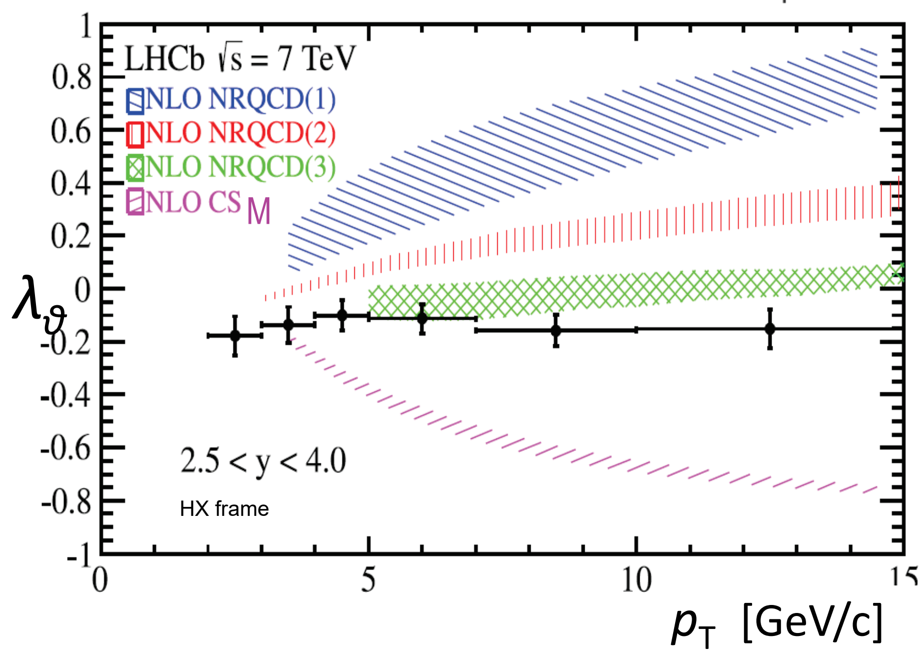


CERN Courier **July/August 2013**

The return of quarkonia

Pietro Faccioli, LIP-Lisbon.

The physics of heavy quark–antiquark bound states is a long-standing puzzle, made more intriguing by results from the LHC.



Solution: more rigorous fits of theory to data

New fit method

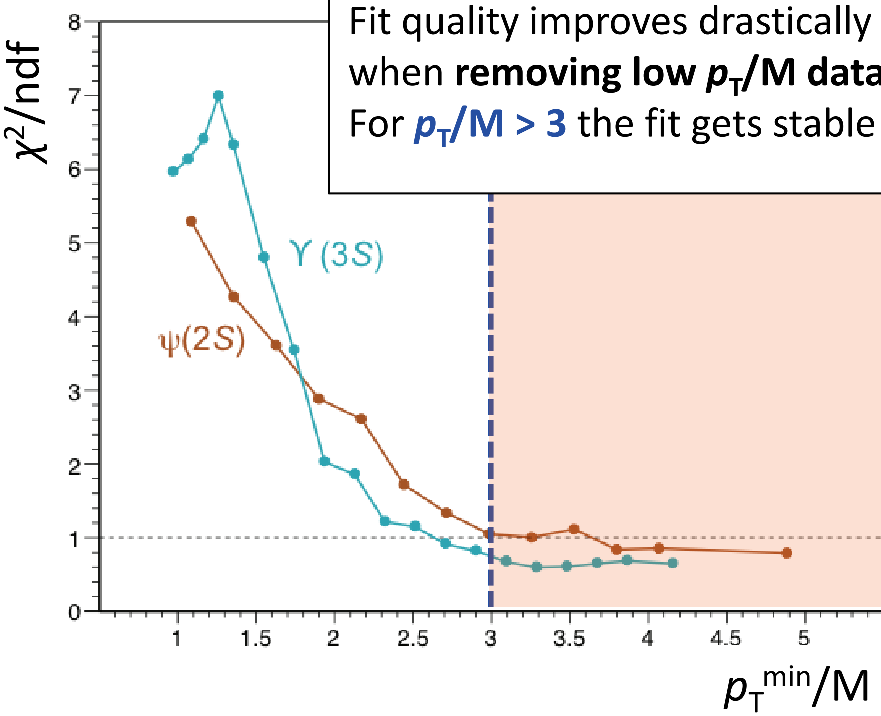
- treating for the first time correlated observables and theoretical uncertainties consistently
- taking into account limitations of the theoretical ingredients

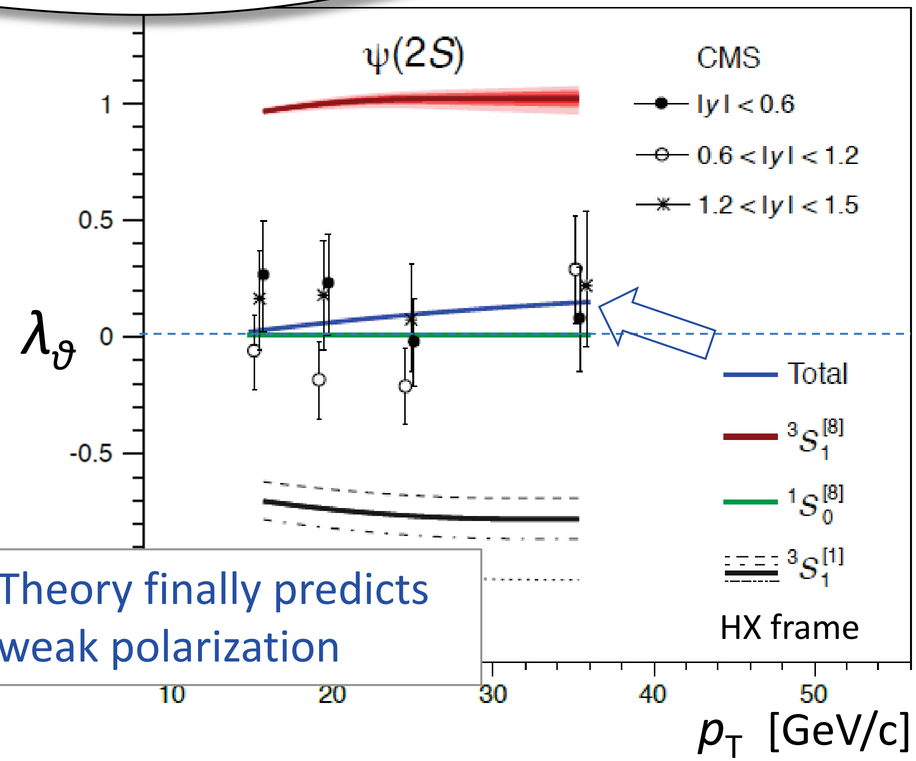
Physics Letters B 736 (2014) 98–109
Contents lists available at ScienceDirect
Physics Letters B
ELSEVIER
www.elsevier.com/locate/physletb

Quarkonium production in the LHC era: A polarized perspective

Pietro Faccioli[a,b,*], Valentin Knünz[c], Carlos Lourenço[d,*], João Seixas[a,b], Hermine K. Wöhri[d]

Fit quality improves drastically when **removing low p_T/M data**
For **p_T/M > 3** the fit gets stable

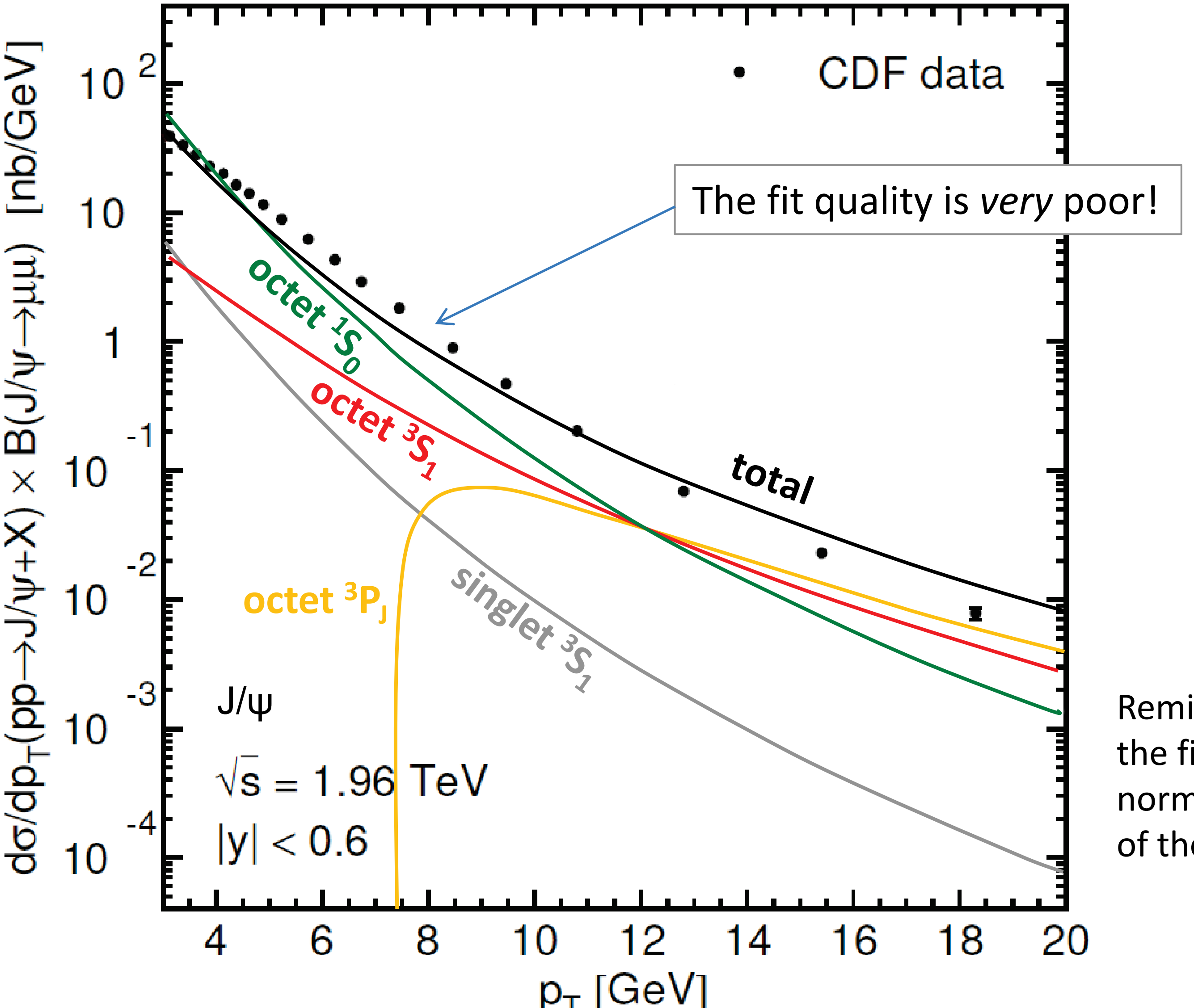




Theory finally predicts weak polarization

A closer look at fits of the past

A representative example of the theory fits
which, until recently, lead to the prediction of transverse polarization at high p_T



Reminder:
the fit freely adjusts the normalizations (LDMEs) of the colour-octet terms

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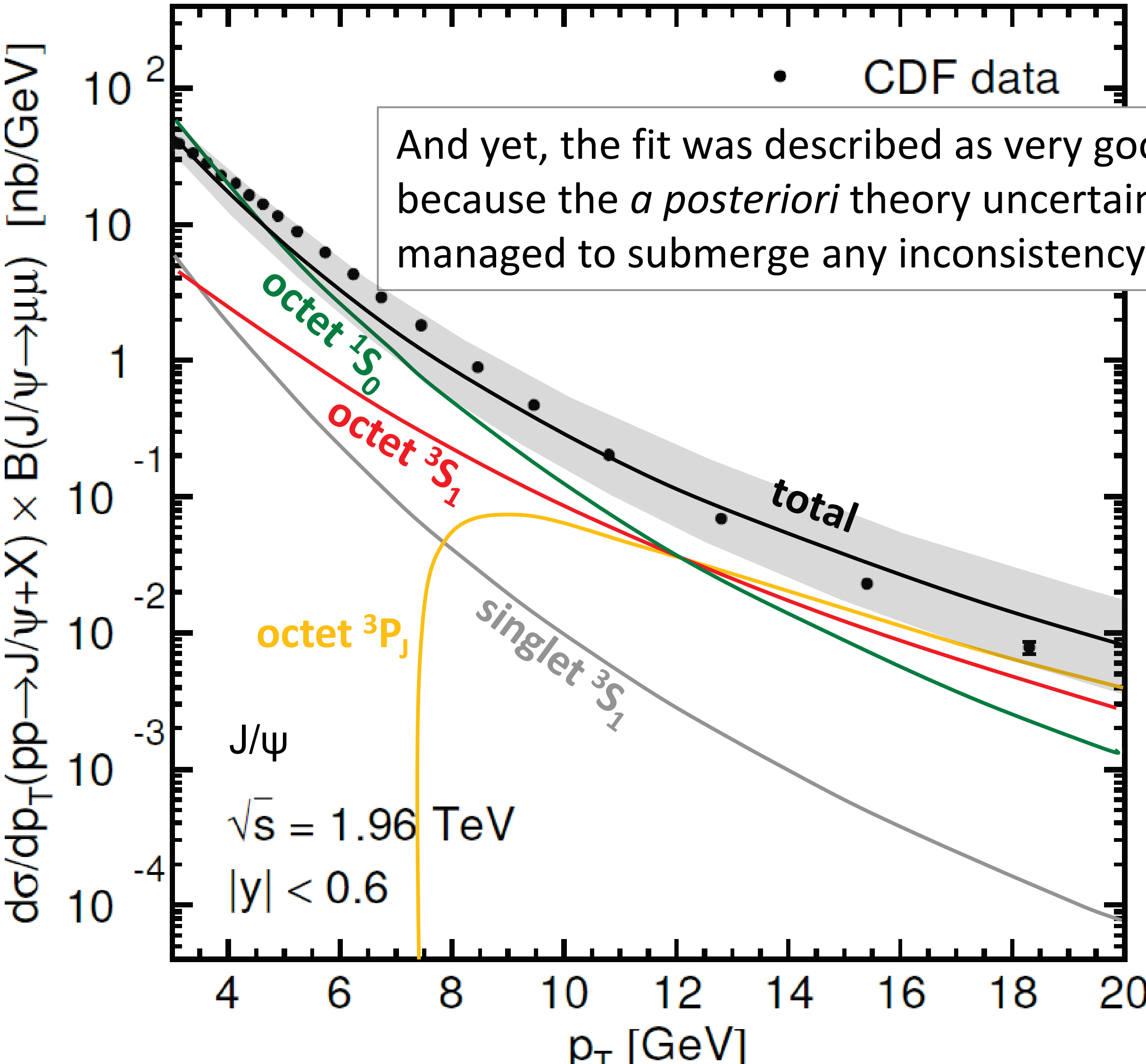

And yet, the fit was described as very good,
because the *a posteriori* theory uncertainty band (from scale variations)
managed to submerge any inconsistency

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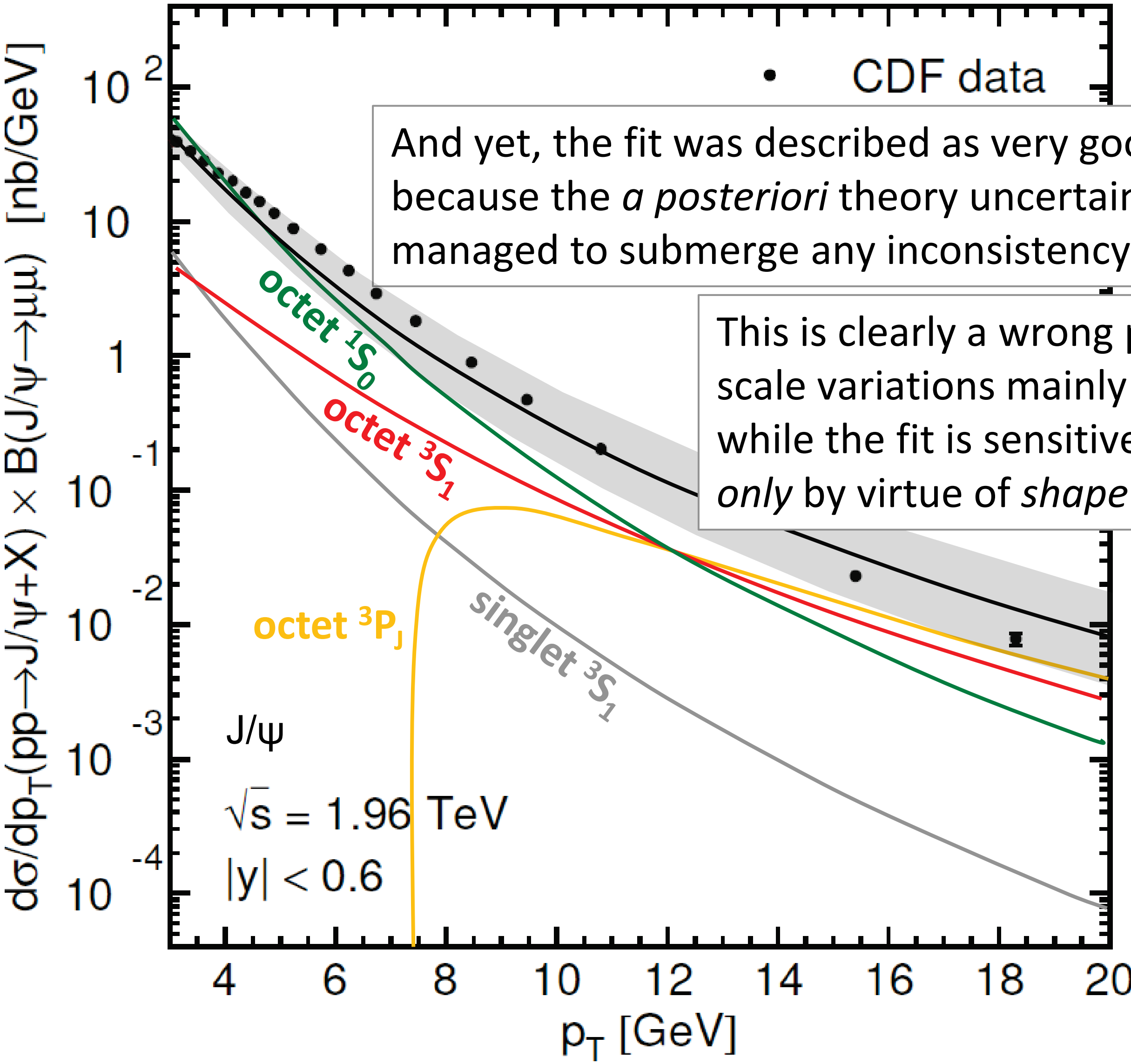

And yet, the fit was described as very good,
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This is clearly a wrong procedure, also because
scale variations mainly affect the global normalization,
while the fit is sensitive to the subprocess composition
only by virtue of *shape* differences

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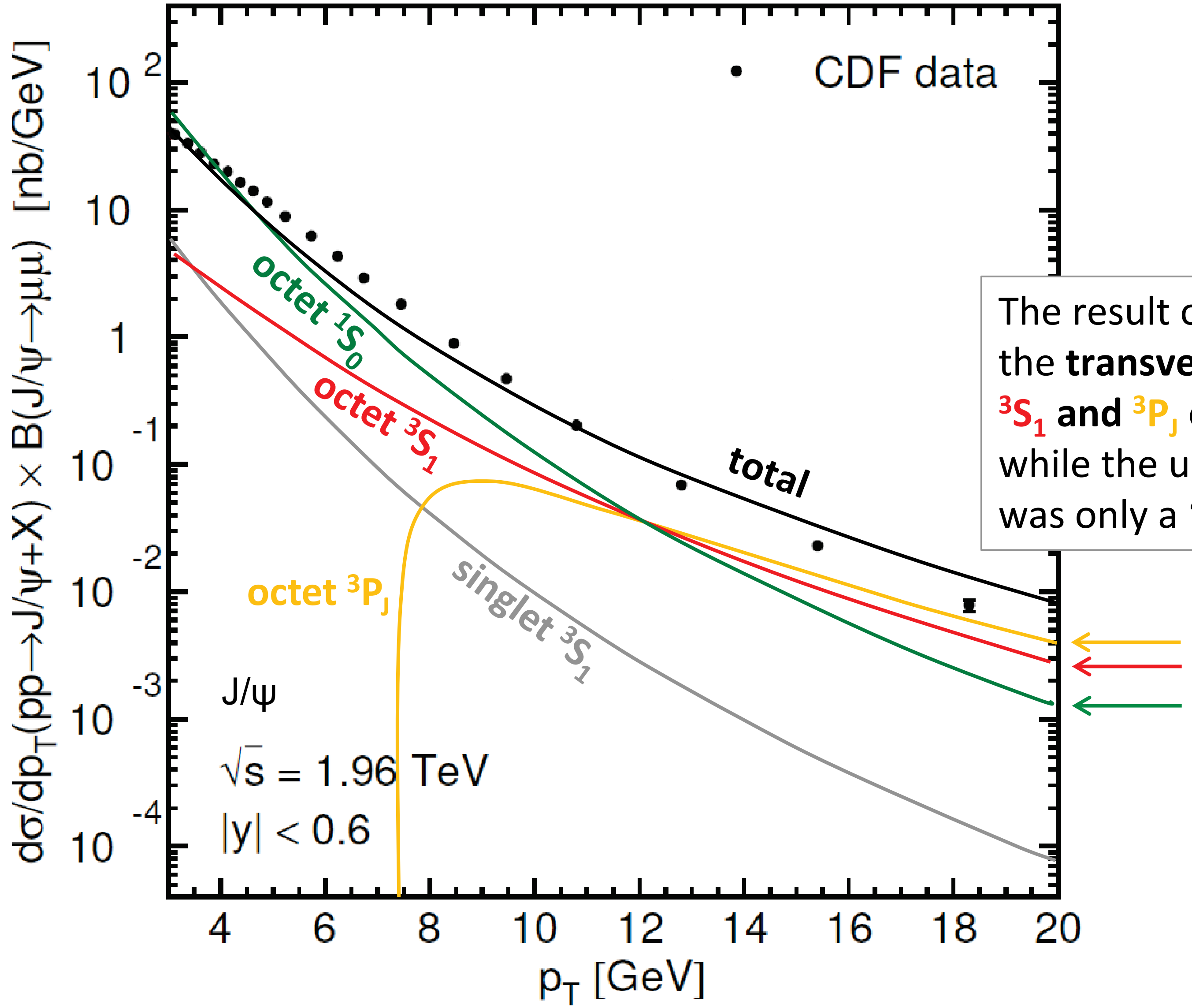
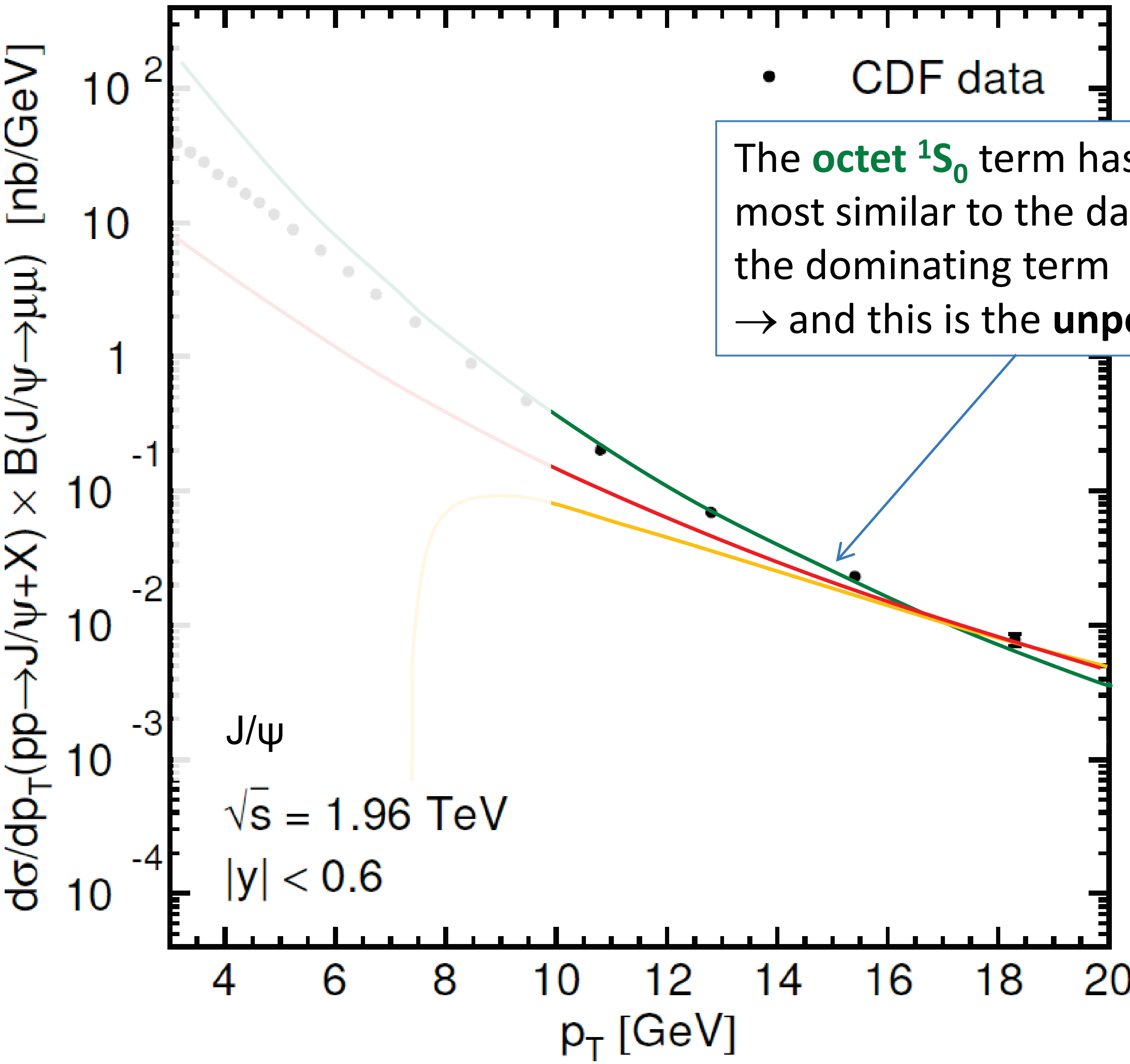


The result of this kind of fit was that the **transversely polarized** $^3S_1$ **and** ^3P_J **octet** terms **dominated**, while the unpolarized $^1S_0$ term was only a "correction"

A closer look at fits of the past

Let's look at the high-p_T behaviors, by normalizing the curves to data points for $p_T/M > 3$



The **octet** $^1S_0$ term has the *shape* most similar to the data and must therefore be the dominating term
→ and this is the **unpolarized** contribution!

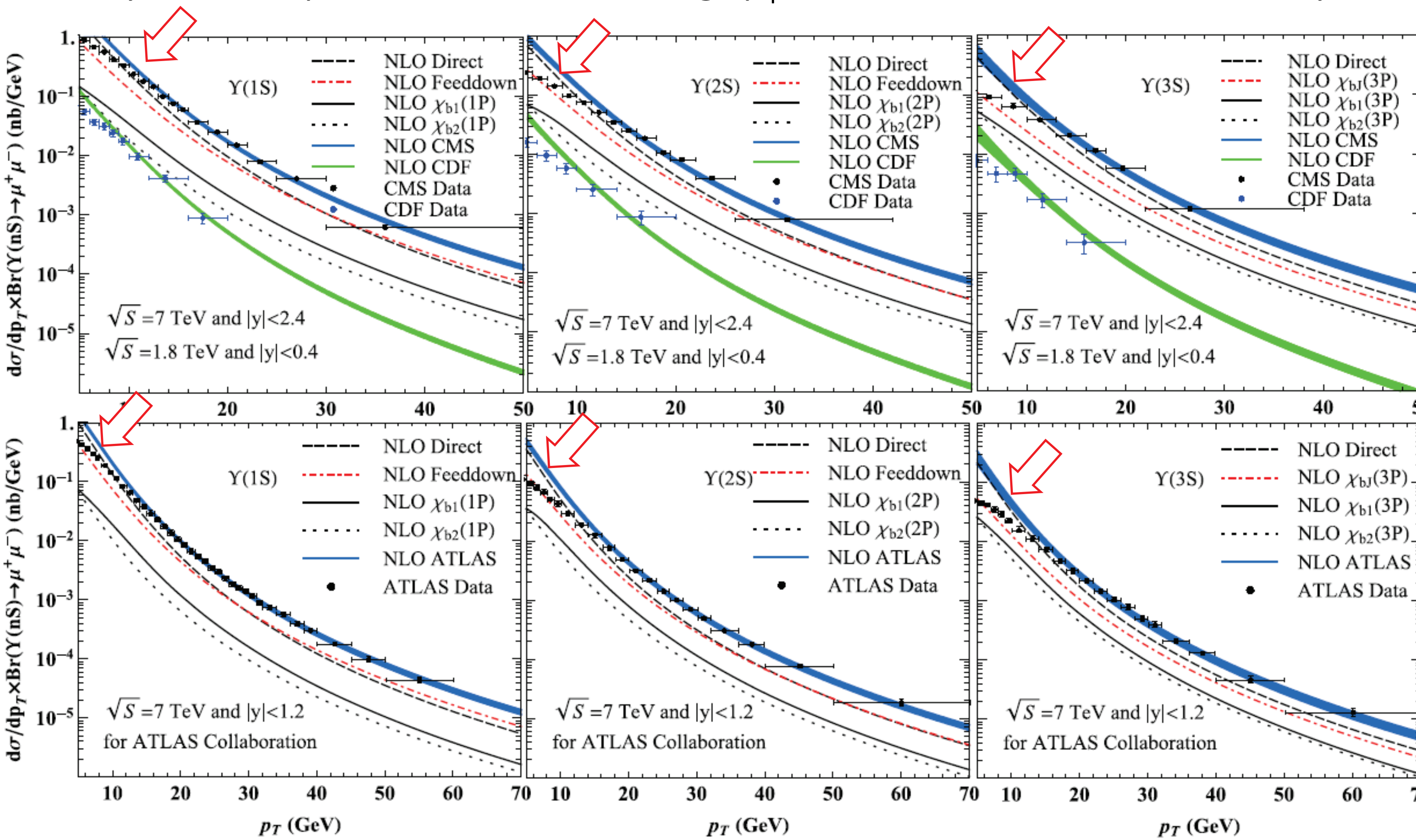
With a proper consideration of current theory limitations, the puzzle of the "unexplained" lack of polarization disappears

The long-neglected culprit

The fact that perturbative calculations for Tevatron and LHC struggle at low p_T has always been "under our eyes".

Most studies now recognize that NLO NRQCD cannot reproduce the curvature shown by data below $p_T \approx$ M.

Theory-data comparisons, restricted to high p_T, show the "no-turn-down" theory flaw



H. Han et al, Phys. Rev. D 94, 014028 (2016)

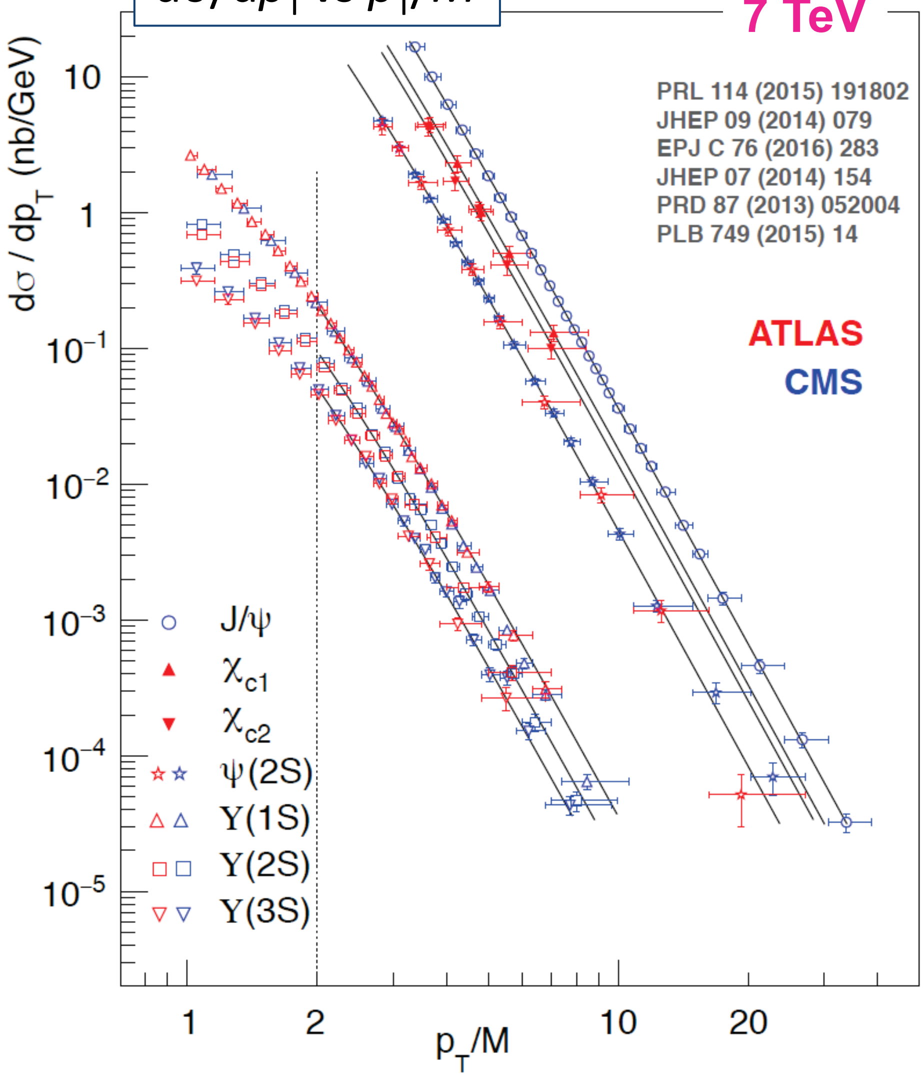
New insights

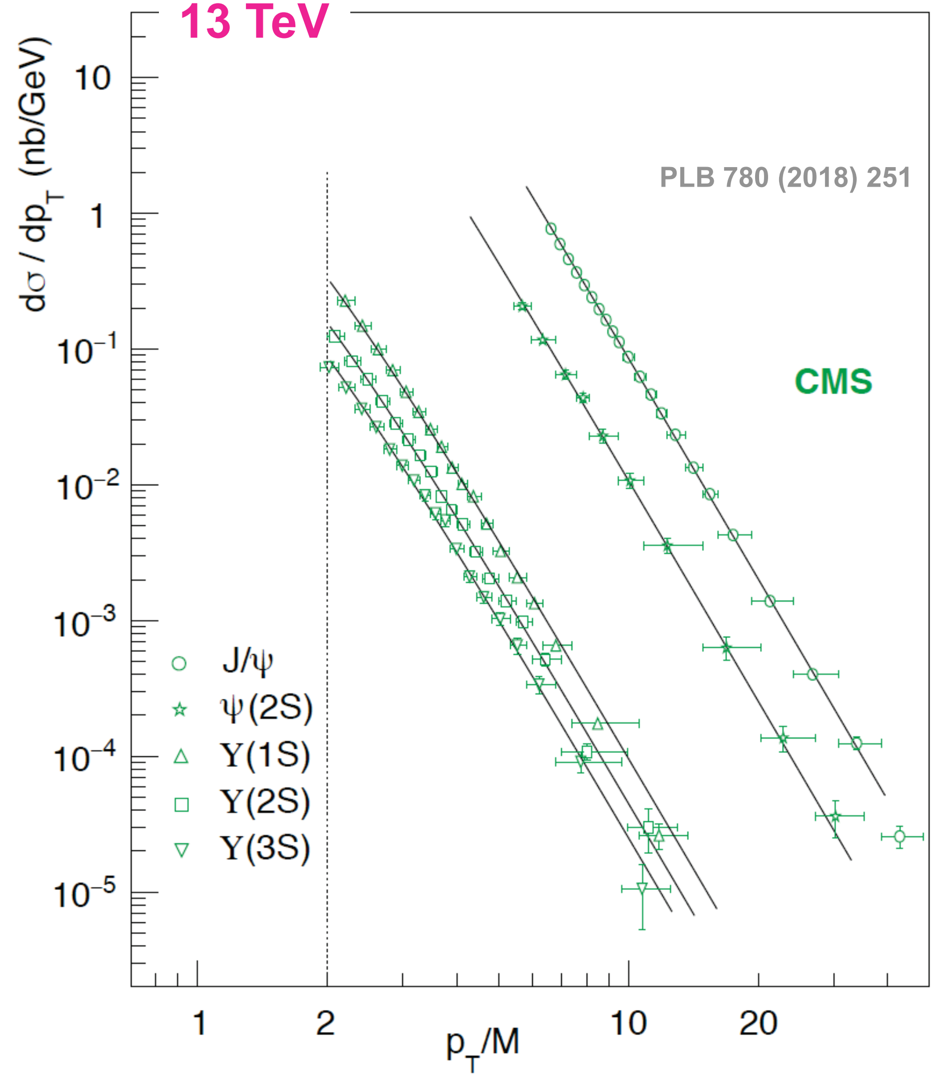
Unexpectedly simple data patterns

P. Faccioli, C. Lourenço, M. Araújo, V. Knünz, I. Krätschmer and J. Seixas,
Quarkonium production at the LHC: A data-driven analysis of remarkably simple experimental patterns,
Phys. Lett. B 773, 476 (2017)

