Unpolarised SIDIS measurements at COMPASS

Andrea Bressan
University of Trieste and INFN
(on behalf of the COMPASS Collaboration)

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Covered Results

- Contribution of exclusive diffractive processes to the measured azimuthal asymmetries in SIDIS, *Nuclear Physics B* 956 (2020) 115039
- Preliminary results from 2016 with a proton target, toward the publication (improving on RCs)
The account of the transverse motion of the quark result in the following general form of the unpolarised semi-inclusive deep inelastic cross-section

\[
\frac{d^5\sigma}{dx dy dz dP_{hT}^2 d\phi_h} = \frac{2\pi\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left( 1 + \frac{2xM^2}{Q^2} \right) \left[ (1-y) + \frac{y^2}{2} \right] \{ F_{UU,T}^h + \epsilon F_{UU,L}^h + \sqrt{2\epsilon}(1+\epsilon) F_{UU}^{\cos \phi_h} \cos \phi_h + \epsilon F_{UU}^{\cos 2\phi_h} \cos 2\phi_h + \ldots \} 
\]

We can then introduce amplitude of the azimuthal asymmetries as

\[
A_{UU}^{\cos \chi \phi_h}(x, z, P_{hT}^2; Q^2) = \frac{F_{UU}^{\cos \chi \phi_h}(x, z, P_{hT}^2; Q^2)}{F_{UU}^h(x, z, P_{hT}^2; Q^2)}
\]

An the angular independent ratio

\[
M_{UU}^h(x, z, P_{hT}^2; Q^2) = \frac{F_{UU}^h(x, z, P_{hT}^2; Q^2)}{F_2(x, Q^2)}
\]

Experimentally these are more difficult measurements than spin asymmetries, since we have to correct for the apparatus acceptance.
Unpolarised Azimuthal Modulation

When looking at the content of the structure functions/modulations in terms of TMD PDFs for the $\cos \phi_h$ and $\cos 2\phi_h$ we can write:

\[
F_{UU}^{\cos \phi_h} = -\frac{2M}{Q} C \left[ \frac{\hat{h} \cdot \vec{k}_\perp}{M} f_1 D_1 - \frac{p_\perp k_\perp}{M} \frac{\vec{P}_{hT} - z(\hat{h} \cdot \vec{k}_\perp)}{zM_h M} h_1^\perp H_1^\perp \right] + \text{twists} > 3
\]

\[
F_{UU}^{\cos 2\phi_h} = C \left[ \frac{(\hat{h} \cdot \vec{k}_\perp)(\hat{h} \cdot \vec{p}_\perp) - \vec{p}_\perp \cdot \vec{k}_\perp}{MM_h} h_1^\perp H_1^\perp \right] + \text{twists} > 3
\]

In the $\cos 2\phi_h$ Cahn effects enters only at twist 4

\[
F_{Cahn}^{\cos 2\phi_h} \approx \frac{2}{Q^2} C \left[ \left\{2(\hat{h} \cdot \vec{k}_\perp)^2 - k_\perp^2 \right\} f_1 D_1 \right]
\]
1. In the case of unpolarized SIDIS the measured rates need to be corrected for the effect of the apparatus (acceptance corrections, including geometrical acceptance, detector efficiencies ...)

2. Events from processes different from SIDIS may be present in the final sample, and we know that charged hadron SIDIS sample at large $z$ and at small $P_{hT}$ contains a non-negligible contribution of hadrons from the decay of vector mesons (VM) produced in exclusive processes

3. Radiative effects change both the LO cross section and the reconstructed event kinematics

   • With the COMPASS data sample increasing over the years we were able to address with improved precision these effects
• Contributions from $\rho^0$, $\omega$, and $\phi$

• Exclusive $\rho^0$ leptoproduction can be viewed as a virtual photon fluctuation into a $q\bar{q}$-pair followed by the scattering of this pair off the nucleon and formation of the final state.

• These are spin-1 objects, i.e. $J = 1$. Decay particles have spin 0, so $L = 1$ for the decay. In words when the VM decays, its spin-state will be reflected in the orbital momentum of the decay particles.

• Due to the nature of the process we can reject some/most, not all, of these hadrons from our sample.

