



October 1, 2024
Yerevan

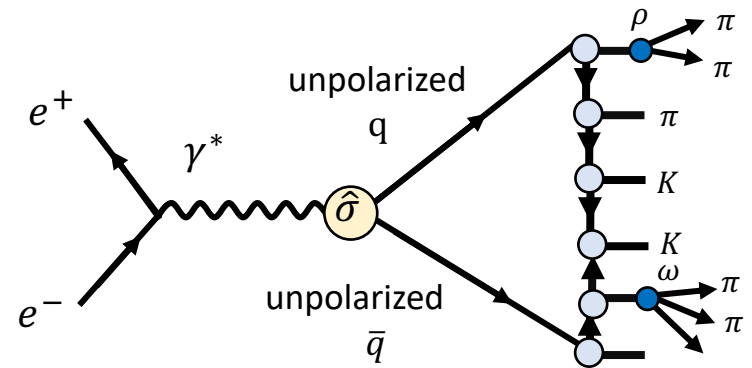
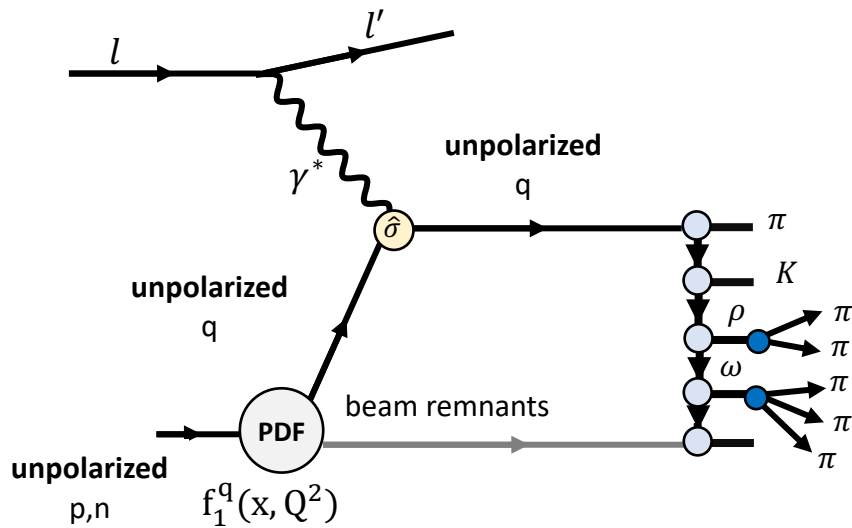
Polarized hadronization in Pythia

Albi Kerbizi
University of Trieste and INFN Trieste



StringSpinner: spin in the Pythia generator

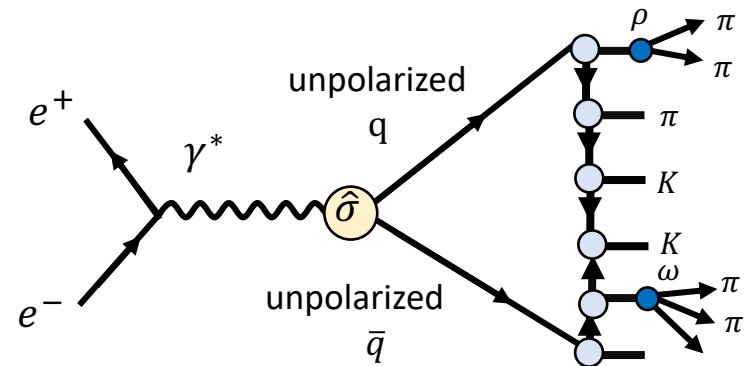
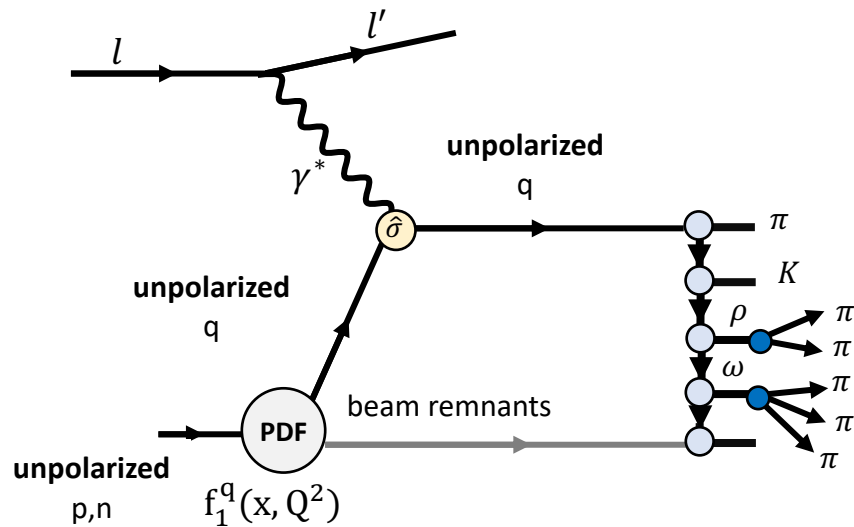
PYTHIA 8 standard tool in particle physics,
capable of simulating several processes: DIS, e+e-, pp, pA, ..
without spin effects



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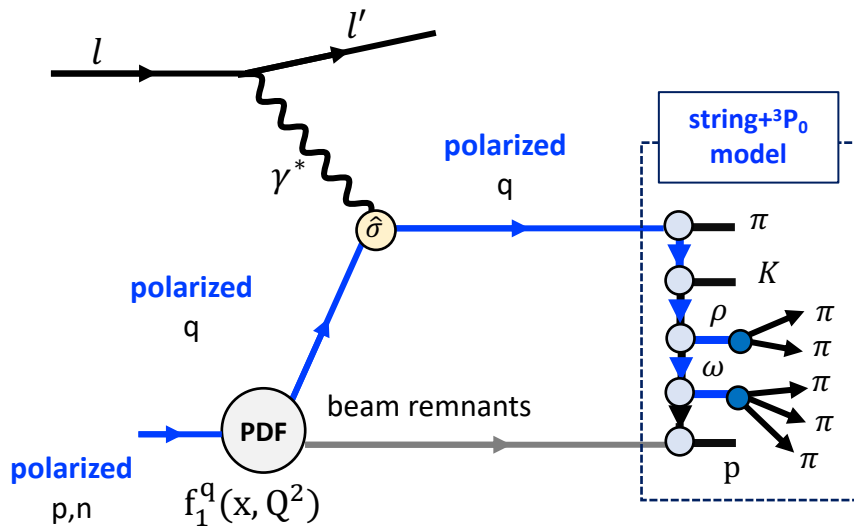
for studies of the nucleon structure and hadronization spin is important
→ implemented in Pythia for DIS and e^+e^- by StringSpinner



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Transversity $h_1^q(x)$ [valence quarks,
sea quarks unpol.]
or arbitrary polarization of quarks

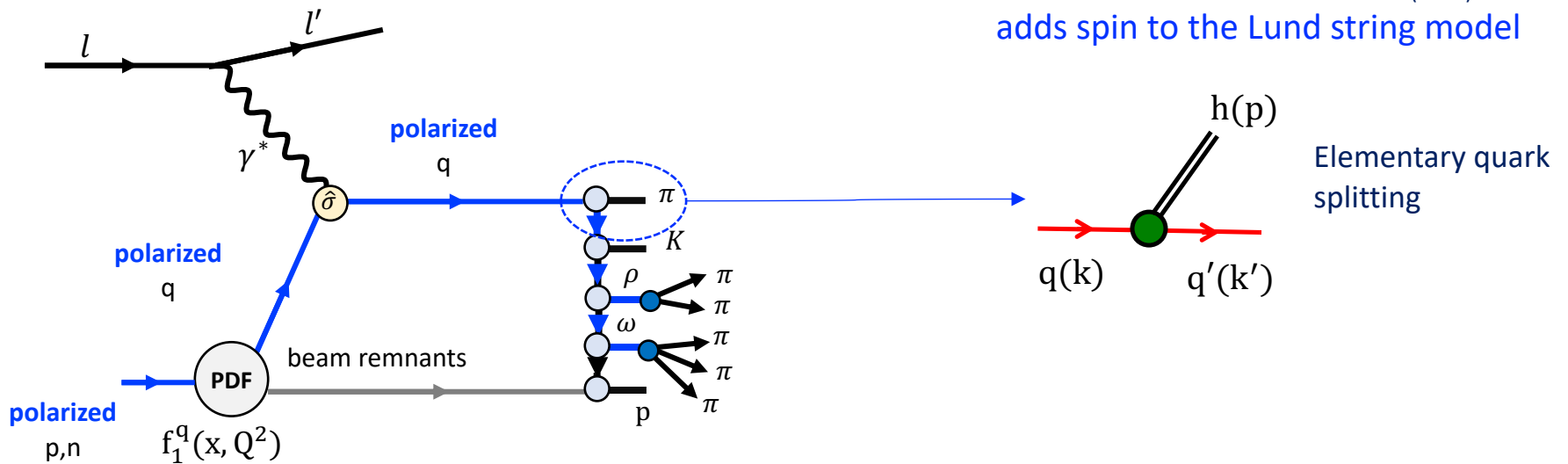
AK, L. Lönnblad, CPC **272** (2022) 108234;
CPC **292** (2023) 108886

StringSpinner: spin in the Pythia generator

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AK, Artru, Martin,
PRD 104, 114038
(2021)