Mid-rapidity cross section measurements show a *common shape pattern* for $p_T/M \gtrsim 2$, *independent of M and quantum numbers*

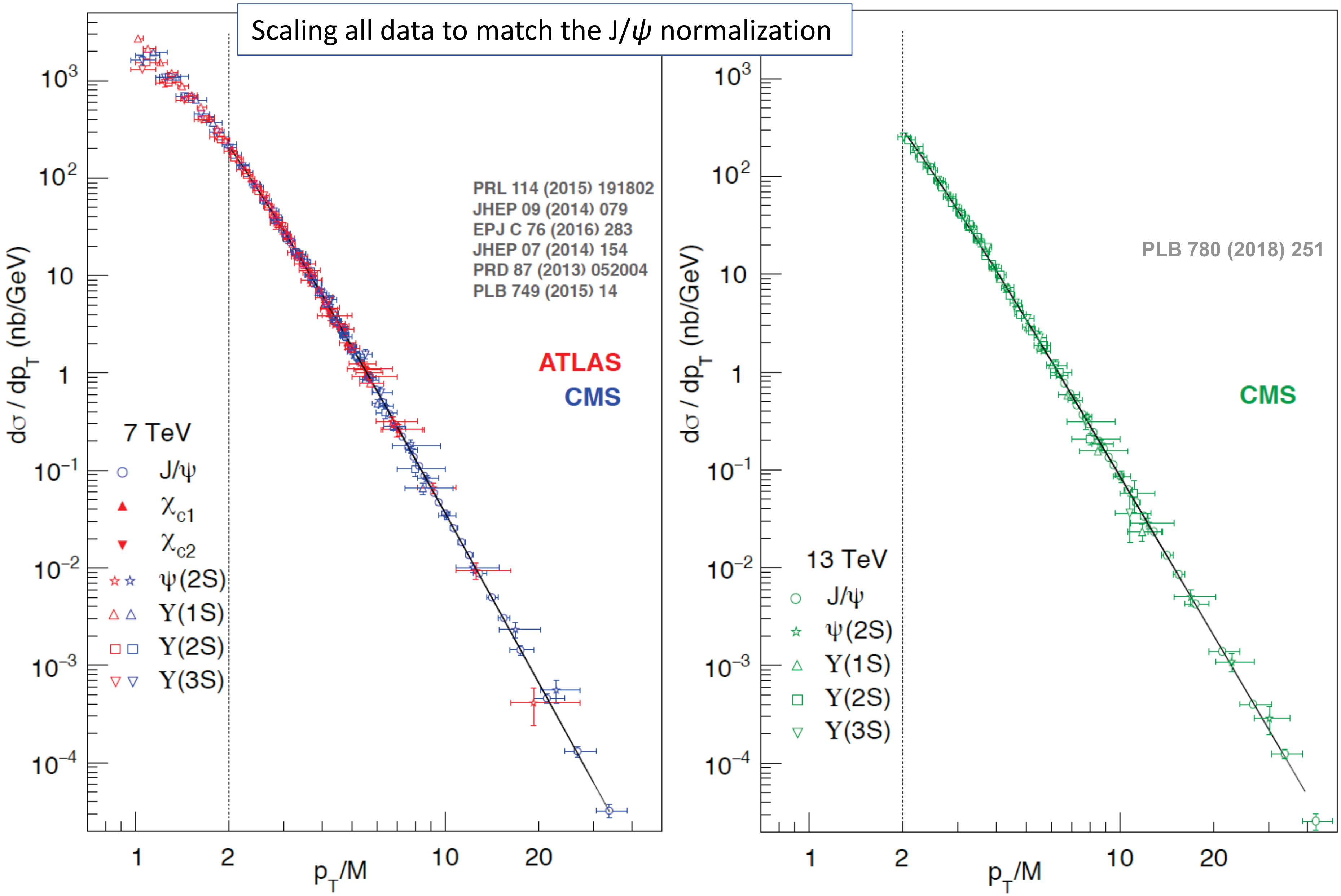
$d\sigma/dp_T$ vs p_T/M



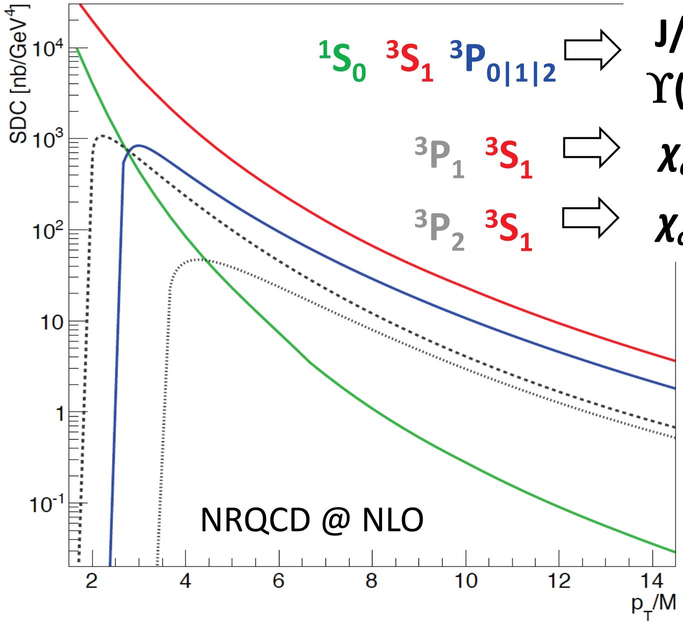


Unexpectedly simple data patterns

Scaling all data to match the J/ψ normalization



A "surprising" agreement with NRQCD

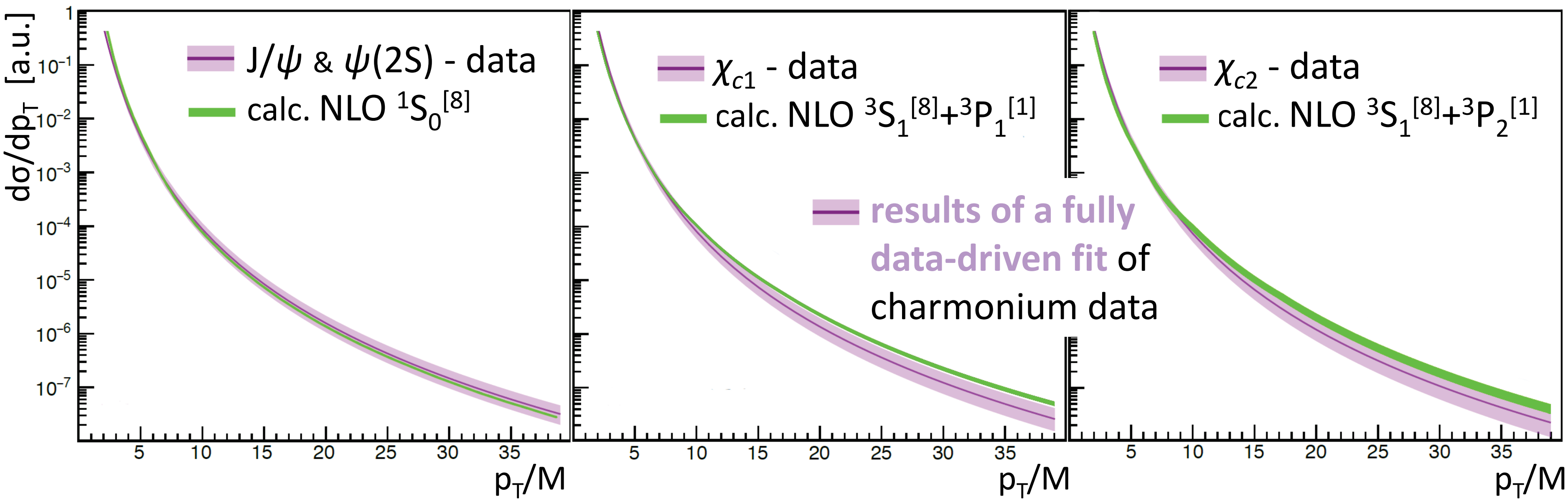


P. Faccioli, C. Lourenço, M. Araújo, V. Knünz, I. Krätschmer and J. Seixas, *From identical S- and P-wave p_T spectra to maximally distinct polarizations: probing NRQCD with χ_c states*, EPJ C 78 (2018) 268

The variety of kinematic behaviours in NRQCD seems **redundant** with respect to the observed "universal" p_T/M scaling and lack of polarization

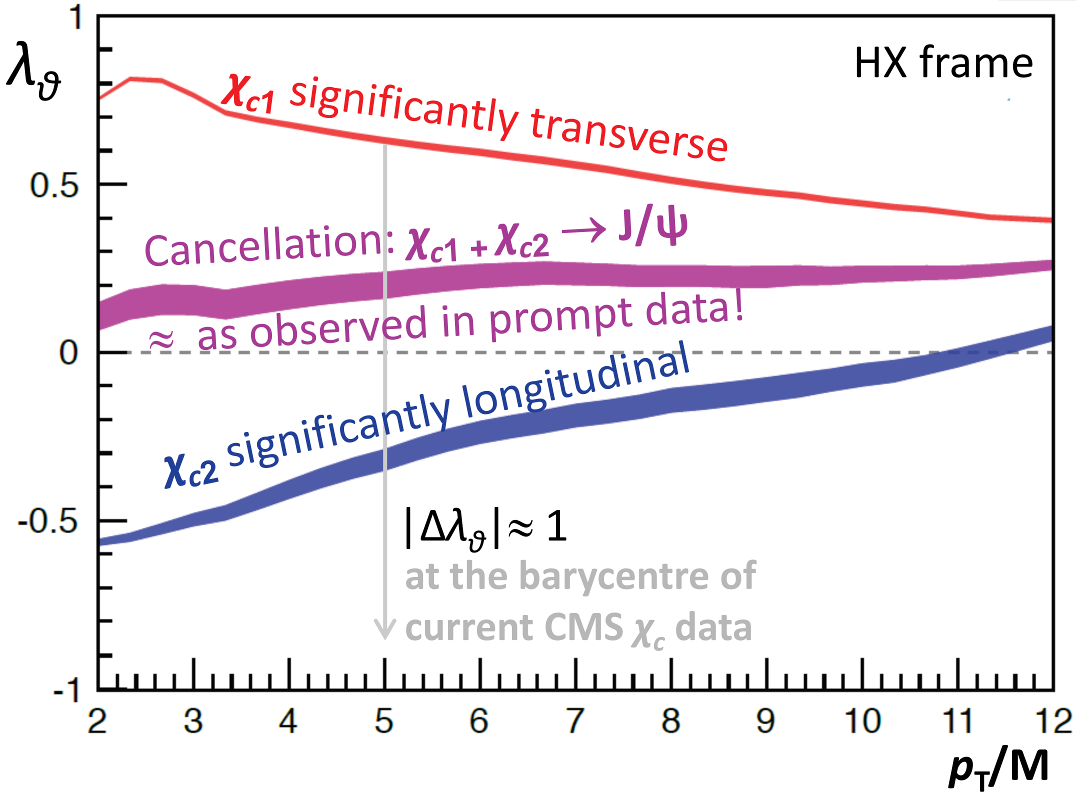
⇒ *cancellations* are needed to reproduce data….

…*and they actually happen*!



Ultimate conspiracy or need for a better NRQCD?

P. Faccioli, C. Lourenço, M. Araújo, V. Knünz, I. Krätschmer and J. Seixas, *From identical S- and P-wave p_T spectra to maximally distinct polarizations: probing NRQCD with χ_c states*, EPJ C 78 (2018) 268



The seeming success of NRQCD uncovers a strong prediction: the unmeasured χ_{c1} and χ_{c2} polarizations must be very different from one another

A potentially striking exception to the uniform picture of mid-rapidity quarkonium production!

χ_c polarization analysis ongoing in the CMS quarkonium group

Will we find ... a large $\chi_{c2} - \chi_{c1}$ polarization difference? $\Rightarrow$ smoking gun!

... weak χ_{c1} and χ_{c2} polarizations as for S-wave states?

$\Rightarrow$ need of improved (simpler?) NRQCD hierarchies or better perturbative calculations

Long-distance scaling: another universal pattern?

P. Faccioli, C. Lourenço, M. Araújo and J. Seixas,
Universal kinematic scaling
as a probe of factorized long-distance effects
in high-energy quarkonium production,
Eur. Phys. J. C 78, 118 (2018)

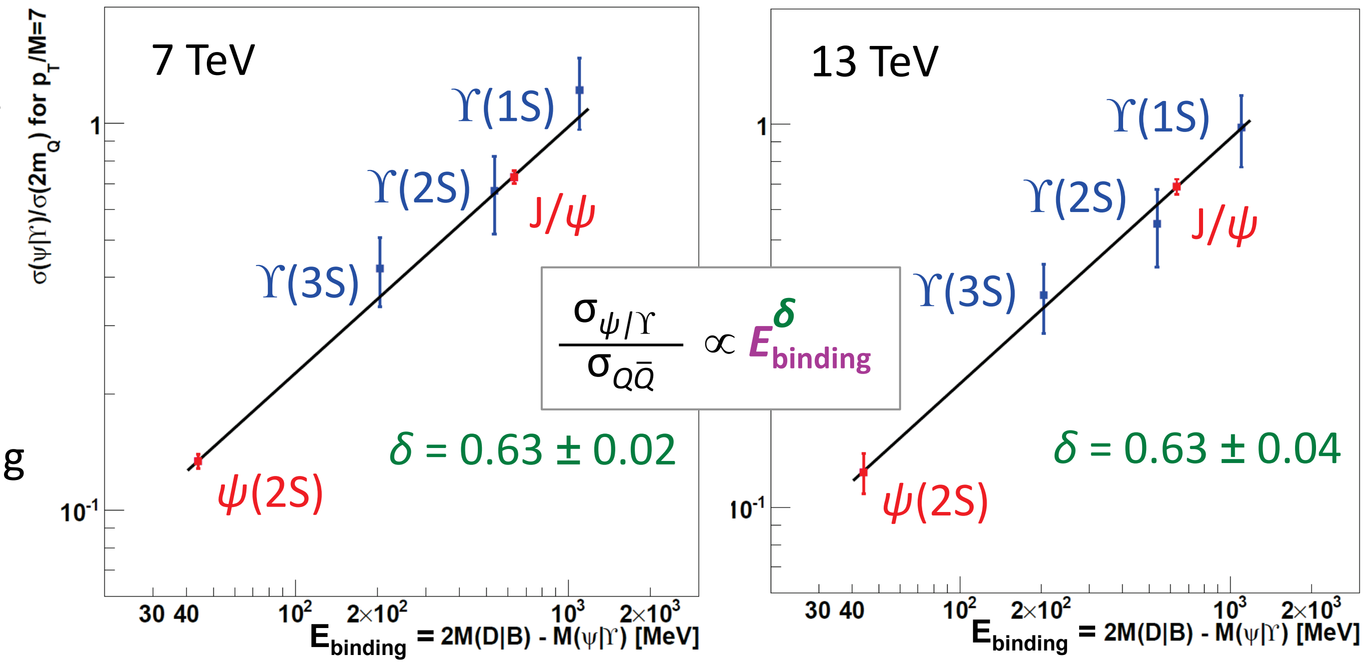
The QQbar→bound-state "transition probabilities" show a clear correlation with **binding energy**,
– common to charmonium and bottomonium,
– identical at 7 and 13 TeV:

Plotted **ratio**:

measured cross sections

$$\frac{\mathbf{d\sigma/d}p_{\mathbf{T}}\mathbf{(quarkonium)}}{\mathbf{d\sigma/d}p_{\mathbf{T}}[M=M(\mathbf{QQbar})]}$$

defined by extrapolating $d\sigma/dp_T(M)$ to $2m_Q = M_{\eta_c(1S)}$ or $M_{\eta_b(1S)}$



$$\frac{\sigma_{\psi/\Upsilon}}{\sigma_{Q\bar{Q}}} \propto E_{\mathrm{binding}}^{\delta}$$

→ an experimental confirmation of the "factorization" ansatz of NRQCD

LHC summary

Long-lasting experimental and theoretical polarization **puzzles have been solved**:
NRQCD can describe well the data.
Unfortunately, available finite-order perturbative calculations are not suited to the **p_T/M < 3 domain**

At the current level of experimental precision, mid-rapidity LHC pp data for charmonium and bottomonium are well described by a simple parametrization reflecting a **surprisingly universal (=*state-independent*) scaling** with two variables:

1. **shapes of the p_T distributions in pp collisions** → p_T/M short distance
2. **pp cross-section scaling with mass** → $E_{binding}$ long distance

This parametrization mirrors well the general idea of **factorization** of NRQCD.
The observed simplicity is in seeming contradiction with the variegated structure of subprocesses composing the NRQCD expansion.
Coincidental cancellations ultimately enable NRQCD to succeed in describing the data...
But does this point to the existence of a simpler (more natural) hierarchy of processes?

Next:
- more precise S-wave polarization measurements: how “fine tuned” are the cancellations?
- first P-wave polarization measurements: will they upset the “simple” picture?

Thoughts about quarkonium production at low p_T

The fundamental physics concepts of NRQCD and the factorized short×long-distance description of quarkonium production can very well be valid at any p_T and, in particular, for quarkonium production at COMPASS.

However, as the polarization puzzle taught us, **fixed-order calculations** of the short-distance QQbar production cross sections (SDCs) have a limited validity domain.
In particular, SDC calculations used for **LHC** analyses are **unreliable below $p_T/M \approx 3$**, where they do not even reproduce the natural turn-down of $d\sigma/dp_T$ towards zero for $p_T \to 0$.
This is actually not surprising: they *explicitly* target the case where the quarkonium p_T reflects the particle's recoil in a **2→2 process**, $i\, j \to \mathcal{Q} + X$.
Moreover, they neglect parton-k_T effects, small at high quarkonium momentum.

For low-p_T studies, the factorized NRQCD description, including colour octet and singlet processes, should be adopted as the most viable and general method currently known.
Specific *short-distance* calculations including **2→1 processes with parton-k_T effects** should be considered → S. Baranov et al. EPJ C 75, 455 (2015)
PRD 93, 094012 (2016)

In any case, as the LHC experience taught us, calculations of quarkonium production must be handled with care! Because of the lack of proper theoretical uncertainties, by blindly relying on calculations we risk to incur in new "puzzles"!
→ Always maintain a **data-driven approach**: look for patterns, differences and similarities (different states, different conditions) *in advance* of any "theory fit".

Thoughts about quarkonium production at low p_T

From high-p_T ***data*** we learned that, for ψ/Υ production,

a) colour-singlet production is negligible
b) lack of polarization $\rightarrow$ the dominating colour-octet channel is $^1S_0$
c) the production kinematics is (also thanks to b) surprisingly independent of final state
d) the observed mass scaling of cross sections strongly indicates that the bound-state-formation process *really* factorizes out of short-distance QQbar production

Without detailed ***measurements***, none of these very strong physical indications would have been reached!

We do not expect identical observations in low-p_T data.
For example, colour-singlet production *may* be important in the 2$\rightarrow$1 regime, leading to a less "universal" kinematic scenario.
Detailed ***measurements***, *not theory*, will tell us if this is the case!

Measurement of:
- p_T and x_F distributions
- polarizations
- for all states in the family: not only J/ψ, but also χ_{c1}, χ_{c2} (if not η_c and χ_{c0})

do not exist at low p_T and theory cannot replace them or integrate missing parts.
$\rightarrow$ an unmissable occasion to build from scratch our knowledge of an unexplored domain!

Thoughts about quarkonium production at low p_T

Cross section ratios and the search of ***mass/momentum scaling patterns***
provide physical indications with minimal use of theory assumptions.

Polarization remains the cleanest probe of the production mechanism.
It is determined by angular momentum conservation and other fundamental symmetries (parity, chirality...).
Just as energy conservation, these symmetries are not affected by perturbative “truncations”.
Polarization measurements are likely to give strong physical indications without any numerically calculated (and approximated) theory input.

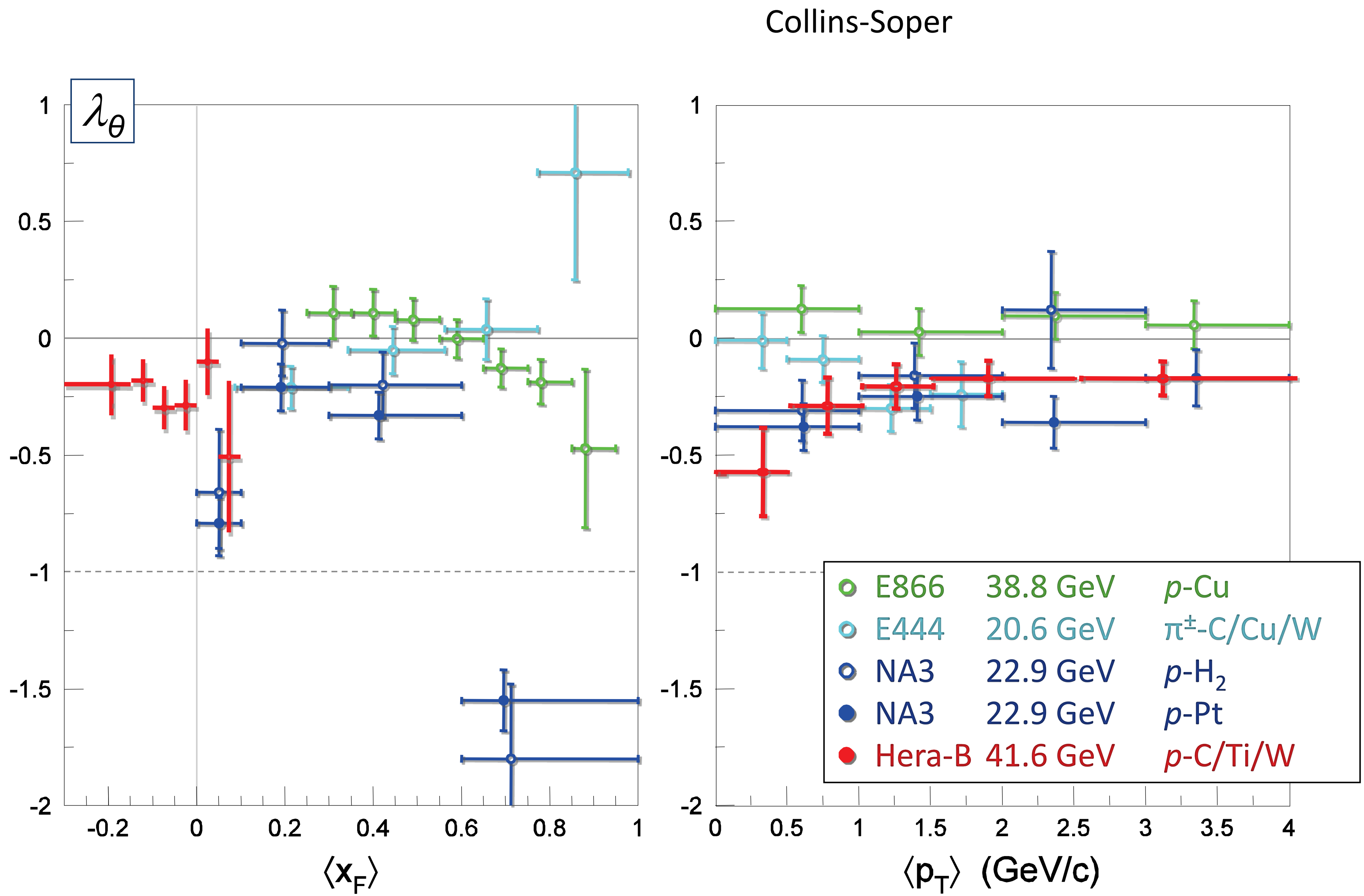
As an example of a possible “workflow” from measurement to understanding,
I will consider the existing low-p_T measurements from fixed-target experiments.

They form a rather complex picture...

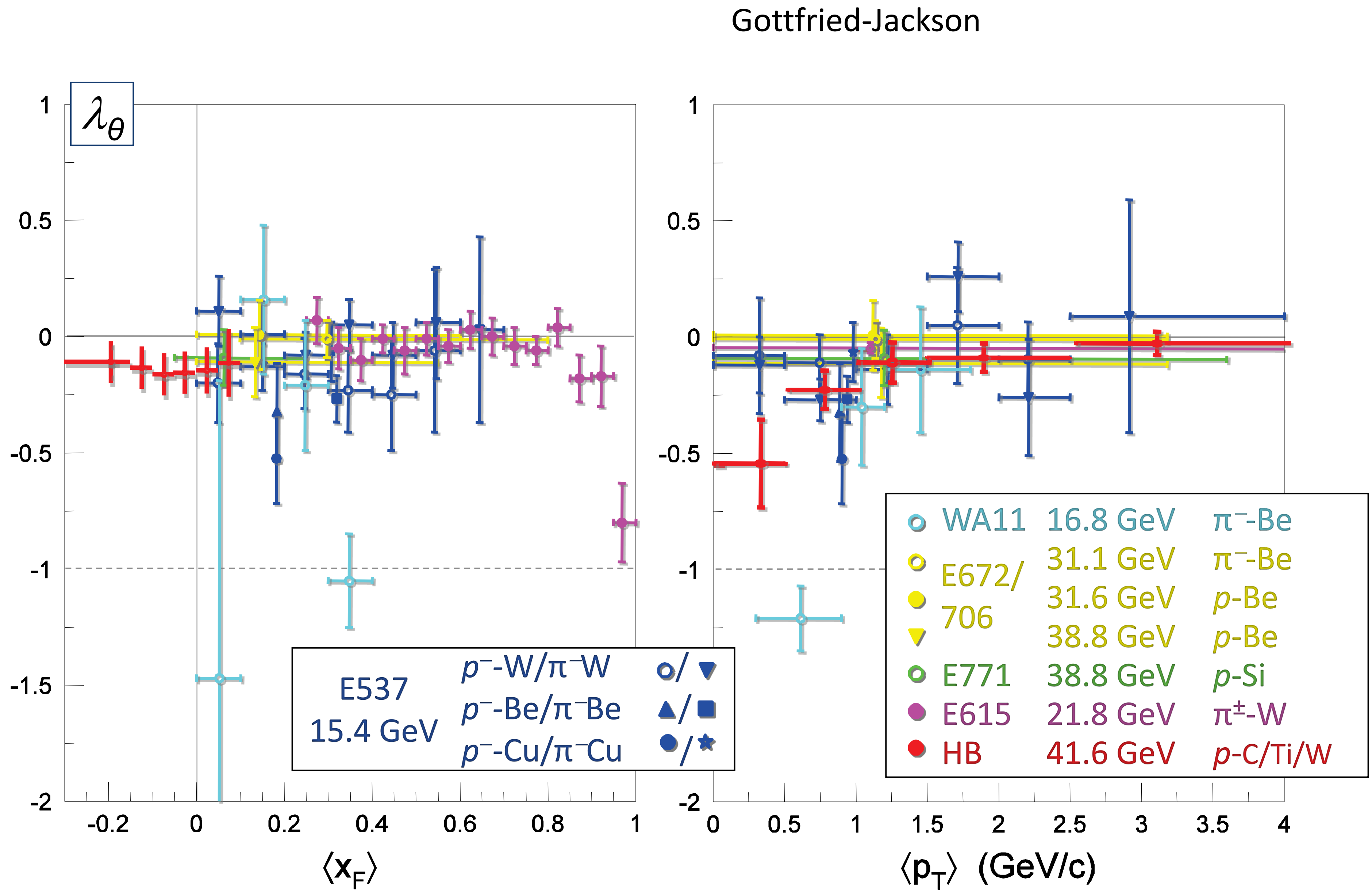
[see “Appendix” for illustrations of basic polarization concepts]

A low-p_T polarization puzzle?

J/ψ polarization in the CS frame

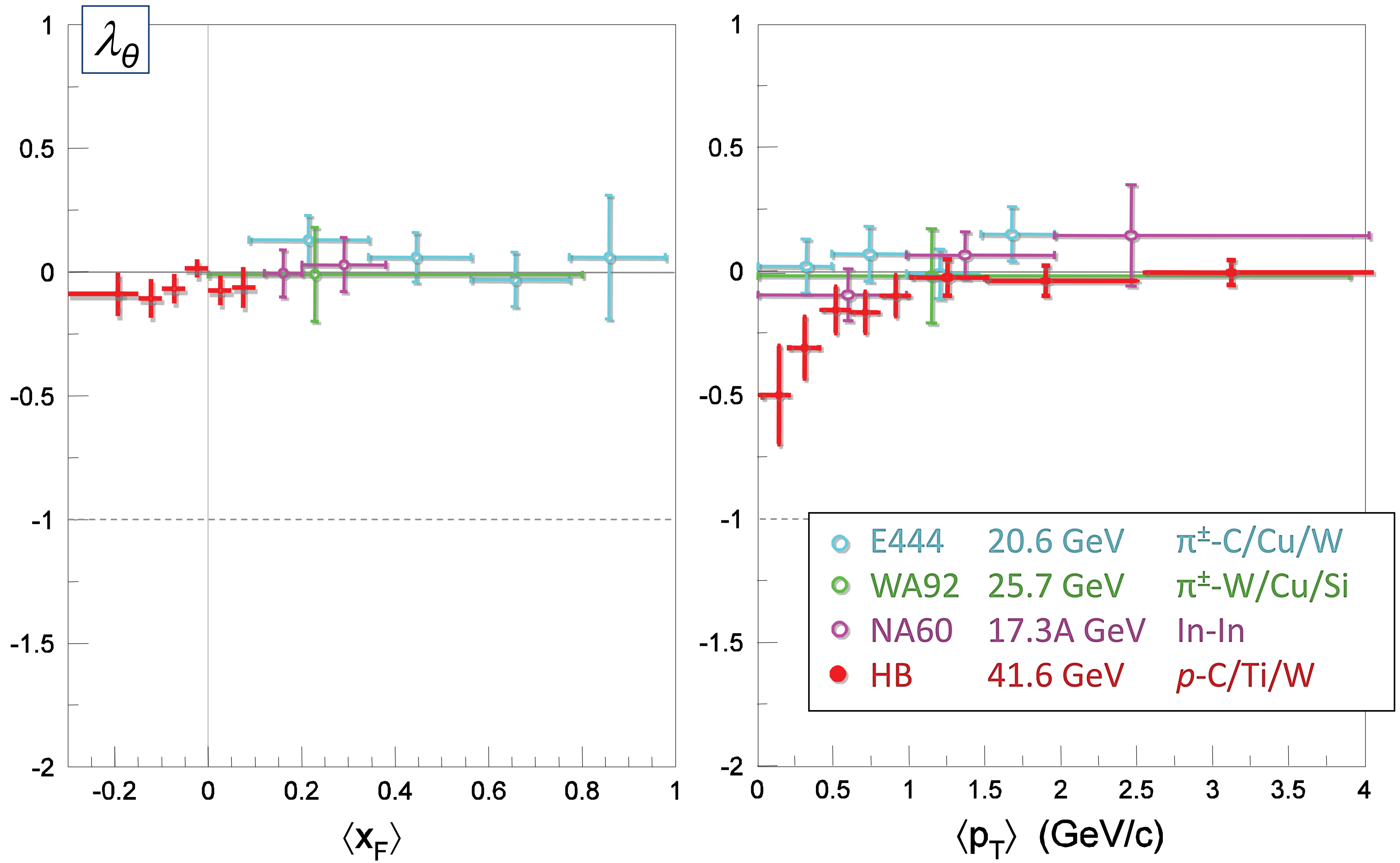


J/ψ polarization in the GJ frame

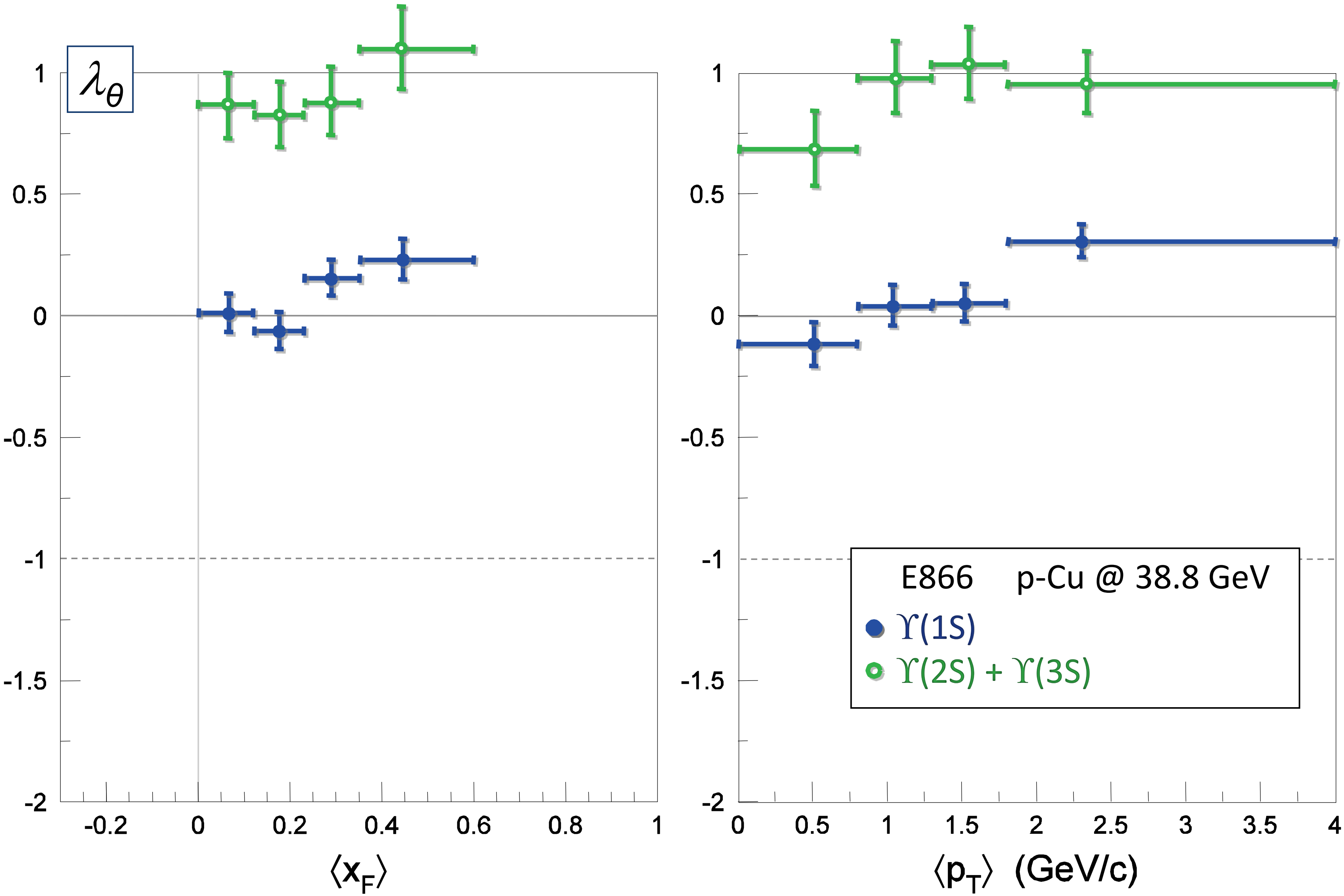


J/ψ polarization in the HX frame

centre-of-mass helicity



Υ Polarization in the CS frame



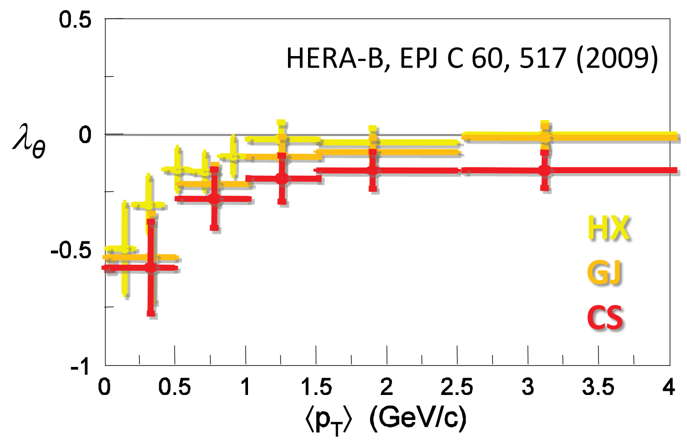
Indications and motivations from existing data

Picture to be observed "with a grain of salt":

- most of these measurements were obtained from 1D analyses (with risks discussed in [P. Faccioli, Mod. Phys. Lett. A Vol. 27, 1230022 (2012)])
- for some of them systematic uncertainties were never evaluated
- some of them exhibit suspicious fluctuations, even reaching unphysical values
- we are mixing different energies and target nuclei (nuclear effects may exist)

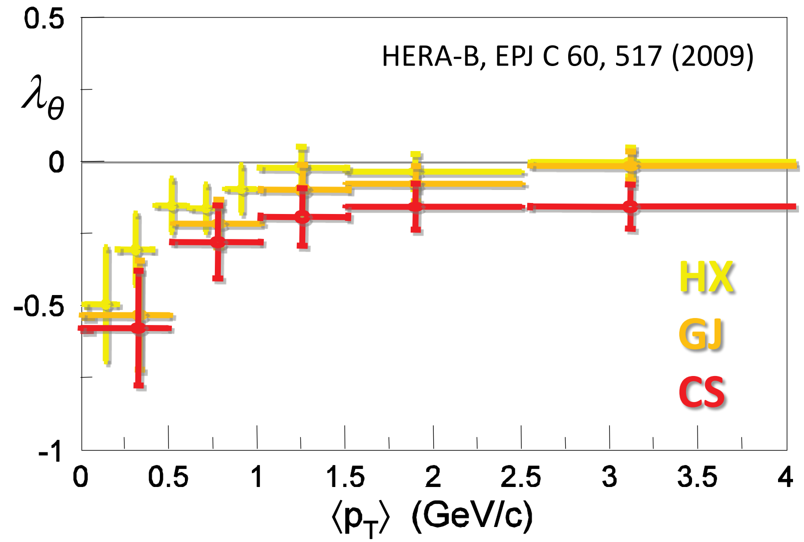
provide accurate (multidimensional) measurements!

