Progress in the Partial-Wave Analysis Methods at COMPASS*

Florian Markus Kaspar [1,2] and Julien Beckers [1] for the COMPASS Collaboration

florian.kaspar@tum.de

in collaboration with Jakob Knollmüller [1,2]

MESON 2023

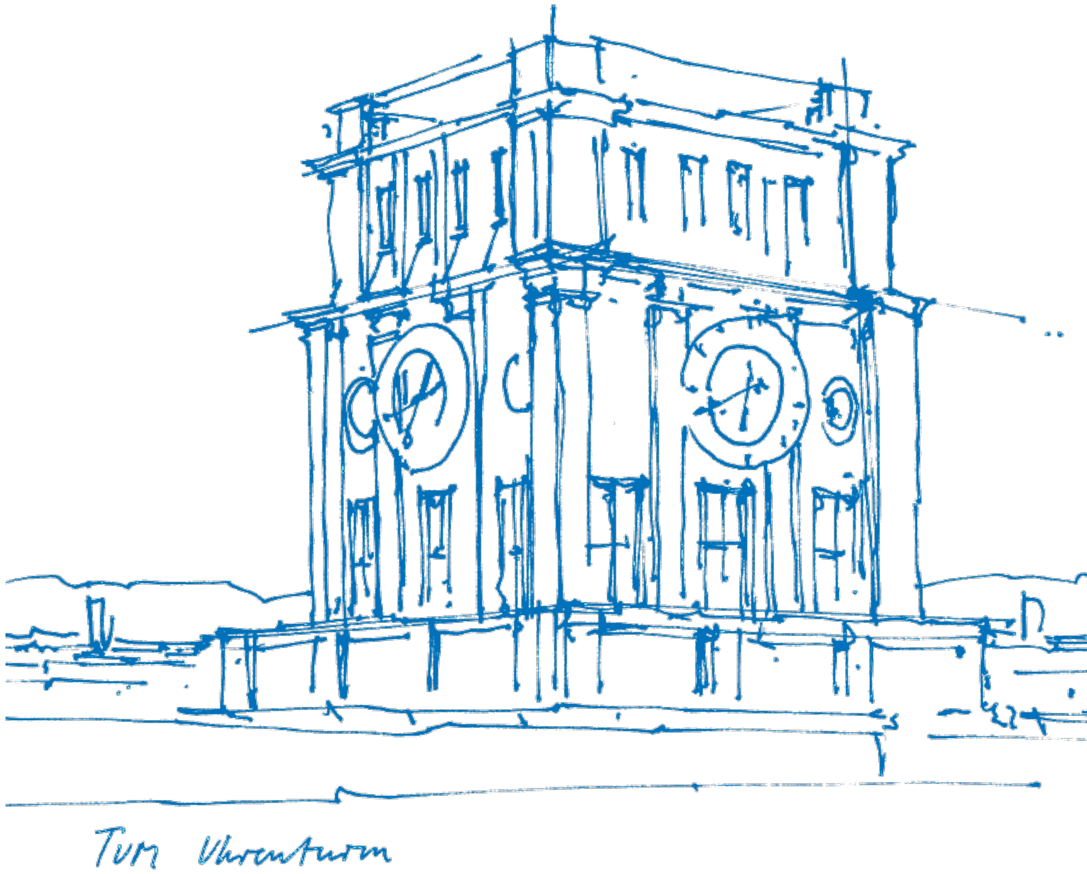
June 23rd 2023 15:40

[1] Technische Universität München (TUM)

