Strange-Meson Spectroscopy with COMPASS

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The Strange-Meson Spectrum

![Graph showing the strange-meson spectrum with mass on the y-axis and parity on the x-axis.]

PDG lists 25 strange mesons

- 16 established states, 9 need further confirmation
- Missing states with respect to quark-model predictions
- Many measurements performed more than 30 years ago
CEDARs
* beam PID

Beam
* 191 GeV
* 2.4 % K⁻

RICH
* final-state PID

H₂ Target
RPD

30 m CEDARs

Strange-Meson Spectroscopy with COMPASS
COMPASS Setup for Hadron Beams
Diffractive scattering of high-energy kaon beam
Strange mesons appear as intermediate resonances $X^-$
Decay to multi-body hadronic final states
$K^-\pi^-\pi^+$ final state
Study in principle all strange mesons
Study a wide mass range
COMPASS measured world's largest data set of about 720 k events
Diffractive scattering of high-energy kaon beam
- Strange mesons appear as intermediate resonances $X^-$
- Decay to multi-body hadronic final states
- $K^- \pi^- \pi^+$ final state
  - Study in principle all strange mesons
  - Study a wide mass range
  - COMPASS measured world’s largest data set of about 720 k events
Partial-Wave Analysis of the $K^-\pi^-\pi^+$ Final State

Partial wave: $J^P M^\varepsilon \xi b^- L$

- $J^P$ spin and parity
- $M^\varepsilon$ spin projection
- $\xi$ isobar resonance
- $b^-$ bachelor particle
- $L$ orbital angular momentum
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Data: 720 k diffractively produced $K^-\pi^-\pi^+$ candidates
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**Data:** 720 k diffractively produced $K^-\pi^-\pi^+$ candidates

(I) **Partial-Wave Decomposition**
Performed independently in narrow $(m_{K\pi\pi}, t')$ cells
No assumption about $K\pi\pi$ resonances

**Partial waves:** Intensities and relative phases as a function of $(m_{K\pi\pi}, t')$
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**Data:** 720 k diffractively produced $K^-\pi^-\pi^+$ candidates

1. **Partial-Wave Decomposition**
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   - No assumption about $K\pi\pi$ resonances

2. **Partial waves:** Intensities and relative phases as a function of $(m_{K\pi\pi}, t')$

3. **Resonance-Model Fit**
   - Model $m_{K\pi\pi}$ dependence of partial waves
   - $K\pi\pi$ resonances and background

4. **Resonance parameters:** Masses and widths of the strange-meson resonances
Partial-Wave Analysis of the $K^-\pi^-\pi^+$ Final State

Partial-Wave Decomposition

$$I(\tau, m_{K\pi\pi}, t') = \sum_{a, b \in \mathbb{W}_z(m_{K\pi\pi}, t')} \Psi_a(\tau) \rho_{ab}(m_{K\pi\pi}, t') [\Psi_b(\tau)]^*$$

- Measure spin-density matrix $\rho_{ab}(m_{K\pi\pi}, t')$ in independently $(m_{K\pi\pi}, t')$ cells
  - No assumption about $K^-\pi^-\pi^+$ resonances
- Wave set $\mathbb{W}_z(m_{K\pi\pi}, t')$ inferred from data using regularization-based model-selection techniques
- Bootstrap resampling to improve uncertainty estimates
  - Performed about 20 M fits
- Detailed Monte Carlo input-output studies

Preliminary
Partial-Wave Analysis of the $K^-\pi^-\pi^+$ Final State

Resonance-Model Fit

\[ \hat{\rho}_{ab}^{K\pi\pi}(m_{K\pi\pi}, t') = \hat{T}_a(m_{K\pi\pi}, t') \left[ \hat{T}_b(m_{K\pi\pi}, t') \right]^* \]

\[ \hat{T}_a(m_{K\pi\pi}, t') = \sum_{k \in S_a} K(m_{K\pi\pi}, t')^k C_a(t') D_k(m_{K\pi\pi}; \zeta_k) \]

- Model $m_{K\pi\pi}$ dependence of partial-wave amplitudes
- Breit-Wigner amplitudes for $K^-\pi^-\pi^+$ resonance components
- Coherent non-resonant component parameterizing other $K^-\pi^-\pi^+$ production mechanisms

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Strange-Meson Spectroscopy with COMPASS

\[ 0.10 \leq t' < 1.00 \, (\text{GeV/c})^2 \]

COMPASS

Intensity $[\text{(GeV/c}^2)^{-1}]$
Partial-Wave Analysis of the $K^-\pi^-\pi^+$ Final State

Resonance-Model Fit

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Incoherent Backgrounds

- **Incoherent background** from $\pi^-$ diffraction to $\pi^-\pi^-\pi^+$ and other reactions (in total about 10%)
- Very good model for dominant $\pi^-\pi^-\pi^+$ background from COMPASS $\pi^-\pi^-\pi^+$ analysis
  - Study background in partial waves by
    - Generate pseudodata from $\pi^-\pi^-\pi^+$ model
    - Apply $K^-\pi^-\pi^+$ reconstruction event selection
    - Project into $K^-\pi^-\pi^+$ partial waves
  - Large in some waves, e.g. with $\rho(770)$ isobar
  - Small in other waves, e.g. with $K^*(892)$ isobar
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Handling of Incoherent Backgrounds

- Challenging to explicitly treat in partial-wave decomposition
  - Effectively taken into account
    \[ \rho_{ab} = \sum_z T_a^z [\bar{T}_b^z]^\ast \]
  - Measured \( \rho_{ab} \) include background
- Explicitly model them in resonance-model fit
  \[ \hat{\rho}_{ab}(m_{K\pi\pi}, t') = \hat{\rho}_{ab}^{K\pi\pi}(m_{K\pi\pi}, t') + \hat{\rho}_{bkg}(m_{K\pi\pi}, t') \]
  - \( \pi^-\pi^-\pi^+ \) background modeled by partial-wave projection of \( \pi^-\pi^-\pi^+ \) pseudodata
  - Yield is only free parameter
  - Incoherent effective background component for other background processes
Partial-Wave Analysis of the $K^-\pi^-\pi^+$ Final State

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Simultaneously included 14 partial waves in resonance-model fit
Modeled by 13 strange-meson resonance components
Using measured intensities and interference terms (relative phases)
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Partial Waves with $J^P = 2^+$

$K_2^*(1430)$ well known resonance
Partial Waves with $J^P = 2^+$

- $K_2^*(1430)$ signal
  - $m_0 = (1430.9 \pm 1.4^{+3.1}_{-1.5})$ MeV/$c^2$
  - $\Gamma_0 = (111 \pm 3^{+4}_{-16})$ MeV/$c^2$
- In different decays
  - $\rho(770) K D$
  - $K^*(892) \pi D$
- In agreement with previous measurements
- Cleaner signal in COMPASS data
- Fitted yield of $\pi^-\pi^-\pi^+$ background consistent with expectation
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$\rho(770)$ $KD$

$0.10 \leq t' < 1.00$ (GeV$/c)^2$

COMPASS

$K\pi\pi$ [GeV$/c^2$]

$2^+ 1^+ \rho(770) KD$

Intensity [(GeV$/c^2)^{-1}] \times 10^5$

$10^5$

$2^+ 1^+$

$0.10 \leq t' < 1.00$ (GeV$/c)^2$

COMPASS

Searching for Exotic Strange Mesons with $J^P = 0^-$

- $K(1460)$ and $K(1830)$
  - $K(1630)$
    - Unexpectedly small width of only 16 MeV/$c^2$
    - $J^P$ of $K(1630)$ unclear
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**COMPASS $K^-\pi^-\pi^+$ data**

- Peak at about 1.4 GeV/c$^2$
  - Established $K(1460)$
  - But, $m_{K\pi\pi} \lesssim 1.5$ GeV/c$^2$ region weakly affected by known analysis artifacts
- Second peak at about 1.7 GeV/c$^2$
  - $K(1630)$ signal with 8.3 $\sigma$ statistical significance
  - Accompanied by rising phase
- Weak signal at about 2.0 GeV/c$^2$
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**Intensity**

$$[(\text{GeV}/c^2)^{-1}] \times 10^5$$

$0 \leq t' < 0.15 \, (\text{GeV}/c)^2$

**COMPASS Preliminary**

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**Additional keywords:**
total resonance model, resonances, non-resonant, $\pi\pi\pi$ background, effective background
Searching for Exotic Strange Mesons with $J^P = 0^-$

