Transverse-Spin Dependent Azimuthal Asymmetries in COMPASS Drell-Yan data

20th International Workshop on Hadron Structure and Spectroscopy and 5th workshop on Correlations in Partonic and Hadronic Interactions (**IWHSS-CPHI 2024**)

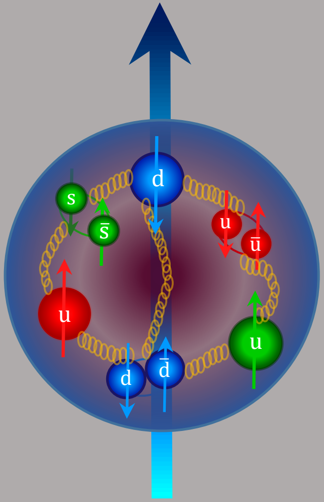
Małgorzata Niemiec on behalf of COMPASS Collaboration
University of Warsaw, Poland

4 X 2024, Yerevan

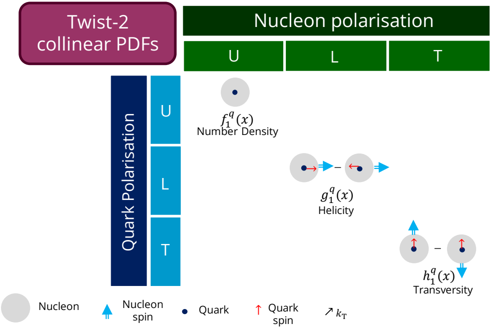
Transverse Momentum Dependent Parton Distribution Functions

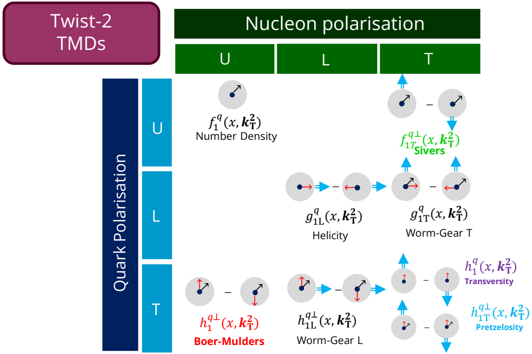
"Well begun is half done."

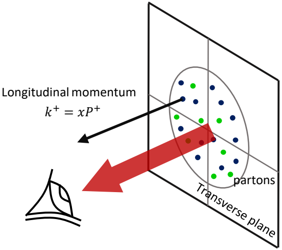
Old Proverb



Nucleon spin structure



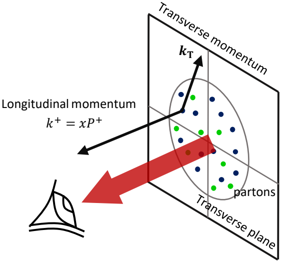




TMD PDFs accessed through target spin dependent azimuthal asymmetries both in SIDIS and Drell-Yan processes.

$$h_1^{q\perp}(\text{SIDIS}) = -h_1^{q\perp}(\text{DY})$$

$$f_{1T}^{q\perp}(\text{SIDIS}) = -f_{1T}^{q\perp}(\text{DY})$$

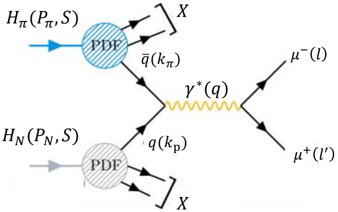


Single polarised Drell-Yan process

"Polarisation data has often been the graveyard of fashionable theories. If theorists had their way, they might well ban such measurements altogether out of self-protection."

James Bjorken

Single polarised Drell-Yan process



Cross-section, LO TMD approach for transversely polarised target:

$$\frac{d\sigma^{LO}}{dx_p dx_\pi d^2 q_T d\varphi_{CS} d(\cos\theta) d\varphi_S} \propto \left\{ \begin{array}{c} 1 + D_{\sin^2\theta} \cos(2\varphi_{CS}) A_U^{\cos(2\varphi_{CS})} \\ + |S_T| \left[\begin{array}{c} \sin(\varphi_S) A_T^{\sin(\varphi_S)} + \\ D_{\sin^2\theta} \left(\begin{array}{c} \sin(2\varphi_{CS} + \varphi_S) A_T^{\sin(2\varphi_{CS}+\varphi_S)} \\ + \\ \sin(2\varphi_{CS} - \varphi_S) A_T^{\sin(2\varphi_{CS}-\varphi_S)} \end{array} \right) \end{array} \right] \end{array} \right\}$$

Boer-Mulders

$$A_U^{\cos(2\varphi_{CS})} \propto h_{1,\pi}^{q\perp} \otimes h_{1,N}^{q\perp}$$

Sivers

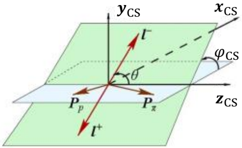
$$A_T^{\sin(\varphi_S)} \propto f_{1,\pi}^{q} \otimes f_{1T,N}^{q\perp}$$

Pretzelosity

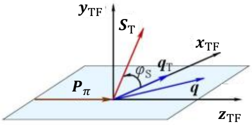
$$A_T^{\sin(2\varphi_{CS}+\varphi_S)} \propto h_{1,\pi}^{q\perp} \otimes h_{1T,N}^{q\perp}$$

Transversity

$$A_T^{\sin(2\varphi_{CS}-\varphi_S)} \propto h_{1,\pi}^{q\perp} \otimes h_{1,N}^{q}$$

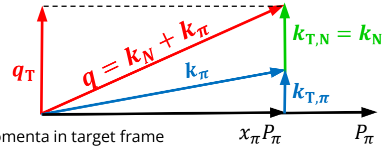


Collin-Soper frame



Target frame

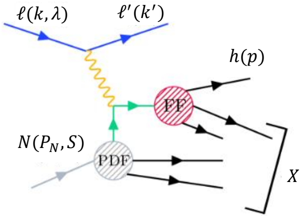
The convolution of TMD PDFs runs over the intrinsic transverse momenta $\boldsymbol{k}_T$.

