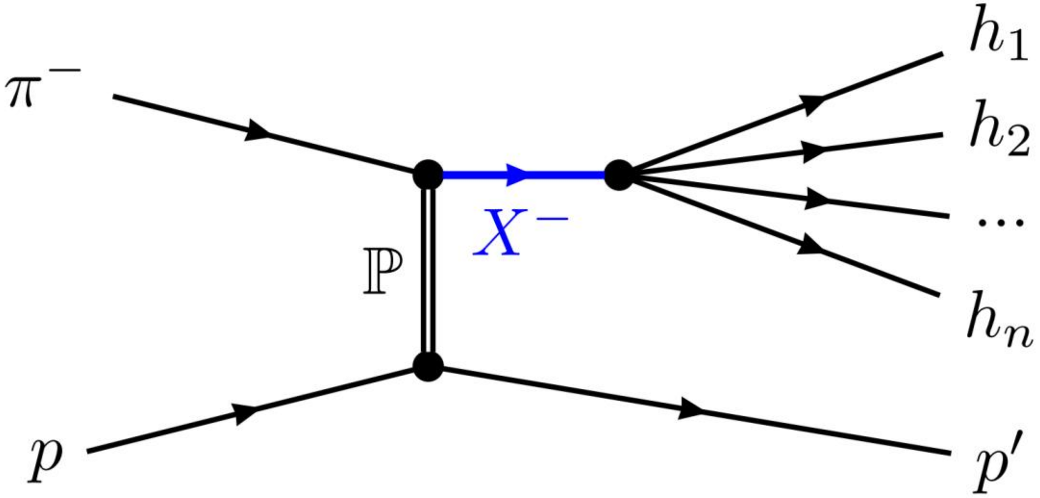
Partial-Wave Analysis of the $K_S^0 K^-$ Final State: Ambiguities and Physics Results

Julien Beckers for the COMPASS Collaboration
julien.beckers@tum.de

PWA/ATHOS 2024
May 28th, 2024

Excited Light Mesons at COMPASS

- Inelastic scattering reactions of high-energetic meson beam



- Strong interaction (Pomeron exchange) between beam meson and target proton
- Intermediate hadronic resonances X^- are created, then decay into n-body final state

  → wide range of allowed (spin) quantum numbers

- Final-state particles measured in the COMPASS spectrometer

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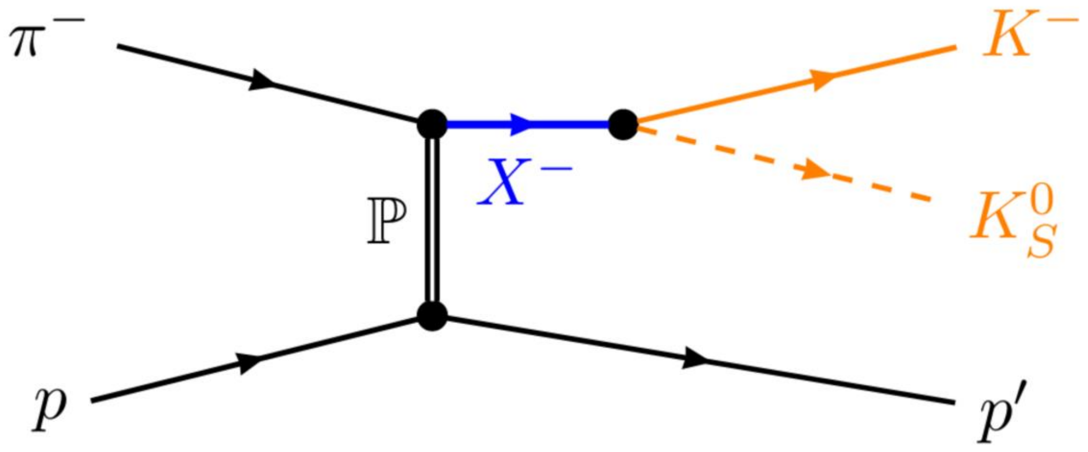

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Large-acceptance magnetic spectrometer @ CERN-SPS

Beam:

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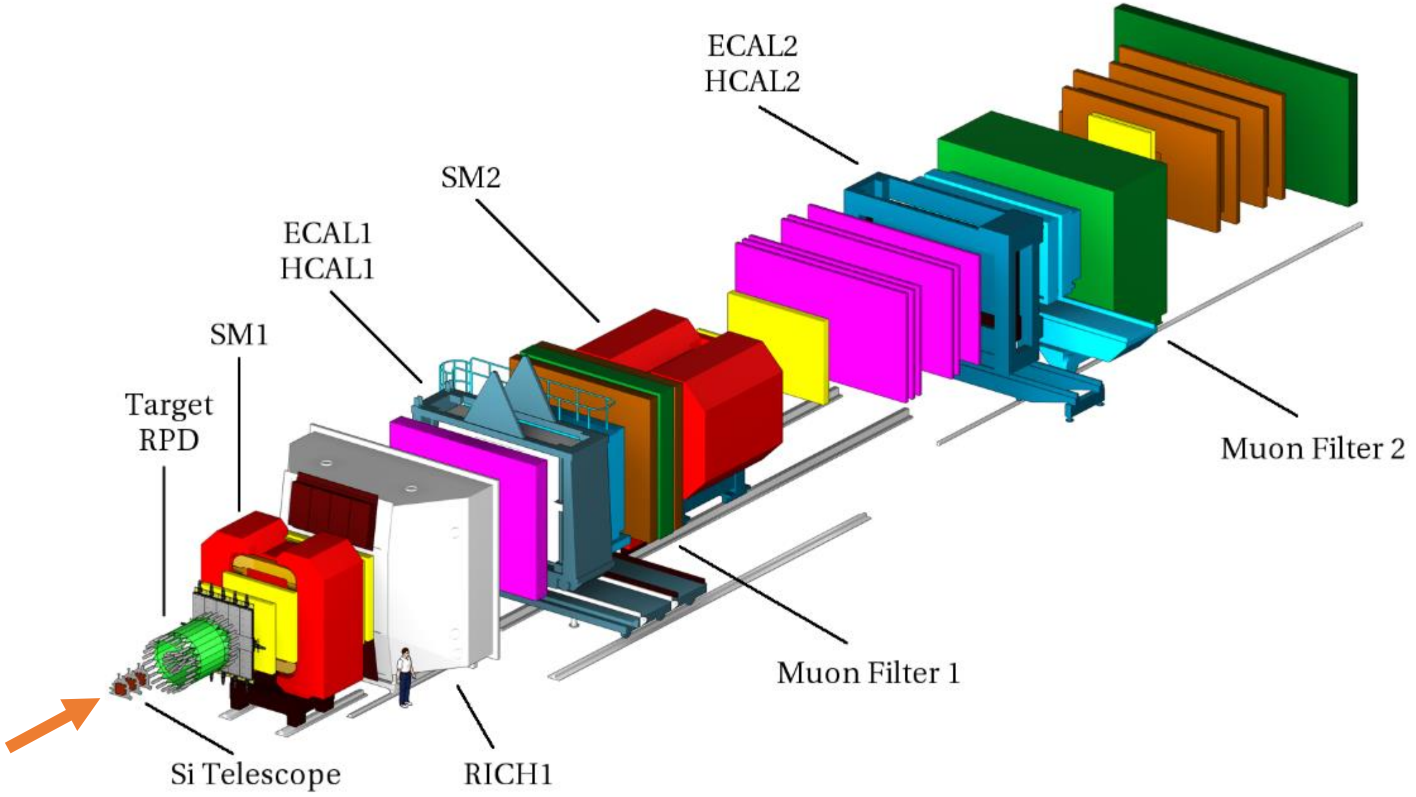
Spectrometer:

- Liquid-hydrogen target
- Two-stage spectrometer setup around two dipole magnets SM1/2



From COMPASS Collab., The COMPASS Setup for Physics with Hadron Beams (Nucl. Instrum. Methods Phys. Res. A 779 (2014), pp. 69–115)

Quantum Numbers of the $K_S^0 K^-$ Final State

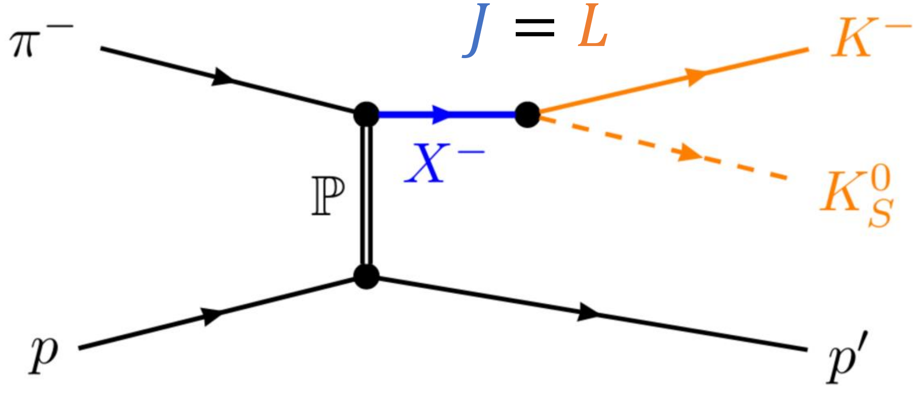
- Quantum numbers of a $K_S^0 K^-$ system

$$I = 1$$

$$G = (-1)^{L+1}$$

$$P = (-1)^L \qquad C = G(-1)^I = (-1)^L$$

all given by L!



$$I^G J^{PC} = 1^+1^{--}, 1^-2^{++}, 1^+3^{--}, 1^-4^{++}, \ldots$$

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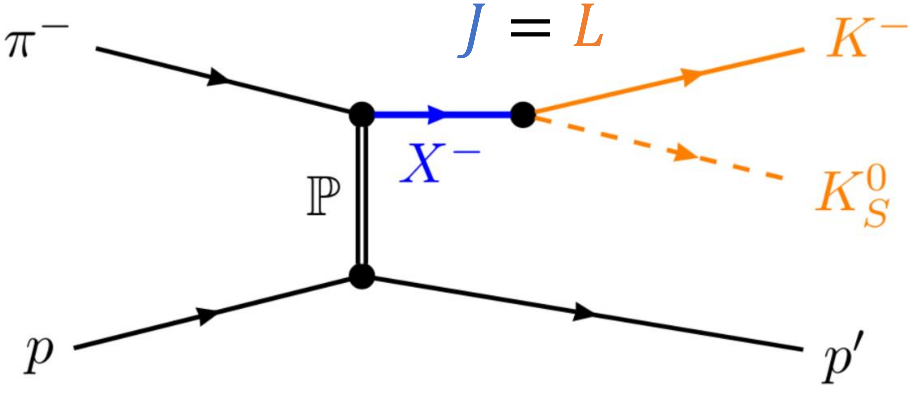
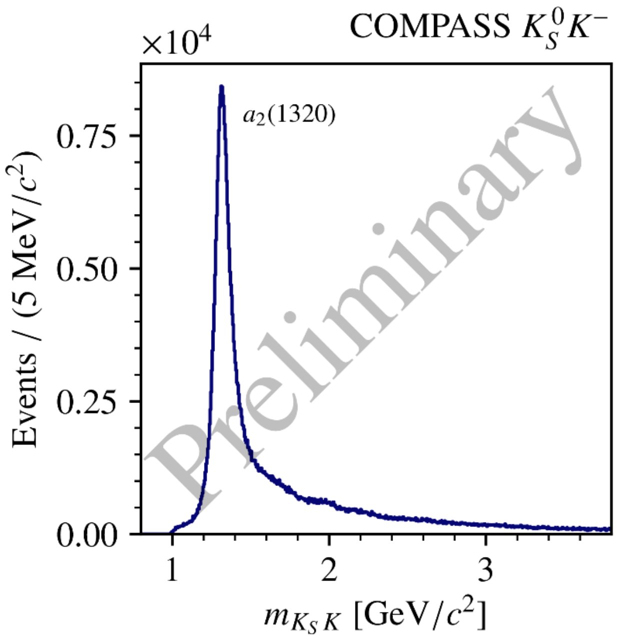


- Quantum numbers of the produced X^-:

$I^G = 1^-$ for Pomeron ($I^G = 0^+$) exchange

→ odd spins J are suppressed!

$$I^G J^{PC} = 1^+1^{--}, 1^-2^{++}, 1^+3^{--}, 1^-4^{++}, \ldots$$



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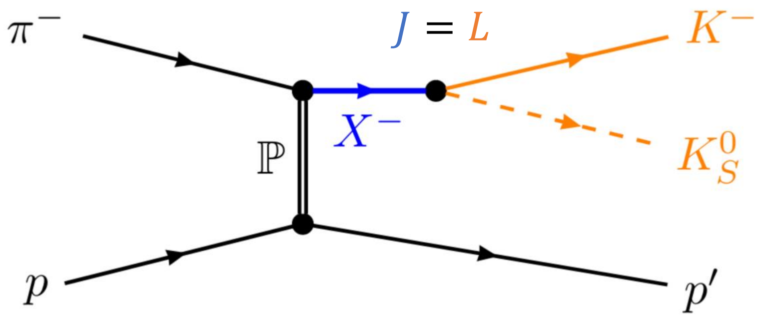
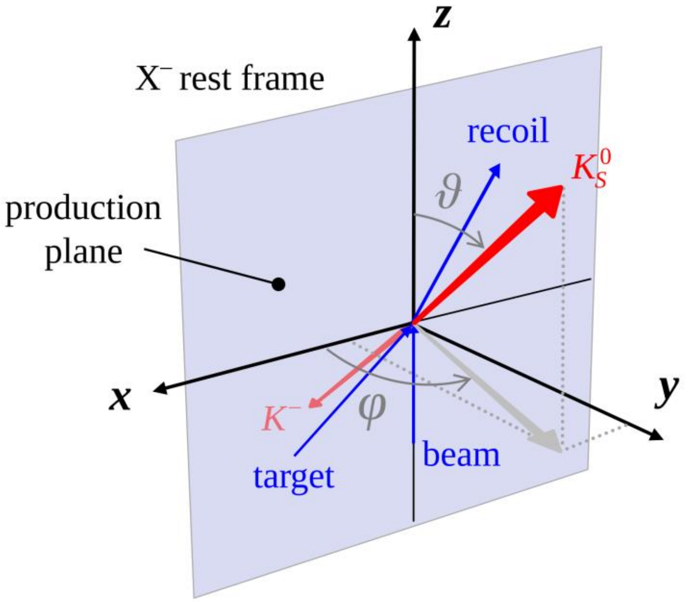
Partial-Wave Analysis Procedure

$$A(m_{KK}, t'; \theta, \phi) = \sum_{J,M} T_{J,M}(m_{KK}, t')\, \psi_{J,M}(\theta, \phi)$$

- Decompose total process amplitude into **partial waves**
  - Depend on spin J and spin-projection M
  - Other quantum numbers fixed by J, M





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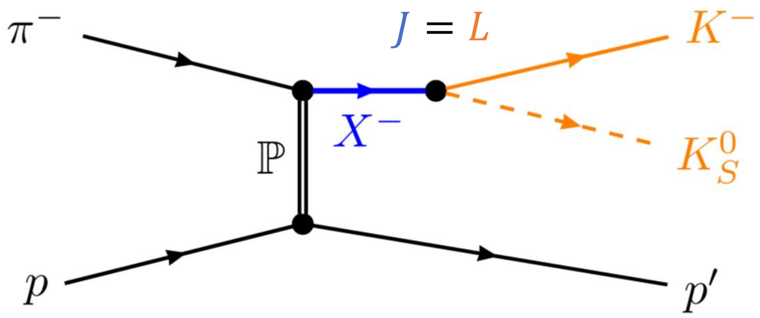