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**Intensity [(GeV/$c^2$)$^{-1}$]$\times 10^5$**

$0^{-+} \rho(770)KP$

$0.10 \leq t' < 0.15$ (GeV/$c$)$^2$

**$m_{Kππ}$ [GeV/$c^2$]**

$1.0$ $1.5$ $2.0$ $2.5$ $3.0$

$0.10 \leq t' < 0.15$ (GeV/$c$)$^2$

Preliminary

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*total resonance model, resonances, non-resonant, πππ background, effective background*
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![Graph showing $\Delta\phi_{ab}$ and $m_{K\pi\pi}$](image-url)

- Preliminary
- S. Wallner Strange-Meson Spectroscopy with COMPASS

Keywords: total resonance model, resonances, non-resonant, $\pi\pi\pi$ background, effective background
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**Graph**

- Intensity $\times 10^5$ vs. $m_{K\pi\pi}$ [GeV/$c^2$]
- $0^{-+}\rho(770)KP$
  - $0.10 \leq t' < 0.15$ (GeV/$c^2$)$^2$
  - COMPASS

---

**References**

- total resonance model, resonances, non-resonant, $\pi\pi\pi$ background, effective background
Searching for Exotic Strange Mesons with $J^P = 0^-$

- $K(1830)$ parameters in good agreement with LHCb measurement [PRL 118 (2017) 022003]
- Expected $K(1630)$ width of about 140 MeV/c^2
Searching for Exotic Strange Mesons with $J^P = 0^-$

- Indications for 3 excited $K$ from a single analysis
- Quark-model predicts only two excited states: potentially $K(1460)$ and $K(1830)$
  - $K(1630)$ supernumerary signal
  - Candidate for exotic non-$q\bar{q}$ state; other explanations possible ($K^*(892)$ $\omega$ threshold nearby)

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Ebert et al., PRD 79 (2009) 114029
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The Strange-Meson Spectrum

- Many strange mesons require further confirmation
- Search for strange partners of exotic non-strange light mesons
**Summary**

<table>
<thead>
<tr>
<th>Mass [GeV/c²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Established</td>
</tr>
<tr>
<td>Not Established</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quark Model</th>
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**COMPASS**

- **World's largest data sample on** $K^-\pi^-\pi^+$  ⇒ Most detailed and comprehensive analysis
- **Candidate for exotic strange-meson signal with** $J^P = 0^-$
Summary

COMPASS

- World's largest data sample on $K^-\pi^-\pi^+$ ⇒ Most detailed and comprehensive analysis
- Candidate for exotic strange-meson signal with $J^P = 0^-$
AMBER: Proposal for High-Precision Strange-Meson Spectroscopy

- Goal: Collect 10 – 20 × 10^6 K^−\pi^−\pi^+ events using high-energy kaon beam
- AMBER is open for interested collaborators to join
World's largest data sample on $K^-\pi^-\pi^+$ $\Rightarrow$ Most detailed and comprehensive analysis

Candidate for exotic strange-meson signal with $J^P = 0^-$
Backup
Partial-Wave Decomposition
- Treating the $\pi^-\pi^-\pi^+$ and Other Backgrounds

Resonance-Model Fit
- Modeling the $K^-\pi^-\pi^+$ Signal
- Modeling the $\pi^-\pi^-\pi^+$ Background
- Modeling the Effective Background
- $\chi^2$ Fit Procedure

Wave-Set Selection
- Regularization: LASSO
- Regularization: Generalized Pareto
- Regularization: Cauchy
- For the $K^-\pi^-\pi^+$ Final State

14-Wave Resonance-Model Fit

- Searching for Exotic Strange Mesons with $J^P = 0^-$
- Partial Waves with $J^P = 2^+$
- Partial Waves with $J^P = 2^-$
- Partial Waves with $J^P = 4^+$

Kinematic Distribution of $K^-\pi^-\pi^+$ Events
- Subsystem
- $m_{K^-\pi^-}$
- $t'$ Spectrum
- Exclusivity

Systematic Studies of the Partial-Wave Decomposition
- 14 Waves
- Leakage Waves

Leakage Effect

Incoherent $\pi^-\pi^-\pi^+$ Background
Partial-Wave Decomposition

Partial wave

\[ J^P M^\varepsilon \xi bL \]

- \( J^P M^\varepsilon \): Spin, parity, and spin projection of \( X^- \)
- \( \xi \): Isobar
- \( b \): Bachelor particle. Here: Spectator \( K^- \)
- \( L \): Angular momentum between bachelor and isobar
Partial-Wave Decomposition

**Model intensity**

\[ I(\tau, m_{K\pi\pi}, t') = \left| \sum_z \sum_{a \in W_z(m_{K\pi\pi}, t')} T^z_a(m_{K\pi\pi}, t') \Psi^z_a(\tau; m_{K\pi\pi}) \right|^2 \]

- **Model intensity distribution**
  - in 5D $K^-\pi^-\pi^+$ phase-space
  - for a given $(m_{K\pi\pi}, t')$ cell
  - as incoherent sum over coherent sectors $z$
    - “Rank” of the partial-wave model = number of coherent sectors

- $\Psi^z_a$ known, assuming the isobar model
- Wave set $W_z(m_{K\pi\pi}, t')$ inferred from data using regularization-based model-selection techniques
- $T^z_a$ extracted in maximum-likelihood fit, independently for each $(m_{K\pi\pi}, t')$ cell

**Spin-Density Matrix**

\[ \rho_{ab} = \sum_z T^z_a [T^z_b]^* \]
Partial-Wave Decomposition

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Spin-Density Matrix

\[ \rho_{ab} = \sum_z T_a^z [T_b^z]^* \]
Effectively take into account in partial-wave decomposition by incoherently adding additional coherent sectors $z$

(Model background by $K^-\pi^-\pi^+$ partial waves)

- Increasing the rank of the spin-density matrix $\rho_{ab}$
- Signal not separated from background in partial-wave decomposition
- Partial-wave amplitudes include background

Model signal and background contributions in resonance-model fit using more constrained signal model

- Separate signal from background

$$\mathcal{I}(\tau, m_{K\pi\pi}, t') = \sum_z \left| \sum_{a \in \mathcal{W}_z} T^z_a (m_{K\pi\pi}, t') \bar{\psi}^z_a (\tau; m_{K\pi\pi}) \right|^2$$

$$\rho_{ab} = \sum_z T^z_a \left[ T^z_b \right]^*$$
Partial-Wave Decomposition

Treating the $\pi^-\pi^-\pi^+$ and Other Backgrounds

**True physics intensity distribution**

\[
I(\tau) = \left| \sum_a T_a \Psi_a(\tau) \right|^2
\]

**Experimentally measured intensity distribution**

\[
I_{\text{measured}}(\tau) = \eta(\tau) I(\tau)
\]

- Take into account different processes $p$
  - Different model intensities $I^p$
  - Different experimental acceptance $\eta^p$
  - Formulated in terms of different phase-space variables $\tau^p$
  - Jacobian terms $J(\tau^{K\pi\pi} \rightarrow \tau^p)$ from variable transformation
Partial-Wave Decomposition
Treating the $\pi^-\pi^-\pi^+$ and Other Backgrounds

True physics intensity distribution for process $p$

$$I^p(\tau) = \left| \sum_a T^p_a \Psi^p_a(\tau) \right|^2$$

Experimentally measured intensity distribution

$$I_{\text{measured}}(\tau) = \sum_p \eta^p(\tau) I^p(\tau)$$

- Take into account different processes $p$
  - Different model intensities $I^p$
  - Different experimental acceptance $\eta^p$
  - Formulated in terms of different phase-space variables $\tau^p$
    - Jacobian terms $J(\tau^{K\pi\pi} \rightarrow \tau^p)$ from variable transformation
### True physics intensity distribution for process $p$

$$I^p(\tau^p) = \left| \sum_a T^a_p \Psi^p_a(\tau^p) \right|^2$$

### Experimentally measured intensity distribution

$$I_{\text{measured}}(\tau^{K\pi\pi}) = \sum_p \eta^p(\tau^p) I^p(\tau^p) J(\tau^{K\pi\pi} \rightarrow \tau^p)$$