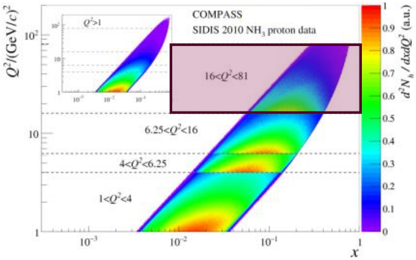


Momenta in target frame

Semi Inclusive Deep Inelastic Scattering

$A_{SIDIS} \propto PDF_N \otimes FF$

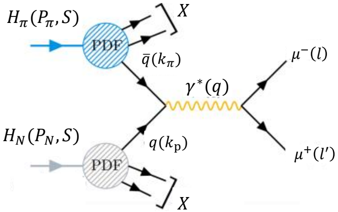


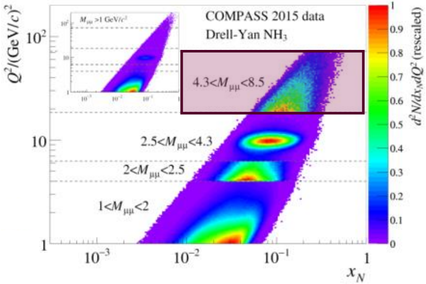


SIDIS vs. DY

Drell-Yan process

$A_{DY} \propto PDF_N \otimes PDF_{\pi^-}$





COMPASS achieves comparable $Q^2 - x$ kinematic coverage

Minimizing possible Q^2 evolution effects

COMPASS Experiment

“Knowledge is of no value unless you put it into practice.”

Anton Chekhov

COMPASS Collaboration

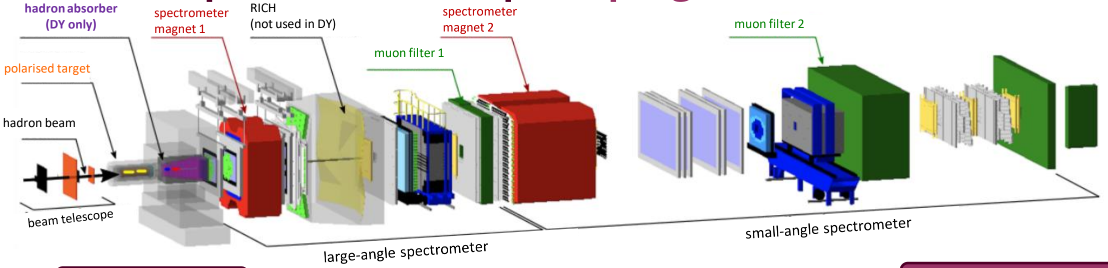
Common **M**uon and **P**roton **A**pparatus for **S**tructure and **S**pectroscopy

- 24 institutions from 14 countries (approximately 220 physicists)
- CERN SPS North Area
- Fixed target experiment

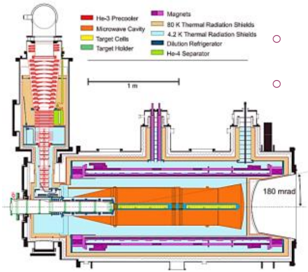
An extensive research programme on the structure of nucleons, including spin and on hadron spectroscopy

Drell Yan data taking
2015 + 2018

COMPASS experimental setup: DY programme

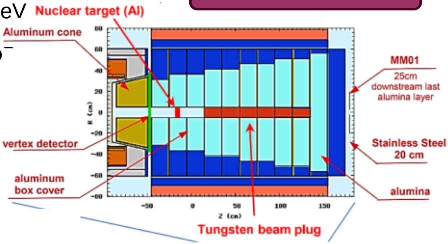


Polarised target



- High energy negative hadron beam: 190 GeV
- Beam composition: **97% π^-**, 2% K^-, 1% p^-
- Targets:
  - polarised NH_3
  - Al, W
  - Hadron absorber
  - Muon identification system
  - 2 spectrometer stages for a wide phase space coverage

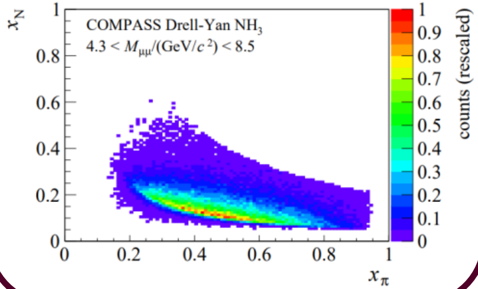
Hadron absorber

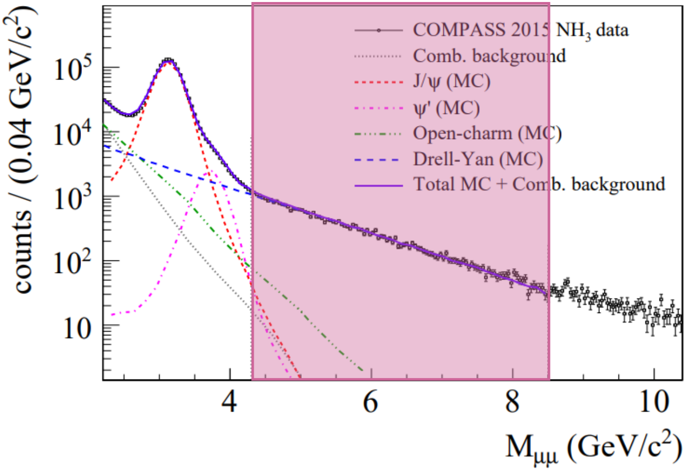


Drell-Yan measurement at COMPASS

96% pure Drell-Yan in dimuon mass range: $4.3 < M_{\mu\mu}/(\mathrm{GeV}/c^2) < 8.5$

Valence region

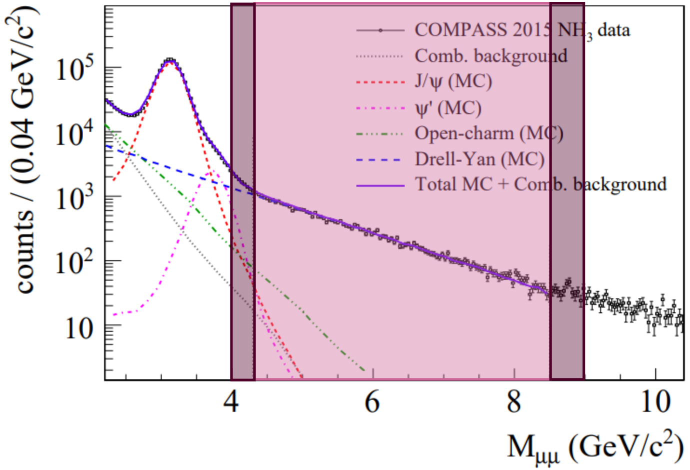




Drell-Yan measurement at COMPASS

96% pure Drell-Yan in dimuon mass range: $4.3 < M_{\mu\mu}/(\mathrm{GeV}/c^2) < 8.5$

