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# NLO QCD analysis of COMPASS'06 data on pion multiplicities

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# OUTLINE

- Why the Fragmentation Functions (FFs) are so important for the determination of the *polarized* PDFs ?

- The present status of fragmentation funcrtions (FFs)

→ The strange quark polarization puzzle

- NLO QCD analysis of the *COMPASS'06* deuteron data on pion multiplicities → new *pion FFs*

- Consistency between COMPASS and HERMES data ?

- Summary

**Due to the lack of the charged current neutrino data *only* the sums of pol. *PDFs* can be determined from the polarized inclusive DIS**

$$(\Delta q + \Delta\bar{q})(\mathrm{x},\, \mathrm{Q}^2)$$

$$g_1(x,Q^2)_{LT} = \frac{1}{2}\sum_q^{N_f} e_q^2 [(\Delta q + \Delta\bar{q}) \otimes (1 + \frac{\alpha_s(Q^2)}{2\pi}\delta C_q) + \frac{\alpha_s(Q^2)}{2\pi}\Delta G \otimes \frac{\delta C_G}{N_f}]$$

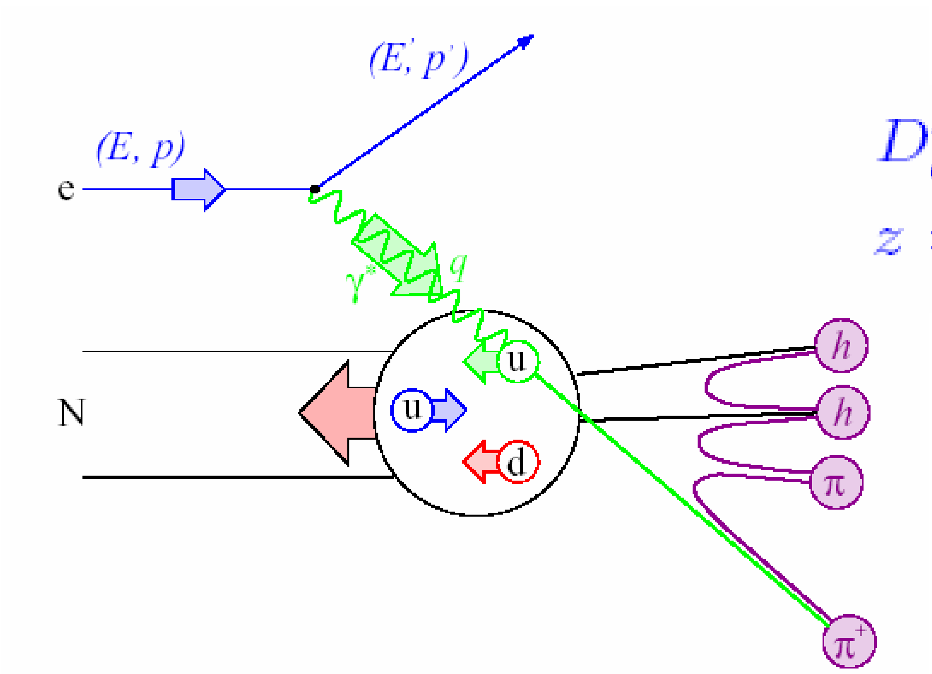
**LT – leading twist QCD contribution**

$\delta C_q, \delta C_G - Wilson$ coefficient functions

**In order to extract separately $\Delta q$ and $\Delta\bar{q}$ another reactions are needed. One possibility is to use the polarized semi-inclusive lepton-hadron processes.**

# Semi-inclusive processes

$\Longrightarrow$ allow to separate $\Delta q$ and $\Delta \overline{q}$




## Fragmentation functions

$D_q^h$ from quark $q$ into hadron $h$

$z = \frac{E_h}{\nu}$ energy fraction carried by $h$

$$A_1^h(x, z, Q^2) = \frac{g_1^h - \gamma^2 g_2^h}{F_1^h}$$

**In LO QCD:** $$A_1^h(x, z, Q^2) \sim \frac{g_1^h}{F_1^h} = \frac{\sum_{q,\bar{q}} e_q^2 \, \Delta q(x,Q^2) \, D_q^h(z,Q^2)}{\sum_{q,\bar{q}} e_q^2 \, q(x,Q^2) \, D_q^h(z,Q^2)}$$

**New physical quantities appear – fragmentation functions** $D^h_{q,\bar{q}}(z,Q^2)$ **. Due to a different fragmentation of** $q$ and $\bar{q}$, $\Delta q$ and $\Delta\bar{q}$ **can be fixed separately.**

**To determine correctly the *polarized* parton densities from a combined analysis of *polarized* DIS and SIDIS data good knowledge of FFs is very important !**

**There are 3 modern versions of FFs based on an analysis of different data sets:**

- **Hirai et al. (HKNS), from SI e+e- annihilation data,** ***PR D75 (2007) 094009***
- **Albino et al. (AKK), from $e^+e^- \oplus$ SI pp (RHIC) data,** ***Nucl. Phys. B803 (2008) 42***
- **De Florian et al. (DSS), from $e^+e^- \oplus$ SI pp $\oplus$ SIDIS unpolarized data (*PR D75, 114010; D76, 074033 (2007)*)**

**Unpublished HERMES'05 which differ from the final HERMES'13 data (*Phys.Rev. D87 (2013) 074029* )**

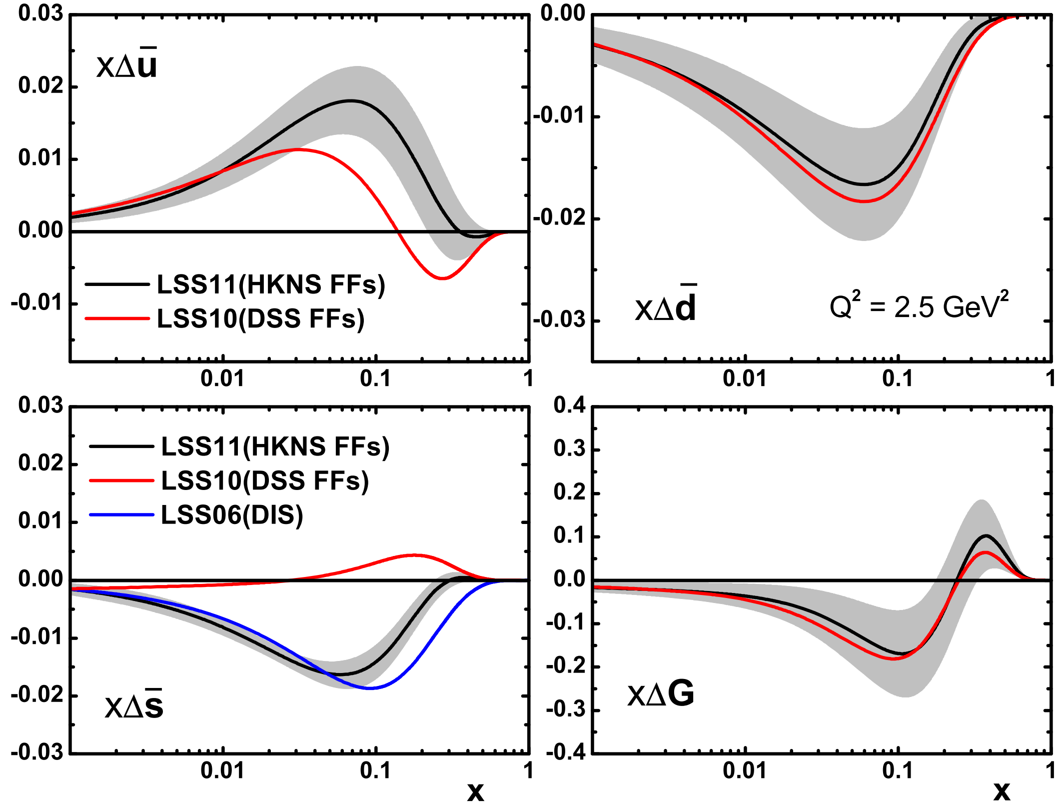
**The unpolarized SIDIS processes are crucial for a reliable determination of FFs $\Longrightarrow$ one can separate** $D_q^h(z,Q^2)$ from $D_{\bar{q}}^h(z,Q^2)$

# Sensitivity of polarized sea quark densities on FFs

**Sea quark densities obtained from NLO QCD analyses of DIS+SIDIS data using DSS and HKNS FFs are compared**

- $\Delta\bar{d}(x)_{DSS}$ and $\Delta\bar{d}(x)_{HKNS}$ **are almost the same**
- **At x > 0.03** $\Delta\bar{u}(x)_{DSS}$ **and** $\Delta\bar{u}(x)_{HKNS}$ **are different**
- **Significant difference between** $\Delta\bar{s}(x)_{DSS}$ (positive) and $\Delta\bar{s}(x)_{HKNS}$ (*negative*)
- **Inconsistency between** $\Delta\bar{s}(x)_{DSS}$ and $\Delta\bar{s}(x)_{DIS}$ **a puzzle which has to be solved**

**Error bands** → $\Delta\chi^2 = 1$




$$err^2 = stat^2 + sys^2$$

*LSS'10, PR D82 (2010); LSS'11, PR D84 (2011)*

**The present sets of pion and kaon FFs are NOT in agreement with the recent HERMES and COMPASS data on multiplicities !!!**

# NLO QCD analysis of COMPASS'06 data on pion multiplicities

**398 exp. points for $\pi^+$ and $\pi^-$ in $[y, x(Q^2), z]$ presentation**

**Statistical and systematic errors are taken in quadrature**

**12 *free* parameters for the input FFs**

**Excellent description of the data**

$$\chi^2/\mathrm{DOF} = 283.12/(398-12) = 0.73$$

**Remark: A fit to the data using only the statistical errors does not practically change the pion fragmentation functions**

$$\chi^2/\mathrm{DOF} = 625.02/(398-12) = 1.62$$

# Hadron multiplicities – theory and experiment

$$M^h(x,Q^2,z)_{\mathrm{exp}} = \frac{d^3N^h(x,Q^2,z)/dxdQ^2dz}{d^2N^{DIS}(x,Q^2)/dxdQ^2} = \frac{\text{Nr of hadrons yelds}}{\text{DIS events yelds}}$$

$$M^h(x,Q^2,z)_{\mathrm{th}} = \frac{d^3\sigma^h(x,Q^2,z)/dxdQ^2dz}{d^2\sigma^{DIS}(x,Q^2)/dxdQ^2}$$

$$= \frac{(1+(1-y)^2)2xF_1^h(x,Q^2,z)+2(1-y)xF_L^h(x,Q^2,z)}{(1+(1-y)^2)2xF_1(x,Q^2)+2(1-y)F_L(x,Q^2)}$$

$F_1^h, \mathrm{F}_L^h$ - SIDIS str. functions, $F_1, \mathrm{F}_L$ - nucleon str. functions, $y = Q^2/2MxE$

**In LO QCD: $\mathbf{F_L^h}$, $\mathbf{F_L = 0}$**

$$M^h(x,Q^2,z) = \frac{F_1^h(x,Q^2,z)_{LO}}{F_1(x,Q^2)_{LO}} = \frac{\sum_{q,\bar{q}} e_q^2 q(x,Q^2) D_q^h(x,Q^2,z)}{\sum_{q,\bar{q}} e_q^2 q(x,Q^2)}$$

# NLO QCD fit to the pion data

## Input FFs at $Q^2_0 = 1$ GeV$^2$

$$zD_u^{\pi+}(z,Q_0^2) = N_u z^{\alpha_u}(1-z)^{\beta_u}(1+\gamma_u(1-z)^{\delta_u})$$ **favored**

$$zD_{\bar{u}}^{\pi+}(z,Q_0^2) = N_{\bar{u}} z^{\alpha_{\bar{u}}}(1-z)^{\beta_{\bar{u}}}(1+\gamma_{\bar{u}}(1-z)^{\delta_{\bar{u}}})$$ **unfavored**

$$zD_g^{\pi+}(z,Q_0^2) = N_g z^{\alpha_g}(1-z)^{\beta_g}$$

→ **13 free parameters** → **12 free parameters** $(\alpha_{\bar{u}} - \mathit{fixed})$

**Additional assumptions for pion FFs:**

- $D_{\bar{d}}^{\pi+}(z,Q_0^2) = D_u^{\pi+}(z,Q_0^2),\ D_d^{\pi+}(z,Q_0^2) = D_{\bar{u}}^{\pi+}(z,Q_0^2)$ ← **from SU(2) symmetry**
- $D_d^{\pi+}(z,Q_0^2) = D_s^{\pi+}(z,Q_0^2) = D_{\bar{s}}^{\pi+}(z,Q_0^2) = D_{\bar{u}}^{\pi+}(z,Q_0^2)$ **for unfavored FFs**

! $\alpha_{\bar{u}} \to 0$ → $\alpha_{\bar{u}} = 1 - \mathit{fixed}$ ($D_{\bar{u}}^{\pi+}$ does not change for $z > 0.2$)