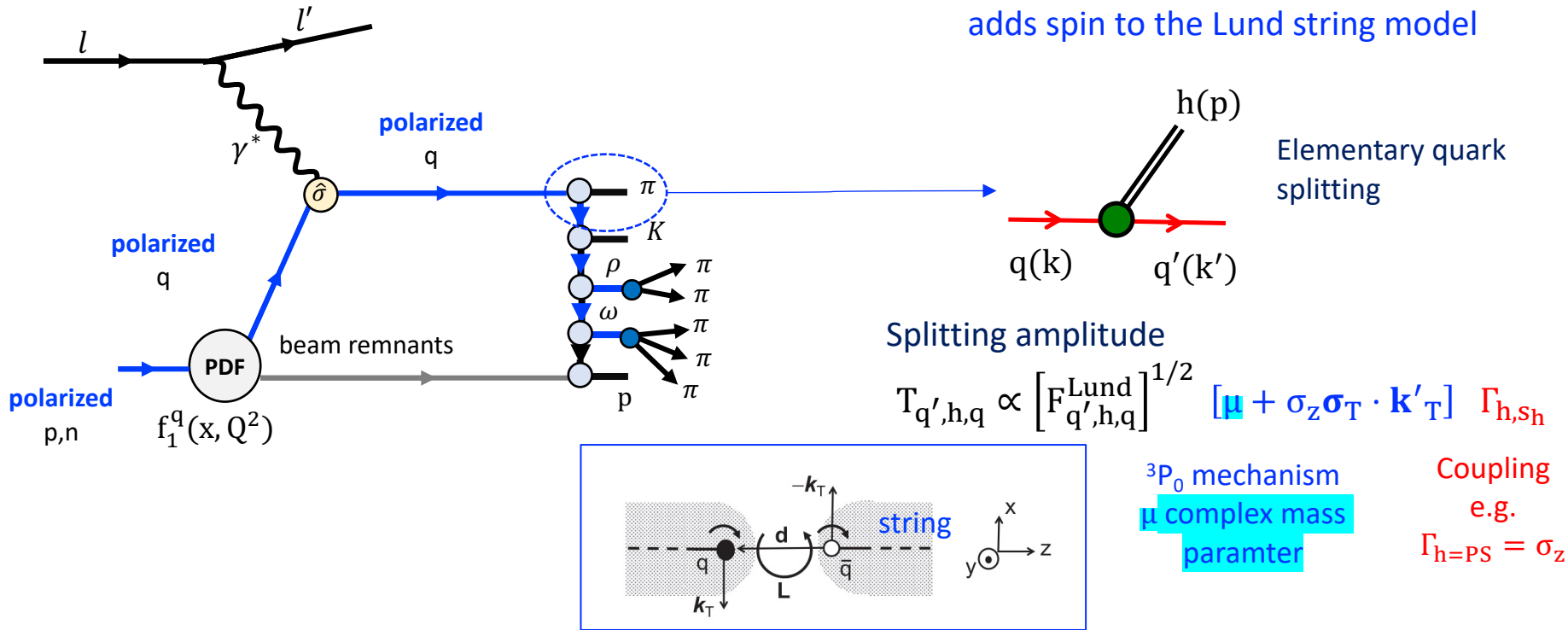


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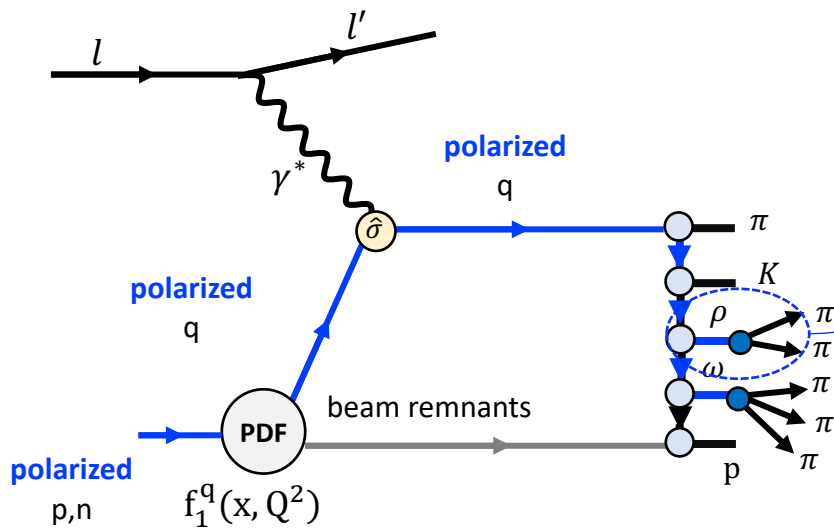


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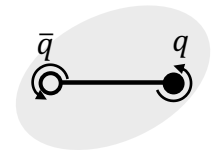
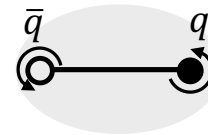
string+³P₀ model:

adds spin to the Lund string model

Coupling of quarks with VMs

f_L
fraction of L pol. VMs

θ_{LT}
oblique polarization
(L/T interference)



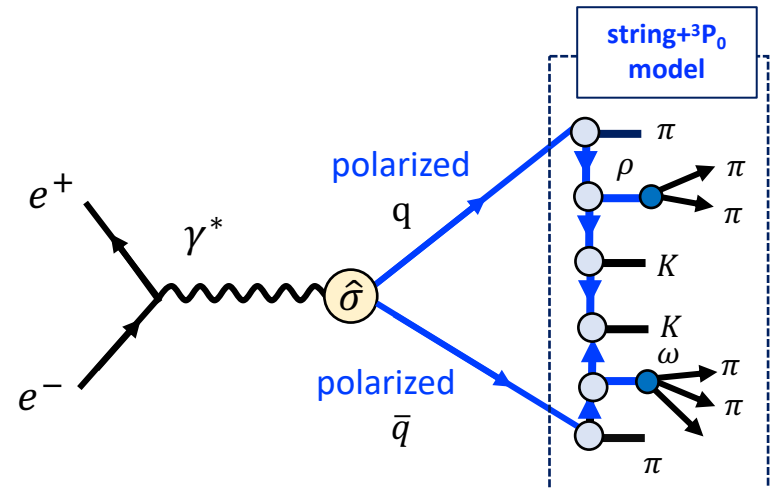
StringSpinner: spin in the Pythia generator

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for studies of the nucleon structure and hadronization **spin** is important
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For e^+e^- splittings from q and \bar{q}

Antiquark splitting amplitude similar to
the quark one



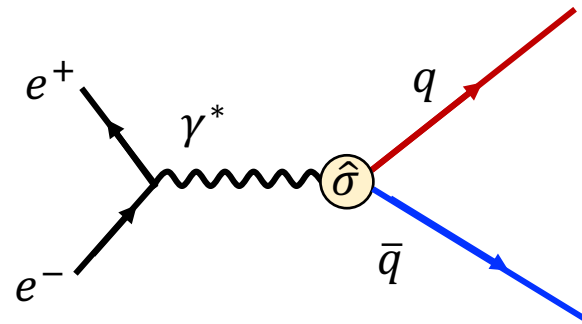
Joint spin density matrix $\rho(q, \bar{q})$
correlations between the spin states of q and \bar{q}

AK, Artru, PRD 109 (2024) 5, 054029
AK, Lönnblad, Martin, arXiv: 2407.07706

Recipe for the simulation of e^+e^- annihilation

AK, X. Artru, PRD 109 (2024) 5, 054029

Recursive recipe for e^+e^-



Steps:

1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from q
4. Update density matrix
5. Hadron emission from \bar{q}
6. Exit condition

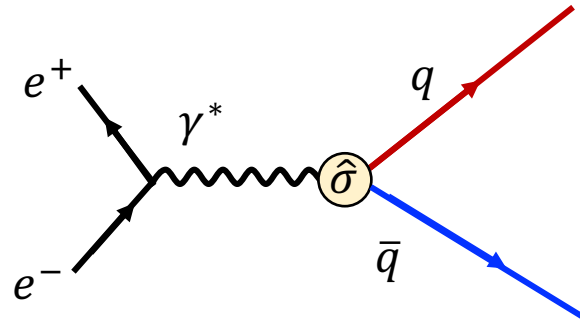
[AK, X. Artru, PRD 109 (2024) 5, 054029]

Recursive recipe for e^+e^-

Steps:

1. Hard scattering

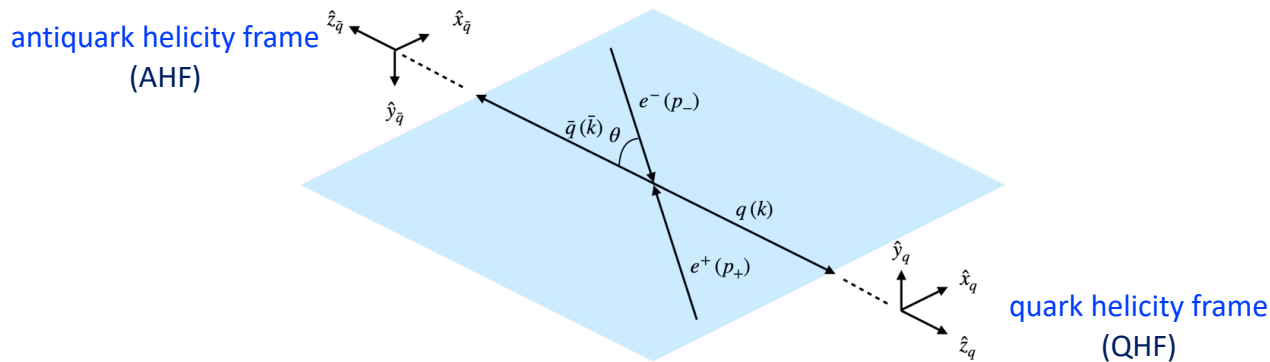
2. Joint spin density matrix
3. Hadron emission from q
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6. Exit condition



[AK, X. Artru, PRD 109 (2024) 5, 054029]

Set up the scattering $e^+e^- \rightarrow q\bar{q}$ in the c.m.s

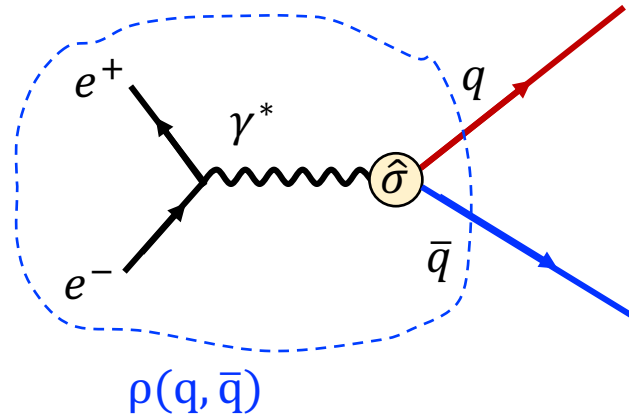
generate the quark flavors and kinematics using differential cross section



Recursive recipe for e^+e^-

Steps:

1. Hard scattering
- 2. Joint spin density matrix**
3. Hadron emission from q
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5. Hadron emission from \bar{q}
6. Exit condition



[AK, X. Artru, PRD 109 (2024) 5, 054029]

□ Set up the **joint spin density matrix** of the $q\bar{q}$ pair

$$\rho(q, \bar{q}) = C_{\alpha\beta}^{q\bar{q}} \sigma_q^\alpha \otimes \sigma_{\bar{q}}^\beta$$

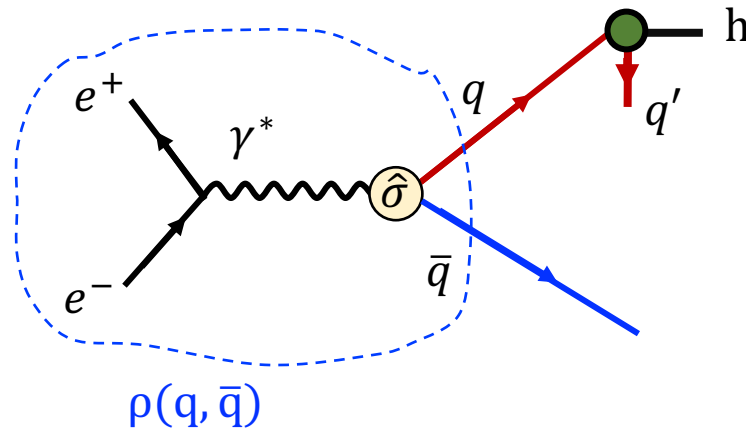
correlation coefficients
Pauli matrices along QHF and AHF

$\alpha = 0, x_q, y_q, z_q$
 $\beta = 0, x_{\bar{q}}, y_{\bar{q}}, z_{\bar{q}}$

For γ^* exchange

$$\rho(q, \bar{q}) \propto 1_q \otimes 1_{\bar{q}} - \sigma_q^z \otimes \sigma_{\bar{q}}^z + \frac{\sin^2\theta}{1+\cos^2\theta} [\sigma_q^x \otimes \sigma_{\bar{q}}^x + \sigma_q^y \otimes \sigma_{\bar{q}}^y]$$

Recursive recipe for e^+e^-



Steps:

1. Hard scattering
2. Joint spin density matrix
- 3. Hadron emission from q**
4. Update density matrix
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6. Exit condition

[AK, X. Artru, PRD 109 (2024) 5, 054029]

- Emit the first hadron using the **splitting function** (emission probability density)