- Take into account different processes $p$
- Different model intensities $I^p(\tau^p)$
- Different experimental acceptance $\eta^p(\tau^p)$
- Formulated in terms of different phase-space variables $\tau^p$
  - Jacobian terms $J(\tau^{K\pi\pi} \rightarrow \tau^p)$ from variable transformation
Partial-Wave Decomposition
Treating the $\pi^-\pi^-\pi^+$ and Other Backgrounds

True physics intensity distribution for process $p$

$$I^p(\tau^p) = \left| \sum_a T^p_a \psi^p_a(\tau^p) \right|^2$$

Experimentally measured intensity distribution

$$I_{\text{measured}}(\tau^{K\pi\pi}) = \sum_p \eta^p(\tau^p) \; I^p(\tau^p) \; J(\tau^{K\pi\pi} \rightarrow \tau^p)$$

- $I^{\pi\pi\pi}$ known by COMPASS analysis
- $\eta^{\pi\pi\pi}$ from detector simulation
- $\eta^{\pi\pi\pi}$ computationally expensive
- Different $m_{3\pi}$ bins enter one $m_{K\pi\pi}$ bin
- Other background channels: $K^-K^-K^+$, ...
  - $I^p$ unknown
  - Unknown background channels
### True Physics Intensity Distribution for Process $p$

$$
Theoretical \ intensity \ distribution: \ \mathcal{I}^p(\tau^p) = \left| \sum_a T^a \psi^a(\tau^p) \right|^2
$$

### Experimentally Measured Intensity Distribution

$$
Experimental \ intensity \ distribution: \ \mathcal{I}_{measured}(\tau^{\pi\pi\pi}) = \sum_p \eta^p(\tau^p) \mathcal{I}^p(\tau^p) J(\tau^{\pi\pi\pi} \rightarrow \tau^p)
$$

- $\eta^{\pi\pi\pi}$ computationally expensive
- Different $m_{3\pi}$ bins enter one $m_{K\pi\pi}$ bin
- Other background channels: $K^-K^-K^+$, ...
  - $\mathcal{I}^p$ unknown
  - Unknown background channels
- $\mathcal{I}^p(\tau^p)$ unknown

- $\mathcal{I}^{\pi\pi\pi}$ known by COMPASS analysis
- $\eta^{\pi\pi\pi}$ from detector simulation
### True physics intensity distribution for process \( p \)

\[
\mathcal{I}^p(\tau^p) = \left| \sum_a T^p_a \Psi^p_a(\tau^p) \right|^2
\]

### Experimentally measured intensity distribution

\[
\mathcal{I}_{\text{measured}}(\tau^{K\pi\pi}) = \sum_p \eta^p(\tau^p) \mathcal{I}^p(\tau^p) J(\tau^{K\pi\pi} \rightarrow \tau^p)
\]

- \( \mathcal{I}^{\pi\pi\pi} \) known by COMPASS analysis
- \( \eta^{\pi\pi\pi} \) from detector simulation
- \( \eta^{\pi\pi\pi} \) computationally expensive
- Different \( m_{3\pi} \) bins enter one \( m_{K\pi\pi} \) bin
- Other background channels: \( K^-K^-K^+, \ldots \)
  - \( \mathcal{I}^p \) unknown
  - Unknown background channels
Approximate model for process $p$ by $K^-\pi^-\pi^+$ partial waves

\[ \eta^p(\tau^p) \left| \sum_a \mathcal{T}_a^p \Psi_a^p(\tau^p) \right|^2 \approx \eta^{K\pi\pi}(\tau^{K\pi\pi}) \left| \sum_a \tilde{\mathcal{T}}_a^p \Psi_a^{K\pi\pi}(\tau^{K\pi\pi}) \right|^2 \]

Total true physics intensity distribution

\[ \mathcal{I}(\tau^{K\pi\pi}) = \sum_p \left| \sum_a \mathcal{T}_a^p \Psi_a^{K\pi\pi}(\tau^{K\pi\pi}) \right|^2 \]

Experimentally measured intensity distribution

\[ \mathcal{I}_{\text{measured}}(\tau^{K\pi\pi}) = \eta^{K\pi\pi}(\tau^{K\pi\pi}) \mathcal{I}(\tau^{K\pi\pi}) \]

- How well can $K^-\pi^-\pi^+$ partial waves approximate the distribution of process $p$?
- Is the set of $K^-\pi^-\pi^+$ partial waves sufficient?
- Automatic wave-set selection using model-selection techniques
Approximate model for process $p$ by $K^-\pi^-\pi^+$ partial waves

$$
\eta^p(\tau^p) \left| \sum_a T^p_a \Psi^p_a(\tau^p) \right|^2 \approx \eta^{K\pi\pi}(\tau^{K\pi\pi}) \left| \sum_a \tilde{T}^p_a \Psi^{K\pi\pi}_a(\tau^{K\pi\pi}) \right|^2
$$

Total true physics intensity distribution

$$
I(\tau^{K\pi\pi}) = \sum_p \left| \sum_a T^p_a \Psi^{K\pi\pi}_a(\tau^{K\pi\pi}) \right|^2
$$

Experimentally measured intensity distribution

$$
I_{\text{measured}}(\tau^{K\pi\pi}) = \eta^{K\pi\pi}(\tau^{K\pi\pi}) I(\tau^{K\pi\pi})
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- How well can $K^-\pi^-\pi^+$ partial waves approximate the distribution of process $p$?
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Partial-Wave Decomposition
Treating the $\pi^+\pi^-\pi^+$ and Other Backgrounds

Approximate model for process $p$ by $K^-\pi^-\pi^+$ partial waves

$$\eta^p(\tau^p) \left| \sum_a T^p_a \Psi^p_a(\tau^p) \right|^2 \approx \eta^{K\pi\pi}(\tau^{K\pi\pi}) \left| \sum_a \tilde{T}^p_a \Psi^{K\pi\pi}_a(\tau^{K\pi\pi}) \right|^2$$

Total true physics intensity distribution

$$\mathcal{I}(\tau^{K\pi\pi}) = \sum_{a,b} \Psi^{K\pi\pi}_a(\tau^{K\pi\pi}) \rho_{a,b} [\Psi^{K\pi\pi}_b(\tau^{K\pi\pi})]^*$$

Spin-density matrix with rank $N_r > 1$

$$\rho_{a,b} = \sum_p T^p_a [T^p_b]^*$$

How well can $K^-\pi^-\pi^+$ partial waves approximate the distribution of process $p$?

Is the set of $K^-\pi^-\pi^+$ partial waves sufficient?

Automatic wave-set selection using model-selection techniques
Partial-Wave Decomposition
Treating the $\pi^-\pi^-\pi^+$ and Other Backgrounds

Approximate model for process $p$ by $K^-\pi^-\pi^+$ partial waves

\[ \eta^p(\tau^p) \bigg| \sum_a \tilde{T}_a^p \Psi_a^p(\tau^p) \bigg|_2^2 \approx \eta^{K\pi\pi}(\tau^{K\pi\pi}) \bigg| \sum_a \tilde{T}_a^p \Psi_a^{K\pi\pi}(\tau^{K\pi\pi}) \bigg|_2^2 \]

Total true physics intensity distribution

\[ \mathcal{I}(\tau^{K\pi\pi}) = \sum_{a,b} \Psi_a^{K\pi\pi}(\tau^{K\pi\pi}) \rho_{a,b} [\Psi_b^{K\pi\pi}(\tau^{K\pi\pi})]^* \]

Spin-density matrix with rank $N_r > 1$

\[ \rho_{a,b} = \sum_p T_a^p [T_b^p]^* \]

- How well can $K^-\pi^-\pi^+$ partial waves approximate the distribution of process $p$?
- Is the set of $K^-\pi^-\pi^+$ partial waves sufficient?
  - Automatic wave-set selection using model-selection techniques
Partial-Wave Decomposition
Treating the $\pi^-\pi^-\pi^+$ and Other Backgrounds

Approximate model for process $p$ by $K^-\pi^-\pi^+$ partial waves

$$
\eta^p(\tau^p) \left| \sum_a T^p_a \Psi^p_a(\tau^p) \right|^2 \approx \eta^{K\pi\pi}(\tau^{K\pi\pi}) \left| \sum_a \tilde{T}^p_a \Psi^{K\pi\pi}_a(\tau^{K\pi\pi}) \right|^2
$$

Total true physics intensity distribution

$$
I(\tau^{K\pi\pi}) = \sum_{a,b} \Psi^{K\pi\pi}_a(\tau^{K\pi\pi}) \rho_{a,b} [\Psi^{K\pi\pi}_b(\tau^{K\pi\pi})]^* 
$$