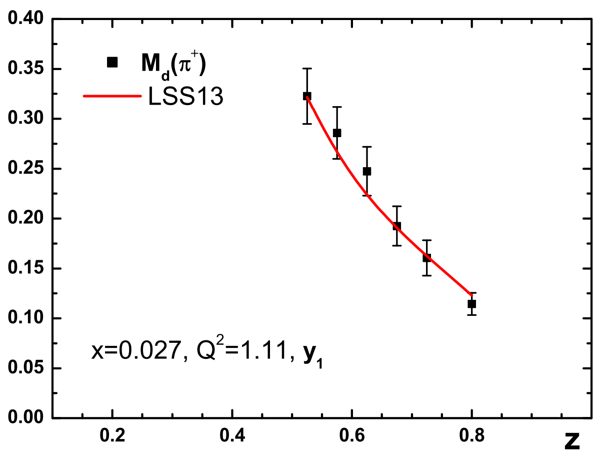
# COMPASS data on pion multiplicities $M_d(\pi^+)$ vs NLO QCD fit curves

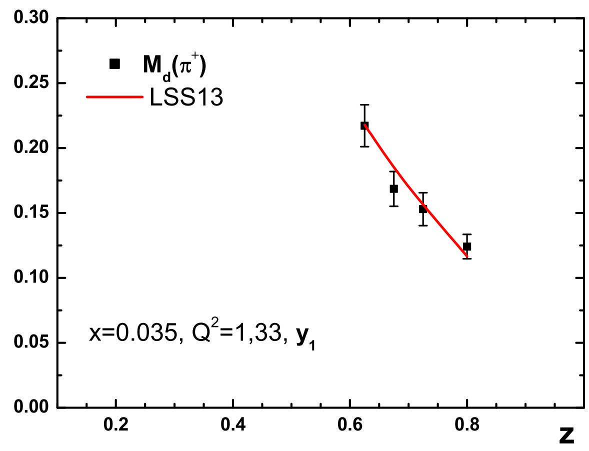
## 199 exp. points in $(y, x(Q^2), z)$ presentation

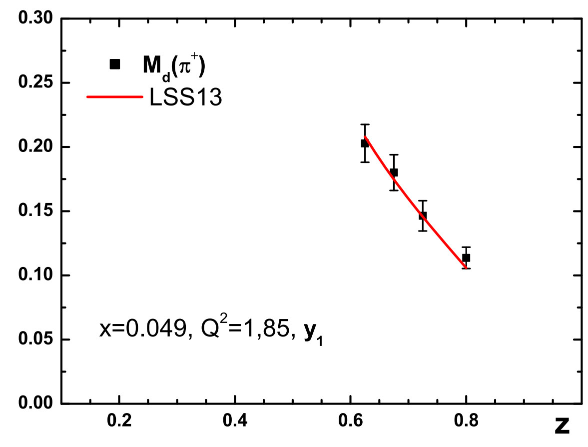
$$\chi^2/\text{point} = 0.61$$

$y = Q^2/2MEx$ E = 160 GeV

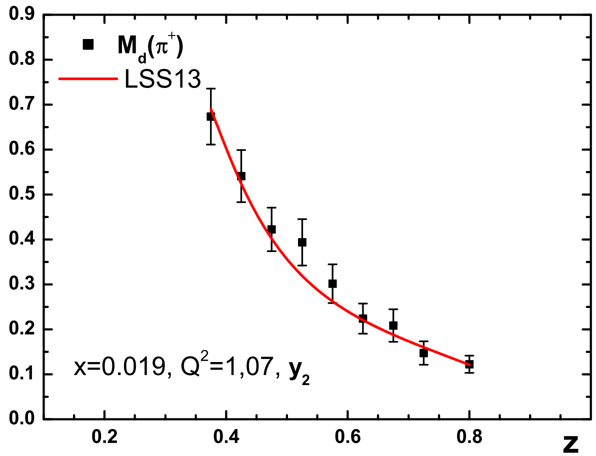
$y_1 = 0.10 - 0.15$



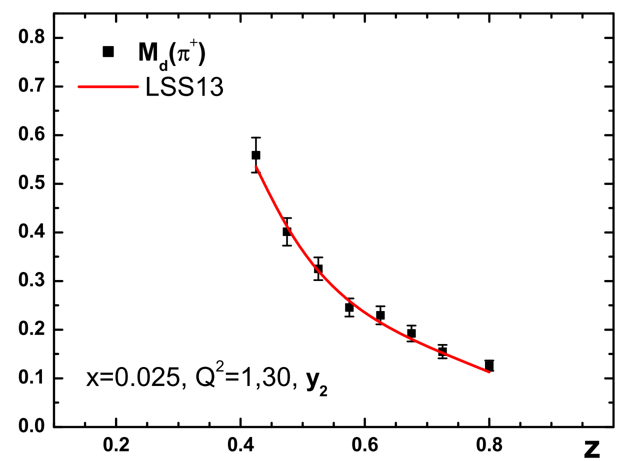



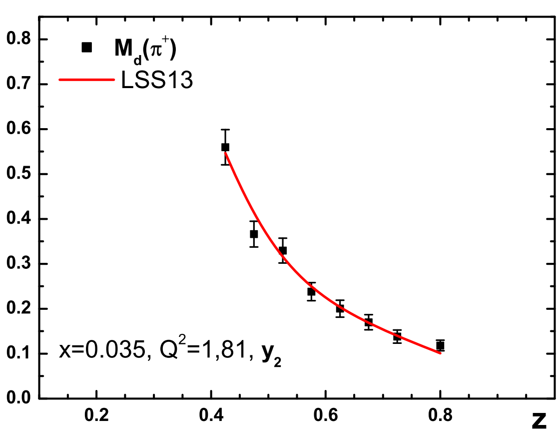
# $y_2 = 0.15 - 0.20$



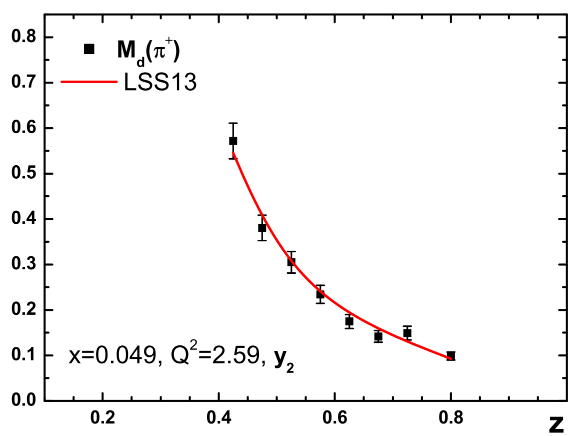
$M_d(\pi^+)$
LSS13
x=0.019, $Q^2$=1,07, $y_2$
z

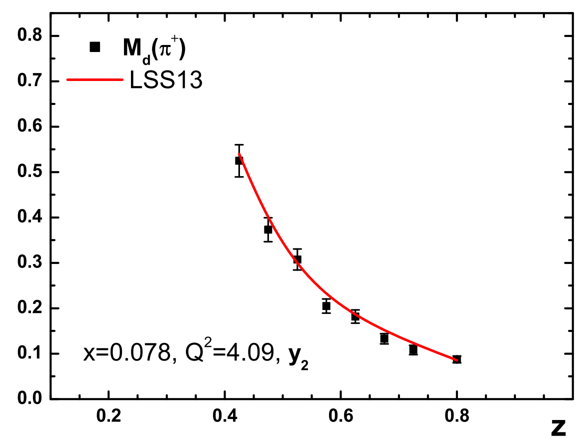
$M_d(\pi^+)$
LSS13
x=0.025, $Q^2$=1,30, $y_2$
z