Applying an appropriate background correction, we can enlarge mass range to $4 < M_{\mu\mu}/(GeV/c^2) < 9$



Transverse Spin Asymmetries (TSA)

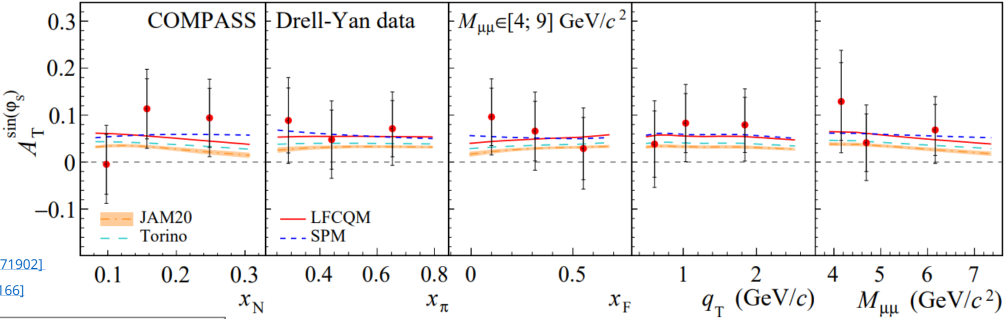
"Nothing in life is to be feared, it is only to be understood. Now is the time to understand more, so that we may fear less."

Maria Skłodowska-Curie

TSA Results: Sivers (SIDIS and DY)

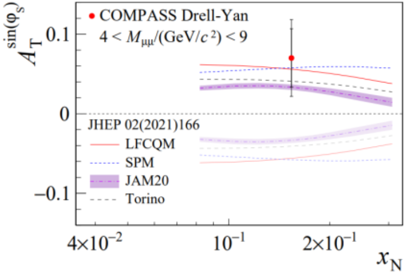
$A_{\mathrm{T}}^{\sin(\varphi_S)} \propto f_{1,\pi}^{q} \otimes f_{1\mathrm{T},N}^{q\perp}$

Sivers DY TSA



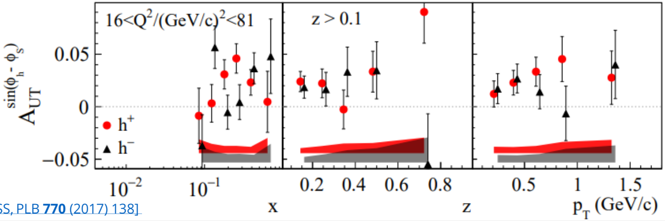
[COMPASS, PRL 133 (2024) 071902]

Theo. Pred.: [JHEP **02** (2021) 166]



SIDIS Sivers TSA

$A_{\mathrm{UT}}^{\sin(\phi_h-\phi_S)} \propto f_{1\mathrm{T},N}^{q\perp} \otimes D_{1q}^{h}$

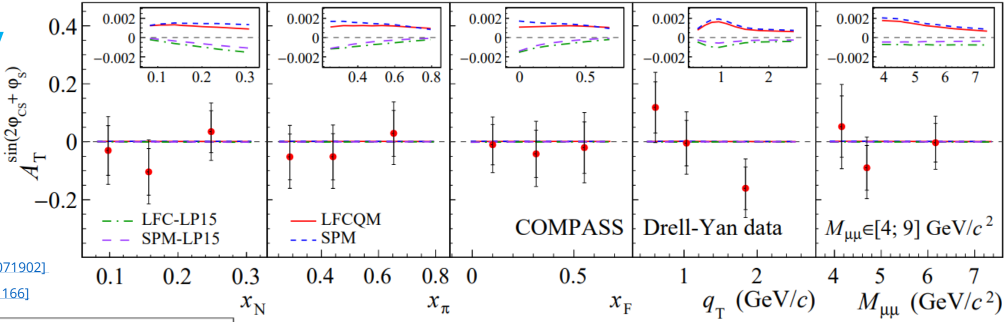


[COMPASS, PLB **770** (2017) 138]

$A_T^{\sin(2\varphi_{CS}+\varphi_S)} \propto h_{1,\pi}^{q\perp} \otimes h_{1T,N}^{q\perp}$

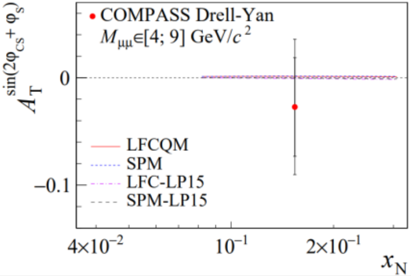
TSA Results: Pretzelosity (SIDIS and DY)

Pretzelosity DY TSA



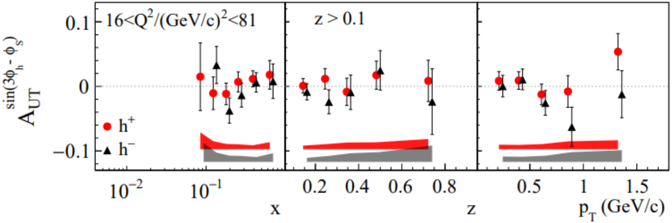
[COMPASS, PRL 133 (2024) 071902]

Theo. Pred.: [JHEP **02** (2021) 166]



Pretzelosity SIDIS TSA

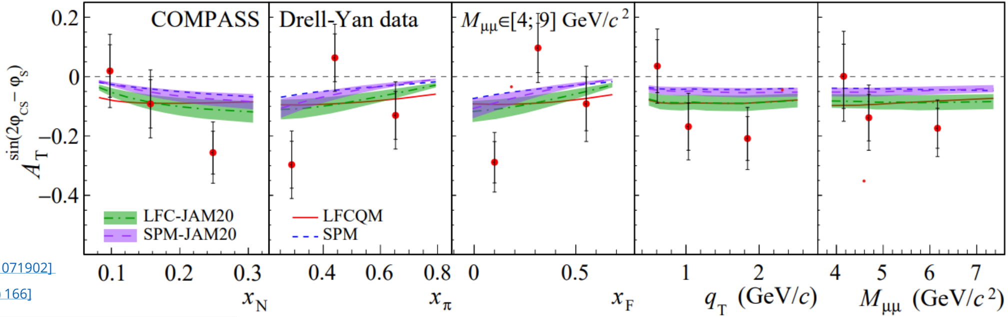
$A_{UT}^{\sin(3\phi_h-\phi_S)} \propto h_{1T,N}^{q\perp} \otimes H_{1q}^{\perp h}$