[2] Excellence Cluster Origins

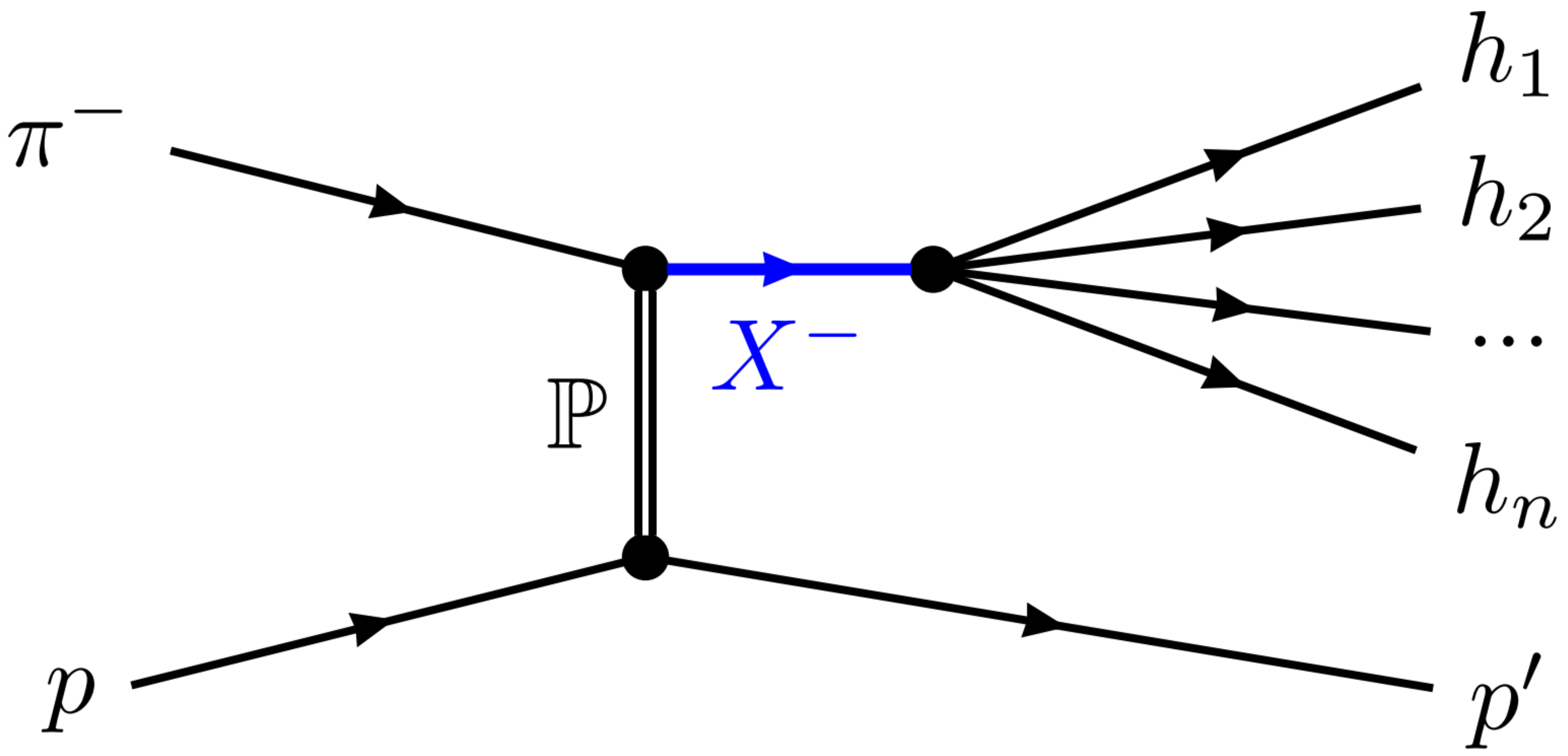
Bundesministerium für Bildung und Forschung

* funded by the DFG under Germany's Excellence Strategy - EXC2094 - 390783311 and BMBF Verbundsforschung 05P21WOCC1 COMPASS

