Further direct extractions of the transversity functions

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Further direct extractions of transversity

point-by-point extractions of $h_1$

- first done in 2015 using the Collins and di-hadron asymmetries in SIDIS measured on p and d by COMPASS, and in $e^+e^-$ annihilation measured by Belle
  to extract $h_{1uv}^u(x), h_{1dv}^d(x), xh_1^u(x) + xh_1^d(x), xh_1^u(x)$ and $xh_1^d(x)$
  
  Extracting the transversity distributions from single-hadron and dihadron production
  A. Martin, F. Bradamante, V. Barone, PRD 91 (2015) 014034

- redone by COMPASS in 2023 for $h_{1uv}^u(x)$ and $h_{1dv}^d(x)$ using the new d measurements from 2022 data
  arXiv:2401.00309 [hep-ex], accepted by PRL, → Athira talk
Transversity from COMPASS SIDIS data

COMPASS 2024 arXiv:2401.00309 [hep-ex], accepted by PRL → Athira talk

extraction of $h_1^{u,v}(x)$ and $h_1^{d,v}(x)$ using all the COMPASS measurements of the Collins asymmetries  p: 2007, 2010, d:2002-2004, 2022

open points: no 2022 data

u: errors smaller by a factor 1.2 to 1.9
d: errors smaller by a factor 1.7 to 3.1

<table>
<thead>
<tr>
<th>data</th>
<th>$\delta u = \int_{0.008}^{0.210} dx h_1^{u,v}(x)$</th>
<th>$\delta d = \int_{0.008}^{0.210} dx h_1^{d,v}(x)$</th>
<th>$g_T = \delta u - \delta d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>no 2022</td>
<td>$0.187 \pm 0.030$</td>
<td>$-0.178 \pm 0.097$</td>
<td>$0.365 \pm 0.078$</td>
</tr>
<tr>
<td>with 2022</td>
<td>$0.214 \pm 0.020$</td>
<td>$-0.070 \pm 0.043$</td>
<td>$0.284 \pm 0.045$</td>
</tr>
</tbody>
</table>

$0 < \rho < 0.1$
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  Extracting the transversity distributions from single-hadron and dihadron production
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• redone by COMPASS in 2023 for $h_1^{uv}(x)$ and $h_1^{dv}(x)$ using the new d measurements from 2022 data
  arXiv:2401.00309 [hep-ex], accepted by PRL, → Athira talk

• also, $h_1^{dv}(x)/h_1^{uv}(x)$ in 2019, using the p and d Collins difference asymmetries measured by COMPASS

  Transversity distributions from difference asymmetries in semi-inclusive DIS.
  V. Barone et el., PRD 99 (2019) 114004
in this talk, point-by-point extractions of

- \( xh_1^u(x) + xh_1^d(x) \)
- \( xh_1^u(x) \) and \( xh_1^d(x) \) as in PRD 91 (2015) 014034
- \( h_1^{dv}(x)/h_1^{uv}(x) \) as in PRD 99 (2019) 114004

new: we used all the Collins asymmetries for \( h^\pm \) measured by COMPASS
namely

- \( p \) from 2007 and 2010 data PLB 717 (2012) 376
- \( d \) from 2002-2004 and 2022 data NPB 765 (2007) 31, ep-ex/2401.00309 PRL
Transversity from Collins asymmetries - reminder

\[ A^h_{Coll}(x, z) = \frac{\sum q\bar{q} e_q^2 x h_1^q (x, k_{Th}^2) \otimes H^h_{1q}(z, p_{1H}^2)}{\sum q\bar{q} e_q^2 x f_1^q (x, k_{Tf}^2) \otimes D^h_{1q}(z, p_{1D}^2)} \]

assumptions:

• Gaussian Ansatz for the TMD PDFs and FF

\[ h_1^q (x, k_{Th}^2) = h_1^q (x) \frac{e^{-k_{Th}^2/\langle k_{Th}^2 \rangle}}{\pi \langle k_{Th}^2 \rangle}, \ldots \]

\[ H^h_{1q}(z) = H^h_{1q}(1/2) (z) \quad \text{half-moment of } H^h_{1q} \]

• all charged hadrons are pions (75% in COMPASS) and the c quark contribution is negligible at COMPASS energy (beam energy 160 GeV)