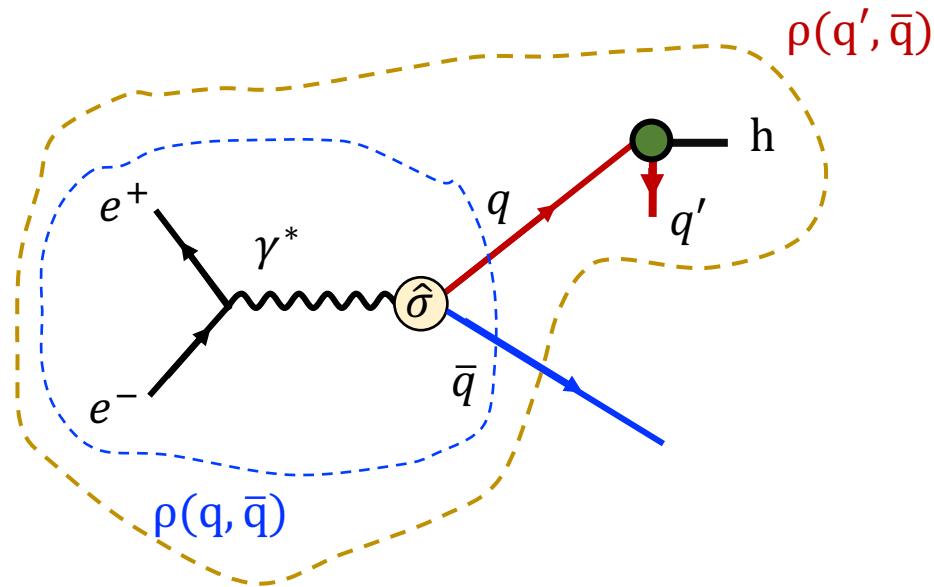
$$\frac{dP(q \rightarrow h + q'; q\bar{q})}{dZ_+ Z_+^{-1} d^2 p_T} = \text{Tr}_{q'\bar{q}} \mathbf{T}_{q',h,q} \rho(q, \bar{q}) \mathbf{T}_{q',h,q}^\dagger = F_{q',h,q}(Z_+, \mathbf{p}_T; \mathbf{k}_T, C^{q\bar{q}})$$

$$\mathbf{T}_{q',h,q} \equiv \mathbf{T}_{q',h,q} \otimes 1_{\bar{q}}$$

in the QHF

- For VM emission need also to handle the polarized decay
→ backup

Recursive recipe for e^+e^-



Steps:

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2. Joint spin density matrix
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- 4. Update density matrix**
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6. Exit condition

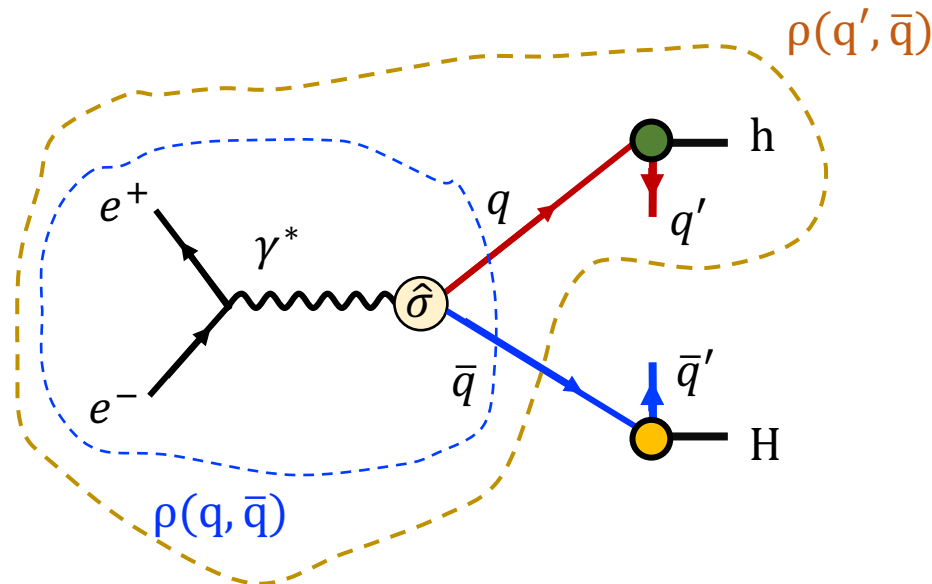
[AK, X. Artru, PRD 109 (2024) 5, 054029]

□ Evaluate the spin density matrix $\rho(q', \bar{q})$

$$\rho(q', \bar{q}) = \mathbf{T}_{q',h,q} \rho(q, \bar{q}) \mathbf{T}_{q',h,q}^\dagger$$

includes the information on the emission of h

Recursive recipe for e^+e^-



Steps:

1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from q
4. Update density matrix
- 5. Hadron emission from \bar{q}**
6. Exit condition

[AK, X. Artru, PRD 109 (2024) 5, 054029]

□ Emit a hadron from the \bar{q} side using the splitting function

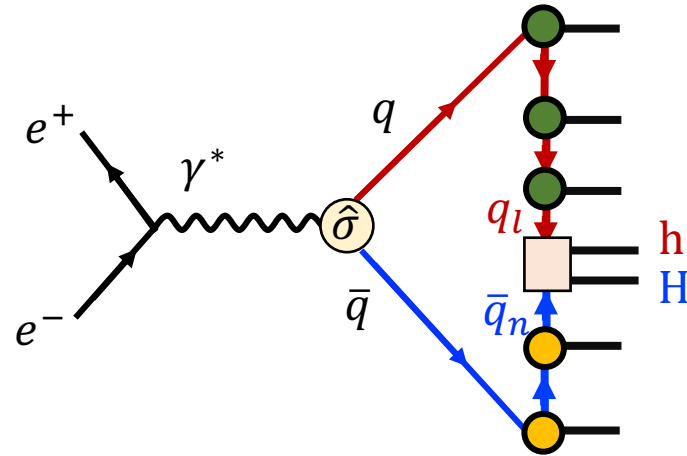
$$\frac{dP(\bar{q} \rightarrow H + \bar{q}'; q'\bar{q})}{dZ_- Z_-^{-1} d^2P_T} = \text{Tr}_{q'\bar{q}'} \mathbf{T}_{\bar{q}', H, \bar{q}} \rho(q', \bar{q}) \mathbf{T}_{q', H, \bar{q}}^\dagger = F_{\bar{q}', H, \bar{q}}(Z_-, P_T; \bar{\mathbf{k}}_T, C^{q'\bar{q}})$$

Depend on the azimuthal angle h
Expressed in the AHF

conditional probability of emitting H , having emitted h
 → correlations between the transverse momenta

[Collins NPB, 304:794–804, 1988, Knowles NPB, 310:571–588, 1988]

Recursive recipe for e^+e^-



Steps:

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3. Hadron emission from q
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- 6. Exit condition**

[AK, X. Artru, PRD 109 (2024) 5, 054029]

- Iterate until the exit condition is called and the last quark pair is hadronized
more details in PRD 109 (2024) 5, 054029

Simulations of e^+e^- annihilation with spin effects using Pythia 8.3 + StringSpinner

□ $\sqrt{s} = 10.6$ GeV, γ^* exchange, quarks produced u, d, s
consistent with BELLE and BABAR data

□ Free parameters

spin-less hadronization

as in standard Pythia 8.3

complex mass μ

as in AK, Lonnblad, CPC 292 (2023) 108886

$f_L = 0.12$

\sim T pol. VMs

$\theta_{LT} = -0.65$

interference between T and L pol. of VMs

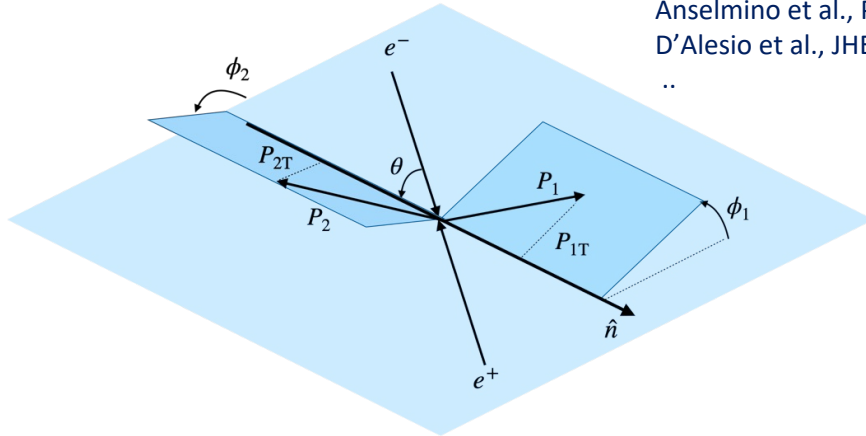
found to give a satisfactory agreement with e^+e^- data,
ok also for SIDIS

□ Compare with Collins asymmetries

The Collins asymmetries in e^+e^- for back-to-back h_1h_2

Thrust axis method

Boer et al., NPB504, 345 (1997).
 Boer, NPB, 806:23–67, 2009
 Anselmino et al., PRD 92, 114023 (2015)
 D'Alesio et al., JHEP 10 (2021) 078
 ..



$$N_{h_1 h_2} \propto 1 + \frac{\langle \sin^2 \theta \rangle}{\langle 1 + \cos^2 \theta \rangle} A_{12} \cos(\phi_1 + \phi_2)$$

Collins asymmetry

$$A_{12} = \frac{\sum_q e_q^2 H_{1q}^{\perp h_1} H_{1\bar{q}}^{\perp h_2}}{\sum_q e_q^2 D_{1q}^{h_1} D_{1\bar{q}}^{h_2}}$$

Measured asymmetry

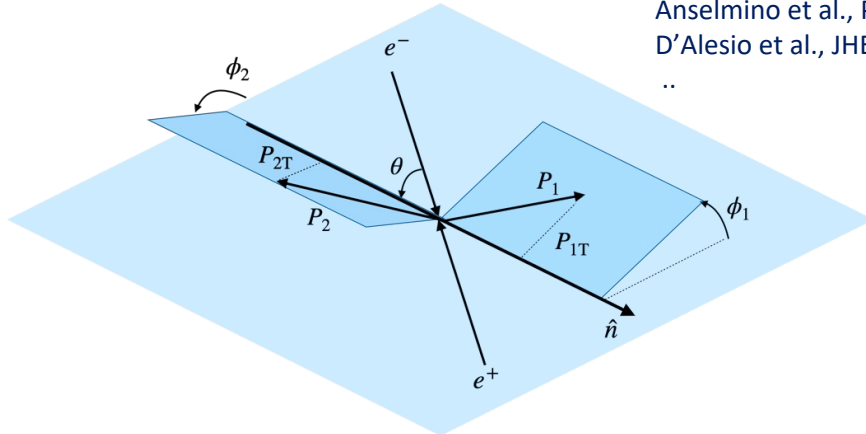
$$A_{12}^{UL(UC)} \simeq A_{12}^U - A_{12}^{L(C)}$$

U unlike sign pair e.g. $\pi^+\pi^- + \pi^-\pi^+$
 L like sign pair e.g. $\pi^+\pi^+ + \pi^-\pi^-$
 C charged pair e.g. $\pi^+\pi^- + \pi^-\pi^+ + \pi^+\pi^+ + \pi^-\pi^-$

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Collins asymmetry

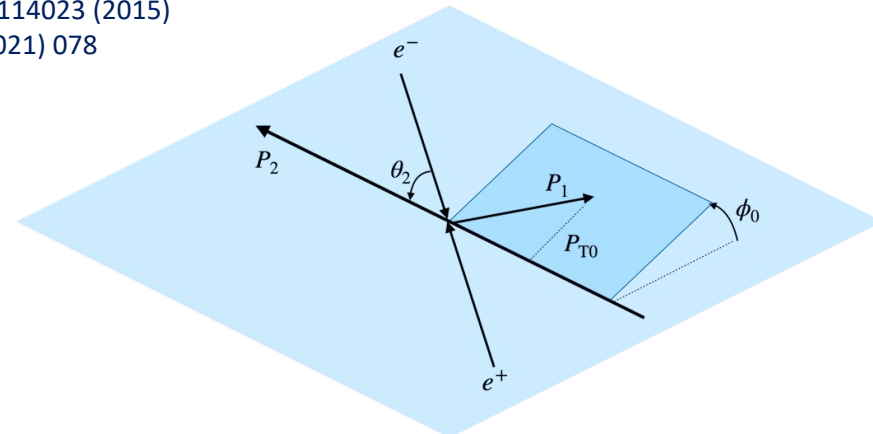
$$A_{12} = \frac{\sum_q e_q^2 H_{1q}^{\perp h_1} H_{1\bar{q}}^{\perp h_2}}{\sum_q e_q^2 D_{1q}^{h_1} D_{1\bar{q}}^{h_2}}$$

