
May 30, 2024, Pavia

# Collins asymmetries in $e^+e^-$ annihilation: a MC event generator study

Albi Kerbizi
University of Trieste and INFN Trieste

in collaboration with Xavier Artru, Leif Lönnblad and Anna Martin

1924 2024

UNIVERSITÀ DEGLI STUDI DI TRIESTE

MUR Ministero dell'Università e della Ricerca

POLFRAG

INFN Istituto Nazionale di Fisica Nucleare

# Studying hadronization

- Phenomenological QCD fits (SIDIS, e+e-, pp)
  parametrize FFs and PDFs, extract from data e.g. Collins FF or IFF, and transversity

- Development of hadronization models

# Studying hadronization

- Phenomenological QCD fits (SIDIS, e+e-, pp)
  parametrize FFs and PDFs, extract from data e.g. Collins FF or IFF, and transversity

- Development of hadronization models
  analytic calculation of FFs e.g. using spectator models Bacchetta et al., PLB 659 (2008) 234-243
  **multiproduction models**
  - **String fragmentation** models
    - Artru, Menessier '74 no spin
    - Lund String Model *'83* no spin
    - **string+$^3P_0$** **spin** [Artru '09; Artru, Belghobsi '13; Kerbizi, Artru, Belghobsi, Bradamante, Martin '18..]
  - Cluster model no spin [Marchesini, Webber '84]
  - quark jet models
    - Field, Feynman '77 no spin
    - *NJL-jet model '17* spin [Matevosyan, Kotzinian, Thomas, '17]

- MC event generators needed to perform calculations with multiproduction models
  general purpose generators (Pythia, Lepto, Herwig).. no spin in hadronization
  for spin effects → develop a model of spin-dependent hadronization
  then implement in MC

# Modeling spin-dependent hadronization: the string+$^3P_0$ model

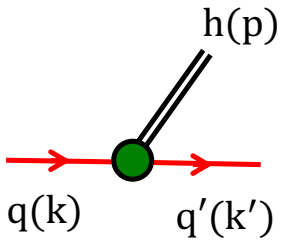
- Extension of the Lund string fragmentation model that includes the quark spin
  based on amplitudes, preserves positivity and entanglement
  different developments

| | | |
|---|---|---|
| Artru, DSPIN-09, arXiv:1001.1061 | 2009 | toy model |
| AK, Artru, Belghobsi, Bradamante, Martin, PRD 97, 074010 (2018) | 2018 | PS mesons |
| AK, Artru, Belghobsi, Martin, PRD 100, 014003 (2019) | 2019 | PS mesons simplified |
| AK, Artru, Martin, PRD 104, 114038 (2021) | 2021 | PS mesons + VM |

# Modeling spin-dependent hadronization: the string+$^3P_0$ model

- Extension of the Lund string fragmentation model to include the quark spin
- Basic quantity – quark (and antiquark) splitting amplitude

AK, Artru, Martin, PRD 104, 114038 (2021)



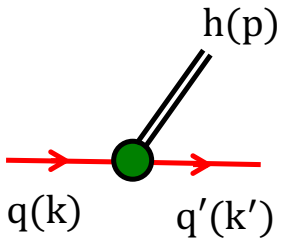

$$\mathbf{k}_T = \mathbf{p}_T + \mathbf{k}'_T$$
$$Z_+ = p^+/k^+$$

$$T_{q',h,q} \propto \left[F^{\text{Lund}}_{q',h,q}(Z_+, \mathbf{p}_T; \mathbf{k}_T)\right]^{1/2} \left[\mu + \sigma_z \boldsymbol{\sigma}_T \cdot \mathbf{k}'_T\right] \Gamma_{h,s_h}$$

# Modeling spin-dependent hadronization: the string+$^3P_0$ model

- Extension of the Lund string fragmentation model to include the quark spin
- Basic quantity – quark (and antiquark) splitting amplitude

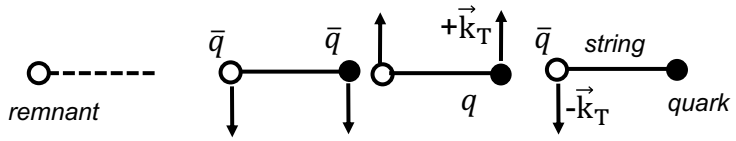
AK, Artru, Martin, PRD 104, 114038 (2021)




$$\mathbf{k}_T = \mathbf{p}_T + \mathbf{k}'_T$$
$$Z_+ = p^+/k^+$$

$$T_{q',h,q} \propto \left[F^{\text{Lund}}_{q',h,q}(Z_+, \mathbf{p}_T; \mathbf{k}_T)\right]^{1/2} \quad [\mu + \sigma_z \boldsymbol{\sigma}_T \cdot \mathbf{k}'_T] \quad \Gamma_{h,s_h}$$

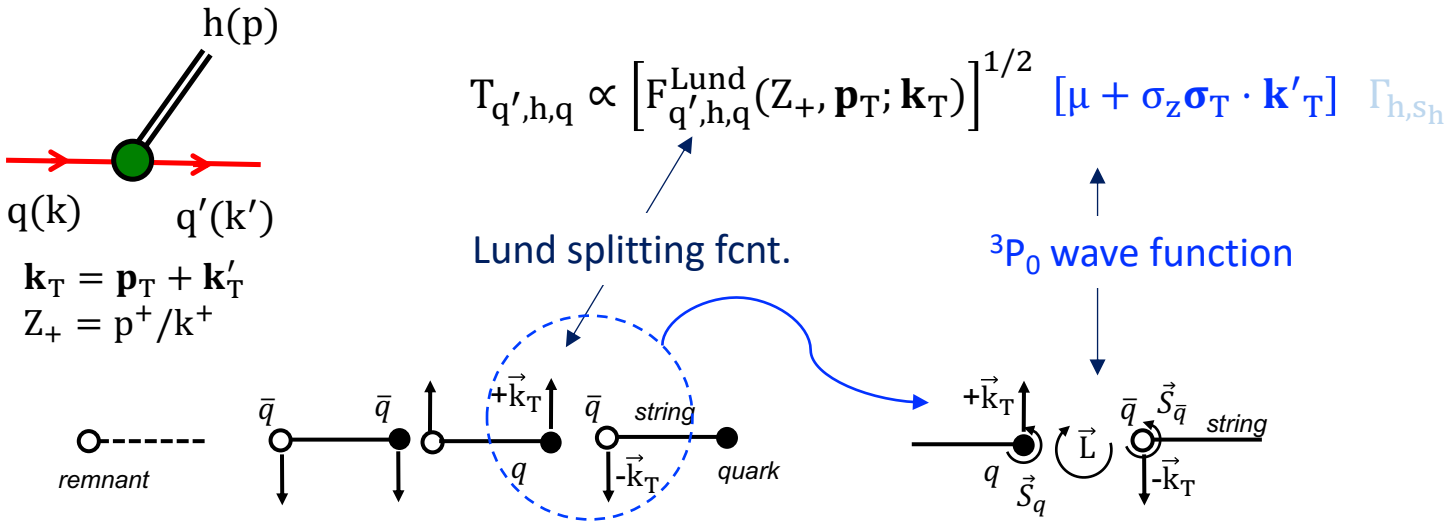
Lund splitting fcnt.



# Modeling spin-dependent hadronization: the string+$^3P_0$ model

- Extension of the Lund string fragmentation model to include the quark spin
- Basic quantity – quark (and antiquark) splitting amplitude

AK, Artru, Martin, PRD 104, 114038 (2021)

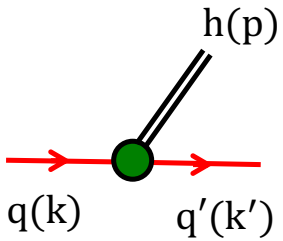


# Modeling spin-dependent hadronization: the string+$^3P_0$ model

- Extension of the Lund string fragmentation model to include the quark spin

- Basic quantity – quark (and antiquark) splitting amplitude

AK, Artru, Martin, PRD 104, 114038 (2021)




$\mathbf{k}_T = \mathbf{p}_T + \mathbf{k}'_T$

$Z_+ = p^+/k^+$

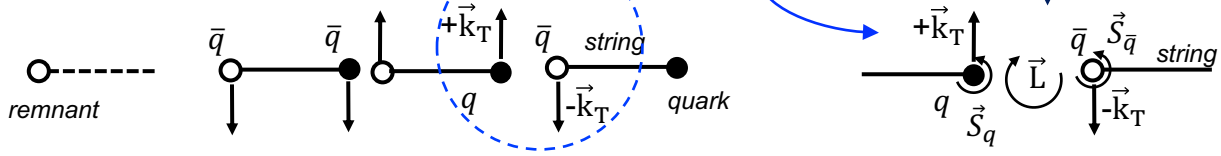
$$T_{q',h,q} \propto \left[F^{\text{Lund}}_{q',h,q}(Z_+, \mathbf{p}_T; \mathbf{k}_T)\right]^{1/2} \; [\mu + \sigma_z \boldsymbol{\sigma}_T \cdot \mathbf{k}'_T] \; \Gamma_{h,s_h}$$

PS meson→ $\sigma_z$

Lund splitting fcnt.

$^3P_0$ wave function

coupling




VM with pol. $\vec{V}$

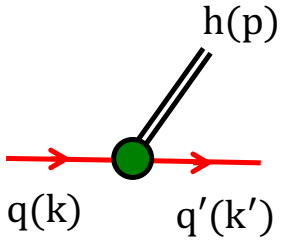
$G_L V_L^* \, 1 + G_T \, \vec{\sigma}_T \cdot \vec{V}_T^* \sigma_z$

# Modeling spin-dependent hadronization: the string+$^3P_0$ model

- Extension of the Lund string fragmentation model to include the quark spin

- Basic quantity – quark (and antiquark) splitting amplitude

AK, Artru, Martin, PRD 104, 114038 (2021)




$\mathbf{k}_T = \mathbf{p}_T + \mathbf{k}'_T$

$Z_+ = p^+/k^+$

$$T_{q',h,q} \propto \left[F^{\text{Lund}}_{q',h,q}(Z_+, \mathbf{p}_T; \mathbf{k}_T)\right]^{1/2} \; [\mu + \sigma_z \boldsymbol{\sigma}_T \cdot \mathbf{k}'_T] \; \Gamma_{h,s_h}$$

Lund splitting fcnt. — $^3P_0$ wave function — coupling

Free parameters: as in Lund Model

complex «mass»

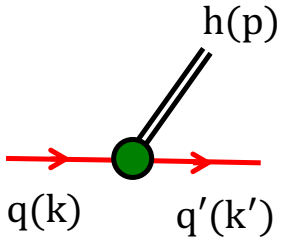
$\mathrm{Im}(\mu)$ responsible for transverse spin effects, e.g. Collins effect

# Modeling spin-dependent hadronization: the string+$^3P_0$ model

- Extension of the Lund string fragmentation model to include the quark spin

- Basic quantity – quark (and antiquark) splitting amplitude

AK, Artru, Martin, PRD 104, 114038 (2021)



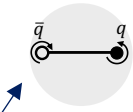

$\mathbf{k}_T = \mathbf{p}_T + \mathbf{k}'_T$

$Z_+ = p^+/k^+$

$$T_{q',h,q} \propto \left[F^{\text{Lund}}_{q',h,q}(Z_+, \mathbf{p}_T; \mathbf{k}_T)\right]^{1/2} \left[\mu + \sigma_z \boldsymbol{\sigma}_T \cdot \mathbf{k}'_T\right] \Gamma_{h,s_h}$$

Lund splitting fcnt. — $^3P_0$ wave function — coupling

PS meson→ $\sigma_z$



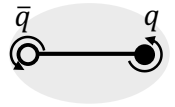

Free parameters: as in Lund Model

complex «mass»

$\mathrm{Im}(\mu)$ responsible for transverse spin effects, e.g. Collins effect

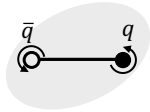
$$f_L = \frac{\left|G_L/G_T\right|^2}{2+\left|G_L/G_T\right|^2}$$

fraction of long. pol. VMs



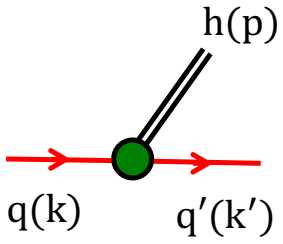

$\theta_{LT} = \arg\left(\frac{G_L}{G_T}\right)$

oblique pol. (LT interference)



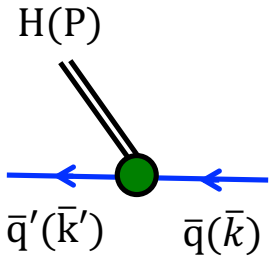
# Modeling spin-dependent hadronization: the string+$^3P_0$ model

- Extension of the Lund string fragmentation model to include the quark spin
- Basic quantity – quark (and antiquark) splitting amplitude




$$\mathbf{k}_T = \mathbf{p}_T + \mathbf{k}'_T$$
$$Z_+ = p^+/k^+$$

$$T_{q',h,q} \propto \left[F^{\text{Lund}}_{q',h,q}(Z_+, \mathbf{p}_T; \mathbf{k}_T)\right]^{1/2} \left[\mu + \sigma_z \boldsymbol{\sigma}_T \cdot \mathbf{k}'_T\right] \Gamma_{h,s_h}$$