$A_T^{\sin(2\varphi_{CS}-\varphi_S)} \propto h_{1,\pi}^{q\perp} \otimes h_{1,N}^{q}$

TSA Results: Transversity (SIDIS and DY)

Transversity DY TSA



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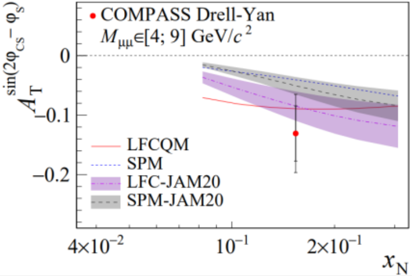
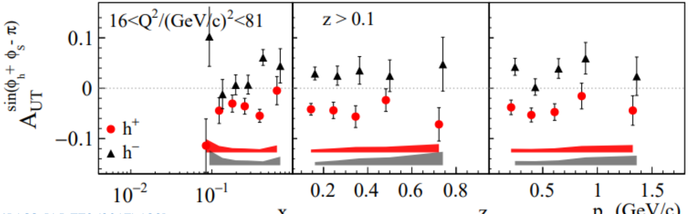


Transversity SIDIS TSA

$A_{UT}^{\sin(\phi_h+\phi_S)} \propto h_{1,N}^{q} \otimes H_{1q}^{\perp h}$

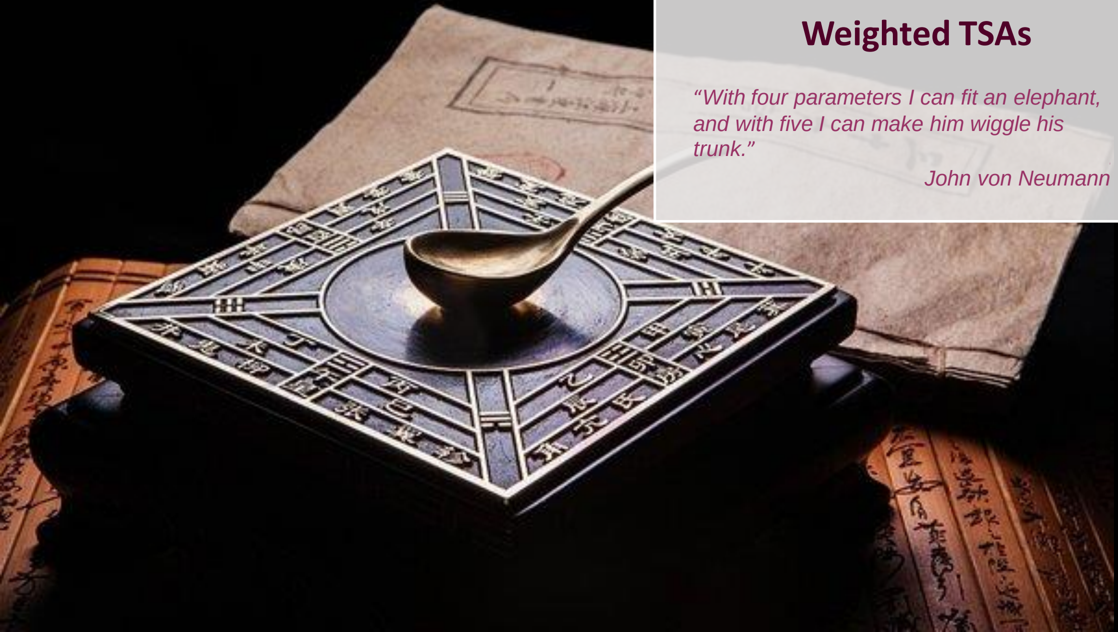


[COMPASS, PLB **770** (2017) 138]

Weighted TSAs

"With four parameters I can fit an elephant, and with five I can make him wiggle his trunk."

John von Neumann



Weighted TSAs in Drell–Yan

The convolution cannot be resolved without assumptions about the dependence of the TMD PDF on the intrinsic transverse momentum.

Weighting with powers of the transverse momentum allows to avoid assumptions on k_{T}.

TSA		**WTSA***
$A_{\mathrm{T}}^{\sin(\varphi_S)} \propto f_{1,\pi}^{q} \otimes f_{1\mathrm{T},N}^{q\perp}$	**Sivers** ⟷	$A_{\mathrm{T}}^{\sin(\varphi_S)\frac{q_T}{M_N}} \propto f_{1,\pi}^{q} \times f_{1\mathrm{T},N}^{q\perp(1)}$
$A_{\mathrm{T}}^{\sin(2\varphi_{CS}+\varphi_S)} \propto h_{1,\pi}^{q\perp} \otimes h_{1\mathrm{T},N}^{q\perp}$	**Pretzelosity** ⟷	$A_{\mathrm{T}}^{\sin(2\varphi_{CS}+\varphi_S)\frac{q_T^3}{2M_N^2 M_\pi}} \propto h_{1,\pi}^{q\perp(1)} \times h_{1\mathrm{T},N}^{q\perp(2)}$
$A_{\mathrm{T}}^{\sin(2\varphi_{CS}-\varphi_S)} \propto h_{1,\pi}^{q\perp} \otimes h_{1,N}^{q}$	**Transversity** ⟷	$A_{\mathrm{T}}^{\sin(2\varphi_{CS}-\varphi_S)\frac{q_T}{M_\pi}} \propto h_{1,\pi}^{q\perp(1)} \times h_{1,N}^{q}$