1) The magnitude of the polarizations is systematically bigger in the CS frame and follows the **hierarchy CS → GJ → HX**



The CS > GJ > HX hierarchy and the "natural" frame

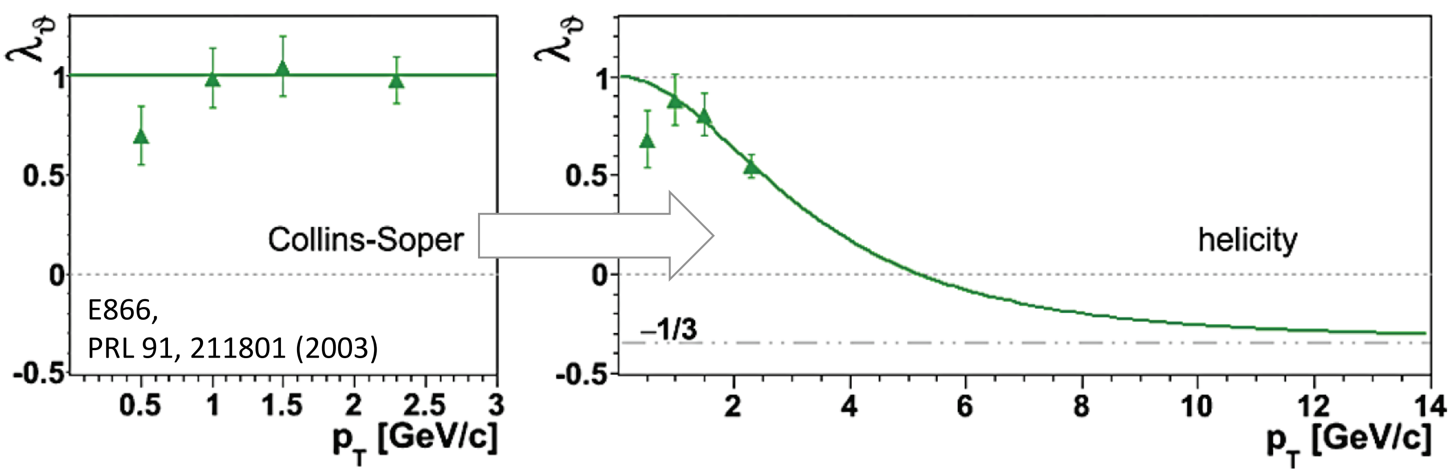
Hierarchy clearly seen in HERA-B measurement in the three frames (where uncertainties are ~100% correlated).



It reflects the geometrical difference between the three frames:
the GJ polarization axis has always an intermediate direction between CS and HX.

The CS axis shows a larger polarization effect
⇒ It more naturally reflects the alignment of the J/ψ angular momentum.

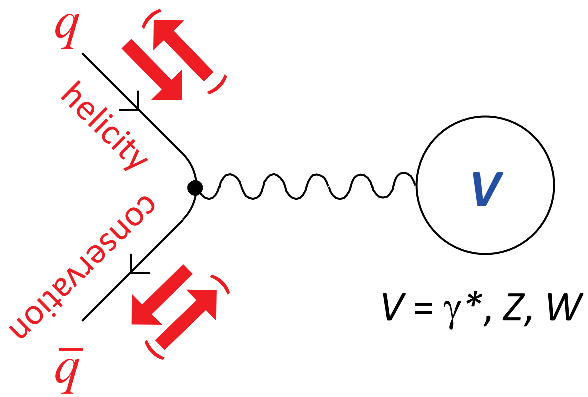
Moreover, the almost maximal Υ(2S)+ Υ(3S) polarization seen by E866 would appear "smeared" in the HX frame:



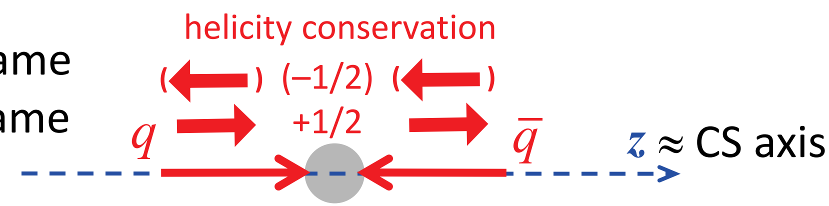
Physical meaning of "CS = natural frame"

As illustrated in the discussion of individual processes contributing to Drell-Yan production (→ Appendix), 2 → 2 processes (t-channel or s-channel) naturally lead to polarizations along the GJ and HX axes.

At the lowest order, DY production is a **2 → 1 process**, as such characterized by a "natural" polarization along the direction of the collision, approximated by the CS axis



q-$\bar{q}$ rest frame
= V rest frame



$\Rightarrow |V\rangle = |1, +1\rangle$ or $|1, -1\rangle$

$|1, 0\rangle$ forbidden

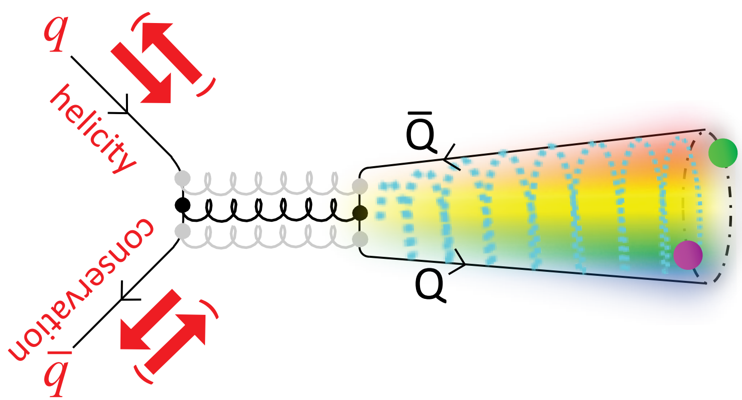
⇒ transverse polarization

The produced object ***inherits by direct angular momentum conservation*** **the polarization state of the system of the colliding partons,** which is polarized **along the direction of the collision**.

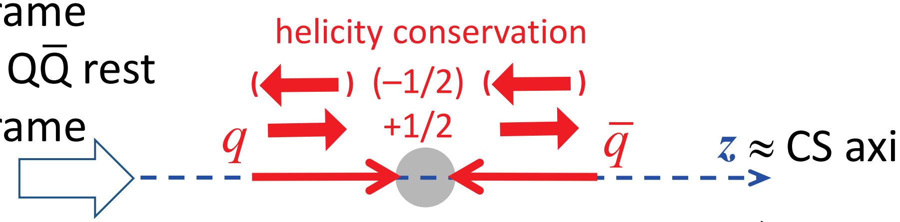
In 2 → 2 processes, instead, the polarization legacy of the partons is *shared* between the two final objects. The angular momentum balance is more complex and depends on the coupling of the final states to the intermediate virtual particles

Quarkonium polarization in 2 → 1 *q*-*q*bar scattering

Fully analogous to the DY case, replacing the virtual photon with **one** or more gluons:



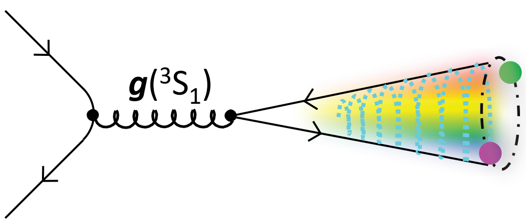
q-$\bar{q}$ rest frame = $Q\bar{Q}$ rest frame



$\Rightarrow |Q\bar{Q}\rangle$ = | 1, **+1** ⟩ or | 1, **−1** ⟩, ~~| 1, 0 ⟩~~

⇒ **transverse polarization**

With only **one gluon**, a ***coloured* QQbar pair in $^3S_1$ state** is produced:



This transversely polarized state transforms to ψ/ϒ by emitting *two* low-energy gluons (to neutralize colour *and* maintain the $^3S_1$ quantum numbers), a process analogous to the ψ(2S) → J/ψ $\pi\pi$ decay.

As indicated by BES data on (e^+e^- →) ψ(2S) → J/ψ $\pi^+\pi^-$ [PRD 62, 032002 (2000)], in such a transition *the c-cbar system maintains its polarization*

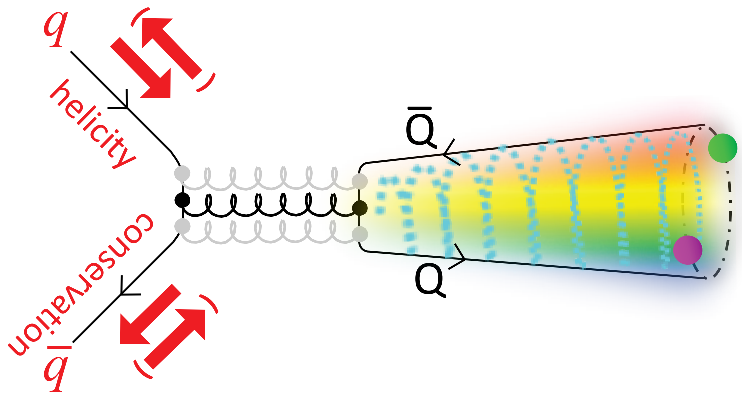
⇒ the $\pi\pi$ system (= the emitted *gg*) has J=0: it does not "carry away" angular momentum

⇒ directly produced ψ/ϒ inherits the **fully transverse polarization** of the $^3S_1$ QQbar

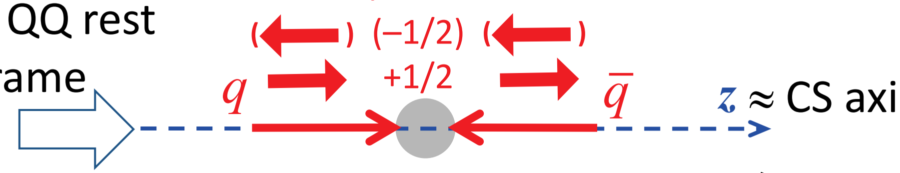
- *Different* ***octet*** *states* (e.g. $^1S_0$, ^3P_J) require ***two*** *gluons* → relatively suppressed

Quarkonium polarization in 2 → 1 *q*-*q*bar scattering

Fully analogous to the DY case, replacing the virtual photon with **one** or more gluons:



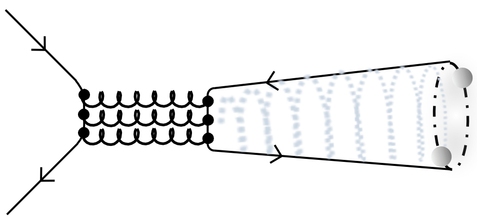
q-$\bar{q}$ rest frame = $Q\bar{Q}$ rest frame



$\Rightarrow |Q\bar{Q}\rangle = |1, +1\rangle$ or $|1, -1\rangle$, ~~$|1, 0\rangle$~~

$\Rightarrow$ **transverse polarization**

Colour singlet ψ/Υ production requires ***3 gluons***
for colour neutralization *and* correct quantum numbers (2 gluons could produce χ states)



The ψ/Υ is produced as an already observable colour-neutral state, without further gluon emissions, and remains, therefore, transverse

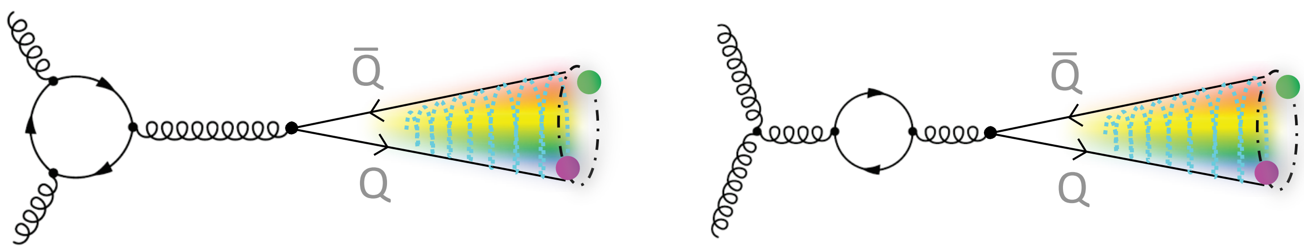
In summary, the ψ/Υ produced directly in 2 → 1 *q*-*q*bar scattering, via the colour octet or colour singlet mechanisms, should be transversely polarized

Quarkonium polarization in 2 → 1 *g-g* scattering

Until recently it was believed that 2 → 1 *g-g* quarkonium production is one of the cases forbidden by the Landau-Yang theorem preventing (J=1) → $\gamma\gamma$ decays and, therefore, $\gamma\gamma$ → (J=1) production (if the initial photons are on shell).

This is true for colour-singlet production: with colours “switched off”, *gg* → (J=1, singlet) is fully analogous to the LY-forbidden $\gamma\gamma$ → (J=1)

However, in ***gg* → (J=1, octet)** the additional colour degrees of freedom are not contemplated in the symmetry properties used by the LY theorem.
It has been recently shown that **2→1 production** is indeed possible in QCD *at NLO* [W. Beenakker et al., arXiv:1508.07115], in particular through processes like these:

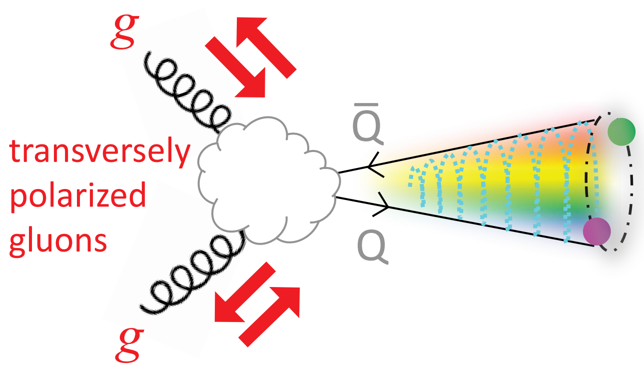


Here the final state comes from a gluon and is therefore a **$^3S_1$ octet**
These are **NLO** processes, while 2→1 q-qbar production happens at LO
⇒ 2→1 *gg* production may have some penalty

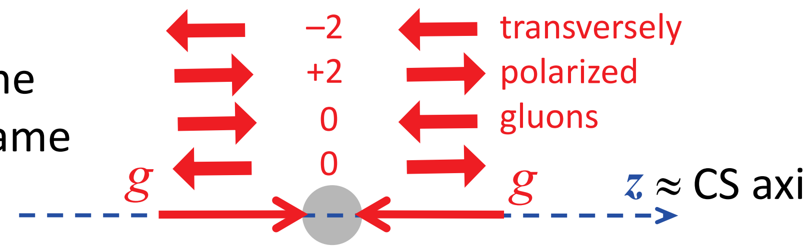
[Even before this result, the k_T-factorization method (S. Baranov et al.) considered *off-shell* scattering gluons, automatically immune to the LY theorem]

Quarkonium polarization in 2 → 1 *g-g* scattering

In 2 → 1 *g-g* scattering quarkonium is produced via the colour-octet mechanism. The transverse nature of the colliding gluons allows only for a subset of the possible angular momentum states of the produced colour-octet state:



g-g rest frame
= $Q\bar{Q}$ rest frame



$\Rightarrow |Q\bar{Q}\rangle = |\,2, \pm 2\,\rangle$ or $|\,J, 0\,\rangle$ (J=0,1,2)

$|\,J, \pm 1\,\rangle$ **forbidden**

$\Rightarrow$ polarized state if J > 0

The polarization of the observed quarkonium will depend on which octet state is produced:

$\mathbf{^1S_0}$, dominating high-p_T 2→2 production at the LHC, is an isotropic state producing *exactly **unpolarized*** (directly produced) J/ψ

$\mathbf{^3S_1}$ with $\mathbf{J_z = 0}$ means **fully longitudinal** (directly produced) J/ψ: a good starting point for describing HERA-B and NA3 data

Indications and motivations from existing data

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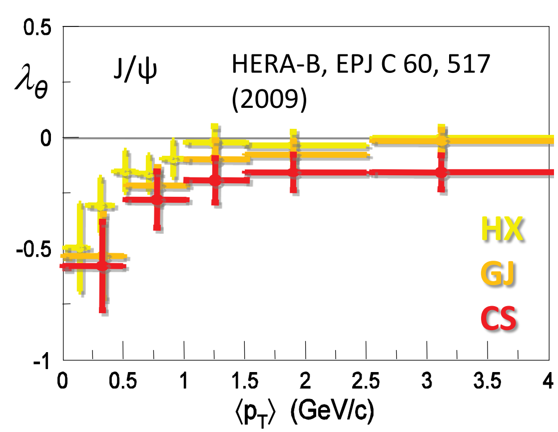
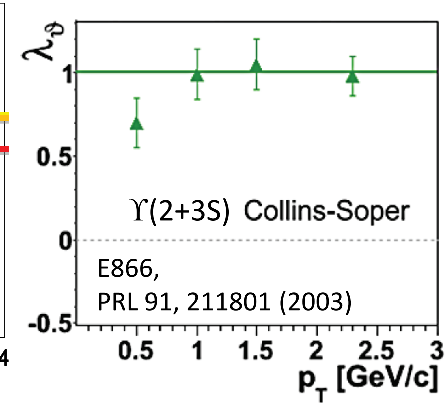
provide accurate (multidimensional) measurements!

1) The magnitude of the polarizations is systematically bigger in the CS frame and follows the **hierarchy CS → GJ → HX**
⇒ probable dominance of **2→1 *q*-*q*bar and *g*-*g* scattering processes**, where the produced state is strongly polarized, directly inheriting the angular momentum state of the system of the colliding partons → we see the **partons' natural polarizations**

always measure in more than one frame!

2) The J/ψ polarization magnitude, but not the ϒ one, seems to decrease quickly with increasing p_T:
does this indicate that in HERA-B, but not in E866, we start seeing 2 → 2 processes "smearing" the polarization?
Rather, what about the parton k_T?





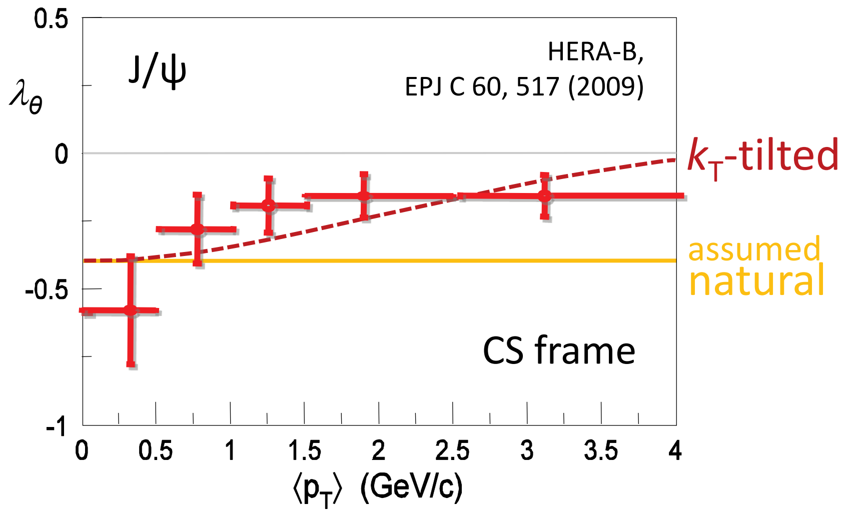
Effect of the intrinsic k_T in 2 → 1 processes

The intrinsic transverse momenta of the partons cause an *event-by-event tilt* between the "natural" polarization axis (relative direction of the colliding partons), and the polarization axis used in the experimental analysis (CS).

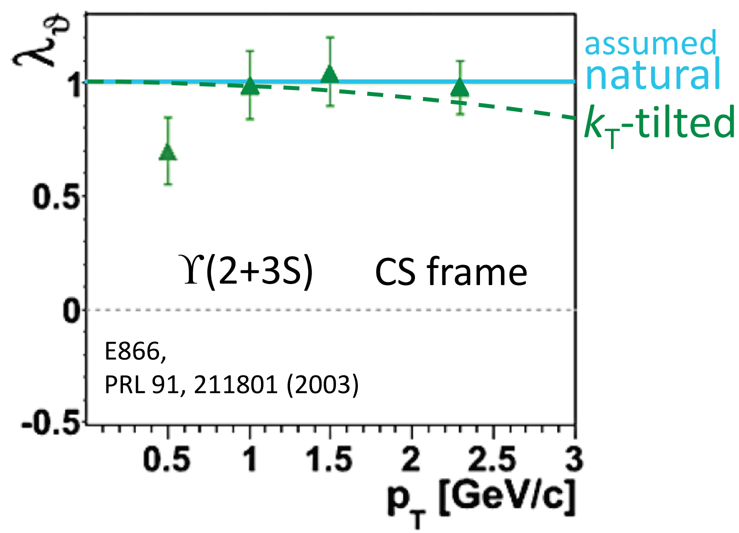
The tilt angle δ satisfies $$\sin^2\delta \approx \frac{2k_T^2}{m_T^2} \approx \frac{p_T^2}{M^2 + p_T^2}$$ [P. Faccioli et al., Eur. Phys. J C 69, 657 (2010)]

and the relation between observed and natural (*) polarizations is $$\lambda_\vartheta = \frac{\lambda_\vartheta^* - \frac{3}{2}\lambda_\vartheta^*\sin^2\delta}{1 + \frac{1}{2}\lambda_\vartheta^*\sin^2\delta}$$

This description approximately accounts for the p_T dependence observed for the $c\bar{c}$...



...and for the lack of a corresponding observation in the $b\bar{b}$ case



The φ dimension

The tilt tends to create an azimuthal anisotropy, but not in a straightforward way as in usual frame transformations (see Appendix).
We can define two extreme cases corresponding to two subcategories of events:

1) events for which δ represents a rotation *in the production plane*:

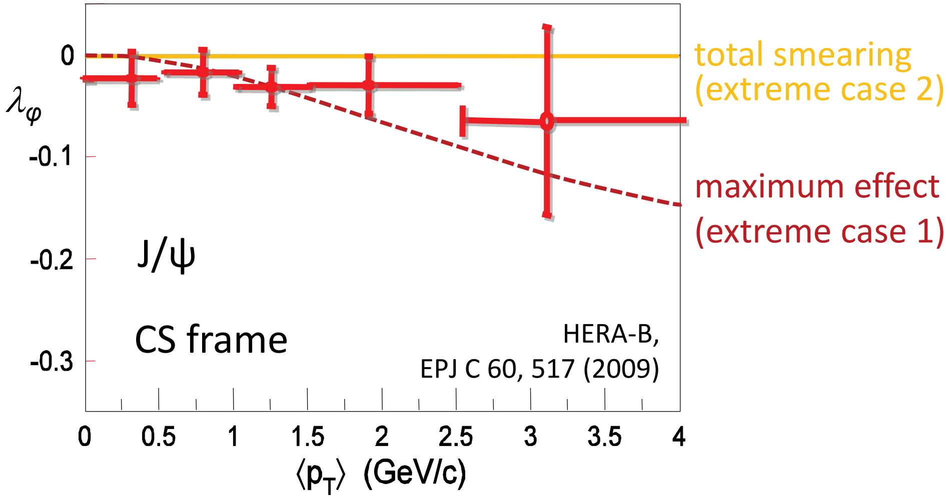
$$\Rightarrow \; \lambda_\varphi = \frac{\frac{1}{2}\lambda^*_\vartheta \sin^2\delta}{1+\frac{1}{2}\lambda^*_\vartheta \sin^2\delta}$$

as in a usual transformation between two frames, e.g. CS $\rightarrow$ HX
These events *preserve* the invariant $\tilde{\lambda}$ (while λ_ϑ is reduced)

2) events for which δ represents a rotation *in the plane $\perp$ to the production plane*:

$\Rightarrow \; \lambda_\varphi = 0$ Because the azimuthal anisotropy is created *with respect to* the production plane and is therefore unobservable (smeared in the sum over all events)

As a test of the hypothesis of a natural constant polarization "tilted" by a k_T effect, we should therefore observe a λ_φ *included between the two cases above,* as indeed we see:



A positive or larger negative λ_φ would tell us that the observed p_T dependence is *not* due to k_T smearing (only)

At the same time, we should observe that the invariant $\tilde{\lambda}$ is *less* affected by the k_T smearing than λ_ϑ *and* does not show an opposite "antismearing" effect

Indications and motivations from existing data

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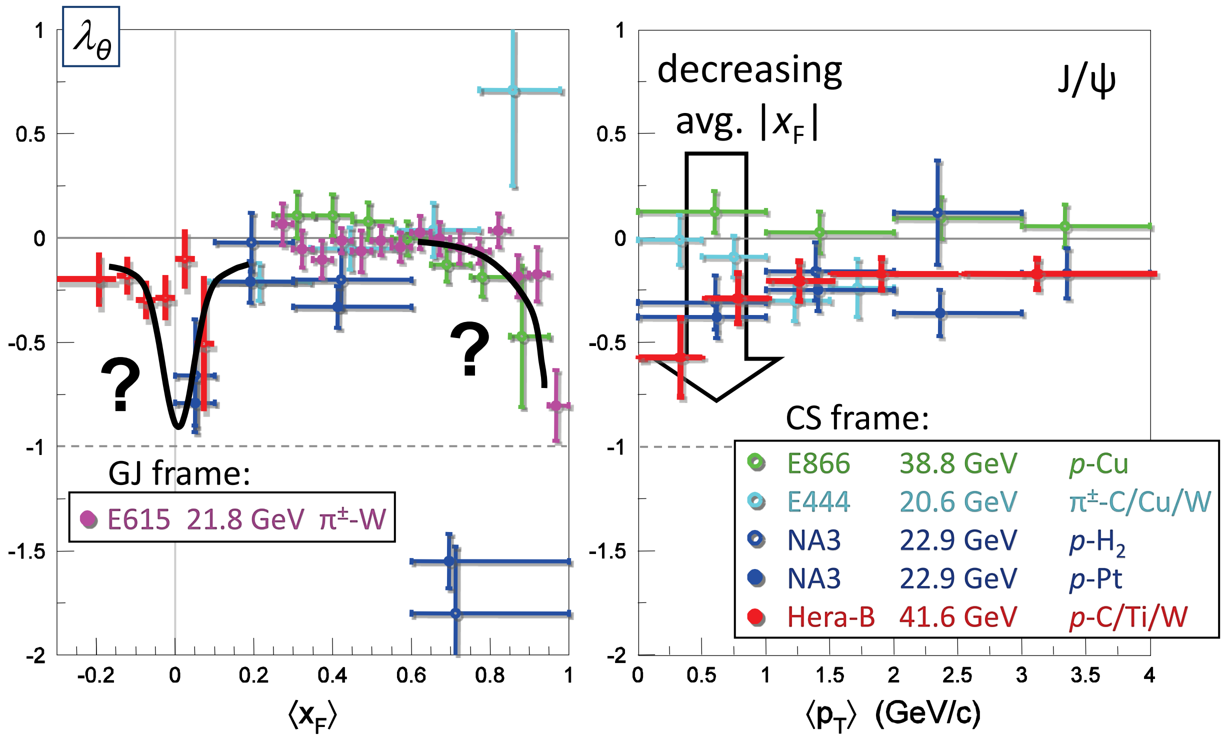
always measure in more than one frame!

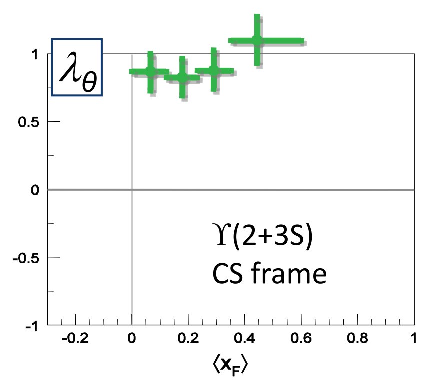
2) The decrease in longitudinal J/ψ polarization with increasing p_T, as observed by HERA-B, is consistent with the **tilt effect** induced by the **parton k_T** on the angular distribution (the effect is negligible for the Υ, as found by E866). The $p_T \to 0$ limit gives, therefore, the most interesting (untilted) polarization measurement. Moreover, the invariant $\tilde{\lambda}$ is less sensitive to the tilt effect than λ_ϑ

measure down to the smallest possible p_T!
measure the invariant polarization!

Indications and motivations from existing data

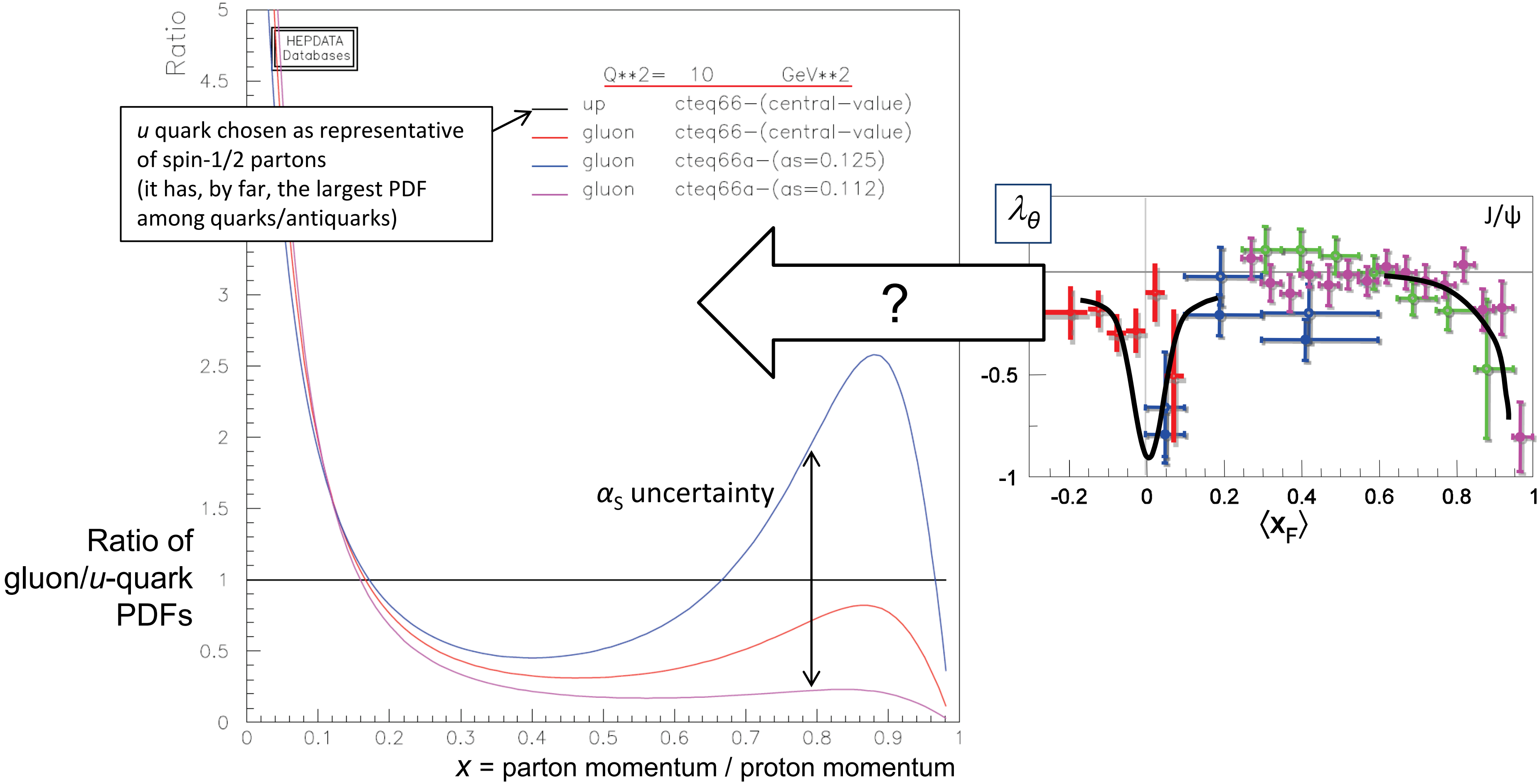
3) We seem to recognize possible **trends** towards **longitudinal** polarization for J/ψ data points approaching $\boldsymbol{x_F = 0}$ and $\boldsymbol{x_F = 1}$...