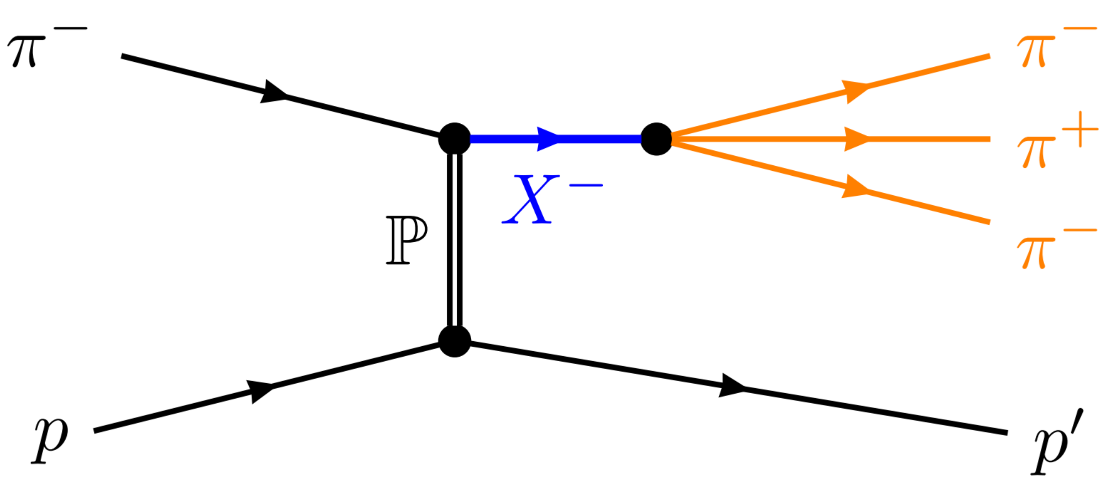


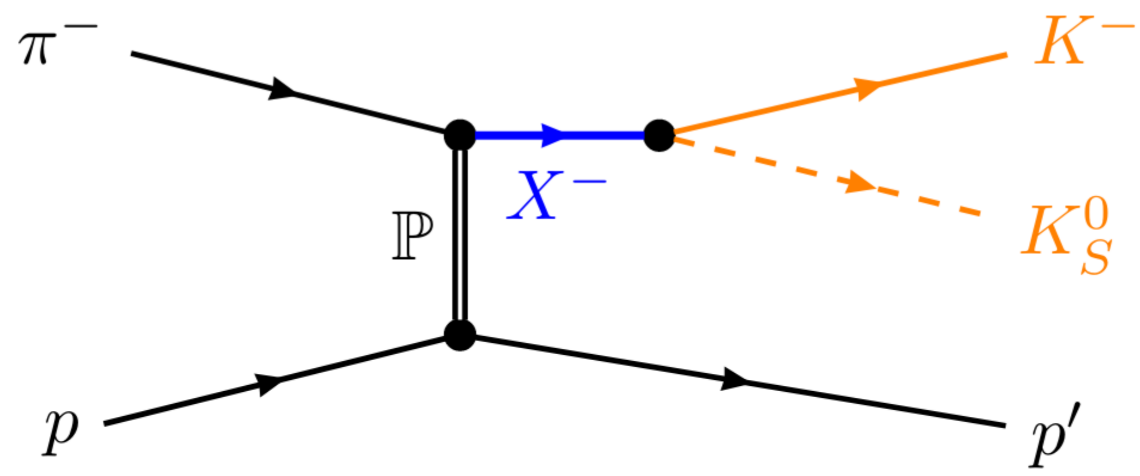
Studies of Excited Light Mesons at COMPASS

Different Hadronic Final States



For this talk:





- COMPASS flagship channel:

> 100 Mio events

→ π_J and a_J resonances

($J^{PC} = 0^{-+}, 1^{-+}, 1^{++}, \ldots$)

highly selective:

Final State: $J^{PC} = 1^{--}, 2^{++}, 3^{--}, \ldots$

Final State + dominant Pomeron exchange

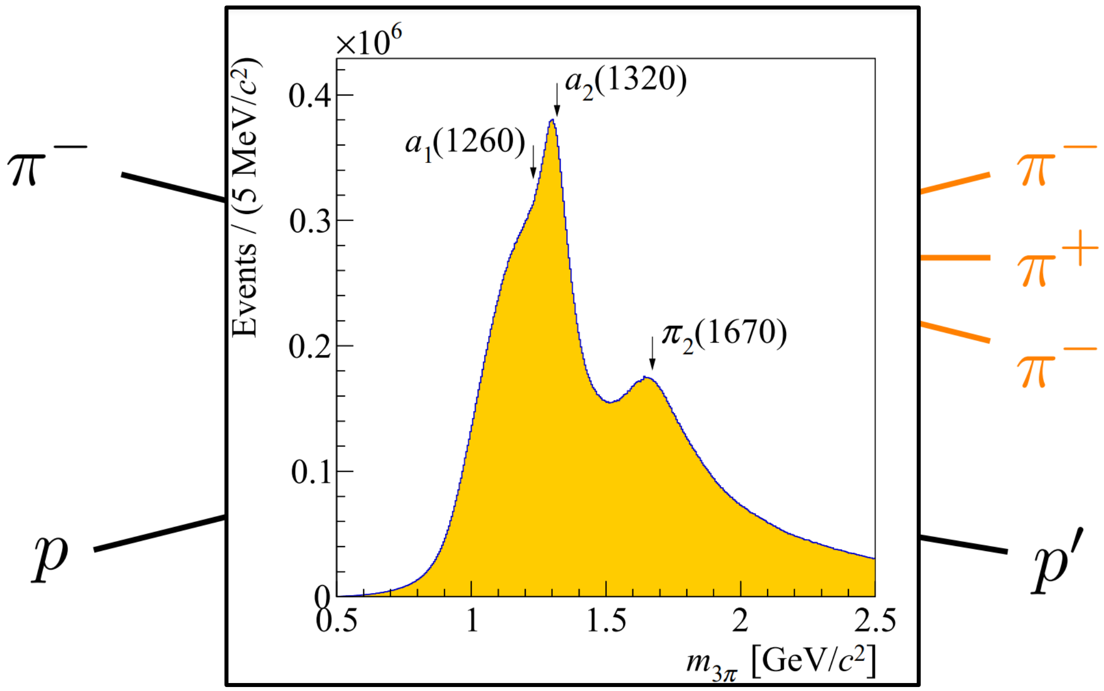
→ a_J for even J

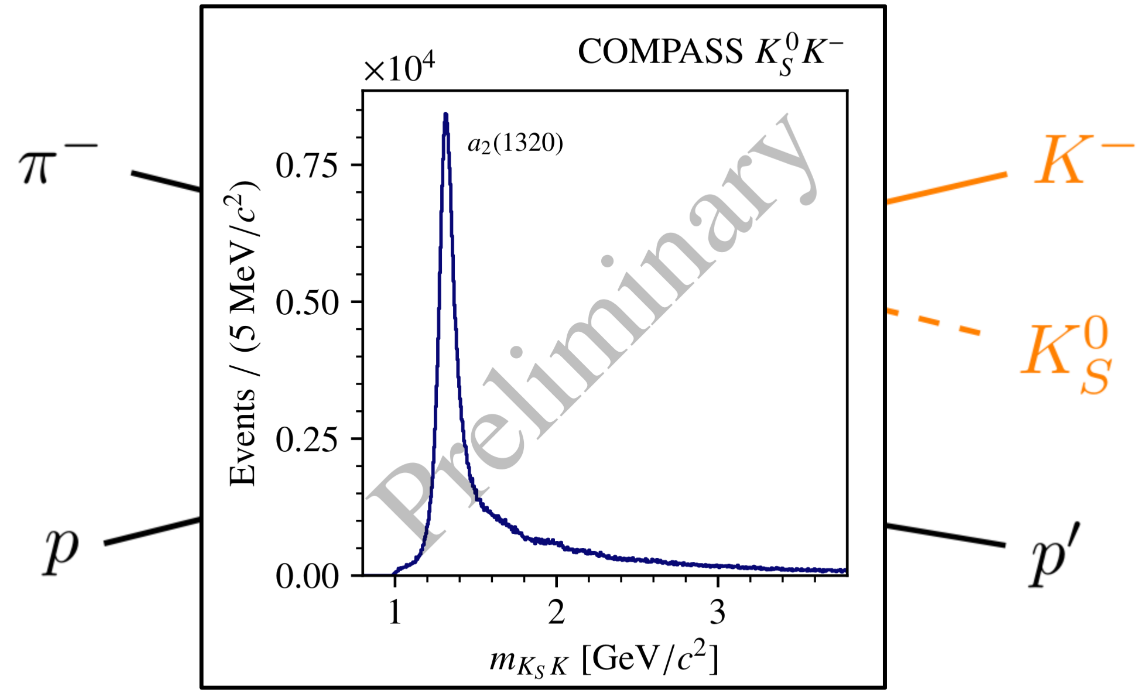
→ search for a_6, a_4'