For anti-quark splitting

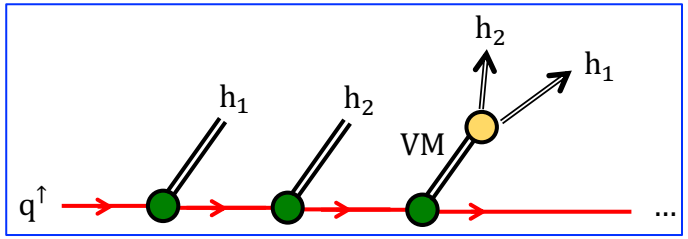
$$\{q, h, q'\} \to \{\bar{q}, H, \bar{q}'\},\ Z_+ \to Z_-,\ \{\mathbf{k}_T, \mathbf{p}_T, \mathbf{k}'_T\} \to \{\bar{\mathbf{k}}_T, \mathbf{P}_T, \bar{\mathbf{k}}'_T\}$$

$$\bar{\mathbf{k}}_T = \mathbf{P}_T + \bar{\mathbf{k}}'_T$$
$$Z_- = P^-/\bar{k}^-$$

AK, Artru, PRD 109 (2024) 5, 054029

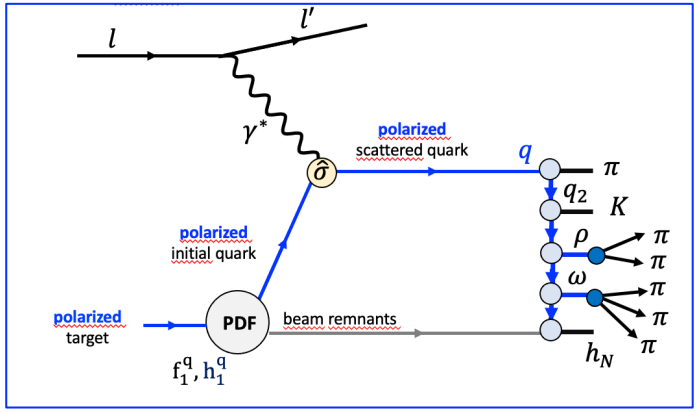
# Applications of the string+$^3P_0$ model

- Standalone MC for polarized quark jets
  - initial studies
  - detailed study of the model predictions
  - (Collins asymmetry, dihadron asymmetry, $G_1^\perp$..)



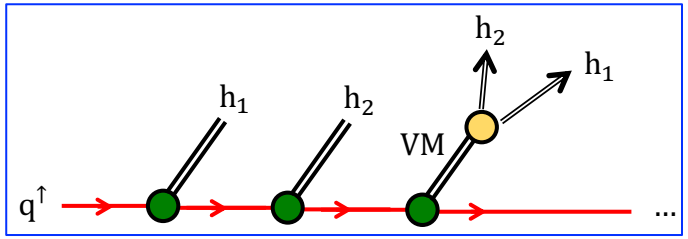

- SIDIS
  - implemented in Pythia via **StringSpinner**
  - complete collisions
  - struck quark polarized, remnant unpolarized
  - parametrization of transversity PDF
    - reproduces TSA data (Collins, 2h)

[AK, L. Lönnblad, CPC **272** (2022) 108234; CPC **292** (2023) 108886]



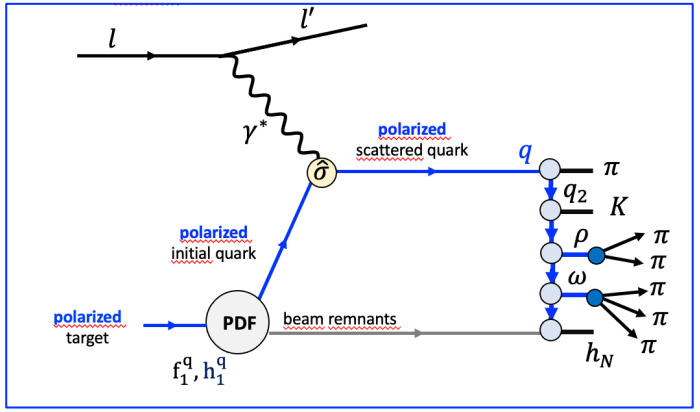
# Applications of the string+$^3P_0$ model

- **Standalone MC** for polarized quark jets
  - initial studies
  - detailed study of the model predictions
  - (Collins asymmetry, dihadron asymmetry, $G_1^\perp$..)



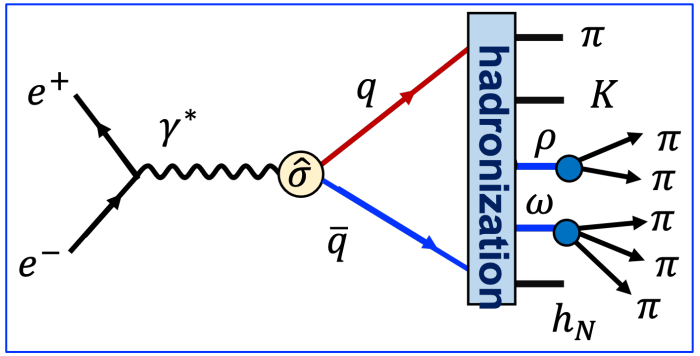

- **SIDIS**
  - implemented in Pythia via **StringSpinner**
  - complete collisions
  - struck quark polarized, remnant unpolarized
  - parametrization of transversity PDF
    - reproduces TSA data (Collins, 2h)

  [AK, L. Lönnblad, CPC **272** (2022) 108234; CPC **292** (2023) 108886]

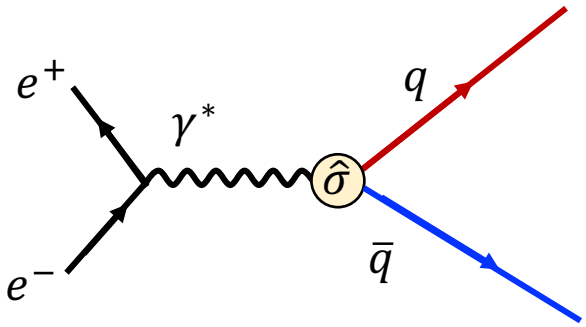



- **$e^+e^-$ annihilation to hadrons** → next slides
  - hadronize $q\bar{q}$ accounting for
  - **i. correlated spin states** of q and $\bar{q}$
  - **ii. quantum mechanical spin-correlations**
    - in fragmentation chain

  AK, X. Artru, PRD 109 (2024) 5, 054029



# Recursive recipe for $e^+e^-$

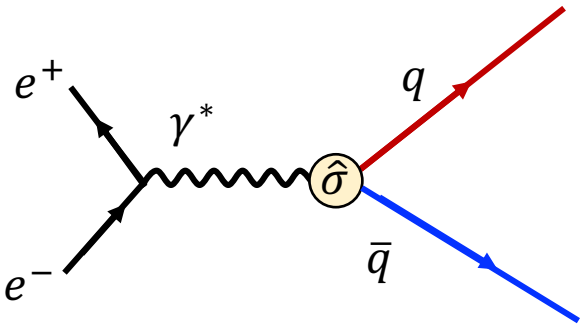



Steps:

1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from $q$
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. Exit condition

[AK, X. Artru, PRD 109 (2024) 5, 054029]

# Recursive recipe for $e^+e^-$




Steps:

**1. Hard scattering**

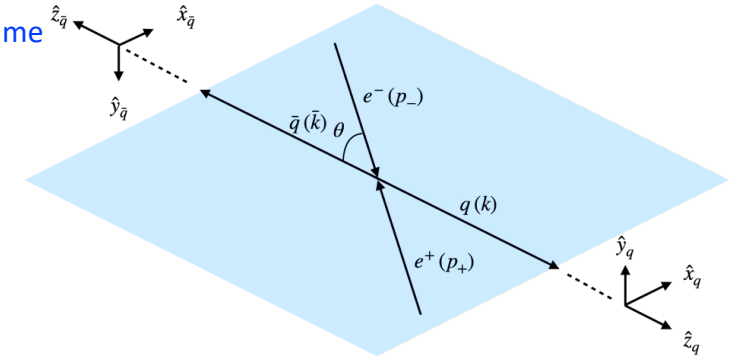
2. Joint spin density matrix
3. Hadron emission from $q$
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. Exit condition

[AK, X. Artru, PRD 109 (2024) 5, 054029]

Set up the scattering $e^+e^- \to q\bar{q}$ in the c.m.s

generate the quark flavors and kinematics using differential cross section

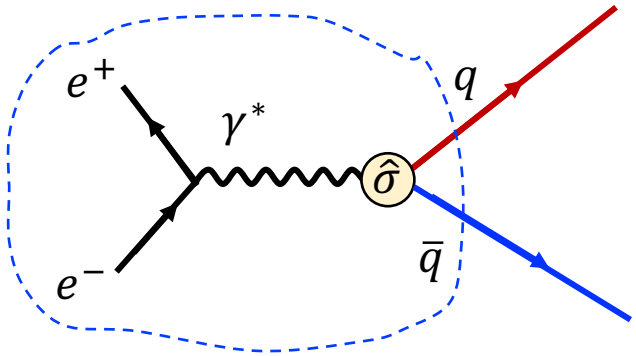
antiquark helicity frame (AHF)




quark helicity frame (QHF)

# Recursive recipe for $e^+e^-$

Steps:
1. Hard scattering
2. **Joint spin density matrix**
3. Hadron emission from $q$
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. Exit condition




$\rho(q,\bar{q})$

[AK, X. Artru, PRD 109 (2024) 5, 054029]

- Set up the joint spin density matrix of the $q\bar{q}$ pair

$$\rho(q,\bar{q}) = C^{q\bar{q}}_{\alpha\beta} \; \sigma^{\alpha}_{q} \otimes \sigma^{\beta}_{\bar{q}}$$

$C^{q\bar{q}}_{\alpha\beta}$: correlation coefficients

$\sigma^{\alpha}_{q} \otimes \sigma^{\beta}_{\bar{q}}$: Pauli matrices along QHF and AHF

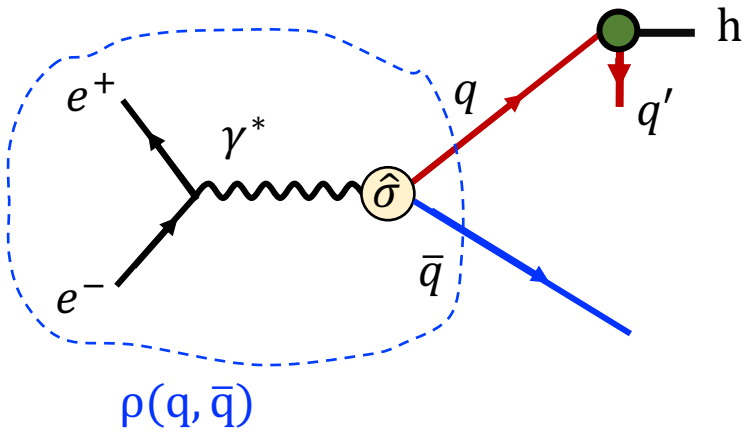
$\alpha = 0, x_q, y_q, z_q$

$\beta = 0, x_{\bar{q}}, y_{\bar{q}}, z_{\bar{q}}$

For $\gamma^*$ exchange

$$\rho(q,\bar{q}) \propto 1_q \otimes 1_{\bar{q}} - \sigma^z_q \otimes \sigma^z_{\bar{q}} + \frac{\sin^2\theta}{1+\cos^2\theta}\left[\sigma^x_q \otimes \sigma^x_{\bar{q}} + \sigma^y_q \otimes \sigma^y_{\bar{q}}\right]$$

# Recursive recipe for $e^+e^-$




Steps:
1. Hard scattering
2. Joint spin density matrix
3. **Hadron emission from $q$**
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. Exit condition

[AK, X. Artru, PRD 109 (2024) 5, 054029]

❑ Emit the first hadron using the splitting function
(emission probability density)

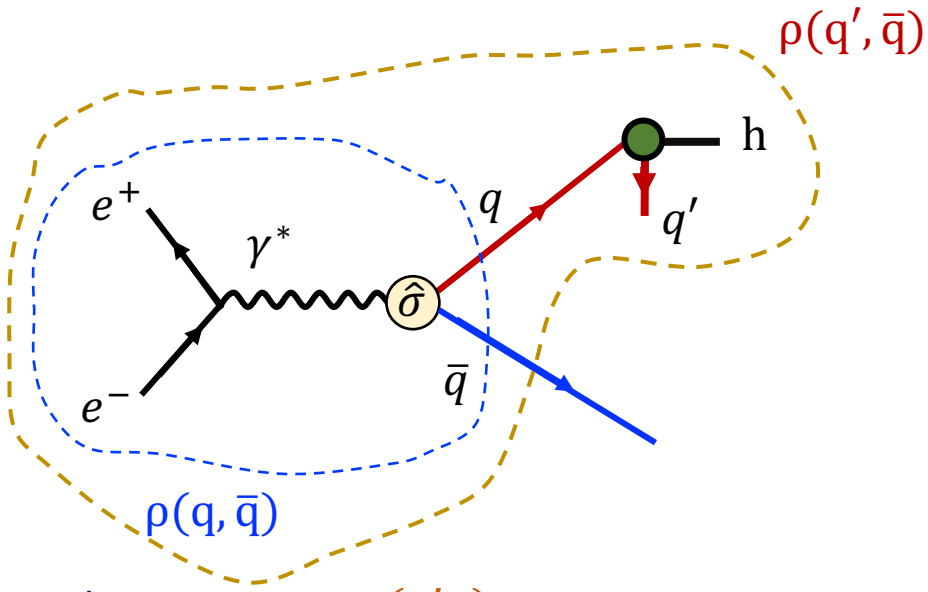
$$\frac{dP(q \to h + q'; q\bar{q})}{dZ_+ Z_+^{-1} d^2p_T} = Tr_{q'\bar{q}} \mathbf{T}_{q',h,q}\, \rho(q,\bar{q})\, \mathbf{T}^{\dagger}_{q',h,q} = F_{q',h,q}(Z_+, \mathbf{p}_T; \mathbf{k}_T, C^{q\bar{q}})$$

$$\mathbf{T}_{q',h,q} \equiv T_{q',h,q} \otimes 1_{\bar{q}}$$

in the QHF

❑ VM emission → backup

# Recursive recipe for $e^+e^-$




Steps:
1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from $q$
4. **Update density matrix**
5. Hadron emission from $\bar{q}$
6. Exit condition