• Exclusive VMs can be removed from the sample when both final hadrons detected (VISIBLE PART). EVM cut: 
  \[ z_t = z_{h^+} + z_{h^-} < 0.95 \]

• If one hadron is miss, this is no longer true (INVISIBLE PART).

• Strategy:
  • have a MC for exclusive VMs with Spin Density Matrix Elements.
  • Compare MC with our exclusive data normalize MCs
  • Use this normalization to subtract the invisible fraction from our data. EVM subtraction
The radiative leptonic tensor $S(\ell, \ell', k)$, include Born + loops at $\mathcal{O}(\alpha_{em}^2)$:
- Gauge invariant
- Infrared finite
- Universal (for $1\gamma$ exchange)
- The kinematic is shifted $\tilde{q}^\mu = q^\mu - k^\mu$
Effects on SIDIS

- Photon radiation from the muon lines changes the DIS kinematics on the event by event basis
- The direction of the virtual photon is changed with respect to the one reconstructed from the muons creating
  - false asymmetries in the azimuthal distribution of hadrons calculated with respect to the virtual photon direction
  - Smearing of the kinematic distributions (f.i. $z$ and $P_{hT}$)
- Due to the energy unbalance, in the lepton plane the true virtual photon direction is always at larger angles with respect to the reconstructed one
- In SIDIS, having an hadron in the final state, only the inelastic part of the radiative corrections plays a role
Observed cross section: convolution of true cross section $\otimes$ radiator function

$$d\sigma^{\text{obs}}(p, q) = \int d^3k \frac{d^3\tilde{k}}{2k^0} R(\ell, \ell', k)d\sigma^{\text{true}}(p, q - k)$$

exp cond's

- Shifted kinematics: $q \rightarrow q - k$, e.g., $Q^2 = -(\ell - \ell')^2 \Rightarrow \tilde{Q}^2 = -(\ell - \ell' - k)^2$
- but integration may be restricted by experimental conditions, also indirectly:
  - leptonic variables: measure $E$ and $\theta$ of scattered lepton $\Rightarrow$ and $Q^2$
  - real photon detection: possibility to reject radiative events by detecting the radiated photon

NOTE the $\tilde{Q}^2 \ll Q^2$ is possible: $\tilde{Q}_{\text{min}}^2 = \frac{x^2}{1-x}M_N^2$

$\Rightarrow$ Difficult to treat radiative and detector effects separately (acceptance cuts, efficiencies, ...)

need full Monte-Carlo treatment DJANGOH
$P_{hT}$-dependent multiplicities
2h Multiplicities (>10 years ago)

\[ Q^2 \in [1.00, 1.70] \]
\[ Q^2 \in [1.70, 4.00] \]
\[ Q^2 \in [4.00, 100.00] \]
1st publication on $P_{hT}$ distributions (2013);
Improved binning

<table>
<thead>
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<th>$x$</th>
<th>$0.003$</th>
<th>$0.008$</th>
<th>$0.013$</th>
<th>$0.02$</th>
<th>$0.032$</th>
<th>$0.055$</th>
<th>$0.1$</th>
<th>$0.21$</th>
<th>$0.4$</th>
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<td>$16$</td>
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<td>$0.6$</td>
<td>$0.8$</td>
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<tr>
<td>$p_{hT}^2$ (GeV/c)$^2$</td>
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<td>$0.04$</td>
<td>$0.06$</td>
<td>$0.08$</td>
<td>$0.10$</td>
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<td>$0.17$</td>
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<td>$1.12$</td>
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<td>$1.38$</td>
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<td>$2.35$</td>
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</tr>
</tbody>
</table>

Subtraction of Diffractive Vector Mesons

![Graphs showing subtraction of diffractive vector mesons](image-url)
2nd publication on $P_{hT}$ distributions;
2nd publication on $P_{hT}$ distributions;
Positive vs Negative charged hadrons ($^6\text{LiD}$)

\[ F_{UU}^h(x, z, P_{\text{HT}}^2; Q^2) = x \sum_q e_q^2 \int d^2 k_\perp d^2 p_\perp \delta(p_\perp - z k_\perp - \vec{p}_{\text{HT}}) \ f_1^q(x, k_\perp^2; Q^2) \ D_1^{q \to h}(z, p_\perp^2; Q^2) \]

\[ \langle Q^2 \rangle = 9.78 \text{ (GeV/c)}^2 \text{ and } \langle x \rangle = 0.149 \]
Positive vs Negative charged hadrons (LH$_2$)

\[ Q^2 (\text{GeV}/c)^2 \]

\[ 0.60 < z < 0.80 \]

\[ h^+ \]
\[ h^- \]