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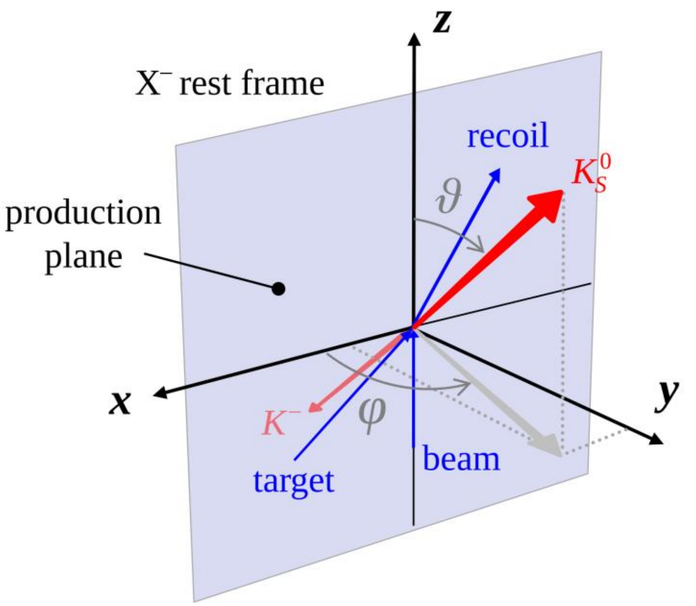

- Partial-wave amplitudes split into
  - production and propagation → $T_{J,M}(m_{KK}, t')$

  and
  - decay of X^- → $\psi_{J,M}(\theta, \phi) = Y_J^M(\theta, \phi)$

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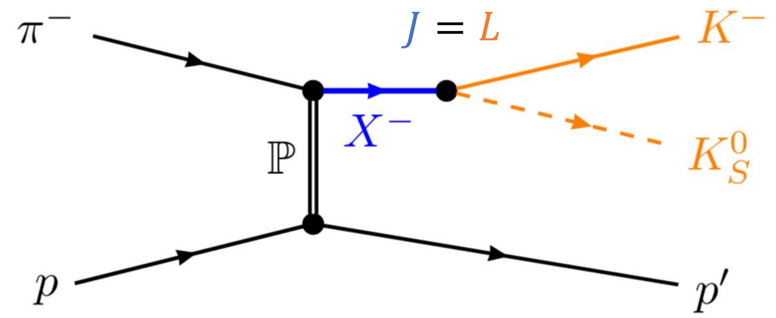

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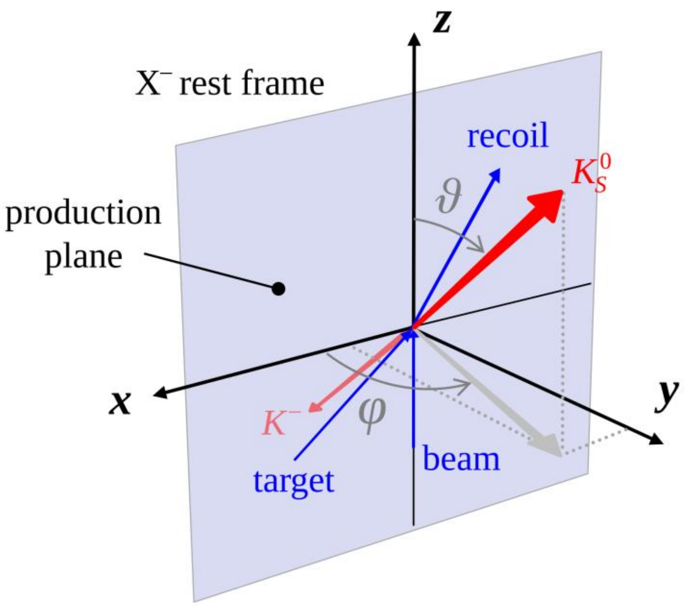
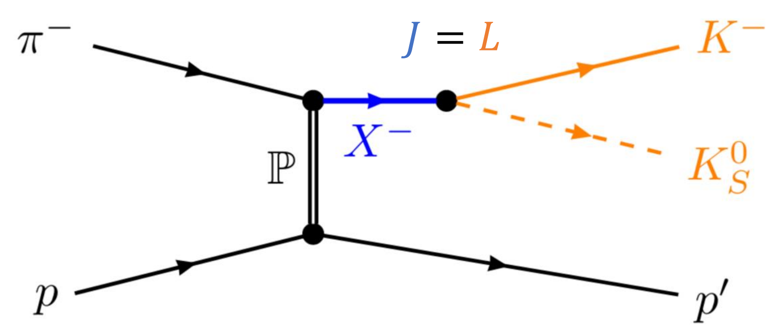


Partial-Wave Analysis Procedure

$$\frac{dN}{d\Phi(\theta,\phi)} \sim I(m_{KK}, t'; \theta, \phi) = \left| \sum_{J,M} T_{J,M}(m_{KK}, t')\, Y_J^M(\theta, \phi) \right|^2$$



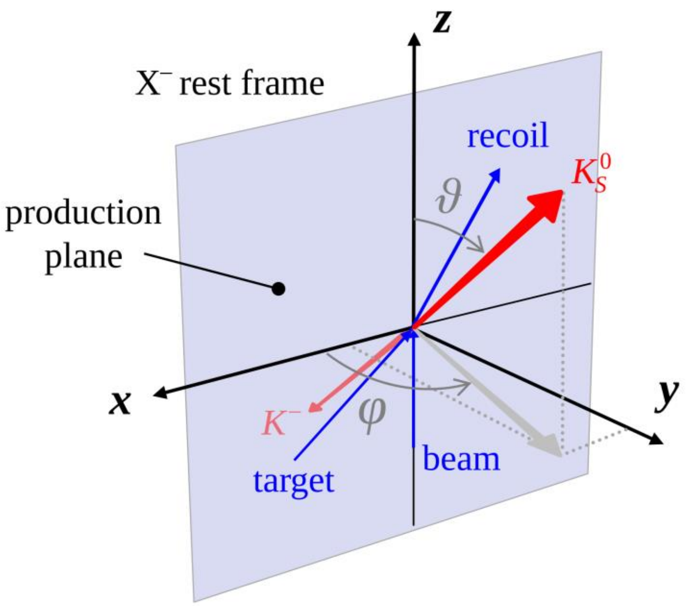
- Decompose total process amplitude into **partial waves**

- Amplitudes split into production/propagation and decay of X^-

- Dependence on (m_{KK}, t') unknown

→ fit $I(m_{KK}, t'; \theta, \phi)$ to data **in** $(\boldsymbol{m_{KK}}, \boldsymbol{t'})$ **bins**

→ extract constant $\{T_{J,M}\}$ in each bin



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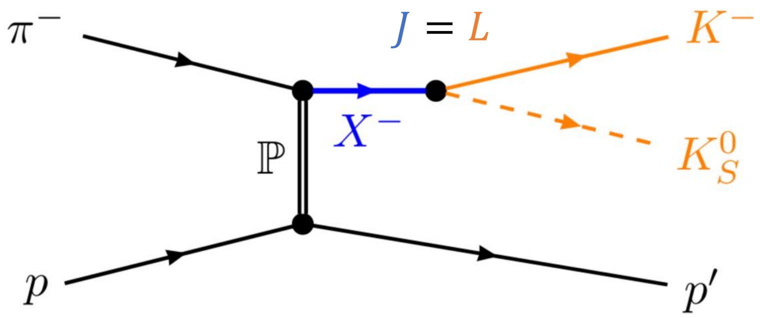

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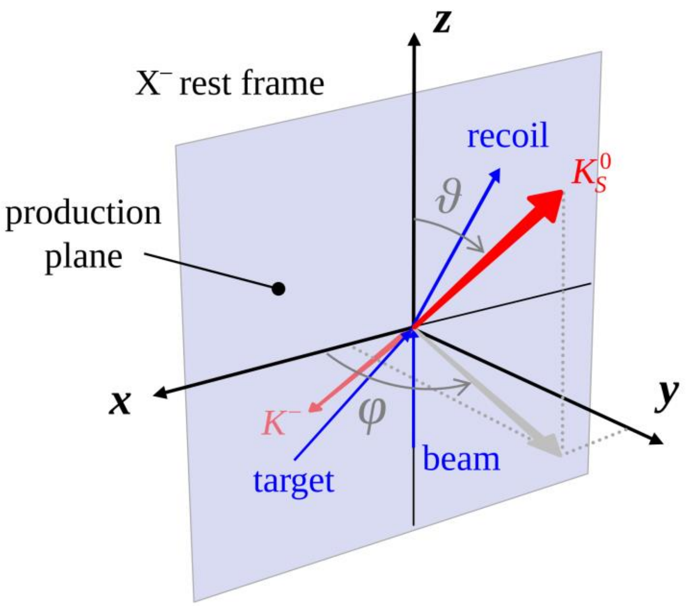

Ambiguities in the Partial-Wave Decomposition

For any final state with **two spinless** particles ($\pi\pi, KK, \eta\pi$, ...):

- Decomposition of intensity into $\{T_{J,M}\}$ is not **unique**

→ Several sets of $\{T_{J,M}\}$ lead to the **same** $\boldsymbol{I(\theta,\phi)}$ in each (m_{KK}, t') bin

$$I(\theta,\phi) = \left|\sum_{J,M} T_{J,M}^{(1)}\, Y_J^M(\theta,\phi)\right|^2 = \left|\sum_{J,M} T_{J,M}^{(2)}\, Y_J^M(\theta,\phi)\right|^2$$

- The fit cannot distinguish between the **mathematically equivalent** solutions!

Chung, PRD 56 7299–7316 (1997)

Barrelet, *Nuov Cim A* **8**, 331–371 (1972)

Ambiguities in the Partial-Wave Decomposition

$$I(\theta,\phi) = \left|\sum_{J,M} T_{J,M}\, Y_J^M(\theta,\phi)\right|^2$$

Assume strong dominance of $|M| = 1$ *

- Pomeron exchange dominant $\rightarrow M \neq 0$
- Higher $|M|$ suppressed

*using reflectivity basis for ψ_{JM} :
doi.org/10.1103/PhysRevD.11.633

Ambiguities in the Partial-Wave Decomposition

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$$I(\theta,\phi) = \left|\sum_J T_J\, Y_J^1(\theta,\phi)\right|^2 = \underbrace{\left|\sum_J T_J\, Y_J^1(\theta,0)\right|^2}_{\boldsymbol{a(\theta)}} |\sin\phi|^2$$

$$Y_J^1(\theta,0) = \sum_{j=0}^{J-1} y_j \tan^{2j}\theta$$

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Polynomial in $\tan^2\theta$

$$a(\theta) = \sum_{j=0}^{\boldsymbol{J_{\max}-1}} c_j(\{T_J\})\, \tan^{2j}(\theta) = c(\{T_J\}) \prod_{k=1}^{J_{\max-1}} \left(\tan^2(\theta) - \boldsymbol{r_k}(\{T_J\})\right)$$

root decomposition

"Barrelet zeros"

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$\sum_J T_J\, Y_J^1(\theta,0) = \boldsymbol{a(\theta)}$

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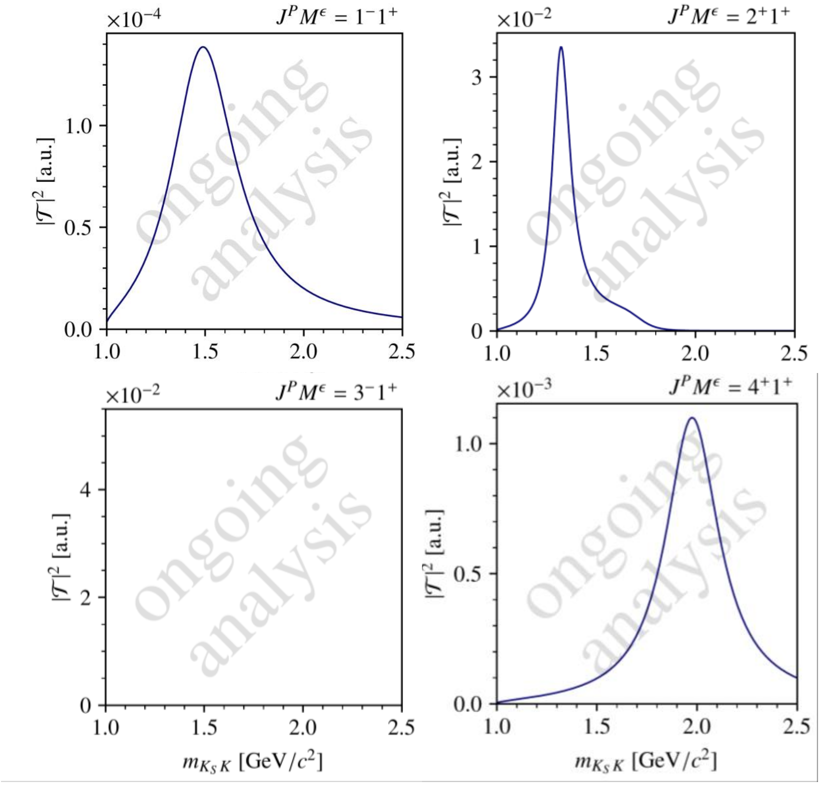
$r_k^* \rightarrow \{c_j'\} \rightarrow \{T_J'\} \neq \{T_J\}$

Study of the Ambiguities

I. Continuous amplitude model

How do the ambiguous solutions look like (**continuity, signals, …**)?

- Create an amplitude model for four partial waves
- Sample points in m_{KK} and calculate ambiguous solutions



Continuous Amplitude Model

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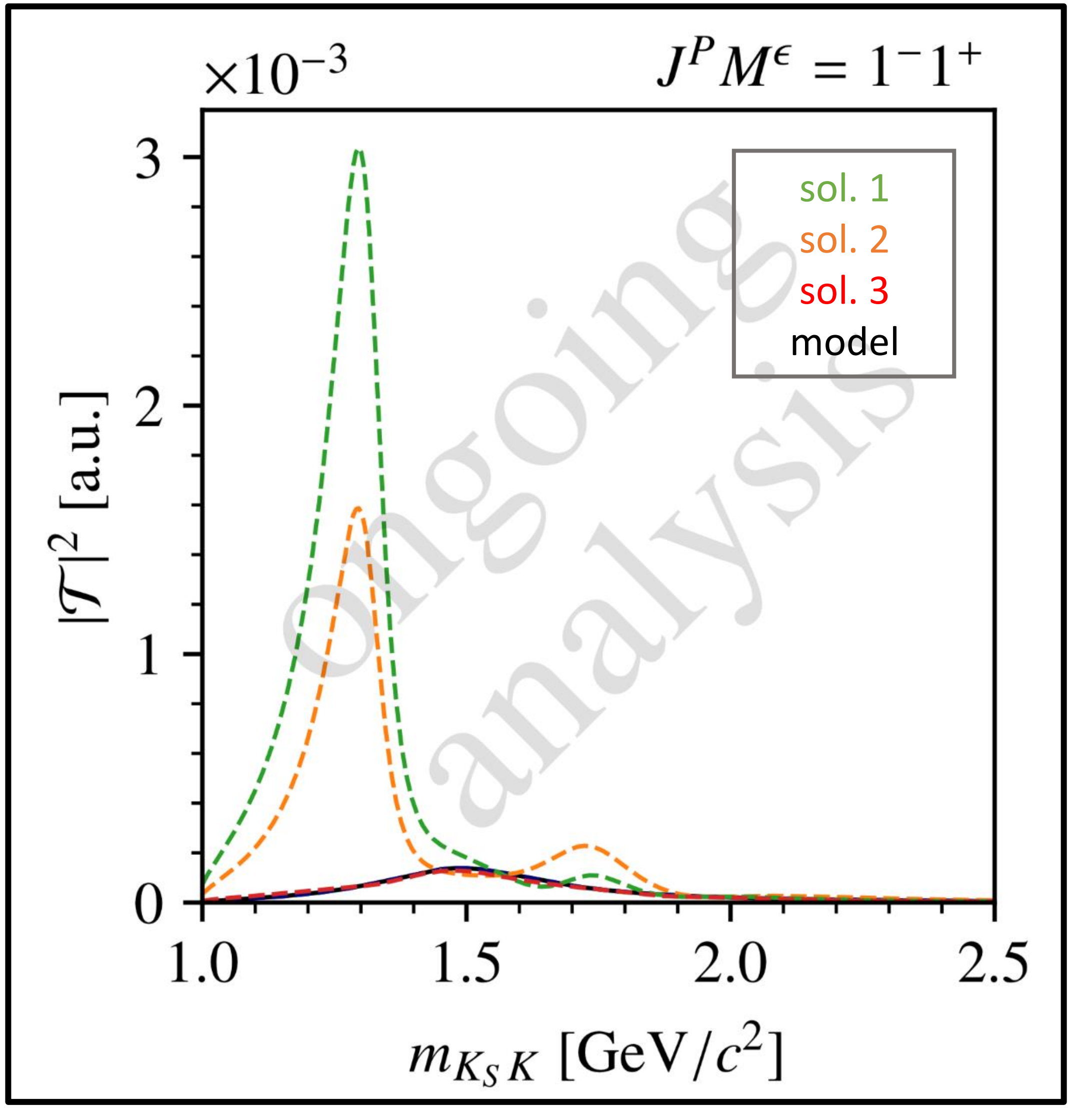
- Not all solutions are different from each other!
- Highest-spin (4^{++}) intensity is invariant!