Measured asymmetry

$$A_{12}^{\text{UL(UC)}} \simeq A_{12}^{\text{U}} - A_{12}^{\text{L(C)}}$$

U unlike sign pair e.g. $\pi^+\pi^- + \pi^-\pi^+$
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 C charged pair e.g. $\pi^+\pi^- + \pi^-\pi^+ + \pi^+\pi^+ + \pi^-\pi^-$

Hadronic plane method



$$N_{h_1h_2} \propto 1 + \frac{\langle \sin^2 \theta_2 \rangle}{\langle 1 + \cos^2 \theta_2 \rangle} A_0 \cos(2\phi_0)$$

Collins asymmetry

$$A_0 = \frac{\sum_q e_q^2 w H_{1q}^{\perp h_1} \otimes H_{1\bar{q}}^{\perp h_2}}{\sum_q e_q^2 D_{1q}^{h_1} \otimes D_{1\bar{q}}^{h_2}}$$

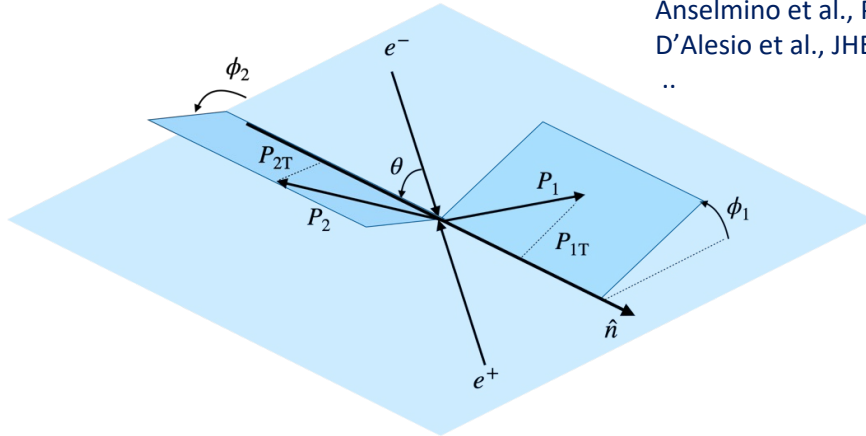
Measured asymmetry

$$A_0^{\text{UL(UC)}} \simeq A_0^{\text{U}} - A_0^{\text{L(C)}}$$

The Collins asymmetries in e^+e^- for back-to-back $h_1 h_2$

Thrust axis method

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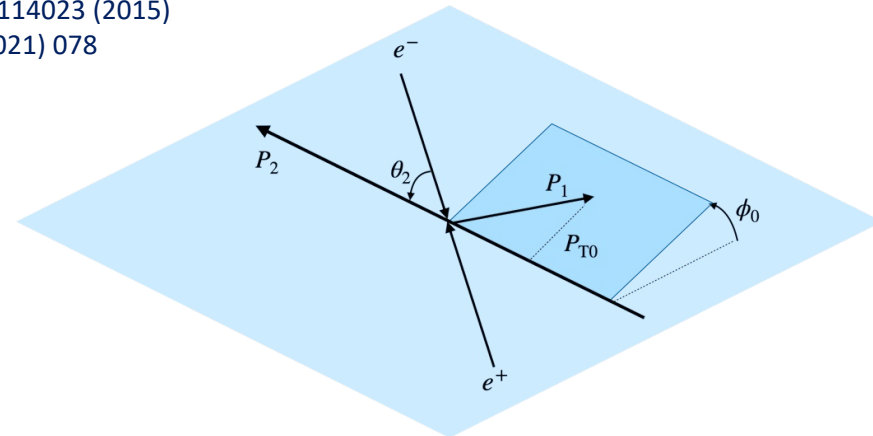


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Measured by	BELLE	2006 ($\pi\pi$), 2008 ($\pi\pi$), 2019 ($\pi\pi, \pi^0\pi, \eta\pi$)
	BABAR	2014 ($\pi\pi$), 2015 ($\pi\pi, \pi K, KK$)
	BESIII	2016 ($\pi\pi$)

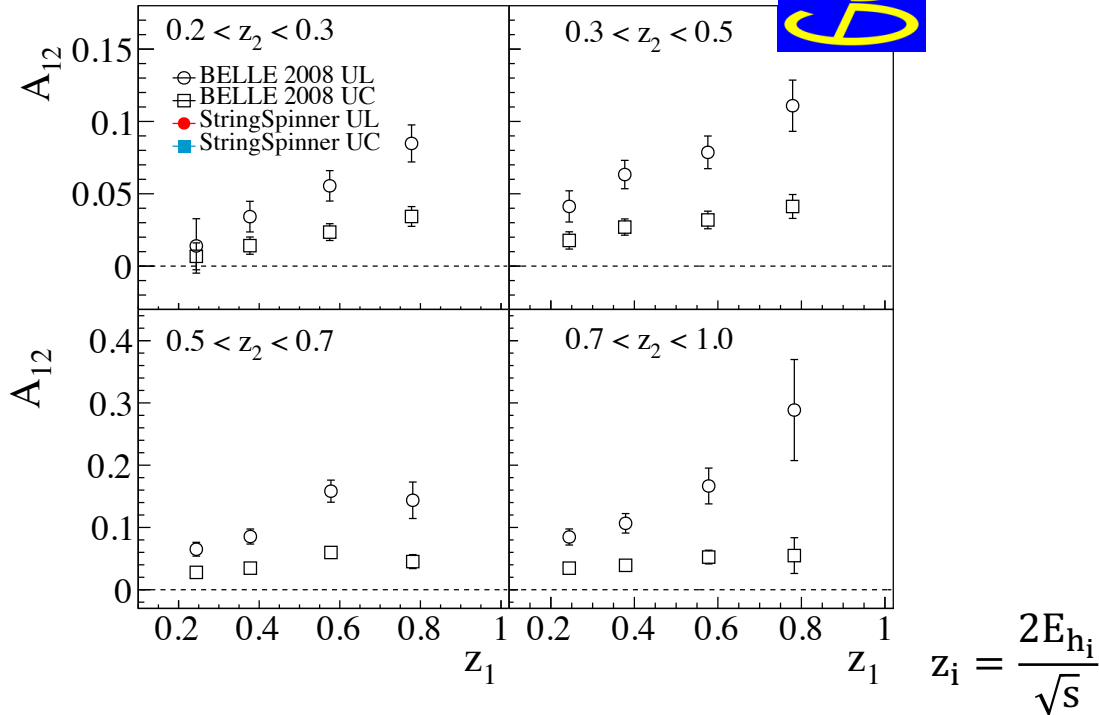
Used in many phenomenological fits to extract h_1 and H_1 , here only the model results

Comparison with the A_{12} asymmetry

AK, L. Lönnblad, A. Martin, arXiv: 2407.07706 (accepted by PRD)

A_{12} asymmetry for charged $\pi\pi$ pairs

PRD 78, 032011 (2008)

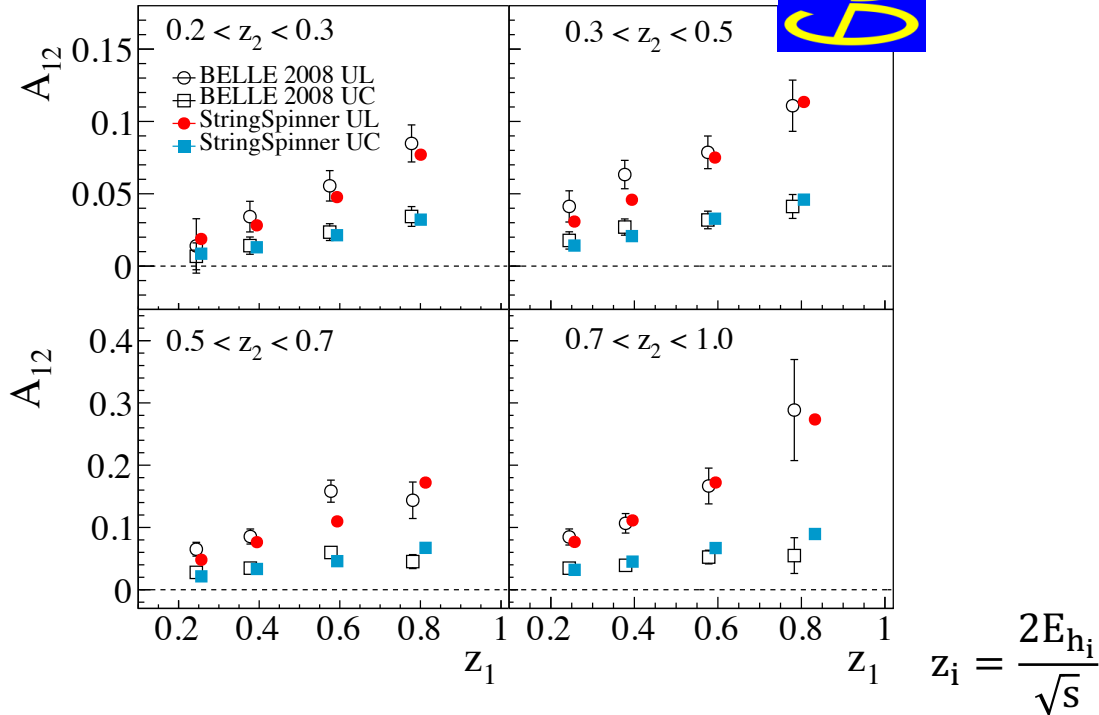


Belle asymmetries corrected for thrust smearing
Cuts:

$$T > 0.8, z > 0.2, Q_T < 3.5 \text{ GeV}$$

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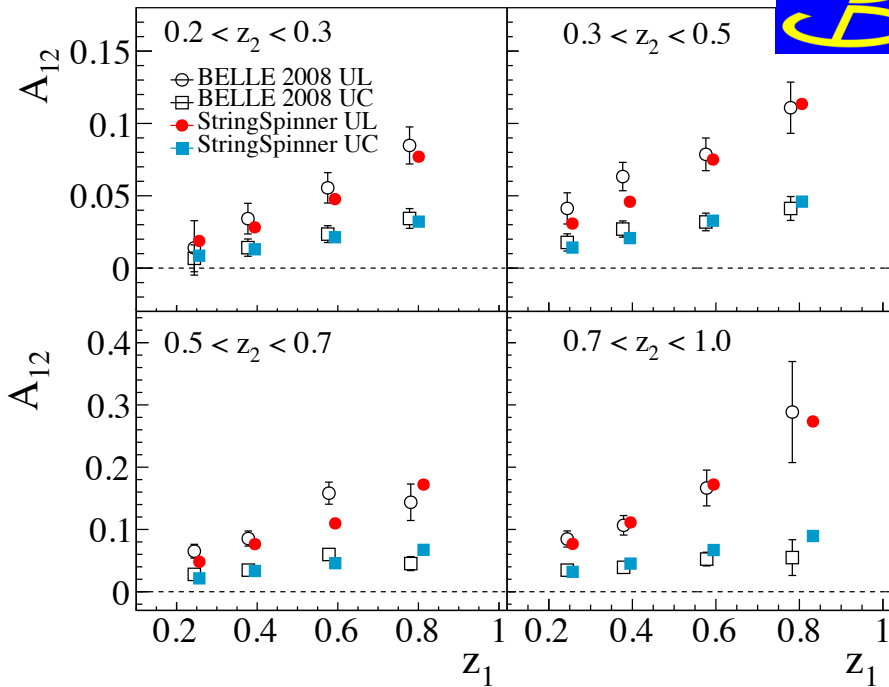
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StringSpinner reproduces trend and size