COMPASS preliminary

\[ P_T^2 (\text{GeV}/c)^2 \]

\[ x \]

\[ 0.003 \quad 0.013 \quad 0.020 \quad 0.055 \quad 0.100 \]
A Gaussian ansatz for $k_\perp$ and $p_\perp$ leads to
\[ \langle P_{hT}^2 \rangle = z^2 \langle k_\perp^2 \rangle + \langle p_\perp^2 \rangle \]
Azimuthal Modulations
An old story

• Cross section for SIDIS process expected to be
  \[ d\sigma \sim \sigma_0 [1 + A \cos \phi_h + B \cos 2\phi_h] \]


• R.N. Cahn [1978]: same modulations can arise due to the quark intrinsic motion \((k_\perp)\) [Phys.Lett.B 78 (1978) 269]

QCD alone

QCD + quark transverse motion

EMC experiment [1987]
Fit: Konig-Kroll model [1982] + Lund String

These effects can be estimated by adopting a model for the transverse momentum distribution of partons in a hadron and for the transverse momentum given to hadrons in the quark decay. Suppose that both these distributions are gaussian:

\[ f(x, p_\perp) \propto e^{-ap_\perp^2}, \quad D(x, p_\perp) \propto e^{-bp_\perp^2}, \]  

(16a, b)

where \(f\) represents the quark distribution and \(D\) the fragmentation function. Let the \(z\)-direction be defined as in fig. 1. Then the longitudinal momentum of the struck parton is \(xp\) and that of the observed hadron is \(zxP\). If the transverse momentum of the struck parton is \(p_{1z}\) and that of the observed hadron is \(p_{1}\), then the momentum of the observed hadron transverse to the parton direction is \((zxP \gg |p_{1z}|, |p_{1}|)\) just \(p_\perp \approx zp_{1\perp}\).
Azimuthal modulations on $^6\text{LiD}$

C. Adolph et al. / Nuclear Physics B 886 (2014) 1046–1077

$N$

$\alpha$

$N_{\text{corr}}$

$0.64 \text{ GeV/c} < p_T < 0.77 \text{ GeV/c}$

$A^{\text{LVLH}, i}_{\text{corr}}$

$A^{\text{LVLH}, j}$

$10^{-2} \quad 10^{-1} \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8$

$N_{\text{corr}}$

$0.64 \text{ GeV/c} < p_T < 0.77 \text{ GeV/c}$

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Azimuthal modulations on $^6\text{LiD}$

NPB 886 (2014)
VM subtraction from $^6\text{LiD}$ results

NPB 956 (2020) 115039
Study of VM contamination $LH_2$

Normalization of HEPGEN
Effect of Exclusive VM subtraction
Radiative effects

COMPASS preliminary $\mu p \rightarrow \mu^+ h^- X$

- $\cos^\phi$
- $\sin^\phi$
- $\alpha_{UI}$

$0.1 < P_T'/(GeV/c) < 1.00$
$0.2 < z < 0.85$

$10^{-2}$ $10^{-1}$ $0.2$ $0.4$ $0.6$ $z$
$P_T$ (GeV/c)

$0.1 < P_T'/(GeV/c) < 1.00$
$0.2 < z < 0.85$

$10^{-2}$ $10^{-1}$ $0.2$ $0.4$ $0.6$ $z$
$P_T$ (GeV/c)
Corrected results

MultiD on LH2, corrected for both VM and RC is coming
Outlook

- In the study of unpolarized multiplicities and azimuthal asymmetries we are able already today to obtain precise multidimensional results.
- This should allow the start for the transition from “exploratory/consolidation” to the “maturity” era that will arrive with the EIC.
- But also offers us the glimpse on the challenges that this “precision” will bring for both the experimentalist and the theoreticians.
Contamination of hadrons from $\rho^0$ and $\phi$

\[ Q^2(\text{GeV/c})^2 \]