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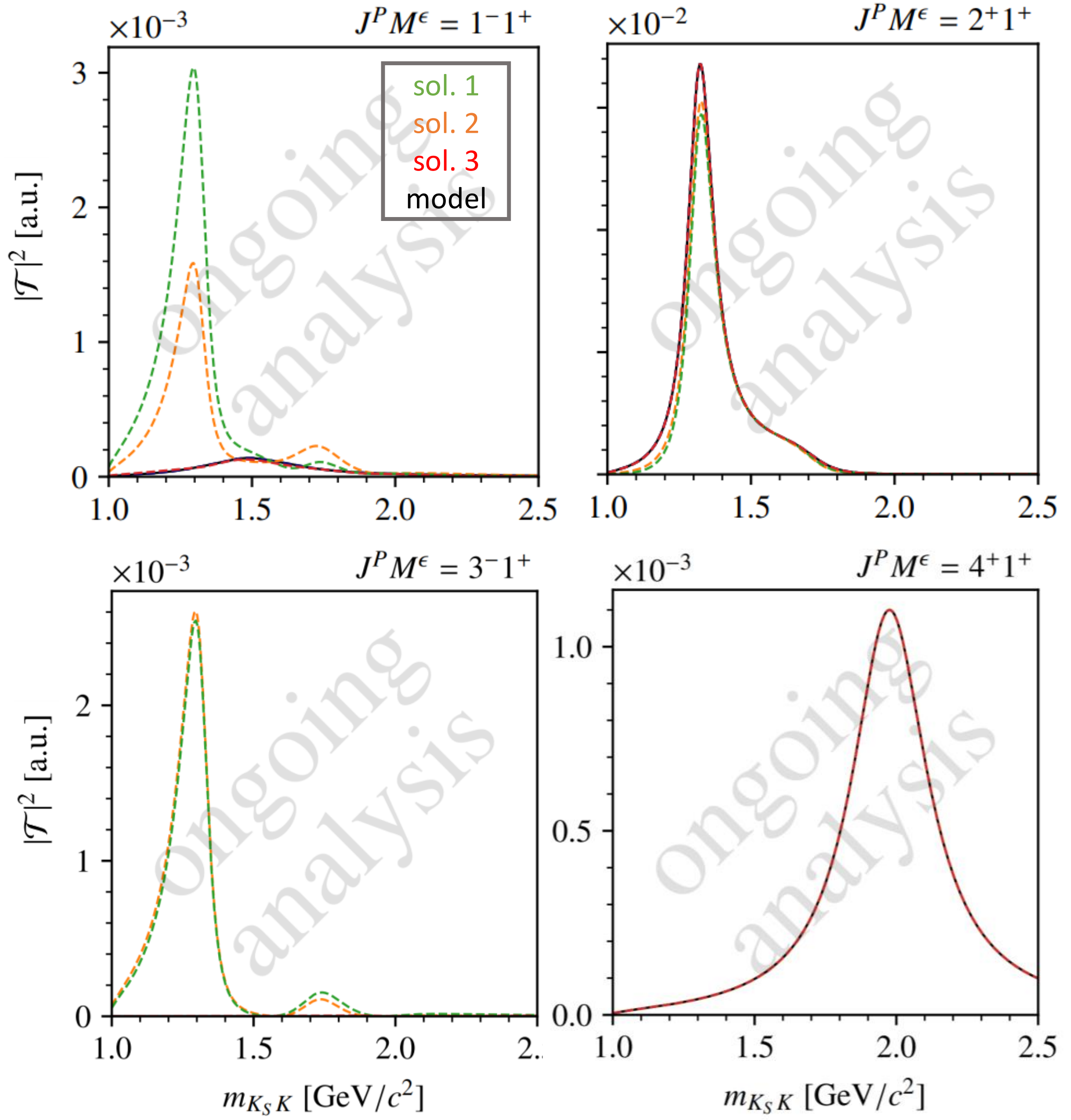

Partial-Wave Decomposition Fits on Pseudodata

II. Finite pseudo-data

- reality: **finite data** and **amplitudes unknown**
- generate pseudo-data according to model
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→ 3000 attempts with random start values

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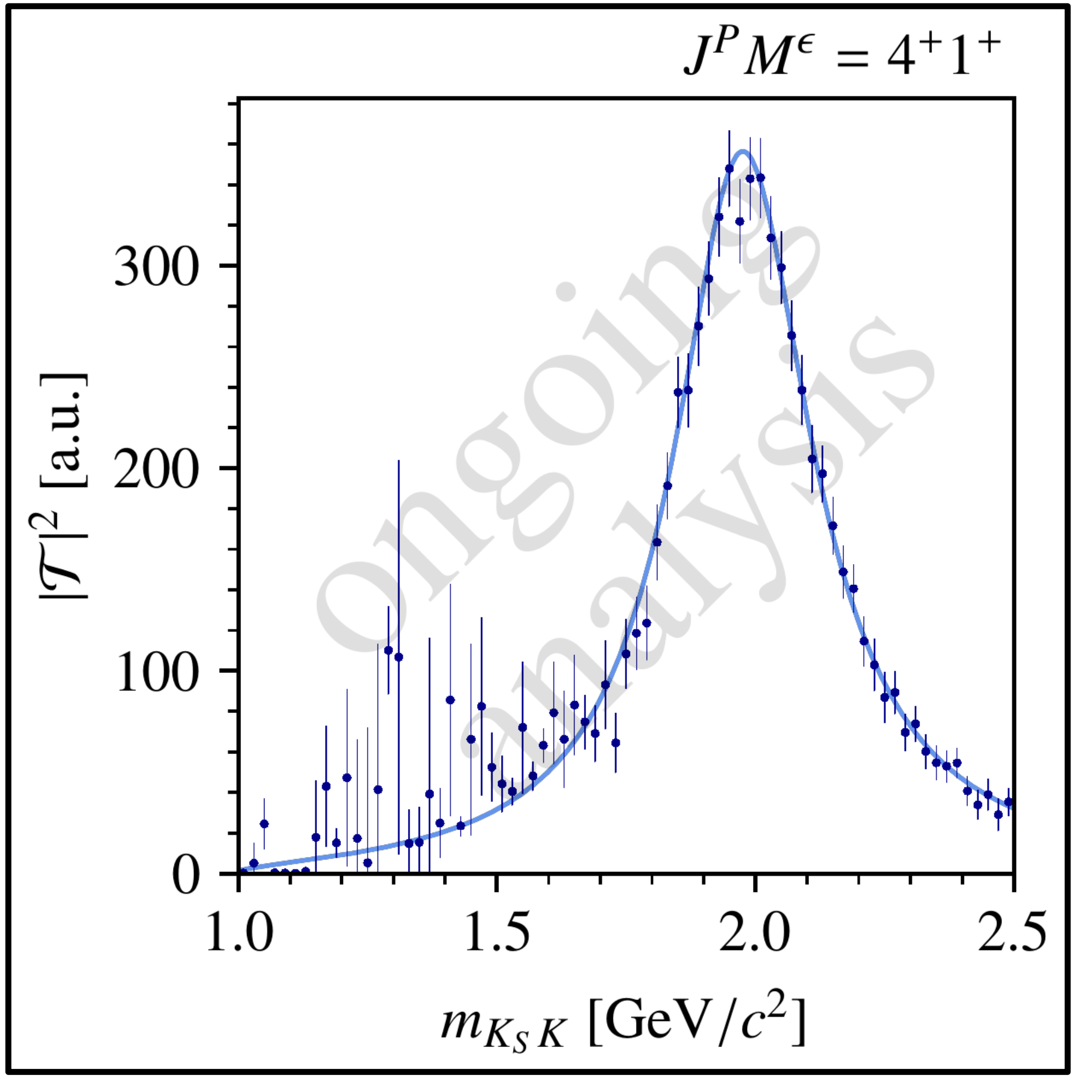
→ PWD fit distorts the intensity distribution!



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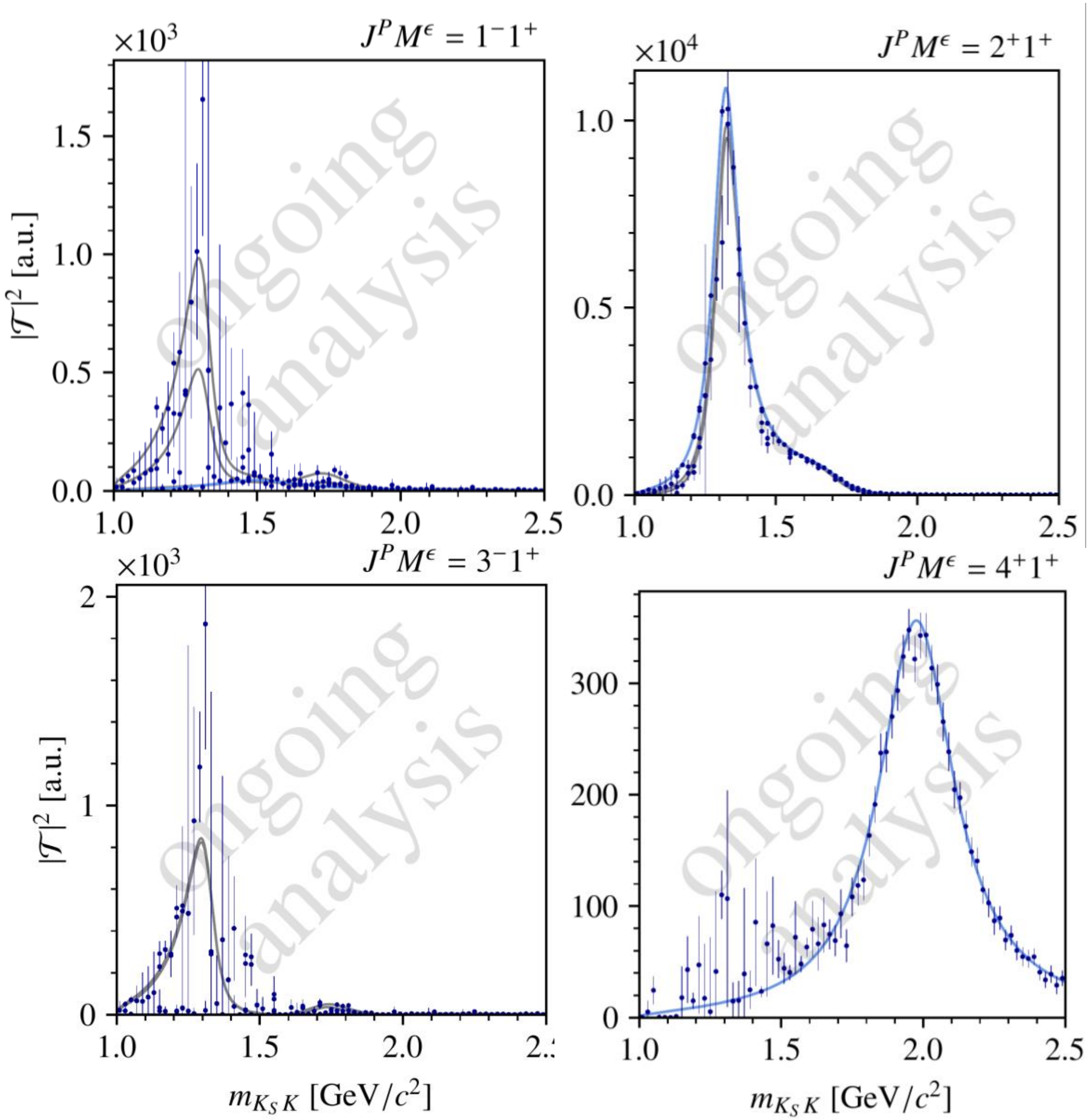

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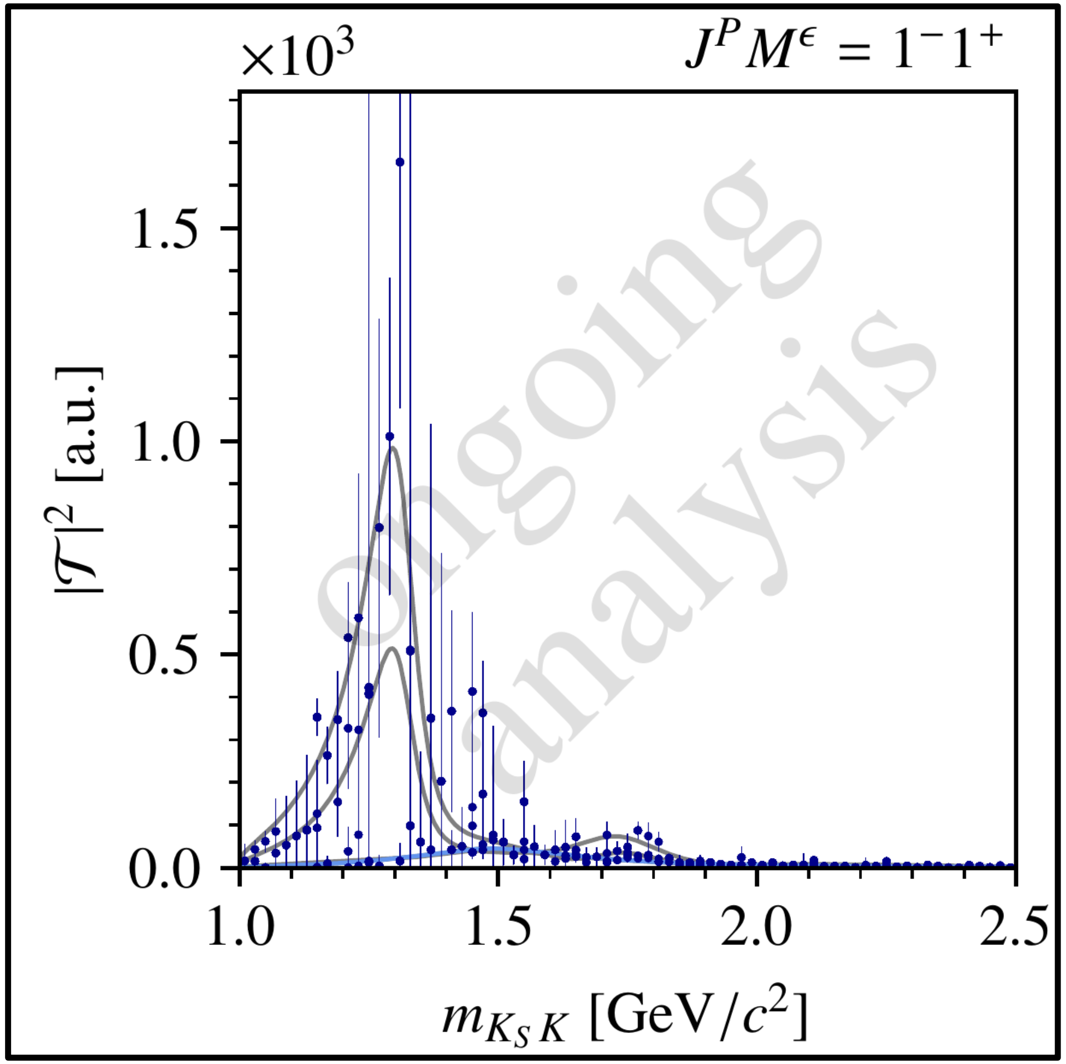