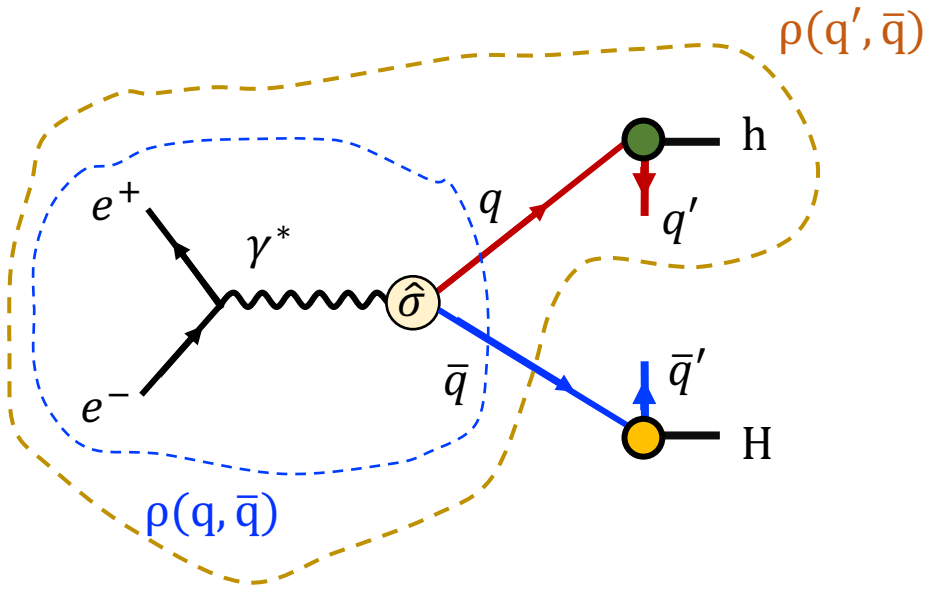
[AK, X. Artru, PRD 109 (2024) 5, 054029]

- Evaluate the spin density matrix $\rho(q'\bar{q})$

$$\rho(q', \bar{q}) = \mathbf{T}_{q',h,q} \;\; \rho(q, \bar{q}) \;\; \mathbf{T}^{\dagger}_{q',h,q}$$

includes the information on the emission of h

# Recursive recipe for $e^+e^-$




Steps:
1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from $q$
4. Update density matrix
5. **Hadron emission from $\bar{q}$**
6. Exit condition

[AK, X. Artru, PRD 109 (2024) 5, 054029]

- Emit a hadron from the $\bar{q}$ side using the splitting function

Depend on the azimuthal angle h

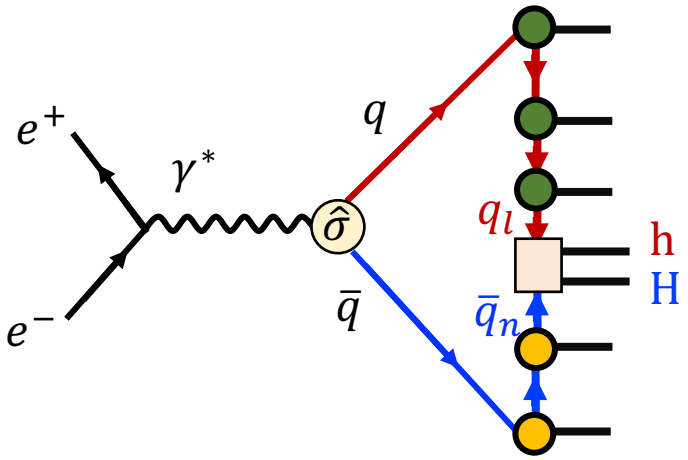
$$\frac{dP(\bar{q} \to H + \bar{q}'; q'\bar{q})}{dZ\_Z\_^{-1} d^2P_T} = \mathrm{Tr}_{q'\bar{q}'} \mathbf{T}_{\bar{q}',H,\bar{q}}\, \rho(q', \bar{q})\, \mathbf{T}^{\dagger}_{\bar{q}',H,\bar{q}} = F_{\bar{q}',H,\bar{q}}\left(Z\_, P_T; \bar{k}_T, C^{q'\bar{q}}\right)$$

Expressed in the AHF

conditional probability of emitting H, having emitted h
→ correlations between the transverse momenta

[Collins NPB, 304:794–804, 1988, Knowles NPB, 310:571–588, 1988]

# Recursive recipe for $e^+e^-$




Steps:
1. Hard scattering
2. Joint spin density matrix
3. Hadron emission from $q$
4. Update density matrix
5. Hadron emission from $\bar{q}$
6. **Exit condition**

[AK, X. Artru, PRD 109 (2024) 5, 054029]

- Iterate until the exit condition is called and the last quark pair is hadronized
  more details in PRD 109 (2024) 5, 054029

# Simulations of $e^+e^-$ with spin effects

- Now possible in Pythia 8.3
  StringSpinner package extended

- Photon exchange, only uds quarks produced, $\sqrt{s} = 10.6\ \mathrm{GeV}$
  consistent with BELLE and BABAR data

# Simulations of $e^+e^-$ with spin effects

- Now possible in Pythia 8.3
  StringSpinner package extended

- Photon exchange, only uds quarks produced, $\sqrt{s} = 10.6$ GeV
  consistent with BELLE and BABAR data

- Free parameters
  spin-less hadronization — as in standard Pythia 8.3
  spin-dependent hadronization
    complex mass $\mu$ — as in AK, Lonnblad, CPC 292 (2023) 108886
    $f_L = 0.12$ — ~ T pol. VMs
    $\theta_{LT} = -0.65$ — interference between T and L pol. of VMs

  found to give a satisfactory agreement with $e^+e^-$ data,
  ok also for SIDIS

- Comparison with Collins asymmetries from BELLE and BABAR
  AK, L. Lönnblad, A. Martin, in preparation

# The $A_{12}$ asymmetry

- Reference plane formed by the thrust axis $\hat{\mathbf{n}}$ and the beam e-
  back-to-back $h_1$ and $h_2$

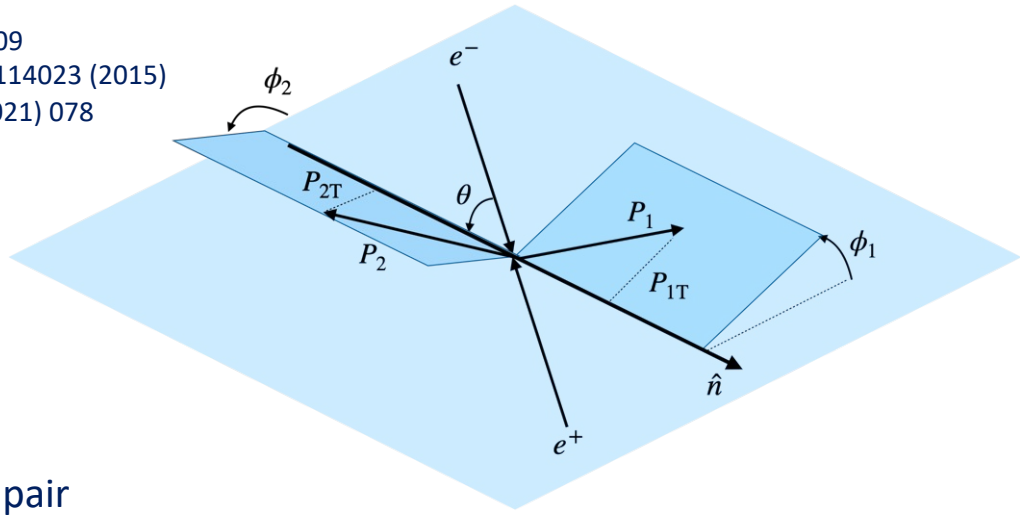
$$N_{h_1h_2} \propto 1 + \frac{\langle \sin^2\theta \rangle}{\langle 1+\cos^2\theta \rangle} A_{12} \cos(\phi_1 + \phi_2)$$

Boer, NPB, 806:23–67, 2009
Anselmino et al., PRD 92, 114023 (2015)
D'Alesio et al., JHEP 10 (2021) 078
..

Collins asymmetry

$$A_{12} = \frac{\sum_q e_q^2 \; H_{1q}^{\perp h_1} \; H_{1\bar{q}}^{\perp h_2}}{\sum_q e_q^2 \; D_{1q}^{h_1} \; D_{1\bar{q}}^{h_2}}$$

$$z_i = \frac{2E_{h_i}}{\sqrt{s}}$$




- Experimental asymmetry

$$R_{12}^{U,L,C} = N_{12}^{U,L,C} / \langle N_{12}^{U,L,C} \rangle$$

U = unlike sign pair
L = like sign pair
C = charged pair

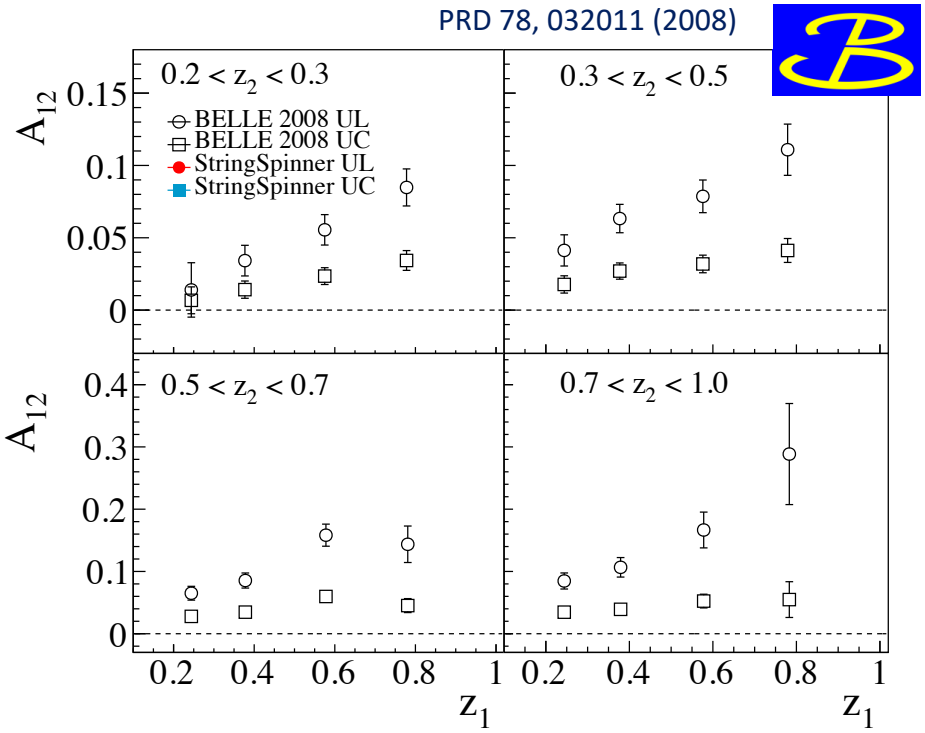
$$R_{12}^{UL(UC)} = \frac{R_{12}^{U}}{R_{12}^{L(C)}} \approx 1 + \frac{\langle \sin^2\theta \rangle}{\langle 1+\cos^2\theta \rangle} A_{12}^{UL(UC)} \cos(\phi_1 + \phi_2)$$

$$A_{12}^{UL(UC)} \simeq A_{12}^{U} - A_{12}^{L(C)}$$

- Measured by BELLE (2008, 2019) and BABAR (2014, 2015)

# $A_{12}$ asymmetry for charged π π pairs

PRD 78, 032011 (2008)