A_{12} asymmetry for charged $\pi\pi$ pairs

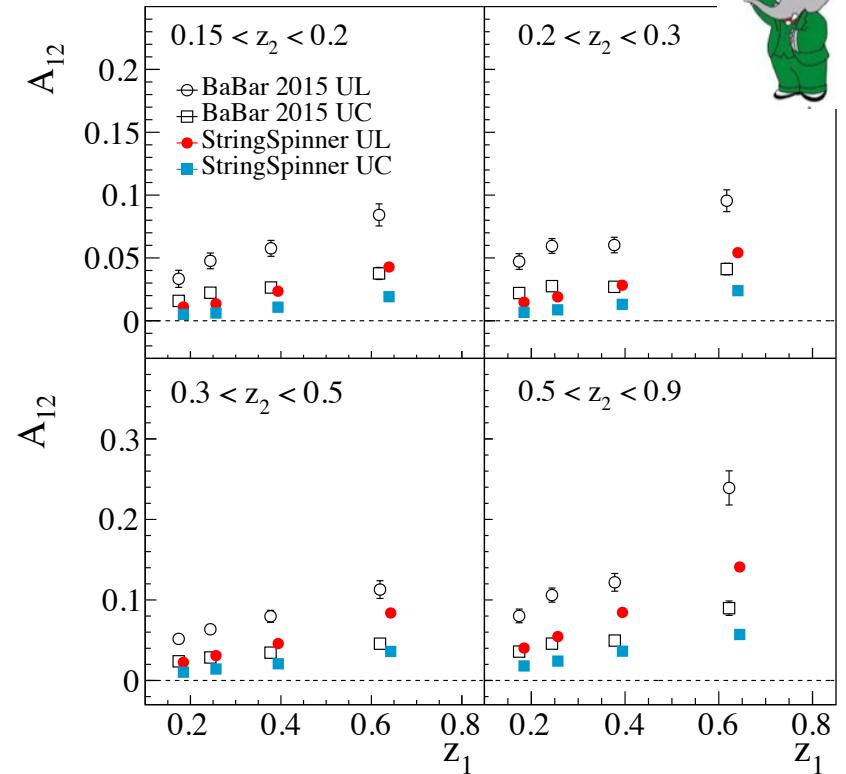
PRD 78, 032011 (2008)



Belle asymmetries corrected for thrust smearing
Cuts:

$$T > 0.8, z > 0.2, Q_T < 3.5 \text{ GeV}$$

PRD 92, 111101(R) (2015)



BaBar asymmetries corrected for thrust smearing
Cuts:

$$T > 0.8, z > 0.15, Q_T < 3.5 \text{ GeV}, \alpha_0 < \pi/4$$

StringSpinner lower than BABAR

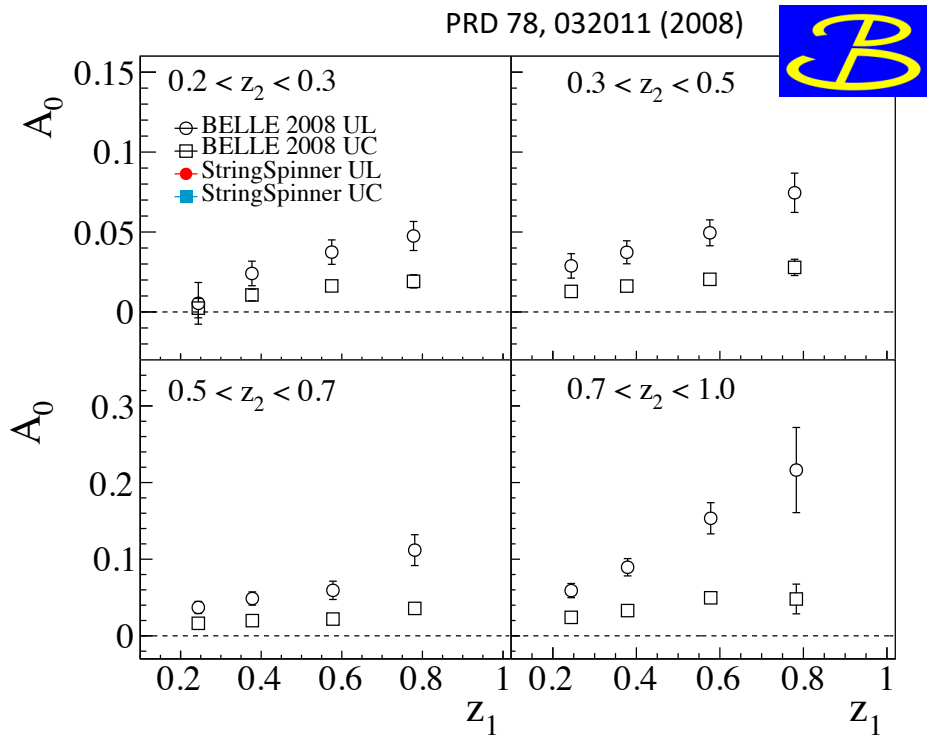
BABAR and BELLE data different, unlike
StringSpinner

PRD 90, 052003 (2014)

Comparison with the A_0 asymmetry

AK, L. Lönnblad, A. Martin, arXiv: 2407.07706 (accepted by PRD)

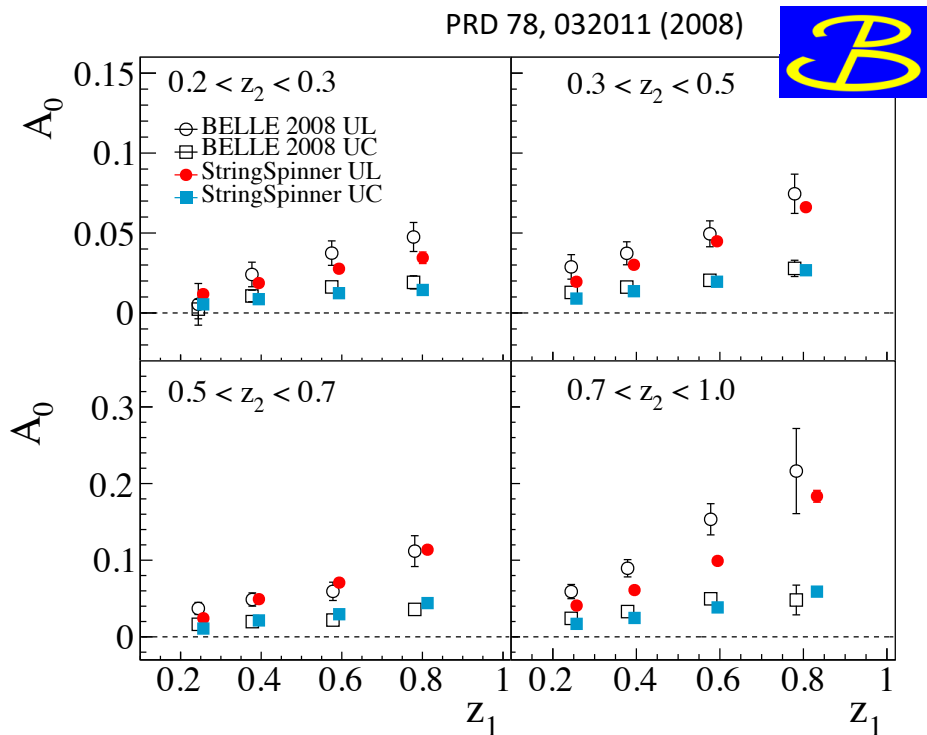
A_0 asymmetry for $\pi\pi$ pairs:



Cuts:

$$T > 0.8, z > 0.2, Q_T < 3.5\text{GeV}$$

A_0 asymmetry for $\pi\pi$ pairs:



Cuts:

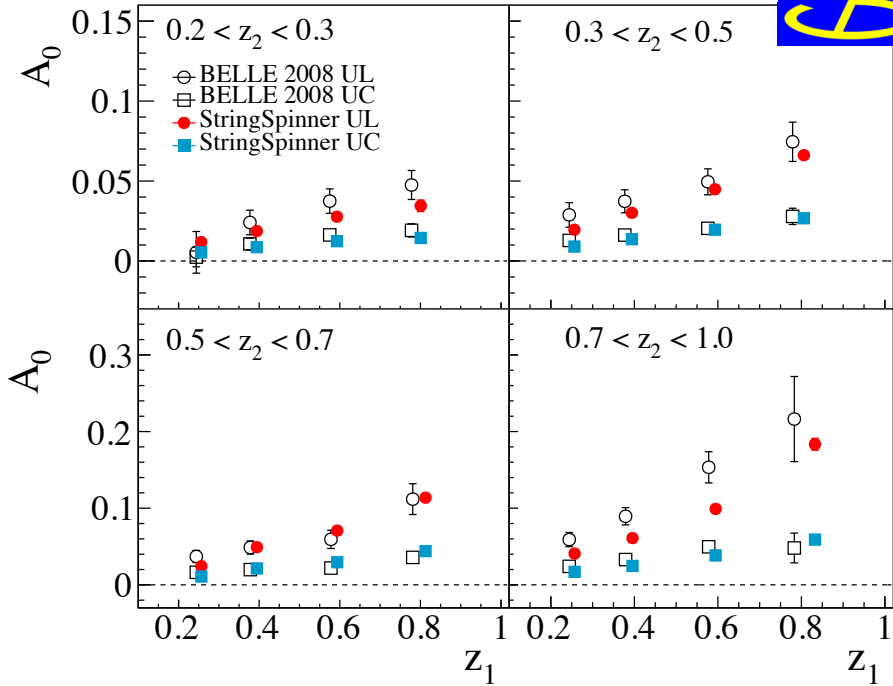
$$T > 0.8, z > 0.2, Q_T < 3.5\text{GeV}$$

Trend reproduced by string+ 3P_0

somewhat lower values in the last z_2 bin

A_0 asymmetry for $\pi\pi$ pairs:

PRD 78, 032011 (2008)

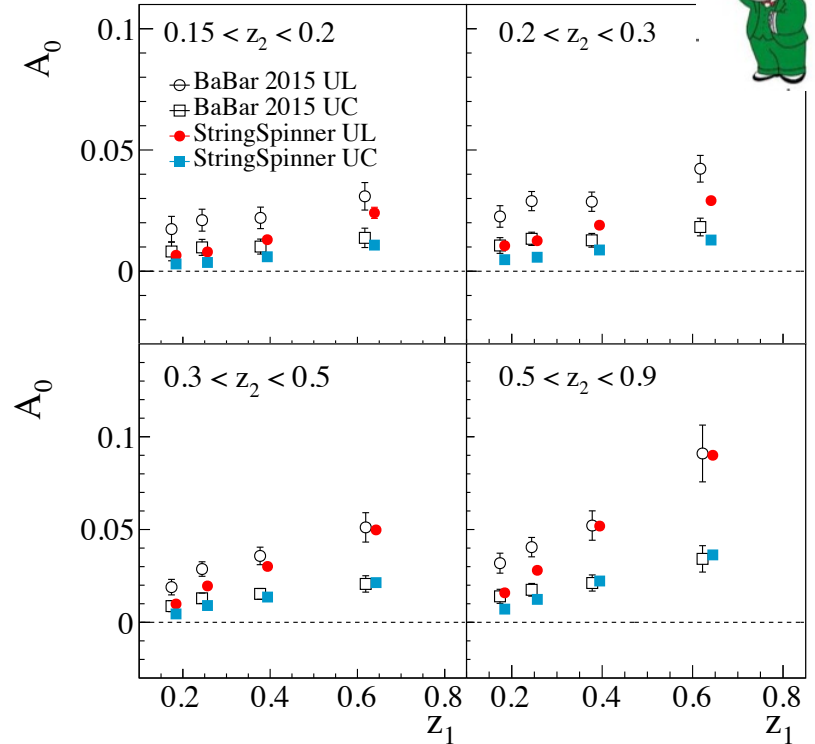


Cuts:

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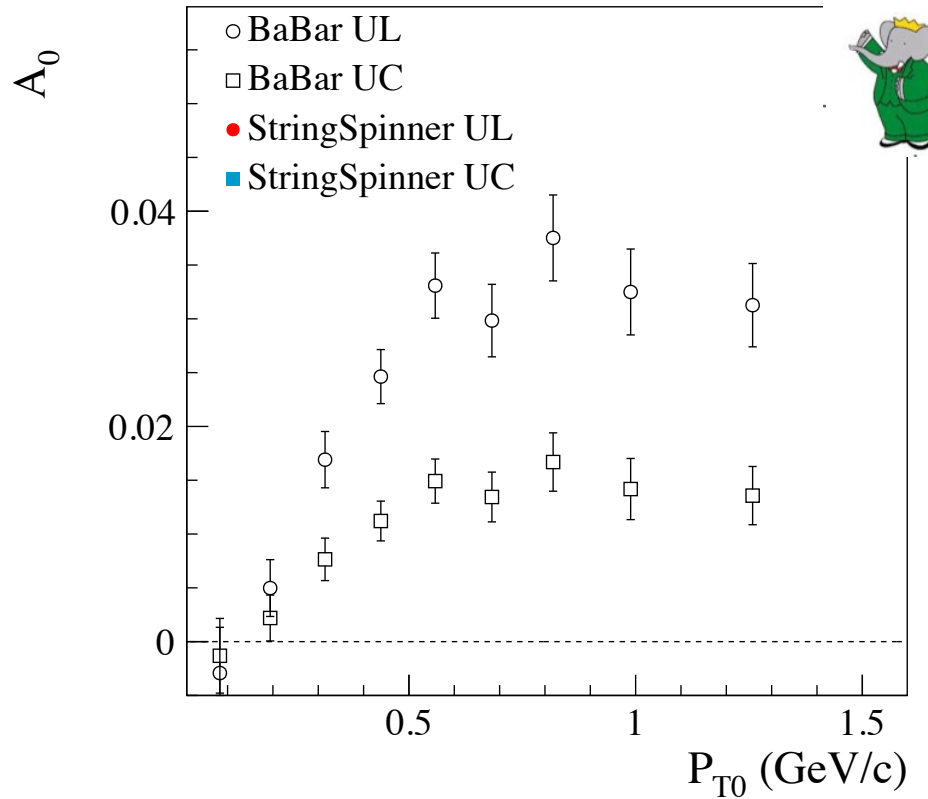
PRD 92, 111101(R) (2015)



BABAR asymmetries also ok

A_0 asymmetry for $\pi\pi$ pairs: P_{T0} dependence

PRD 90, 052003 (2014)

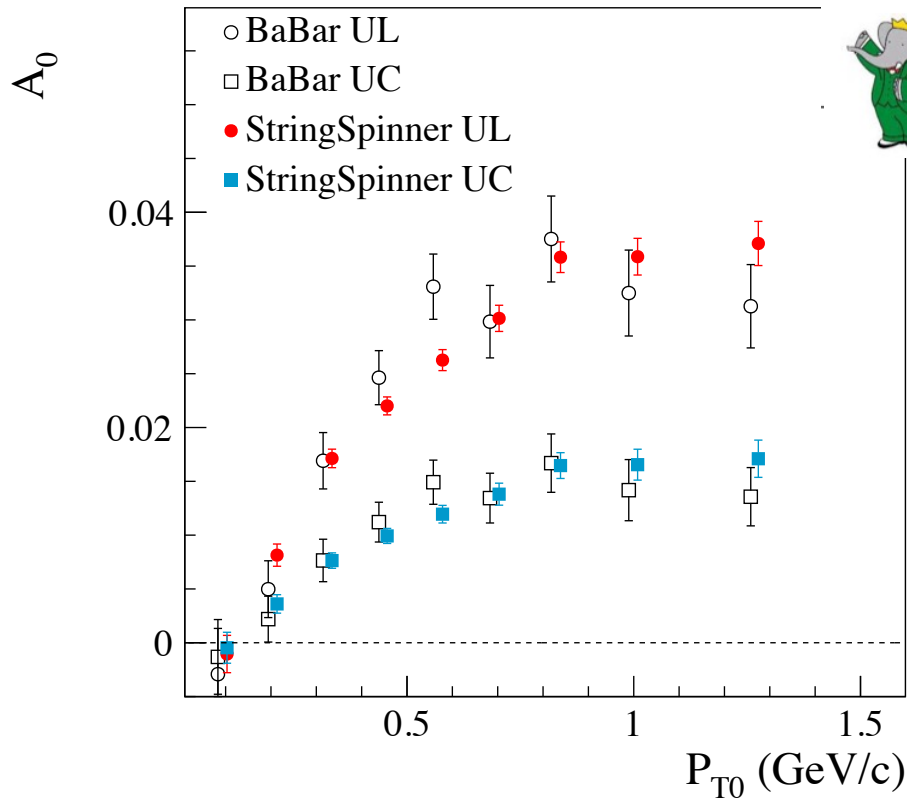


Cuts:

$T > 0.8, z > 0.15, Q_T < 3.5\text{GeV}$

A_0 asymmetry for $\pi\pi$ pairs: P_{T0} dependence

PRD 90, 052003 (2014)



Cuts:

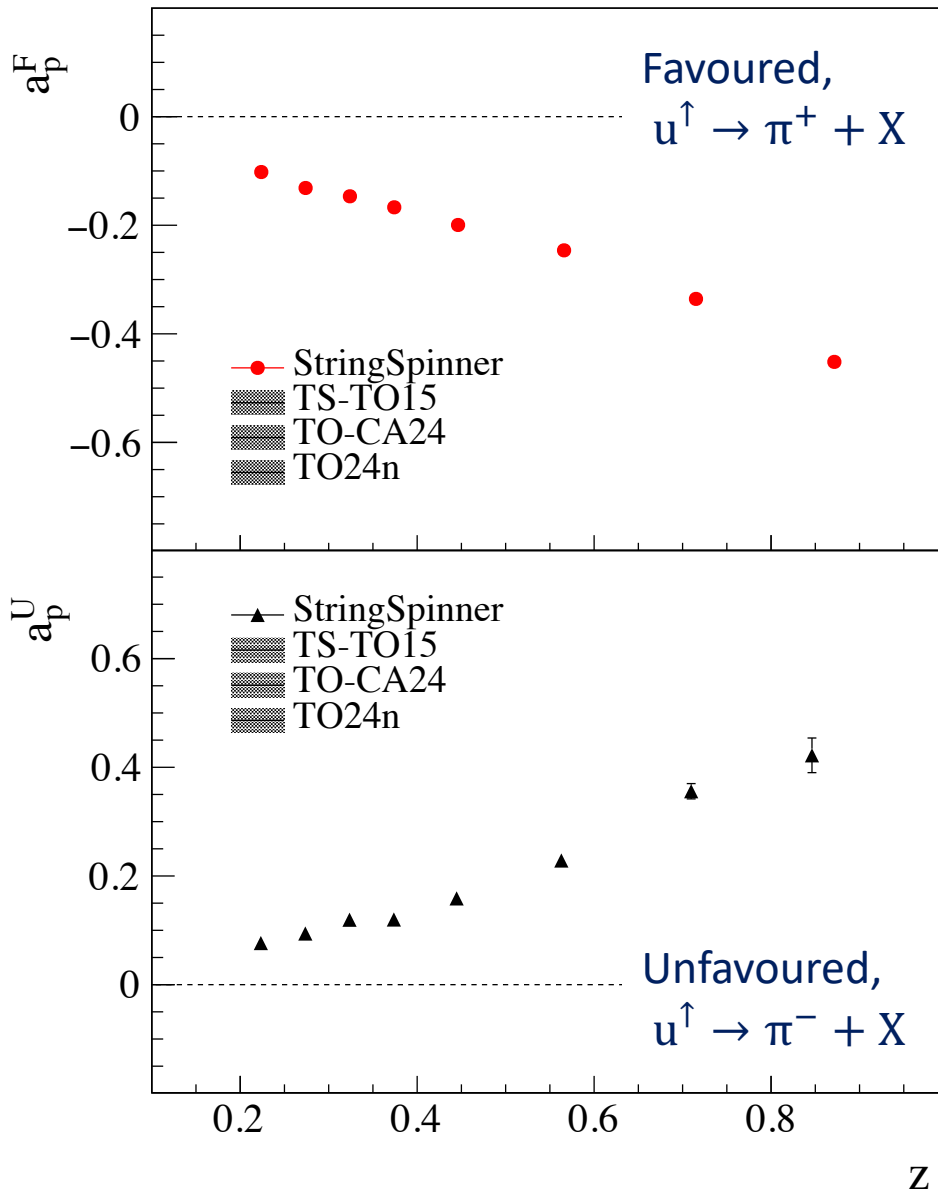
$T > 0.8, z > 0.15, Q_T < 3.5\text{GeV}$

Transverse-momentum dependence reproduced by string+ 3P_0 !

Comparison with phenomenology

AK, L. Lönnblad, A. Martin, arXiv: 2407.07706 (accepted by PRD)

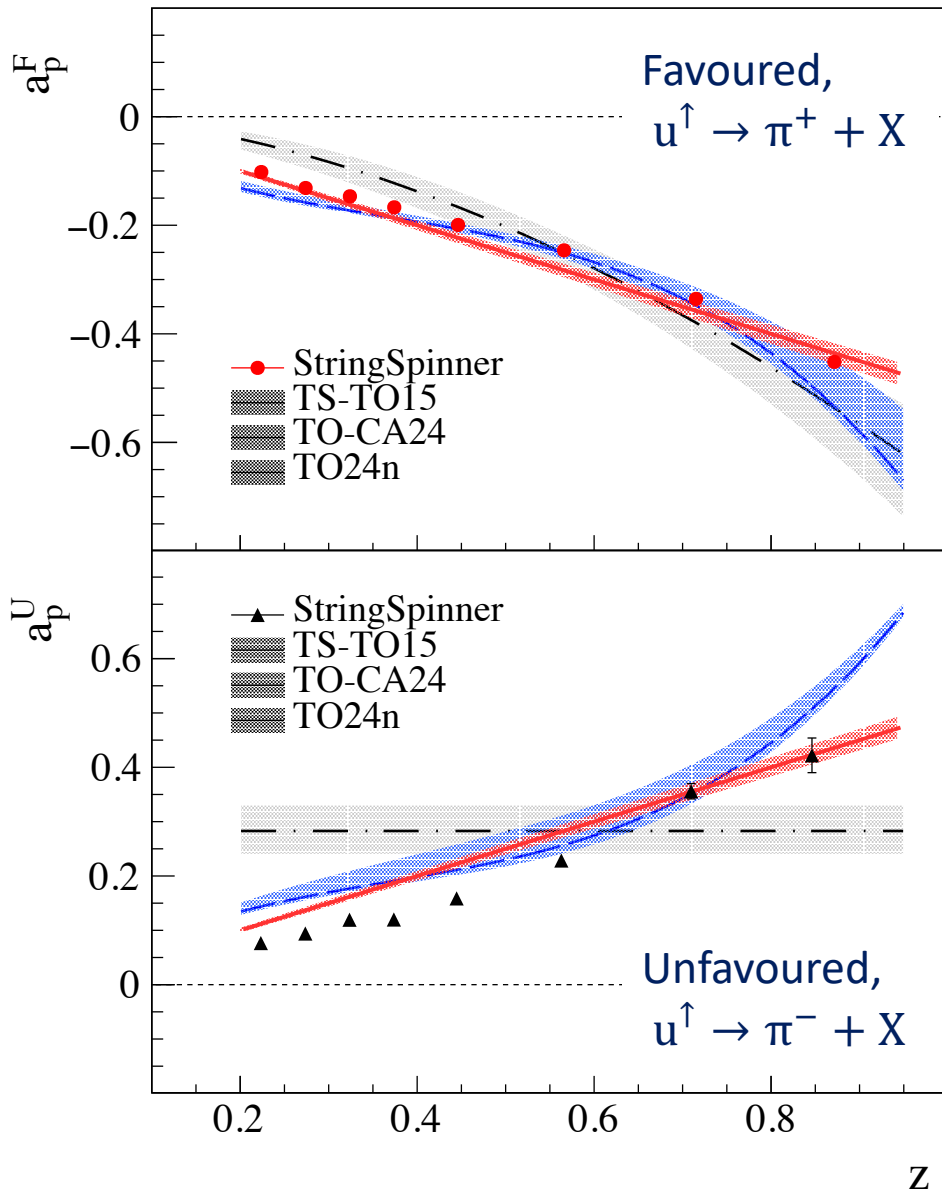
Comparison with phenomenology



Collins analysing power

$$a_p = -\frac{p_\perp}{zM_h} \frac{H_{1q}^{h\perp}}{D_{1q}^h}$$

Comparison with phenomenology



Collins analysing power

$$a_p = -\frac{p_\perp}{zM_h} \frac{H_{1q}^{h\perp}}{D_{1q}^h}$$

TS-TO15 Martin, Bradamante, Barone
PRD 91 (2015) 1, 014034

point-by-point extraction from Belle A_{12}^{UL}

TO-CA24 Boglione et al., PLB 854 (2024) 138712
fit of SIDIS, e^+e^- and pp

TO24n Boglione, Flore (2024, private comm.)
like in TO-CA24 + Collins asymm. COMPASS
2022 data on d

string+ $^3P_0 \rightarrow$ rising trend with z for fav. and unfav.