$M_d(\pi^+)$
LSS13
x=0.035, $Q^2$=1,81, $y_2$
z

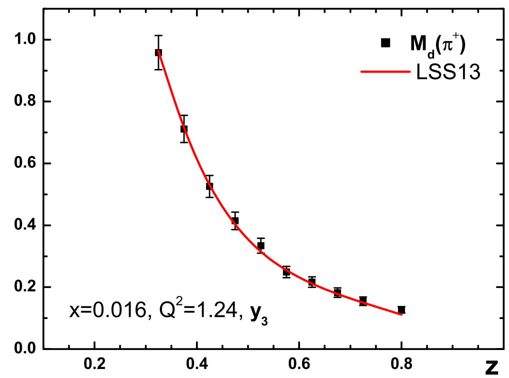



$M_d(\pi^+)$
LSS13
x=0.049, $Q^2$=2.59, $y_2$
z

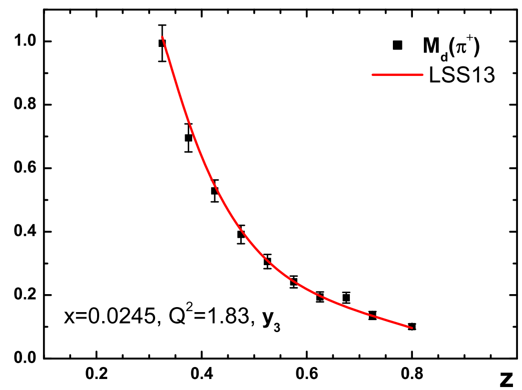



$M_d(\pi^+)$
LSS13
x=0.078, $Q^2$=4.09, $y_2$
z

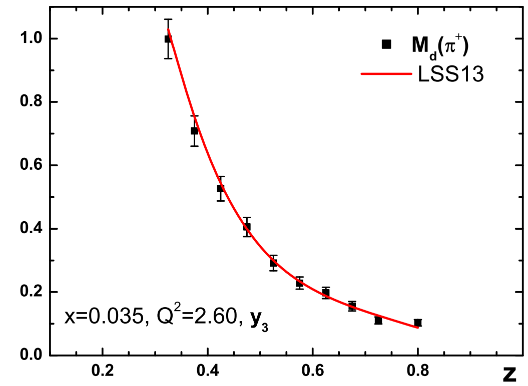
# $y_3 = 0.20 – 0.30$



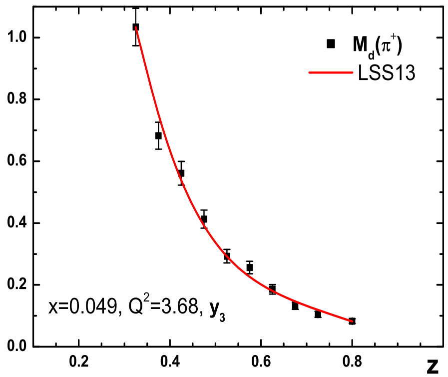
Md(π+)
LSS13
x=0.016, Q2=1.24, y3
z

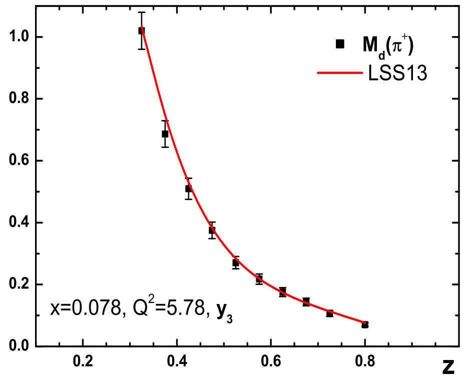
Md(π+)
LSS13
x=0.0245, Q2=1.83, y3
z

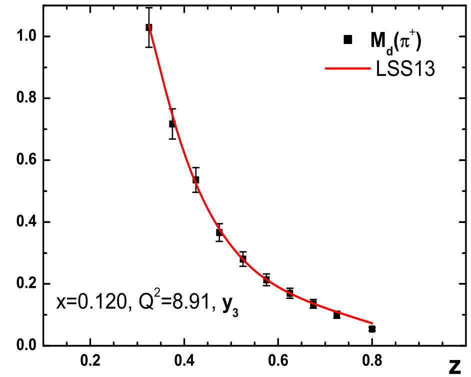
Md(π+)
LSS13
x=0.035, Q2=2.60, y3
z

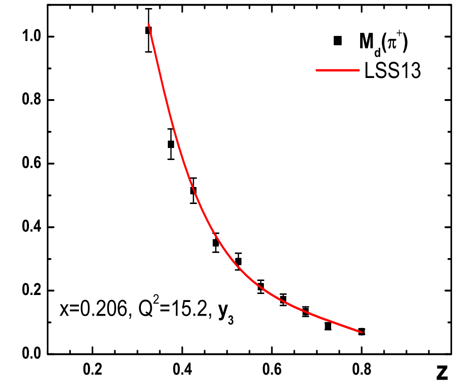
Md(π+)
LSS13
x=0.049, Q2=3.68, y3
z

Md(π+)
LSS13
x=0.078, Q2=5.78, y3
z

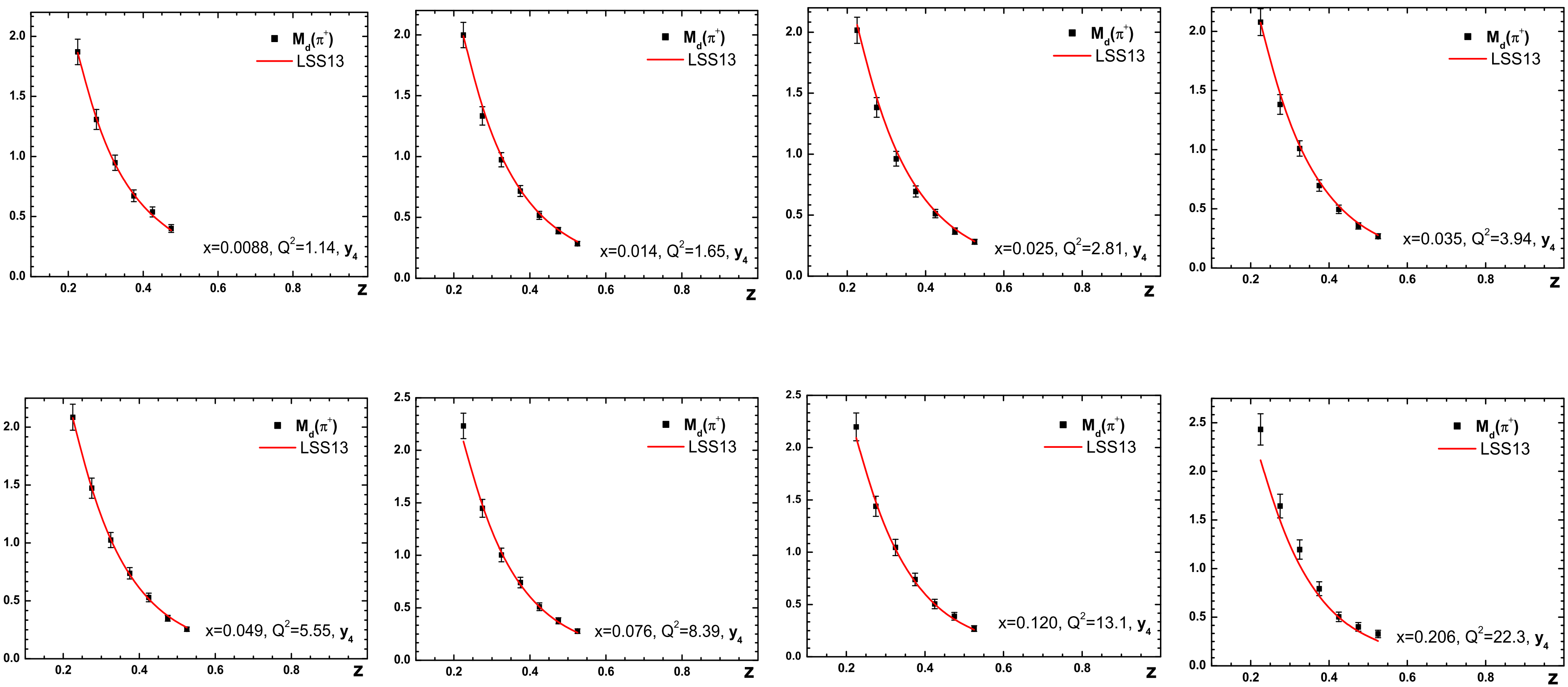



Md(π+)
LSS13
x=0.120, Q2=8.91, y3
z

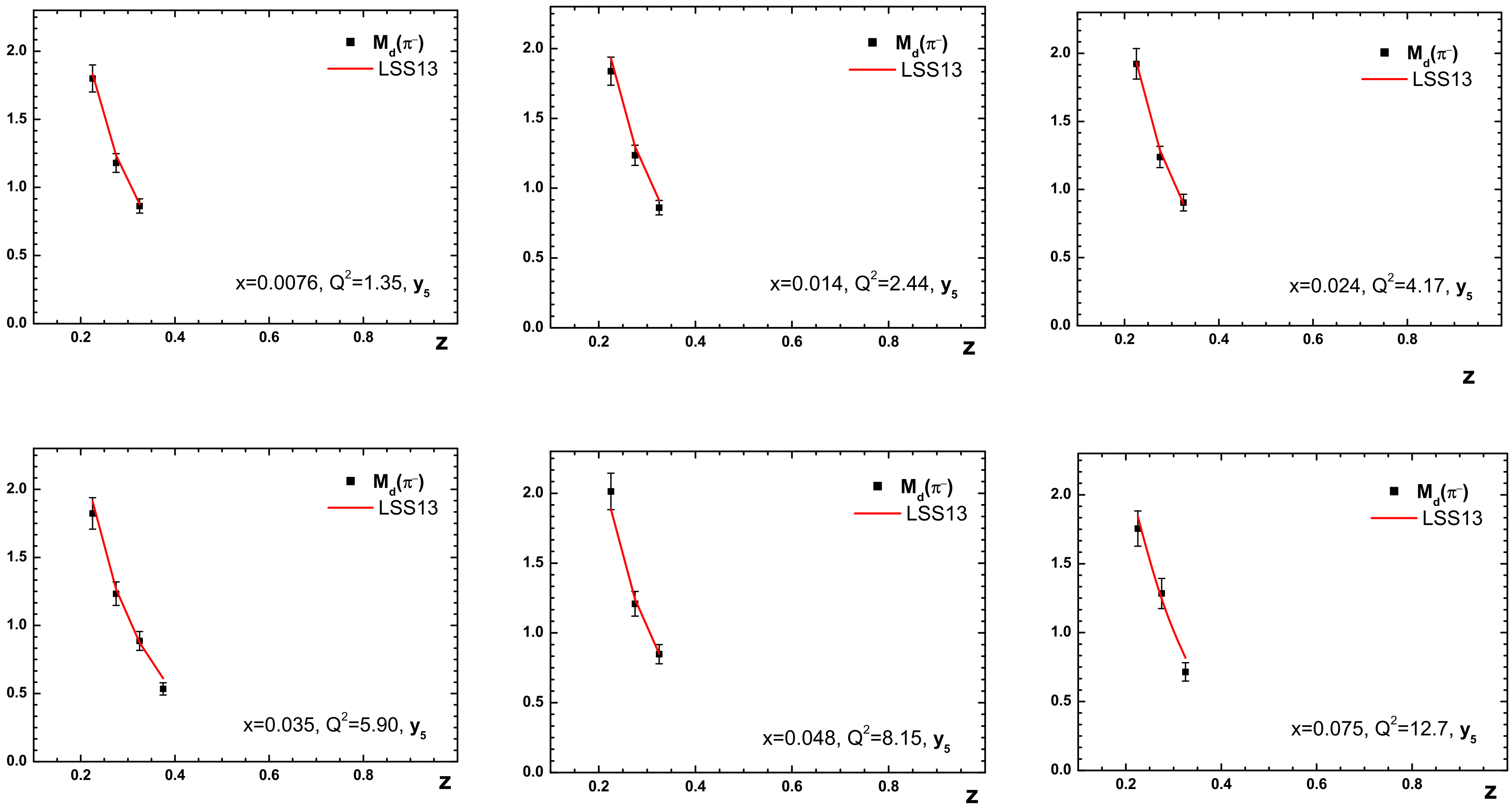
Md(π+)
LSS13
x=0.206, Q2=15.2, y3
z

# $y_4 = 0.30 - 0.50$



M_d(π+)
LSS13
x=0.0088, Q2=1.14, y4
x=0.014, Q2=1.65, y4
x=0.025, Q2=2.81, y4
x=0.035, Q2=3.94, y4
x=0.049, Q2=5.55, y4
x=0.076, Q2=8.39, y4
x=0.120, Q2=13.1, y4
x=0.206, Q2=22.3, y4
z

# $y_5 = 0.50 – 0.70$



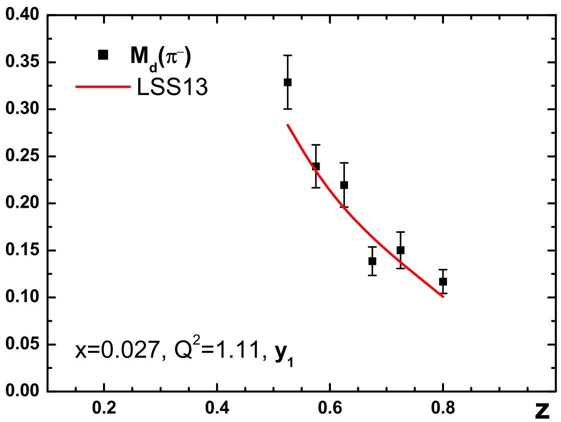
$M_d(\pi^-)$
LSS13
x=0.0076, $Q^2$=1.35, $y_5$
x=0.014, $Q^2$=2.44, $y_5$
x=0.024, $Q^2$=4.17, $y_5$
x=0.035, $Q^2$=5.90, $y_5$
x=0.048, $Q^2$=8.15, $y_5$
x=0.075, $Q^2$=12.7, $y_5$
z

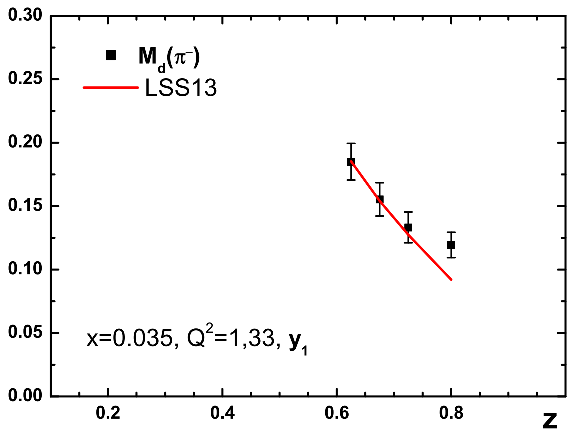
**COMPASS data on pion multiplicities $M_d(\pi^-)$ vs NLO QCD fit curves**

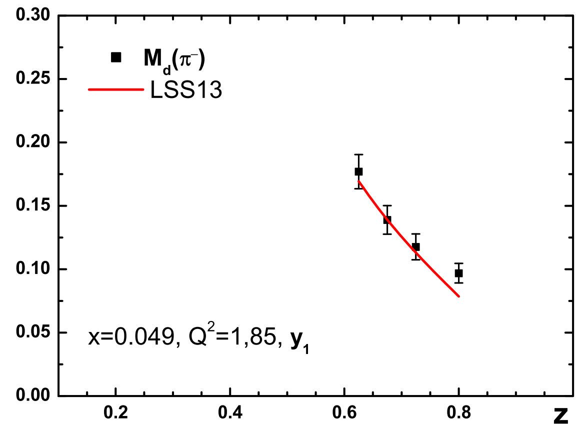
**199 exp. points in $(y, x(Q^2), z)$ presentation**