Spin-density matrix with rank $N_r > 1$

$$
\rho_{a,b} = \sum_r T^r_a [T^r_b]^* 
$$

▶ Experimentally measurable quantities are spin-density matrix elements

⇒ Transition amplitudes $T^p_a$ are only effective parameters
⇒ Cannot determine $T^p_a$ of individual processes
⇒ Cannot separate different processes
**Partial-Wave Decomposition**

**Treating the $\pi^-\pi^-\pi^+$ and Other Backgrounds**

### Approximate model for process $p$ by $K^-\pi^-\pi^+$ partial waves

$$
\eta^p(\tau^p) \left| \sum_a T_a^p \Psi_a^p(\tau^p) \right|^2 \approx \eta^{K\pi\pi}(\tau^{K\pi\pi}) \left| \sum_a \tilde{T}_a^p \Psi_a^{K\pi\pi}(\tau^{K\pi\pi}) \right|^2
$$

### Total true physics intensity distribution

$$
I(\tau^{K\pi\pi}) = \sum_{a,b} \Psi_a^{K\pi\pi}(\tau^{K\pi\pi}) \rho_{a,b} \left[ \Psi_b^{K\pi\pi}(\tau^{K\pi\pi}) \right]^* 
$$

### Spin-density matrix with rank $N_r > 1$

$$
\rho_{a,b} = \sum_{r} T_a^r \left[ T_b^r \right]^*
$$

- **Large number of fit parameters:** $N_{\text{para}} = N_r (2N_{\text{waves}} - N_r)$
- **Sufficient rank of spin-density matrix must be determined**
  - Rank two needed to describe pure $\pi^-\pi^-\pi^+$ Monte Carlo sample using $K^-\pi^-\pi^+$ partial waves
  - Used rank three to model $K^-\pi^-\pi^+$ sample
**Resonance-Model Fit**

- **Data**
  - 720k diffractively produced $K^-\pi^-\pi^+$ candidates

- **(I) Partial-Wave Decomposition**
  - Partial Waves
    - Intensities and relative phases of the partial waves

- **(II) Resonance-Model Fit**
  - Resonance Parameters
    - Masses and widths of the meson resonances
Spin-density matrix $\rho_{ab}(m_{K\pi\pi}, t')$ measured in partial-wave decomposition

Model spin-density matrix in resonance-model fit

$$\hat{\rho}_{ab}(m_{K\pi\pi}, t') = \hat{\rho}_{ab}^{K\pi\pi}(m_{K\pi\pi}, t') + \hat{\rho}_{ab}^{3\pi}(m_{K\pi\pi}, t') + \hat{\rho}_{ab}^{\text{Bkg}}(m_{K\pi\pi}, t')$$
Model transition amplitudes as coherent sum over various components

\[ \hat{T}_z(m_{K\pi\pi}, t') = \sum_{k \in S_a} K(m_{K\pi\pi}, t')^k C_{aK\pi\pi}^{K\pi\pi}(t') D_k(m_{K\pi\pi}; \zeta_k) \]

- **Dynamic functions** \( D_k(m_{K\pi\pi}; \zeta_k) \)
  - For resonances: rel. Breit-Wigner
  - For non-resonant terms: \( D^{NR}_k(m_{K\pi\pi}; a_k, c_k) = (m_{K\pi\pi} - m_{thr})^{a_k} e^{-b(c_k)} q^2_k(m_{K\pi\pi}) \)

- “Coupling amplitudes”: \( k C_{aK\pi\pi}^{K\pi\pi}(t') \)
  - Independent coupling amplitude for each \( t' \) bin

- Kinematic factor \( K(m_{K\pi\pi}, t') \)

- Coherently summed over all assumed model components
Model transition amplitudes as coherent sum over various components

\[ \hat{F}_a^z(m_{K\pi\pi}, t') = \sum_{k \in S_a} K(m_{K\pi\pi}, t')^k C_{a}^{K\pi\pi}(t') D_k(m_{K\pi\pi}; \zeta_k) \]

- Dynamic functions \( D_k(m_{K\pi\pi}; \zeta_k) \)
  - For resonances: rel. Breit-Wigner
  - For non-resonant terms: \( D_{k}^{NR}(m_{K\pi\pi}; a_k, c_k) = (m_{K\pi\pi} - m_{\text{thr}})^{a_k} e^{-b(c_k)} q_k^2(m_{K\pi\pi}) \)
- "Coupling amplitudes": \( k C_{a}^{z}(t') \)
  - Independent coupling amplitude for each \( t' \) bin
- Kinematic factor \( K(m_{K\pi\pi}, t') \)
- Coherently summed over all assumed model components
Model transition amplitudes as coherent sum over various components

\[ \hat{T}_a^z(m_{K\pi\pi}, t') = \sum_{k \in S_a} K(m_{K\pi\pi}, t')^{k} C_{a}^{K_{\pi\pi}}(t') D_k(m_{K\pi\pi}; \zeta_k) \]

- Dynamic functions \(D_k(m_{K\pi\pi}; \zeta_k)\)
  - For resonances: rel. Breit-Wigner
  - For non-resonant terms: \(D_k^{NR}(m_{K\pi\pi}; a_k, c_k) = (m_{K\pi\pi} - m_{thr})^{a_k} e^{-b(c_k) q_k^2(m_{K\pi\pi})}\)
- “Coupling amplitudes”: \(k C_{a}^z(t')\)
  - Independent coupling amplitude for each \(t'\) bin
- Kinematic factor \(K(m_{K\pi\pi}, t')\)
- Coherently summed over all assumed model components
Resonance-Model Fit
Modeling the $K^-\pi^-\pi^+$ Signal

Model transition amplitudes as coherent sum over various components

$$\hat{\mathcal{T}}^z_a(m_{K\pi\pi}, t') = \sum_{k \in S_a} K(m_{K\pi\pi}, t')^k C^Z_a (t') D_k(m_{K\pi\pi}; \zeta_k)$$

- Dynamic functions $D_k(m_{K\pi\pi}; \zeta_k)$
  - For resonances: rel. Breit-Wigner
  - For non-resonant terms: $D^{NR}_k(m_{K\pi\pi}; a_k, c_k) = (m_{K\pi\pi} - m_{thr})^{a_k} e^{-b(c_k) q^2_k(m_{K\pi\pi})}$
- “Coupling amplitudes”: $C^Z_a (t')$
  - Independent coupling amplitude for each $t'$ bin
- Kinematic factor $K(m_{K\pi\pi}, t')$
- Coherently summed over all assumed model components
Resonance-Model Fit
Modeling the $\pi^-\pi^-\pi^+$ Background

$3\pi$ spin-density matrix

$$\hat{\rho}_{ab}^{\pi\pi\pi}(m_{K\pi\pi}, t') = \left| C^{\pi\pi\pi} \right|^2 \rho_{ab}^{\pi\pi\pi}(m_{K\pi\pi}, t')$$

- $\rho_{ab}^{\pi\pi\pi}(m_{K\pi\pi}, t')$ obtained from PWD of $\pi^-\pi^-\pi^+$ pseudodata sample
  - $m_{K\pi\pi}$ dependence fixed
  - $t'$ dependence fixed
  - Rel. strength between partial waves fixed (freed in a study)
- One global real-valued yield parameter $|C^{\pi\pi\pi}|^2$
Background spin-density matrix

- Additional incoherent contribution form other processes: $K^- K^- K^+$, ...
- Transition amplitudes modeled by non-resonant parameterizations for each partial wave

$$\hat{f}_a^{\text{eBKG}} (m_{K\pi\pi}, t') = K(m_{K\pi\pi}, t') \ C_a^{\text{eBKG}} (t') \ D_{ka}^{\text{eBKG}} (m_{K\pi\pi}; a_k, c_k)$$
Resonance-Model Fit

$\chi^2$ Fit Procedure

- $\chi^2$ fit of the real and imaginary parts of the spin-density matrix
- Taking into account correlations between spin-density matrix elements
- Shape parameters ($m_0$, $\Gamma_0$, ...) and coupling amplitudes are free parameters

- For the main fit, we performed 2000 fit attempts with random start-parameter values for the shape parameters, e.g. mass and width parameters, and the coupling and branching amplitudes.
- Start-parameter ranges for the shape parameters are chosen according to previous measurements (see note)
- The best result is the one which yielded the smallest $\chi^2$ value
χ² Fit of the real and imaginary parts of the spin-density matrix
- Taking into account correlations between spin-density matrix elements
- Shape parameters \((m_0, \Gamma_0, \ldots)\) and coupling amplitudes are free parameters

For the main fit, we performed 2000 fit attempts with random start-parameter values for the shape parameters, e.g. mass and width parameters, and the coupling and branching amplitudes.