Reducing the Ambiguities

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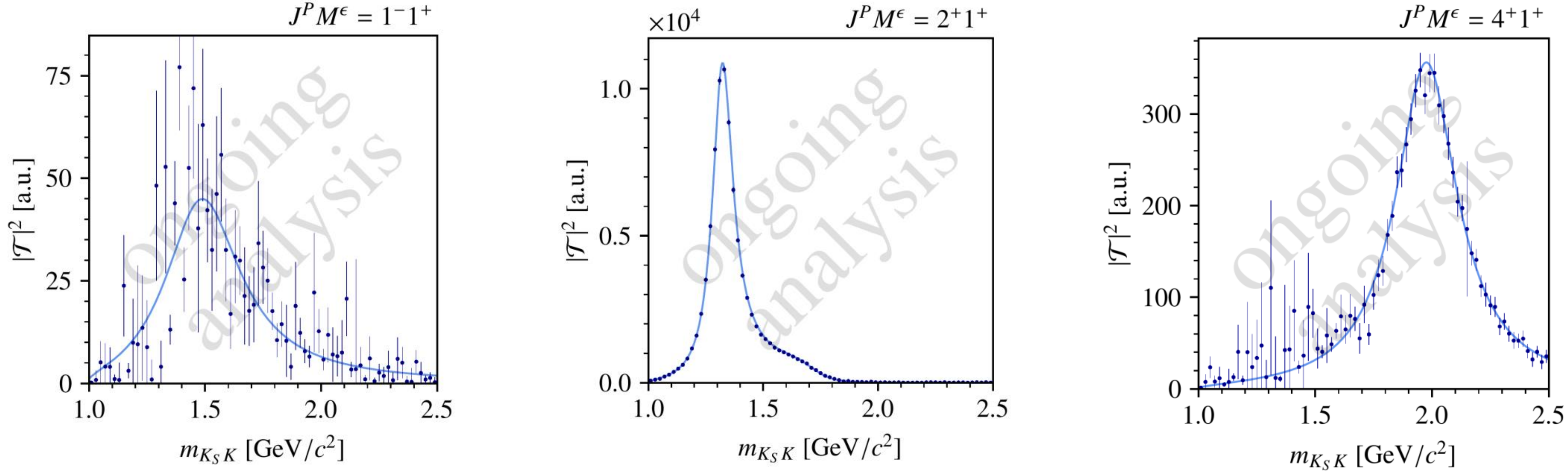
- Remove one wave with $J < J_{\max}$ → **resolves ambiguities**



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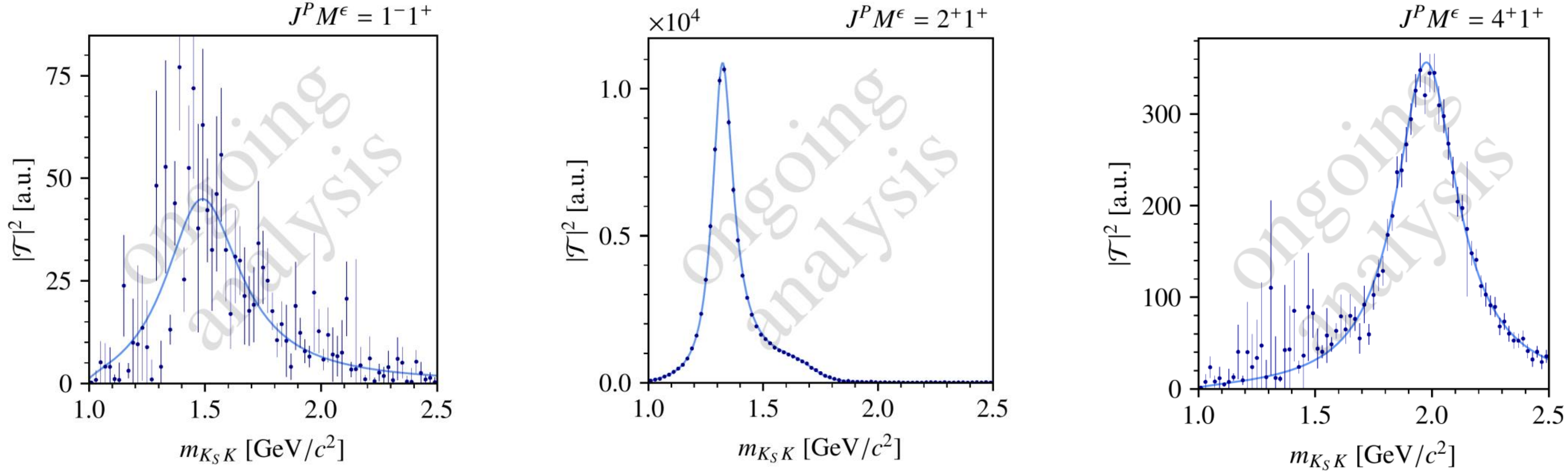


At COMPASS: **odd-spin waves strongly suppressed**!

→ <u>assume no ambiguities in the decomposition!</u>

- Except: Two solutions in the phases of the partial waves

$$I(\theta,\phi) = \left|\sum_J T_J \, Y_J^1(\theta,\phi)\right|^2 = \left|\sum_J T_J^* \, Y_J^1(\theta,\phi)\right|$$

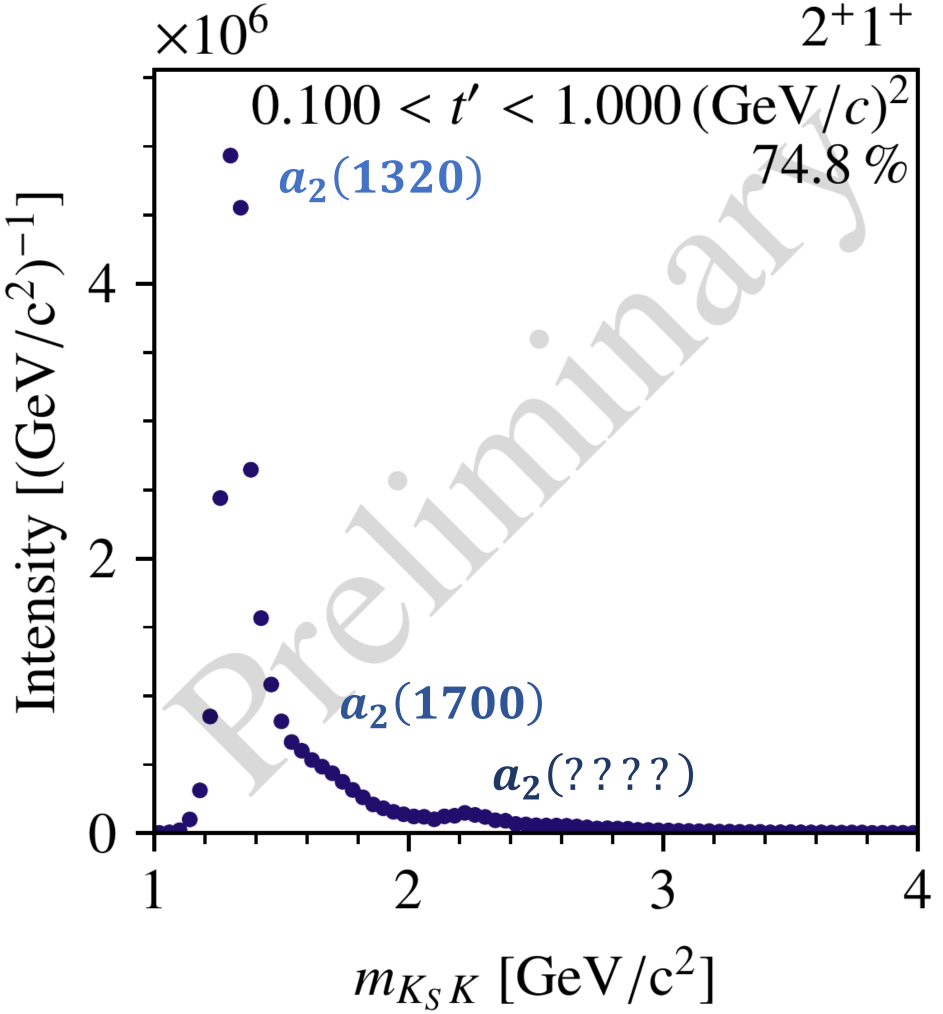
Partial-Wave Analysis of the $K_S^0 K^-$ Final State at COMPASS: Physics Results

J^{PC}	M^{ϵ}	Resonances
$\mathbf{2}^{++}$	1^+	a_2-like
	2^+	
$\mathbf{4}^{++}$	1^+	a_4-like
$\mathbf{6}^{++}$	1^+	a_6-like
$(\mathbf{1}^{--}$	1^+	ρ-like)

→ later

Previous study by . E. Cleland et al. “Resonance Production in the Reaction π±p →K0K±p at 30 GeV/c and 50 GeV/c”. In: Nucl. Phys. B 208 (1982)