$\chi^2$/point = 0.81

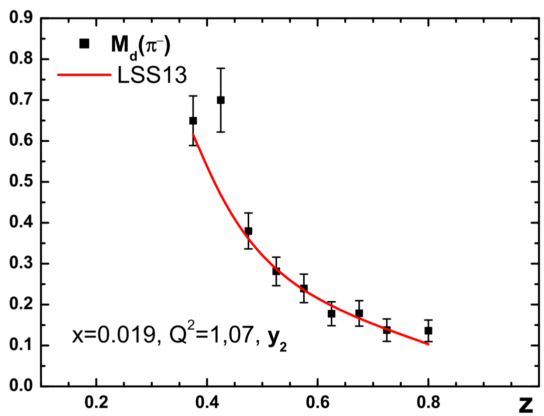
$\mathbf{y_1 = 0.10 – 0.15}$

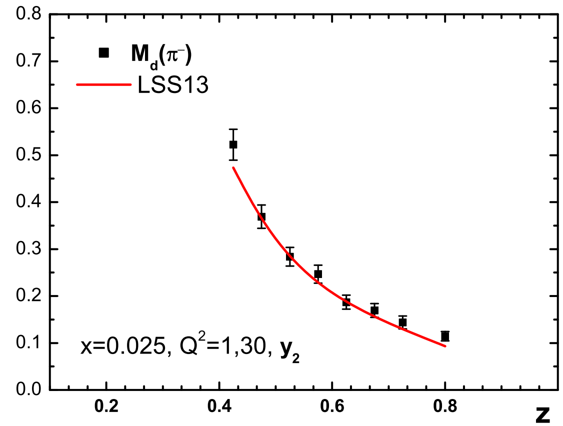


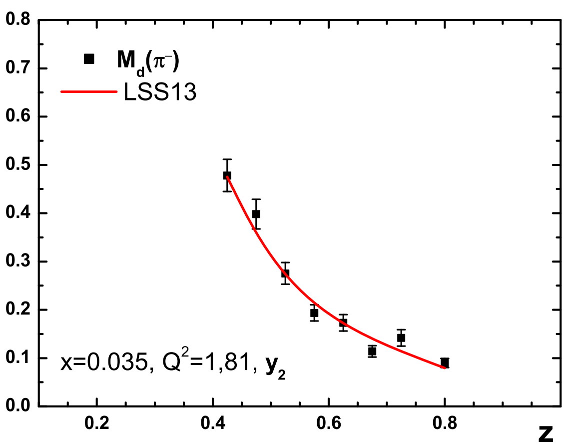



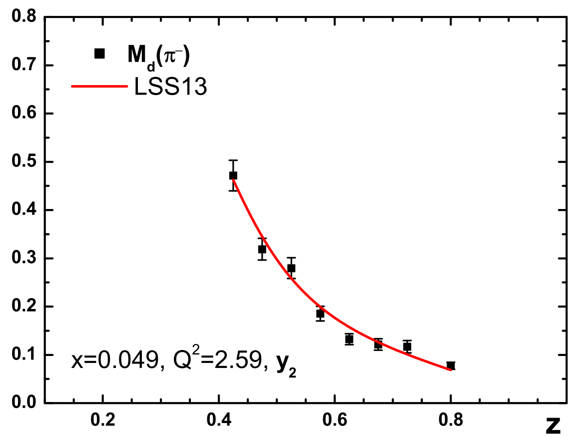



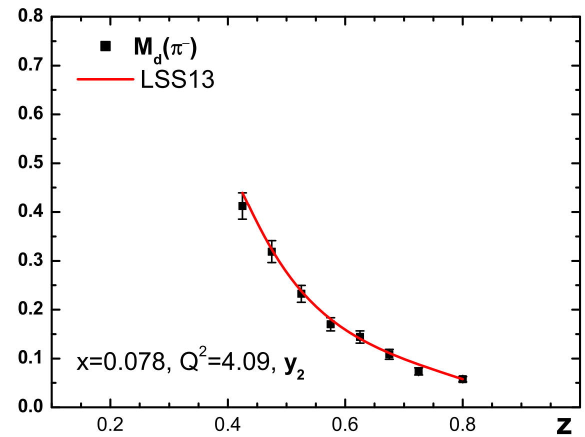
# $y_2 = 0.15 – 0.20$


$M_d(\pi^-)$
LSS13
x=0.019, $Q^2$=1,07, $\mathbf{y_2}$
z

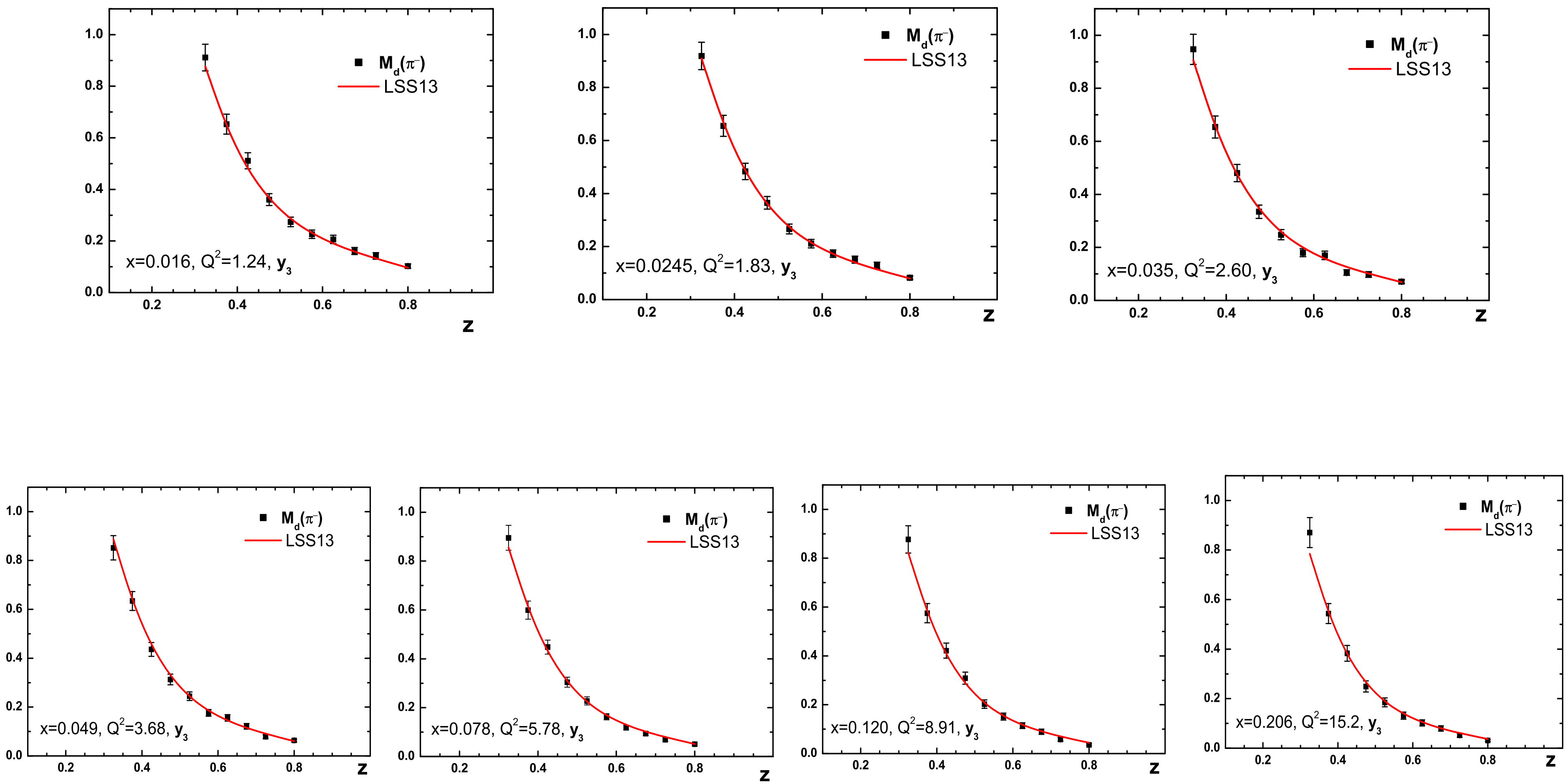

$M_d(\pi^-)$
LSS13
x=0.025, $Q^2$=1,30, $\mathbf{y_2}$
z


$M_d(\pi^-)$
LSS13
x=0.035, $Q^2$=1,81, $\mathbf{y_2}$
z


$M_d(\pi^-)$
LSS13
x=0.049, $Q^2$=2.59, $\mathbf{y_2}$
z

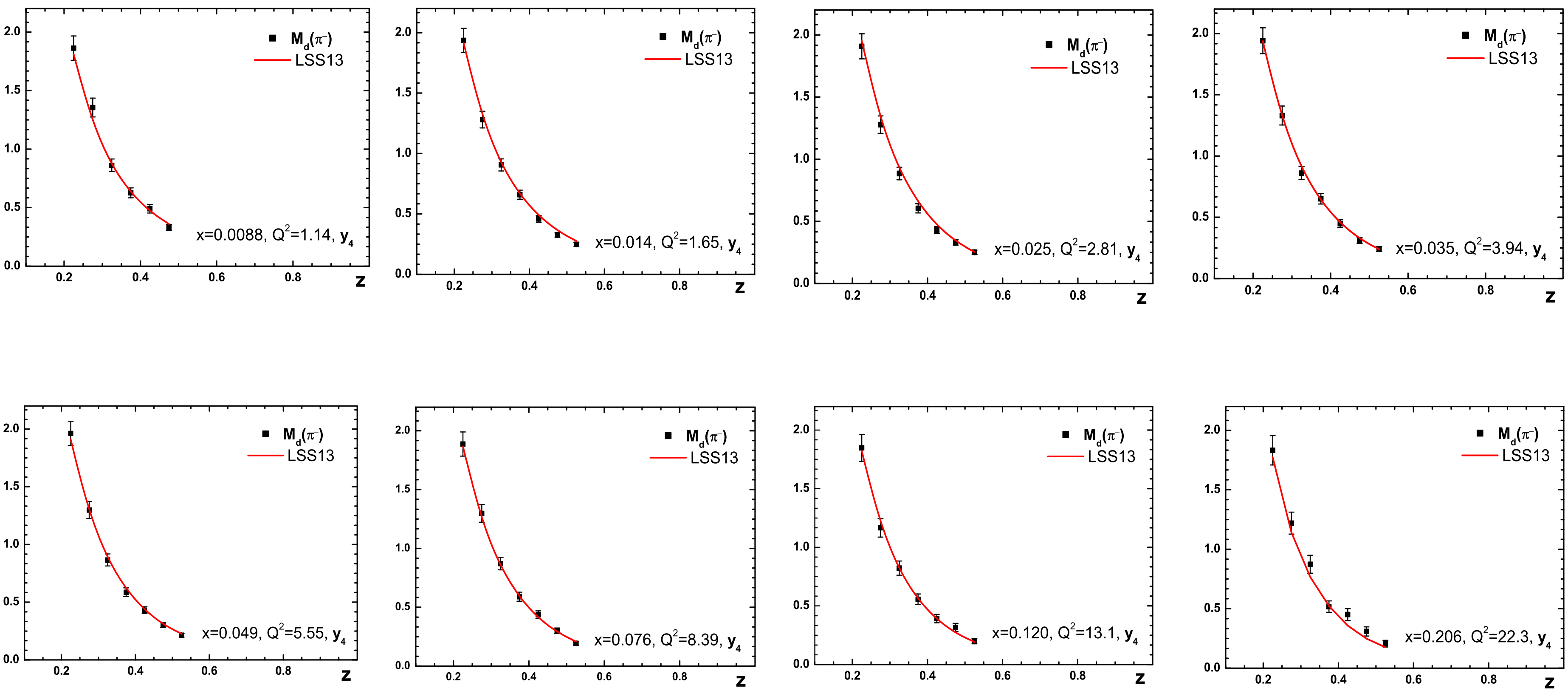

$M_d(\pi^-)$
LSS13
x=0.078, $Q^2$=4.09, $\mathbf{y_2}$
z

# $y_3 = 0.20 - 0.30$



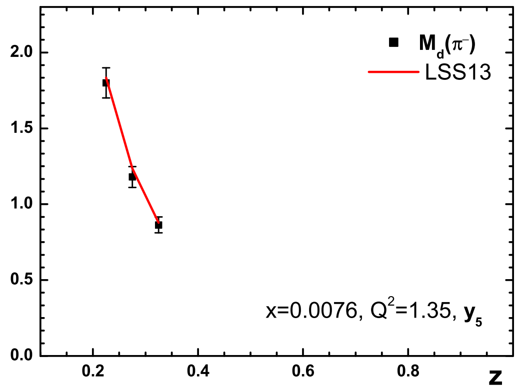
$M_d(\pi^-)$
LSS13
x=0.016, $Q^2$=1.24, $y_3$
$M_d(\pi^-)$
LSS13
x=0.0245, $Q^2$=1.83, $y_3$
$M_d(\pi^-)$
LSS13
x=0.035, $Q^2$=2.60, $y_3$
$M_d(\pi^-)$
LSS13
x=0.049, $Q^2$=3.68, $y_3$
$M_d(\pi^-)$
LSS13
x=0.078, $Q^2$=5.78, $y_3$
$M_d(\pi^-)$
LSS13
x=0.120, $Q^2$=8.91, $y_3$
$M_d(\pi^-)$
LSS13
x=0.206, $Q^2$=15.2, $y_3$
z

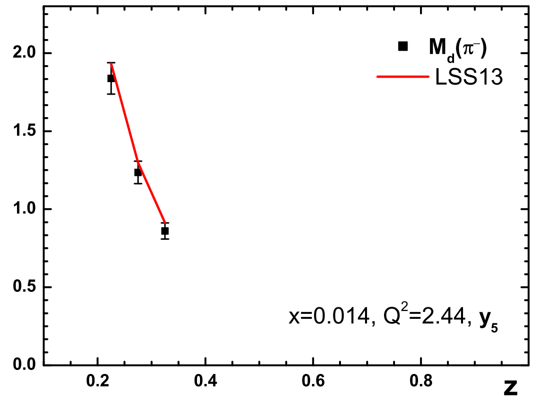
# $y_4 = 0.30 - 0.50$

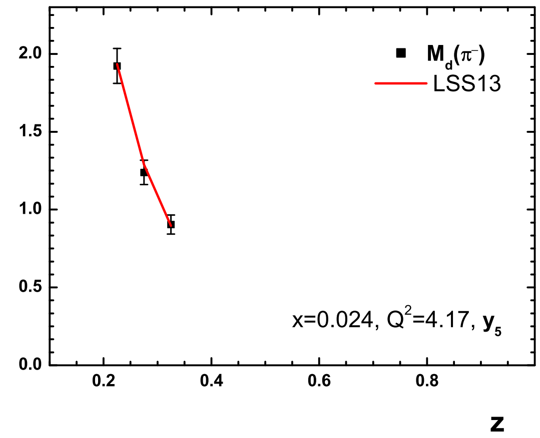


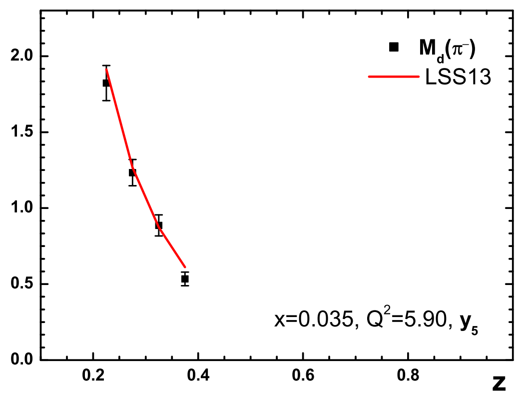
$M_d(\pi^-)$
LSS13
x=0.0088, $Q^2$=1.14, $y_4$
x=0.014, $Q^2$=1.65, $y_4$
x=0.025, $Q^2$=2.81, $y_4$
x=0.035, $Q^2$=3.94, $y_4$
x=0.049, $Q^2$=5.55, $y_4$
x=0.076, $Q^2$=8.39, $y_4$
x=0.120, $Q^2$=13.1, $y_4$
x=0.206, $Q^2$=22.3, $y_4$
z
0.0
0.5
1.0
1.5
2.0
0.2
0.4
0.6
0.8