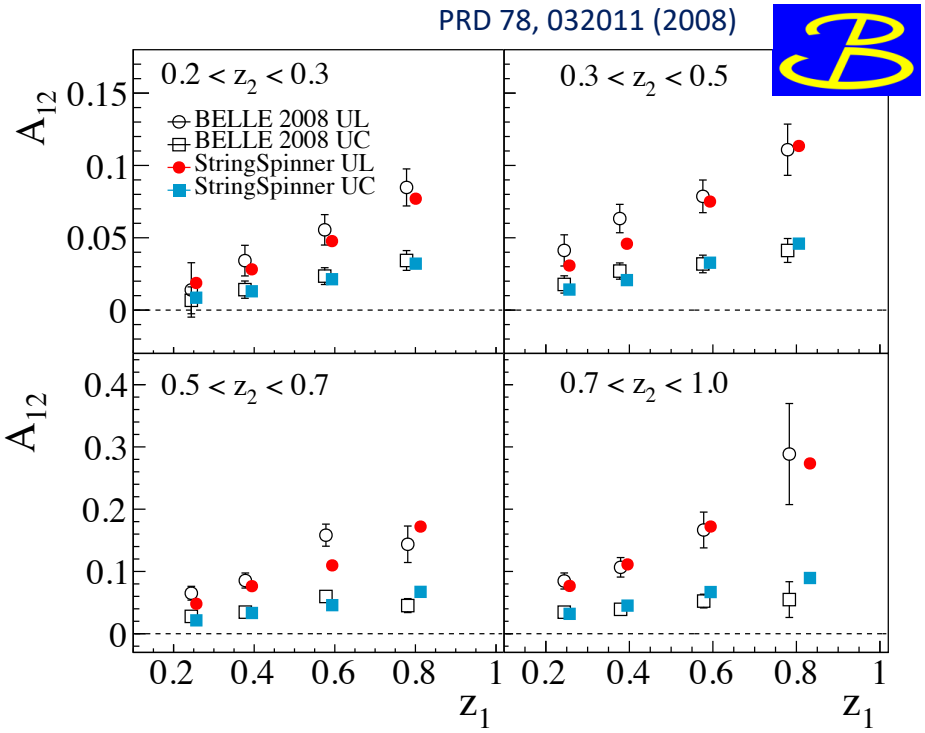

Belle asymmetries corrected for thrust smearing

Cuts:

$$T > 0.8, z > 0.2, Q_T < 3.5\ \text{GeV}$$

# $A_{12}$ asymmetry for charged π π pairs

PRD 78, 032011 (2008)




Belle asymmetries corrected for thrust smearing

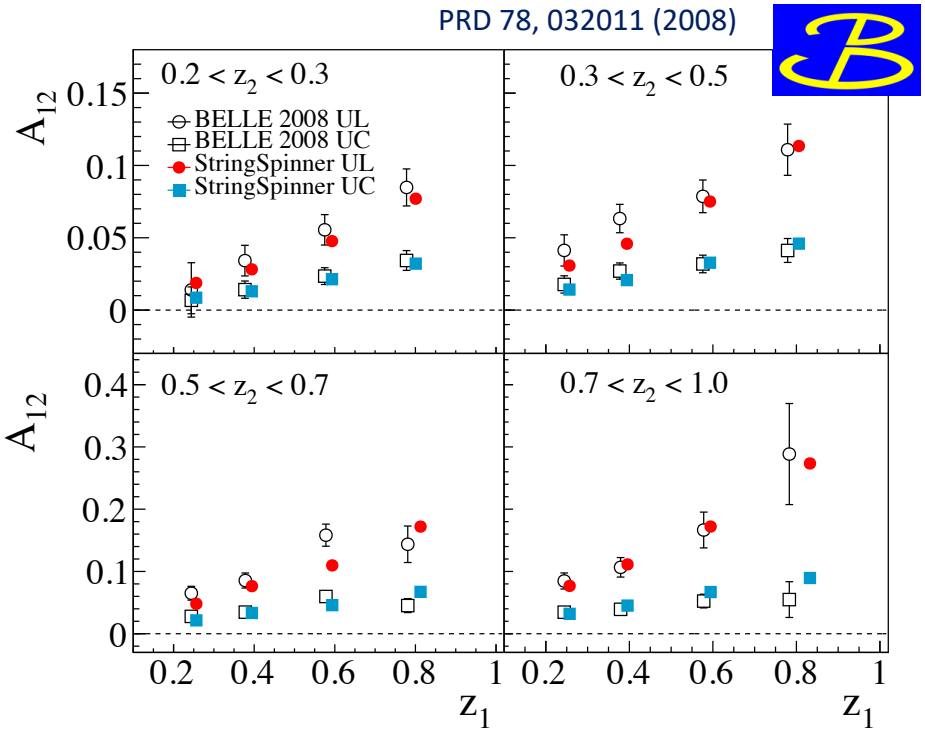
Cuts:

$$T > 0.8, z > 0.2, Q_T < 3.5\ \text{GeV}$$

StringSpinner reproduces trend and size

# $A_{12}$ asymmetry for charged π π pairs

PRD 78, 032011 (2008)

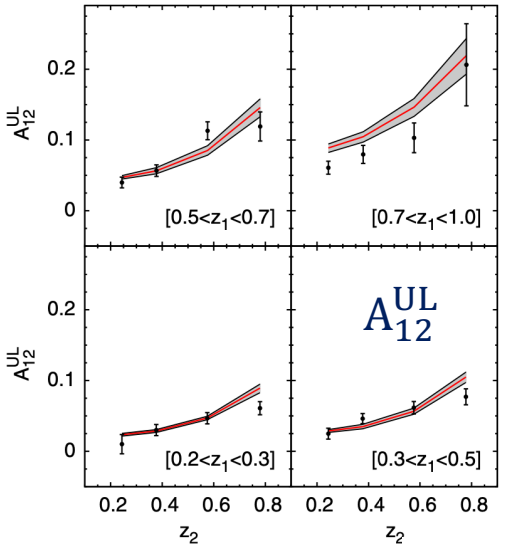



Belle asymmetries corrected for thrust smearing

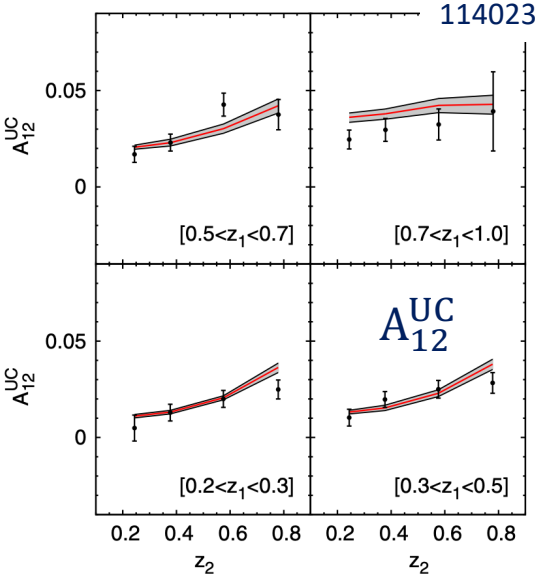
Cuts:

$$T > 0.8, z > 0.2, Q_T < 3.5\ \text{GeV}$$

StringSpinner reproduces trend and size

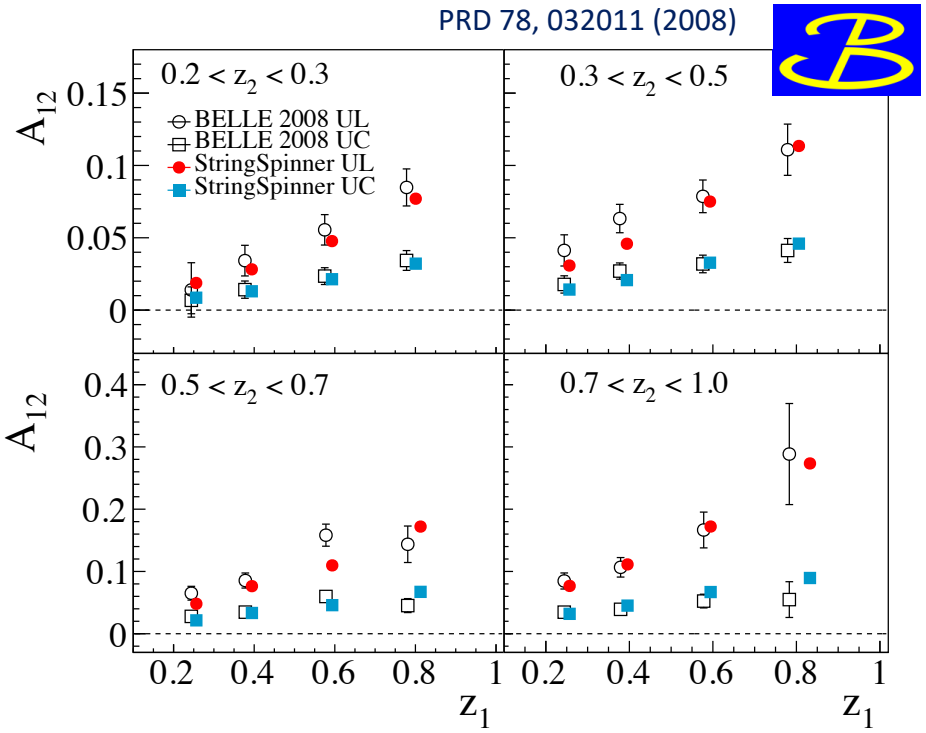


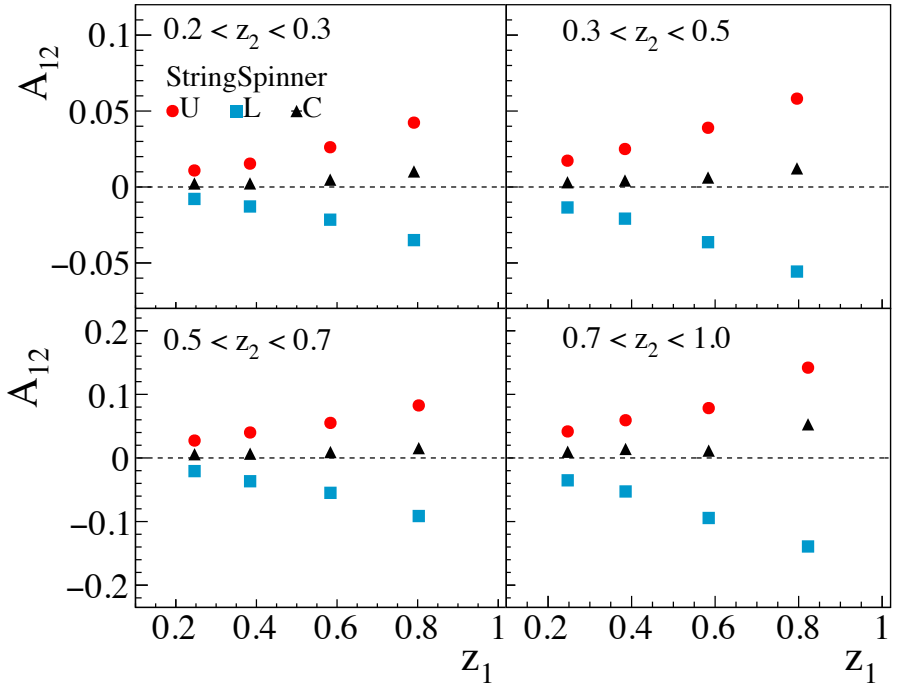

Anselmino et al., PRD 92, 114023 (2015)



# $A_{12}$ asymmetry for charged π π pairs

PRD 78, 032011 (2008)




Belle asymmetries corrected for thrust smearing

Cuts:

$$T > 0.8, z > 0.2, Q_T < 3.5\ \text{GeV}$$

StringSpinner reproduces trend and size

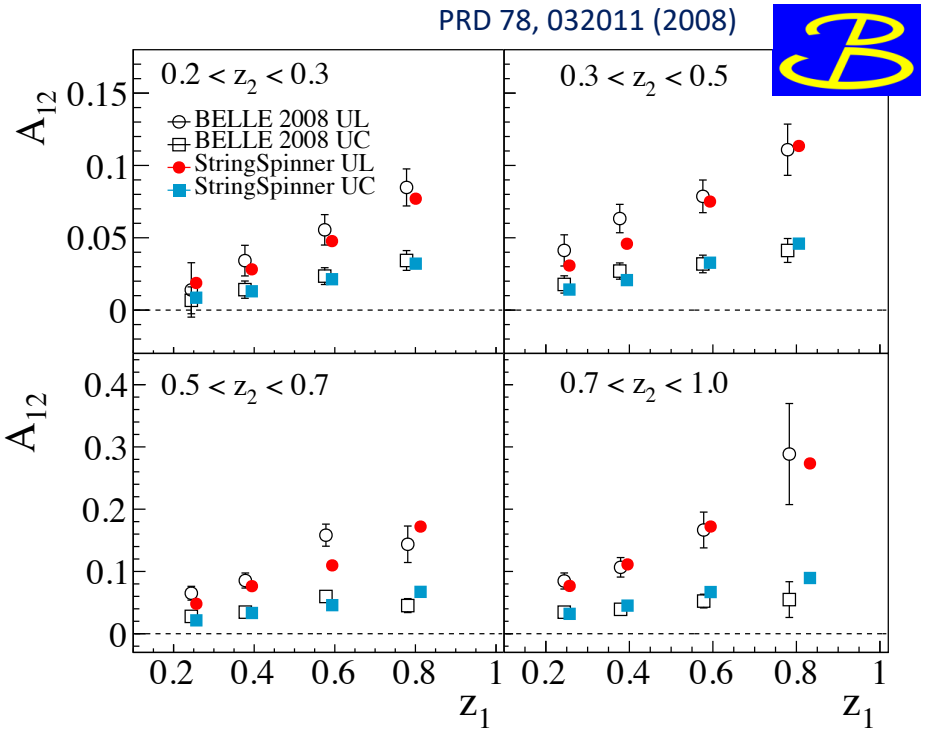
Opposite sign for $A_{12}^{U}$ and $A_{12}^{L}$, as expected