COMPASS preliminary

The diffractive $\rho^0$ production and decay.
Azimuthal modulations on $(\text{LH}_2)$ – 1D

COMPASS preliminary

$A_{\cos \phi}$

$A_{\sin \phi}$
Contamination on (LH$_2$) – 1D

- Determined from $z_1 + z_2 > 0.95$
- Selecting $\rho^0$, $\omega$ and $\phi$

The diffractive $\rho^0$ production and decay.
$P_{hT}$ distributions vs W

\[ R = \frac{dN/dP_T^2 \text{ (W>12 GeV/c$^2$)}}{dN/dP_T^2 \text{ (W<12 GeV/c$^2$)}} \]

0.30 < z < 0.40

- h$^+$
- h$^-$
Unpolarised Transverse Momentum dependent PDFs

• When we consider the transverse momentum of the quark in the calculation of the cross section Transverse Momentum Dependent parton distribution (TMDs)

• The unpolarised number density of the quarks gains a dependence from the intrinsic transverse momentum $k_\perp$

\[ f_1^q(x, k_\perp) \]

• New parton densities arise: the Boer-Mulders functions $h_{1L}^{q} (x, k_\perp)$, describing the correlation between the intrinsic quark transverse momentum and the spin of the quark in an unpolarised nucleon

\[ f_{q\uparrow}(x, k_\perp, \vec{s}) = f_1^q(x, k_\perp) - \frac{1}{M} h_{1L}^{q} (x, k_\perp) \vec{s} \cdot (\hat{p} \times \vec{k}_\perp) \]
Unpolarised Azimuthal Modulation

The cross-section is
\[ d\sigma^{\ell p \rightarrow \ell'hX} = \sum_q f_q(x, Q^2) \otimes d\sigma^{\ell q \rightarrow \ell'q} \otimes D_q^h(z, Q^2) \]

with the partonic process given by
\[ d\sigma^{\ell q \rightarrow \ell'q} = \hat{s}^2 + \hat{u}^2 \]

\[ \hat{s} := (\ell + k)^2 \sim 2\ell \cdot k \]
\[ \hat{u} := (\ell - k)^2 \sim -2\ell \cdot k \]

In collinear PM
\[ d\sigma^{\ell q \rightarrow \ell'q} = \hat{s}^2 + \hat{u}^2 = x[1 + (1 - y)^2], \text{ i.e. no } \phi_h \text{ dependence.} \]
Unpolarised Azimuthal Modulation

When $k_\perp$ is taken into account:

$$k \approx (xP, k_\perp \cos \phi, k_\perp \sin \phi, xP)$$

$$k_\perp \approx (0, k_\perp \cos \phi, k_\perp \sin \phi, 0)$$

\[ \hat{s} = sx \left[ 1 - \frac{2k_\perp}{Q} \sqrt{1 - y \cos \phi} \right] + \sigma \left( \frac{k_\perp^2}{Q} \right) \]

\[ \hat{u} = sx (1 - y) \left[ 1 - \frac{2k_\perp}{Q \sqrt{1 - y}} \cos \phi \right] + \sigma \left( \frac{k_\perp^2}{Q} \right) \]

and

\[ d\sigma^{\ell q \to \ell' q} \propto \hat{s}^2 + \hat{u}^2 \propto \left[ 1 - \frac{2k_\perp}{Q} \sqrt{1 - y \cos \phi} \right]^2 + (1 - y)^2 \left[ 1 - \frac{2k_\perp}{Q \sqrt{1 - y}} \cos \phi \right]^2, \]

Resulting in the $\cos \phi_h$ and $\cos 2\phi_h$ modulations observed in the azimuthal distributions.
The asymmetries are:

\[ A_{UU}^{w(\phi_h)}(x, z, P_{hT}^2; Q^2) = \frac{F_{UU}^{w(\phi_h)}}{F_2(x, Q^2)} \]