# $y_5 = 0.50 - 0.70$


$M_d(\pi^-)$
LSS13
x=0.0076, $Q^2$=1.35, $y_5$
z


$M_d(\pi^-)$
LSS13
x=0.014, $Q^2$=2.44, $y_5$
z


$M_d(\pi^-)$
LSS13
x=0.024, $Q^2$=4.17, $y_5$
z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x=0.035, $Q^2$=5.90, $y_5$
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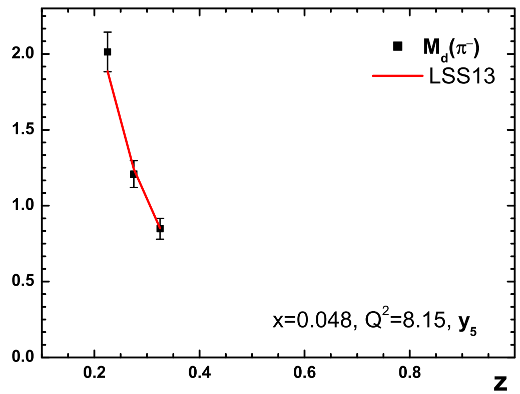
$M_d(\pi^-)$
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x=0.048, $Q^2$=8.15, $y_5$
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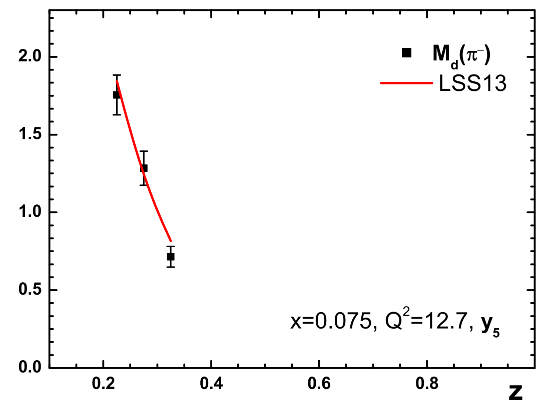

$M_d(\pi^-)$
LSS13
x=0.075, $Q^2$=12.7, $y_5$
z

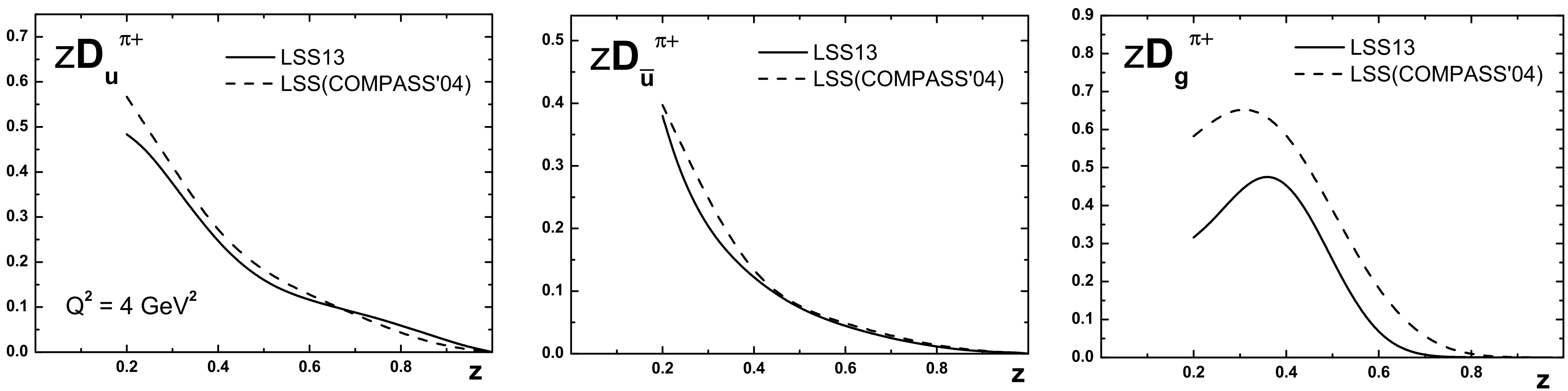
# Comparison between the pion FFs (*COMPASS'06*) and FFs (*COMPASS'04*)

## COMPASS'04: [$x(Q^2)$, $z$] presentation

**94 exp. points, 13 free parameters** $\sigma_{\exp} = [\sigma_{stat}^2 + (0.01 * centr.value)^2]^{1/2}$

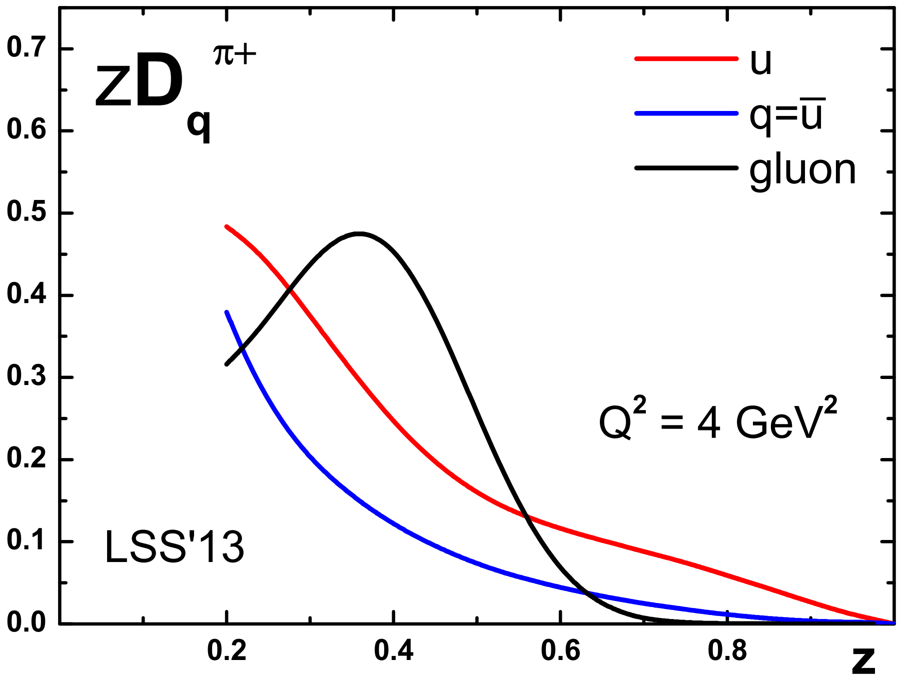
a part of syst errors

$$\chi^2 / DOF = 192.5/(94\text{-}13) = 2.38$$

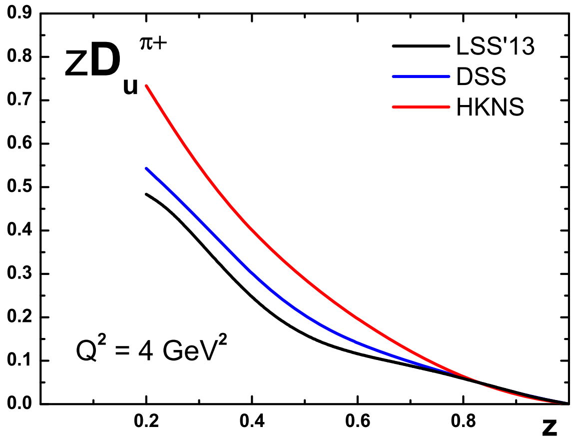


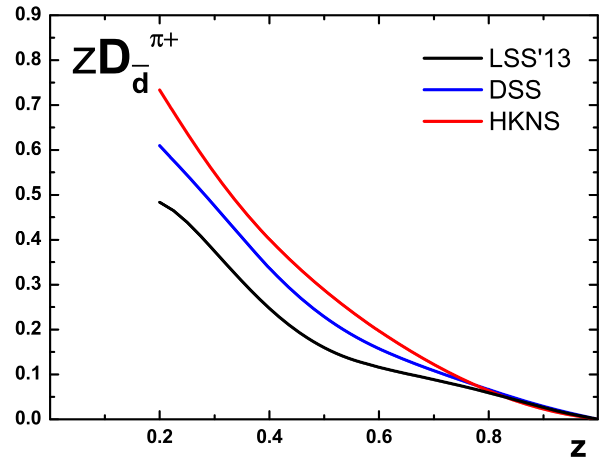

**Excepting $D_g^{\pi+}$ the new pion FFs are in a good agreement with the old ones**

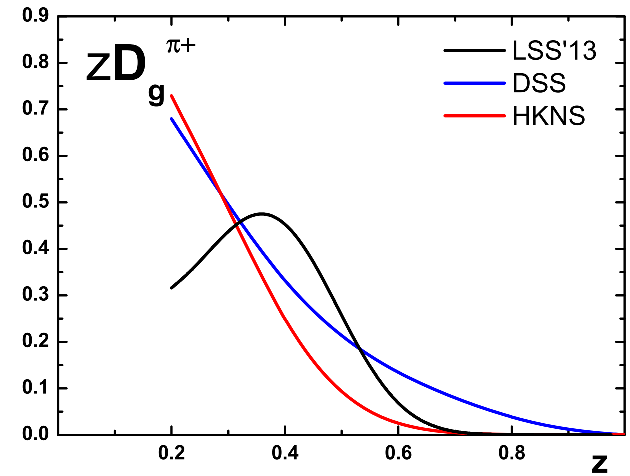
# NLO LSS'2013 pion FFs

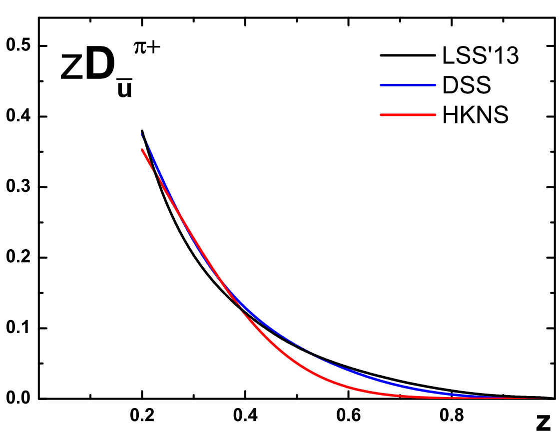


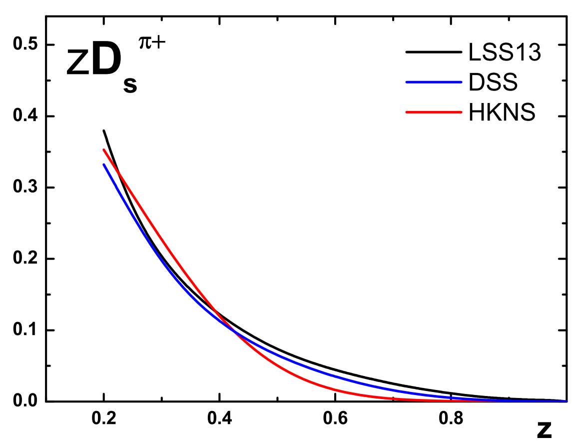
# Comparison between the new pion FFs and those of DSS and HKNS




**DSS**

$$D_u^{\pi+} \neq D_{\bar{d}}^{\pi+}, \; D_{\bar{u}}^{\pi+} = D_d^{\pi+}$$

$$D_{\bar{u}}^{\pi+} \neq D_{\bar{s}}^{\pi+}, \; D_{\bar{s}}^{\pi+} = D_s^{\pi+}$$

**LSS, HKNS**

$$D_u^{\pi+} = D_{\bar{d}}^{\pi+}$$

$$D_{\bar{u}}^{\pi+} = D_d^{\pi+} = D_{\bar{s}}^{\pi+} = D_s^{\pi+}$$

## HERMES/(p,d) data on pion multiplicities (*Phys.Rev. D87 (2013) 074029* )

72 $\pi^+$ and $\pi^-$ data points for a proton as well as for a deuteron target

Total: 144 exp. points in $[x(Q^2), z]$ presentation

! Up to now we can not find a reasonable fit to the HERMES pion data !