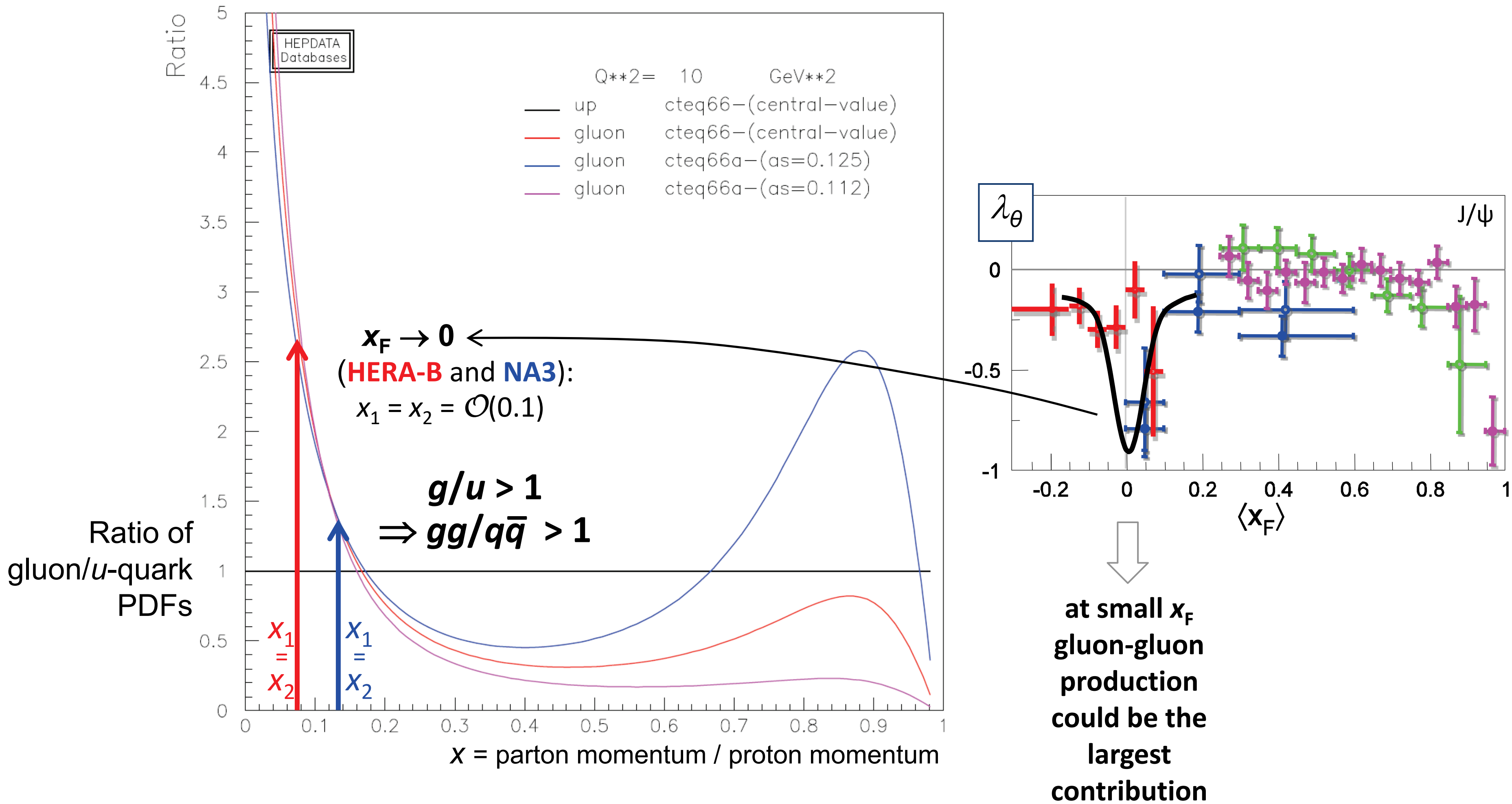
...but ϒ data seem to belong to another story...

Map of data in the parton/proton *x* space



$$x_1 - x_2 = x_{\mathrm{F}}$$
$$x_1 \cdot x_2 = (M_{Q\bar{Q}}/\sqrt{s})^2$$

Map of data in the parton/proton x space

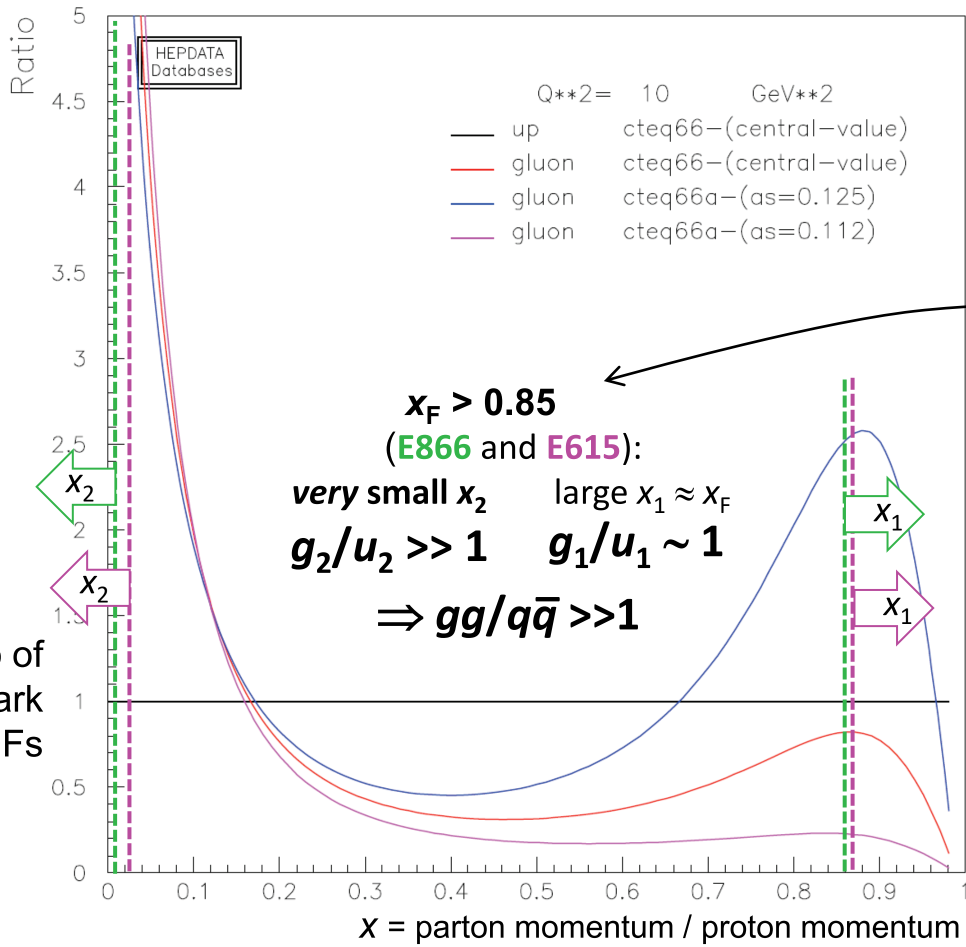


at small x_F gluon-gluon production could be the largest contribution

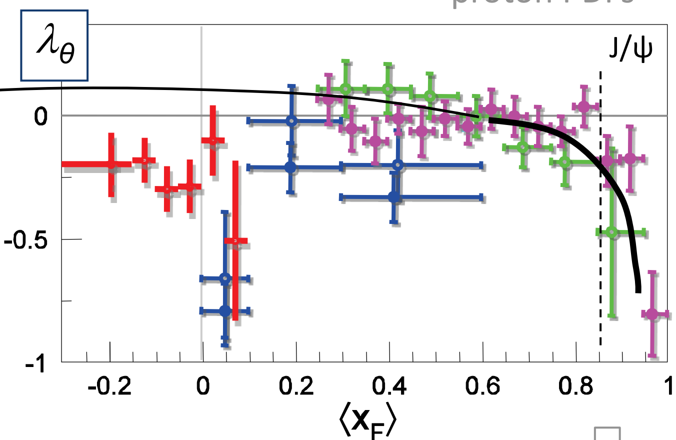
$$x_1 - x_2 = x_F$$
$$x_1 \cdot x_2 = (M_{Q\bar{Q}}/\sqrt{s})^2$$

Map of data in the parton/proton x space

Ratio of gluon/u-quark PDFs



For E615 (*pion*-nucleus) we assume similar qualitative behaviour between pion and proton PDFs

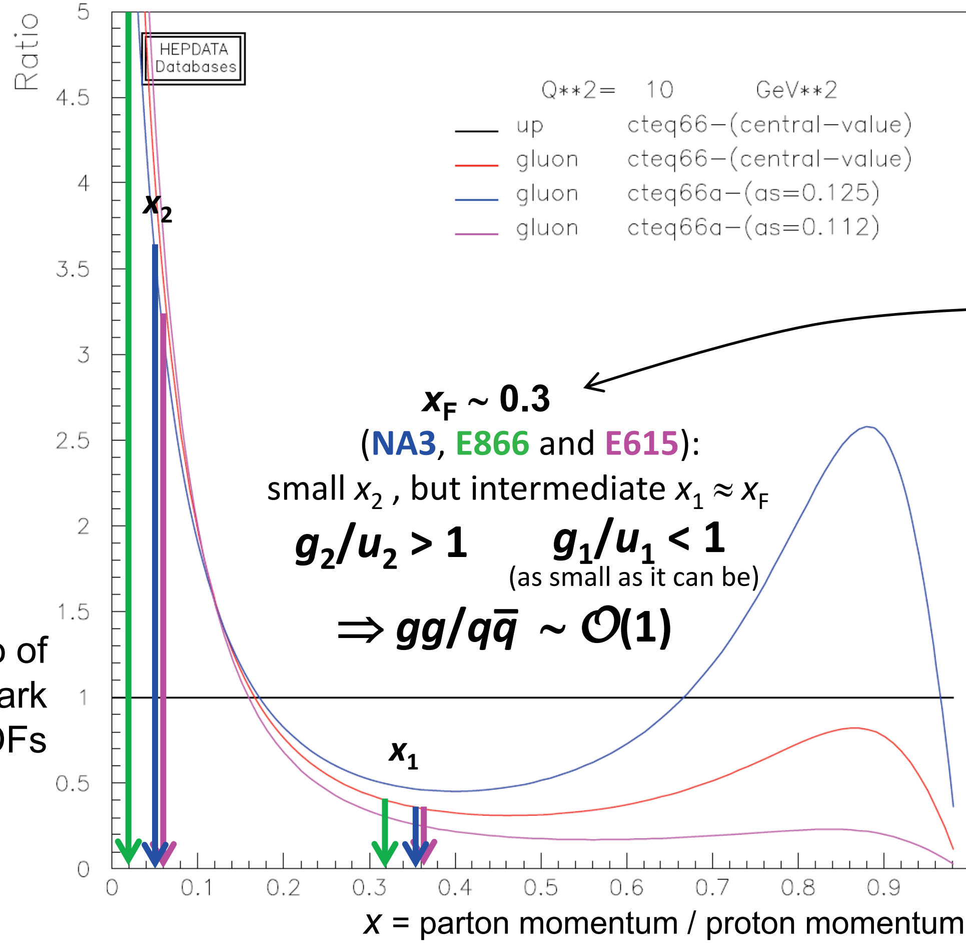


gluon-gluon production should dominate at high x_F

$$x_1 - x_2 = x_F$$
$$x_1 \cdot x_2 = (M_{Q\bar{Q}}/\sqrt{s})^2$$

Map of data in the parton/proton *x* space

Ratio of gluon/*u*-quark PDFs



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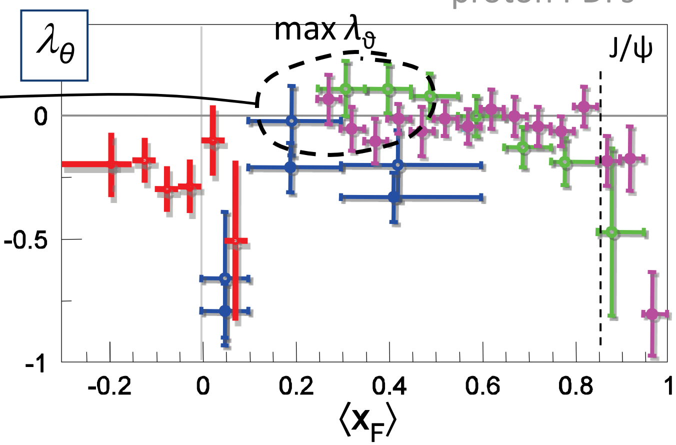


at intermediate x_F quark-antiquark and gluon-gluon production should compete

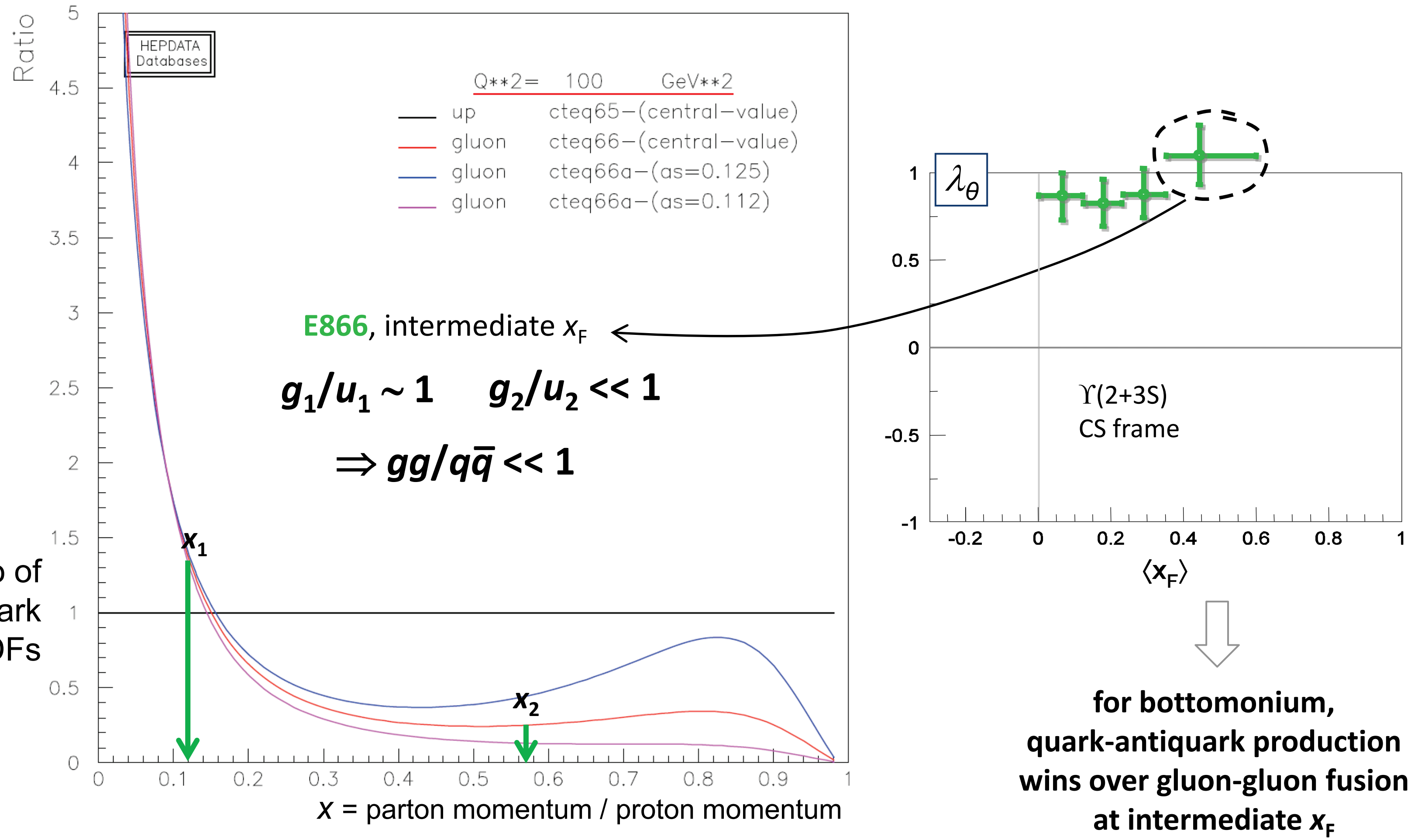
$$x_1 - x_2 = x_F$$

$$x_1 \cdot x_2 = (M_{Q\bar{Q}}/\sqrt{s})^2$$

Map of data in the parton/proton *x* space

What about the "very different" Υ(2+3S) result?

Ratio of gluon/*u*-quark PDFs



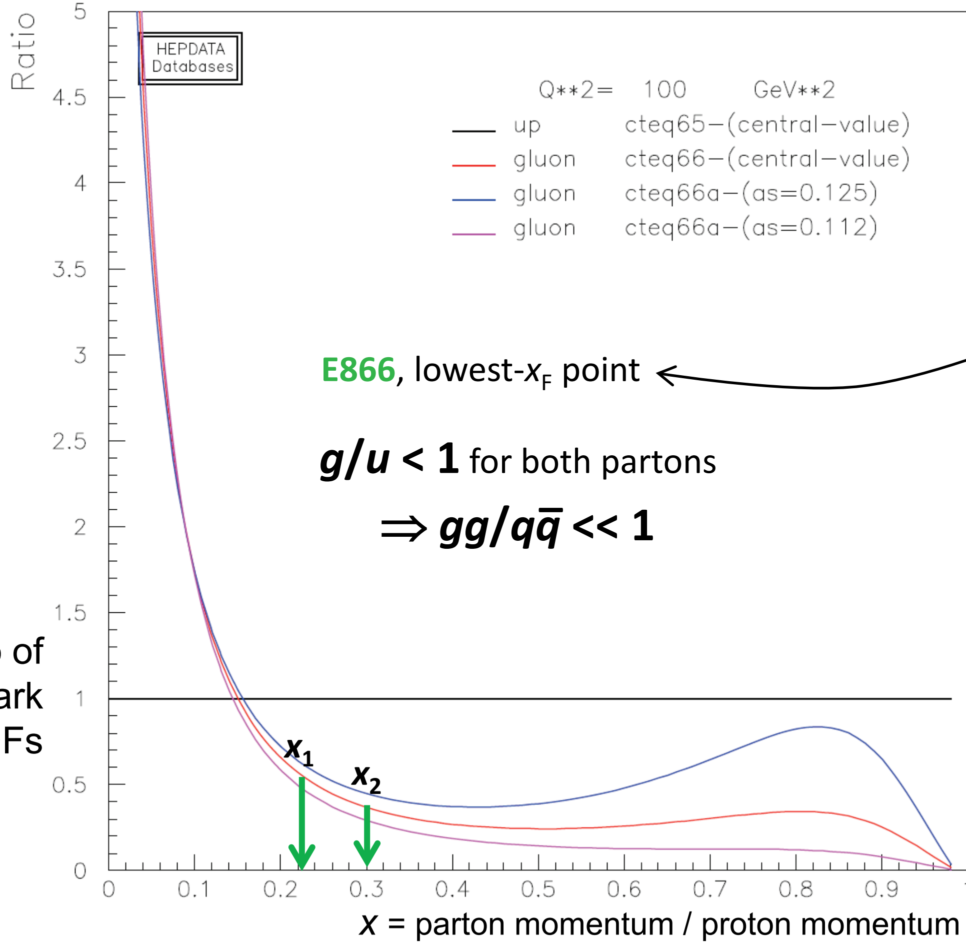
for bottomonium, quark-antiquark production wins over gluon-gluon fusion at intermediate x_F

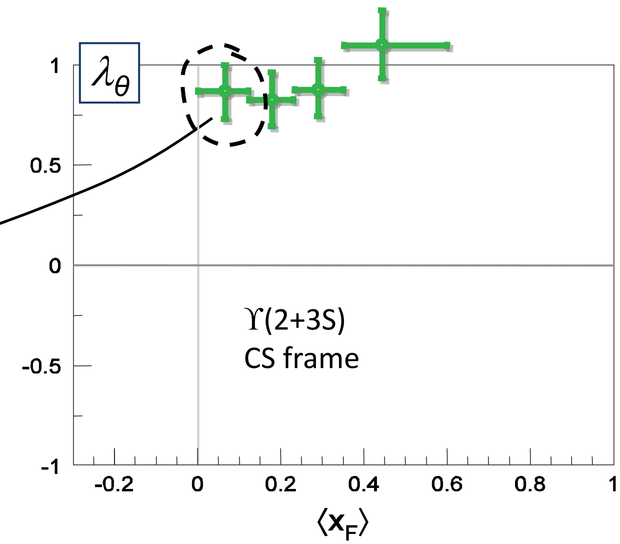
$$x_1 - x_2 = x_F$$
$$x_1 \cdot x_2 = (M_{Q\bar{Q}}/\sqrt{s})^2$$

Map of data in the parton/proton *x* space

What about the "very different" ϒ(2+3S) result?

Ratio of gluon/*u*-quark PDFs

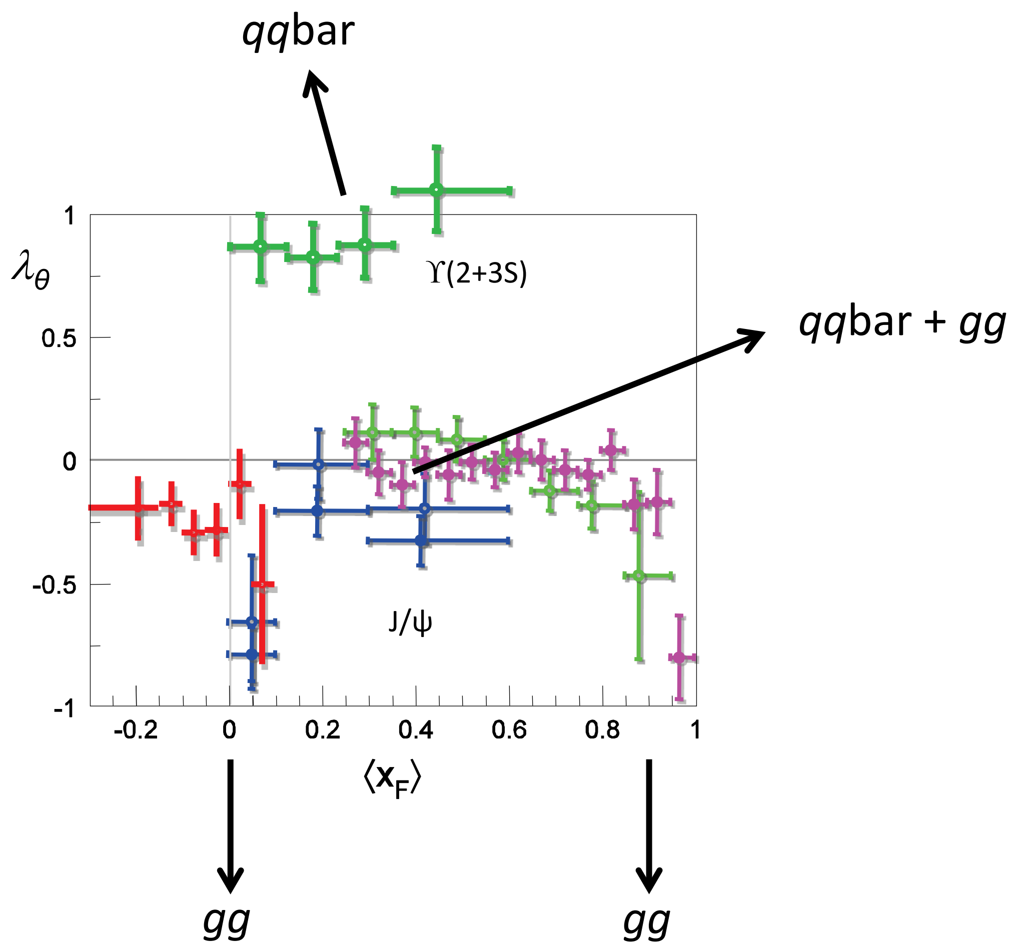




for bottomonium, quark-antiquark production dominates even at small x_F

$$x_1 - x_2 = x_F$$
$$x_1 \cdot x_2 = (M_{Q\bar{Q}}/\sqrt{s})^2$$

(Longitudinal vs transverse) = (*gg* vs *qq*bar)



By measuring quarkonium polarization in the low-p_T 2 → 1 domain
(and comparing with corresponding theoretical predictions for the *gg* and *qq*bar cases)
we can probe the identity of the colliding partons!

Indications and motivations from existing data

3) A consistency can be recognized in the perplexing scenario of J/ψ and Υ polarizations vs $x_F = x_1 - x_2$ when we correlate the observed **longitudinal** polarizations with the dominance of ***gg* processes** and **transverse** polarizations with the dominance of ***qq*bar processes**.
This correlation is in agreement with the corresponding expectations for polarizations in $2 \rightarrow 1$ processes, hinting at *gg* production via $^3S_1$-octet (the longitudinal case).

polarization measurements are a unique probe of elementary partonic processes!
measure as much "differentially" as possible vs $|x_F|$!

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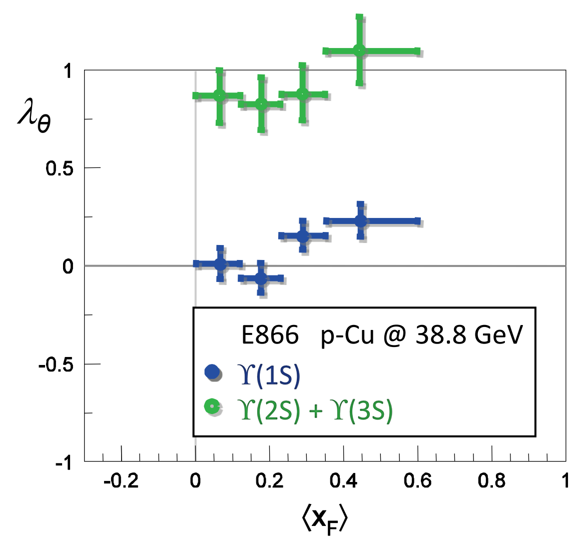
polarization measurements are a unique probe of elementary partonic processes!
measure as much "diffentially" as possible vs $|x_F|$!

4) In all previous considerations we neglected a crucial detail:
the **feed-down from χ states**,
~20% for the J/ψ (HERA-B*), probably larger for the ϒ(1S).
In fact, the ϒ(2S)+ϒ(3S) measurement by E866 stands apart as the strongest *transverse* polarization,
while ϒ(1S) is almost *unpolarized*.
Feed-down from χ states must be the culprit...



*PRD 79, 012001

The role of feed-down decays

As long as heavier and lighter S-states have the same production mechanism, feed-down **from heavier S-states** is “**invisible**” from the polarization point of view, as shown by the mentioned BES ψ(2S) $\rightarrow$ J/ψ $\pi^+\pi^-$ result.

Feed-down **from χ** is more complex:

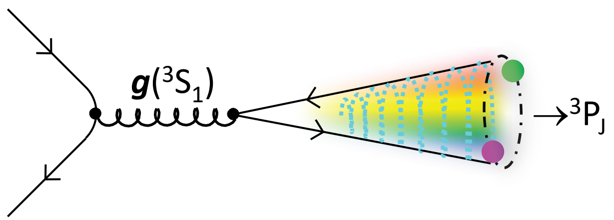
a) P-wave states can have different **production mechanisms** with respect to S waves
b) they decay to J/ψ and Υ with the emission of a transversely polarized photon, which alters the **spin-alignment** of the QQbar [Faccioli et al., Phys. Rev. D 83, 096001 (2011)]

As a result, we can expect **different polarizations** for ψ/Υ from χ decays with respect to the directly produced ones

χ polarization in 2→1 *qq*bar production

All polarizations are referred to the parton-parton collision direction (~CS)

Colour-octet:

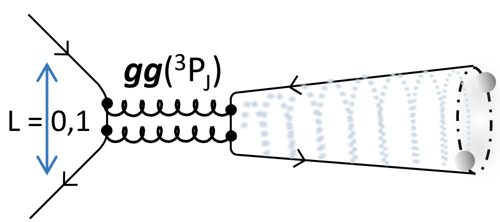


The coloured $^3S_1$ QQbar has $J_z = \pm1$.
It transforms to χ_J by emitting *one* low-energy gluon (to neutralize colour *and* produce a ^3P_J state).
The χ_J then decays radiatively to ψ/ϒ

The resulting ψ/ϒ has: λ_θ = **+1/5** (χ_1)
λ_θ = **+21/73** (χ_2)

(directly produced ψ/ϒ has λ_θ = **+1**)

Colour-singlet:



Two gluons are sufficient to produce a colourless ^3P_J already with the χ_J quantum numbers.
The χ_J has $J_z = \pm1$ and decays radiatively to ψ/ϒ

The resulting ψ/ϒ has: λ_θ = **−1/3** (χ_1)
λ_θ = **−1/3** (χ_2)

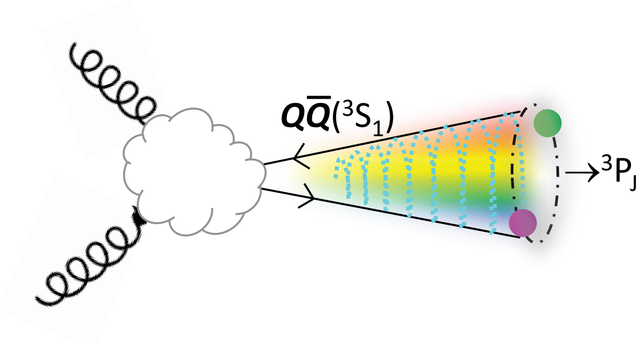
(directly produced ψ/ϒ has λ_θ = **+1**)

⇒ expect a strong reduction of the transverse ψ/ϒ polarization from *direct* to *prompt* if χ feed-down is large

χ polarization in 2→1 *gg* production

All polarizations are referred to the parton-parton collision direction (~CS)

Colour-octet:

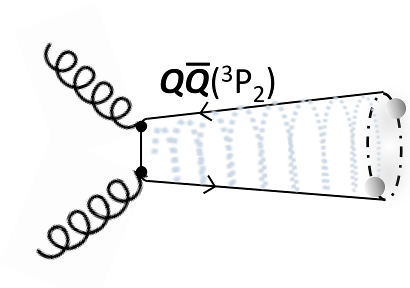


$^3S_1$ is *the* favoured octet for χ_J production
(only requires one emitted gluon to give ^3P_J)
The QQbar has $J_z = 0$ (transverse colliding gluons)
The χ_J then decays radiatively to ψ/Υ

The resulting ψ/Υ has: $\lambda_\theta = \mathbf{-1/3}$ (χ_1)
$\lambda_\theta = \mathbf{-21/47}$ (χ_2)

directly produced ψ/Υ has $\lambda_\theta = \mathbf{-1}$ (from $^3S_1$ octet)
$\lambda_\theta = \mathbf{0}$ (from $^1S_0$ octet)

Colour-singlet:



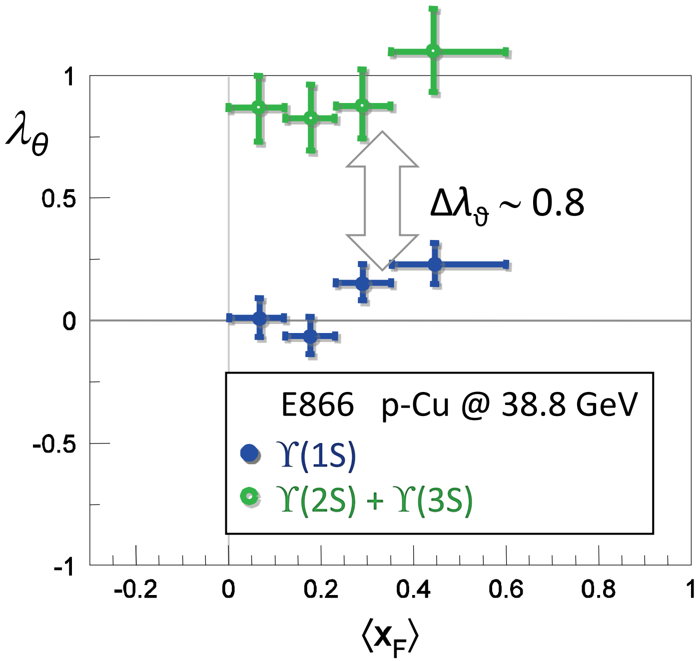
χ_2 (like η and χ_0) can be produced directly as colourless QQbar, with $J_z = 0$ or $J_z = \pm 2$.
This decays radiatively to ψ/Υ

The resulting ψ/Υ has $\lambda_\theta = \mathbf{-3/5}$ or $\mathbf{+1}$ (χ_2)

(no χ_1 or direct ψ/Υ in this channel)
(χ_1 allowed from off-shell gluons → $\lambda_\theta = \mathbf{+1}$ **?**)

⇒ longitudinal prompt-ψ/Υ polarization scenario favoured with minimum smearing from χ feed-down

χ feed-down and the E866 "puzzle"

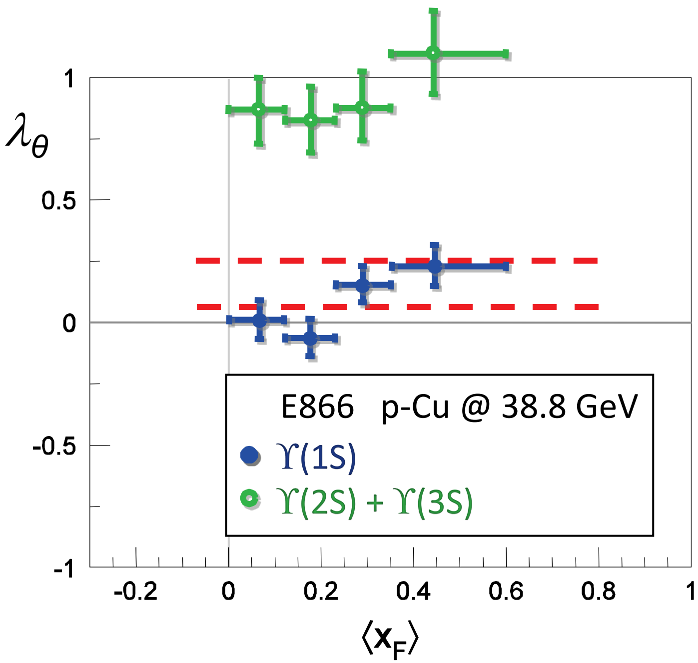


1S, 2S and 3S states should have the same polarization when **directly produced** $\rightarrow$ $\boldsymbol{\lambda_\vartheta \approx +1}$
(or when coming from heavier ϒ)

To justify the large difference between 2-3S and 1S, we must assume that **χ feed-down**:
a) is **negligible for 2-3S states** and **large for 1S**
b) tends to be **longitudinal**

No measurements of *how many ϒ come from* χ_b exist for low-p_T quarkonium production.
χ production may be large with respect to ψ production, since, for example,
it is easier to produce a singlet χ (2 intermediate gluons) than a singlet ψ/ϒ (3 gluons).

χ feed-down and the E866 "puzzle"



1S, 2S and 3S states should have the same polarization when **directly produced** → $\boldsymbol{\lambda_\vartheta}$ **≈ +1** (or when coming from heavier Υ)

To justify the large difference between 2-3S and 1S, we must assume that **χ feed-down**:

a) is **negligible for 2-3S states** and **large for 1S**

b) tends to be **longitudinal**

No measurements of *how many Υ come from* χ_b exist for low-p_T *qq*bar production.
χ production may be large with respect to ψ production, since, for example,
it is easier to produce a singlet χ (2 intermediate gluons) than a singlet ψ/Υ (3 gluons).

If...

- **50–60%** of the Υ(1S) come from χ_b
- χ states are produced in the colour-singlet channel → λ_ϑ = –1/3

⇒ the observed Υ(1S) would have λ_ϑ in the range 1/13 – 1/4 = 0.08 – 0.25

Note: sum rule for polarizations of different samples:

$$\lambda_\alpha(1+2) = \frac{\frac{f_1 \cdot \lambda_{\alpha,1}}{3+\lambda_{\vartheta,1}} + \frac{f_2 \cdot \lambda_{\alpha,2}}{3+\lambda_{\vartheta,2}}}{\frac{f_1}{3+\lambda_{\vartheta,1}} + \frac{f_2}{3+\lambda_{\vartheta,1}}}$$

$$\alpha = \vartheta, \varphi, \vartheta\varphi$$

Indications and motivations from existing data

3) A consistency can be recognized in the perplexing scenario of J/ψ and ϒ polarizations vs $x_F = x_1 - x_2$ when we correlate the observed **longitudinal** polarizations with the dominance of ***gg* processes** and **transverse** polarizations with the dominance of ***qq*bar processes**.
This correlation is in agreement with the corresponding expectations for polarizations in $2 \rightarrow 1$ processes, hinting at *gg* production via $^3S_1$-octet (the longitudinal case)

polarization measurements are a unique probe of elementary partonic processes!
measure as much "diffentially" as possible vs $|x_F|$!