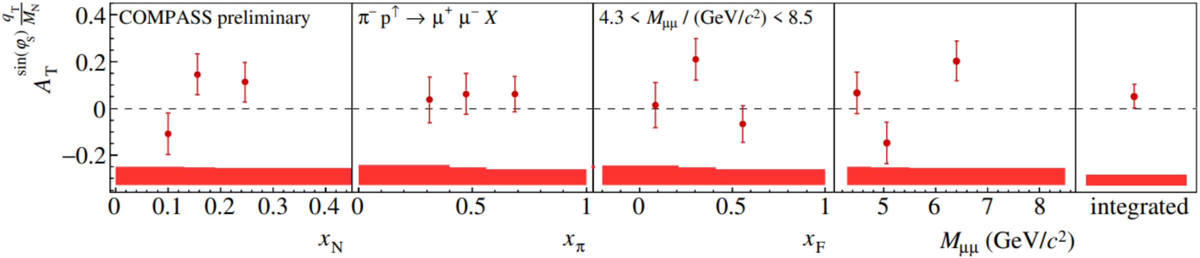
* The n-th moment of a TMD PDF of a pion or proton:

$$f^{(n)}(x) = \int \mathrm{d}^2\boldsymbol{k}_{\mathrm{T}} \left(\frac{k_{\mathrm{T}}^2}{2M^2}\right)^n f(x, k_{\mathrm{T}}^2)$$

WTSA Results: Sivers (SIDIS and DY)

Sivers DY WTSA

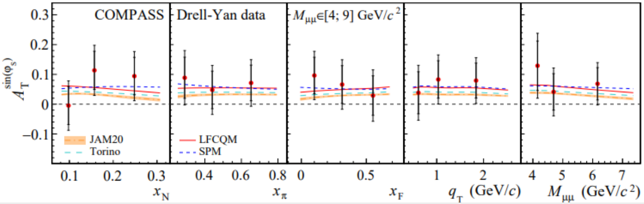
$$A_{\mathrm{T}}^{\sin(\varphi_S)\frac{q_T}{M_N}} \propto f_{1,\pi}^{q} \times f_{1T,N}^{q\perp(1)}$$



1σ positive **Sivers WTSA** compatible with **Sivers TSA**

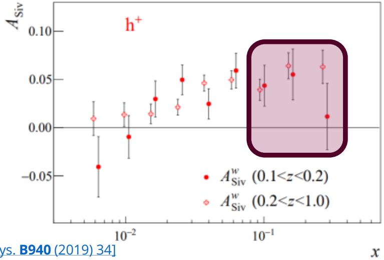
[COMPASS, PRL 133 (2024) 071902] **Sivers DY TSA**

$$A_{\mathrm{T}}^{\sin(\varphi_S)} \propto f_{1,\pi}^{q} \otimes f_{1T,N}^{q\perp}$$



Sivers SIDIS WTSA

$$A_{\mathrm{UT}}^{\sin(\phi_h-\phi_S)\frac{P_T}{zM_N}} \propto f_{1T,N}^{q\perp(1)} \times D_{1q}^{h}$$



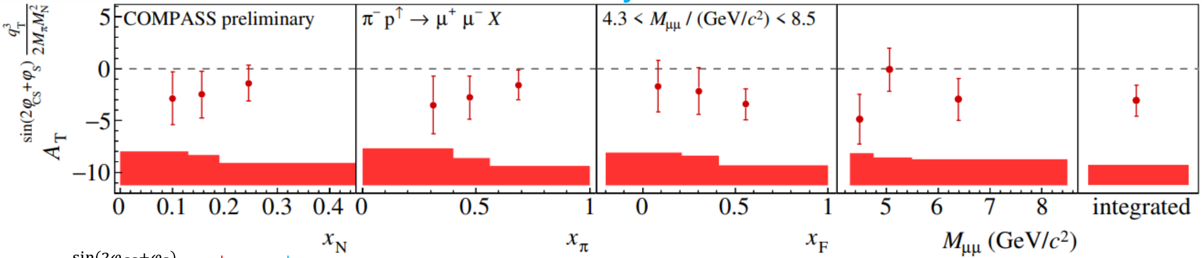
[COMPASS, Nucl.Phys. **B940** (2019) 34]

WTSA Results: Pretzelosity (DY)

Comparison of WTSA and TSA for **Pretzelosity** asymmetry

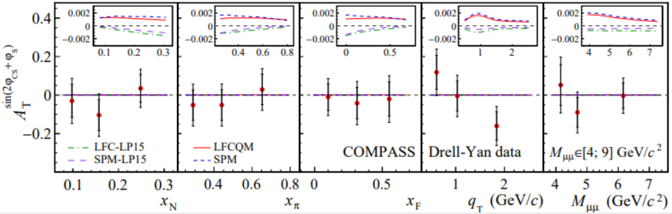
$$A_T^{\sin(2\varphi_{CS}+\varphi_S)\frac{q_T^3}{2M_N^2M_\pi}} \propto h_{1,\pi}^{q\perp(1)} \times h_{1T,N}^{q\perp(2)}$$

DY Pretzelosity WTSA



$$A_T^{\sin(2\varphi_{CS}+\varphi_S)} \propto h_{1,\pi}^{\perp} \otimes h_{1T,N}^{\perp}$$

DY Pretzelosity TSA



Pretzelosity is expected to be zero

2σ negative **Pretzelosity WTSA**

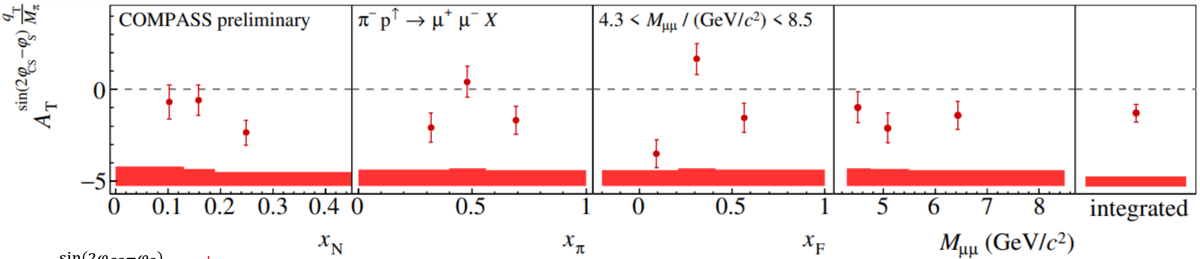
[COMPASS, PRL 133 (2024) 071902]

WTSA Results: Transversity (DY)

Comparison of WTSA and TSA for **Transversity** asymmetry

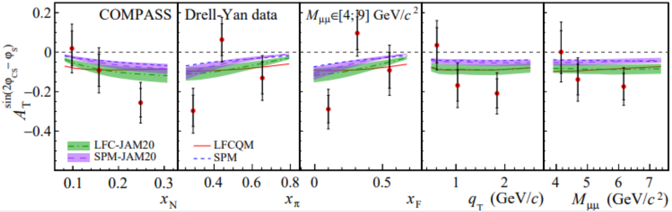
DY Transversity WTSA

$$A_T^{\sin(2\varphi_{CS}-\varphi_S)\frac{q_T}{M_\pi}} \propto h_{1,\pi}^{q\perp(1)} \times h_{1,N}^{q}$$



$$A_T^{\sin(2\varphi_{CS}-\varphi_S)} \propto h_{1,\pi}^{\perp} \otimes h_{1,N}$$