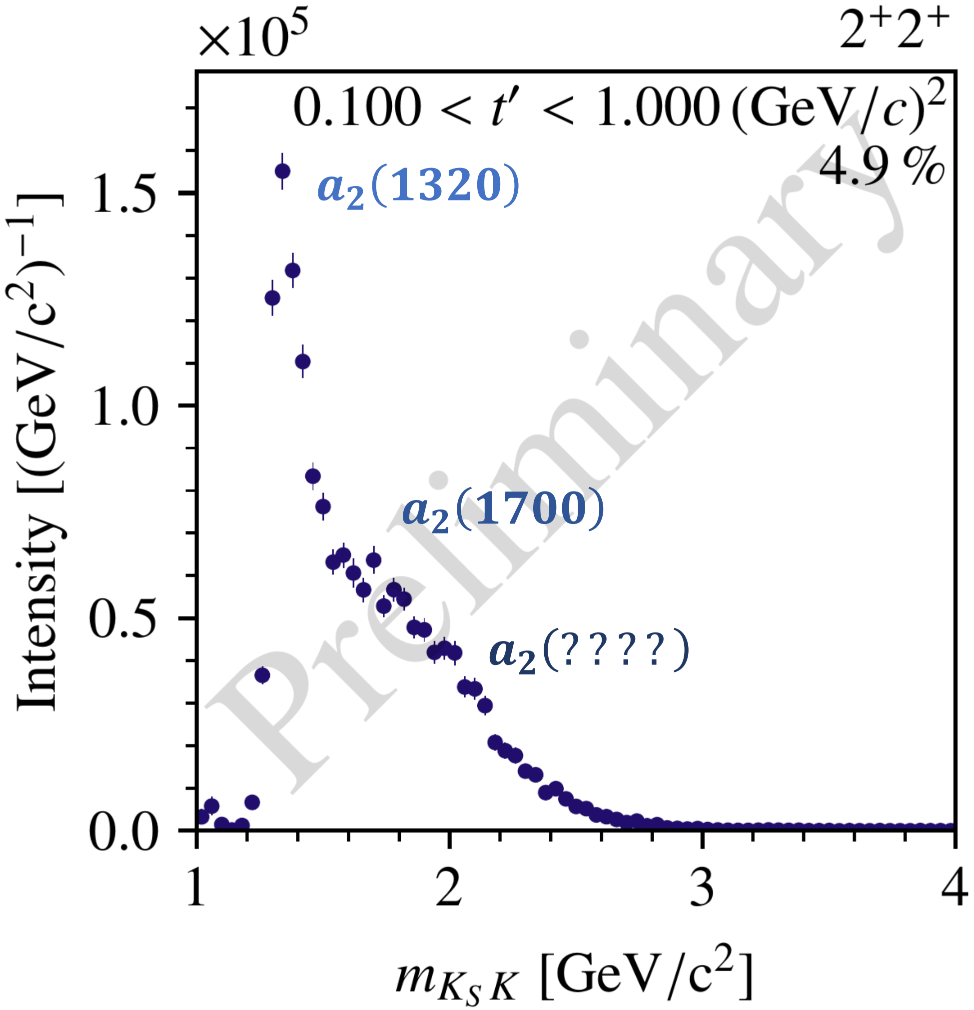
Results: Intensities and Phases



$a_2(1320)$
$m_{PDG} = 1.32$ GeV
$\Gamma_{PDG} = 0.10$ GeV

$a_2(1700)$
$m_{PDG} = 1.71$ GeV
$\Gamma_{PDG} = 0.38$ GeV

$a_2(????)$



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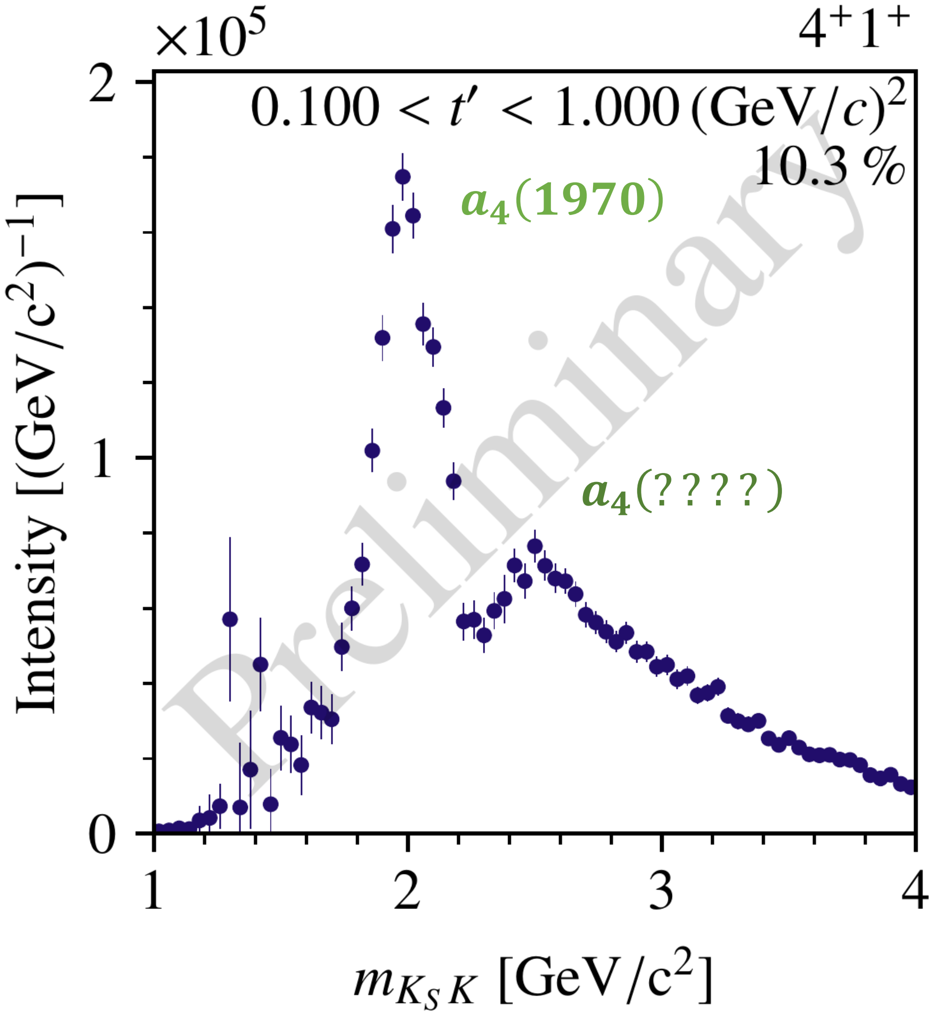
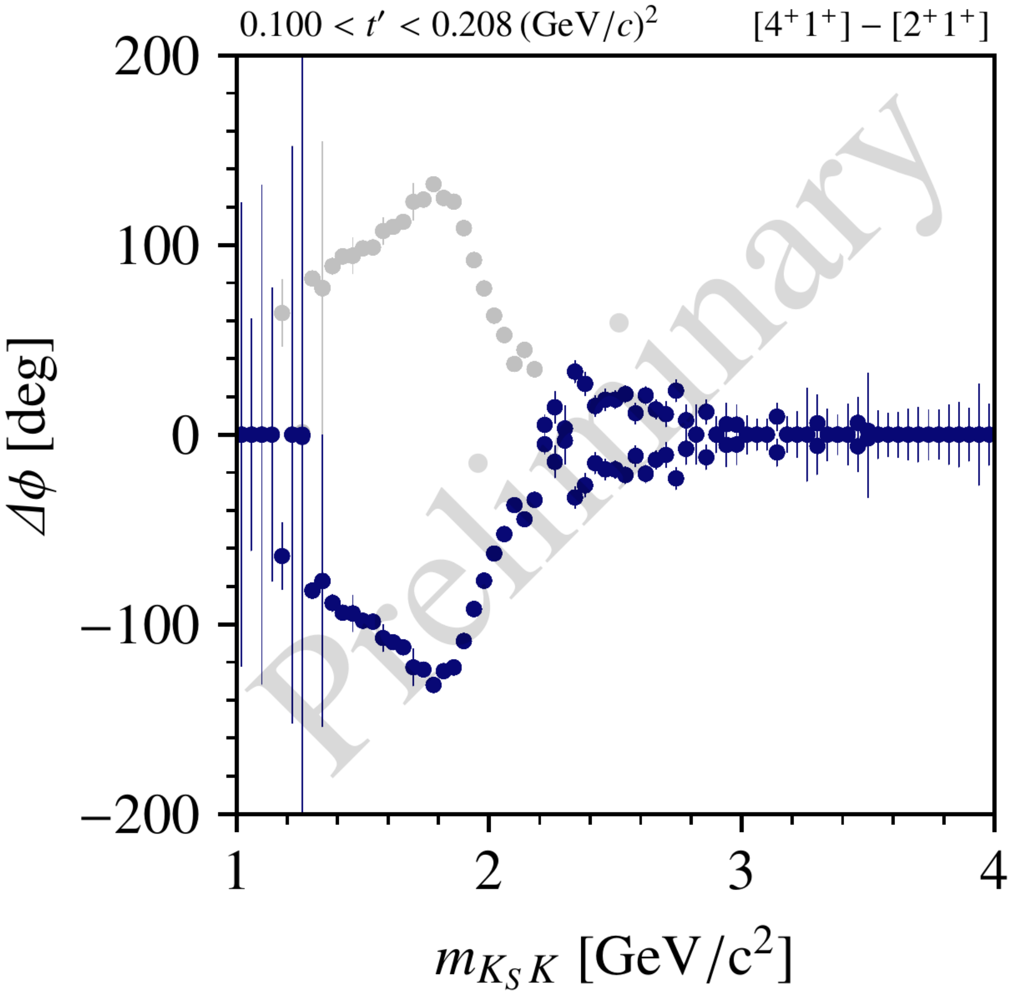


$a_4(1970)$
$m_{PDG} = 1.97$ GeV
$\Gamma_{PDG} = 0.32$ GeV

$a_4(????)$



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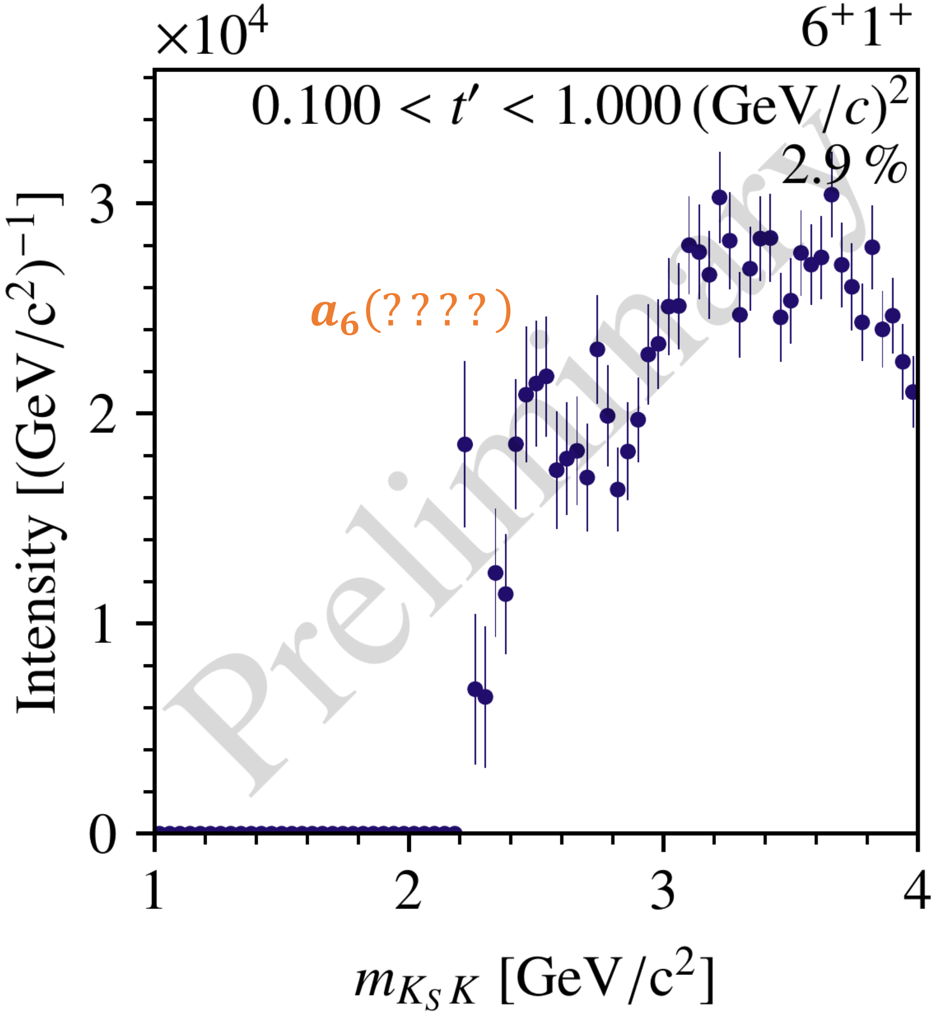
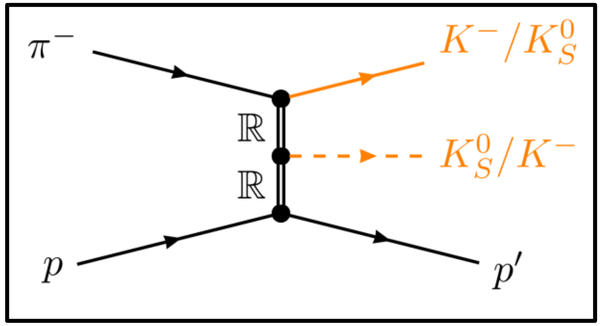
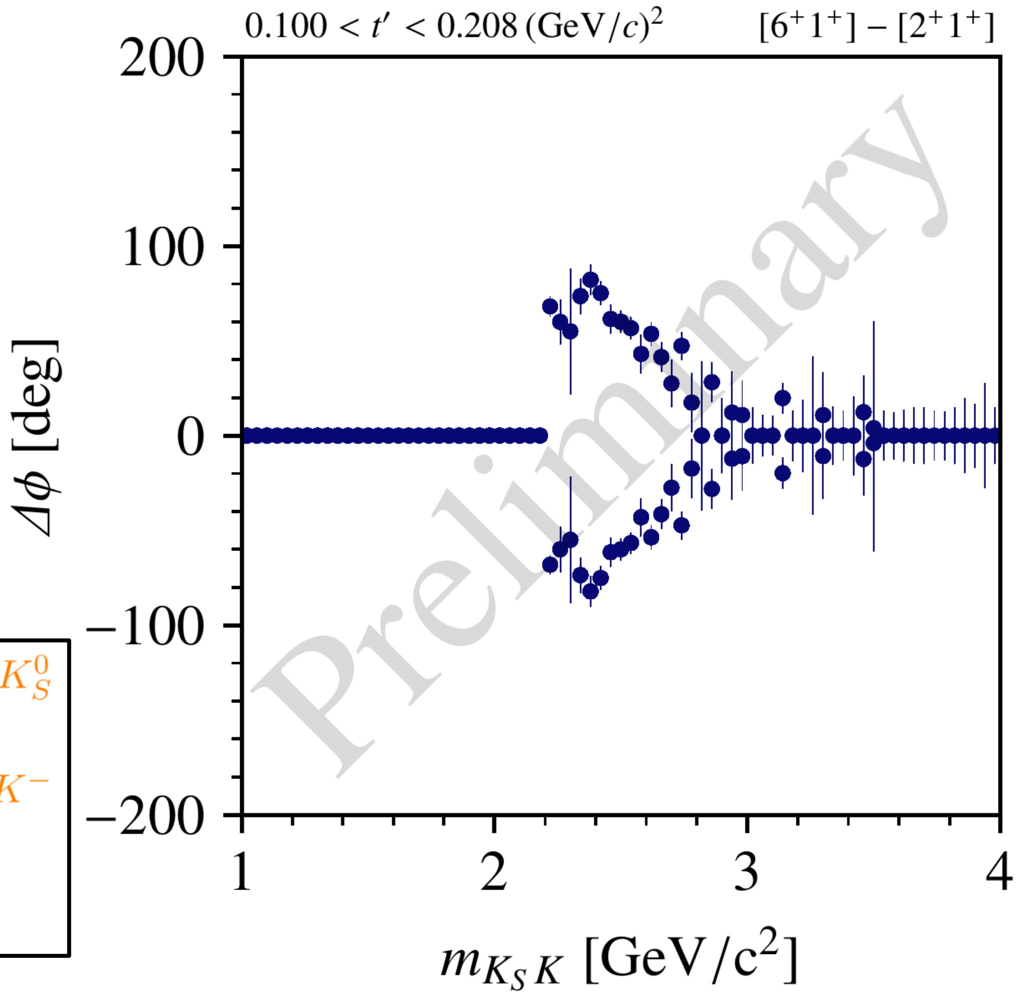
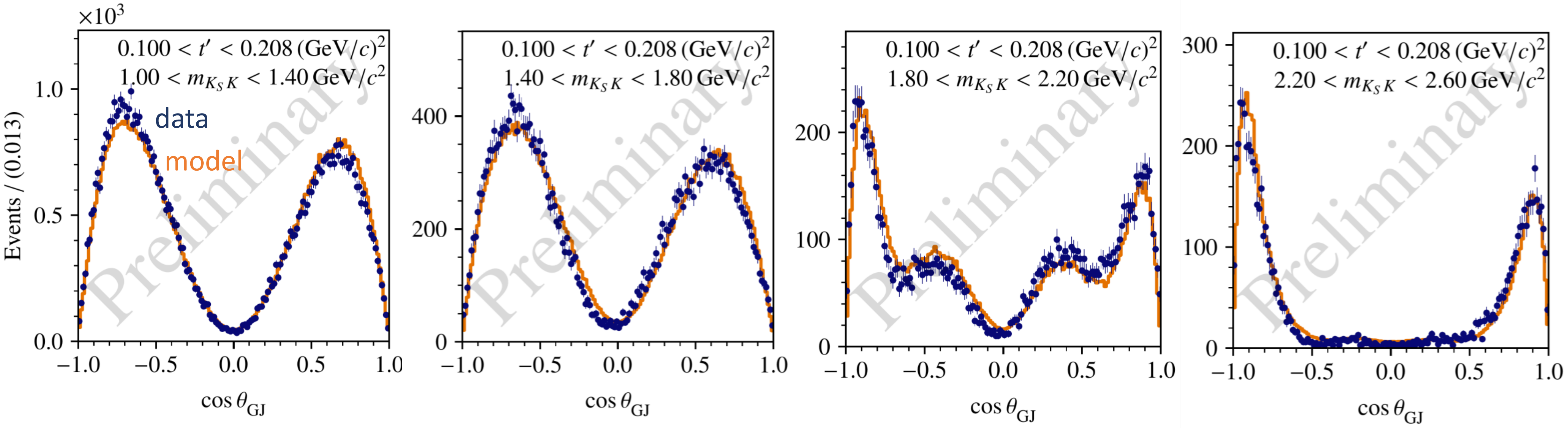


$a_6(????)$





Agreement Between Model and Data



- Angular distributions as predicted by the PWA model after the fit vs real data
- Good agreement, but data exhibits larger asymmetry than predicted in the model

Adding an Odd-Spin Wave

How can we introduce asymmetry in the model?

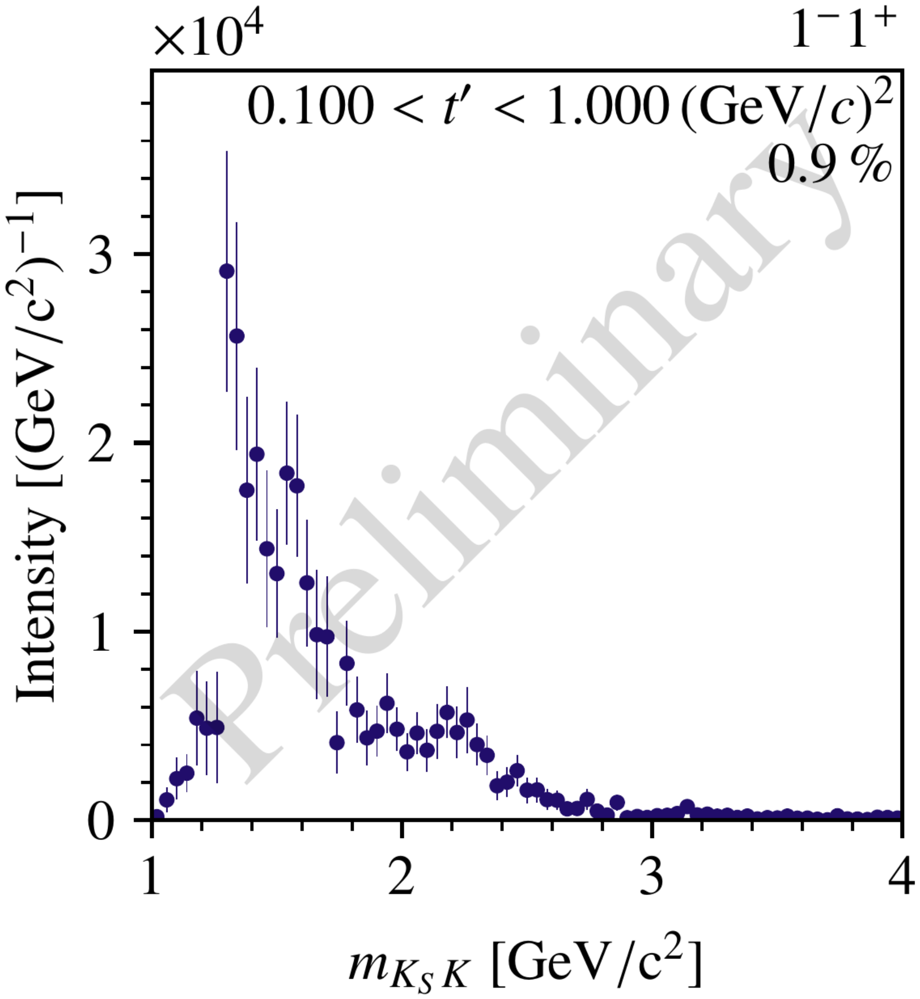
- Effects of the detector acceptance
- Introduce partial wave(s) with odd spin J

$$I^G J^{PC} = 1^+ 1^{--}$$

- Cannot be produced via Pomeron exchange!