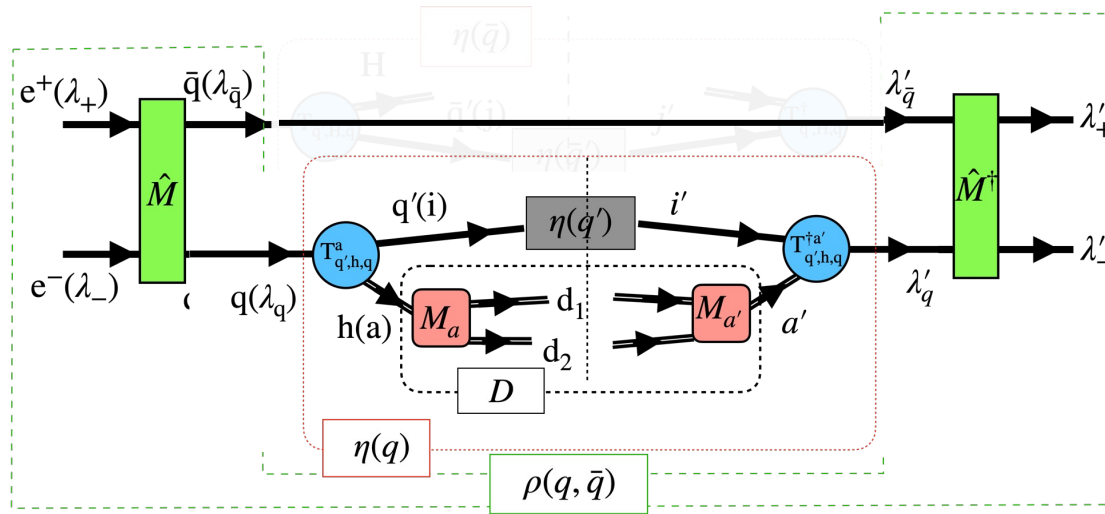
A guide for the choice of the parametrizations of Collins FFs used in phenomenology?

Conclusions

- ❑ Spin effects implemented in Pythia 8.3 for DIS and e^+e^-
StringSpinner \rightarrow string+ 3P_0 in Pythia public soon for e^+e^-
- ❑ Encouraging results on Collins asymmetries in e^+e^-
the predicted Collins effect can be used to guide parametrizations
in phenomenological extractions
- ❑ More developments of the string+ 3P_0 model possible and foreseen
baryon production, pQCD effects, inclusion of TMDs in DIS, ..
- ❑ Several steps towards an event generator systematically implementing spin effects done!

Backup

The recursive recipe for simulating e^+e^- annihilation: VM emission



For a vector meson $h=VM$

$$\rightarrow \eta(q) = \mathbf{T}_{q',h=VM,q}^{a'\dagger} \eta(q') \mathbf{T}_{q',h=VM,q}^a D_{a'a}, \quad \eta(q') = 1_{q'}, \text{ and } \eta(\bar{q}) = 1_{\bar{q}}$$

Steps:

i) Emission probability density (summing over decay information, i.e. $D_{a'a} = \delta_{a'a}$)

$$\frac{dP(q \rightarrow h = VM + q'; q\bar{q})}{dM^2 dZ_+ Z_+^{-1} d^2 p_T} = \text{Tr}_{q'\bar{q}} \mathbf{T}_{q',h,q}^a \rho(q, \bar{q}) \mathbf{T}_{q',h,q}^{a'\dagger} = F_{q',h,q}(M^2, Z_+, p_T; k_T, C^{q\bar{q}})$$

ii) Calculate the spin density matrix of $h=VM$, and decay the meson

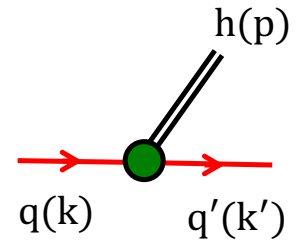
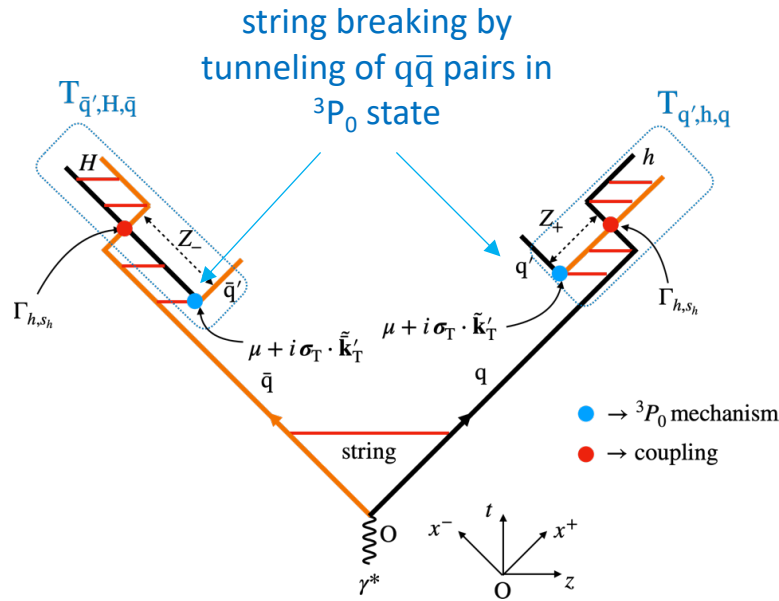
$$\rho_{aa'}(h) = \text{Tr}_{q'\bar{q}} \mathbf{T}_{q',h,q}^a \rho(q, \bar{q}) \mathbf{T}_{q',h,q}^{a'\dagger}$$

iii) Decay the meson $p \rightarrow p_1 p_2 \dots$

$$dN(p_1, p_2 \dots) / d\Omega \propto M_{\text{dec}}^a(p \rightarrow p_1 p_2 \dots) \rho_{aa'}(h) M_{\text{dec}}^{a'\dagger}(p \rightarrow p_1 p_2 \dots)$$

iv) Build the decay matrix $D_{a'a}(p_1, p_2, \dots) = M_{\text{dec}}^{a'\dagger}(p \rightarrow p_1 p_2 \dots) M_{\text{dec}}^a(p \rightarrow p_1 p_2 \dots)$

The hadronization model: string+ 3P_0



quark splitting $q \rightarrow h + q'$

Relevant variables:

$$\mathbf{k}_T = \mathbf{p}_T + \mathbf{k}'_T$$

$$Z_+ = p^+ / k^+$$

$$\varepsilon_h^2 = M^2 + p_T^2$$

Transverse vectors defined w.r.t. string axis

Quark splitting amplitude in the string+ 3P_0 model

$$T_{q',h,q} \propto C_{q',h,q} D_h(M^2) \underbrace{\left(\frac{1 - Z_+}{\varepsilon_h^2} \right)^{\frac{a}{2}} \exp \left[-\frac{\mathbf{b}_L \varepsilon_h^2}{2Z_+} \right]}_{\text{longitudinal momentum}} \underbrace{N_a^{-\frac{1}{2}} (\varepsilon_h^2) e^{-\frac{\mathbf{b}_T k'^2_T}{2}}}_{\text{transverse momentum (w.r.t string axis)}}$$

$[\mu + \sigma_z \sigma_T \cdot \mathbf{k}'_T]$
 3P_0 mechanism
 $[\mu \text{ complex mass parameter}]$

Γ_{h,s_h}
 Coupling
 e.g.
 $\Gamma_{h=PS} = \sigma_z$

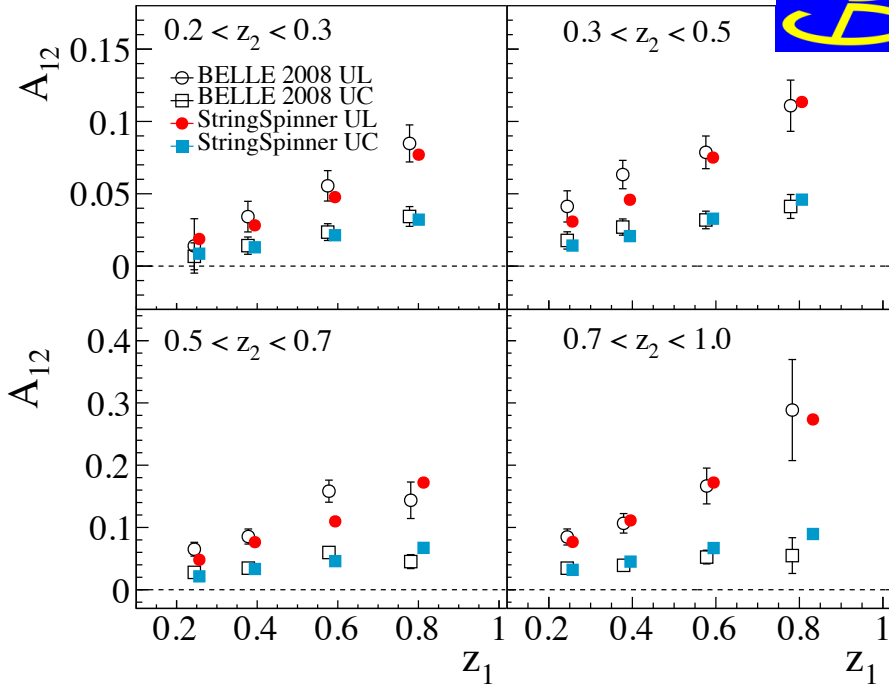
Free param. Lund

Free param. string+ 3P_0

AK, Artru, Martin, PRD 104, 114038 (2021)

A_{12} asymmetry for charged $\pi\pi$ pairs

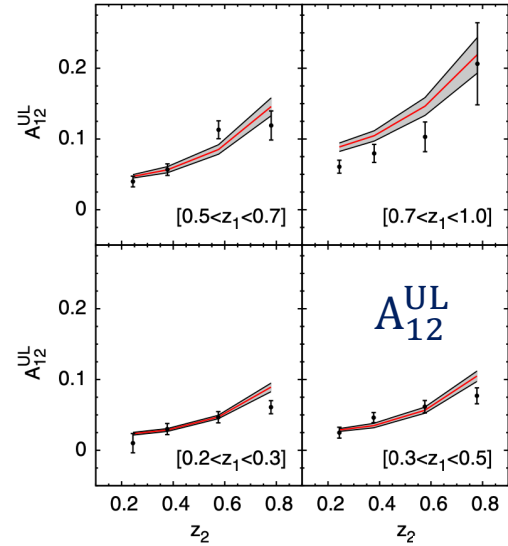
PRD 78, 032011 (2008)