$A_{12}^{C} \simeq 0 \rightarrow$ explains $A_{12}^{UC} < A_{12}^{UL}$

$$A_{12}^{UL(UC)} \simeq A_{12}^{U} - A_{12}^{L(C)}$$

# $A_{12}$ asymmetry for charged π π pairs

PRD 78, 032011 (2008)

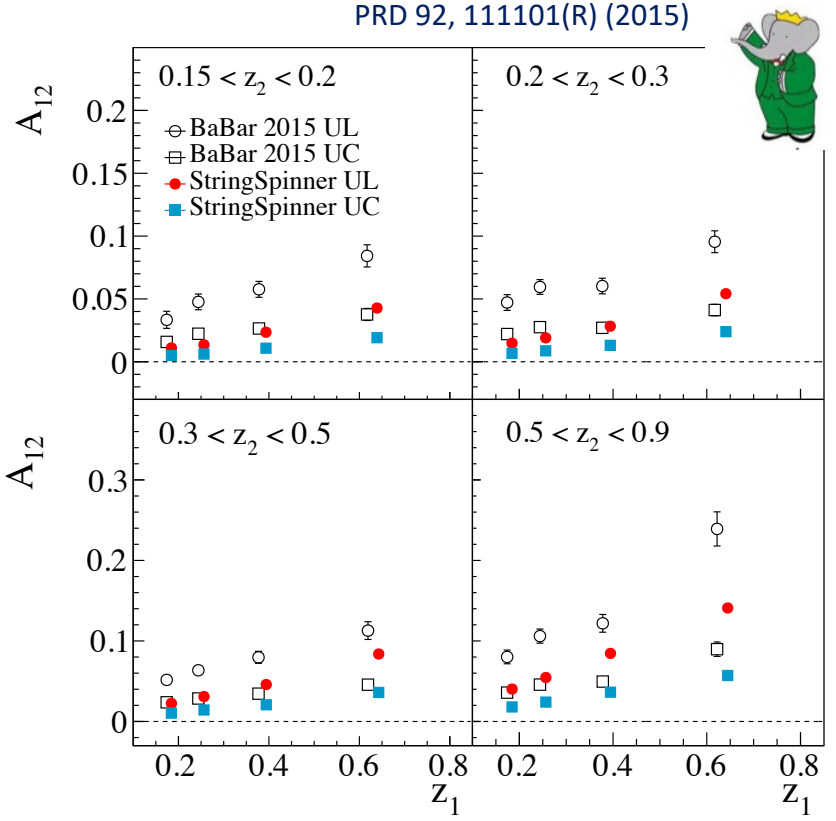



Belle asymmetries corrected for thrust smearing

Cuts:

$$T > 0.8, z > 0.2, Q_T < 3.5\ \text{GeV}$$

PRD 92, 111101(R) (2015)




BaBar asymmetries corrected for thrust smearing

Cuts:

$$T > 0.8, z > 0.15, Q_T < 3.5\ \text{GeV}, \alpha_o < \pi/4$$

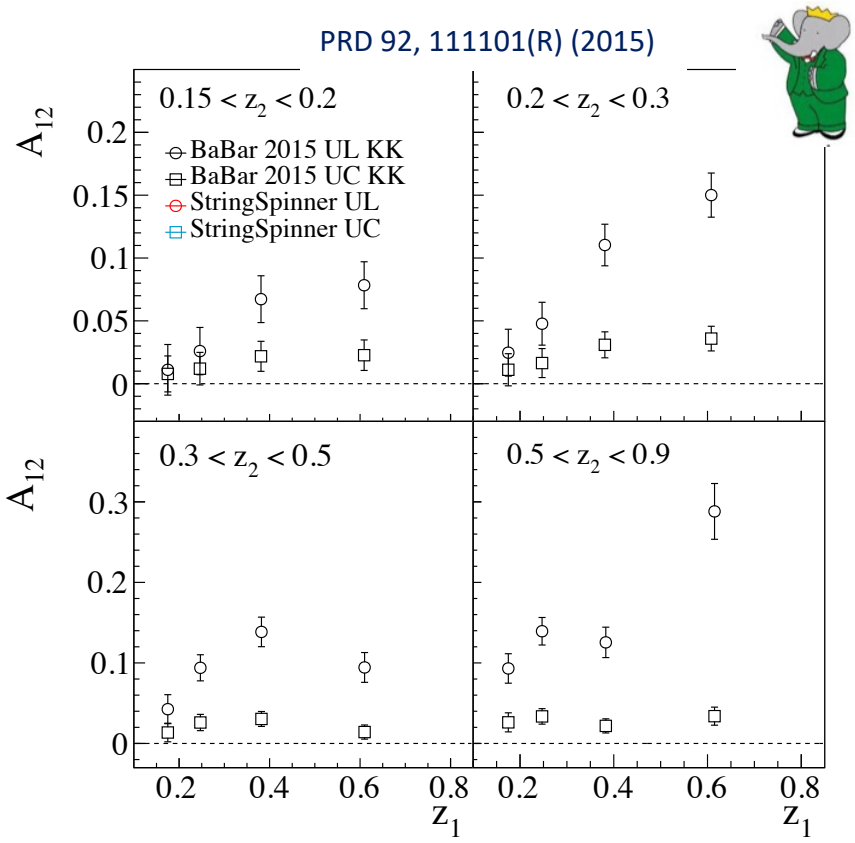
StringSpinner lower than BABAR

difference between BABAR and BELLE known

PRD 90, 052003 (2014)

# $A_{12}$ asymmetry for charged KK pairs

PRD 92, 111101(R) (2015)



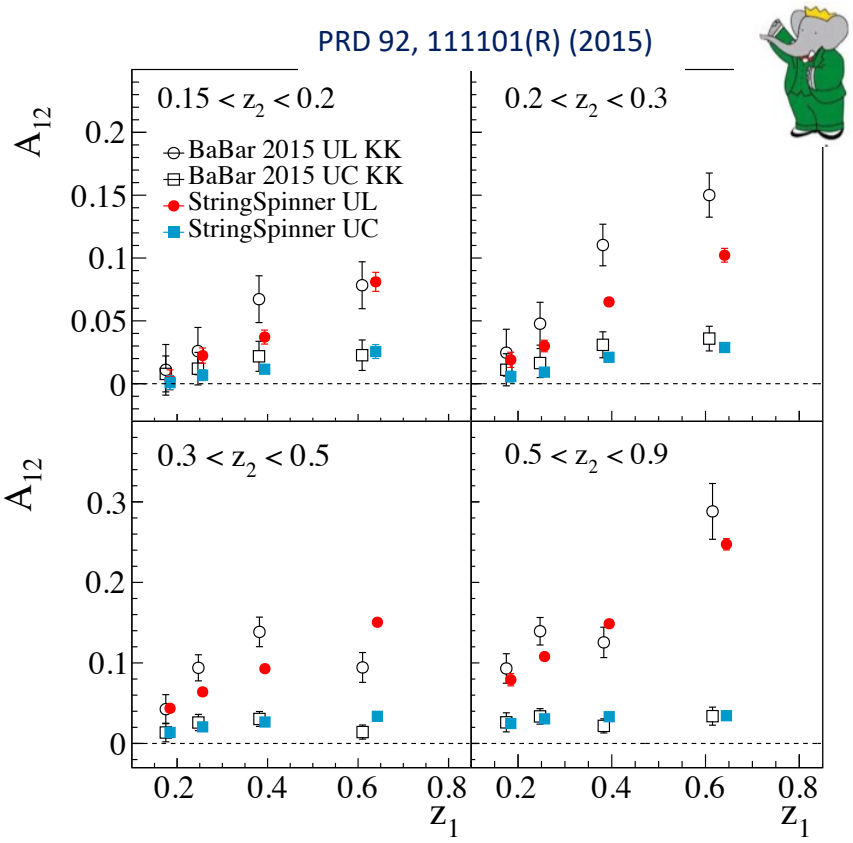

Corrected for thrust smearing

Cuts

$T > 0.8, z > 0.15, Q_T < 3.5\ \text{GeV}, \alpha_O < \pi/4$

# $A_{12}$ asymmetry for charged KK pairs

PRD 92, 111101(R) (2015)




Corrected for thrust smearing

Cuts

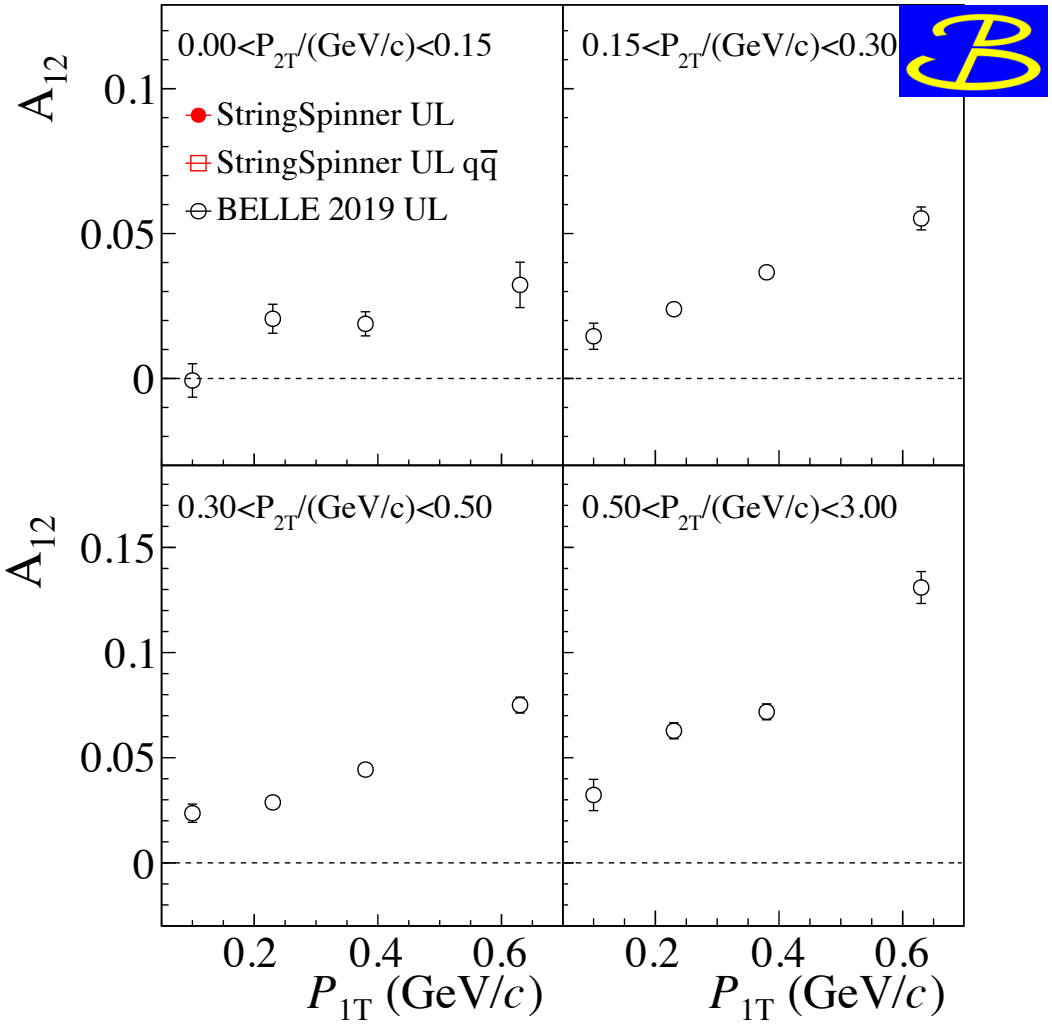
$T > 0.8, z > 0.15, Q_T < 3.5\ \text{GeV}, \alpha_O < \pi/4$

$A_{12}^{UC}$ much smaller than $A_{12}^{UL}$ at large z

reproduced by string+$^3P_0$

# $A_{12}^{UL}$ asymmetry for charged π π pairs

## $P_{T1} \times P_{T2}$ - dependence w.r.t thrust




PRD 100, 092008 (2019)