Start-parameter ranges for the shape parameters are chosen according to previous measurements (see note)

The best result is the one which yielded the smallest \(\chi^2\) value
Wave-Set Selection

\[ I(\tau, m_{K\pi\pi}, t') = \left| \sum_{a \in W(\tau, m_{K\pi\pi}, t')} T_a(m_{K\pi\pi}, t') \psi_a(\tau; m_{K\pi\pi}) \right|^2 \]

**Challenge:** Find the “best” set of waves that describes the data

- If the wave set is too large
  - Starting to describe statistical fluctuations
- If waves that contribute to the data are missing
  - Intensity can be wrongly attributed to other waves
  - Model leakage
**Wave-Set Selection**

**Infer wave set from data**

- **Systematically construct** large set of allowed partial waves
  - “Wave pool”
- Fit wave pool to data
  - Impose penalty on $|T_a|^2 \Rightarrow \text{regularization}$
  - Suppress insignificant waves
- **Select waves** that significantly contribute to data
  - “Best” subset of waves that describe the data
Wave-Set Selection

$\pi^–\pi^–\pi^+$ Monte Carlo mock data set with 126 partial waves

- Fitting wave pool of 753 waves
  - Massive overfitting
  - Almost all waves pick up intensity

Courtesy F. Kaspar, TUM
Wave-Set Selection

- $\pi^–\pi^–\pi^+$ Monte Carlo mock data set with 126 partial waves
- Fitting wave pool of 753 waves
  - Massive overfitting
  - Almost all waves pick up intensity

Courtesy F. Kaspar, TUM
Wave-Set Selection

Regularization: LASSO

\[
\ln \mathcal{L}_{\text{fit}} = \ln \mathcal{L}_{\text{extended}} + \sum_a \ln \mathcal{L}_{\text{reg}}(|T_a|; \{c_{\text{para}}\})
\]

LASSO/L1 regularization\(^1\)

\[
\ln \mathcal{L}_{\text{reg}}(|T_a|; \lambda) = -\lambda |T_a|
\]

- Maximum at \(|T_a| = 0\)
- Well established\(^2\)
- “Smoothing” at \(|T_a| = 0\)
  \[
  |T_a| \to \sqrt{|T_a|^2 + \epsilon}
  \]

---


\(^2\) Baptiste Guegan et al. "Model selection for amplitude analysis". In: JINST 10.09 (2015), P09002
Wave-Set Selection
Regularization: LASSO

\[
\ln L_{\text{fit}} = \ln L_{\text{extended}} + \sum_{a}^{\text{waves}} \ln L_{\text{reg}}(|T_a|; \{c_{\text{para}}\})
\]

LASSO/L1 regularization\(^1\)

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|T_a| \rightarrow \sqrt{|T_a|^2 + \epsilon}
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Wave-Set Selection
Regularization: LASSO

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LASSO/L1 regularization\(^1\)

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\[
|T_a| \rightarrow \sqrt{|T_a|^2 + \varepsilon}
\]


\(^2\) Baptiste Guegan et al. "Model selection for amplitude analysis". In: JINST 10.09 (2015), P09002
Wave-Set Selection
Regularization: LASSO

- Bias also on large transition amplitudes
- Some additional waves
- Some waves missing

\[ \lambda = 0.3 \]
\[ \varepsilon = 10^{-5} \]

Courtesy F. Kaspar, TUM
Wave-Set Selection
Regularization: Generalized Pareto

Generalized Pareto\(^1\)

\[
\ln \mathcal{L}_{\text{reg}}(|T_a|; \Gamma, \zeta) = -\frac{1}{\zeta} \ln \left[ 1 + \zeta \frac{|T_a|}{\Gamma} \right]
\]

- Wave intensities spread over orders of magnitudes
- Use logarithmic prior
  - Heavy-tailed
  - Less bias on large waves
- LASSO-like for \(|T_a| \to 0\)
- “Smoothing” at \(|T_a| = 0\)

\[
|T_a| \to \sqrt{|T_a|^2 + \varepsilon}
\]

---

Wave-Set Selection
Regularization: Generalized Pareto

- Less bias on large transition amplitudes
- Clear kink in intensity distribution to smoothing scale $\Rightarrow$ Selection
- Less additional waves
- Some small waves missing

Courtesy F. Kaspar, TUM
Wave-Set Selection
Regularization: Cauchy

“Cauchy”

\[
\ln \mathcal{L}_{\text{reg}}(|\mathcal{T}_a|; \Gamma) = -\ln \left[ 1 + \frac{|\mathcal{T}_a|^2}{\Gamma^2_a} \right]
\]

- Logarithmic prior
- L2-like for $|\mathcal{T}_a| \to 0$
Wave-Set Selection
Regularization: Cauchy

Less bias on large transition amplitudes
Clear kink in intensity distribution
Few additional waves
Few small waves missing

Courtesy F. Kaspar, TUM
Wave-Set Selection
For the $K^-\pi^-\pi^+$ Final State

Wave pool

- Spin $J \leq 7$
- Angular momentum $L \leq 7$
- Positive naturality of exchange particle
- 12 isobars
  - $[K\pi]_S^K\pi$, $[K\pi]_S^K\eta$, $K^*(892)$, $K^*(1680)$, $K_2^*(1430)$, $K_3^*(1780)$
  - $[\pi\pi]_S$, $f_0(980)$, $f_0(1500)$, $\rho(770)$, $f_2(1270)$, $\rho_3(1690)$

$\Rightarrow$ “Wave pool” of 596 waves

“only” 720 k events
Wave-Set Selection
For the $K^\pi^\pi^+$ Final State

Wave pool

- Spin $J \leq 7$
- Angular momentum $L \leq 7$
- Positive naturality of exchange particle
- 12 isobars
  - $[K\pi]^K\pi, [K\pi]^K\eta, K^*(892), K^*(1680), K_2^*(1430), K_3^*(1780)$
  - $[\pi\pi], f_0(980), f_0(1500), \rho(770), f_2(1270), \rho_3(1690)$

⇒ “Wave pool” of 596 waves

“only” 720 k events
Wave-Set Selection
For the $K^−\pi^−\pi^+$ Final State

Regularization

$\ln \mathcal{L}_{\text{reg}} (|T_a|; \Gamma) = -\ln \left[ 1 + \frac{|T_a|^2}{\Gamma_a^2} \right]$  

- Use Cauchy regularization
- Scale of $|T_a|$ depends on experimental acceptance
  - Apply penalty on expected number $\bar{N}_a$ of observed events
    \[ \Gamma_a = \frac{\Gamma}{\sqrt{\bar{N}_a}} \implies \frac{|T_a|^2}{\Gamma_a^2} = \frac{\bar{N}_a}{\Gamma^2} \]
- $\Gamma$ is a universal parameter
Wave-Set Selection
For the $K^-\pi^-\pi^+$ Final State

Regularization

$$\ln \mathcal{L}_{\text{reg}}(|T_a|; \Gamma) = -\ln \left[ 1 + \frac{|T_a|^2}{\Gamma_a^2} \right]$$

- Use Cauchy regularization
- Scale of $|T_a|$ depends on experimental acceptance
  - Apply penalty on expected number $\tilde{N}_a$ of observed events
    $$\Gamma_a = \frac{\Gamma}{\sqrt{\eta_a}} \Rightarrow \frac{|T_a|^2}{\Gamma_a^2} = \frac{\tilde{N}_a}{\Gamma^2}$$
  - $\Gamma$ is a universal parameter

$\ln \mathcal{L}_{\text{reg}}(|T_a|; \Gamma) = -\ln \left[ 1 + \frac{|T_a|^2}{\Gamma_a^2} \right]$
Wave-Set Selection
For the $K^-\pi^-\pi^+$ Final State

Regularization

\[ \ln L_{\text{reg}}(|T_a|; \Gamma) = -\ln \left[ 1 + \frac{|T_a|^2}{\Gamma_a^2} \right] \]