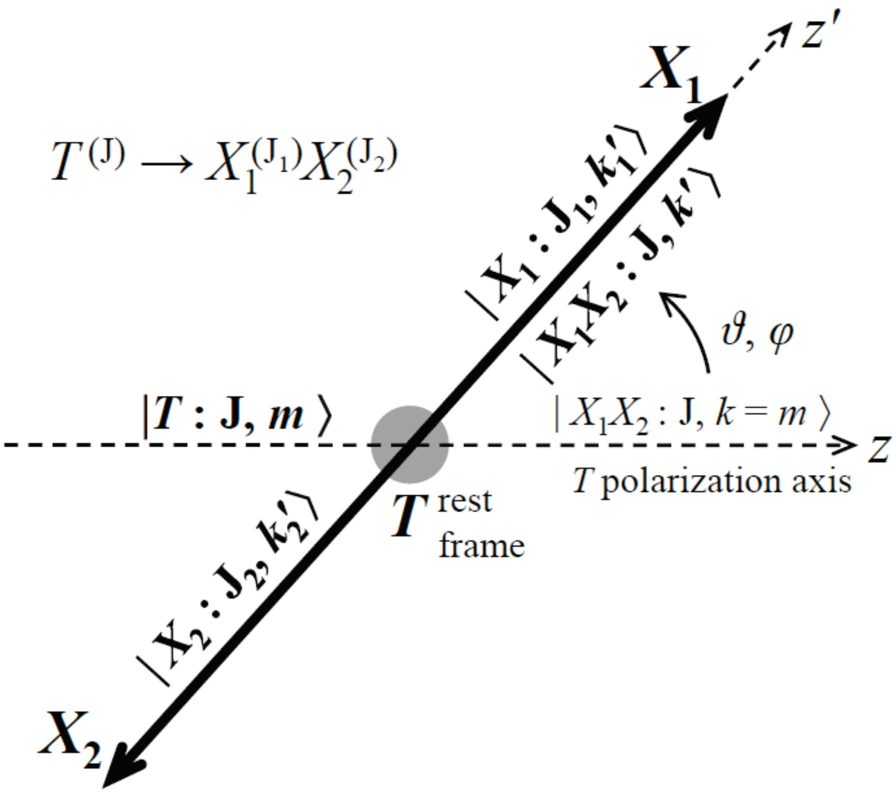
4) E866 data clearly show how sensitive the observed polarizations of J/ψ and ϒ can be to the **feed-down from χ states**.
So much that strong hypotheses on the (unknown) feed-down fractions and polarizations of χ_b states are required to describe the striking difference between ϒ(2S)+ϒ(3S) and ϒ(1S) observations.
χ production is not a second-order correction for J/ψ and ϒ yields and polarizations and it is equally interesting for the understanding of the fundamental processes

measure production properties of χ states
(feed-down fractions, polarizations)!

“Offline” appendix: polarization basics, invariants and the Lam-Tung relation

Polarization basics

Measure **polarization** of a particle = measure the (average) **angular momentum composition** in which the particle is produced, by studying the **angular distribution** of its **decay** in its rest frame



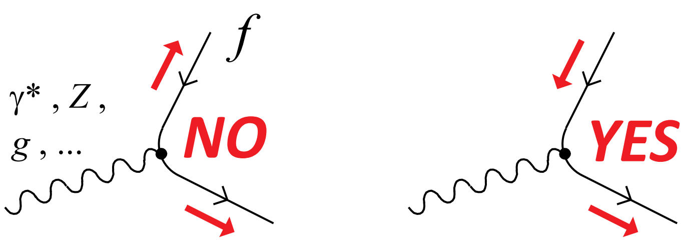
Example: polarization of vector particles

$J = 1$ → three J_z eigenstates $|1,+1\rangle$, $|1,0\rangle$, $|1,-1\rangle$ wrt a certain z

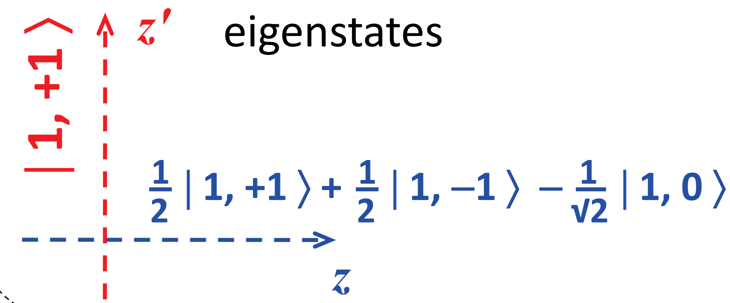
The decay **into a fermion-antifermion pair** is an especially clean case to be studied

The shape of the observable angular distribution is determined by
a few basic principles:

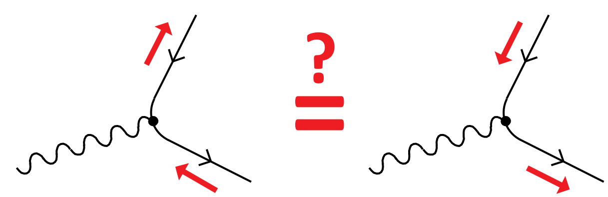
1) elementary coupling properties
E.g.: "helicity conservation"



2) rotational covariance
of angular momentum
eigenstates

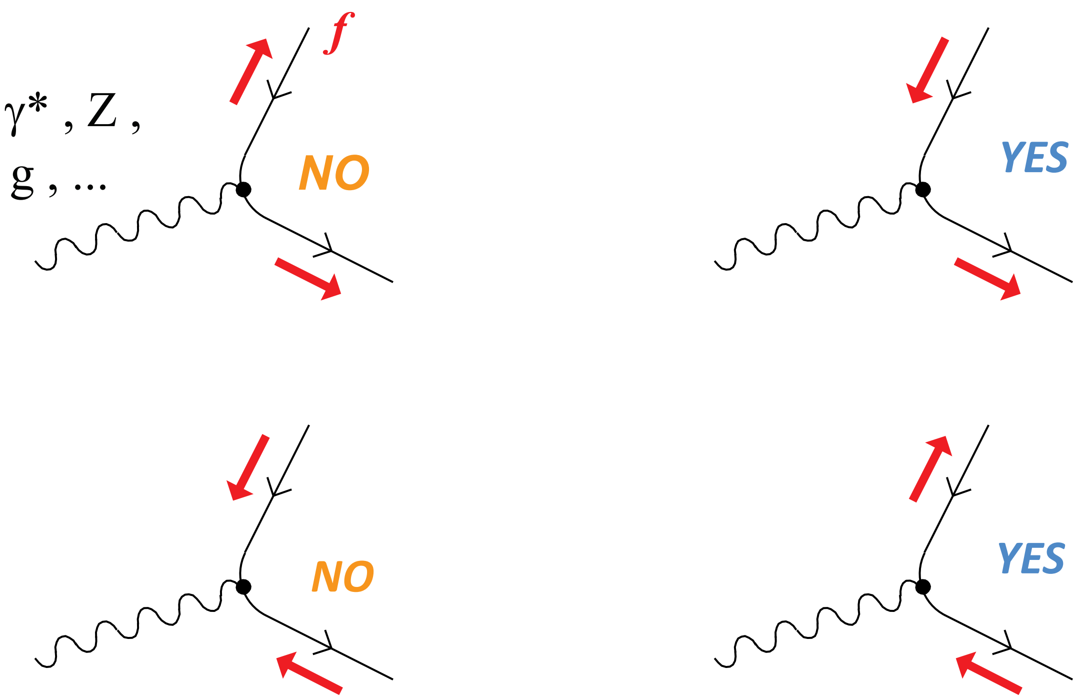


3) parity properties

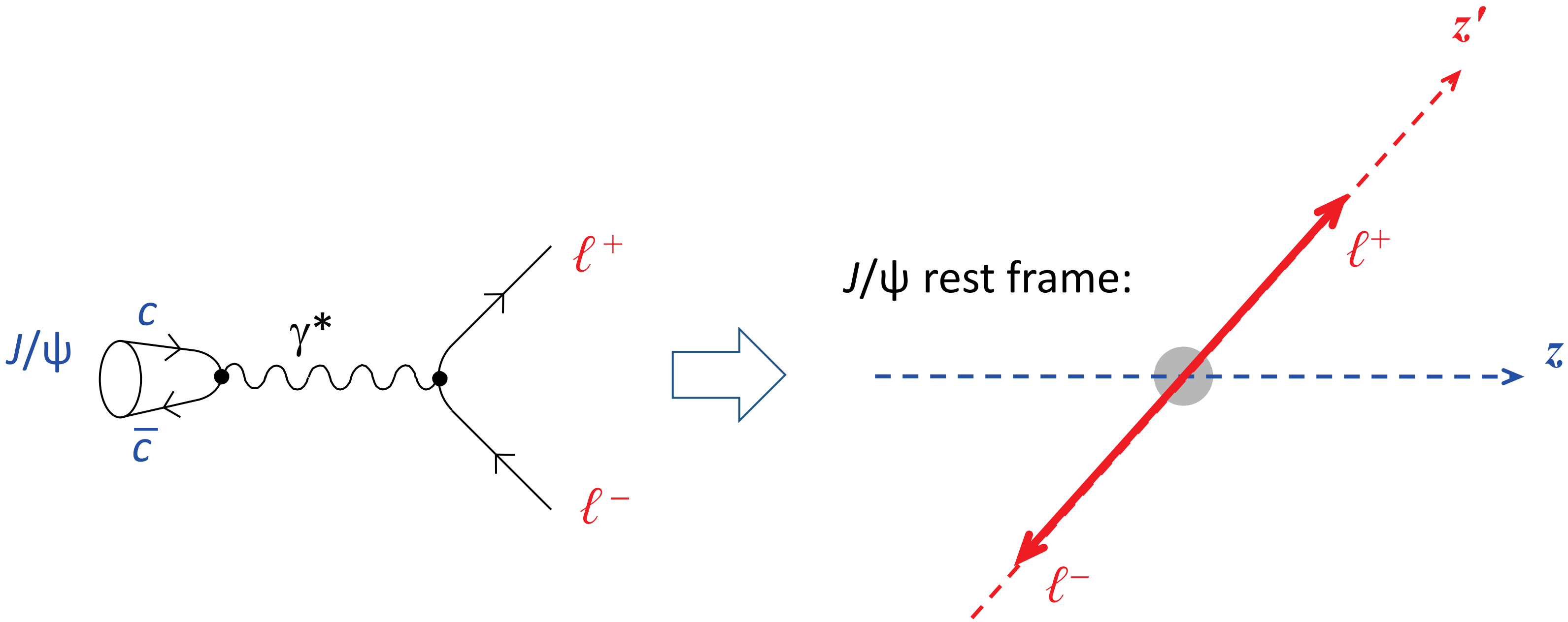


1: elementary coupling properties

Relevant property for cases considered here:
EW and strong forces preserve the *chirality* (L/R) of fermions.
In the relativistic (massless) limit, *chirality* = ***helicity*** **= spin-momentum alignment**
→ the **fermion spin never flips** in the coupling to gauge bosons:



example: dilepton decay of *J*/ψ



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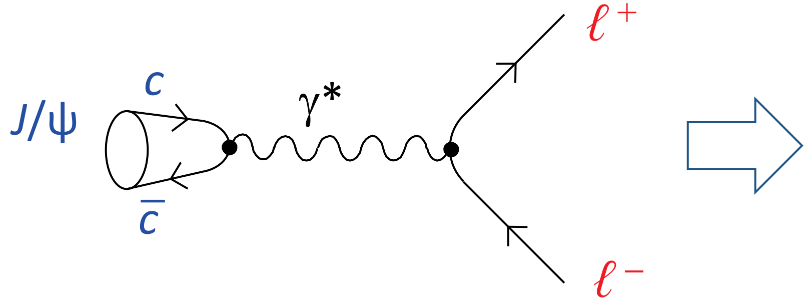
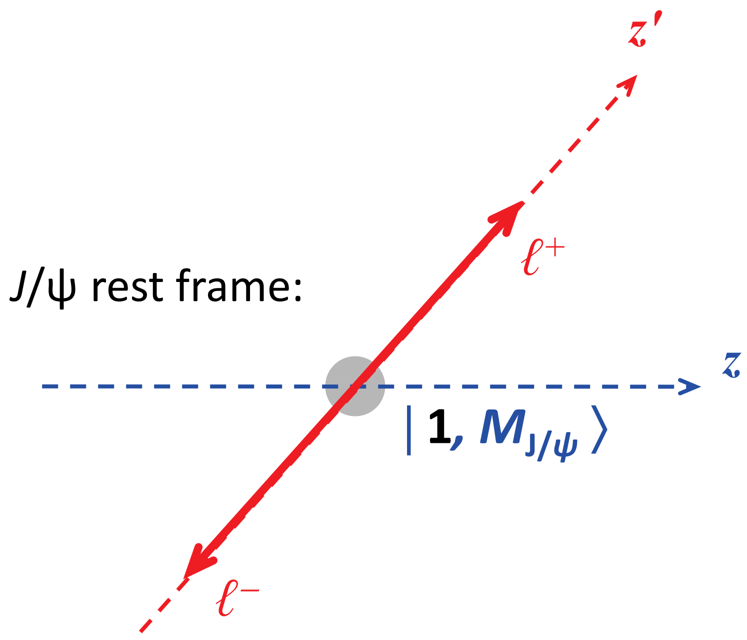
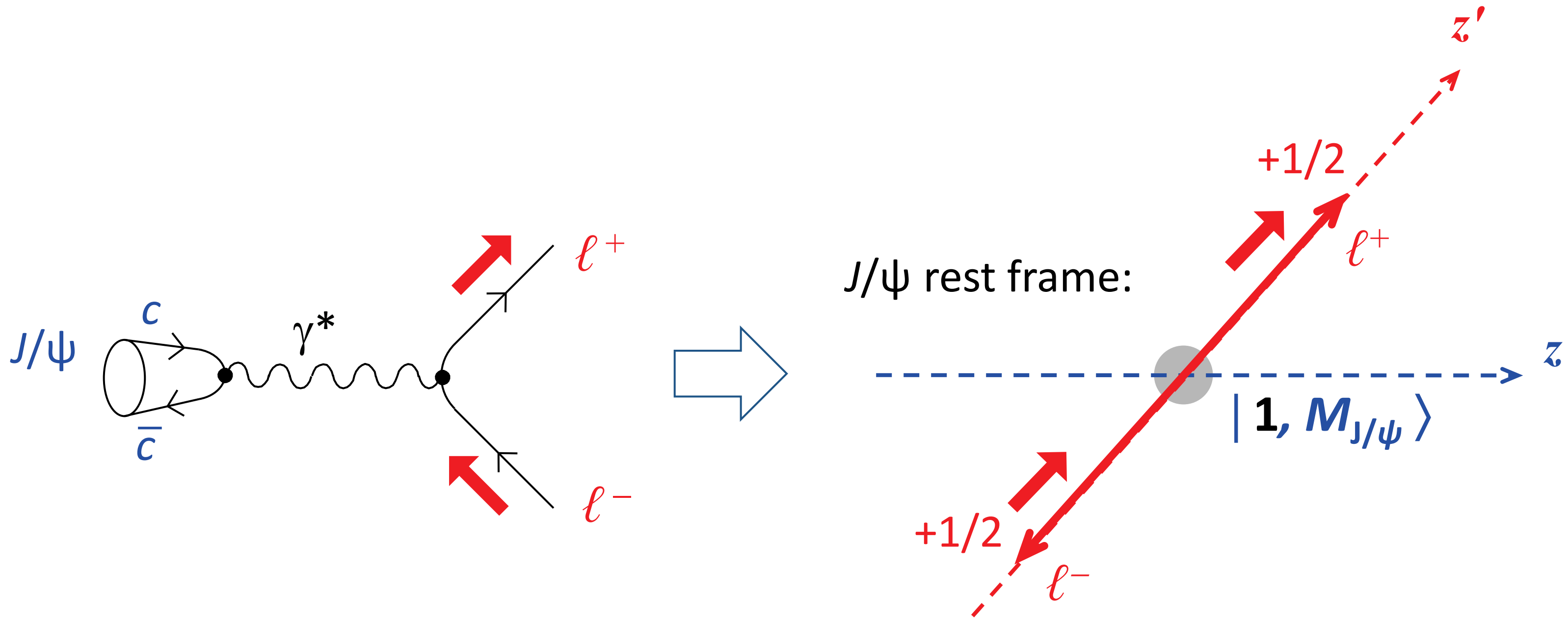




***J*/ψ** angular momentum component along the polarization axis ***z*:**

$M_{J/\psi}$ **= -1, 0, +1** (determined by *production mechanism*)

example: dilepton decay of *J*/ψ



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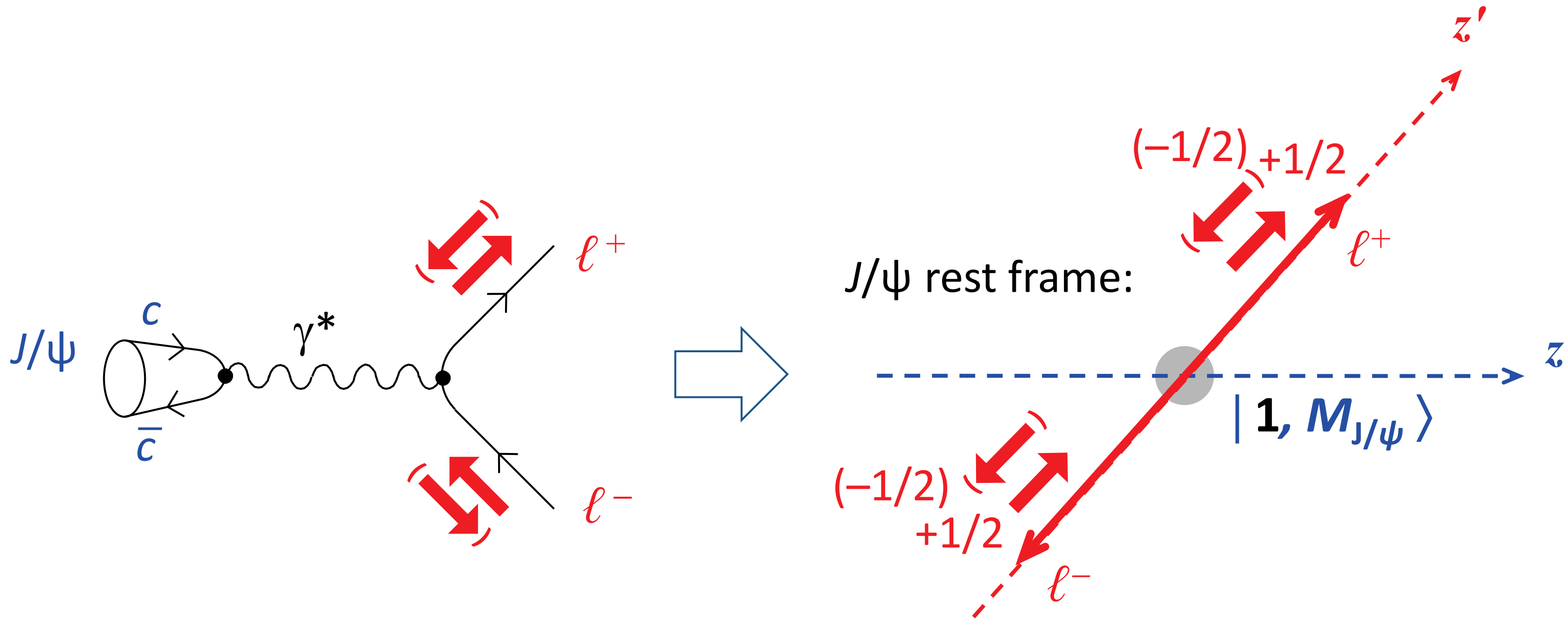

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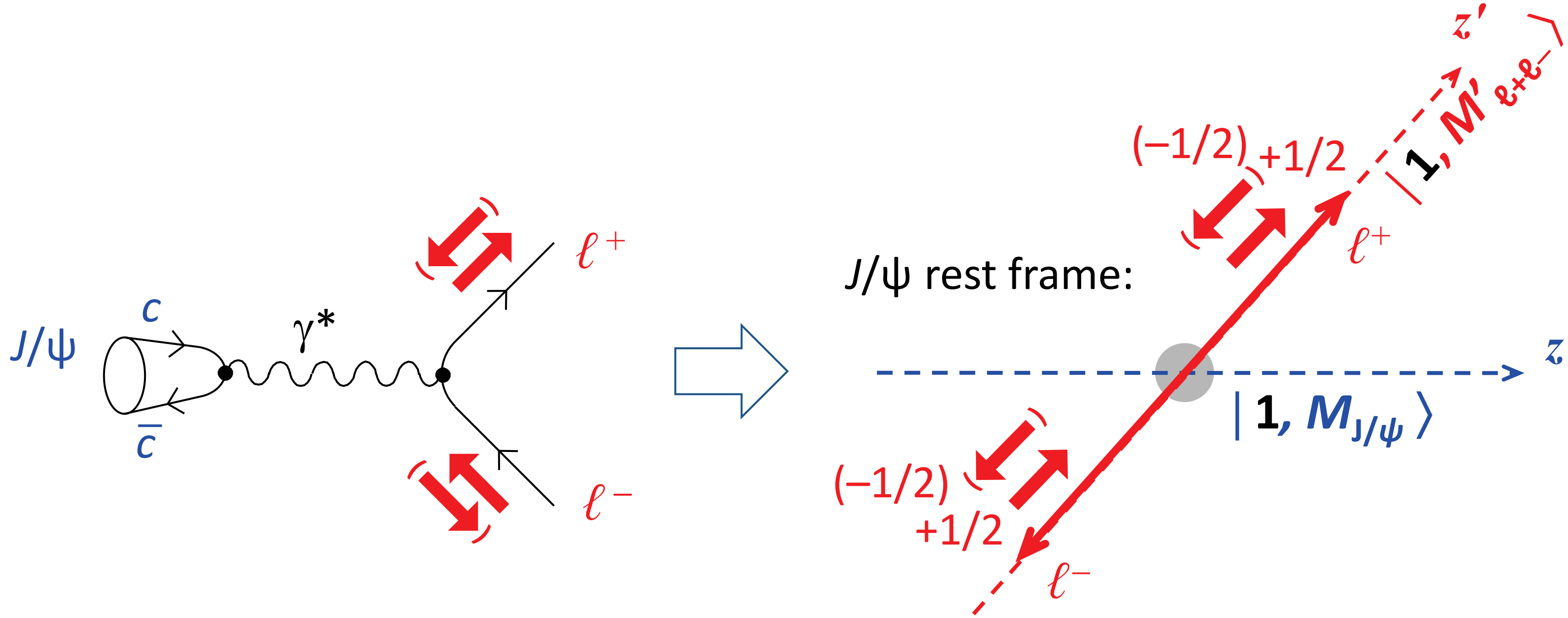


J/ψ angular momentum component along the polarization axis ***z*:**

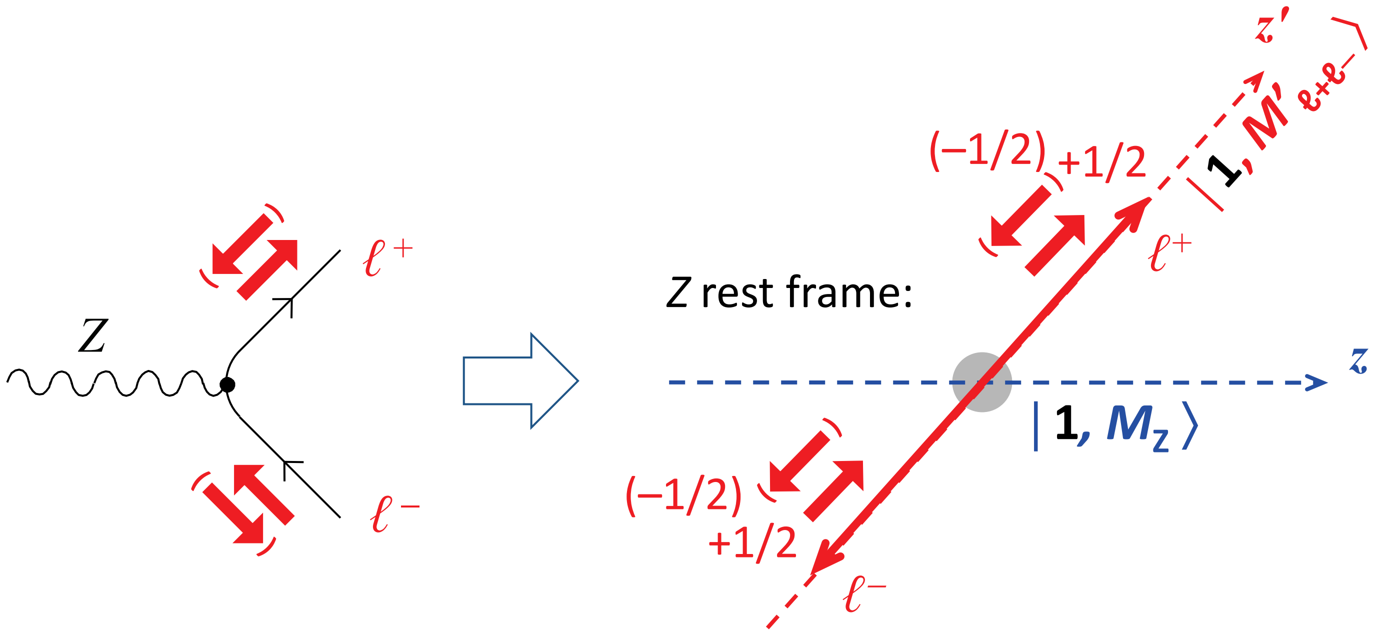
$$M_{J/\psi} = -1,\ 0,\ +1$$ (determined by *production mechanism*)

The **two leptons** can only have total angular momentum component

$$M'_{\ell^+\ell^-} = +1 \text{ or } -1$$ along their common direction ***z′***

0 is forbidden

example: dilepton decay of *Z* (or DY)



Z angular momentum component along the polarization axis ***z*:**

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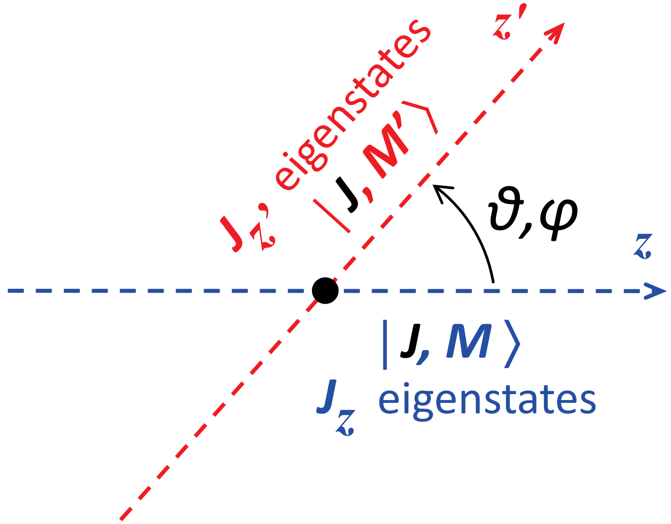
2: rotation of angular momentum eigenstates



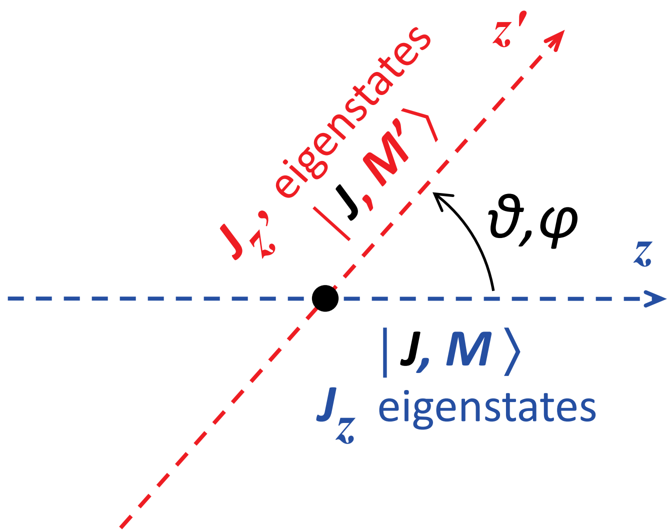
change of quantization frame:

$R(\vartheta,\varphi)$: $z \to z'$
$y \to y'$
$x \to x'$

$$|J, M'\rangle = \sum_{M=-J}^{+J} D^J_{MM'}(\vartheta,\varphi)\ |J, M\rangle$$

Wigner D-matrices

2: rotation of angular momentum eigenstates



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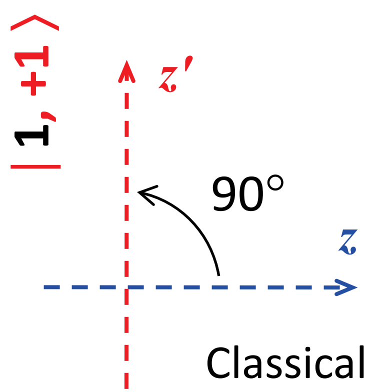
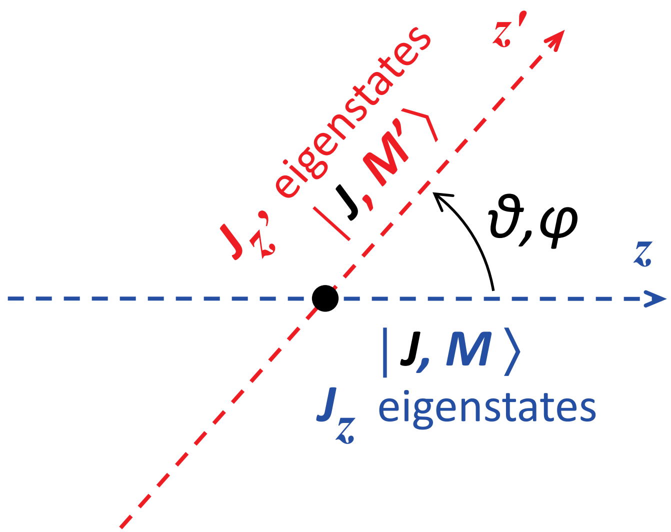


Classically, we would expect $|1, 0\rangle$

2: rotation of angular momentum eigenstates



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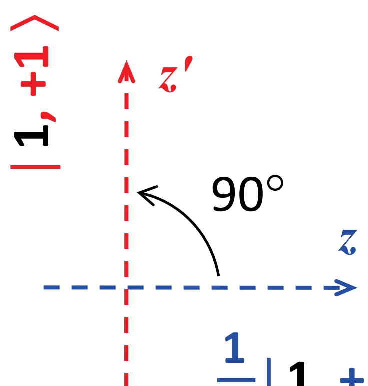
Wigner D-matrices

Example:



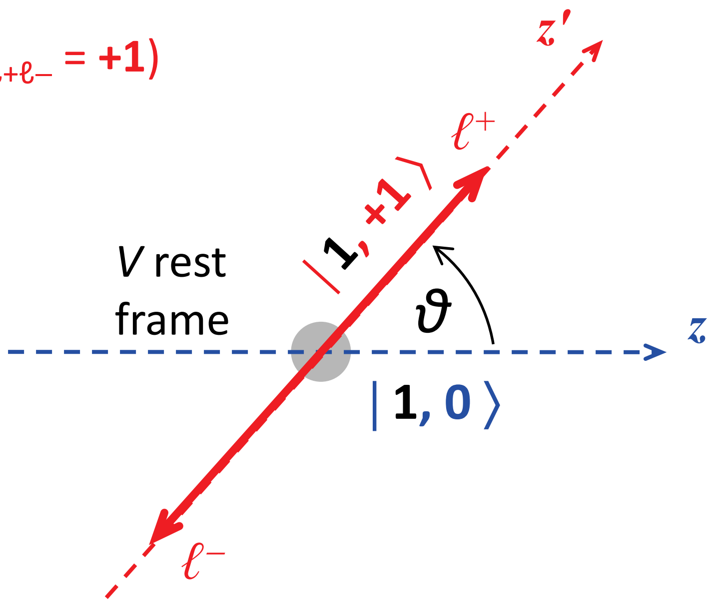
$$\frac{1}{2}\,|1, +1\rangle + \frac{1}{2}\,|1, -1\rangle - \frac{1}{\sqrt{2}}\,|1, 0\rangle$$

example: $M = 0$

$V\,(M_V = \mathbf{0}) \rightarrow \ell^+\ell^-(M'_{\ell+\ell-} = \mathbf{+1})$

$\downarrow$

$V = J/\psi$ | Z



$$|\,\mathbf{1},\mathbf{+1}\,\rangle \;=\; D^{1}_{-1,+1}(\vartheta,\varphi)\;|\,\mathbf{1},\mathbf{-1}\,\rangle + D^{1}_{0,+1}(\vartheta,\varphi)\;|\,\mathbf{1},\mathbf{0}\,\rangle + D^{1}_{+1,+1}(\vartheta,\varphi)\;|\,\mathbf{1},\mathbf{+1}\,\rangle$$

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$V\,(M_V = 0) \rightarrow \ell^+\ell^-(M'_{\ell^+\ell^-} = +1)$

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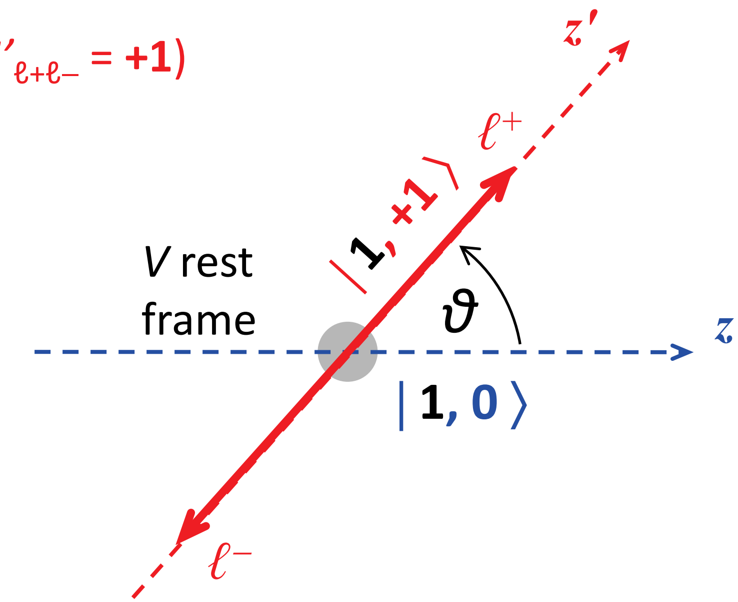
$V = J/\psi$ | Z



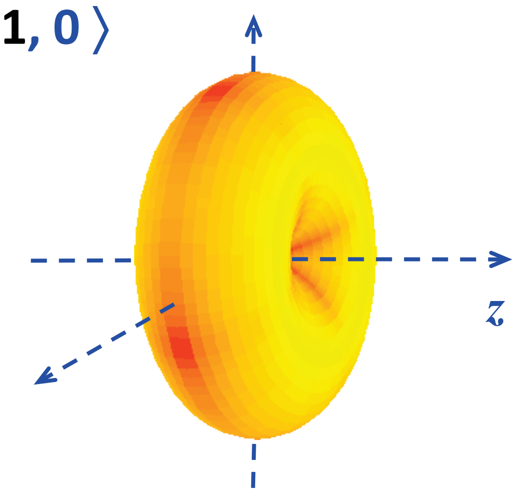
$$|\,1, +1\,\rangle = D^1_{-1,+1}(\vartheta,\varphi)\;|\,1, -1\,\rangle + D^1_{0,+1}(\vartheta,\varphi)\;|\,1, 0\,\rangle + D^1_{+1,+1}(\vartheta,\varphi)\;|\,1, +1\,\rangle$$

→ the $J_{z'}$ eigenstate $|\,1, +1\,\rangle$ "contains" the J_z eigenstate $|\,1, 0\,\rangle$ with component amplitude $D^1_{0,+1}(\vartheta,\varphi)$

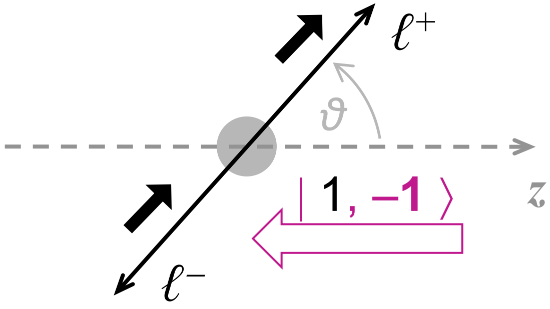
→ the decay distribution is

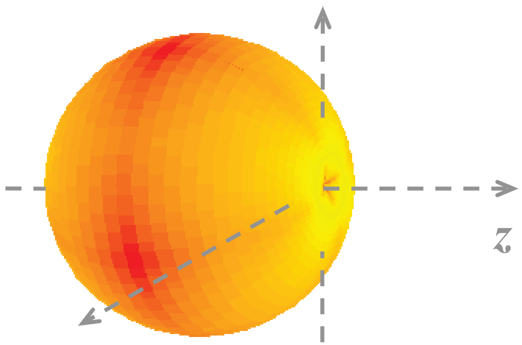
$$|\langle\, 1, +1\,|\,\mathcal{O}\,|\,1, 0\,\rangle|^2 \;\propto\; |D^{1*}_{0,+1}(\vartheta,\varphi)|^2 = \frac{1}{2}\,(\,1 - \cos^2\vartheta\,)$$

$\ell^+\ell^- \leftarrow J/\psi$



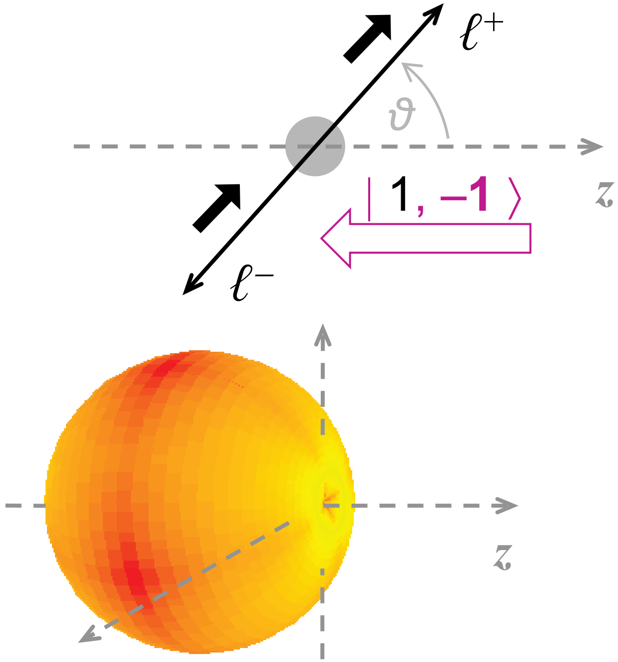
3: parity



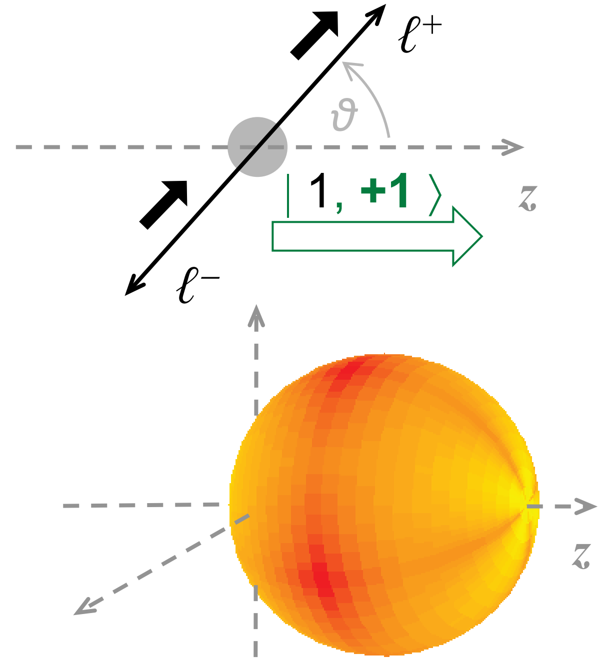


$$\frac{dN}{d\Omega} \propto |D^{1*}_{-1,+1}(\vartheta,\varphi)|^2 \propto 1 + \cos^2\vartheta - \mathbf{2\cos\vartheta}$$

3: parity



$|\,1, -1\,\rangle$ and $|\,1, +1\,\rangle$
distributions
are mirror reflections
of one another

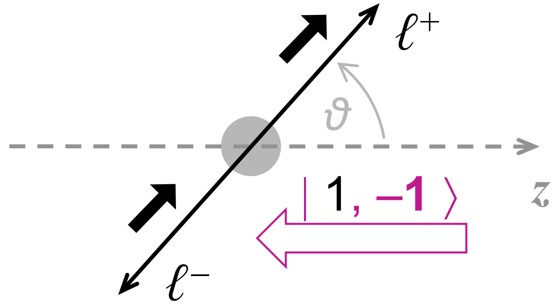


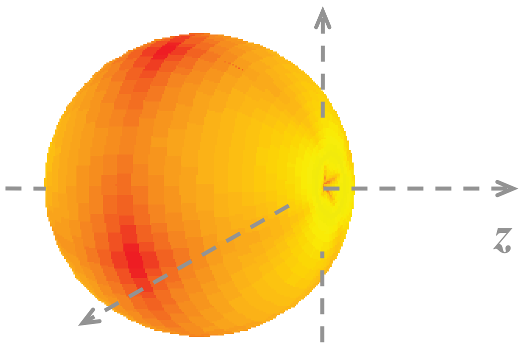
$$\frac{dN}{d\Omega} \propto |D^{1*}_{-1,+1}(\vartheta,\varphi)|^2 \propto 1 + \cos^2\vartheta - \mathbf{2\cos\vartheta}$$

$$\frac{dN}{d\Omega} \propto |D^{1*}_{+1,+1}(\vartheta,\varphi)|^2 \propto 1 + \cos^2\vartheta + \mathbf{2\cos\vartheta}$$

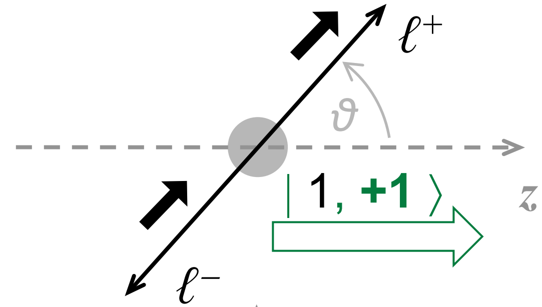
Are they equally probable?