Asymmetries using thrust axis,
not corrected for thrust smearing

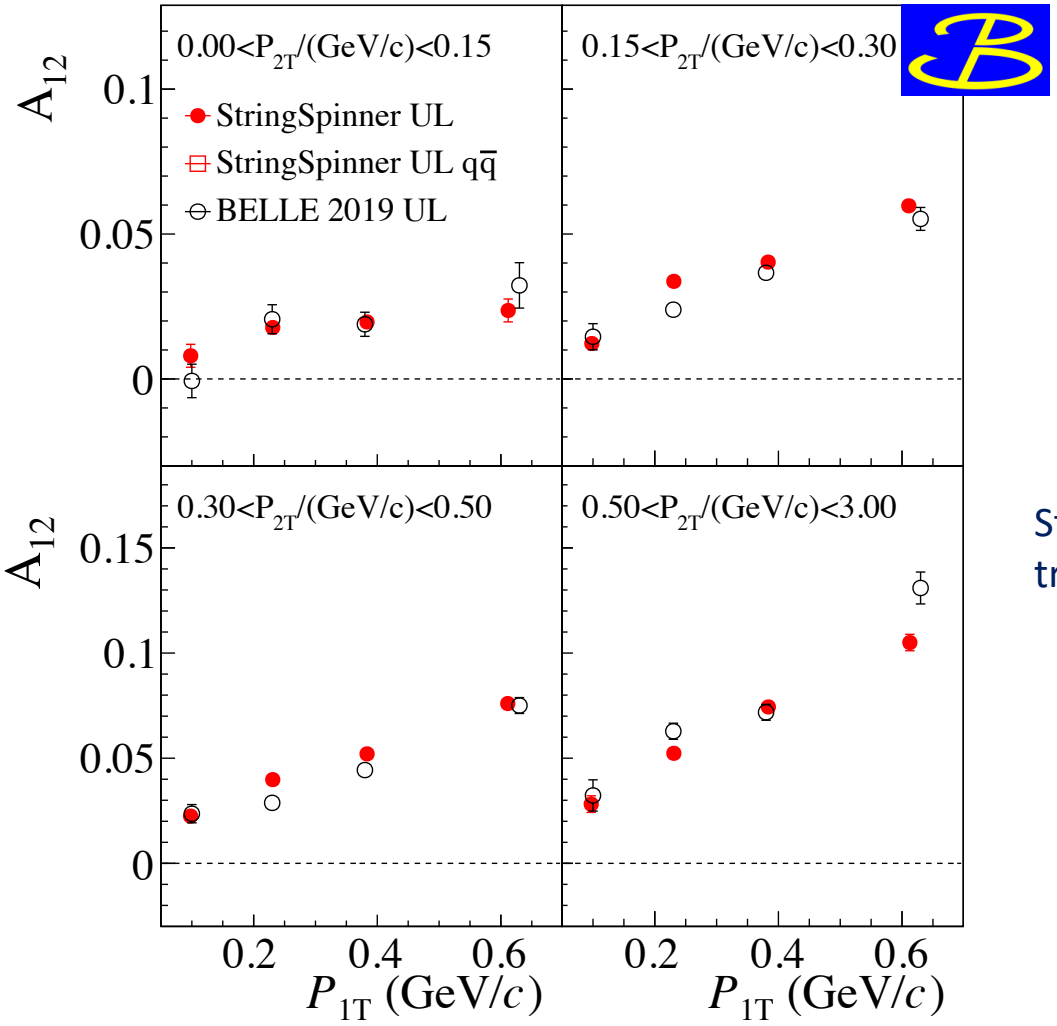
$T > 0.8$

$z > 0.2, P_T < 3.0$ GeV/c

$\alpha_O < 0.3$ rad

# $A_{12}^{UL}$ asymmetry for charged π π pairs

## $P_{T1} \times P_{T2}$ - dependence w.r.t thrust




PRD 100, 092008 (2019)

Asymmetries using thrust axis,
not corrected for thrust smearing

$T > 0.8$

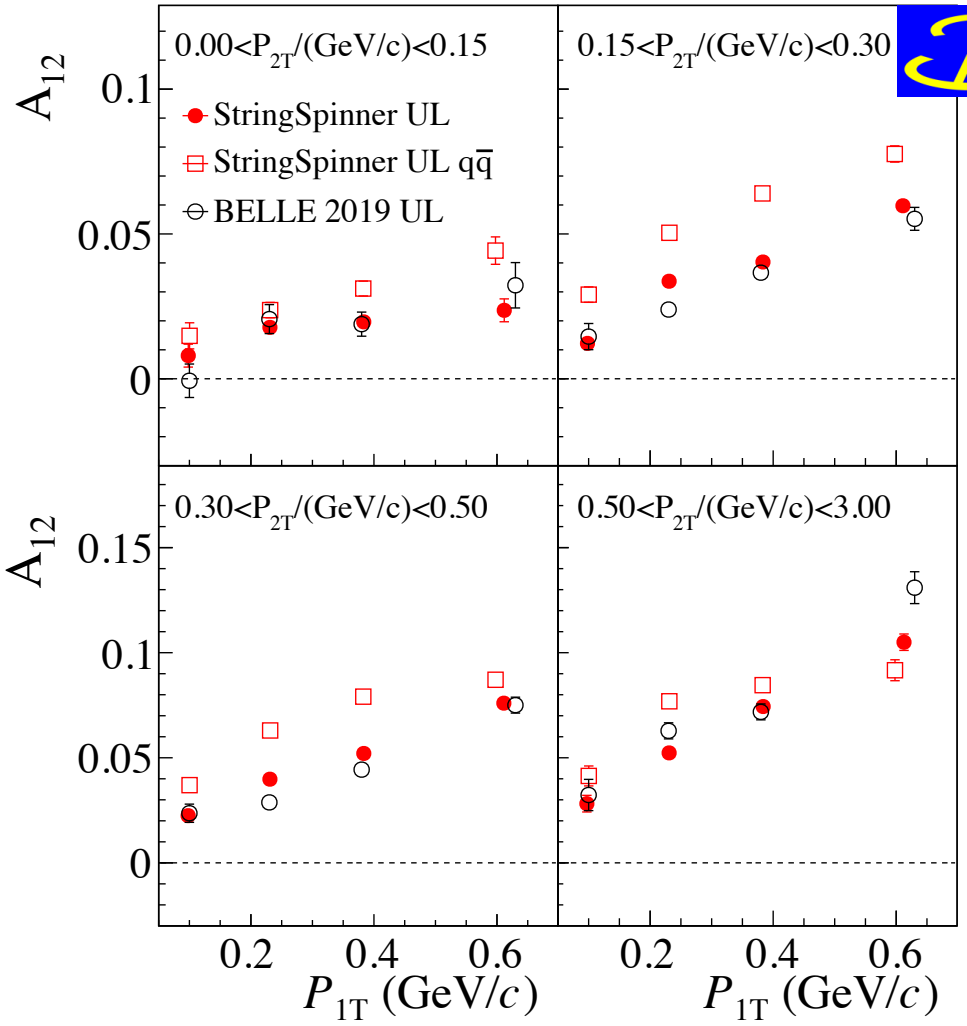
$z > 0.2, P_T < 3.0$ GeV/c

$\alpha_O < 0.3$ rad

StringSpinner reproduces the nearly linear trend observed by BELLE

# $A_{12}^{UL}$ asymmetry for charged π π pairs

## $P_{T1} \times P_{T2}$ - dependence w.r.t thrust




PRD 100, 092008 (2019)

Asymmetries using thrust axis,
not corrected for thrust smearing

$T > 0.8$

$z > 0.2, P_T < 3.0$ GeV/c

$\alpha_O < 0.3$ rad

StringSpinner reproduces the nearly linear trend observed by BELLE

Linear trend shows up as an effect of the misalignment between thrust and $q\bar{q}$ axis

strong effect by thrust

# The $A_0$ asymmetry

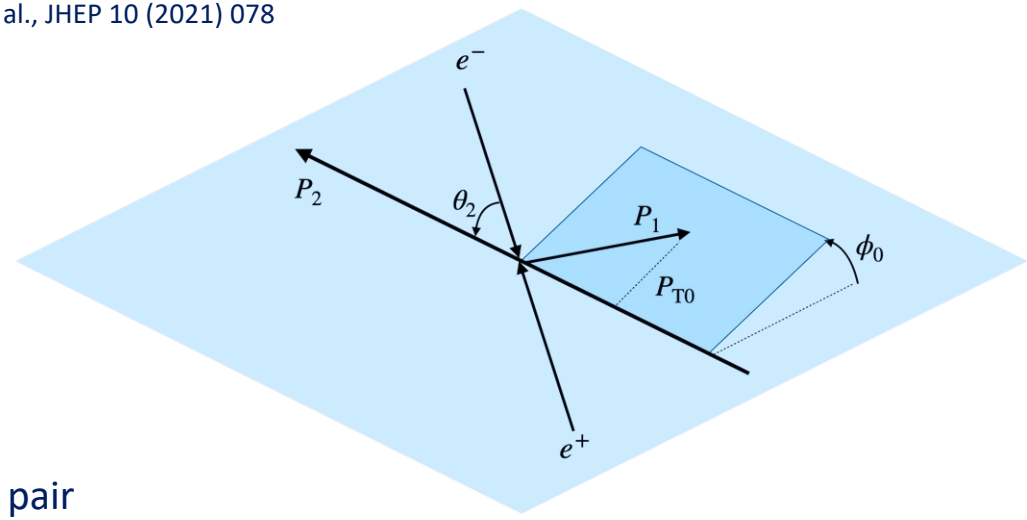
- Reference plane formed by the momentum $\mathbf{p_2}$ of $h_2$ and the beam e-

$$N_0 \propto 1 + \frac{\langle \sin^2 \theta_2 \rangle}{\langle 1+\cos^2 \theta_2 \rangle} A_0 \cos(2\phi_0)$$

Boer, NPB, 806:23–67, 2009
Anselmino et al., PRD 92, 114023 (2015)
D'Alesio et al., JHEP 10 (2021) 078
..

Collins asymmetry

$$A_0 = \frac{\sum_q e_q^2 \; w H_{1q}^{\perp h_1} \otimes H_{1\bar{q}}^{\perp h_2}}{\sum_q e_q^2 \; D_{1q}^{h_1} \otimes D_{1\bar{q}}^{h_2}}$$




- Experimental asymmetry

$$R_0^{U,L,C} = N_0^{U,L,C} / \langle N_0^{U,L,C} \rangle$$

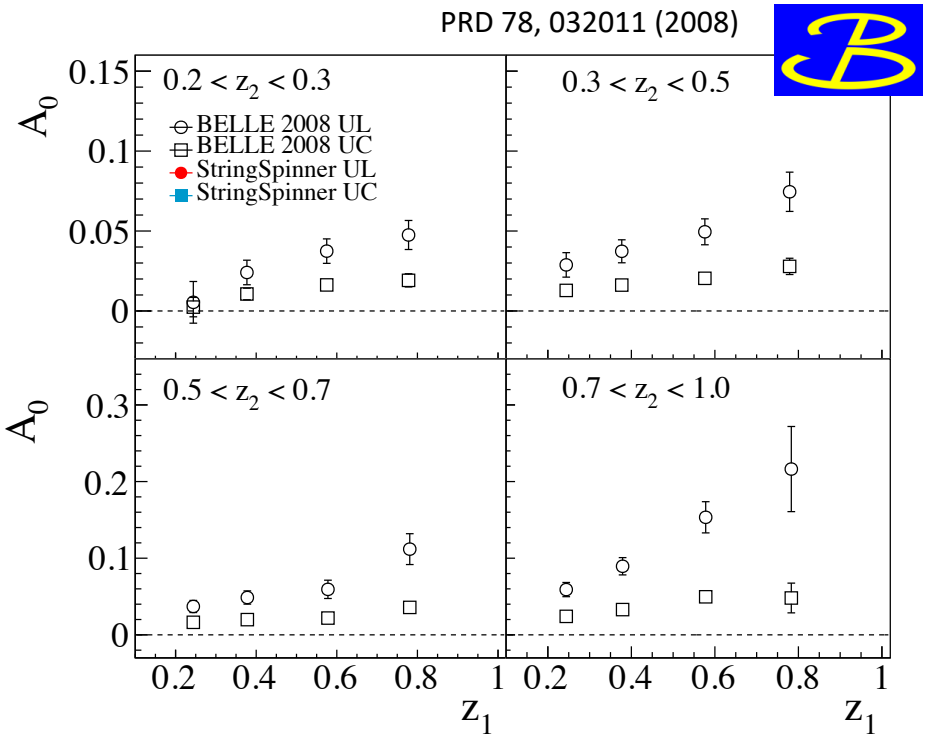
U = unlike sign pair
L = like sign pair
C = charged pair

$$R_0^{UL(UC)} = \frac{R_0^U}{R_0^{L(C)}} \approx 1 + \frac{\langle \sin^2 \theta_2 \rangle}{\langle 1 + \cos^2 \theta_2 \rangle} A_0^{UL(UC)} \cos(2\phi_0)$$

$$A_0^{UL(UC)} \simeq A_0^U - A_0^{L(C)}$$

- Measured by BELLE (2008) and BABAR (2014, 2015)

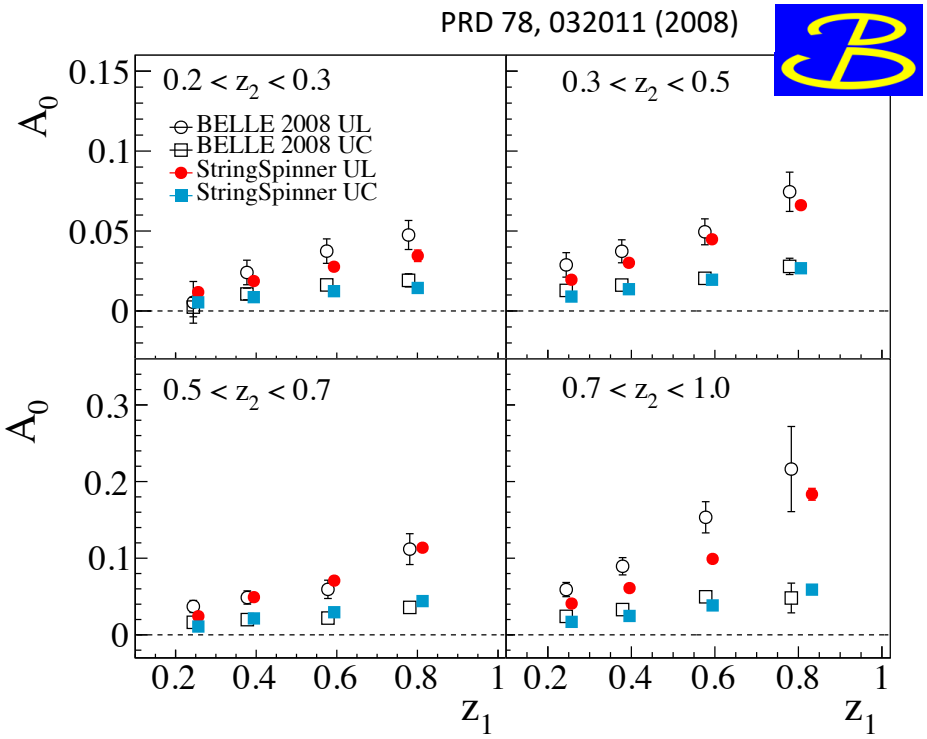
# $A_0$ asymmetry for charged pions




Cuts:

$$T > 0.8, z > 0.2, Q_T < 3.5\text{GeV}$$

# $A_0$ asymmetry for charged pions




Cuts:

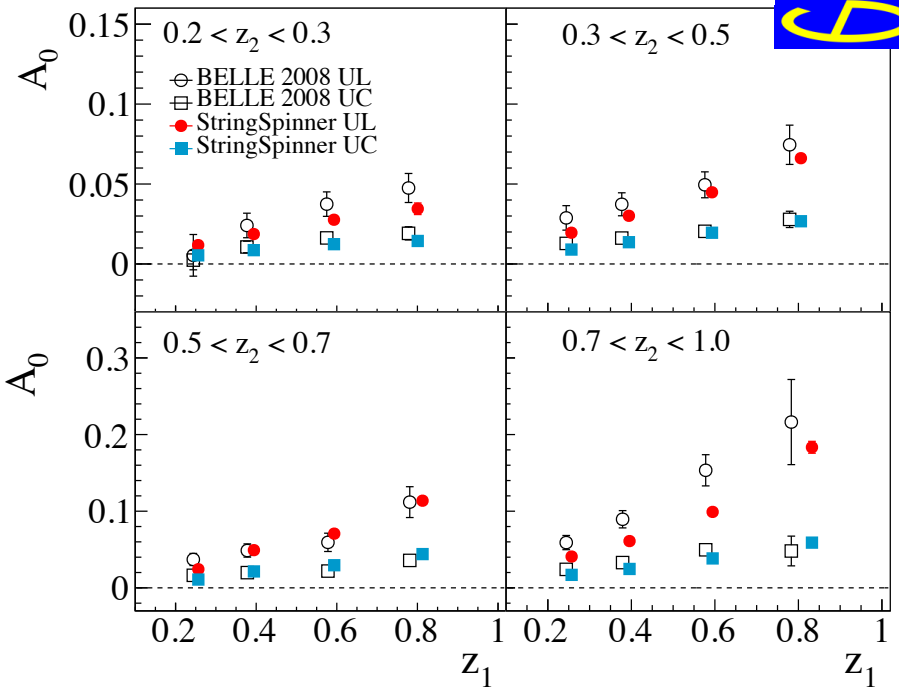
$$T > 0.8, z > 0.2, Q_T < 3.5\text{GeV}$$

Trend reproduced by string+$^3P_0$

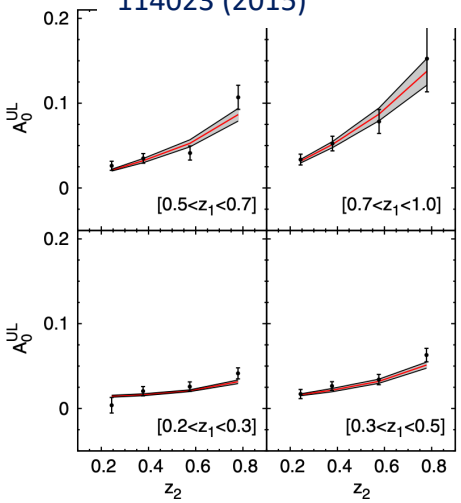
somewhat lower values in the last $z_2$ bin

# $A_0$ asymmetry for charged pions

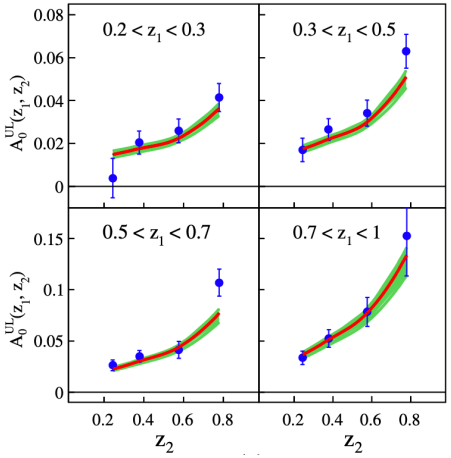
PRD 78, 032011 (2008)




Anselmino et al., PRD 92, 114023 (2015)




Kang et al., PRD 93, 014009 (2016)

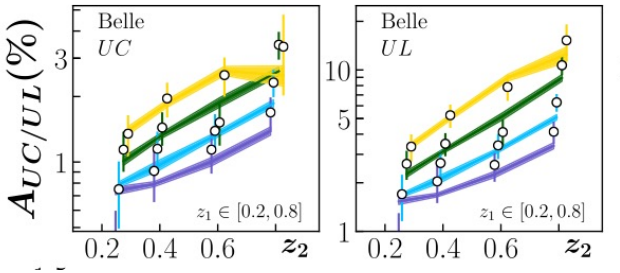



Cuts:

$$T > 0.8, z > 0.2, Q_T < 3.5\text{GeV}$$

Trend reproduced by string+$^3P_0$

somewhat lower values in the last $z_2$ bin

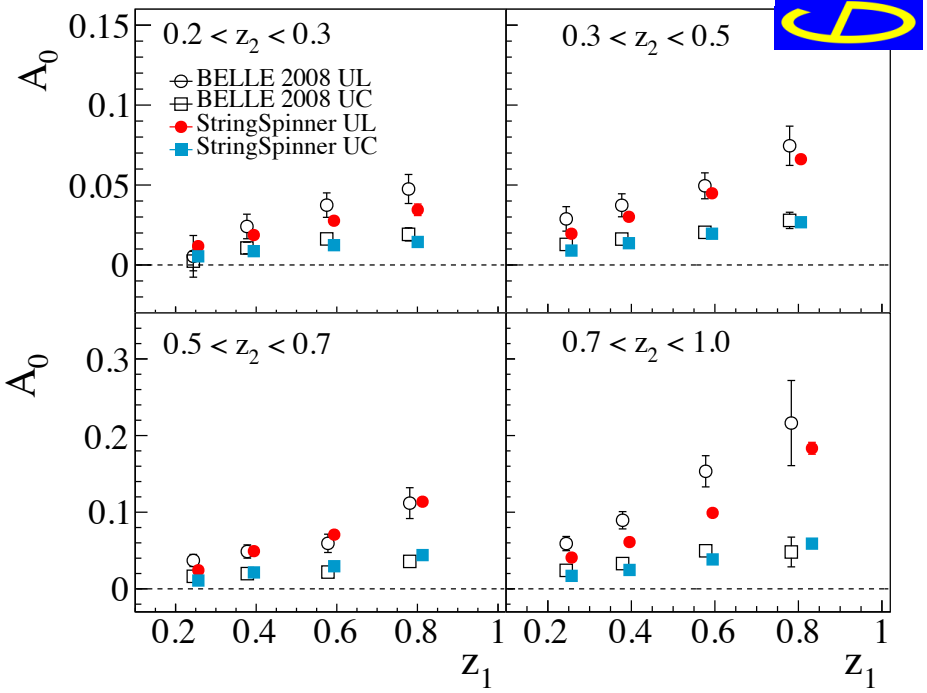



JAM, PRD 102, 054002 (2020)

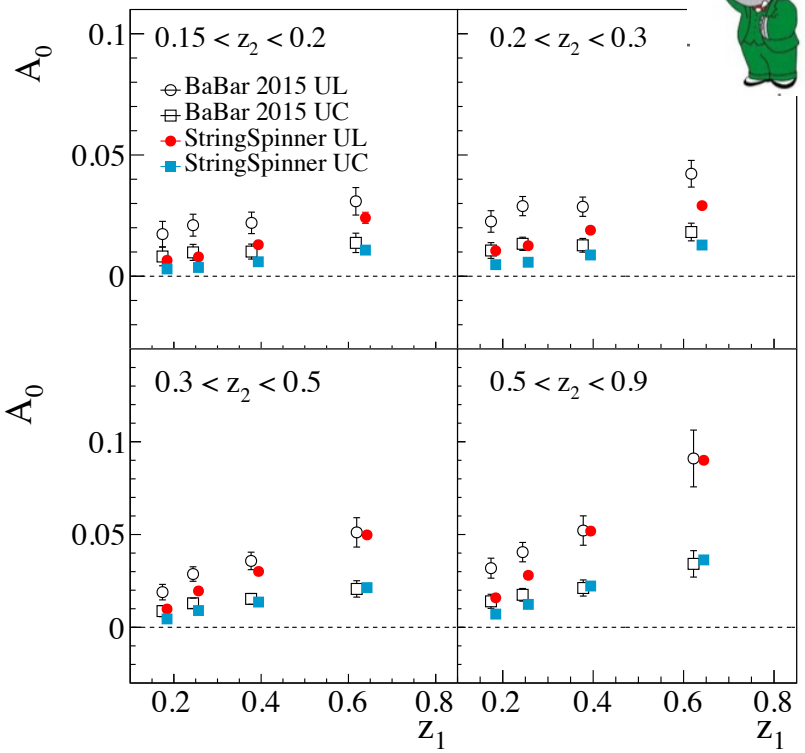
$A_0$ asymmetry included in phenomenological fits

# $A_0$ asymmetry for charged pions

PRD 78, 032011 (2008)




PRD 92, 111101(R) (2015)




Cuts:

$$T > 0.8, z > 0.2, Q_T < 3.5\text{GeV}$$

Trend reproduced by string+$^3P_0$

somewhat lower values in the last $z_2$ bin

BABAR asymmetries higher for $z_1$<0.3 $z_2$<0.3

# Conclusions

- The string+$^3P_0$ model applied to the fragmentation of a string stretched between quarks with correlated spin states
  - these kind of strings are produced also in pp collisions..

- Implemented in Pythia 8.3 for $e^+e^-$
  - extension of StringSpinner to be published

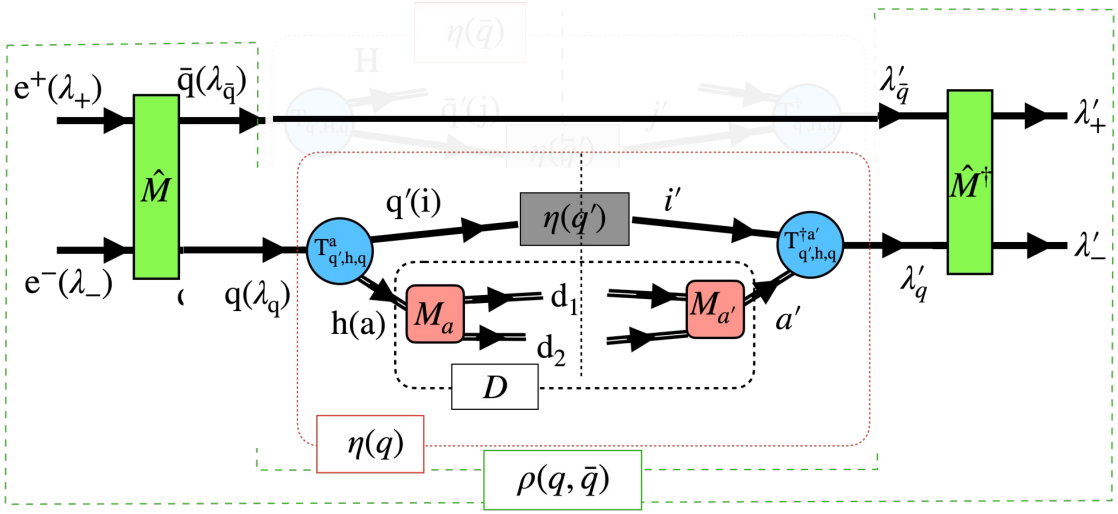
- Encouraging results on Collins asymmetries in $e^+e^-$
  - study of Artru-Collins asymmetries ongoing

- More developments of the string+$^3P_0$ model foreseen
  - baryon production
  - connection to parton shower
  - …

Still a long way to an event generator fully implementing spin effects, but several steps done!