• favoured and unfavored FFs

\[ D_{1, fav} = D_{1u}^+ = D_{1d}^- = D_{1\bar{u}}^- = D_{1\bar{d}}^+ \quad D_{1, unf} = D_{1u}^- = D_{1d}^+ = D_{1\bar{u}}^+ = D_{1\bar{d}}^- = D_{1s}^\pm = D_{1\bar{s}}^\pm \]

\[ H_{1, fav} = H_{1u}^+ = H_{1d}^- = H_{1\bar{u}}^- = H_{1\bar{d}}^+ \quad H_{1, unf} = H_{1u}^- = H_{1d}^+ = H_{1\bar{u}}^+ = H_{1\bar{d}}^- \]

\[ H_{1s}^\pm = H_{1\bar{s}}^\pm = 0 \]
with these assumptions the Collins asymmetries measured with p and d targets can be written as

\[ A_p^+ (x) = \tilde{\alpha}_p \frac{4 \left( x h_1^u (x) + \tilde{\alpha} x h_1^{\bar{u}} (x) \right) + (\tilde{\alpha} x h_1^d (x) + x h_1^\bar{d} (x))}{x f_p^+ (x)} \]

\[ A_p^- (x) = \tilde{\alpha}_p \frac{4 \left( \tilde{\alpha} x h_1^u (x) + x h_1^{\bar{u}} (x) \right) + (x h_1^d (x) + \tilde{\alpha} x h_1^\bar{d} (x))}{x f_p^- (x)} \]

\[ A_d^+ (x) = \tilde{\alpha}_p \frac{(4 + \tilde{\alpha}) \left( x h_1^u (x) + x h_1^d (x) \right) + (1 + 4 \tilde{\alpha}) \left( x h_1^{\bar{u}} (x) + x h_1^\bar{d} (x) \right)}{x f_d^+ (x)} \]

\[ A_d^- (x) = \tilde{\alpha}_p \frac{\left( 1 + 4 \tilde{\alpha} \right) \left( x h_1^u (x) + x h_1^d (x) \right) + \left( 4 + \tilde{\alpha} \right) \left( x h_1^{\bar{u}} (x) + x h_1^\bar{d} (x) \right)}{x f_d^- (x)} \]
with these assumptions the Collins asymmetries measured with p and d targets can be written as

\[
A_p^+(x) = \tilde{a}_p \frac{4(xh_1^u(x) + \tilde{a}xh_1^u(x)) + (\tilde{a}xh_1^d(x) + xh_1^d(x))}{xf_p^+(x)}
\]

\[
A_p^-(x) = \tilde{a}_p \frac{4(\tilde{a}xh_1^u(x) + xh_1^u(x)) + (xh_1^d(x) + \tilde{a}xh_1^d(x))}{xf_p^-(x)}
\]

\[
A_d^+(x) = \tilde{a}_p \frac{(4 + \tilde{a})(xh_1^u(x) + xh_1^d(x)) + (1 + 4\tilde{a})(xh_1^u(x) + xh_1^d(x))}{xf_d^+(x)}
\]

\[
A_d^-(x) = \tilde{a}_p \frac{(1 + 4\tilde{a})(xh_1^u(x) + xh_1^d(x)) + (4 + \tilde{a})(xh_1^u(x) + xh_1^d(x))}{xf_d^-(x)}
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\[ A_d^+(x) = \tilde{\alpha}_p \frac{(4 + \tilde{\alpha})(xh_1^u(x) + xh_1^d(x)) + (1 + 4\tilde{\alpha})(xh_1^\bar{u}(x) + xh_1^\bar{d}(x))}{xf_d^+(x)} \]

\[ A_d^-(x) = \tilde{\alpha}_p \frac{(1 + 4\tilde{\alpha})(xh_1^u(x) + xh_1^d(x)) + (4 + \tilde{\alpha})(xh_1^\bar{u}(x) + xh_1^\bar{d}(x))}{xf_d^-(x)} \]

\[ f_T^\pm(x) \] are combinations of unpolarized PDFs and FFs

f.i. \( f_p^+ = [4(f_1^u + \tilde{\beta}f_1^\bar{u}) + (\tilde{\beta}f_1^d + f_1^\bar{d})] + \tilde{\beta}(f_1^s + f_1^\bar{s}) \]

\[ \tilde{\beta} = \frac{\int dz D_{1unf}(z)}{\int dz D_{1fav}(z)} \]

known, evaluated using CTEQ5D (PDFs) and DSS LO (FFs)
with these assumptions the Collins asymmetries measured with p and d targets can be written as

$$A_p^+(x) = \tilde{\alpha}_p \frac{4(x h_1^u(x) + \tilde{\alpha} x h_1^u(x)) + (\tilde{\alpha} x h_1^d(x) + x h_1^d(x))}{x f_p^+(x)}$$