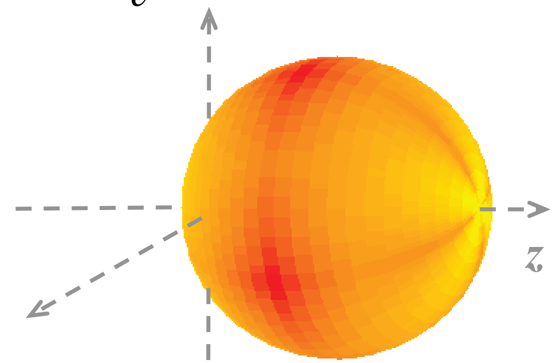
3: parity





$|\,1,\,-1\,\rangle$ and $|\,1,\,+1\,\rangle$
distributions
are mirror reflections
of one another

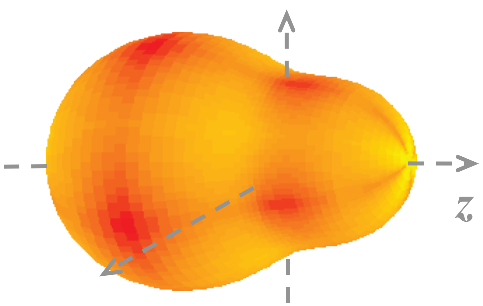




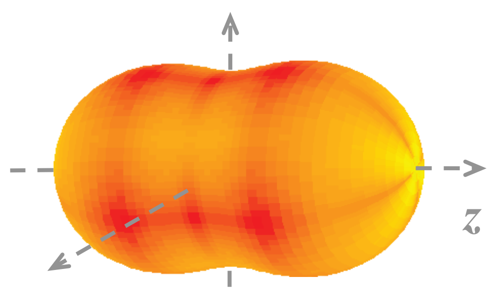
$$\frac{dN}{d\Omega} \propto |D^{1*}_{-1,+1}(\vartheta,\varphi)|^2 \propto 1 + \cos^2\vartheta - 2\cos\vartheta$$

$$\frac{dN}{d\Omega} \propto |D^{1*}_{+1,+1}(\vartheta,\varphi)|^2 \propto 1 + \cos^2\vartheta + 2\cos\vartheta$$

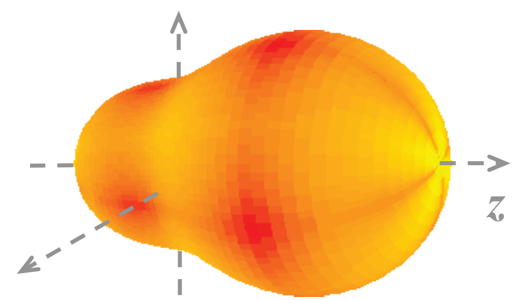
Are they equally probable?



$\mathcal{P}(-1) > \mathcal{P}(+1)$



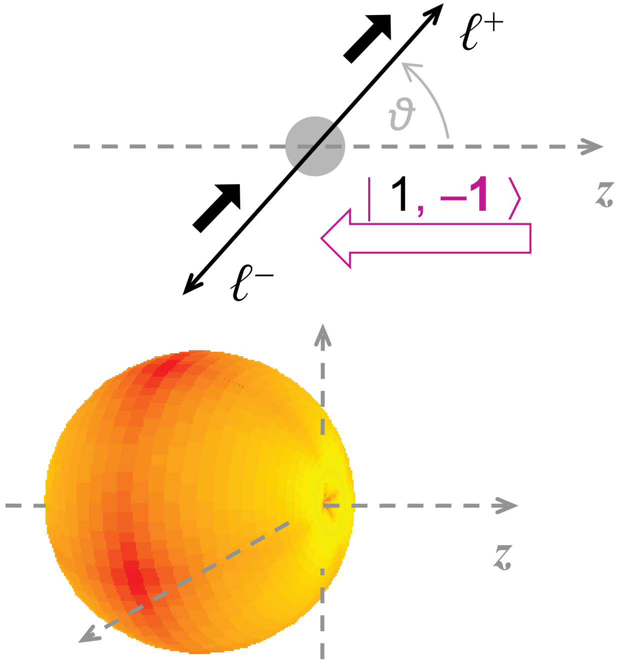
$\mathcal{P}(-1) = \mathcal{P}(+1)$



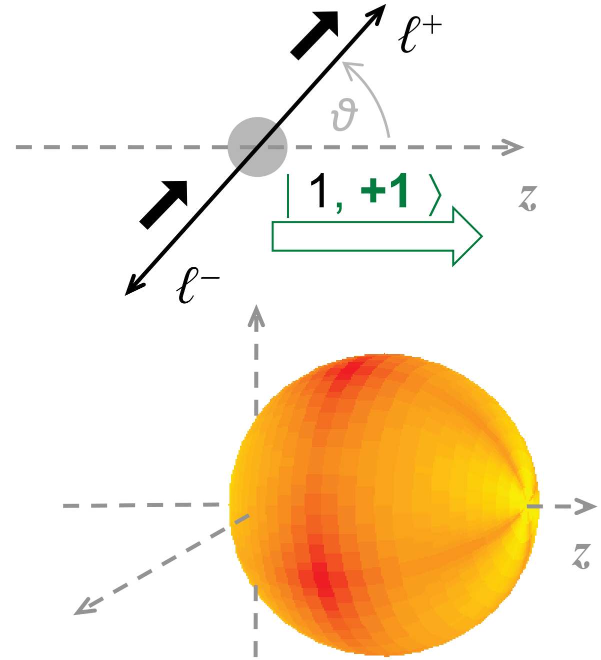
$\mathcal{P}(-1) < \mathcal{P}(+1)$

$$\frac{dN}{d\Omega} \propto 1 + \cos^2\vartheta + 2[\mathcal{P}(+1) - \mathcal{P}(-1)]\cos\vartheta$$

3: parity



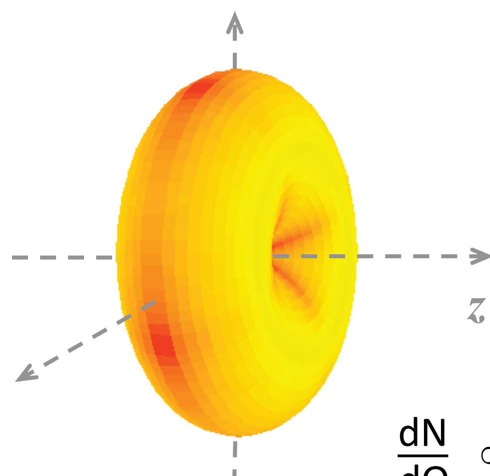
| 1, −1 ⟩ and | 1, +1 ⟩ distributions are mirror reflections of one another



$$\frac{dN}{d\Omega} \propto |D^{1*}_{-1,+1}(\vartheta,\varphi)|^2 \propto 1 + \cos^2\vartheta - \mathbf{2\cos\vartheta}$$

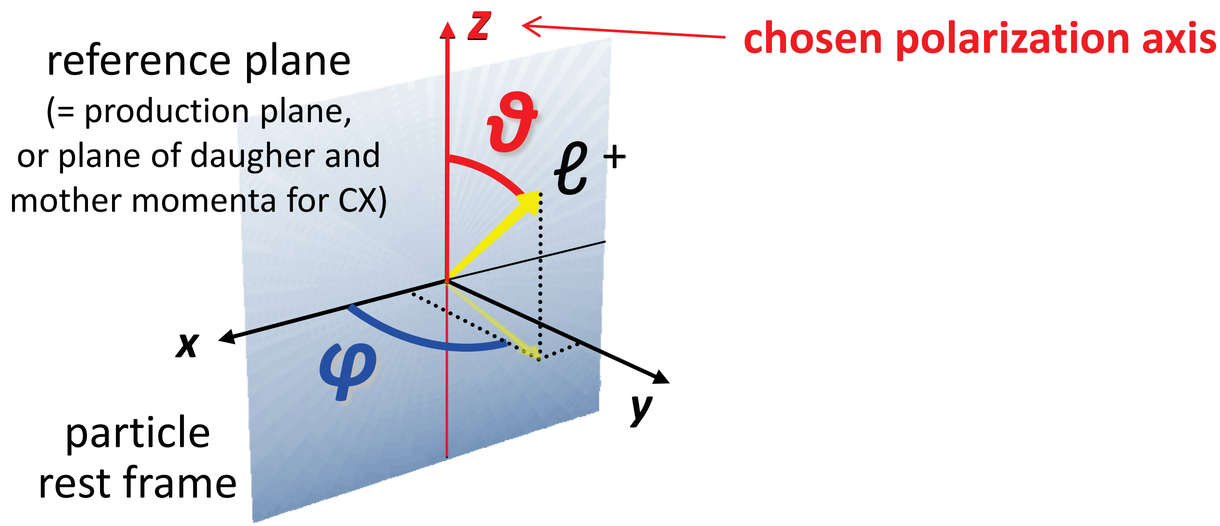
$$\frac{dN}{d\Omega} \propto |D^{1*}_{+1,+1}(\vartheta,\varphi)|^2 \propto 1 + \cos^2\vartheta + \mathbf{2\cos\vartheta}$$

Decay distribution of | 1, **0** ⟩ state is always parity-symmetric:



$$\frac{dN}{d\Omega} \propto |D^{1*}_{0,+1}(\vartheta,\varphi)|^2 \propto 1 - \cos^2\vartheta$$

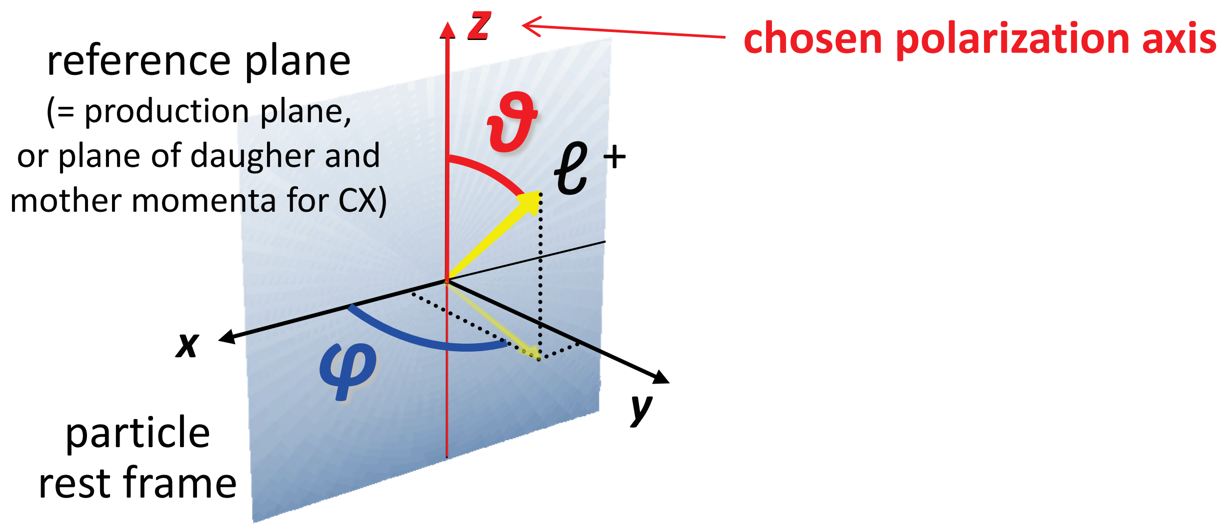
General two-body decay distribution



$$\frac{dN}{d\Omega} \propto 1 + \lambda_\vartheta \cos^2\vartheta + \lambda_\varphi \sin^2\vartheta \cos 2\varphi + \lambda_{\vartheta\varphi} \sin 2\vartheta \cos\varphi$$

$$+ 2A_\vartheta \cos\vartheta + 2A_\varphi \sin\vartheta \cos\phi \quad + \ldots$$

General two-body decay distribution

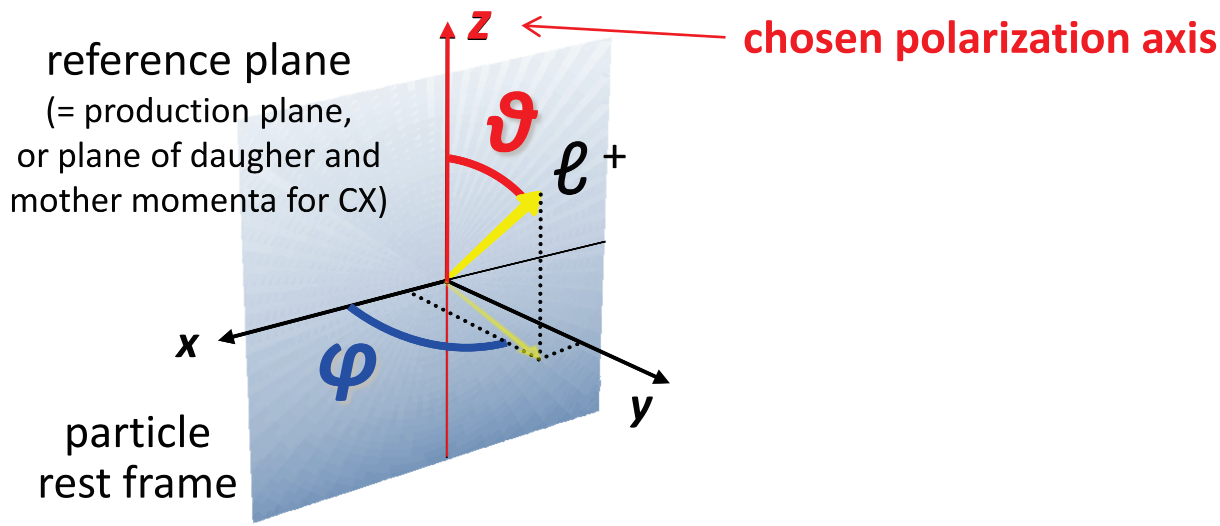


$$\frac{dN}{d\Omega} \propto 1 + \lambda_\vartheta \cos^2\vartheta + \lambda_\varphi \sin^2\vartheta \cos 2\varphi + \lambda_{\vartheta\varphi} \sin 2\vartheta \cos\varphi$$

$$+ 2A_\vartheta \cos\vartheta + 2A_\varphi \sin\vartheta \cos\phi + \ldots$$

parity violating

General two-body decay distribution



average polar anisotropy

average azimuthal anisotropy

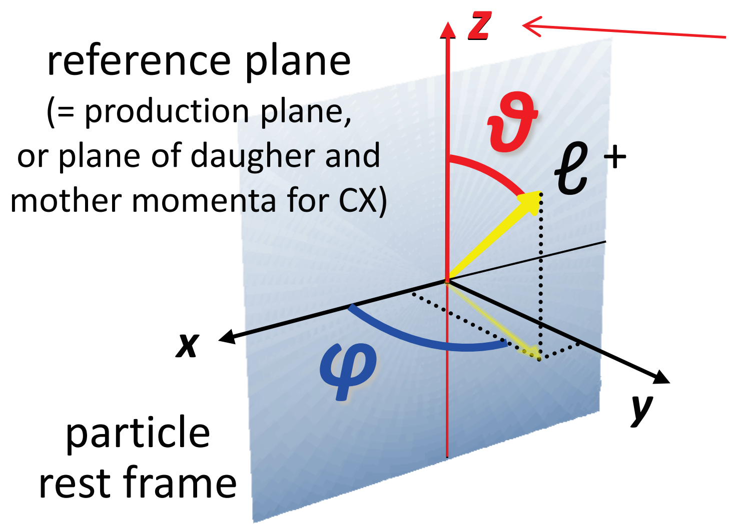
correlation polar - azimuthal

$$\frac{dN}{d\Omega} \propto 1 + \lambda_\vartheta \cos^2\vartheta + \lambda_\varphi \sin^2\vartheta \cos 2\varphi + \lambda_{\vartheta\varphi} \sin 2\vartheta \cos\varphi$$

$$+ 2A_\vartheta \cos\vartheta + 2A_\varphi \sin\vartheta \cos\phi + \ldots$$

parity violating

General two-body decay distribution



chosen polarization axis

pp c.o.m. helicity (**HX**): particle direction wrt pp c.o.m.

Gottfried-Jackson (**GJ**): direction of one or the other beam

Collins-Soper (**CS**): average of the two beam directions

perpendicular helicity (**PX**): perpendicular to CS

cascade helicity (**CX**): (in cascade decays) decaying particle's momentum direction in rest frame of its mother

etc.

average polar anisotropy — average azimuthal anisotropy — correlation polar - azimuthal

$$\frac{dN}{d\Omega} \propto 1 + \lambda_\vartheta \cos^2\vartheta + \lambda_\varphi \sin^2\vartheta \cos 2\varphi + \lambda_{\vartheta\varphi} \sin 2\vartheta \cos\varphi$$

$$+ 2A_\vartheta \cos\vartheta + 2A_\varphi \sin\vartheta \cos\phi + \ldots$$

parity violating

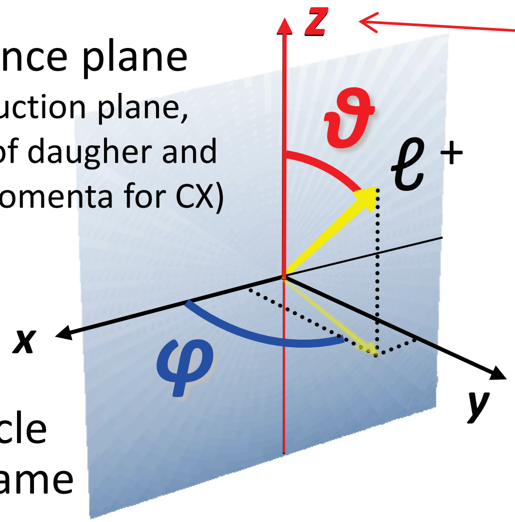
λ_ϑ, λ_φ, $\lambda_{\vartheta\varphi}$, etc. *depend* on the chosen frame [Faccioli et al., Eur. Phys. J C 69, 657 (2010)],

while $\mathcal{F} = \frac{1 + \lambda_\vartheta + 2\lambda_\varphi}{3 + \lambda_\vartheta}$ (as well as its functions, e.g. $\tilde{\lambda}$) does not

[Faccioli et al., Phys. Rev. Lett. 105, 061601 (2010)]

Alternative notation

reference plane
(= production plane, or plane of daugher and mother momenta for CX)



particle rest frame

chosen polarization axis

pp c.o.m. helicity (HX): particle direction wrt pp c.o.m.

Gottfried-Jackson (GJ): direction of one or the other beam

Collins-Soper (CS): average of the two beam directions

perpendicular helicity (PX): perpendicular to CS

cascade helicity (CX): (in cascade decays) decaying particle's momentum direction in rest frame of its mother

etc.

average polar anisotropy

average azimuthal anisotropy

correlation polar - azimuthal

$$\frac{dN}{d\Omega} \propto 1 + \frac{A_0}{2} + \left(1 - \frac{3}{2}A_0\right)\cos^2\vartheta + \frac{A_2}{2}\sin^2\vartheta\cos 2\varphi + A_1 \sin 2\vartheta\cos\varphi$$

$$+ A_4\cos\vartheta + A_3\sin\vartheta\cos\phi + \ldots$$

parity violating

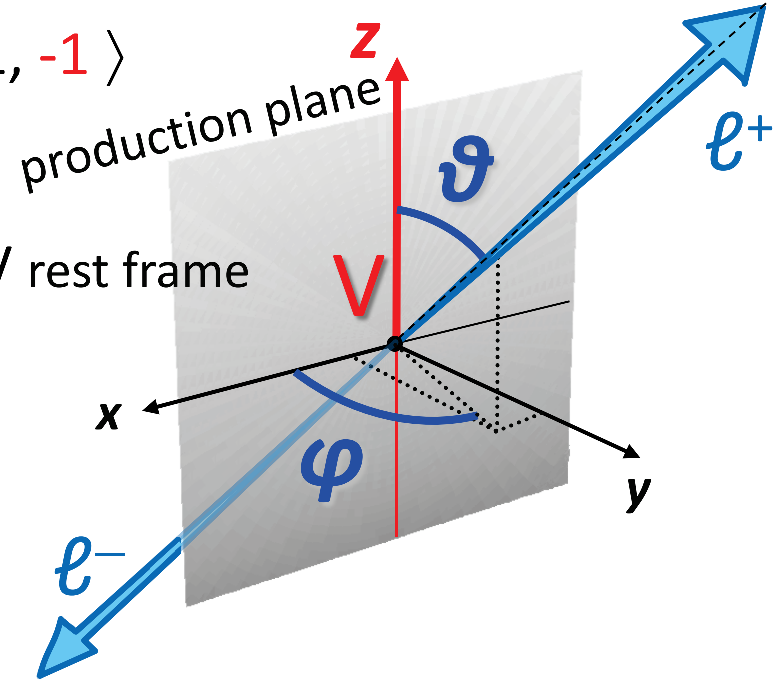
A_0, A_1, A_2, etc. *depend* on the chosen frame,

while $\mathcal{F} = \frac{1}{2}\left(1 + \frac{A_2 - A_0}{2}\right)$ (as well as its functions, e.g. A_2-A_0) does not

Relation to production mechanism

$$|V\rangle = b_{+1}\,|1,+1\rangle + b_0\,|1,0\rangle + b_{-1}\,|1,-1\rangle$$

V production mechanism



polarization measurement:

$dN/d\Omega(\cos\vartheta, \varphi) \rightarrow b_{+1}, b_0, b_{-1}$

These amplitudes are frame-dependent!

$$\frac{dN}{d\Omega} \propto 1 + \frac{A_0}{2} + \left(1 - \frac{3}{2}A_0\right)\cos^2\vartheta + \frac{A_2}{2}\sin^2\vartheta\cos 2\varphi + A_1 \sin 2\vartheta\cos\varphi + \ldots$$

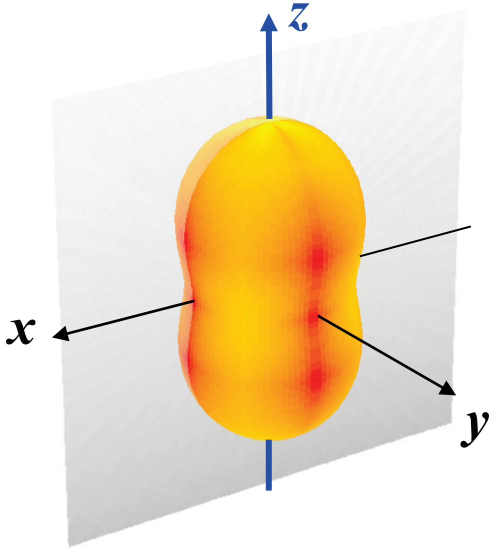
for decay to $\ell^+\ell^-$

$$A_0 = 2|b_0|^2 \qquad A_2 = 4\,\mathrm{Re}(b_{+1}^* b_{-1})$$

$$A_1 = \sqrt{2}\,\mathrm{Re}[b_0^*(b_{+1} - b_{-1})]$$

these relations depend on the decay channel

“Transverse” and “longitudinal”

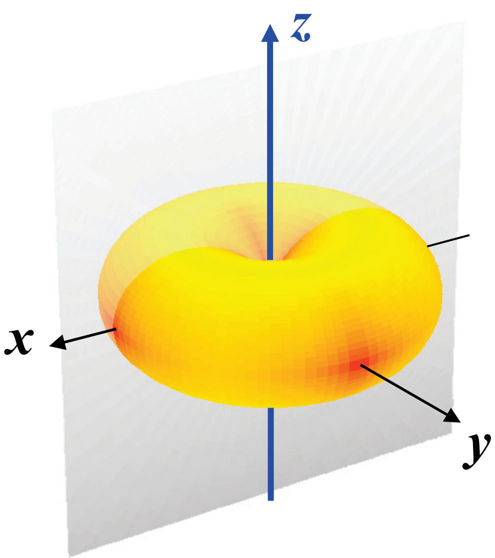


$|J/\psi\rangle = |1, +1\rangle$
or $|1, -1\rangle$

$$\frac{dN}{d\Omega} \propto 1 + \cos^2\vartheta$$

(parity-conserving case)

“Transverse” polarization, like for *real photons*. The word refers to the alignment of the *field* vector, not to the *spin* alignment!



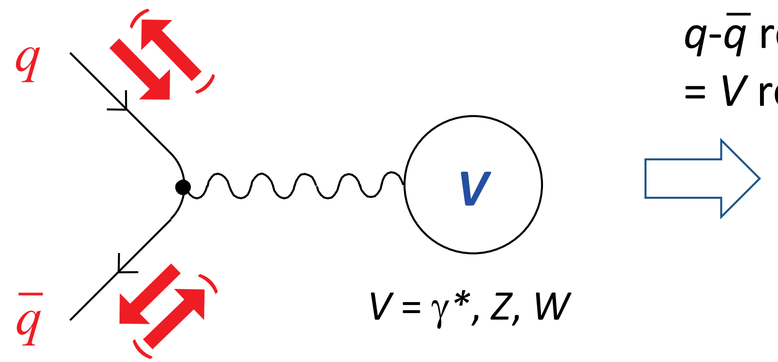
$|J/\psi\rangle = |1, 0\rangle$

$$\frac{dN}{d\Omega} \propto 1 - \cos^2\vartheta$$

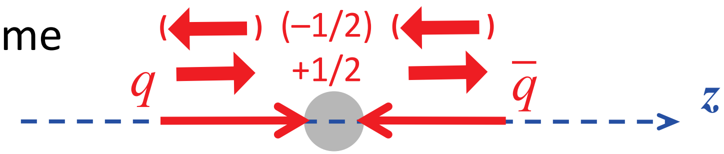
“Longitudinal” polarization

Why "photon-like" polarizations are common

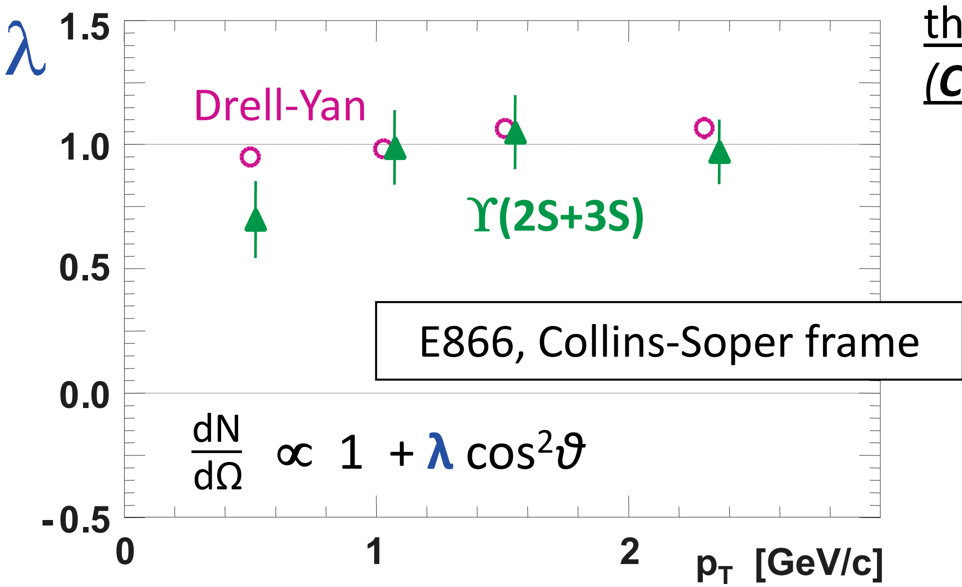
We can apply **helicity conservation at the *production* vertex** to predict that all *vector* states produced in ***fermion-antifermion annihilations* (q-$\bar{q}$ or e^+e^-)** at Born level have *transverse* polarization



q-$\bar{q}$ rest frame
= V rest frame



$|V\rangle = |1, +1\rangle$
$(|1, -1\rangle)$

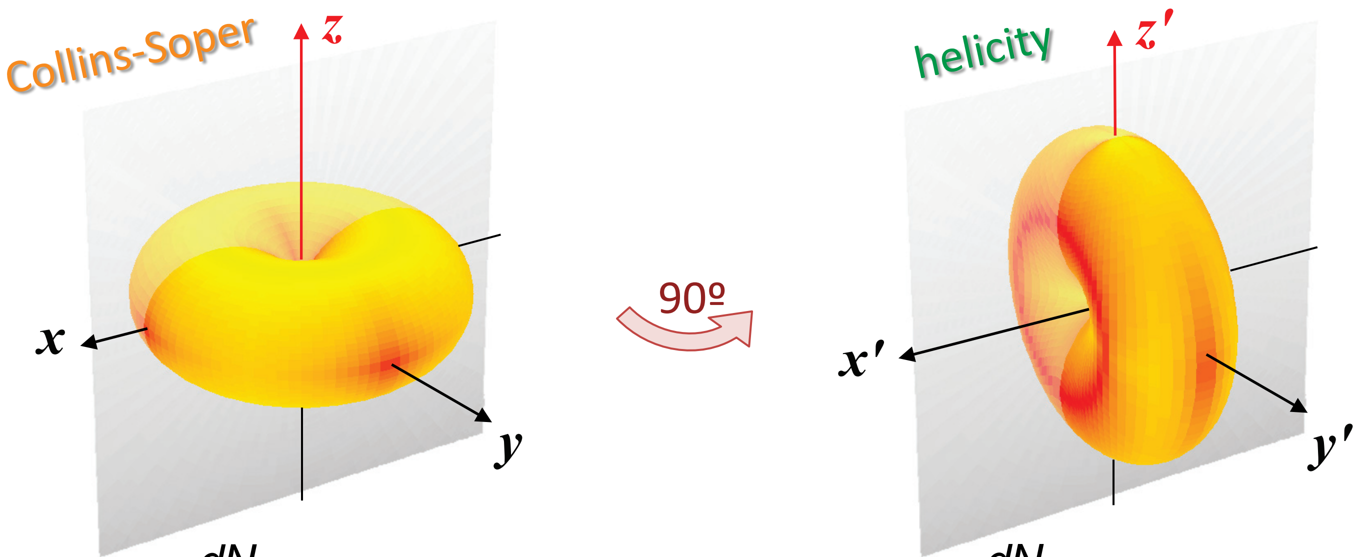


The "natural" polarization axis in this case is the *relative direction of the colliding fermions* ***(Collins-Soper axis)***

Drell-Yan is a paradigmatic case
But not the only one

The observed polarization depends on the frame

For $|p_L| << p_T$, the CS and HX frames differ by a rotation of 90º



$$\frac{dN}{d\Omega} \propto 1 - \cos^2\theta$$

longitudinal

$$|\psi\rangle = |0\rangle$$

(pure state)

$$\frac{dN}{d\Omega} \propto 1 + \cos^2\theta - \sin^2\theta \cos 2\varphi$$

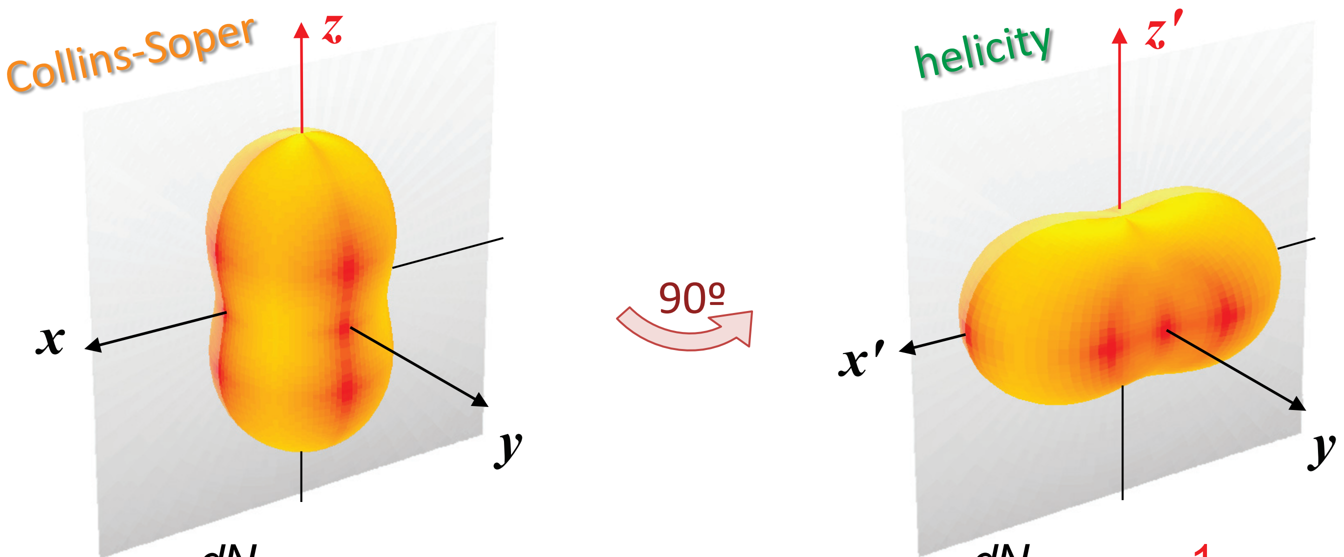
"transverse"

$$|\psi\rangle = \frac{1}{\sqrt{2}}|+1\rangle - \frac{1}{\sqrt{2}}|-1\rangle$$

(mixed state)

The observed polarization depends on the frame

For $|p_L| << p_T$, the CS and HX frames differ by a rotation of 90º



$$\frac{dN}{d\Omega} \propto 1 + \cos^2\theta$$

transverse

$$\frac{dN}{d\Omega} \propto 1 - \frac{1}{3}\cos^2\theta + \frac{1}{3}\sin^2\theta \cos 2\varphi$$

moderately "longitudinal"

$$|\psi\rangle = |+1\rangle \; or \; |-1\rangle$$

(pure state)

$$|\psi\rangle = \frac{1}{2}|+1\rangle + \frac{1}{2}|-1\rangle \, \mathrm{m} \frac{1}{\sqrt{2}}|0\rangle$$

(mixed state)

All reference frames are equal… but some are more equal than others

What do different detectors measure with *arbitrary* frame choices?