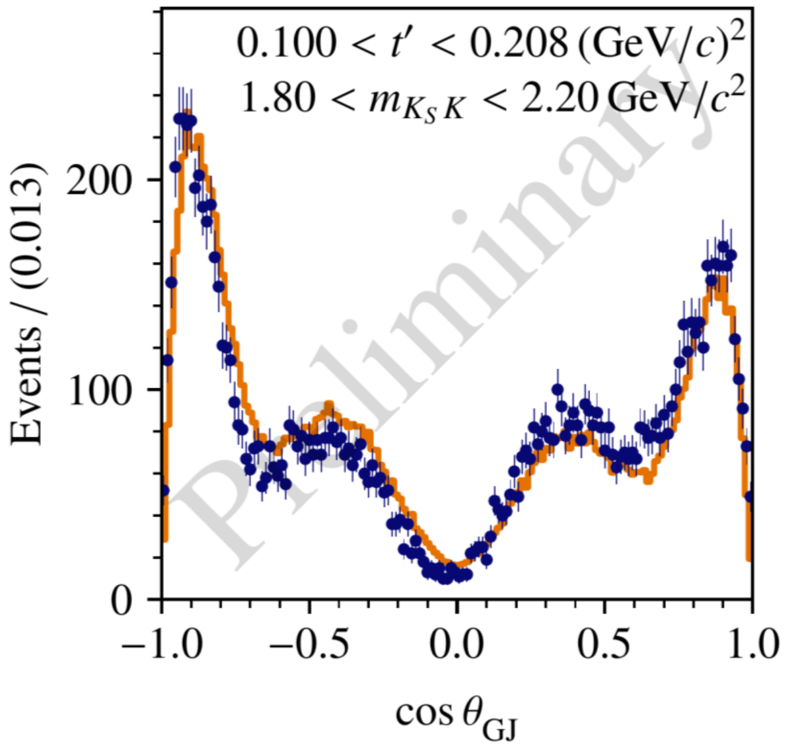
  (but e.g. by ω exchange)

$\boldsymbol{\rho(1450)}$	$\boldsymbol{\rho(1700)}$
• $m = 1.47$ GeV	• $m = 1.73$ GeV
• $\Gamma = 0.40$ GeV	• $\Gamma = 0.25$ GeV

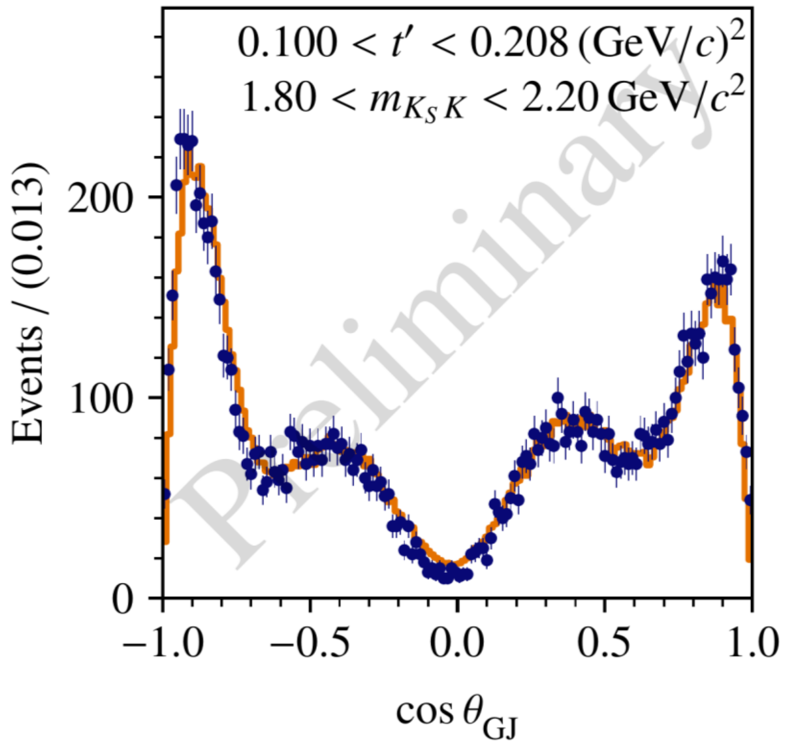


Agreement Between Model and Data II

Without 1^- wave



With 1^- wave



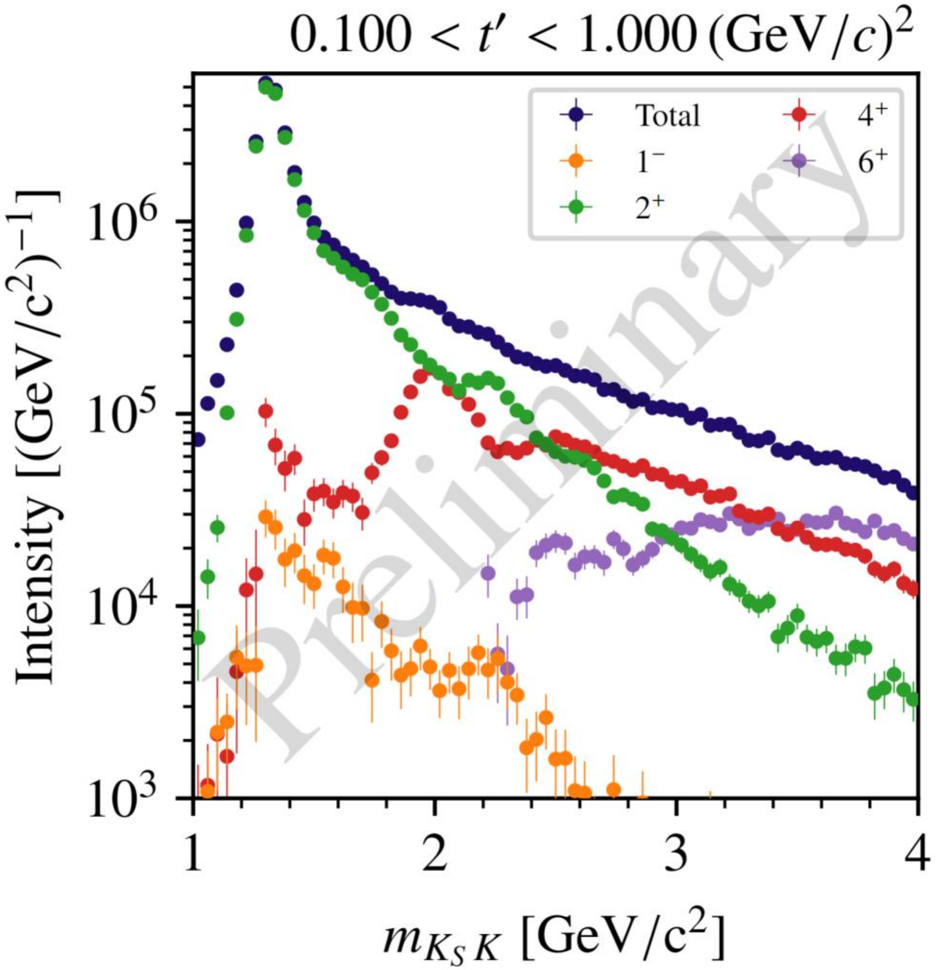
- Interference of $J = 1$ and even J could explain the forward/backward asymmetry

Conclusion and Outlook

- Resonances appearing in the final state:
  $J^{PC} = 2^{++}, 4^{++}, \ldots \rightarrow\ a_J$ states (Pomeron exchange)
  other exchanges may yield $J^{PC} = 1^{--}, 3^{--}, \ldots$

- Performed the **partial-wave decomposition**:
  - Clear signals of $a_2(1320)$ and $a_4(1970)$
  - Possible higher-lying a_2, a_4' and a_6
  - Indications of small intensity in 1^{--} partial wave

- Next step: **Resonance-model fit**
  Extract resonance parameters by modelling m_{KK}, t' dependences



Thank you for your attention!

BACKUP

Quantum Numbers of the $K_S^0 K^-$ Final State

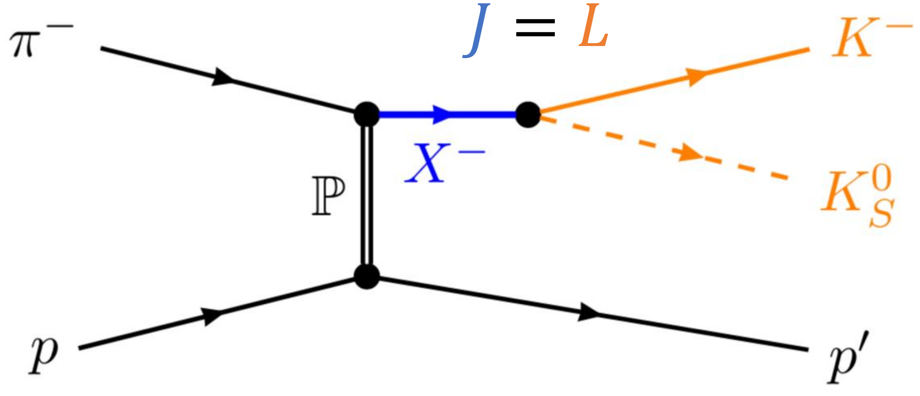
- Quantum numbers of a $K_S^0 K^-$ system

$$I = 1$$

$$G = (-1)^{L+1}$$

$$P = (-1)^L \qquad C = G(-1)^I = (-1)^L$$

all given by L!



- Quantum numbers of the produced X^-:

$I^G = 1^-$ for Pomeron ($I^G = 0^+$) exchange

→ odd spins J are suppressed!

$$I^G J^{PC} = 1^+1^{--}, 1^-2^{++}, 1^+3^{--}, 1^-4^{++}, \ldots$$

- Reflectivity ($\epsilon = \pm 1$) basis:

eigenvalue of $\hat{\epsilon} = \hat{P} R_\perp(\pi)$

at high $\sqrt{s}$, $\epsilon \approx \eta$ — η: naturality of the exchange particle

→ $\boldsymbol{\epsilon = +1}$ for Pomeron exchange

S. U. Chung, doi.org/10.1103/PhysRevD.11.633
S. U. Chung, C- and G-parity: A New Definition and Applications. https://suchung.web.cern.ch/Cparity7b.pdf

Ambiguities in Incoherent Sectors

$$\boldsymbol{\varepsilon = \pm 1}: \quad I(m_X, t'; \theta, \phi) = \left| \sum_{JM} T_{JM}^{+}(m_X, t')\, \psi_{JM}^{+}(\theta, \phi) \right|^2 + \left| \sum_{JM} T_{JM}^{-}(m_X, t')\, \psi_{JM}^{-}(\theta, \phi) \right|^2$$

$$a_1^+ = \sum_{J=1}^{J_{\max}^+} T_{J1}^+\, Y_J^1(\theta, 0) \qquad \varepsilon = +1, M = 1$$

$$a_0^- = \sum_{J=0}^{J_{\max}^-} T_{J0}^-\, Y_J^0(\theta, 0) \qquad \varepsilon = -1, M = 0$$

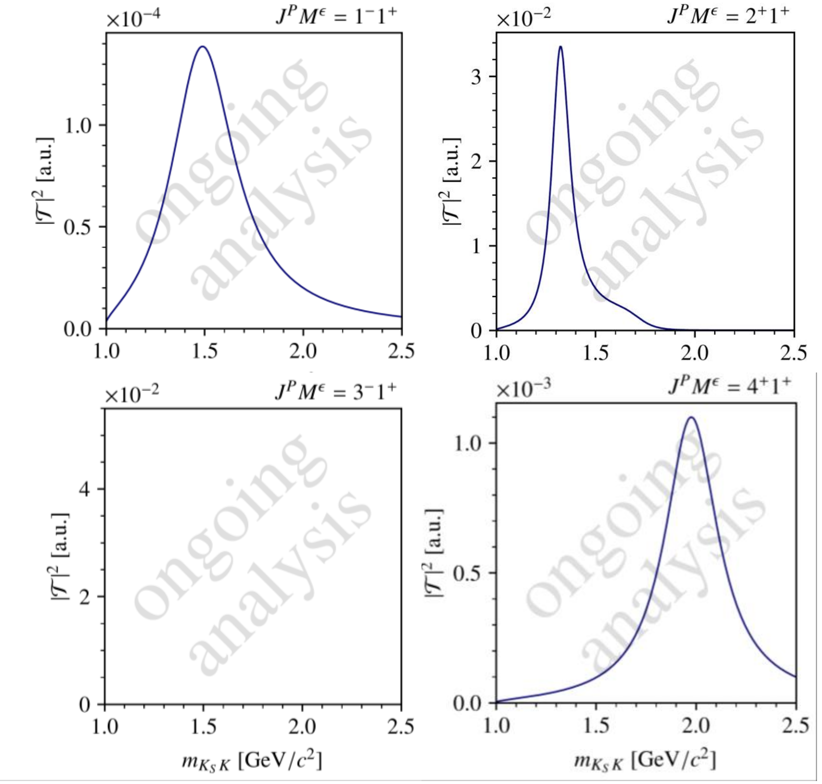
$$a_1^- = \sum_{J=1}^{J_{\max}^-} T_{J1}^-\, Y_J^1(\theta, 0) \qquad \varepsilon = -1, M = 1$$

- Define $a_s^- = a_0^- + a_1^-$, then same procedure as for only positive-reflectivity sector

- New amplitudes for $\varepsilon = +1$: $|a_1^+|^2 = |a_1^-|^2 - \text{const.}$ $\rightarrow$ **positivity requirement**!

Study of the Ambiguities

- How do the ambiguous solutions look like (**continuity, signals, …**)?
- What are the effects of the **partial-wave decomposition fit** on **finite data** on the ambiguities?