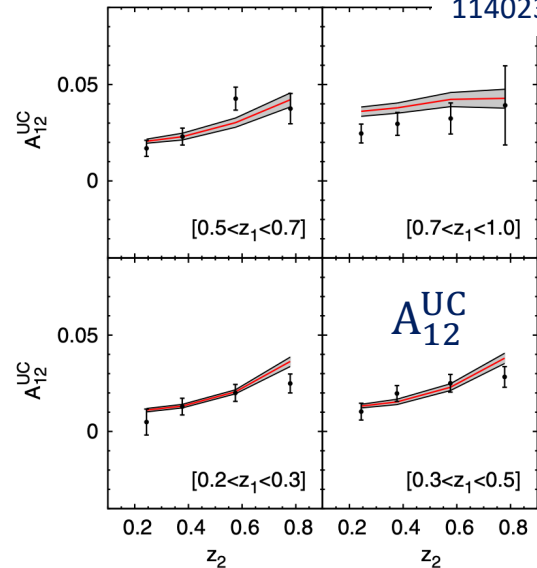
Belle asymmetries corrected for thrust smearing
Cuts:

$$T > 0.8, z > 0.2, Q_T < 3.5 \text{ GeV}$$

StringSpinner reproduces trend and size

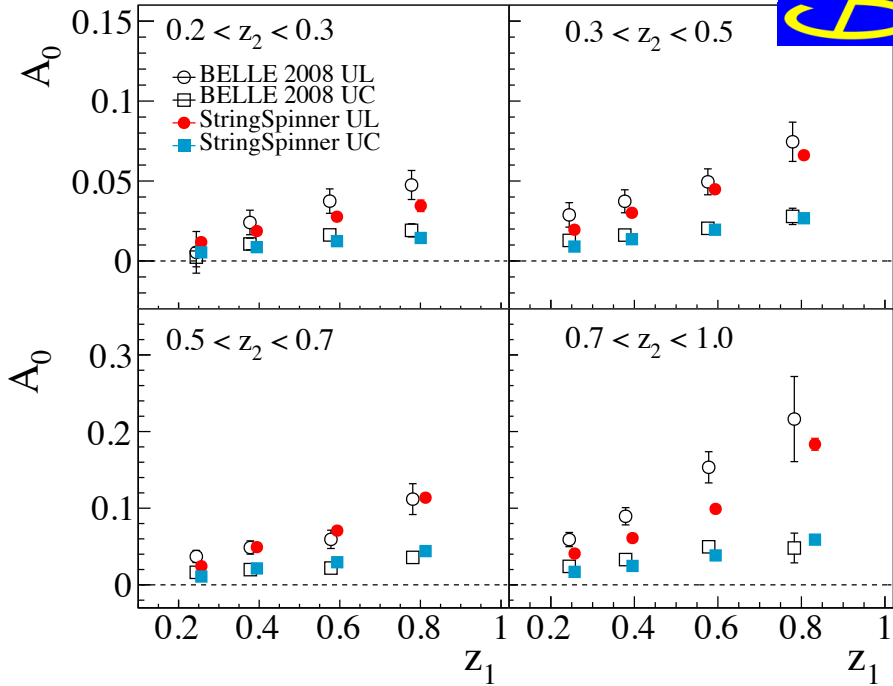


Anselmino et al., PRD 92,
114023 (2015)



A_0 asymmetry for charged pions

PRD 78, 032011 (2008)

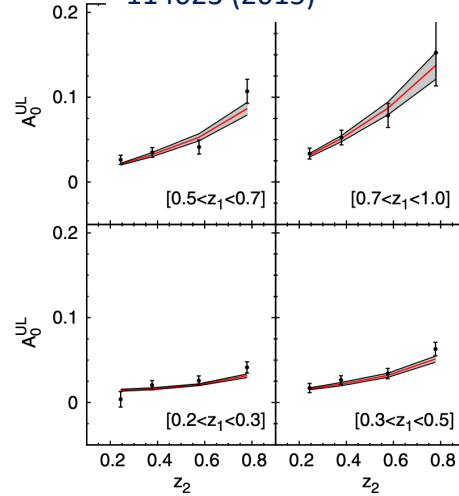


Cuts:

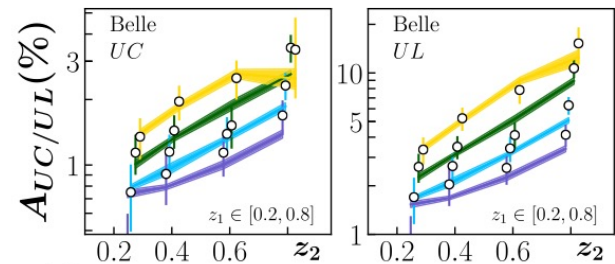
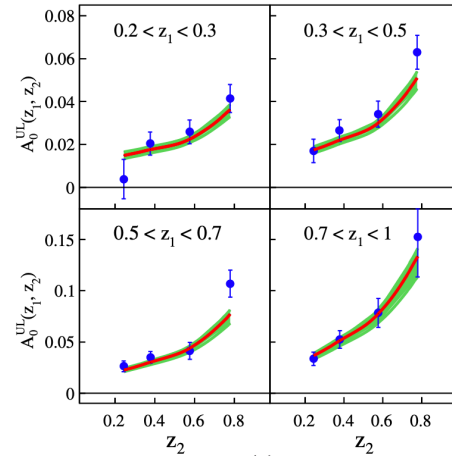
$$T > 0.8, z > 0.2, Q_T < 3.5\text{GeV}$$

Trend reproduced by string+ 3P_0
somewhat lower values in the last z_2 bin

Anselmino et al., PRD 92, 114023 (2015)



Kang et al., PRD 93, 014009 (2016)

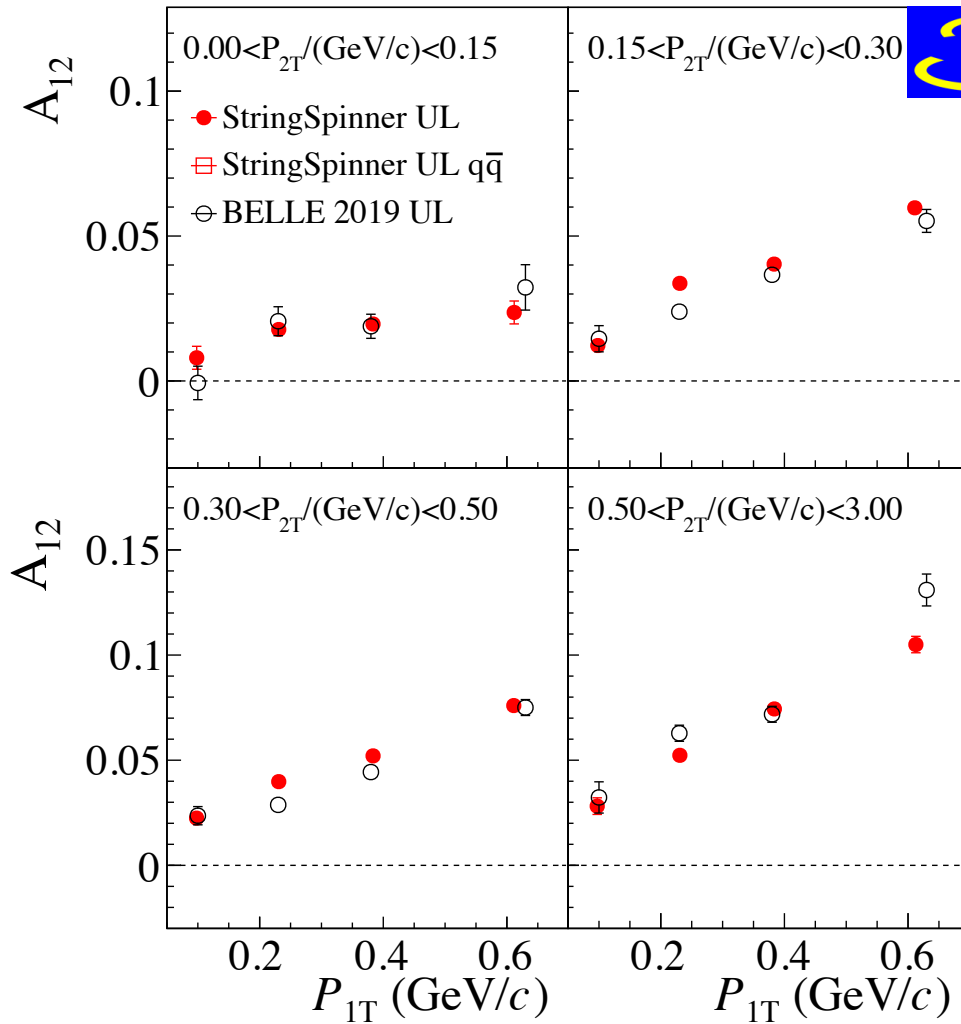


JAM, PRD 102, 054002 (2020)

A_0 asymmetry essential observable
included in phenomenological fits

A_{12}^{UL} asymmetry for charged $\pi \pi$ pairs

$P_{T1} \times P_{T2}$ - dependence w.r.t thrust



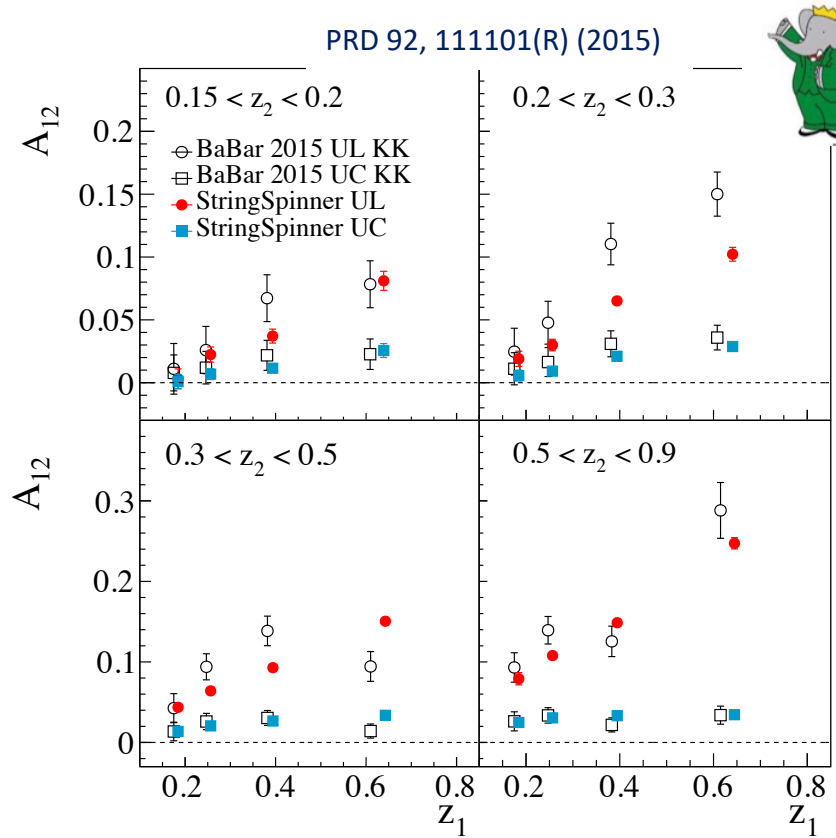
PRD 100, 092008 (2019)

Asymmetries using thrust axis,
not corrected for thrust smearing

$T > 0.8$
 $z > 0.2, P_T < 3.0 \text{ GeV}/c$
 $\alpha_0 < 0.3 \text{ rad}$

StringSpinner reproduces the nearly linear trend observed by BELLE

A_{12} asymmetry for charged KK pairs



Corrected for thrust smearing

Cuts

$$T > 0.8, z > 0.15, Q_T < 3.5 \text{ GeV}, \alpha_0 < \pi/4$$

A_{12}^{UC} much smaller than A_{12}^{UL} at large z
reproduced by string+ 3P_0