DY Transversity TSA



2σ negative **Transversity**

Results compatible with **Transversity TSA**

[COMPASS, PRL 133 (2024) 071902]

Conclusions

- **COMPASS probes 3-dimensional structure of nucleon**
- COMPASS SIDIS and Drell-Yan TSAs measurements represent a unique experimental input to study the universality of TMD PDFs

Drell-Yan TSAs

$$A_{DY} \propto PDF_N \otimes PDF_{\pi^-}$$

- 1σ positive **Sivers TSA**
- **Pretzelosity TSA** found to be small and compatiblewith zero
- 2σ negative **Transversity TSA**
- Results agree with theoretical predictions and consistent with analogous measurements for SIDIS

Transverse momentum weighted Drell-Yan TSA

$$A_{DY}^{W} \propto PDF_N \times PDF_{\pi^-}$$

- A way to overcome the convolution over intrinsic k_{T}
- A direct access to the k_{T}^2-moments of TMD PDFs
- $\sim 1\sigma$ positive **Sivers WTSA** compatible with DY TSA and SIDIS P_{T}-weighted TSA
- $\sim 2\sigma$ negative **Pretzelosity WTSA** effect
- $\sim 2\sigma$ negative **Transversity WTSA** consistent with TSAs

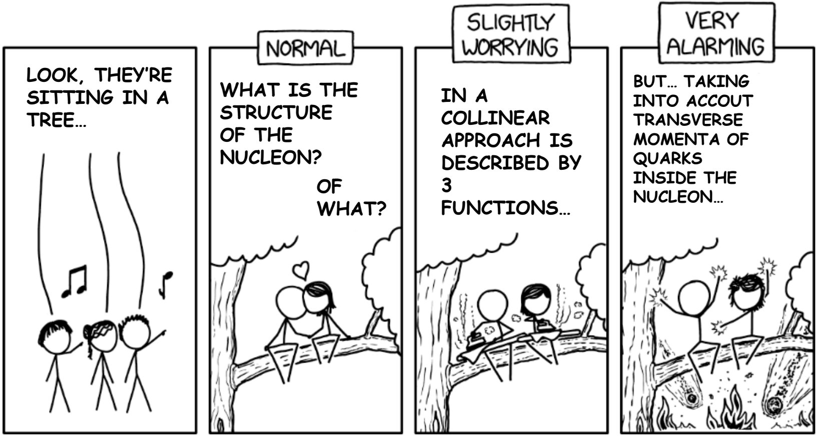
Prospects

- Analysis of a WTSA ongoing, paper in preparation

Thank you for attention!

"All of physics is either impossible or trivial. It is impossible until you understand it, and then it becomes trivial."

Ernest Rutherford

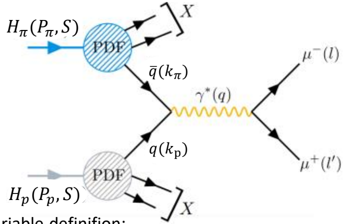


Backup slides

Single polarised Drell-Yan process

Cross-section, LO TMD approach for transversely polarised target in terms of structure functions:

$$\frac{d\sigma^{LO}}{dx_p dx_\pi d^2 q_T d\varphi_{CS} d(\cos\theta) d\varphi_S} = C_0 \left\{ \begin{array}{c} (1+\cos^2\theta)F_U^1 + \sin^2\theta \cos 2\varphi F_U^{\cos 2\varphi_{CS}} \\ + |S_T| \left[\begin{array}{c} (1+\cos^2\theta)\sin(\varphi_S)F_T^{\sin\varphi_S} + \\ \sin^2\theta \left(\begin{array}{c} \sin(2\varphi+\varphi_S)F_T^{\sin(2\varphi_{CS}+\varphi_S)} \\ + \\ \sin(2\varphi-\varphi_S)F_T^{\sin(2\varphi_{CS}-\varphi_S)} \end{array} \right) \end{array} \right] \end{array} \right\}$$

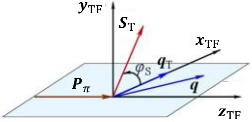


Variable definifion:

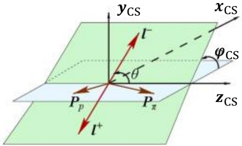
$$M_{\mu\mu} = (p_{\mu^+} + p_{\mu^-})^2$$

q_L^*- γ^* longitudinal momentum in $\pi - p$ rest frame

$$x_F = \frac{2q_L^*}{\sqrt{s}}, x_{\pi,p} = \frac{1}{2}\left(\sqrt{x_F^2 + 4\frac{M_{\mu\mu}^2}{s}} \pm x_F\right)$$



Target frame



Collin-Soper frame

Boer-Mulders

$$F_U^{\cos 2\varphi_{CS}} \propto h_{1,\pi}^{q\perp} \otimes h_{1,N}^{q\perp}$$

Sivers

$$F_T^{\sin\varphi_S} \propto f_{1,\pi}^{q} \otimes f_{1T,N}^{q\perp}$$

Pretzelosity

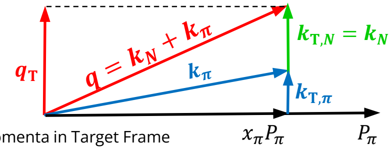
$$F_T^{\sin(2\varphi_{CS}+\varphi_S)} \propto h_{1,\pi}^{q\perp} \otimes h_{1T,N}^{q\perp}$$

Transversity

$$F_T^{\sin(2\varphi_{CS}-\varphi_S)} \propto h_{1,\pi}^{q\perp} \otimes h_{1,N}^{q}$$

Single polarised Drell-Yan process

The convolution of TMD PDFs runs over the intrinsic transverse momenta $\boldsymbol{k}_{\mathrm{T}}$.



TMD PDFs are accessed through measurement of target spin dependent azimuthal asymmetries TSA.