I. Continuous intensity model

- create an amplitude model for selected partial waves
- calculate exact ambiguities

II. Finite pseudo-data

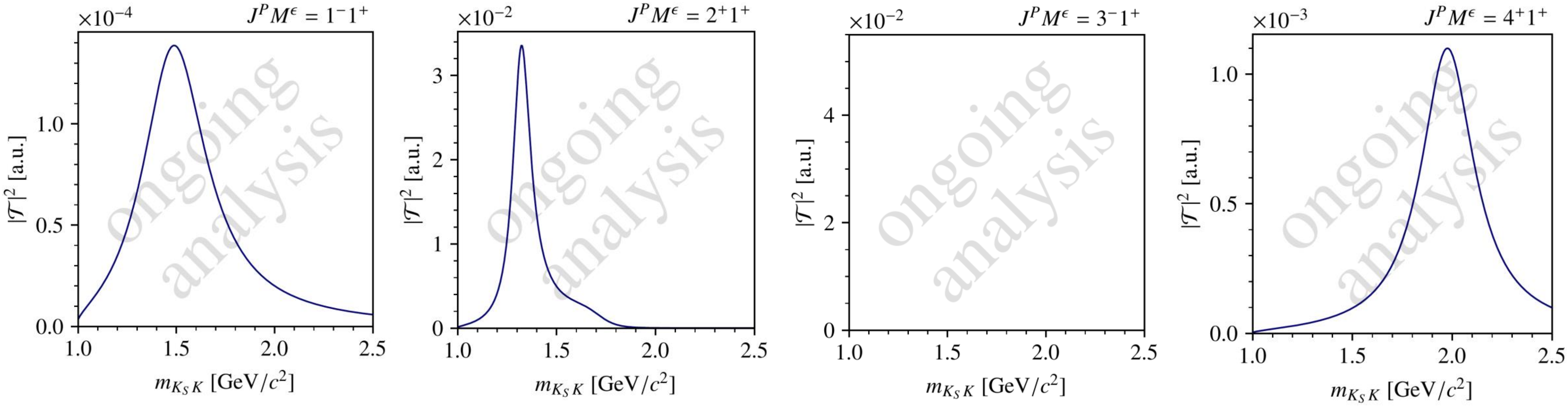
- generate pseudo-data according to model
- perform partial-wave decomposition

Continuous Amplitude Model

I. Continuous intensity model

J^{PC}	Resonances
$\mathbf{1^{--}}$	$\rho(1450)$
$\mathbf{2^{++}}$	$a_2(1320), a_2'(1700)$
$\mathbf{3^{--}}$	None
$\mathbf{4^{++}}$	$a_4(1970)$

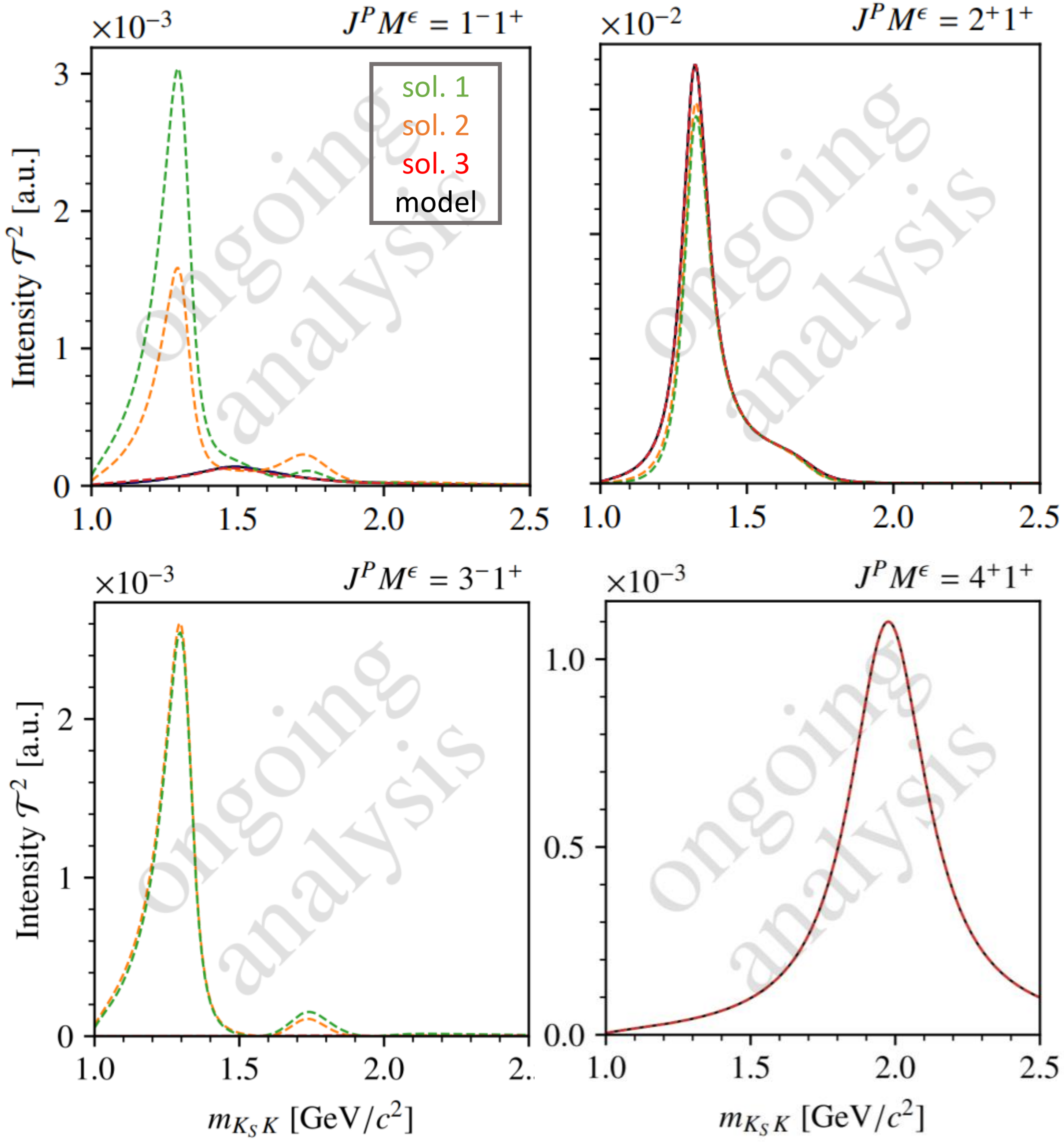
- create an amplitude model for four selected partial waves
- In $1.0 < m_X < 2.5\ \mathrm{GeV}/c^2$
- m_X-dependence by Breit-Wigner amplitudes (PDG parameters)



Continuous Amplitude Model

I. Continuous intensity model $N_a = 3$

- Sample points in m_X and calculate ambiguous solutions
- Ambiguous intensities are also continuous
- Not all solutions are different from each other!
- Highest-spin (4^{++}) intensity is invariant!

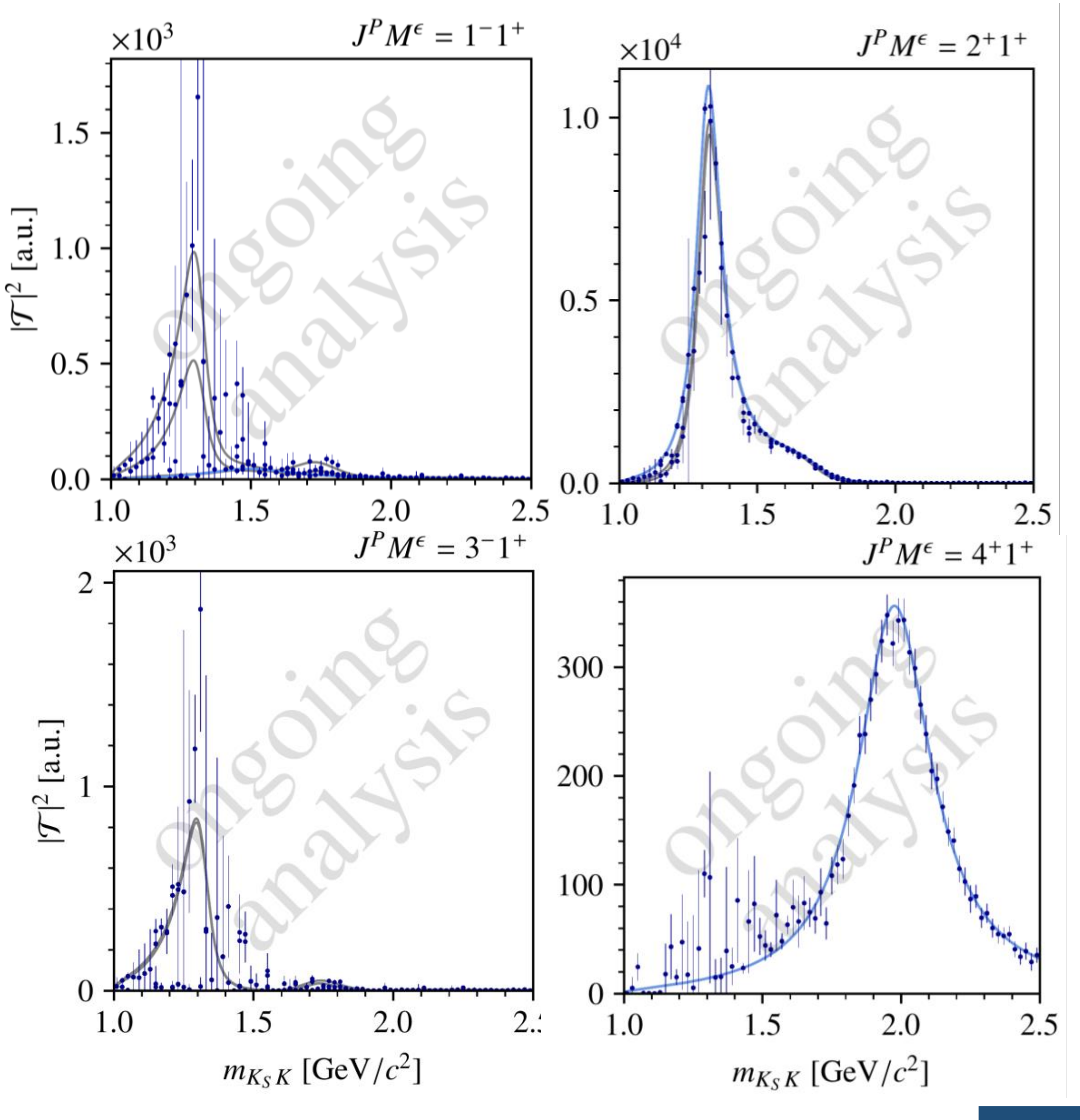


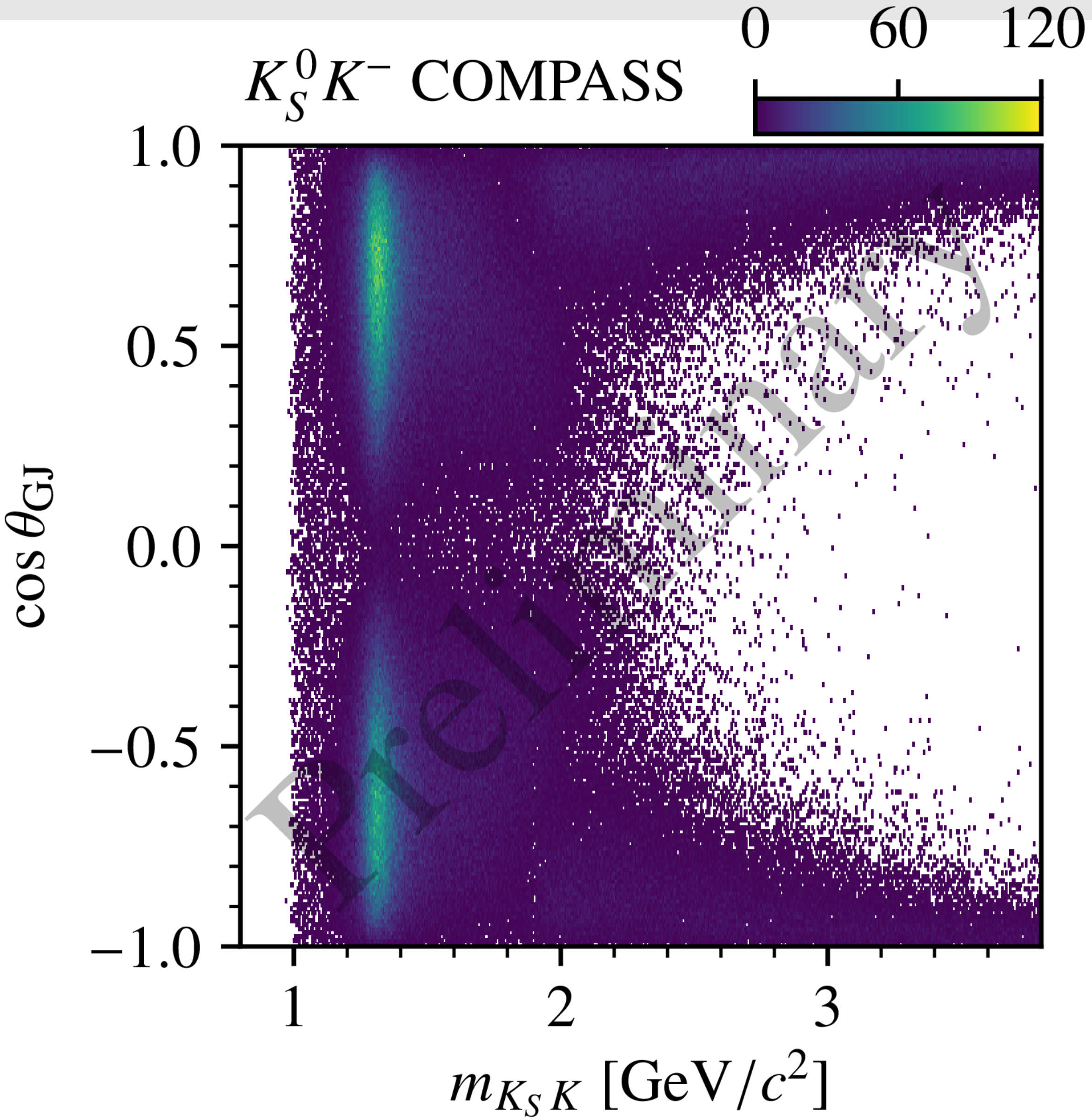
Partial-Wave Decomposition Fits on Pseudodata

II. Finite pseudo-data

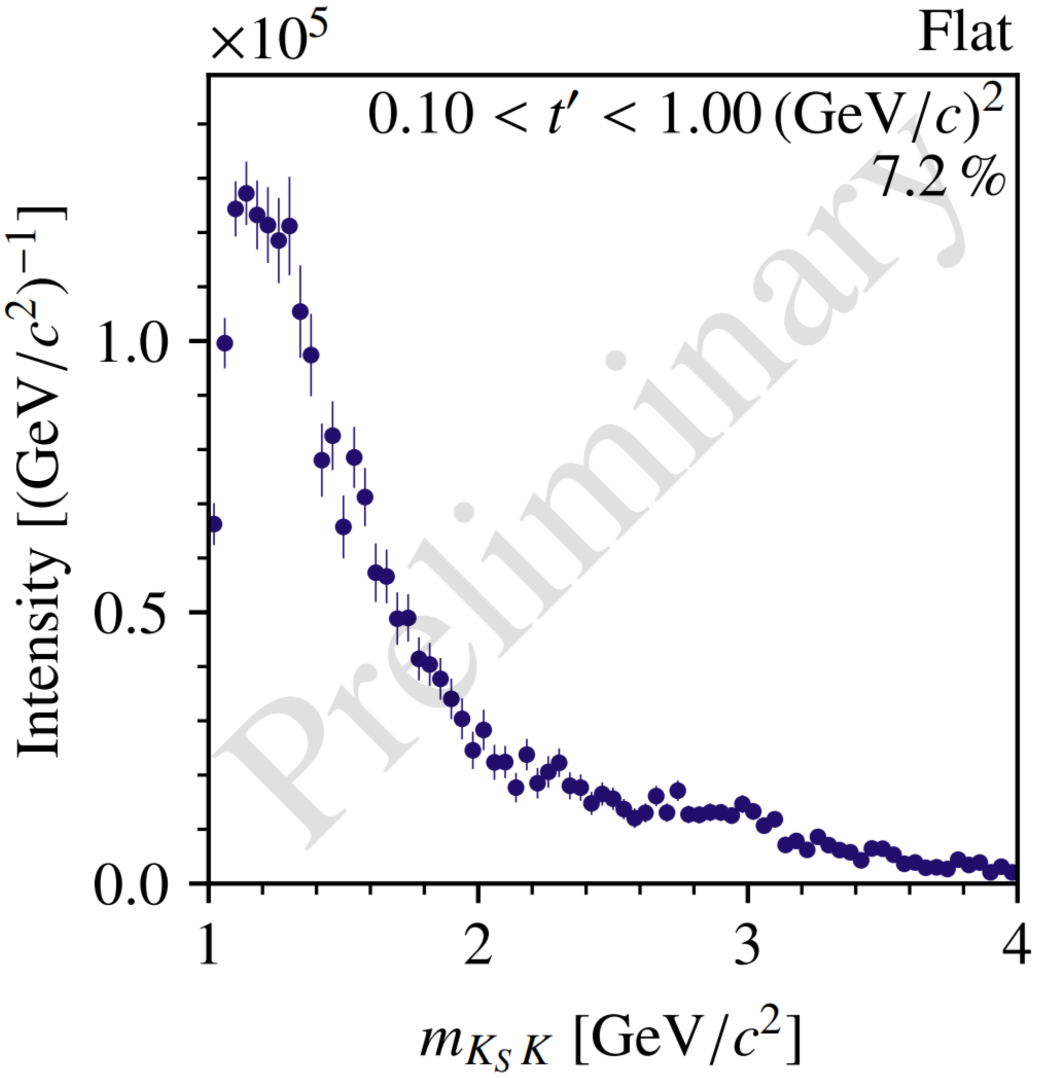
- 4^{++} intensity is still invariant!
- Overall, amplitude values found by the fit follow the calculated distributions
- Not all solutions are found in each m_X bin

→ PWD fit distorts the intensity distribution!