# Backup

# The recursive recipe for simulating $e^+e^-$ annihilation: VM emission




For a vector meson h=VM

$$\rightarrow \eta(q) = \mathbf{T}^{a'\dagger}_{q',h=VM,q}\,\eta(q')\,\mathbf{T}^{a}_{q',h=VM,q} D_{a'a},\ \eta(q') = 1_{q'} \text{ and } \eta(\bar{q}) = 1_{\bar{q}}$$

Steps:

i) Emission probability density (summing over decay information, i.e. $D_{a'a} = \delta_{a'a}$)

$$\frac{dP(q \to h = VM + q'; q\bar{q})}{dM^2 dZ_+ Z_+^{-1} d^2p_T} = Tr_{q'\bar{q}}\,\mathbf{T}^{a}_{q',h,q}\ \rho(q,\bar{q})\ \mathbf{T}^{a\dagger}_{q',h,q} = F_{q',h,q}(M^2, Z_+, p_T; k_T, C^{q\bar{q}})$$

ii) Calculate the spin density matrix of h=VM, and decay the meson

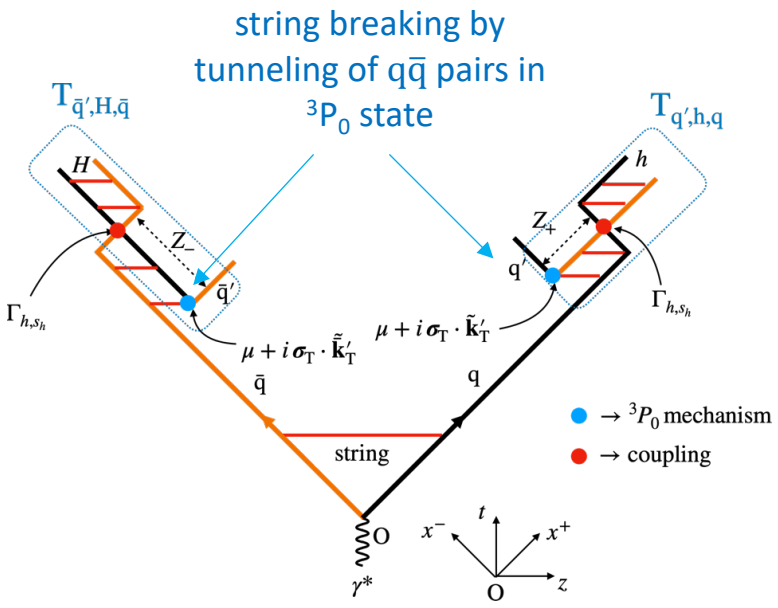
$$\rho_{aa'}(h) = Tr_{q'\bar{q}}\,\mathbf{T}^{a}_{q',h,q}\,\rho(q,\bar{q})\,\mathbf{T}^{a'\dagger}_{q',h,q}$$

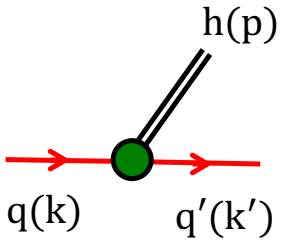
iii) Decay the meson $p \to p_1\, p_2..$

$$dN(p_1, p_2..)/d\Omega \propto M^{a}_{dec.}(p \to p_1 p_2..)\,\rho_{aa'}(h) M^{\dagger a'}_{dec.}(p \to p_1 p_2..)$$

iv) Build the decay matrix $D_{a'a}(p_1, p_2, ..) = M^{\dagger a'}_{dec.}(p \to p_1 p_2..)\, M^{a}_{dec.}(p \to p_1 p_2..)$

# The hadronization model: string+$^3P_0$




quark splitting $q \rightarrow h + q'$

Relevant variables:
$\mathbf{k}_T = \mathbf{p}_T + \mathbf{k}'_T$
$Z_+ = p^+/k^+$
$\varepsilon_h^2 = M^2 + p_T^2$

Transverse vectors defined w.r.t. string axis

## Quark splitting amplitude in the string+$^3P_0$ model

$$T_{q',h,q} \propto C_{q',h,q}\; D_h(M^2) \left(\frac{1-Z_+}{\varepsilon_h^2}\right)^{\frac{a}{2}} \exp\left[-\frac{b_L \varepsilon_h^2}{2Z_+}\right] N_a^{-\frac{1}{2}}(\varepsilon_h^2)\; e^{-\frac{b_T k'^2_T}{2}}$$

flavor — mass — longitudinal momentum — transverse momentum (w.r.t string axis)

$[\mu + \sigma_z \boldsymbol{\sigma}_T \cdot \mathbf{k}'_T]$ — $^3P_0$ mechanism [$\mu$ complex mass paramter]

$\Gamma_{h,s_h}$ — Coupling e.g. $\Gamma_{h=PS} = \sigma_z$

Free param. Lund
Free param. string+$^3P_0$

AK, Artru, Martin, PRD 104, 114038 (2021)

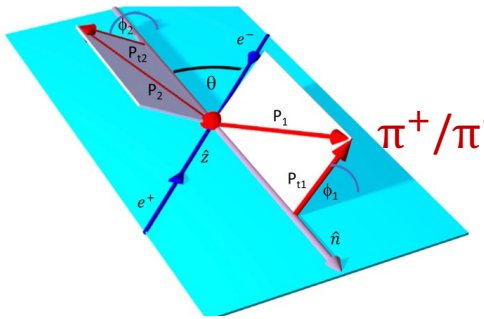
# Steps for the extraction of Collins asymmetries

## Example of $e^+e^- \to \pi^+\pi^- X$

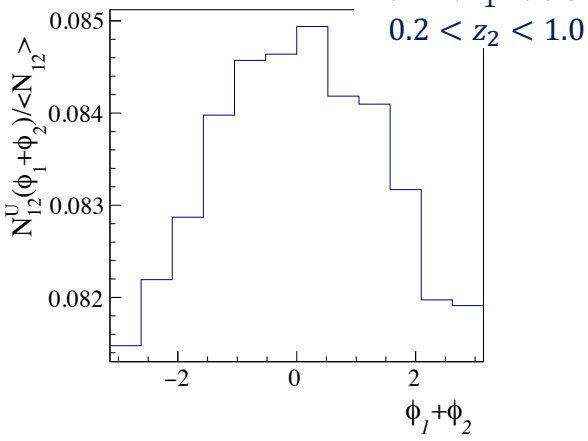
i) Evaluate normalized yields for $\pi^\pm - \pi^\mp$ "Unlike pairs"

$$R_{12}^{U} = \frac{N_{12}^{U}(\phi_1 + \phi_2)}{\langle N_{12} \rangle}$$

$\pi^-/\pi^+$



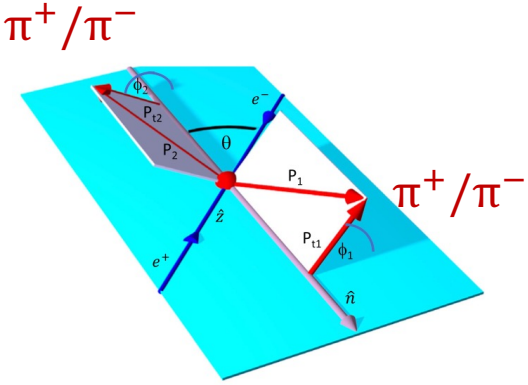

$0.4 < z_1 < 0.5$
$0.2 < z_2 < 1.0$

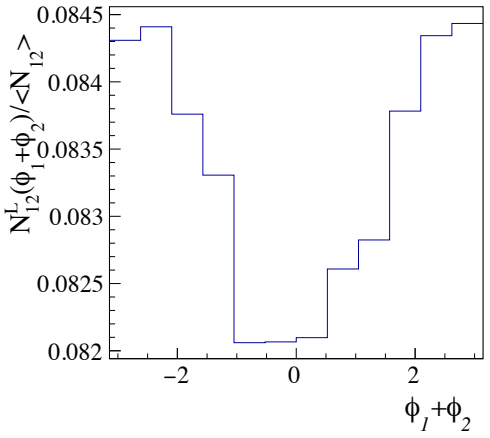



ii) Evaluate normalized yields for $\pi^\pm - \pi^\pm$ "Like pairs"

$$R_{12}^{L} = \frac{N_{12}^{L}(\phi_1 + \phi_2)}{\langle N_{12} \rangle}$$

$\pi^+/\pi^-$



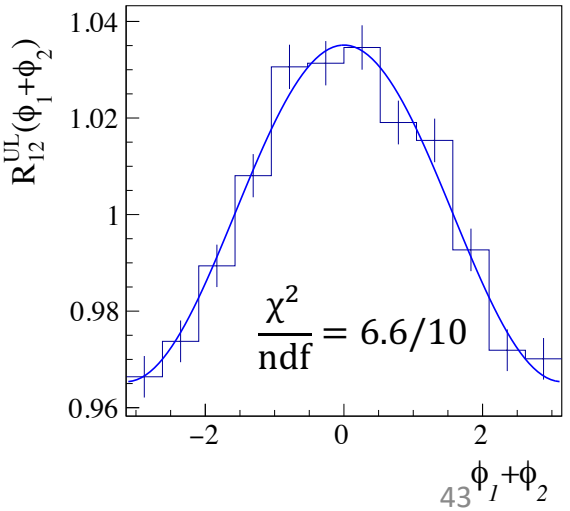




ii) Evaluate the ratio $\frac{R_{12}^{U}}{R_{12}^{L}}$ and fit the asymmetry

$$R_{12}^{UL} = \frac{R_{12}^{U}}{R_{12}^{L}} \approx 1 + A_{12}^{UL}\cos(\phi_1 + \phi_2)$$

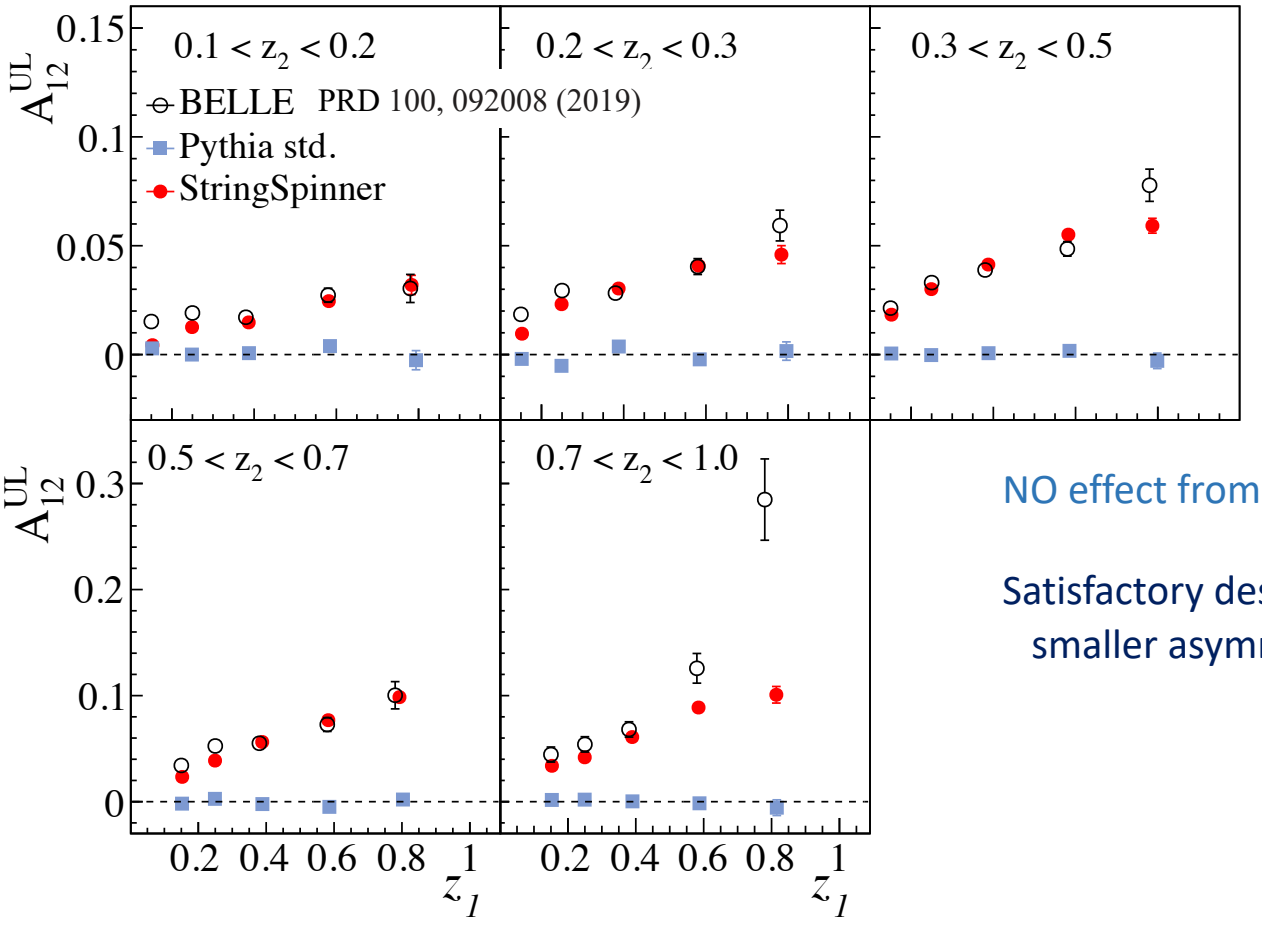
Fit function

$$f(\phi_1 + \phi_2) = p_0 + p_1\cos(\phi_1 + \phi_2)$$



# $A_{12}^{UL}$ asymmetry for back-to-back $\pi^{\pm} - \pi^{\mp}$

## $z_1 \times z_2$ - dependence




Asymmetries w.r.t thrust axis (not $q\bar{q}$ axis)

$T > 0.8$

$z > 0.1, P_T < 3.0$ GeV/c

$\alpha_O < 0.3$

NO effect from Pythia

Satisfactory description from StringSpinner

smaller asymmetries at large z