Sivers Asymmetries $A_{\mathrm{T}}^{\sin(\varphi_S)} = \dfrac{F_{\mathrm{T}}^{\sin(\varphi_S)}}{F_{\mathrm{U}}^1}$ **Sivers** for nucleon, number density for π^-

Pretzelosity Asymmetries $A_{\mathrm{T}}^{\sin(2\varphi_{CS}+\varphi_S)} = \dfrac{F_{\mathrm{T}}^{\sin(2\varphi_{CS}+\varphi_S)}}{2F_{\mathrm{U}}^1}$ **Pretzelosity** for nucleon, Boer–Mulders for π^-

Transversity Asymmetries $A_{\mathrm{T}}^{\sin(2\varphi_{CS}-\varphi_S)} = \dfrac{F_{\mathrm{T}}^{\sin(2\varphi_{CS}-\varphi_S)}}{2F_{\mathrm{U}}^1}$ **Transversity** for nucleon, Boer–Mulders for π^-

Backup: Single Polarised Drell-Yan Process

Each structure function can be written as a TMD PDF convolution over the instrinsic transverse momenta.

$$F_{\mathrm{T}}^{\sin(\varphi_S)} = \mathcal{C}\left[\frac{\boldsymbol{q_T k_{T,N}}}{q_T M_N} f_{1,\pi} f_{1T,p}^{\perp}\right]$$

Sivers for proton, number density for π^-

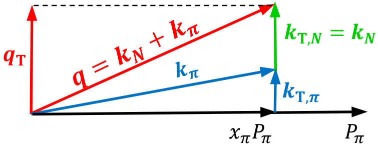
$$F_{\mathrm{T}}^{\sin(2\varphi_{CS}+\varphi_S)} = -\mathcal{C}\left[\frac{2(\boldsymbol{q_T k_{T,N}})[2(\boldsymbol{q_T k_{T,N}})(\boldsymbol{q_T k_{T,\pi}}) - q_T^2(\boldsymbol{k_{T,N} k_{T,\pi}})] - q_T^2 k_{T,N}^2(\boldsymbol{q_T k_{T,\pi}})}{2 q_T^3 M_N^2 M_\pi} h_{1,\pi}^{\perp} h_{1T,N}^{\perp}\right]$$

Pretzelosity for proton, Boer–Mulders for π^-

$$F_{\mathrm{T}}^{\sin(2\varphi_{CS}-\varphi_S)} = -\mathcal{C}\left[\frac{\boldsymbol{q_T k_{T,\pi}}}{q_T M_\pi} h_{1,\pi}^{\perp} h_{1.N}\right]$$

Transversity for proton, Boer–Mulders for π^-

$$\mathcal{C}[w(\boldsymbol{k_{T,\pi}};\boldsymbol{k_{T,N}};\boldsymbol{q_T})\mathrm{f}_\pi \mathrm{f}_N] = \frac{1}{\mathrm{N_c}}\sum_q \left\{ \begin{array}{c} e_q^2 \int \mathrm{d}^2\boldsymbol{k_{T,\pi}} \mathrm{d}^2\boldsymbol{k_{T,N}} \delta^{(2)}(\boldsymbol{q_T} - \boldsymbol{k_{T,\pi}} - \boldsymbol{k_{T,N}}) \\ \times\, w(\boldsymbol{k_{T,\pi}};\boldsymbol{k_{T,N}};\boldsymbol{q_T}) \left[\begin{array}{c} f_\pi^{\bar{q}}(x_\pi, k_{T,\pi}^2) f_N^q(x_N, k_{T,N}^2) \\ + \\ f_\pi^q(x_\pi, k_{T,\pi}^2) f_p^{\bar{q}}(x_p, k_{T,p}^2) \end{array}\right] \end{array} \right\}$$

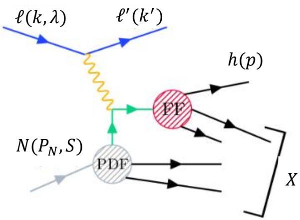


Momenta in Target Frame

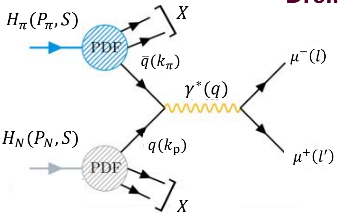
TMD PDFs can be accessed through measurement of target spin (in)dependent azimuthal asymmetries

$$A_{\mathrm{U}}^{\cos(\varphi_{\mathrm{CS}})} = \frac{F_{\mathrm{U}}^{\cos(\varphi_{\mathrm{CS}})}}{F_{\mathrm{U}}^1} \quad A_{\mathrm{T}}^{\sin(\varphi_S)} = \frac{F_{\mathrm{T}}^{\sin(\varphi_S)}}{F_{\mathrm{U}}^1} \quad A_{\mathrm{T}}^{\sin(2\varphi_{\mathrm{CS}}+\varphi_S)} = \frac{F_{\mathrm{T}}^{\sin(2\varphi_{\mathrm{CS}}+\varphi_S)}}{2F_{\mathrm{U}}^1} \quad A_{\mathrm{T}}^{\sin(2\varphi_{CS}-\varphi_S)} = \frac{F_{\mathrm{T}}^{\sin(2\varphi_{\mathrm{CS}}-\varphi_S)}}{2F_{\mathrm{U}}^1}$$

SIDIS



Drell-Yan



SIDIS vs. DY

$$A_{SIDIS} \propto PDF_N \otimes FF$$

$$A_{DY} \propto PDF_N \otimes PDF_{\pi^-}$$

$$\frac{d\sigma^{LO}_{SIDIS}}{dx\,dy\,dz\,d^2p_\text{T}\,d\phi_h\,d\phi_S} \propto \left\{ \begin{array}{c} 1 + \cos(2\phi_\text{h})\varepsilon A_\text{UU}^{\cos(2\phi_\text{h})} \\ +|S_\text{T}| \left[\begin{array}{c} \sin(\phi_\text{h}-\phi_\text{S}) A_\text{UT}^{\sin(\phi_\text{h}-\phi_\text{S})} + \\ \sin(\phi_\text{h}+\phi_\text{S}) \varepsilon A_\text{UT}^{\sin(\phi_\text{h}+\phi_\text{S})} + \\ \sin(3\phi_\text{h}-\phi_\text{S}) \varepsilon A_\text{UT}^{\sin(3\phi_\text{h}-\phi_\text{S})} + \end{array} \right] \end{array} \right\}$$

$$\frac{d\sigma^{LO}_{DY}}{d\Omega\,d^4q} \propto \left\{ \begin{array}{c} 1 + D_{\sin^2\theta} \cos(2\varphi_\text{CS}) A_\text{U}^{\cos(2\varphi_\text{CS})} \\ +|S_\text{T}| \left[\begin{array}{c} \sin(\varphi_\text{S}) A_\text{T}^{\sin(\varphi_\text{S})} + \\ D_{\sin^2\theta} \left(\begin{array}{c} \sin(2\varphi_\text{CS}+\varphi_\text{S}) A_\text{T}^{\sin(2\varphi_\text{CS}+\varphi_\text{S})} \\ + \\ \sin(2\varphi_\text{CS}-\varphi_\text{S}) A_\text{T}^{\sin(2\varphi_\text{CS}-\varphi_\text{S})} \end{array} \right) \end{array} \right] \end{array} \right\}$$