$$A_p^-(x) = \tilde{\alpha}_p \frac{4(\tilde{\alpha} x h_1^u(x) + x h_1^u(x)) + (x h_1^d(x) + \tilde{\alpha} x h_1^d(x))}{x f_p^-(x)}$$

$$A_d^+(x) = \tilde{\alpha}_p \frac{(4 + \tilde{\alpha})(x h_1^u(x) + x h_1^d(x)) + (1 + 4\tilde{\alpha})(x h_1^u(x) + x h_1^d(x))}{x f_d^+(x)}$$

$$A_d^-(x) = \tilde{\alpha}_p \frac{(1 + 4\tilde{\alpha})(x h_1^u(x) + x h_1^d(x)) + (4 + \tilde{\alpha})(x h_1^u(x) + x h_1^d(x))}{x f_d^-(x)}$$

$f_T^\pm(x)$ are combinations of unpolarized PDFs and FFs, known

$$\tilde{\alpha}_p = \frac{\int dz \ H_{1fav}(z)}{\int dz \ D_{1fav}(z)} \quad \text{and} \quad \tilde{\alpha} = \frac{\int dz \ H_{1unf}(z)}{\int dz \ H_{1fav}(z)}$$

are obtained from $e^+e^- \rightarrow h_1 h_2 X$
\( \tilde{\alpha}_P \) and \( \tilde{\alpha} \) from Collins asymmetries in \( e^+e^- \) - reminder

used data:
Belle results on the asymmetry \( A_{12} \)
in \( e^+e^- \rightarrow h_1h_2X \) with the two hadrons in different hemispheres, corrected for charm contribution
in the bins in the bins \( z_1 = z_2 = z \)

same assumptions on the FFs as for SIDIS
(Belle data are subtracted for charm)

\[ \alpha(z) = \frac{H_{1\text{unf}}(z)}{H_{1\text{fav}}(z)} = -\beta(z) = -\frac{D_{1\text{unf}}(z)}{D_{1\text{fav}}(z)} \]
i.e. \[ \frac{H_{1\text{fav}}(z)}{D_{1\text{fav}}(z)} = -\frac{H_{1\text{unf}}(z)}{D_{1\text{unf}}(z)} \]

\( \Rightarrow \quad a_P(z) = \frac{H_{1\text{fav}}(z)}{D_{1\text{fav}}(z)} = N'z, \quad N' = 0.501 \pm 0.011 \]

\( \Rightarrow \quad a_P(z) = \frac{H_{1\text{unf}}(z)}{D_{1\text{unf}}(z)} = \frac{H_{1\text{fav}}(z)}{D_{1\text{fav}}(z)} = 0.173 \)

\( \tilde{a}_P = \frac{\int dz H_{1\text{fav}}(z)}{\int dz D_{1\text{fav}}(z)} = \frac{\int dz a_P(z) D_{1\text{fav}}(z)}{\int dz D_{1\text{fav}}(z)} = 0.173 \)

at Belle

\( \alpha(z) = \frac{\int dz H_{1\text{unf}}(z)}{\int dz H_{1\text{fav}}(z)} = -\frac{\int dz zD_{1\text{unf}}(z)}{\int dz zD_{1\text{fav}}(z)} \)

ranges from -0.43 (highest \( x \))
to -0.34 (lowest \( x \))
those values were used to extract

\[
xh_{1u} = \frac{1}{5} \frac{1}{\bar{a}_p(1-\bar{\alpha})} \left[ (xf_p^+ A^+_p - xf_p^- A^-_p) + \frac{1}{3} (xf_d^+ A^+_d - xf_d^- A^-_d) \right]
\]

\[
xh_{1d} = \frac{1}{5} \frac{1}{\bar{a}_p(1-\bar{\alpha})} \left[ \frac{4}{3} (xf_p^+ A^+_p - xf_p^- A^-_p) + (xf_d^+ A^+_d - xf_d^- A^-_d) \right]
\]

from the COMPASS Collins asymmetries available at the time


extractions also using the di-hadron asymmetries (open points):

good agreement
going back to the expressions for the four measured Collins asymmetries

\[
A_p^+(x) = \tilde{\alpha}_p \frac{4 \left( x h_1^u(x) + \tilde{\alpha} x h_1^d(x) \right) + \left( \tilde{\alpha} x h_1^q(x) + x h_1^\bar{q}(x) \right)}{x f_p^+(x)}
\]

\[
A_p^-(x) = \tilde{\alpha}_p \frac{4 \left( \tilde{\alpha} x h_1^u(x) + x h_1^d(x) \right) + \left( x h_1^q(x) + \tilde{\alpha} x h_1^\bar{q}(x) \right)}{x f_p^-(x)}
\]