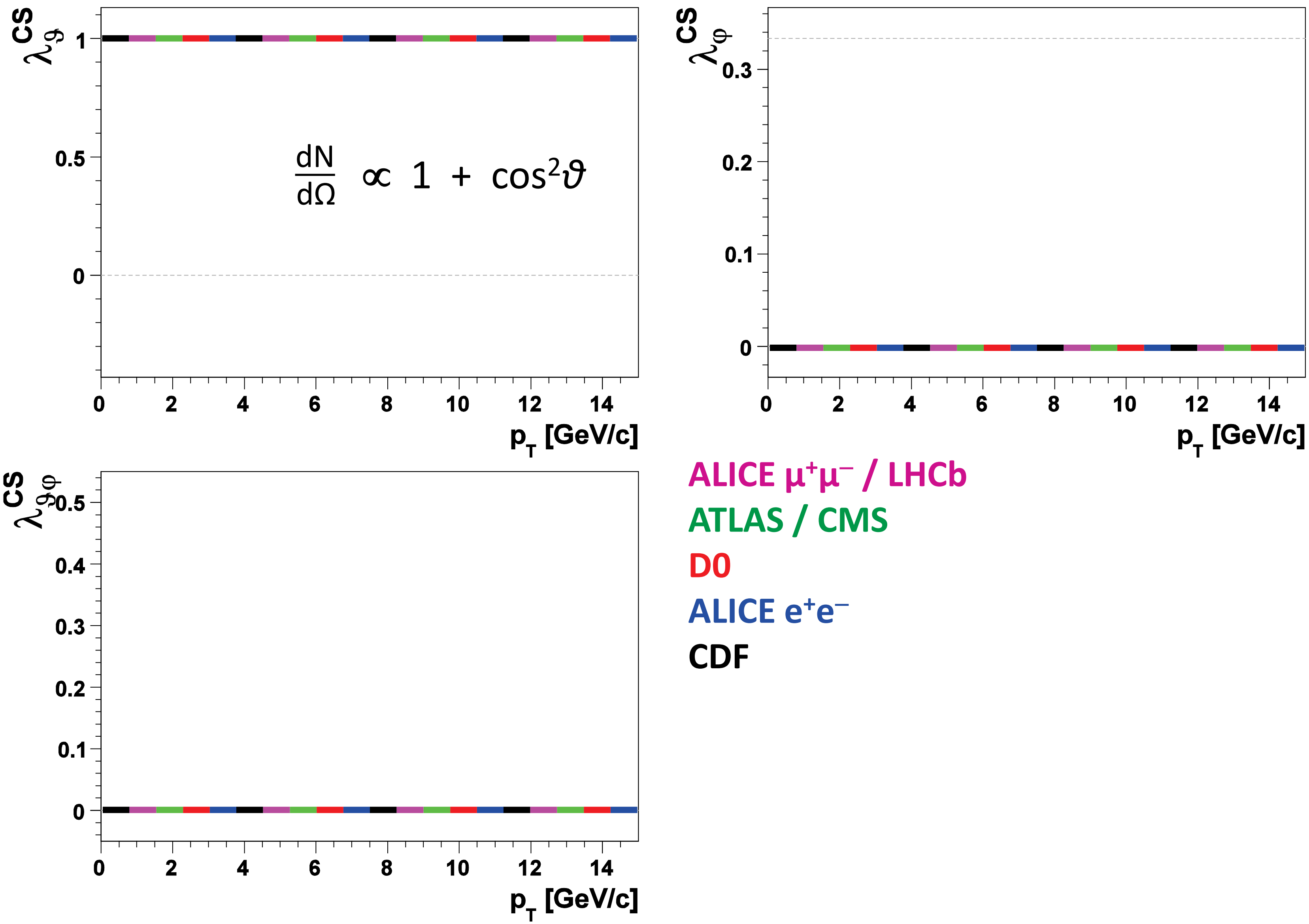
Gedankenscenario:

- **dileptons are fully transversely polarized in the CS frame**
- the decay distribution is measured at the $\Upsilon(1S)$ mass by 6 detectors with different **dilepton acceptances**:

CDF	$|y| < 0.6$
D0	$|y| < 1.8$
ATLAS & CMS	$|y| < 2.5$
ALICE e^+e^-	$|y| < 0.9$
ALICE $\mu^+\mu^-$	$2.5 < y < 4$
LHCb	$2 < y < 4.5$

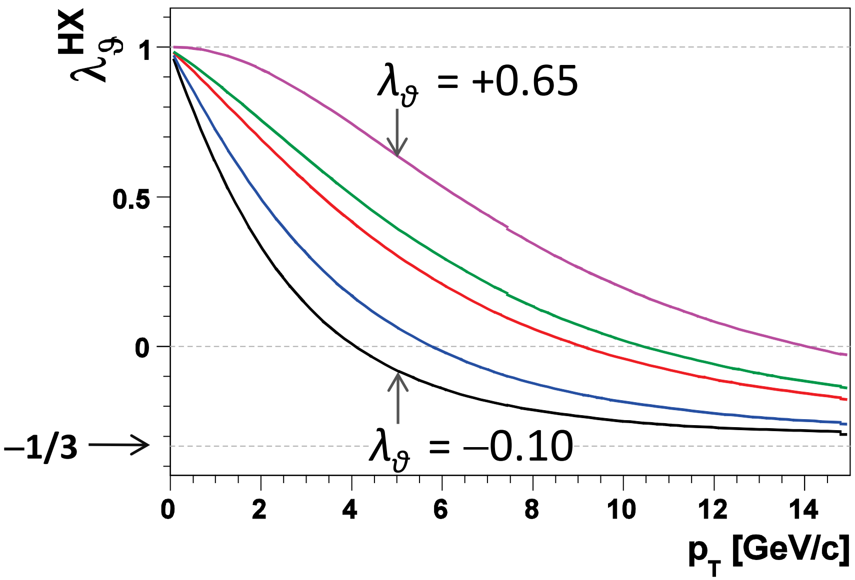
The lucky frame choice

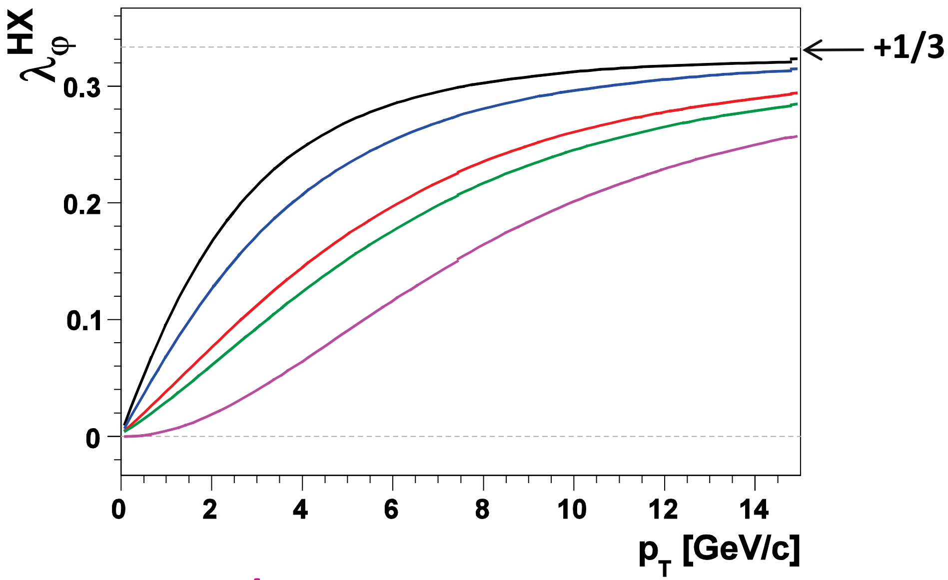
(CS in this case)

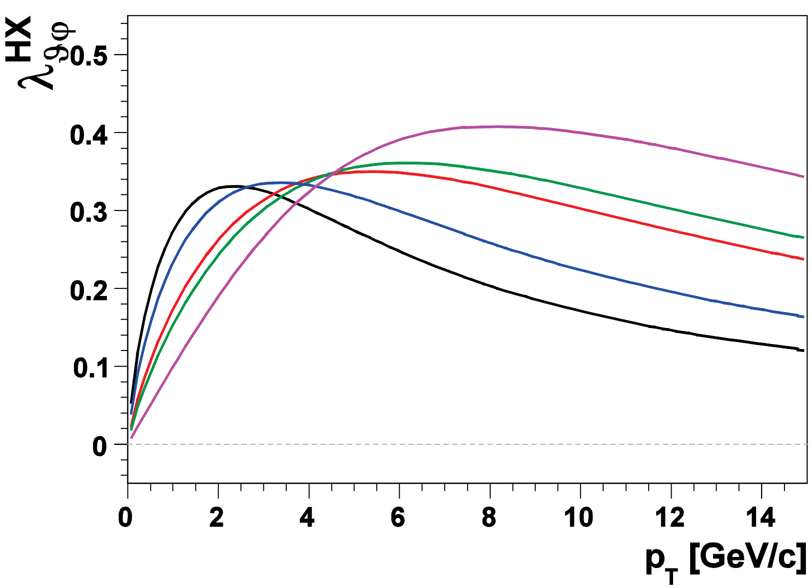


Less lucky choice

(HX in this case)







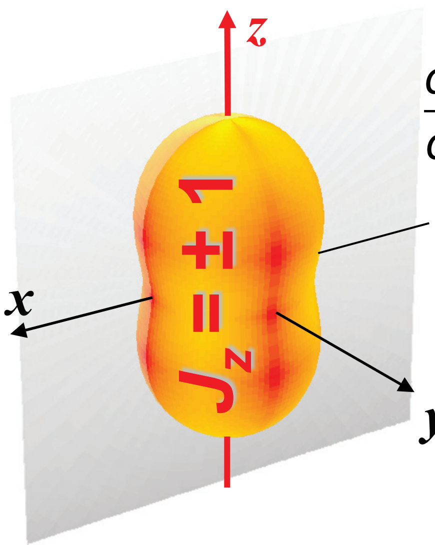
ALICE $\mu^+\mu^-$ / LHCb
ATLAS / CMS
D0
ALICE e^+e^-
CDF

artificial (experiment-dependent!) kinematic behaviour
→ measure in more than one frame!

The azimuthal anisotropy is not a detail

Quarkonium measurements used to ignore the azimuthal component of the distribution. This is a mutilation of the measurement!

Case 1: natural transverse polarization

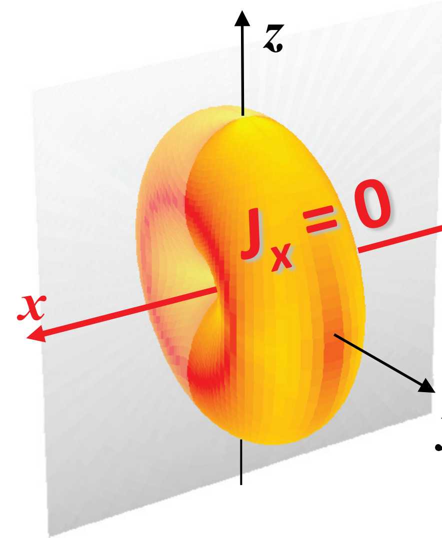


$$\frac{dN}{d\Omega} \propto 1 + \cos^2\theta$$

$\lambda_\vartheta = +1$

$\lambda_\varphi = 0$

Case 2: natural longitudinal polarization, observation frame ⊥ to the natural one



$$\frac{dN}{d\Omega} \propto 1 + \cos^2\theta - \sin^2\theta \cos 2\varphi$$

$\lambda_\vartheta = +1$

$\lambda_\varphi = -1$

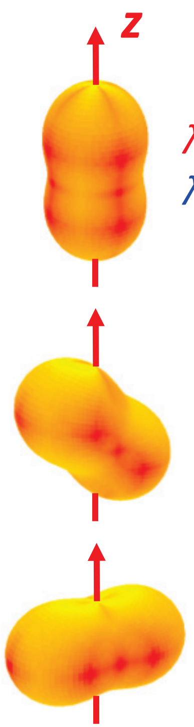
- Two very different (opposite) physical cases, with same λ_ϑ
- distinguishable only by measuring λ_φ (no integration over φ !)

A complementary approach: frame-independent polarization

The *shape* of the distribution is (obviously) frame-invariant (= invariant by rotation)

→ it can be characterized by a frame-independent parameter, writeable e.g. as

$$\tilde{\lambda} = \frac{\lambda_\vartheta + 3\lambda_\varphi}{1 - \lambda_\varphi} \quad \text{or} \quad \mathcal{F} = \frac{1 + \lambda_\vartheta + 2\lambda_\varphi}{3 + \lambda_\vartheta} \quad \left(\mathcal{F} = \frac{1 + \tilde{\lambda}}{3 + \tilde{\lambda}} \right)$$



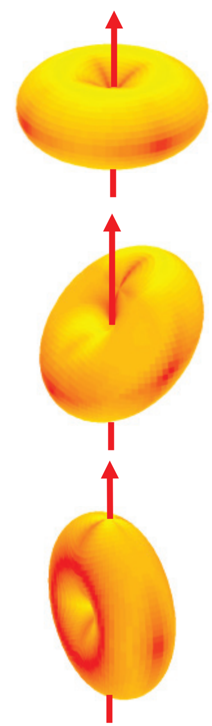
$\lambda_\vartheta = +1$, $\lambda_\varphi = 0$

$\lambda_\vartheta = +1/5$, $\lambda_\varphi = +1/5$

$\lambda_\vartheta = -1/3$, $\lambda_\varphi = +1/3$

$\tilde{\lambda} = +1$

$\mathcal{F} = 1/2$



$\lambda_\vartheta = -1$, $\lambda_\varphi = 0$

$\lambda_\vartheta = -1/3$, $\lambda_\varphi = -1/3$

$\lambda_\vartheta = +1$, $\lambda_\varphi = -1$

$\tilde{\lambda} = -1$

$\mathcal{F} = 0$

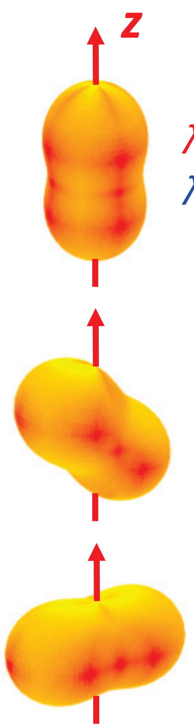
rotations *in the production plane!*

A complementary approach: frame-independent polarization

The *shape* of the distribution is (obviously) frame-invariant (= invariant by rotation)

→ it can be characterized by a frame-independent parameter, writeable e.g. as

$$\tilde{\lambda} = \frac{\lambda_\vartheta + 3\lambda_\varphi}{1 - \lambda_\varphi} \quad \text{or} \quad \mathcal{F} = \frac{1 + \lambda_\vartheta + 2\lambda_\varphi}{3 + \lambda_\vartheta} \quad \left(\mathcal{F} = \frac{1 + \tilde{\lambda}}{3 + \tilde{\lambda}} \right)$$



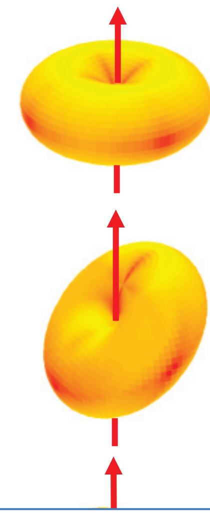
$\lambda_\vartheta = +1$
$\lambda_\varphi = 0$

$\lambda_\vartheta = +1/5$
$\lambda_\varphi = +1/5$

$\lambda_\vartheta = -1/3$
$\lambda_\varphi = +1/3$

$\tilde{\lambda}$ **= +1**

$\mathcal{F}$ **= 1/2**



$\lambda_\vartheta = -1$
$\lambda_\varphi = 0$

$\lambda_\vartheta = -1/3$
$\lambda_\varphi = -1/3$

$\tilde{\lambda}$ **= –1**

$\mathcal{F}$ **= 0**

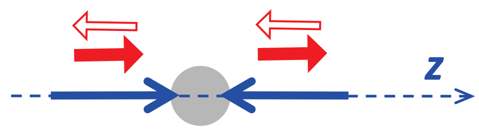
N.B: $\tilde{\lambda}$ is convenient because it is "homogeneous" to λ_ϑ , but **$-1 < \tilde{\lambda} < +\infty$** (no upper bound!), while $\mathcal{F}$ is more conveniently normalized in the range **$0 < \mathcal{F} < 1$**

rotations *in the production plane!*

Frames for Drell-Yan, *Z* and *W* polarizations

- polarization is *always fully* ***transverse***...

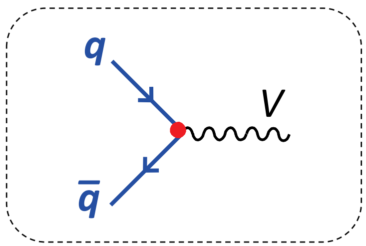
$V = \gamma^*, Z, W$



Due to helicity conservation at the q-$\bar{q}$-V (q-q^*-V) vertex,
$J_z = \pm 1$ along the q-$\bar{q}$ (q-q^*) scattering direction ***z***

- ...but with respect to a ***subprocess-dependent quantization axis***

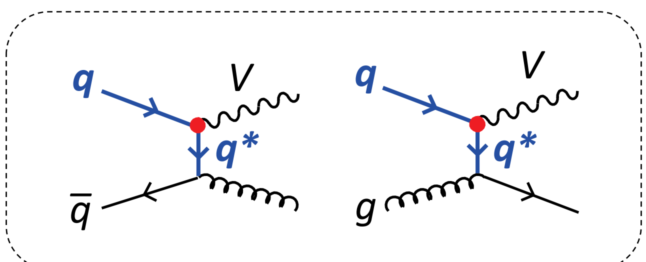
$O(\alpha_S^0)$



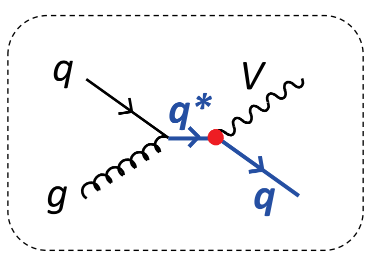
z = relative dir. of incoming q and $q$bar
(~ **Collins-Soper frame**)

important only up to $p_T = \mathcal{O}$(parton k_T)

$O(\alpha_S^1)$
QCD
corrections



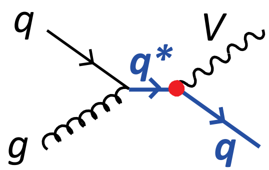
z = dir. of *one* incoming quark
(~ **Gottfried-Jackson frame**)



z = dir. of outgoing q
(= **parton-cms-helicity** $\approx$ **lab-cms-helicity**)

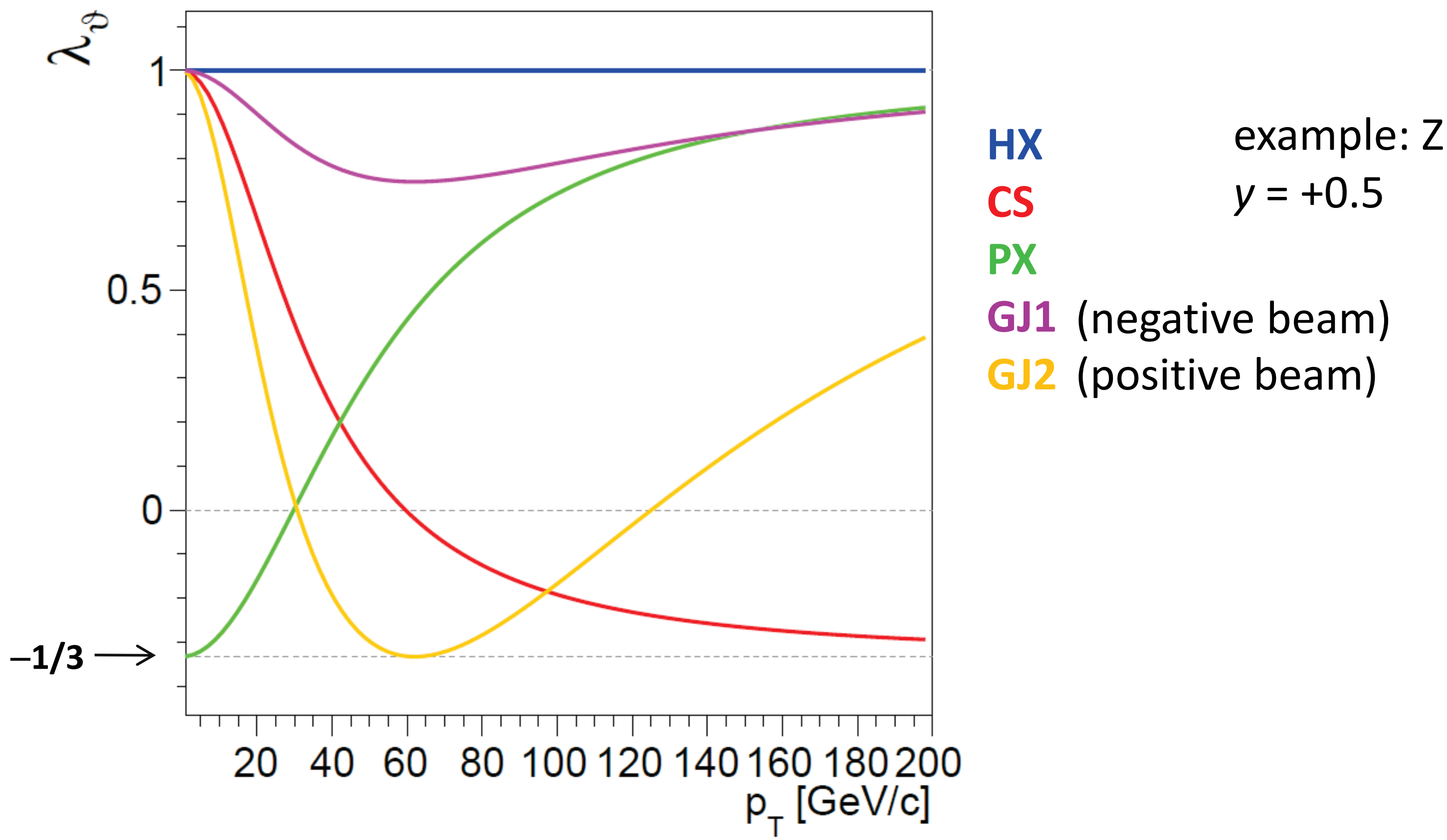
"Optimal" frames for Drell-Yan, *Z* and *W* polarizations

Different subprocesses have different "natural" quantization axes



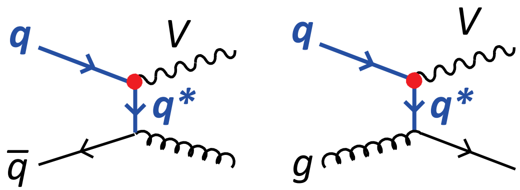
For ***s*-channel processes** the natural axis is the direction of the outgoing quark (= direction of dilepton momentum)

→ optimal frame (= maximizing polar anisotropy): **HX** (neglecting parton-parton-cms vs proton-proton-cms difference!)



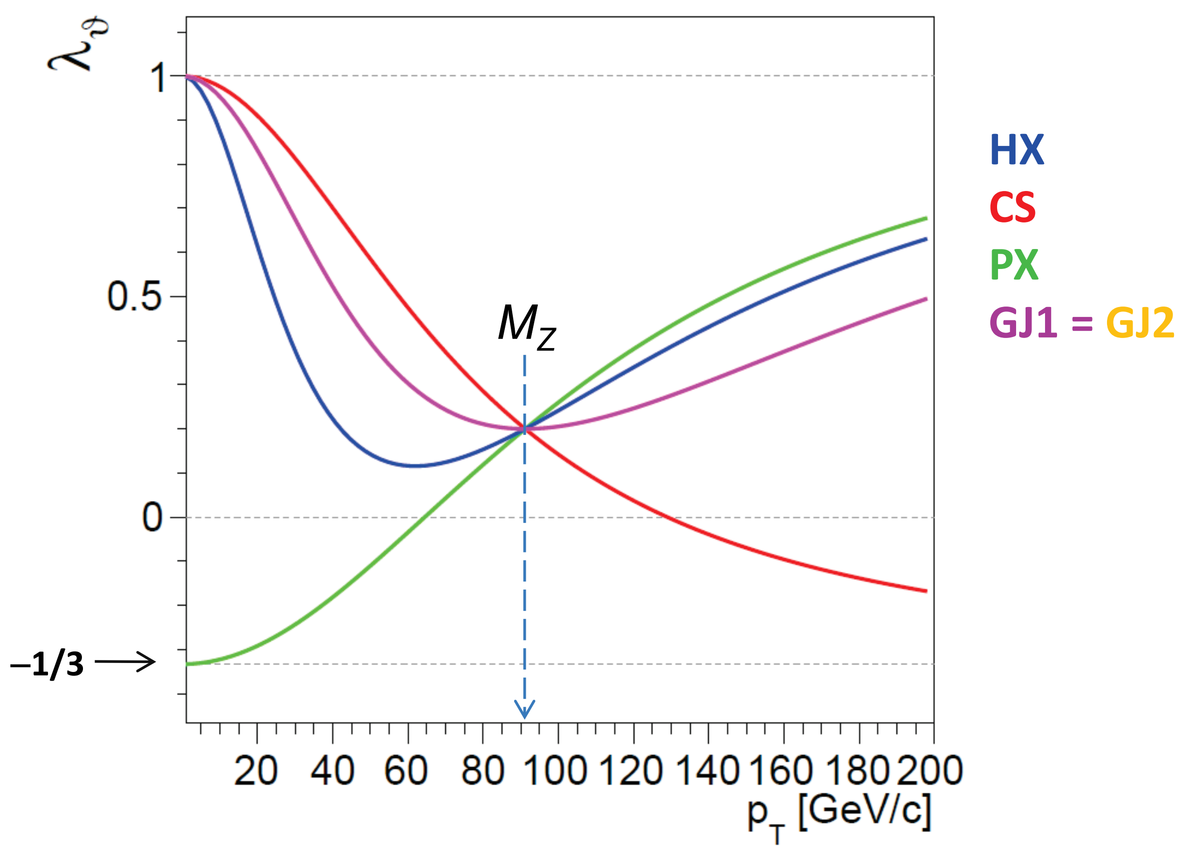
“Optimal” frames for Drell-Yan, *Z* and *W* polarizations

Different subprocesses have different “natural” quantization axes



For ***t*- and *u*-channel processes** the natural axis is the direction of either one or the other incoming parton (~ “Gottfried-Jackson” axes)

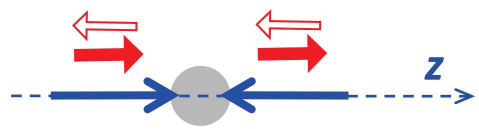
→ optimal frame: geometrical average of GJ1 and GJ2 axes = **CS ($p_T < M$)** and **PX ($p_T > M$)**



Rotation-invariant Drell-Yan, *Z* and *W* polarizations

- polarization is *always fully* ***transverse***...

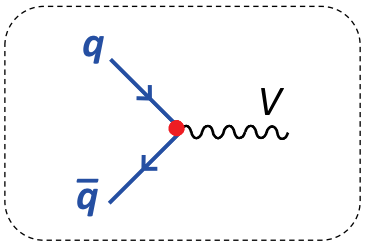
$V = \gamma^*, Z, W$



Due to helicity conservation at the q-$\bar{q}$-V (q-q^*-V) vertex,
$J_z = \pm 1$ along the q-$\bar{q}$ (q-q^*) scattering direction ***z***

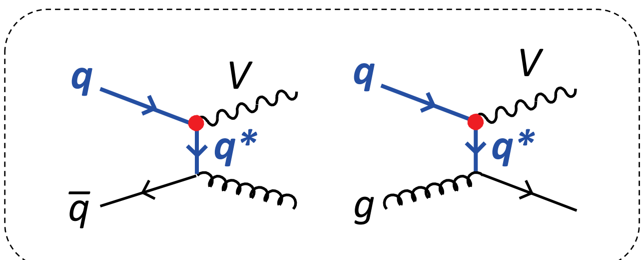
- ...but with respect to a ***subprocess-dependent quantization axis***

$O(\alpha_S^0)$

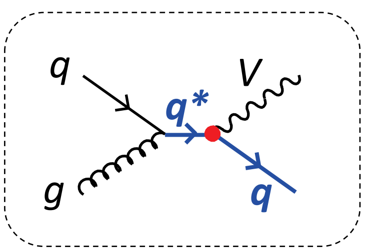


"natural" z = relative dir. of *q* and *q*bar
→ λ_ϑ("CS") = +1
wrt ***any*** axis: $\tilde{\lambda} = +1$

$O(\alpha_S^1)$
(LO) QCD corrections



z = dir. of *one* incoming quark
→ λ_ϑ("GJ") = +1
$\tilde{\lambda} = +1$



z = dir. of outgoing *q*
→ λ_ϑ("HX") = +1
$\tilde{\lambda} = +1$

N.B.: $\tilde{\lambda} = +1$ in both pp-HX and qg-HX frames!