When we measure on 1D, i.e. as a function of \( x \), we integrate over the phase space of the other variables

\[ A_{UU}^{w(\phi_h)}(x) = \frac{\int_{Q_{min}^2}^{Q_{max}^2} dQ^2 \int_{z_{min}}^{z_{max}} dz \int_{p_{T,min}}^{p_{T,max}} dP_{hT}^2 F_{UU}^{w(\phi_h)}}{\int_{Q_{min}^2}^{Q_{max}^2} dQ^2 \int_{z_{min}}^{z_{max}} dz \int_{p_{T,min}}^{p_{T,max}} dP_{hT}^2 (F_2(x, Q^2))} \]
Comparison with the publish deuteron

\[ Q^2 (\text{GeV}/c)^2 \]

- \( h^- \)
- \( 0.20 < z < 0.30 \)
- \( 0.30 < z < 0.40 \)
- \( 0.40 < z < 0.60 \)
- \( 0.60 < z < 0.80 \)
- \( \text{deuteron, } \text{PRD67}(2018) \)

COMPASS preliminary

\[ P_T^2 (\text{GeV}/c)^2 \]

- \( x \)

6/5/2024
$q_T$ distributions

COMPASS preliminary

$Q^2(\text{GeV}/c)^2$

- $h^{-}$
  - $0.20 < z < 0.30$
  - $0.30 < z < 0.40$
  - $0.40 < z < 0.60$
  - $0.60 < z < 0.80$
  - from $P_T^2$ dist.

$dN/dq_T$

$q_T$ (GeV/c) vs. $x$

$y$-axis: $dN/dq_T$

$x$-axis: $q_T$ (GeV/c)

$y$-axis: $Q^2(\text{GeV}/c)^2$

$z$-axis: $h^{-}$

Legend:
- ● $0.20 < z < 0.30$
- ■ $0.30 < z < 0.40$
- ▲ $0.40 < z < 0.60$
- ▼ $0.60 < z < 0.80$
- ○ from $P_T^2$ dist.
Phenomenological fits

Azimuthal modulations on $(\text{LH}_2) – 3D$
Contamination on (LH$_2$) – 3D

![Graphs showing data points and plots with axes labeled](image-url)
$Q^2$ behavior

COMPASS preliminary
The account of the transverse motion of the quark result in the following general form of the unpolarised semi-inclusive deep inelastic cross-section

$$\frac{d^5 \sigma}{dx dy dz dP_{hT}^2 d\phi_h} = \frac{\alpha^2}{xyQ^2} \left[ (1 - y) + \frac{y^2}{2} \right] F_2(x, Q^2) \times \left\{ 1 + \frac{2(2 - y)\sqrt{1 - y}}{1 + (1 - y)^2} A_{UU}^{\cos \phi_h} \cos \phi_h + \frac{2(1 - y)}{1 + (1 - y)^2} A_{UU}^{\cos 2\phi_h} \cos 2\phi_h \right\}$$

Where we have introduced the amplitude of the azimuthal asymmetries as

$$A_{UU}^{\cos X \phi_h}(x, z, P_{hT}^2; Q^2) = \frac{F_{UU}^{\cos X \phi_h}(x, z, P_{hT}^2; Q^2)}{F_{UU}^h(x, z, P_{hT}^2; Q^2)}$$

An the angular independent ratio

$$M_{UU}^h(x, z, P_{hT}^2; Q^2) = \frac{F_{UU}^h(x, z, P_{hT}^2; Q^2)}{F_2(x, Q^2)}$$

Experimentally these are more difficult measurements than spin asymmetries, since we have to correct for the apparatus acceptance.
RADIATIVE CORRECTIONS

• Measure FFs, PDFs, etc. by comparing data with theoretical predictions:
  \[ \sigma_{\text{exp}} = \sigma_{\text{theory}}[PDFs, TMDs \ldots] \]

• High precision requires knowledge of higher-order corrections
  \[ \sigma_{\text{theory}} = \sigma^{(0)}[\equiv \sigma_{\text{Born}}] + \alpha_{em} \sigma^{(1)} + \alpha_{em}^2 \sigma^{(2)} + \ldots \]

• Emission of real photons
  • experimentally often not distinguished from non-radiative processes: soft photons, collinear photons
  \( \rightarrow \) ”radiative corrections”

• Virtual corrections: loop diagrams
  • needed to cancel infrared divergences