- Use Cauchy regularization
- Scale of $|T_a|$ depends on experimental acceptance
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    \[ \Gamma_a = \frac{\Gamma}{\sqrt{\eta_a}} \Rightarrow \frac{|T_a|^2}{\Gamma_a^2} = \frac{\bar{N}_a}{\Gamma^2} \]
- $\Gamma$ is a universal parameter

COMPASS

\[ m_{K\pi\pi} \text{ [GeV/c}^2\text{]} \]

Graph showing various lines representing different states.
Wave-Set Selection
For the $K^-\pi^-\pi^+$ Final State

Regularization

\[ \ln \mathcal{L}_{\text{reg}}(|T_a|; \Gamma) = -\ln \left( 1 + \frac{|T_a|^2}{\Gamma_a^2} \right) \]

- Use Cauchy regularization
- Scale of $|T_a|$ depends on experimental acceptance
  - Apply penalty on expected number $\tilde{N}_a$ of observed events

\[ \Gamma_a = \frac{\Gamma}{\sqrt{\eta}_a} \Rightarrow \frac{|T_a|^2}{\Gamma_a^2} = \frac{\tilde{N}_a}{\Gamma^2} \]

- $\Gamma$ is a universal parameter

COMPASS

0$^-0^+ \bar{K}^*(892)\pi P$
1$^+0^+ \bar{K}^*(892)\pi S$
2$^+1^+ \rho(770) K D$
Flat

S. Wallner Strange-Meson Spectroscopy with COMPASS
Wave-Set Selection
For the $K^-\pi^-\pi^+$ Final State

Imposing continuity of the wave set

- Wave-set inferred independently for each $(m_{K\pi\pi}, t')$ cell
- Impose continuity of the wave set in $m_{K\pi\pi}$ by adding additional regularization term

$$\ln \mathcal{L}_{\text{cont}}(\{T_a(m_{K\pi\pi}, t')\}; \lambda) = \sum_{j=i-3}^{j=i+3} \lambda \left| T_a(m_{K\pi\pi}, t')(m_{K\pi\pi}^{j+1}) - T_a(m_{K\pi\pi}, t')(m_{K\pi\pi}^j) \right|^2,$$

which suppresses fluctuations among neighboring $m_{K\pi\pi}$ bins.
Wave-Set Selection
For the $K^-\pi^-\pi^+$ Final State

Wave-set size

- 5 to 90 waves per $(m_{K\pi\pi}, t')$ cell
- Larger wave set for larger binning in $m_{K\pi\pi}$
- Larger wave set in $t'$ bins with more events
Wave-Set Selection
For the $K^-\pi^-\pi^+$ Final State

- Selection of large signals
- as well as of signals at per-mil level

![Graph showing intensity vs. $m_{K\pi\pi}$]
Selection of large signals
as well as of signals at per-mil level
14-Wave Resonance-Model Fit
Searching for Exotic Strange Mesons with $J^P = 0^-$

- **K(1460) and K(1830)**
  - **K(1630)**
    - Unexpectedly small width of only 16 MeV/c²
    - $J^P$ of K(1630) unclear

PDG (2022)

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14-Wave Resonance-Model Fit
Searching for Exotic Strange Mesons with $J^P = 0^-$

PDG

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- Peak at about 1.4 GeV/$c^2$
  - Potentially from established $K(1460)$
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- Second peak at about 1.7 GeV/$c^2$
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14-Wave Resonance-Model Fit
Searching for Exotic Strange Mesons with $J^P = 0^-$

COMPASS
$0.10 \leq t' < 0.15\ (GeV/c)^2$

**Total model**

**Resonance components**

**Non-resonant component**

$\pi^-\pi^-\pi^+$ background

Effective background
14-Wave Resonance-Model Fit
Searching for Exotic Strange Mesons with $J^P = 0^-$

COMPASS
$0.15 \leq t' < 0.24 \text{(GeV/c)}^2$

Total model
Resonance components
Non-resonant component
$\pi^-\pi^-\pi^+$ background
Effective background
14-Wave Resonance-Model Fit
Searching for Exotic Strange Mesons with $J^P = 0^-$

COMPASS
$0.24 \leq t' < 0.34 \text{ (GeV/c)}^2$

Total model
Resonance components
Non-resonant component
$\pi^-\pi^+\pi^+$ background
Effective background
14-Wave Resonance-Model Fit
Searching for Exotic Strange Mesons with $J^P = 0^-$

COMPASS
$0.34 \leq t' < 1.00$ (GeV/c)$^2$

- Total model
- Resonance components
- Non-resonant component
- $\pi^-\pi^-\pi^+$ background
- Effective background
$K(1830)$ parameters in good agreement with LCHb measurement [PRL 118 (2017) 022003]

Realistic $K(1630)$ width of about 140 MeV/$c^2$
Indications for 3 excited $K$ from a single analysis

- Quark-model predicts only two excited states: potentially $K(1460)$ and $K(1830)$
- $K(1630)$ supernumerary signal
- Candidate for exotic non-$q\bar{q}$ state; other explanations possible ($K^*(892)\omega$ threshold nearby)
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14-Wave Resonance-Model Fit
Searching for Exotic Strange Mesons with $J^P = 0^-$

$0^{-0^+} \rho(770) K P$

$0.10 \leq t' < 1.00 \ (GeV/c)^2$

COMPASS
14-Wave Resonance-Model Fit
Searching for Exotic Strange Mesons with $J^P = 0^-$
14-Wave Resonance-Model Fit
Searching for Exotic Strange Mesons with $J^P = 0^-$

$K^-\pi^-\pi^+$ from ACCMOR

- Potential $K(1630)$ signal already in ACCMOR analysis

$K^-\pi^-\pi^+$ from LHCb

- Measurement of $D^0 \rightarrow K^\mp\pi^\pm\pi^\pm\pi^\mp$ at LHCb
- Study strange mesons in $K\pi\pi$ subsystem
- MIPWA of $J^P = 0^-$ amplitude
- Potential signal above 1.6 GeV/$c^2$
- Limited by kinematic range

14-Wave Resonance-Model Fit
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14-Wave Resonance-Model Fit
Partial Waves with $J^P = 2^+$

$K^*_2(1430)$ well known resonance
Partial Waves with $J^P = 2^+$

- Signal in $K_2^*(1430)$ mass region
- In different decays
  - $\rho(770)\ K\ D$
  - $K^*(892)\ \pi\ D$
- In agreement with previous measurements
- Cleaner signal in COMPASS data

**Total Resonance Model, resonances, non-resonant, $\pi\pi\pi$ background, effective background**
14-Wave Resonance-Model Fit
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- $\rho(770) K D$
- $K^*(892) \pi D$

In agreement with previous measurements

Cleaner signal in COMPASS data
14-Wave Resonance-Model Fit
Partial Waves with $J^P = 2^+$

- $K_2^*(1430)$ parameters consistent with previous observations
- Better agreement with PDG average values for neutral $K_2^*(1430)$
14-Wave Resonance-Model Fit
Partial Waves with $J^P = 2^-$

PDG

- Established $K_2(1770)$ and $K_2(1820)$
- $K_2(2250)$ need further confirmation
Simultaneously fit 4 waves with $J^P = 2^-$
- 1.8 GeV/$c^2$ peak modeled by $K_2(1770)$, $K_2(1820)$
- High-mass shoulder modeled by $K_2(2250)$
- Different intensity spectra and large phase motions among $2^-$ waves
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Different intensity spectra and large phase motions among $2^-$ waves
14-Wave Resonance-Model Fit
Partial Waves with \( J^P = 2^- \)

**\( K_2(1770) \) and \( K_2(1820) \)**

- Two states were considered by only three measurements ACCMOR, LASS, LHCb
- Only LHCb measurement could confirm two states (3 \( \sigma \) statistical significance)
- We observe two states with 11 \( \sigma \) statistical significance
14-Wave Resonance-Model Fit
Partial Waves with $J^P = 2^-$

- Studied so far mainly in $\Lambda(\bar{p})$ final states
- First simultaneous measurement of $K_2(1770)$, $K_2(1820)$, and $K_2(2250)$
- Resonance parameters consistent with previous observations

$K_2(2250)$
14-Wave Resonance-Model Fit

Partial Waves with $J^P = 2^-$

Mass [GeV/$c^2$]