\[
A_d^+(x) = \tilde{\alpha}_p \frac{(4 + \tilde{\alpha}) \left( x h_1^u(x) + x h_1^d(x) \right) + \left( 1 + 4\tilde{\alpha} \right) \left( x h_1^\bar{u}(x) + x h_1^\bar{d}(x) \right)}{x f_d^+(x)}
\]

\[
A_d^-(x) = \tilde{\alpha}_p \frac{(1 + 4\tilde{\alpha}) \left( x h_1^u(x) + x h_1^d(x) \right) + \left( 4 + \tilde{\alpha} \right) \left( x h_1^\bar{u}(x) + x h_1^\bar{d}(x) \right)}{x f_d^-(x)}
\]

other combinations which can be used to extract the sea quarks transversity functions:

\[
x h_1^\bar{u} + x h_1^\bar{d} = \frac{1}{15} \frac{1}{\tilde{\alpha}_p(1 - \tilde{\alpha}^2)} \left[ (4 + \tilde{\alpha}) x f_d^− A_d^- - (4\tilde{\alpha} + 1) x f_d^+ A_d^+ \right]
\]

\[
x h_1^\bar{u} = \frac{1}{15} \frac{1}{\tilde{\alpha}_p(1 - \tilde{\alpha}^2)} \left[ (1 - 4\tilde{\alpha}) x f_p^+ A_p^+ + (4 - \tilde{\alpha}) x f_p^- A_p^- - x f_d^+ A_d^+ + \tilde{\alpha} x f_d^- A_d^- \right]
\]

\[
x h_1^\bar{d} = \frac{1}{15} \frac{1}{\tilde{\alpha}_p(1 - \tilde{\alpha}^2)} \left[ (4\tilde{\alpha} - 1) x f_p^+ A_p^+ - (4 - \tilde{\alpha}) x f_p^- A_p^- - 4\alpha x f_d^+ A_d^+ + \tilde{\alpha} x f_d^- A_d^- \right]
\]

they were evaluated using the same numerical values as for the valence quark transversity today: results using the new COMPASS data
\( xh_1^{\bar{u}}(x) + xh_1^{\bar{d}}(x) \) from Collins asymmetries

COMPASS data

deuteron 2002-2004

deuteron 2002-2004 + 2022

mean value \( 0.045 \pm 0.020 \)
compatible with zero \( \chi^2 = 9.7 \)

expected to vanish in the large \( N_C \) limit

mean value \( 0.014 \pm 0.009 \)
compatible with zero \( \chi^2 = 13.2 \)

errors reduced by a factor 1.8 to 3.4


**COMPASS data**

- Proton 2007+2010
- Deuteron 2002-2004

\[ x h_{1}^{\bar{u}}(x) \text{ and } x h_{1}^{\bar{d}}(x) \text{ from Collins asymmetries} \]

- **\( \bar{u} \):** Mean value \(+0.003 \pm 0.011\)
  - Compatible with zero \( \chi^2 = 2.3 \)

- **\( \bar{d} \):** Mean value \(+0.042 \pm 0.025\)
  - Compatible with zero \( \chi^2 = 7.2 \)

- **\( \bar{u} \):** Mean value \(+0.017 \pm 0.009\)
  - Compatible with zero \( \chi^2 = 11.9 \)

- **\( \bar{d} \):** Mean value \(-0.003 \pm 0.014\)
  - Compatible with zero \( \chi^2 = 7.8 \)

- \( \bar{u} \) errors reduced by a factor 1.1 to 1.5
- \( \bar{d} \): errors reduced by a factor 1.6 to 2.8
\( h_1^{d_v}(x)/h_1^{u_v}(x) \) from difference asymmetries

use of difference asymmetries \( A^{h^+-h^-} \)

main advantage:

\[ h_1^{d_v}(x)/h_1^{u_v}(x) \]

is obtained from the p and d Collins asymmetries in SIDIS only:

the Collins FF is not needed

the method is not new:

used by EMC \( NPB 321 \) (1989) 541

quoted in HELP proposal for transversity

and in COMPASS proposal for L and T spin asymmetries

…

E. Christova and E. Leader, \( NPB 607 \) (2001) 369, …

V. Barone et al., \( PRD 99 \) (2019) 114004

M. Anselmino, R. Kishore and A. Mukherjee, \( PRD 102 \) (2020) 9, 096012
the difference asymmetries

cross section for $h^{+}, h^{-}$

$$\sigma_t^{\pm}(\Phi_C) = \sigma_{0t}^{\pm} + f P_t D_{NN} \sigma_{Ct}^{\pm} \sin \Phi_C , \quad t = p, d$$