$A_\text{UU}^{\cos(2\phi_\text{h})} \propto h_{1,N}^{q\perp} \otimes H_{1q}^{\perp h}$ ⟵ **Boer-Mulders** ⟶ $A_\text{U}^{\cos(2\varphi_\text{CS})} \propto h_{1,\pi}^{q\perp} \otimes h_{1,\text{N}}^{q\perp}$

$A_\text{UT}^{\sin(\phi_\text{h}-\phi_\text{S})} \propto f_{1\text{T},N}^{q\perp} \otimes D_{1q}^{h}$ ⟵ **Sivers** ⟶ $A_\text{T}^{\sin(\varphi_\text{S})} \propto f_{1,\pi}^{q} \otimes f_{1\text{T},N}^{q\perp}$

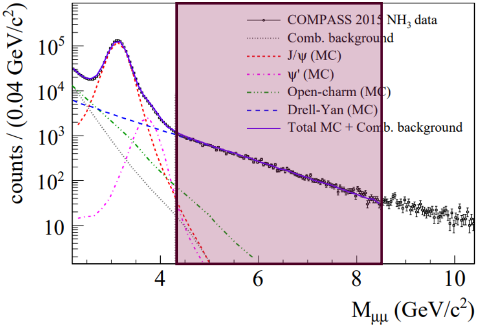
$A_\text{UT}^{\sin(3\phi_\text{h}-\phi_\text{S})} \propto h_{1\text{T},N}^{q\perp} \otimes H_{1q}^{\perp h}$ ⟵ **Pretzelosity** ⟶ $A_\text{T}^{\sin(2\varphi_\text{CS}+\varphi_\text{S})} \propto h_{1,\pi}^{q\perp} \otimes h_{1\text{T},N}^{q\perp}$

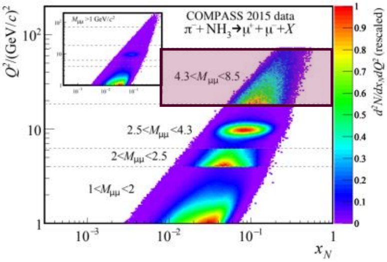
$A_\text{UT}^{\sin(\phi_\text{h}+\phi_\text{S})} \propto h_{1\text{T},N}^{q} \otimes H_{1q}^{\perp h}$ ⟵ **Transversity** ⟶ $A_\text{T}^{\sin(2\varphi_\text{CS}-\varphi_\text{S})} \propto h_{1,\pi}^{q\perp} \otimes h_{1,N}^{q}$

Drell-Yan measurement at COMPASS

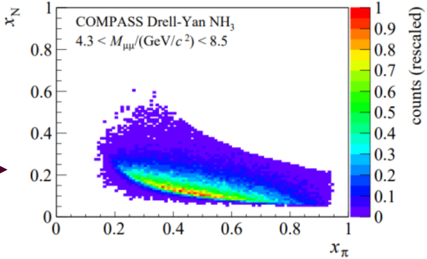
The dimuon mass range $4.3 < M_{\mu\mu}/(\mathrm{GeV}/c^2) < 8.5$ **is 96% pure Drell-Yan.**

- Low background
- Valence region





Valence region of x_N and x_π in a given $Q^2 - x$ kinematic coverage



Weighted TSAs in Drell–Yan

The convolution cannot be resolved without assumptions about the dependence of the TMD PDF on the intrinsic transverse momentum.

Weighting with powers of the transverse momentum allows to avoid assumptions on k_{T}.

Asymmetries in terms of structure functions:

$$A_{\mathrm{T}}^{\sin\Phi W_{\Phi}} = \frac{\int d^2\boldsymbol{q_T} W_{\Phi} F_{\mathrm{T}}^{\sin\Phi}}{\int d^2\boldsymbol{q_\mathrm{T}} F_{\mathrm{U}}^{1}},$$

where W_{Φ} is weight for $\Phi = \varphi_S, 2\varphi_{CS} + \varphi_S, 2\varphi_{CS} - \varphi_S$.

The n-th moment of a TMD PDF of a pion or proton:

$$f^{(n)}(x) = \int d^2\boldsymbol{k}_{\mathrm{T}} \left(\frac{k_{\mathrm{T}}^2}{2M^2}\right)^n f(x, k_{\mathrm{T}}^2).$$

TSA		WTSA
$A_{\mathrm{T}}^{\sin(\varphi_S)} \propto f_{1,\pi}^{q} \otimes f_{1\mathrm{T},N}^{q\perp}$	Sivers	$A_{\mathrm{T}}^{\sin(\varphi_S)\frac{q_T}{M_N}} \propto f_{1,\pi}^{q} \times f_{1\mathrm{T},N}^{q\perp(1)}$
$A_{\mathrm{T}}^{\sin(2\varphi_{CS}+\varphi_S)} \propto h_{1,\pi}^{q\perp} \otimes h_{1\mathrm{T},N}^{q\perp}$	Pretzelosity	$A_{\mathrm{T}}^{\sin(2\varphi_{CS}+\varphi_S)\frac{q_T^3}{2M_N^2 M_\pi}} \propto h_{1,\pi}^{q\perp(1)} \times h_{1\mathrm{T},N}^{q\perp(2)}$
$A_{\mathrm{T}}^{\sin(2\varphi_{CS}-\varphi_S)} \propto h_{1,\pi}^{q\perp} \otimes h_{1,N}^{q}$	Transversity	$A_{\mathrm{T}}^{\sin(2\varphi_{CS}-\varphi_S)\frac{q_T}{M_\pi}} \propto h_{1,\pi}^{q\perp(1)} \times h_{1,N}^{q}$