$K$, $K_0^*$, $K^*$, $K_1$, $K_2$, $K_2^*$, $K_3^*$, $K_3$, $K_4$, $K_4^*$, $K_5^*$

- Established
- Not Established
- Quark Model

[Ebert et al., PRD 79 (2009) 114029]
14-Wave Resonance-Model Fit
Partial Waves with $J^P = 2^-$
14-Wave Resonance-Model Fit
Partial Waves with $J^P = 4^+$

$K^*_4(2045)$ known resonance
Signal $K_4^*(2045)$ signal in $K^*(892)\pi$ and $\rho(770)K$ decays
14-Wave Resonance-Model Fit
Partial Waves with $J^P = 4^+$

- Signal $K_4^*(2045)$ signal in $K^*(892)\,\pi$ and $\rho(770)\,K$ decays
Partial Waves with \( J^P = 4^+ \)

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14-Wave Resonance-Model Fit
Partial Waves with $J^P = 4^+$

\[ m_0 [\text{MeV}/c^2] \]

\[ \Gamma_0 [\text{MeV}/c^2] \]

\( K_4^*(2045) \) COMPASS
\( K_4^*(2045) \) PDG average
\( K_4^*(2045) \) Prev. exp.
14-Wave Resonance-Model Fit
Partial Waves with \( J^P = 4^+ \)

- Imperfect description of magnitude of intensity,

- Also, real and imaginary parts of interference terms described well, including their magnitude

- Intensities and real and imaginary parts of interference terms not directly related as \( \text{Rank}[\rho_{ab}] > 1 \)
  \( |\rho_{ab}| \neq \sqrt{|\rho_{aa}| |\rho_{bb}|} \)
  - Analysis artifacts in intensities of small waves, which are the least constrained by data

- Results validated by Monte Carlo input-output and systematic studies

- Imperfections considered in systematic uncertainties

- Results in agreement with previous experiments

---

**total resonance model, resonances, non-resonant, \( \pi \pi \) background, effective background**
14-Wave Resonance-Model Fit
Partial Waves with $J^P = 4^+$

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**Graph:**
- Real part of the interference term $\Re(\rho_{ab})$ for $m_{K\pi\pi} \in [1, 3] \text{ GeV}/c^2$
- $0.10 \leq t' < 0.15 \text{ (GeV}/c)^2$

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**Keywords:**
- total resonance model
- resonances
- non-resonant
- $\pi\pi\pi$ background
- effective background
14-Wave Resonance-Model Fit
Partial Waves with \( J^P = 4^+ \)

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\[ \Imag([4^+1^+ K^* (892) \pi G][2^{-0^+} K^*_2 (1430) \pi S]^*) \times 10^4 \]

\[ S(\rho_{ab}) \text{ [GeV/c}^2\text{]}^{-1} \]

$0.10 \leq t' < 0.15 \text{ (GeV/c)}^2$

$1.0 \leq m_{K\pi\pi} \text{ [GeV/c}^2\text{]} \leq 3.0$

total resonance model, resonances, non-resonant, $\pi\pi\pi$ background, effective background
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$\Im([4^+1^+ K^*(892)\pi G][2^0^+ K^*_2(1430)\pi S]^*)$

$0.10 \leq t' < 0.15 \text{ (GeV/c)}^2$

$\Im(\rho_{ab}) \times 10^4$

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$\Im(\rho_{ab}) \times 10^4$
Kinematic Distribution of $K^-\pi^-\pi^+$ Events

Subsystem

Also structure in $\pi^-\pi^+$ and $K^-\pi^+$ subsystems

- Successive 2-body decay via $\pi^-\pi^+ / K^-\pi^+$ resonance called isobar

Also structure in angular distributions
Kinematic Distribution of $K^-\pi^-\pi^+$ Events

Subsystem

![Diagram of K^-pi^-pi+ subsystem with an X- meson and K* resonance]

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  - Successive 2-body decay via $\pi^-\pi^+$ / $K^-\pi^+$ resonance called isobar
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Kinematic Distribution of $K^-\pi^-\pi^+$ Events

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Kinematic Distribution of $K^-\pi^-\pi^+$ Events

Subsystem

$1.2 < m_{K\pi\pi} < 1.4 \text{ GeV}/c^2$

$1.5 < m_{K\pi\pi} < 2.0 \text{ GeV}/c^2$

$K^*(892)$

$K^*_0(1430)$

$K^*_2(1430)$

$\rho(770)$

S. Wallner Strange-Meson Spectroscopy with COMPASS 47 / 57
Kinematic Distribution of $K^-\pi^-\pi^+$ Events

$\Delta m_{K^-\pi^-}$

No dominant resonant structures

Preliminary

S. Wallner
Strange-Meson Spectroscopy with COMPASS

Events / (4 MeV/$c^2$) $\times 10^3$
Kinematic Distribution of $K^-\pi^-\pi^+$ Events

t' Spectrum

- Exponential shape
- Shallower for larger $t'$
Kinematic Distribution of $K^{-}\pi^{-}\pi^{+}$ Events

Exclusivity

$E_{\text{beam}}$ [GeV]

$\Delta\phi_{\text{recoil}}$ [deg]

Events / (0.1 GeV) $	imes 10^4$

Events / (0.11 deg) $	imes 10^4$
Kinematic Distribution of $K^-\pi^-\pi^+$ Events

Exclusivity

- $E_{\text{beam}}$ [GeV]
- Events / (0.1 GeV) $\times 10^4$

$\Delta \phi_{\text{recoil}}$ [deg]

Preliminary
$\times 10^5 \quad 0^{-0^+} \rho(770) K\pi$

$0.10 \leq t' < 1.00 \text{ (GeV/c)}^2$

COMPASS Main Studies

Preliminary
Systematic Studies of the Partial-Wave Decomposition

14 Waves

\[ 1^+0^+ \rho(770) K S \]

\[ 0.10 \leq t' < 1.00 \text{ (GeV/c)}^2 \]

COMPASS Main Studies

Preliminary
Systematic Studies of the Partial-Wave Decomposition

14 Waves

$1^+ 1^+ \rho(770) KS$

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COMPASS Main Studies

S. Wallner Strange-Meson Spectroscopy with COMPASS
Systematic Studies of the Partial-Wave Decomposition

14 Waves

$1.0 \ 1.5 \ 2.0 \ 2.5 \ 3.0$

$\times 10^6 \ 2^{+1} K^\ast(892) \pi D$

$0.10 \leq t' < 1.00 \text{ (GeV/c)}^2$

COMPASS Main Studies

Preliminary

S. Wallner Strange-Meson Spectroscopy with COMPASS

Intensity $[(\text{GeV/c}^2)^{-1}] \times 10^6$

$2+1+K^*(892)\pi D$

$0.10 \leq t' < 1.00 \text{ (GeV/c)}^2$

COMPASS Main Studies

Int. $[(\text{GeV/c}^2)^{-1}] \times 10^6$

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COMPASS

Main Studies

Preliminary

S. Wallner Strange-Meson Spectroscopy with COMPASS
Systematic Studies of the Partial-Wave Decomposition

14 Waves

$m_{K\pi\pi}$ [GeV/c$^2$]

Intensity [(GeV/c$^2$)$^{-1}$] $\times 10^5$

$2^{-0^+} K^*(892)\pi F$

$0.10 \leq t' < 1.00$ (GeV/c$^2$)$^2$

COMPASS Main Studies

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14 Waves

\[ 2^{-0^+} \rho(770) KF \]

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COMPASS Main Studies

Preliminary

S. Wallner
Strange-Meson Spectroscopy with COMPASS

Intensity \([\text{GeV}/c^2]^{-1}\) \times 10^5

\[ m_{K\pi\pi} \text{ [GeV}/c^2] \]
Systematic Studies of the Partial-Wave Decomposition

14 Waves

$14 \text{ Waves}$

$1.0 \ 1.5 \ 2.0 \ 2.5 \ 3.0$

$m_{K\pi\pi} \ [\text{GeV/c}^2]$

$0 \ 2 \ 4$

Intensity $[(\text{GeV/c}^2)^{-1}] \times 10^5$

$2^{-0^+} K^*_2(1430) \pi S$

$0.10 \leq t' < 1.00 \ (\text{GeV/c})^2$

COMPASS Main Studies

Preliminary

S. Wallner Strange-Meson Spectroscopy with COMPASS 51 / 57
Systematic Studies of the Partial-Wave Decomposition

14 Waves

\[ \times 10^5 \quad 2^{-0^+} f_2(1270) K S \]

\[ 0.10 \leq t' < 1.00 \text{ (GeV/c)}^2 \]