As the HERMES data are very precise one has:

- to account for the uncertainties of the unpolarized PDFs
- to check if the isospin SU(2) symmetry is satisfied $(D_u^{\pi+} = D_{\bar{d}}^{\pi+}\ ?)$
- to try to involve $1/Q^2$ higher twist effects ?

**Pion multiplicities at HERMES kinematics computed with the FFs determined from the analysis of COMPASS data (LSS'13)**

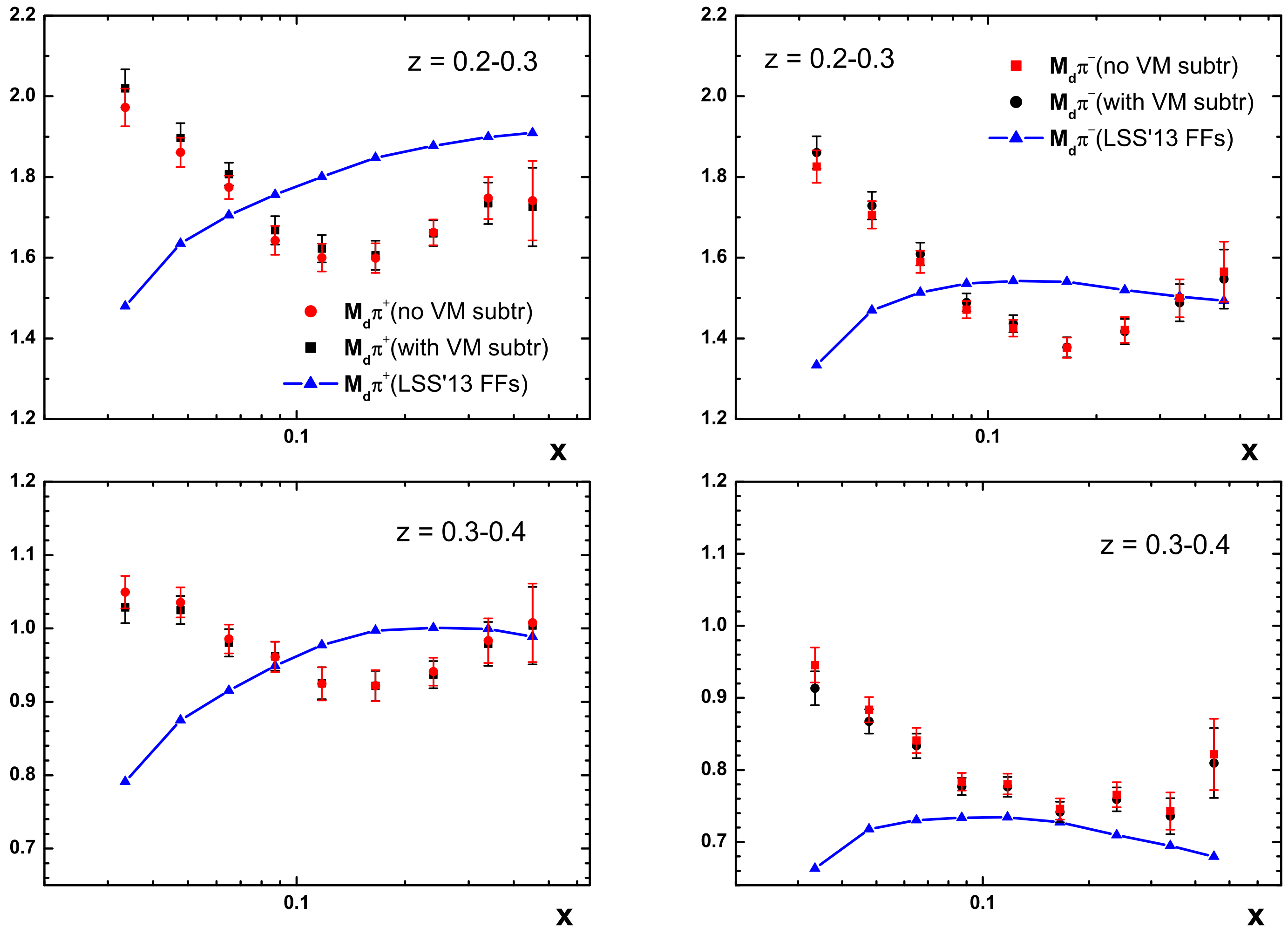
$$\chi^2 = 3231.8 \;!!!$$

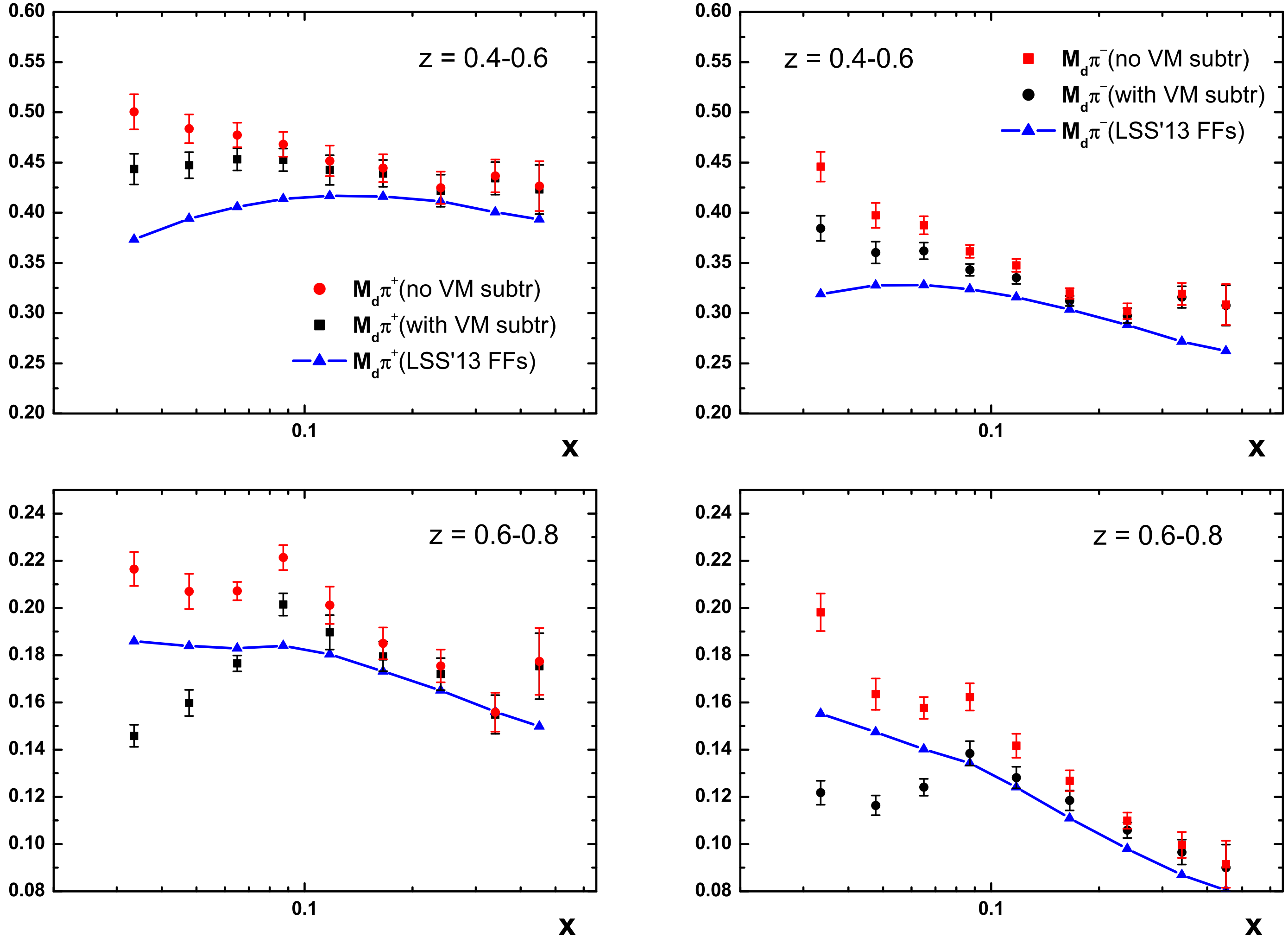
$\chi^2/\text{point}\ (M_p^{\pi+}) = 25.2$

$\chi^2/\text{point}\ (M_d^{\pi+}) = 21.1$

$\chi^2/\text{point}\ (M_p^{\pi-}) = 20.5$

$\chi^2/\text{point}\ (M_d^{\pi-}) = 23.0$





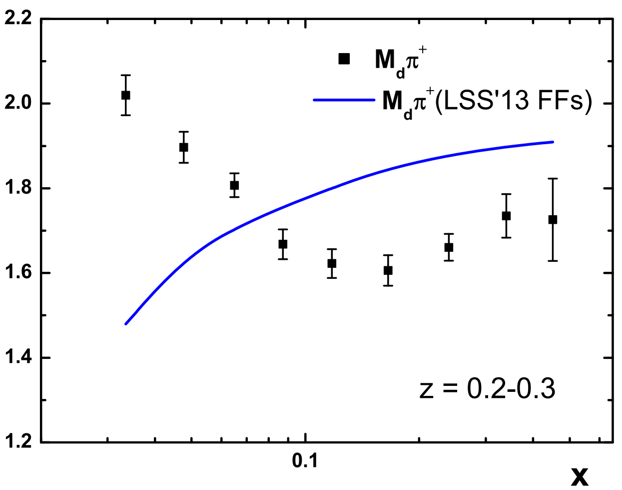

**The corrections coming from the exclusive vector meson production are NOT negligible for $z > 0.4$ at $x < 0.1$.**

# Influence of the scale on graphs used to present results

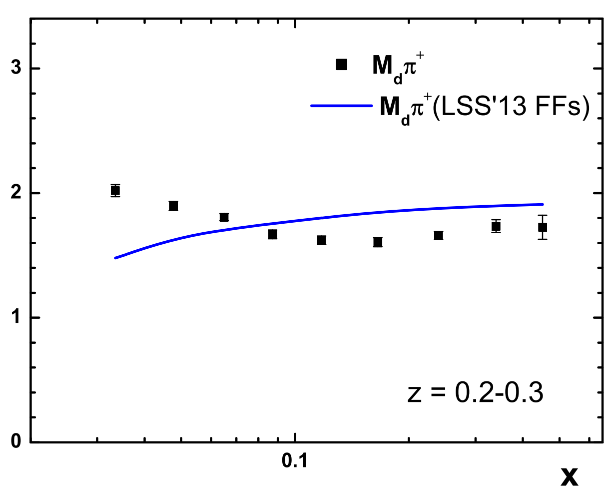
**The HERMES $M_d^{\pi+}$ data for the z-bin [0.2-0.3] presented in different scales**

$$\chi^2/\text{point}\ (M_d^{\pi+}) = 35.8$$

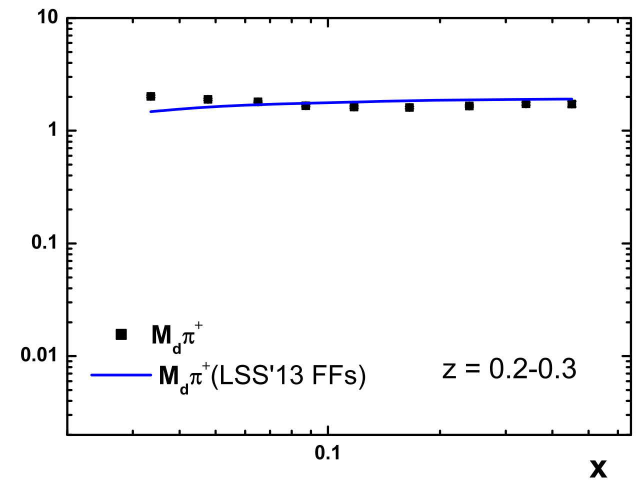
***This talk***




***HERMES paper***




***a log scale***




**Some time the use of a log scale is not convenient (a good way to hide the problems)**

***M. Stratmann* (DSS) - QCD Landscape of the Nucleon and Atomic Nuclei, Berkeley, August 2013:**

“***Perfect global fit for pions***

***No tension between COMPASS and HERMES***”

**Remark: To illustrate the quality of the fit only figures are presented (the figures for the HERMES data in a log scale !) NO numbers for $\chi^2$ at all.**