Collin asymmetries

$$A_{c,t}^{\pm} = \frac{\sigma_{ct}^{\pm}}{\sigma_{0t}^{\pm}}$$

difference asymmetries

$$A_{d,t} = \frac{\sigma_{ct}^{+} - \sigma_{ct}^{-}}{\sigma_{0t}^{+} + \sigma_{0t}^{-}}$$
the difference asymmetries

cross section for $h^+, h^-$

$$\sigma_t^\pm (\Phi_C) = \sigma_{0t}^\pm + f P_t D_{NN} \sigma_{Ct}^\pm \sin \Phi_C,$$

$\quad t = p, d$

Collin asymmetries

$$A_{C,t}^\pm = \frac{\sigma_{Ct}^\pm}{\sigma_{0t}^\pm}$$

difference asymmetries

$$A_{D,t} = \frac{\sigma_{Ct}^+ - \sigma_{Ct}^-}{\sigma_{0t}^+ + \sigma_{0t}^-}$$

with the usual assumptions on FFs, difference asymmetries for proton and for deuteron can be written as

$$A_{D,p} = \frac{1}{9} \frac{H_{1 fav} - H_{1 unf}}{\sigma_{0p}^+ + \sigma_{0p}^-} \left(4 h_1^{uv} - h_1^{dv}\right)$$

$$A_{D,d} = \frac{1}{3} \frac{H_{1 fav} - H_{1 unf}}{\sigma_{0d}^+ + \sigma_{0d}^-} \left(h_1^{uv} + h_1^{dv}\right)$$

in the ratio, the Collins FFs cancel and it is

$$\frac{A_{D,d}}{A_{D,p}} = 3 \left[\frac{(4 f_1^{uv} + 4 f_1^{ud} + f_1^{dv} + f_1^{dd})(D_{1,fav} + D_{1,unf}) + 2(f_1^s + f_1^t)D_{1,s}}{5(f_1^{uv} + f_1^{ud} + f_1^{dv} + f_1^{dd})(D_{1,fav} + D_{1,unf}) + 4(f_1^s + f_1^t)D_{1,s}}\right] \frac{h_1^{uv} + h_1^{dv}}{4 h_1^{uv} - h_1^{dv}}$$

thus, from the ratio of the difference asymmetries on p and d, one obtains $h_1^{dv}(x)/h_1^{uv}(x)$
how to measure \[ A_{D,t} = \frac{\sigma^+_c - \sigma^-_c}{\sigma^+_0 + \sigma^-_0} \]
in principle, one should measure the difference of the cross-section \[ \sigma_t^D(\Phi_C) = +fP_tD_{NN}\sigma^+_c(\sigma^+_c - \sigma^-_c)\sin \Phi_C \]
and extract the amplitude of the \( \sin \Phi_C \) modulation

experimentally it is easier, if the acceptances for positive and negative hadrons are about the same, as in COMPASS, use the ratios of number of events and obtain the difference asymmetries from

\[ A_{D,t} = \frac{\text{var}(A^-_c)}{\text{var}(A^+_c) + \text{var}(A^-_c)} A^+_c - \frac{\text{var}(A^+_c)}{\text{var}(A^+_c) + \text{var}(A^-_c)} A^-_c \]

this is the method used in 2019, and it is what we have done now, with the new COMPASS data
$h_1^{dv}(x)/h_1^{uv}(x)$ from difference asymmetries

COMPASS data only

proton 2007+2010
deuteron 2002-2004

proton 2007+2010
deuteron 2002-2004 + 2022

Transversity 2024

F. Bradamante
\( h_1^{d_v}(x)/h_1^{u_v}(x) \) from difference asymmetries

from COMPASS data only

proton 2007+2010
deuteron 2002-2004

proton 2007+2010
deuteron 2002-2004 + 2022

mean value \(-0.79 \pm 0.53\)

mean value \(-0.45 \pm 0.16\)

errors reduced by a factor 2 to 5
Summary

• in a simple and direct model-independent way we have extracted the u and d quark transversity distributions, both valence and sea, from the COMPASS and the Belle data, and their ratio from the difference asymmetries

• thanks to the new COMPASS results, transversity of the valence d quark
  • turns out to be compatible but smaller than that previously estimated and different from zero
  • no hints for violation of the Soffer bound

• sea-quark transversity functions are better determined and compatible with zero
Thank you