$K_S^0 K^-$ COMPASS
0
60
120
1.0
0.5
0.0
−0.5
−1.0
$\cos\theta_{\mathrm{GJ}}$
1
2
3
$m_{K_S K}$ [GeV/c^2]
Preliminary

Results: Flat-Wave Intensity

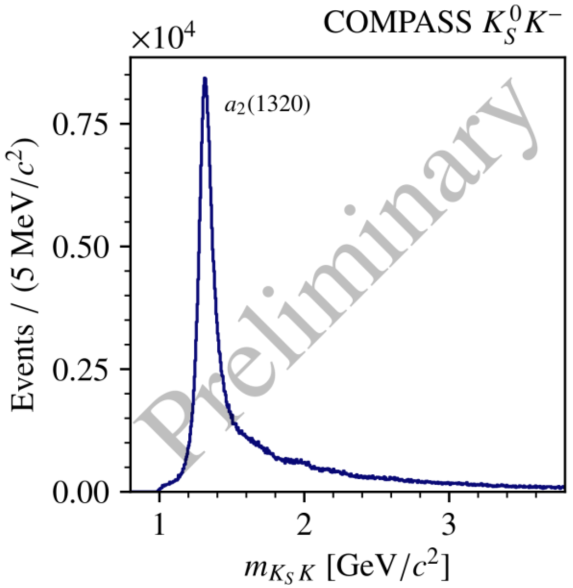


Kinematic Binning

Analyzed invariant mass range:

$$1.0 < m_{K_S K} < 4.0 \text{ GeV}/c^2$$

divided into 75 bins of $40 \text{ MeV}/c^2$ width

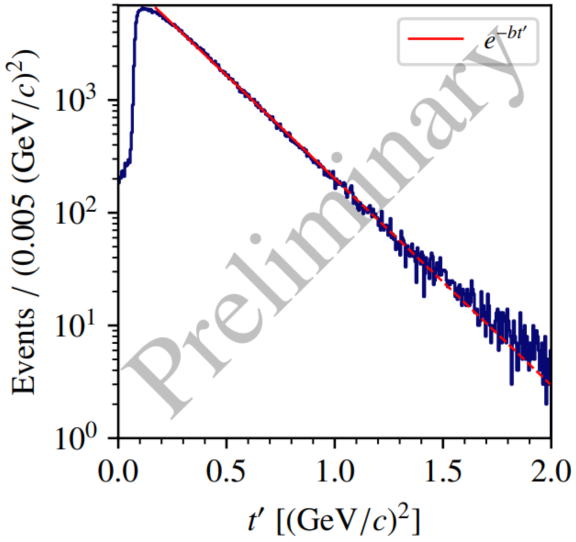


Analyzed t' range

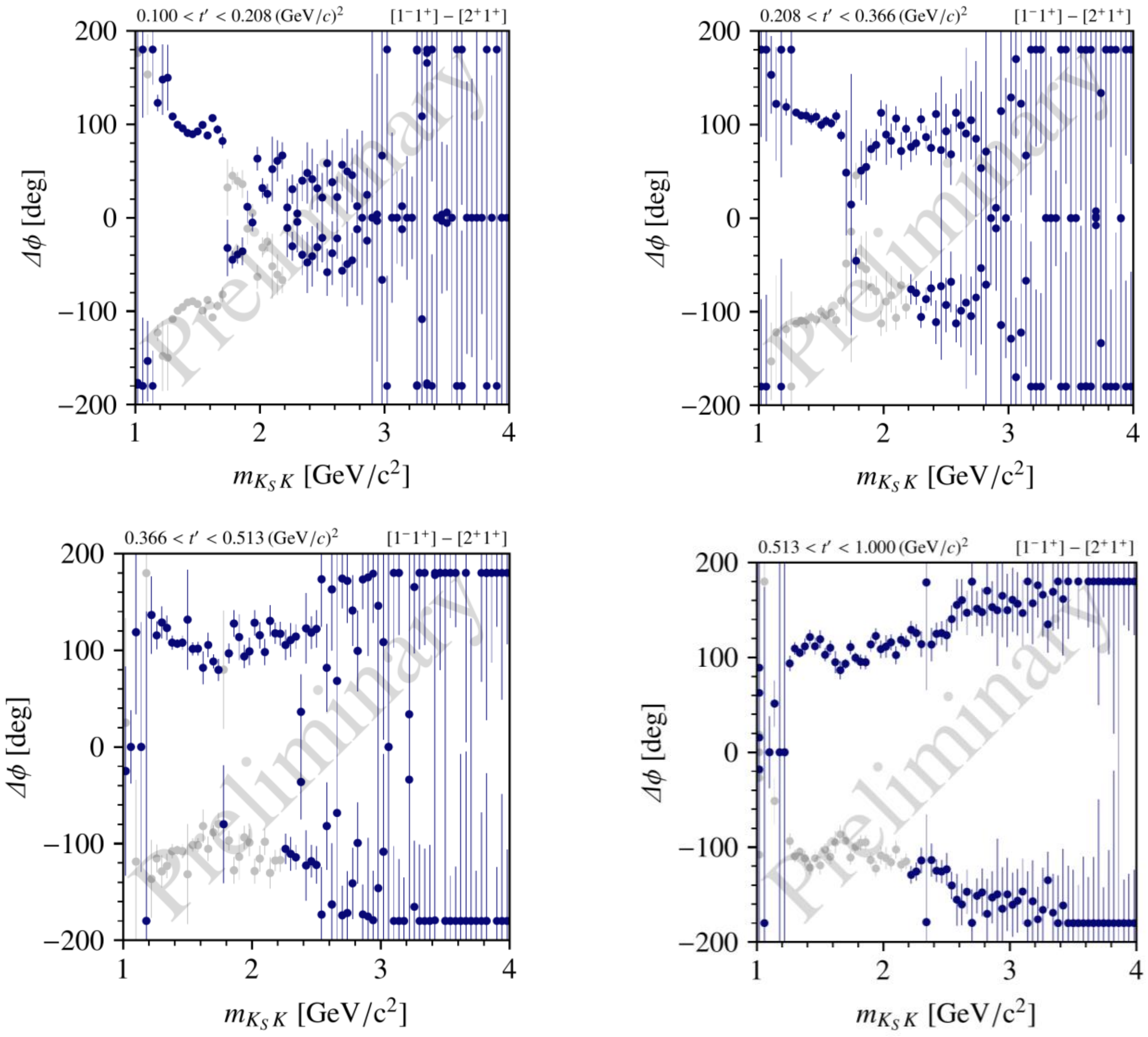
$$0.1 < t' < 1.0 \text{ GeV}^2/c^2$$

divided into four bins:

t' bin borders in $(\text{GeV}/c)^2$				
0.100	0.208	0.366	0.513	1.000



$J^{PC} = 1^{--}$ Partial Wave



Resonance-Model Fit

Second step: **extract resonance parameters**

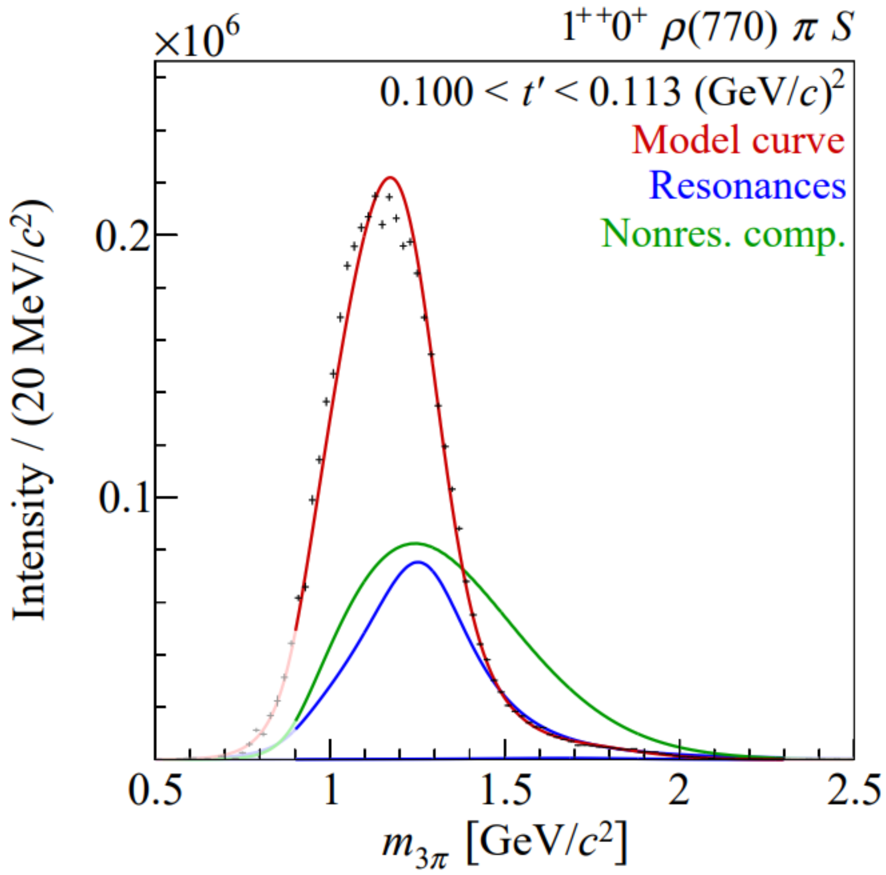
- Build model for **mass dep. of partial-wave amplitudes**:

  resonant (e.g. Breit-Wigner distribution)

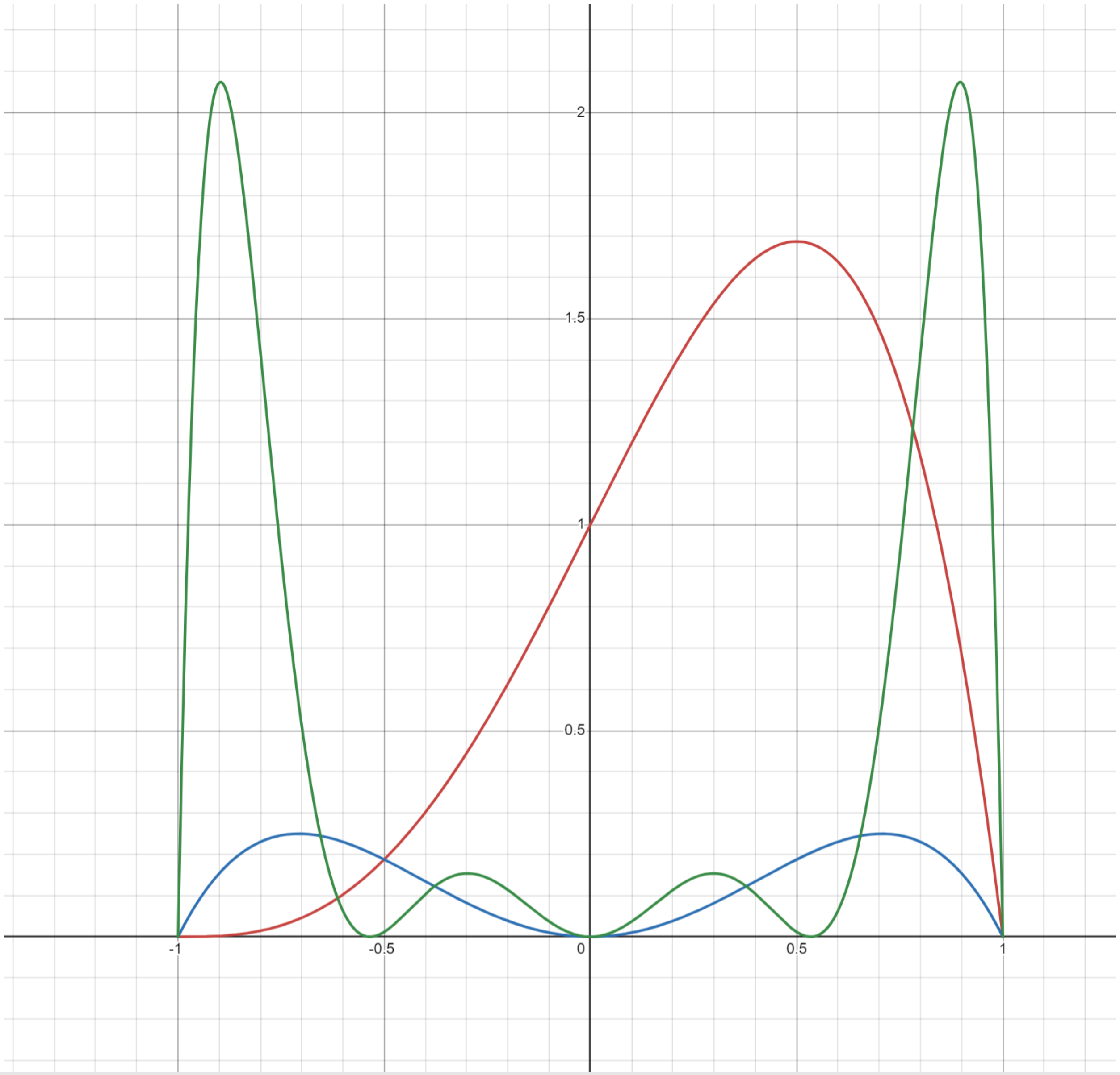
  \+ **non-resonant background** components

- χ^2 fit to output of partial-wave decomposition

→ get masses and widths of parameterized resonances



COMPASS PRD 98 (2018) 092003


2
1.5
1
0.5
-1
-0.5
0
0.5
1