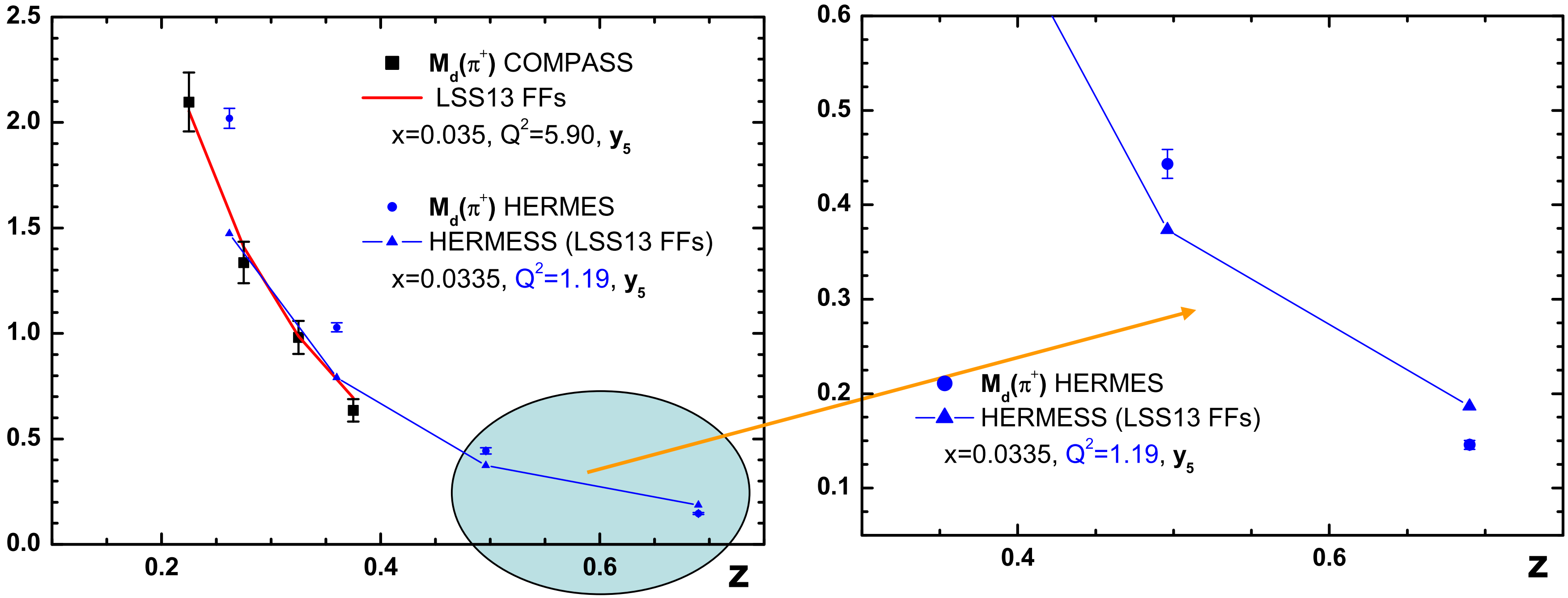
# Comparison between HERMES/d and COMPASS/d pion data

**The experimental points at almost the same *x* and *y* are presented**

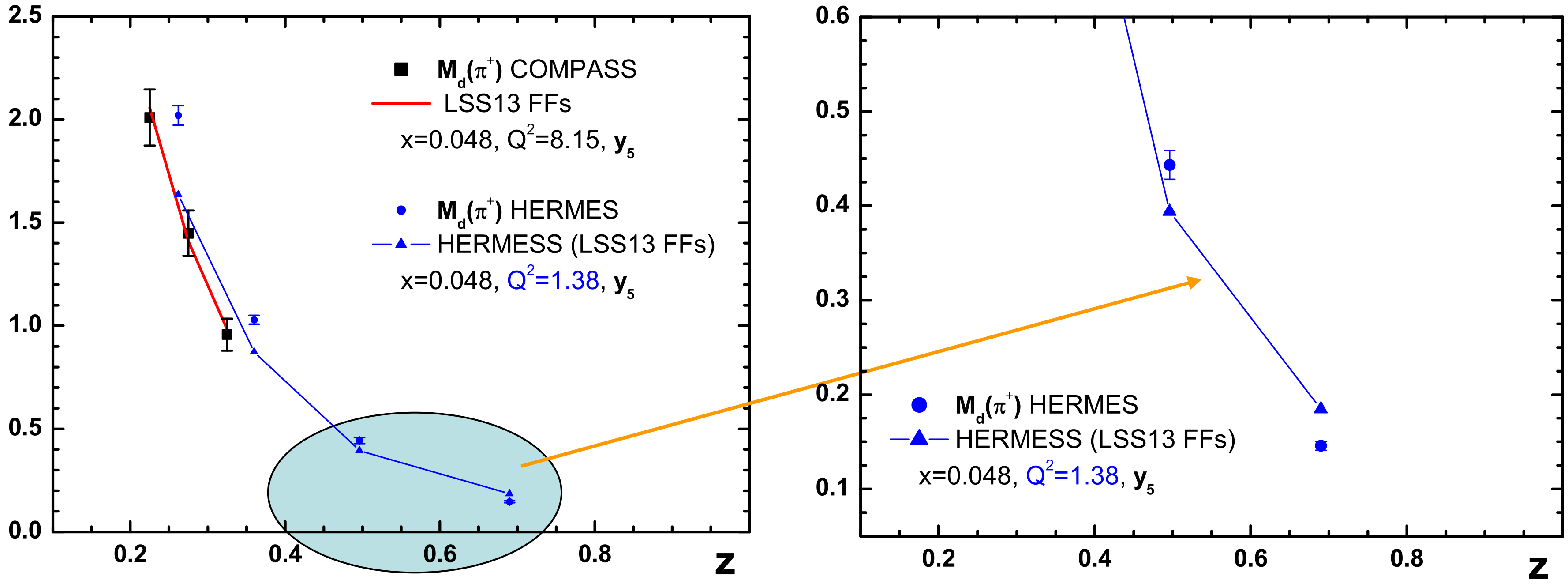
**Due to the different energy beams the corresponding $Q^2$ are different**

**The red curves correspond to the best NLO QCD fit (LSS) to the COMPASS data**

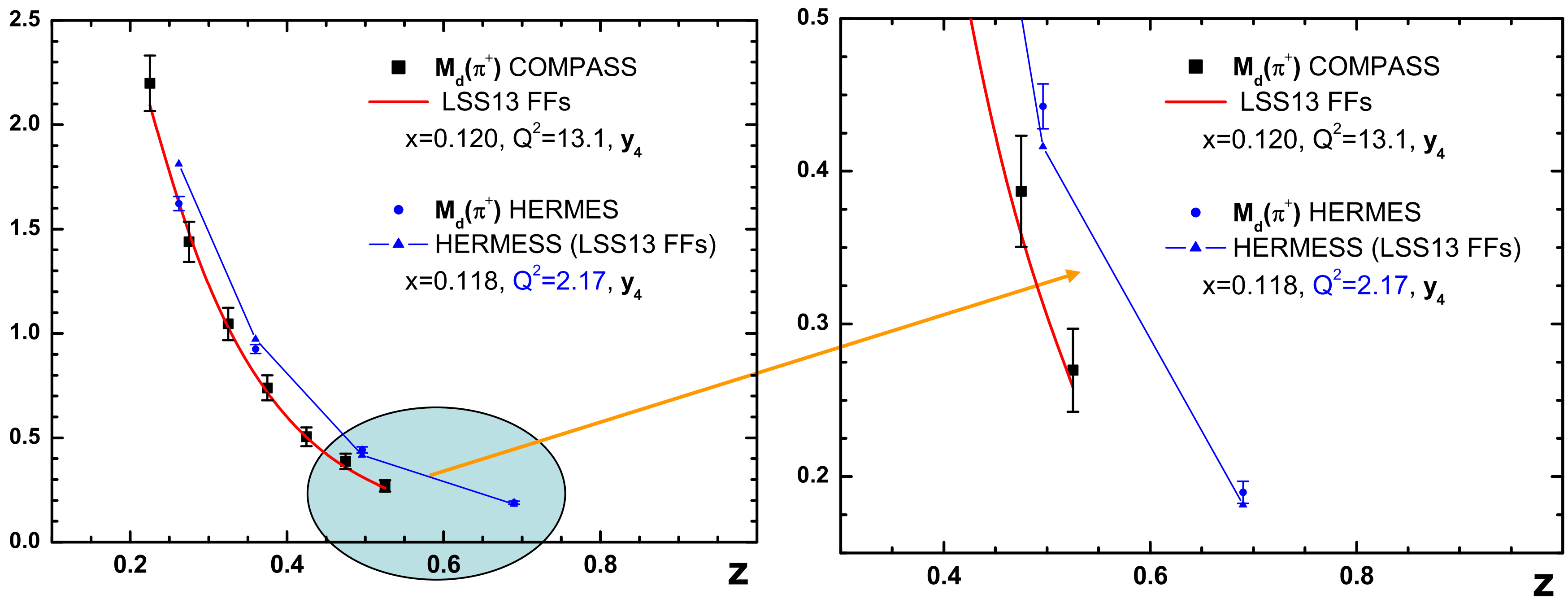
**The blue triangles correspond to the predictions for the HERMES points calculated by LSS'13 pion FFs obtained from the fit to the COMPASS data**




$\chi^2_1 = 132.7$ $\chi^2_2 = 125.6$ $\chi^2_3 = 21.1$ $\chi^2_4 = 75.7$




$\chi^2_1 = 50.7$ $\chi^2_2 = 62.6$ $\chi^2_3 = 17.0$ $\chi^2_4 = 20.4$



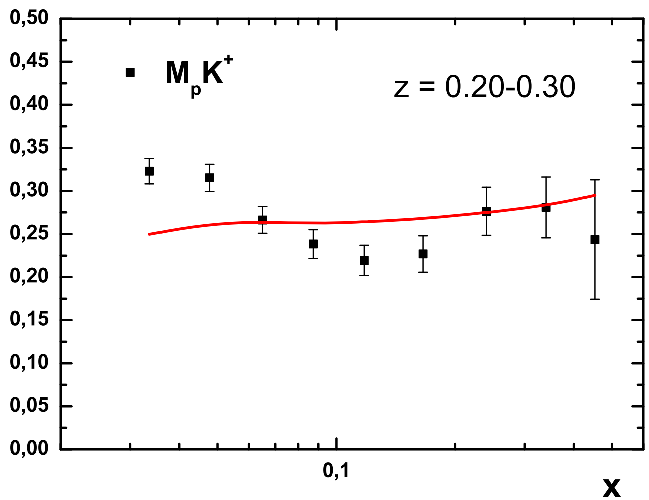

$\chi^2{}_1 = 30.9$ $\chi^2{}_2 = 4.7$ $\chi^2{}_3 = 3.2$ $\chi^2{}_4 = 1.3$

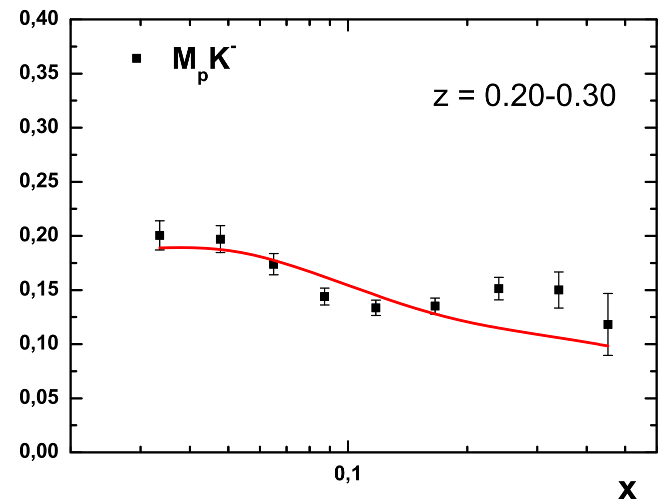
*Preliminary*

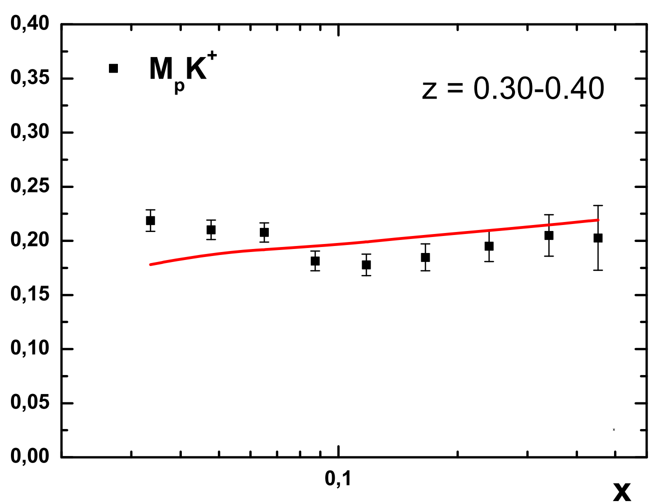
# Comparison between the HERMES (x,z) proton data on kaon multiplicities with the best fit NLO QCD LSS curves