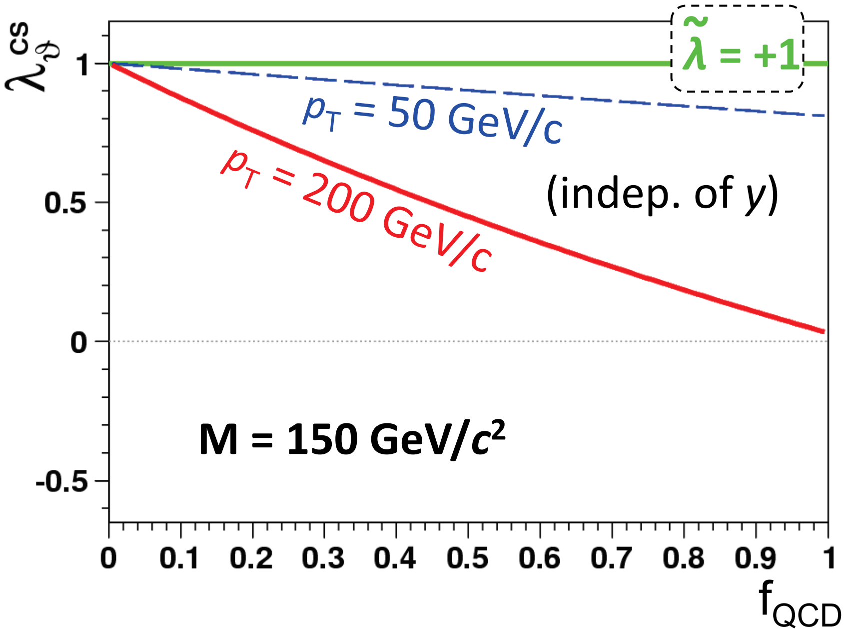
$\tilde{\lambda} = +1$
any frame

In all these cases the *q*-*q*-*V* lines are in the production plane (***"planar" processes***)
The CS, GJ, pp-HX and qg-HX axes only differ by a rotation in the production plane

λ_ϑ vs $\tilde{\lambda}$

Example: ***Z/γ*/W* polarization** (CS frame) as a function of contribution of LO QCD corrections:

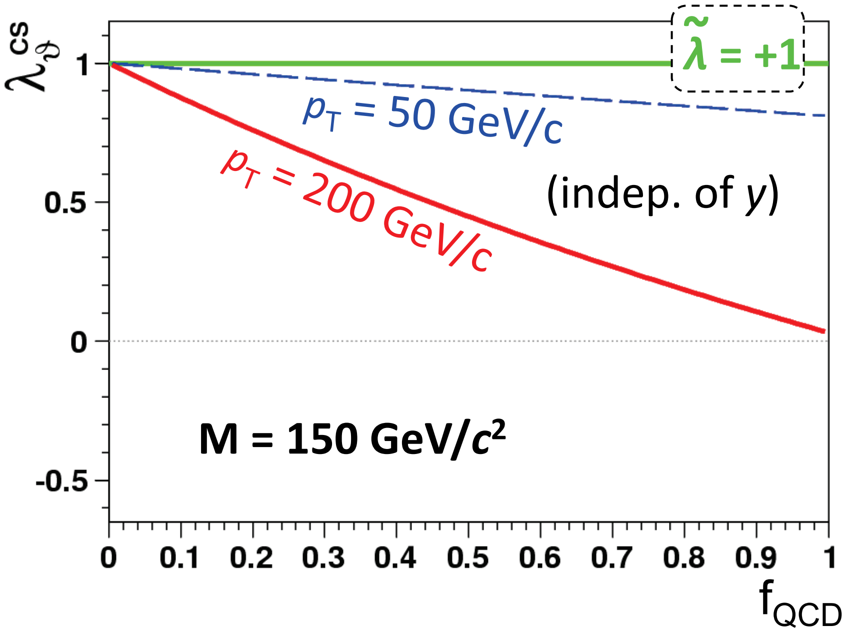
Case 1: dominating ***q*-*q*bar** QCD corrections



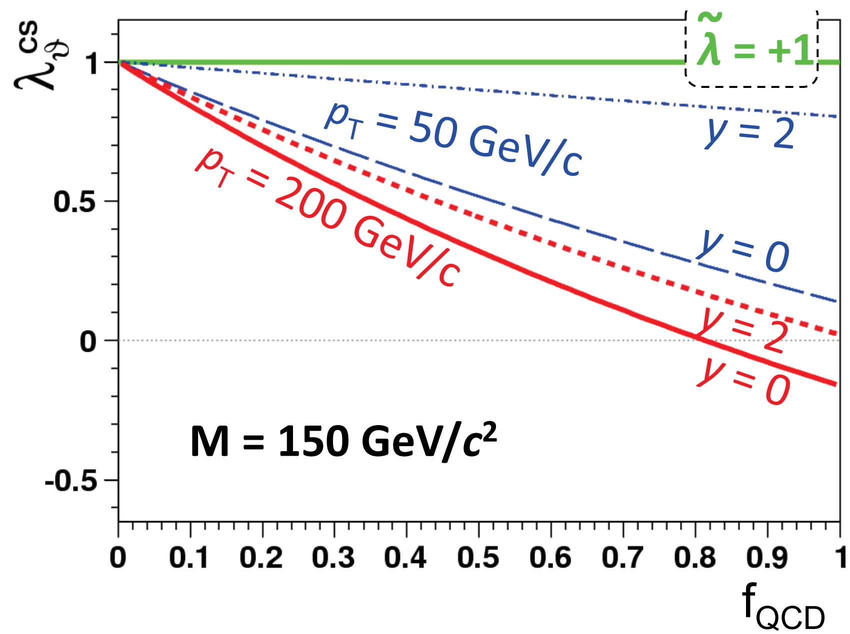
λ_ϑ vs $\tilde{\lambda}$

Example: ***Z/γ*/W* polarization** (CS frame) as a function of contribution of LO QCD corrections:

Case 1: dominating ***q-qbar*** QCD corrections



Case 2: dominating ***q-g*** QCD corrections

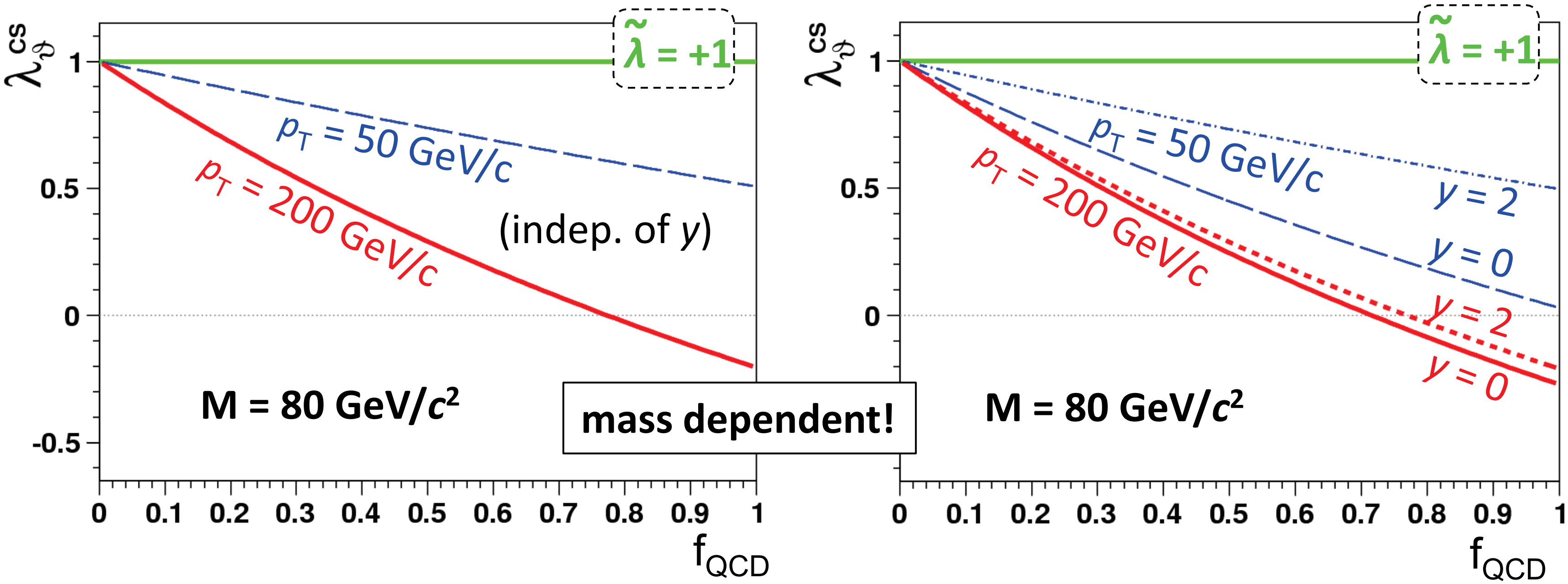


λ_ϑ vs $\tilde{\lambda}$

Example: ***Z/γ*/W* polarization** (CS frame) as a function of contribution of LO QCD corrections:

Case 1: dominating ***q-qbar*** QCD corrections

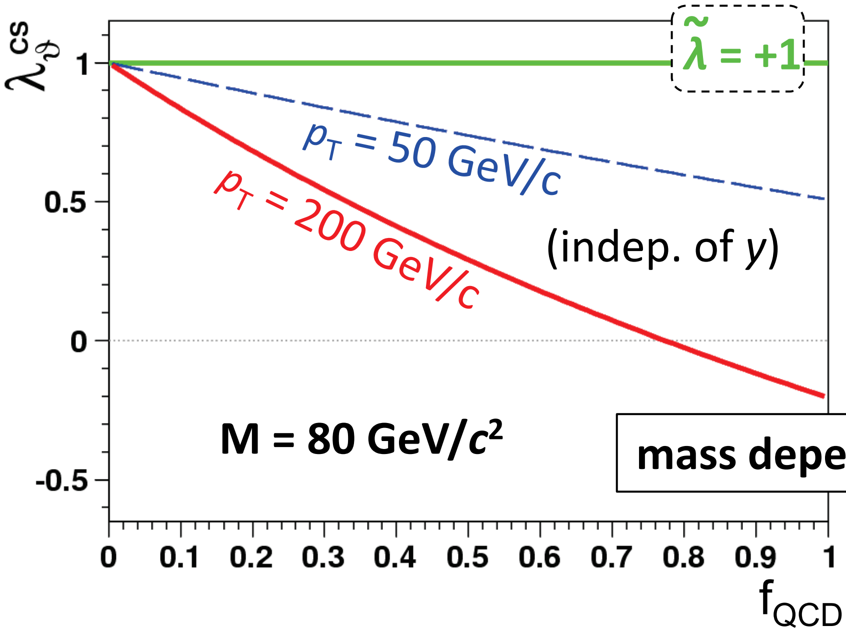
Case 2: dominating ***q-g*** QCD corrections



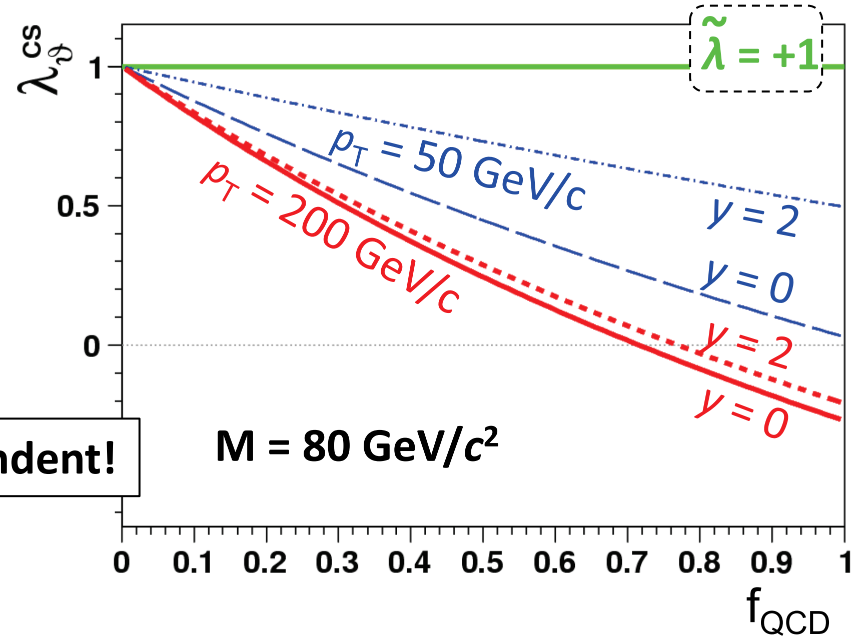
λ_ϑ vs $\tilde{\lambda}$

Example: ***Z/γ*/W* polarization** (CS frame) as a function of contribution of LO QCD corrections:

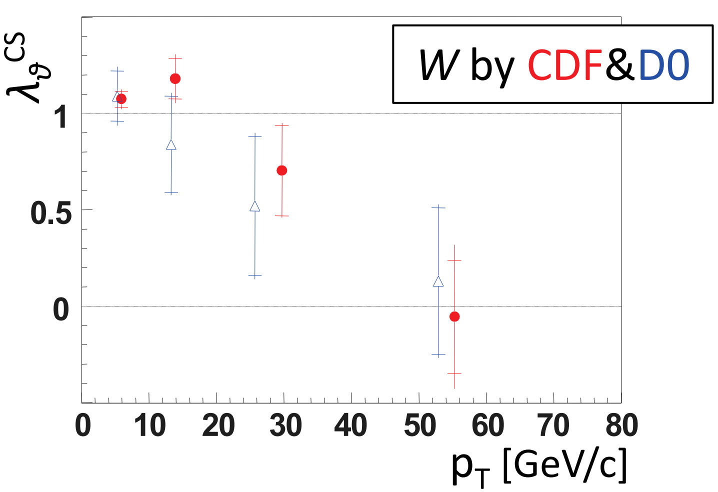
Case 1: dominating ***q-qbar*** QCD corrections



Case 2: dominating ***q-g*** QCD corrections



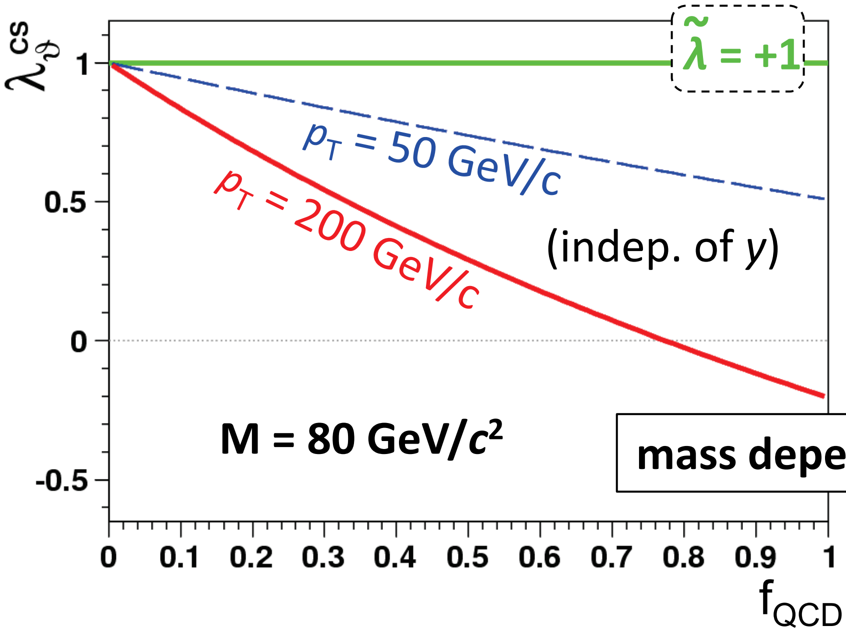
mass dependent!



λ_ϑ vs $\tilde{\lambda}$

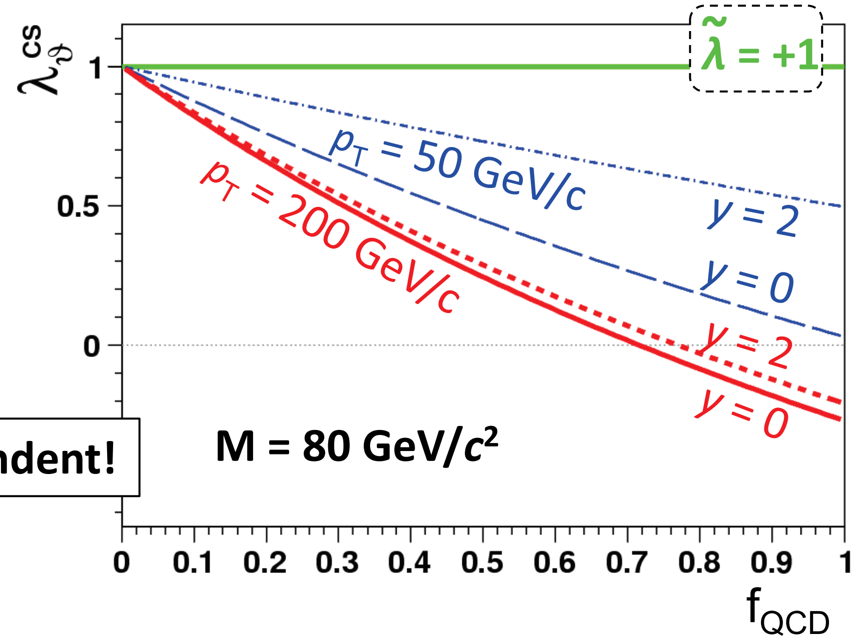
Example: ***Z/γ*/W* polarization** (CS frame) as a function of contribution of LO QCD corrections:

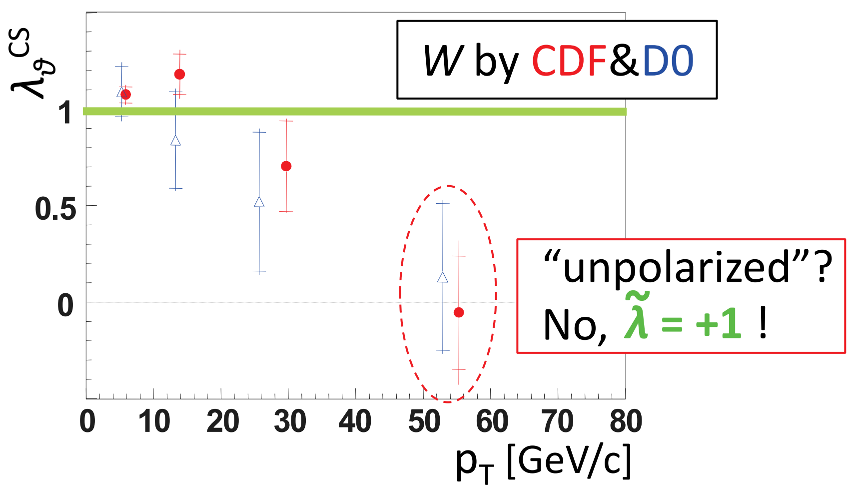
Case 1: dominating ***q-qbar*** QCD corrections



mass dependent!

Case 2: dominating ***q-g*** QCD corrections

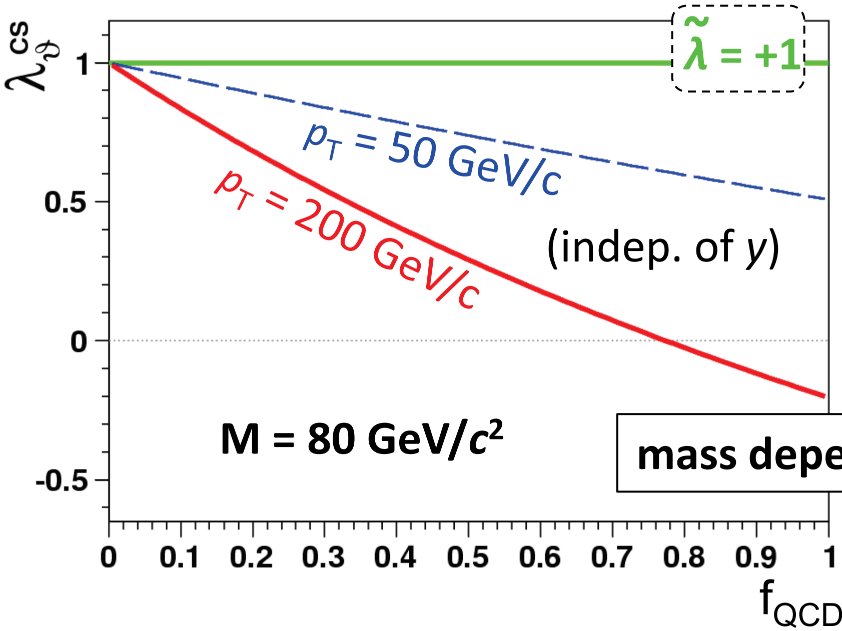




λ_ϑ vs $\tilde{\lambda}$

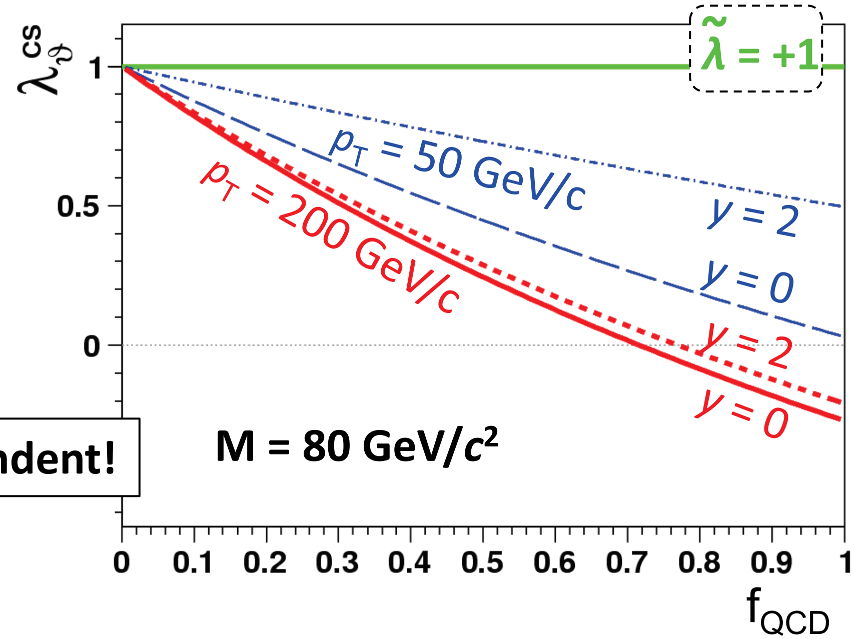
Example: ***Z/γ*/W* polarization** (CS frame) as a function of contribution of LO QCD corrections:

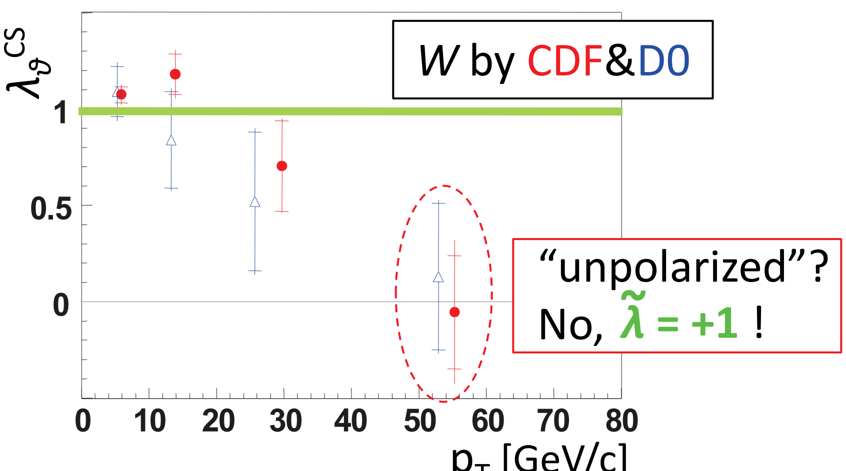
Case 1: dominating ***q-qbar*** QCD corrections



mass dependent!

Case 2: dominating ***q-g*** QCD corrections





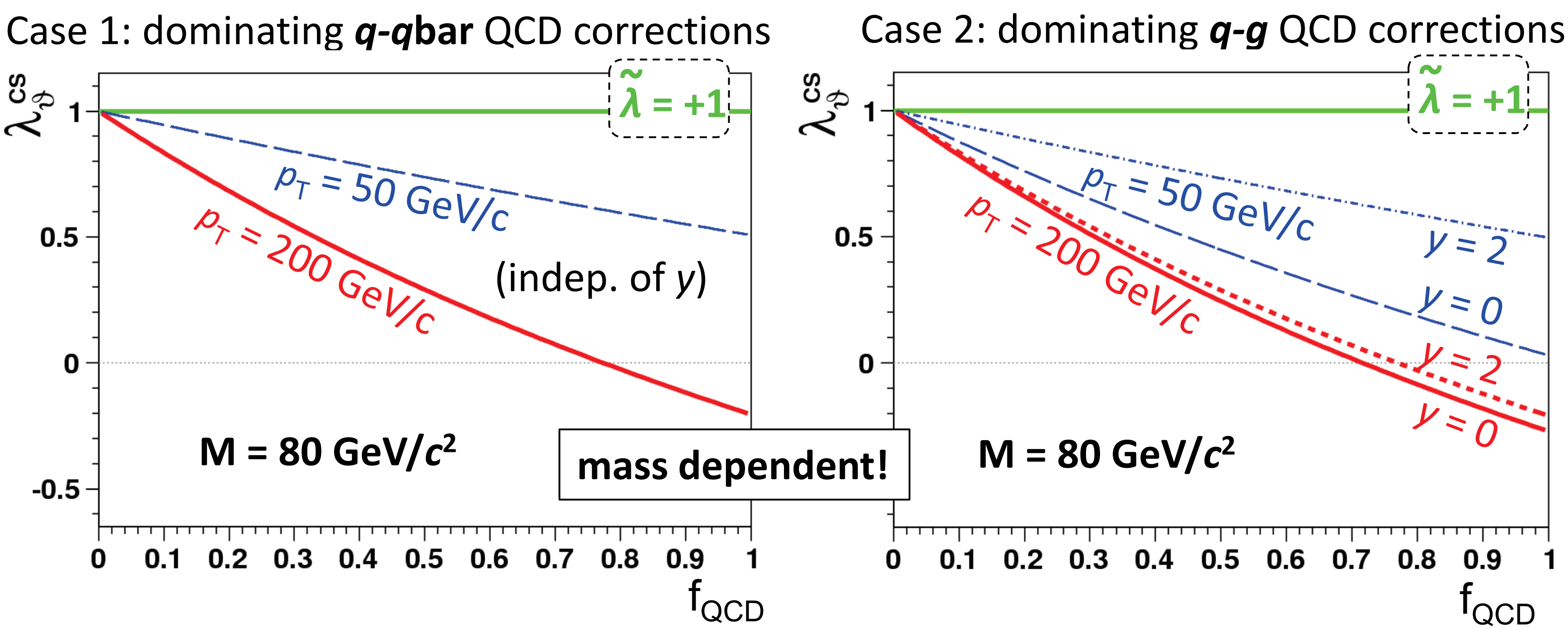
λ_ϑ

- depends on p_T , y and mass
  → by integrating we lose significance
- is far from being maximal
- depends on process admixture
  → need pQCD and PDFs

$\tilde{\lambda}$ is constant, maximal and independent of process admixture

λ_ϑ vs $\tilde{\lambda}$

Example: ***Z/γ*/W* polarization** (CS frame) as a function of contribution of LO QCD corrections:



On the other hand, $\tilde{\lambda}$ forgets about the direction of the quantization axis.
This information is crucial if we want to **disentangle the *qg* contribution**, the only one resulting in a ***rapidity-dependent*** $\boldsymbol{\lambda_\vartheta}$

Measuring λ_ϑ(CS) *as a function of rapidity* gives information on the gluon content of the proton

The Lam-Tung relation

PHYSICAL REVIEW D VOLUME 21, NUMBER 9 1 MAY 1980

Parton-model relation without quantum-chromodynamic modifications in lepton pair production

C. S. Lam
Wu-Ki Tung

...ch more to the quar... QCD modifications in LPP than jus... ...egrated Drell-Yan cross-section formula. ...lepton angular distributions are controlled by structure functions which obey parton-model relations[3,4] similar to those between F_1 and F_2 in deep-inelastic scattering (DIS). How are these relations affected by perturbative QCD corrections? The answer to this question is quite surprising: At least one of these relations—the exact counterpart of the Callan-Gross[5] relations—is not modified at all by first-order QCD corrections, although individual terms in this relation ...y be subject to large corrections. In the re... ...note, we spell out explicitly the pa... ...as the contrast betw...

...cross-section formula [essentially W_μ^μ, c... Eq. (2)]. This appears to be a rather remarkable result; we are not aware of any other parton-model result which is not affected by QCD corrections. For this reason, we sketch in the ...endix a derivation of Eq. (5) from the dia... ...which is more di...

...rms of helicity structure fu... ...lation takes the form $W_L = 2W_{\Delta\Delta}$, Eq. (7). ...though for LPP, the helicity structure functions depend on the choice of coordinate axes[4] (e.g., Gottfried-Jackson, Collins-Soper, etc.), *this relation remains frame independent*—i.e., if the QCD–quark-parton model is correct, the two structure functions W_L and $W_{\Delta\Delta}$ must be related by Eq. (7), for *any* choice of axes in the lepton-pair center-of-mass frame. This strong result again demonstrates the significance of this relation.

We know the angular distribution of the le...

The Lam-Tung relation

PHYSICAL REVIEW D **76**, 074006 (2007)

Transverse momentum dependence of the angular distribution of the Drell-Yan process

Edmond L. Berger,[1,*] Jian-Wei Qiu,[1,2,†] and Ricardo A. Rodriguez-Pedraza[2,‡]

We calculate the transverse momentum $Q_\perp$ dependence of the helicity structure functions for the hadroproduction of a massive pair of leptons with pair invariant mass Q. These structure functions determine the angular distribution of the leptons in the pair rest frame. Unphysical behavior in the region $Q_\perp \to 0$ is seen in the results of calculations done at fixed order in QCD perturbation theory. We use current conservation to demonstrate that the unphysical inverse-power and $\ln(Q/Q_\perp)$ logarithmic divergences in three of the four independent helicity structure functions share the same origin as the divergent terms in fixed-order calculations of the angular-integrated cross section. We show that the resummation of these divergences to all orders in the strong coupling strength α_s can be reduced to the solved problem of the resummation of the divergences in the angular-integrated cross section, resulting in well-behaved predictions in the small $Q_\perp$ region. Among other results, we show the resummed part of the helicity structure functions preserves the Lam-Tung relation between the longitudinal and double spin-flip structure functions as a function of $Q_\perp$ to all orders in α_s.

The Lam-Tung relation

A fundamental result of the theory of vector-boson polarizations (Drell-Yan, directly produced *Z* and *W*) is that, at leading order in perturbative QCD,

$$\lambda_\vartheta + 4\lambda_\varphi = 1$$ independently of the polarization frame

Lam-Tung relation

This identity was considered as a surprising result

Today we know that it is only a *special* case of general frame-independent polarization relations, corresponding to a *transverse* intrinsic polarization:

$$\tilde{\lambda} = \frac{\lambda_\vartheta + 3\lambda_\varphi}{1 - \lambda_\varphi} = +1 \quad \Rightarrow \lambda_\vartheta + 4\lambda_\varphi = 1$$

It is, therefore, **not properly a "QCD" relation**, but a consequence of

1) rotational invariance

2) properties of the **quark-photon/Z/W couplings** (helicity conservation)

Beyond the Lam-Tung relation

Even when the Lam-Tung relation is violated,

$\tilde{\lambda}$ can always be defined and is *always frame-independent*

$\rightarrow$ any violation, $\tilde{\lambda} - 1 \neq 0$, is quantitatively frame-independent

N.B.:

the quantity $4\lambda_\varphi - (1 - \lambda_\theta)$, or $2\nu - (1 - \lambda)$ in an alternative notation, often adopted to estimate violations of the LT relation, is *not* frame independent!

$A_2 - A_0$ is frame independent:

$$A_2 - A_0 = 4\mathcal{F} - 2 = 2\,\frac{\tilde{\lambda} - 1}{3 + \tilde{\lambda}}$$

Beyond the Lam-Tung relation

Even when the Lam-Tung relation is violated,
$\tilde{\lambda}$ can always be defined and is *always frame-independent*
$\rightarrow$ any violation, $\tilde{\lambda} - 1 \neq 0$, is quantitatively frame-independent

$$\mathcal{F} = \frac{1+\tilde{\lambda}}{3+\tilde{\lambda}}$$

$\tilde{\lambda} = +1$ ($\mathcal{F} = 1/2$) → Lam-Tung. New interpretation: only ***vector boson – quark – quark*** couplings (in planar processes) $\rightarrow$ automatically verified in DY at QED & LO QCD levels and in several higher-order QCD contributions

$\tilde{\lambda} = +1 - \mathcal{O}(0.1)$ ($\mathcal{F} = 1/2 - \mathcal{O}(0.1)$) with $\tilde{\lambda} \rightarrow +1$ for $p_T \rightarrow 0$ ($\mathcal{F} \rightarrow 1/2$) → same, "ordinary" vector-boson – quark – quark couplings, but in ***non-planar* processes** (*higher-order* contributions)

OR

smearing due to intrinsic **parton k_T**

$-1 < \tilde{\lambda} << +1$ ($0 < \mathcal{F} << 1/2$)

$+1 < \tilde{\lambda} < +\infty$ ($1/2 < \mathcal{F} < 1$)

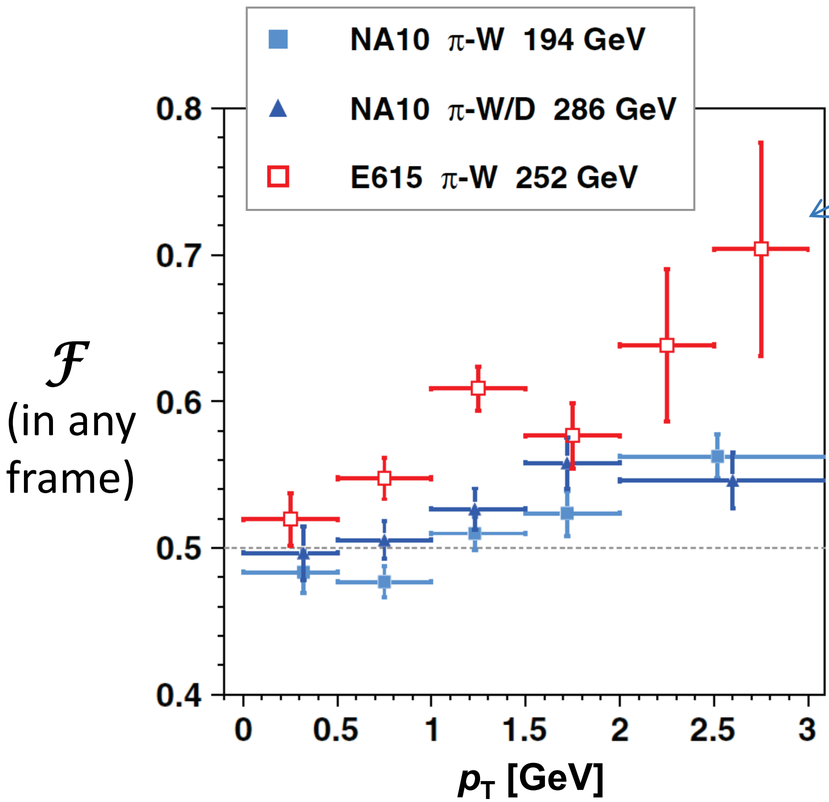
→ contribution of ***different/new couplings or processes*** (e.g.: Z from Higgs, W from top, triple $ZZ\gamma$ coupling, higher-twist effects in DY production, etc…)

$\tilde{\lambda} < -1$ ($\mathcal{F} < 0$ or $\mathcal{F} > 1$) → experimental mistake

Example 1

$+1 < \tilde{\lambda} < +\infty$
($1/2 < \mathcal{F} < 1$)

$\rightarrow$ higher-twist effects in DY production



very *large* effect,
progressively approaching the
physical limit $\mathcal{F}$=1

[$\mathcal{F}$=1 represents a very peculiar case:
fully *longitudinal* polarization
along the axis
perpendicular to the production plane]

$\rightarrow$ not a mere
"higher-order correction"!

See discussion in
P. Faccioli et al., Phys. Rev. D 83, 056008 (2011)

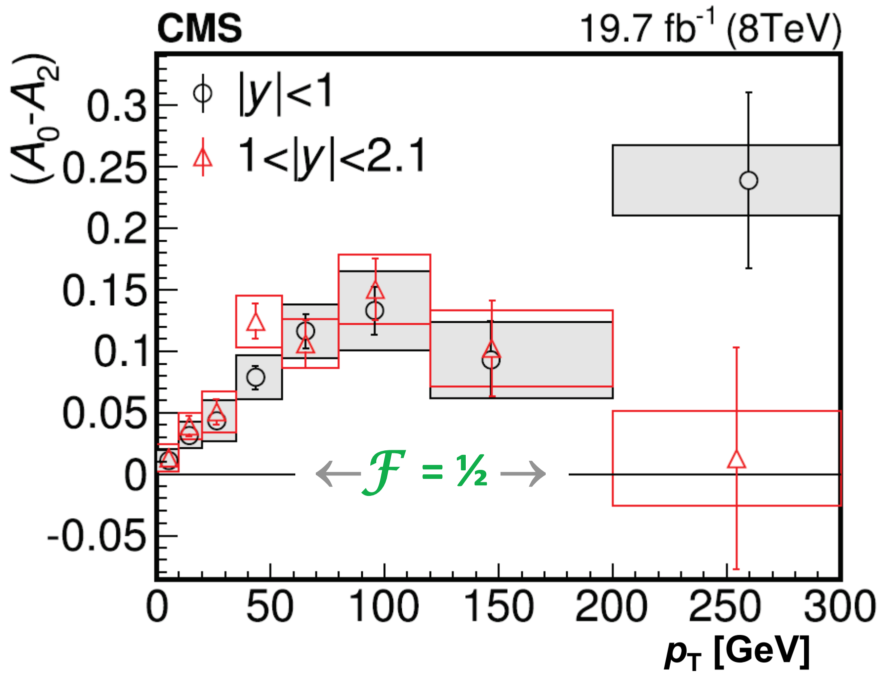
Example 2

CMS measurement of $Z \to \ell^+\ell^-$ [PLB 750, 154 (2015)], using parametrization of slide 11

$$A_2 = \frac{8\lambda_\varphi}{3+\lambda_\vartheta} \qquad A_0 = 2\,\frac{1-\lambda_\vartheta}{3+\lambda_\vartheta}$$

$$A_0 - A_2 = 2 - 4\mathcal{F}$$

$$A_0 - A_2 > 0 \quad \to \quad \mathcal{F} < 1/2$$



This is the case

$\mathcal{F}$ = **1/2** – $\mathcal{O}(0.1)$

with $\mathcal{F} \to$ **1/2** for $p_T \to 0$

Parton-k_T effects are negligible for Z production: partons in the Z rest frame have k_L > 50 GeV [see quantitative description in EPJ C 69, 657 (2010)]

The formulation of the LT-violation in terms of $\mathcal{F}$, with its known physical limits, allows us to *quantify* the magnitude of the effect:

$A_0 - A_2 \approx 0.1$ corresponds to $\mathcal{F} - ½ \approx 0.025$

relatively small!

→ we are "seeing" the contribution of higher-order, non-planar processes

Further reading

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- P. Faccioli, C. Lourenço and J. Seixas, *New approach to quarkonium polarization studies*, Phys. Rev. D 81, 111502(R) (2010)
- P. Faccioli, C. Lourenço, J. Seixas and H.K. Wöhri, *Towards the experimental clarification of quarkonium polarization*, Eur. Phys. J. C 69, 657 (2010)
- P. Faccioli, C. Lourenço, J. Seixas and H. K. Wöhri, *Rotation-invariant observables in parity-violating decays of vector particles to fermion pairs*, Phys. Rev. D 82, 096002 (2010)
- P. Faccioli, C. Lourenço, J. Seixas and H. K. Wöhri, *Model-independent constraints on the shape parameters of dilepton angular distributions*, Phys. Rev. D 83, 056008 (2011)
- P. Faccioli, C. Lourenço, J. Seixas and H. K. Wöhri, *Determination of χ_c and χ_b polarizations from dilepton angular distributions in radiative decays*, Phys. Rev. D 83, 096001 (2011)
- P. Faccioli and J. Seixas, *Observation of χ_c and χ_b nuclear suppression via dilepton polarization measurements*, Phys. Rev. D 85, 074005 (2012)
- P. Faccioli, *Questions and prospects in quarkonium polarization measurements from proton-proton to nucleus-nucleus collisions*, invited "brief review", Mod. Phys. Lett. A Vol. 27 N. 23, 1230022 (2012)
- P. Faccioli and J. Seixas, *Angular characterization of the Z Z → 4ℓ background continuum to improve sensitivity of new physics searches*, Phys. Lett. B 716, 326 (2012)
- P. Faccioli, C. Lourenço, J. Seixas and H. K. Wöhri, *Minimal physical constraints on the angular distributions of two-body boson decays*, Phys. Rev. D 88, 031901(R) (2013)