COMPASS Main Studies

Preliminary

S. Wallner
Strange-Meson Spectroscopy with COMPASS

51 / 57
Systematic Studies of the Partial-Wave Decomposition

$14$ Waves

\[
\times 10^4 \quad 3^+ 0^+ K_3^*(1780) \pi S
\]

$0.10 \leq t' < 1.00 \text{ (GeV}/c)^2$

COMPASS Main Studies

\[\text{Intensity } [(\text{GeV}/c^2)^{-1}] \times 10^4 \]

\[3^+ 0^+ K_3^*(1780) \pi S \]

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COMPASS Main Studies

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\[3^+ 0^+ K_3^*(1780) \pi S \]

$0.10 \leq t' < 1.00 \text{ (GeV}/c)^2$

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$0.10 \leq t' < 1.00 \text{ (GeV}/c)^2$
Systematic Studies of the Partial-Wave Decomposition

$14 \text{ Waves}$

$3^{+} 1^{+} K_{2}^{*}(1430) \pi P$

$0.10 \leq t' < 1.00 \text{ (GeV/c)}^2$

COMPASS

Main Studies

$\times 10^4$

Intensity $[(\text{GeV/c}^2)^{-1}]$

$0 \leq m_{K\pi\pi} \text{ [GeV/c}^2]\leq 3.0$

$3+1+K_2^*(1430)\pi P$

$0.10 \leq t' < 1.00 \text{ (GeV/c)}^2$

COMPASS

Main Studies
Systematic Studies of the Partial-Wave Decomposition

14 Waves

\[ 4^{+}1^{+} K^{*}(892) \pi G \]

\[ 0.10 \leq t' < 1.00 \text{ (GeV/c)}^2 \]

COMPASS Main Studies

Preliminary
Systematic Studies of the Partial-Wave Decomposition

14 Waves

\[ \times 10^4 \quad 4^+1^+ \rho(770) KG \]

\[ 0.10 \leq t' < 1.00 \ (\text{GeV/c})^2 \]

COMPASS Main Studies

\[ \text{Intensity} \quad [(\text{GeV/c}^2)^{-1}] \times 10^4 \]

\[ 4^+1^+ \rho(770) KG \]

\[ m_{K\pi\pi} \quad [\text{GeV/c}^2] \]
Systematic Studies of the Partial-Wave Decomposition

14 Waves

\[ 4^{-0+} K_2^*(1430) \pi D \]

\[ 0.10 \leq t' < 1.00 \text{ (GeV/c)}^2 \]

COMPASS Main Studies

Preliminary

S. Wallner Strange-Meson Spectroscopy with COMPASS
Systematic Studies of the Partial-Wave Decomposition

Leakage Waves

\[ m_{K\pi\pi} \ [\text{GeV}/c^2] \]

Intensity \[ (\text{GeV}/c^2)^{-1} \times 10^6 \]

0.10 \leq t' < 1.00 (GeV/c)^2

COMPASS Main Studies

S. Wallner Strange-Meson Spectroscopy with COMPASS
Systematic Studies of the Partial-Wave Decomposition

Leakage Waves

\[ \times 10^7 \quad 0^{-0^+} K^*(892) \pi P \]

\[ 0.10 \leq t' < 1.00 \text{ (GeV/c)}^2 \]

COMPASS

Main Studies

Preliminary

Intensity \([(\text{GeV/c}^2)^{-1}] \times 10^7 \]

\[ 0^{-0^+} K^*(892) \pi P \]

\[ 0.10 \leq t' < 1.00 \text{ (GeV/c)}^2 \]

COMPASS

Main Studies
Systematic Studies of the Partial-Wave Decomposition

Leakage Waves

\[ 1.0 \ 1.5 \ 2.0 \ 2.5 \ 3.0 \]

\[ m_{K\pi\pi} \ [\text{GeV}/c^2] \]

\[ \times 10^7 \]

\[ 1^{+0+} K^*(892) \pi S \]

\[ 0.10 \leq t' < 1.00 \ (\text{GeV}/c)^2 \]

COMPASS Main Studies

Preliminary
Systematic Studies of the Partial-Wave Decomposition
Leakage Waves

\[ 3^{+1+} K^*(892) \pi D \]

\[ 0.10 \leq t' < 1.00 \text{ (GeV/c)}^2 \]

COMPASS Main Studies

Preliminary

S. Wallner
Strange-Meson Spectroscopy with COMPASS
Leakage Effect

- Unexpected low-mass enhancement in $3^+ 1^+ K^*(892)\pi D$ wave
- Similar to dominant $1^+$ wave
- Sensitive to systematic effects
- Decay amplitudes of different $J^P$ are orthogonal
- Event selection requires to identify one of the two negative particles
  - Limited acceptance due to limited kinematic range of final-state PID
- Loss of orthogonality taking acceptance into account
  - Reduced differentiability of certain partial waves
- Only a sub-set of partial waves affected
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![](image.png)

- $m_{K \pi \pi}$ [GeV/c$^2$]
- $1^{0+} K^{*}(892) \pi S$
- $0.10 \leq t' < 1.00$ (GeV/c$^2$)

COMPASS Main Studies

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\[
\bar{s} \to \bar{p} l^- l^+
\]

\[
\sum_{i=1}^{3} I_{\text{int}}(m_{K\pi\pi}) \times 10^7
\]

\[
0.10 \leq t' < 1.00 \text{(GeV/c)}^2
\]

COMPASS Main Studies

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\[ I_{a,b} = \int d\varphi_3(\tau) \Psi_a(\tau)\Psi^*_b(\tau) \]
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Incoherent $\pi^- \pi^- \pi^+$ Background

- $K^- \pi^- \pi^+$ and $\pi^- \pi^- \pi^+$ similar experimental footprint

- Distinguishable only by
  - Beam particle identification
  - Final-state particle identification

- Excellent beam PID:
  - Expect small contamination from beam $\pi^-$

- Final-state PID does not suppress $\pi^- \pi^- \pi^+$ background
  - Non-negligible $\pi^- \pi^- \pi^+$ background in $K^- \pi^- \pi^+$ sample of about 7% 
  - Dominant background in $K^- \pi^- \pi^+$ sample
Incoherent $\pi^-\pi^-\pi^+$ Background

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Incoherent $\pi^-\pi^-\pi^+$ Background

- Well established model for $\pi^- + p \rightarrow \pi^-\pi^-\pi^+ + p$
  - From very same data set
  - Measured with high precision
  - Acceptance corrected
- Generate $\pi^-\pi^-\pi^+$ Monte Carlo sample
- Mis-interpret $\pi^-\pi^-\pi^+$ Monte Carlo events as $K^-\pi^-\pi^+$
  - Apply wrong mass assumption
  - Same event reconstruction and selection as for $K^-\pi^-\pi^+$
- Perform partial-wave decomposition of mis-interpreted $\pi^-\pi^-\pi^+$ Monte Carlo sample
  - Using the same PWA model as for measured $K^-\pi^-\pi^+$ sample
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- Study $\pi^-\pi^-\pi^+$ background in individual $K^-\pi^-\pi^+$ partial waves
Incoherent $\pi^-\pi^-\pi^+$ Background

- Significant contribution to waves with $\rho(770)$ isobar
- $\pi^-\pi^-\pi^+$ produces peaking structures
- Largest relative contribution to $2^+ 1^+ \rho(770) K D$ wave
- Small contribution to waves with $K^*(892)$ isobar
- Also significant contribution to waves with $f_2(1270)$ and $K^*_2(1430)$ isobars
- No contribution to flat wave

\[ K^-\pi^-\pi^+ \text{ data, } \pi^-\pi^-\pi^+ \text{ pseudo data} \]
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![Graph showing $2^{-0} f_2(1270) K S$ intensity vs. $m_{K\pi\pi}$ and $t'$ range]

$K^-\pi^-\pi^+$ data, $\pi^-\pi^-\pi^+$ pseudo data
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Incoherent $\pi^- \pi^- \pi^+$ Background

- 238-wave set can describe main features of $\pi^- \pi^- \pi^+$ pseudodata sufficiently well
- Largest deviation for $K^- \pi^+$ isobar system at thigh $m_{K\pi\pi}$

$\pi^- \pi^- \pi^+$ pseudo data, prediction (weighted-MC) of $K^- \pi^- \pi^+$ PWD to $\pi^- \pi^- \pi^+$ pseudo data