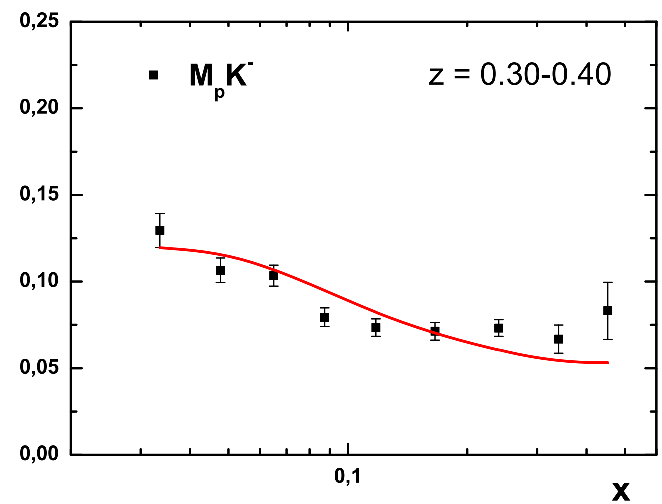
$\chi^2$/Nrpoints **(K+) = 4.84**

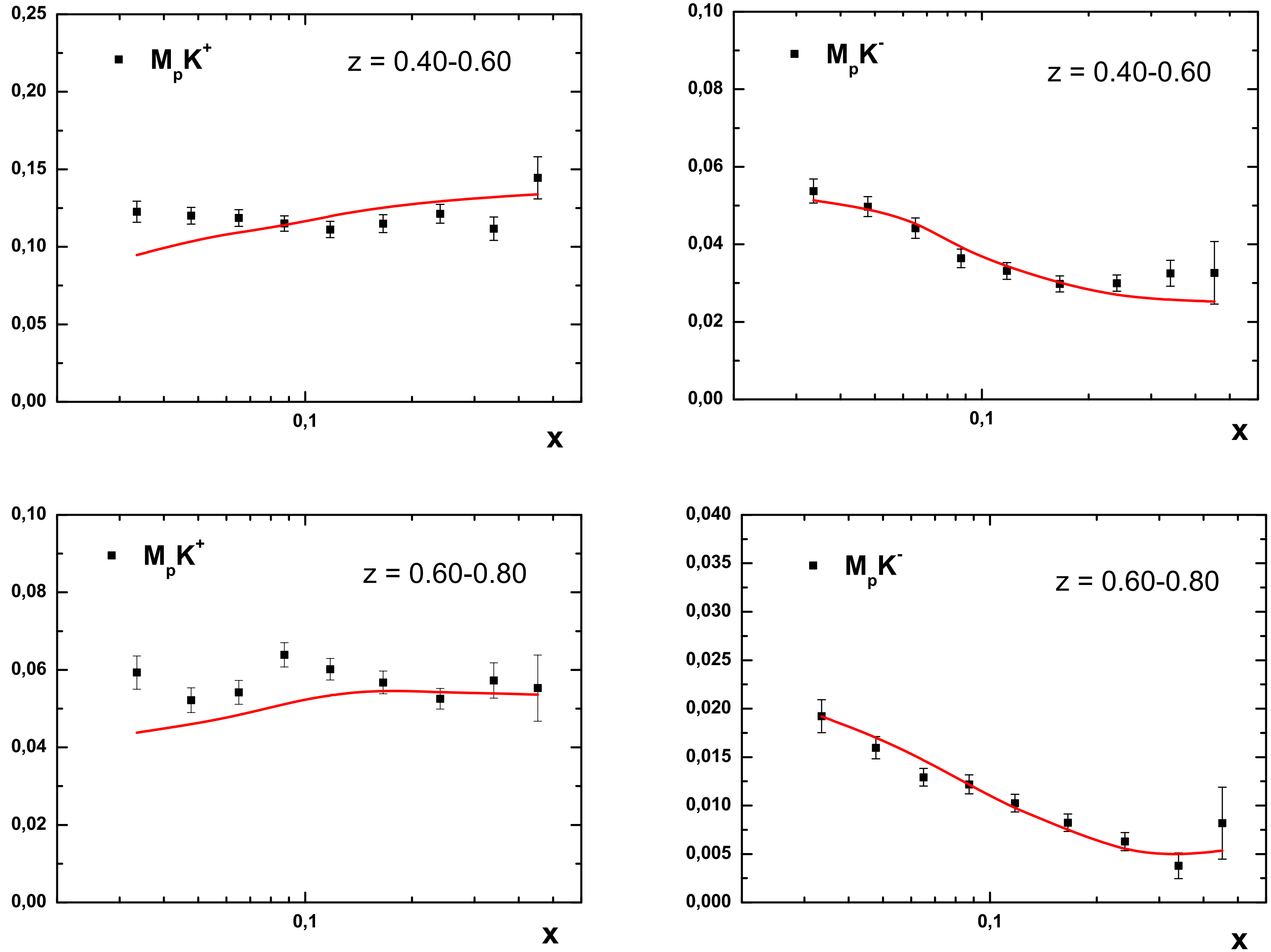
$\chi^2$/Nrpoints **(K-) = 2.16**



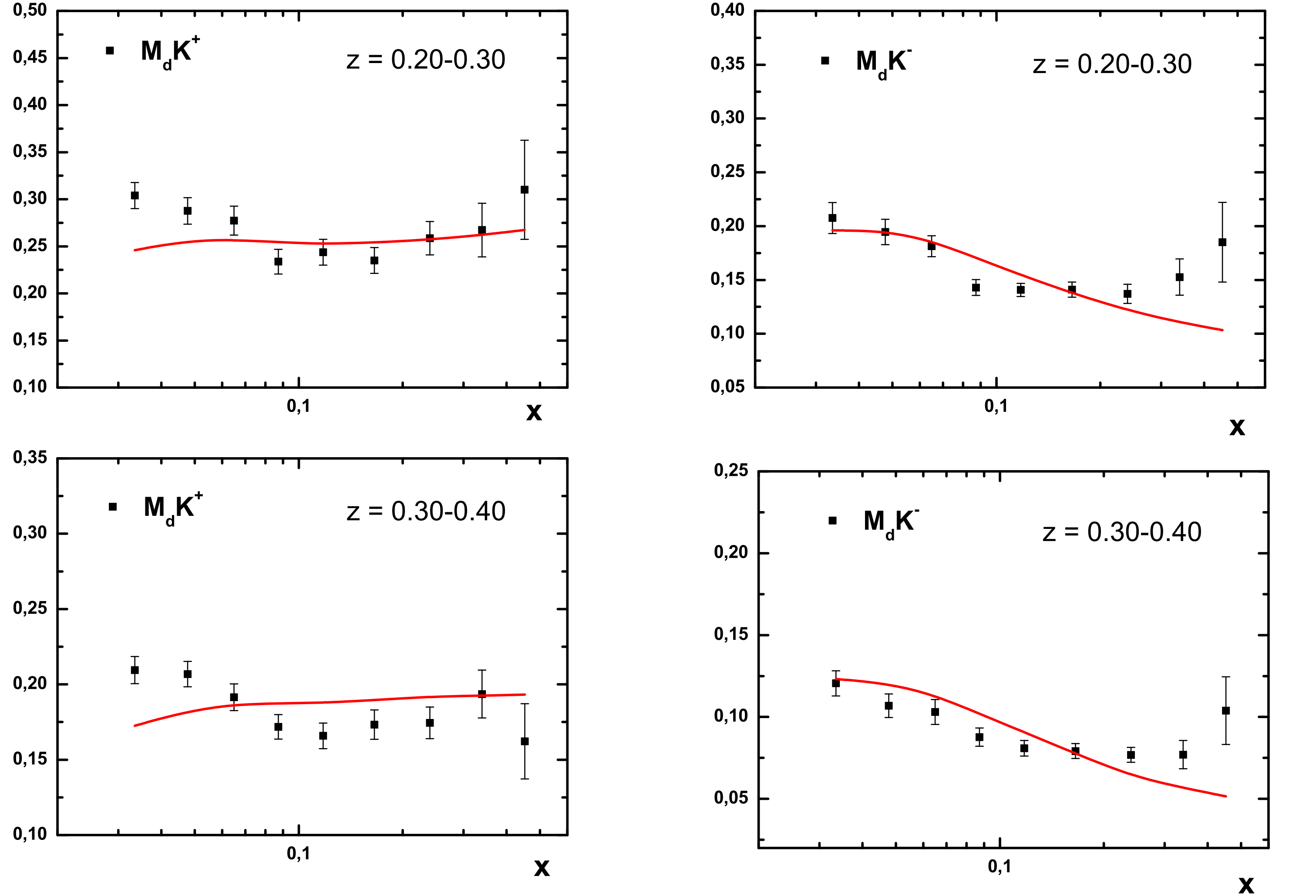


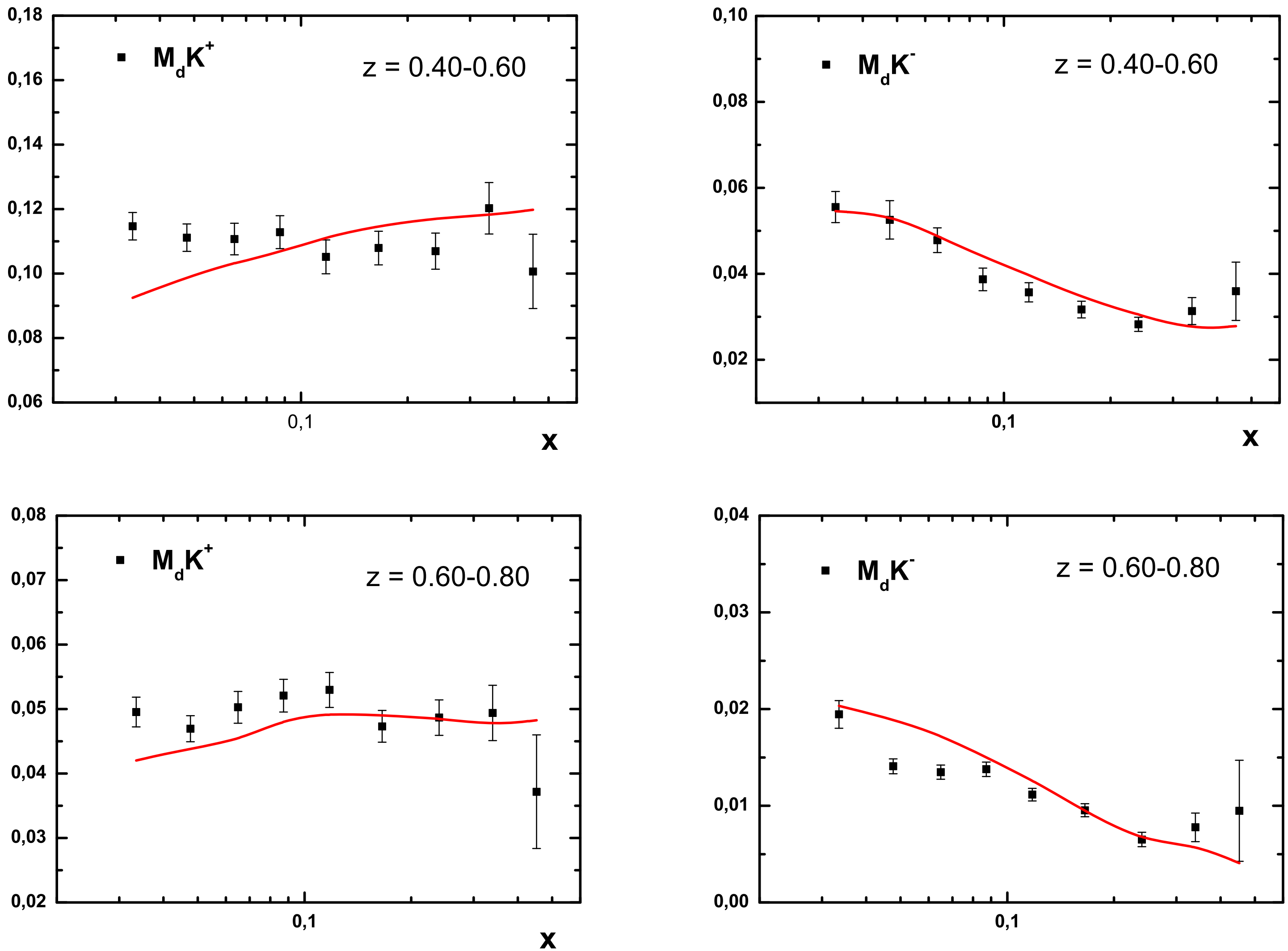

**The description of the $K^{(-)}$ data is much better**

*Preliminary*

# Comparison between the HERMES (x,z) deuteron data on kaon multiplicities with the best fit NLO QCD LSS curves

$\chi^2$/Nrpoints (K+) = 4.04 $\chi^2$/Nrpoints (K-) = 4.55




0,18
0,16
0,14
0,12
0,10
0,08
0,06
0,1
x
$M_dK^+$
z = 0.40-0.60
0,10
0,08
0,06
0,04
0,02
0,1
x
$M_dK^-$
z = 0.40-0.60
0,08
0,07
0,06
0,05
0,04
0,03
0,02
0,1
x
$M_dK^+$
z = 0.60-0.80
0,04
0,03
0,02
0,01
0,00
0,1
$M_dK^-$
z = 0.60-0.80

# SUMMARY

- A good NLO QCD fit to COMPASS pion data is achieved

  → a new set of pion FFs

- Up to now we can not find a reasonable fit to the HERMES pion data
  (the fits under way)

There is an indication that the HERMES and COMPASS data are not consistent

OR

Possibly the theory treatment is not correct