→ Probe for same resonances in different channels: Systematics!

For this talk:

PRD 95 (2017) 032004





- COMPASS flagship channel:

> 100 Mio events

$\rightarrow$ $\boldsymbol{\pi_J}$ and $\boldsymbol{a_J}$ resonances

($J^{PC} = 0^{-+}, 1^{-+}, 1^{++}, \dots$)

highly selective:

Final State: $J^{PC} = 1^{--}, 2^{++}, 3^{--}, \dots$

Final State + dominant Pomeron exchange

$\rightarrow$ $\boldsymbol{a_J}$ for even J

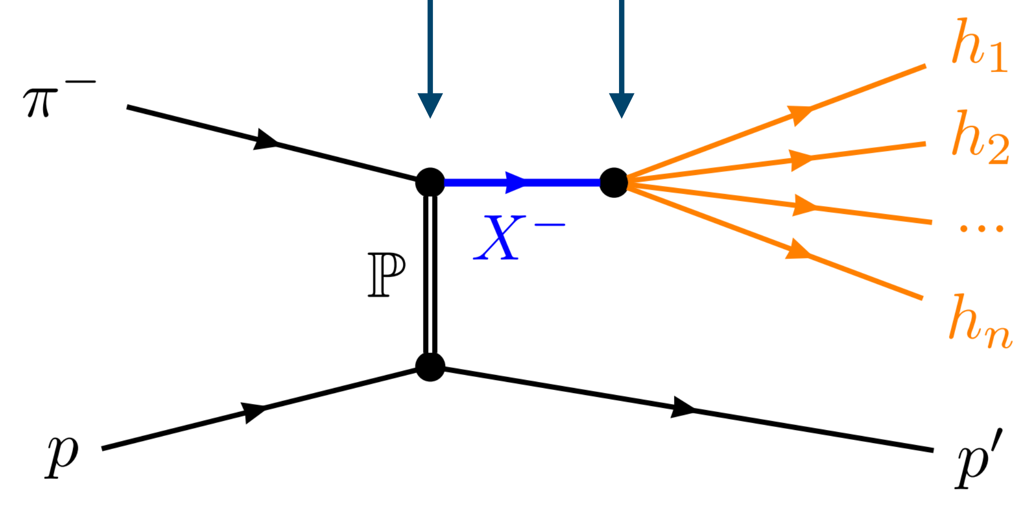
$\rightarrow$ search for a_6, a_4'

$\rightarrow$ Probe for same resonances in different channels: Systematics!

Partial-Wave Analysis

Two Steps:

$$I(\tau; m_X, t') = \left| \sum_a T_a(m_X, t')\, \psi_a(\tau; m_X) \right|^2$$



1) mass-independent fit

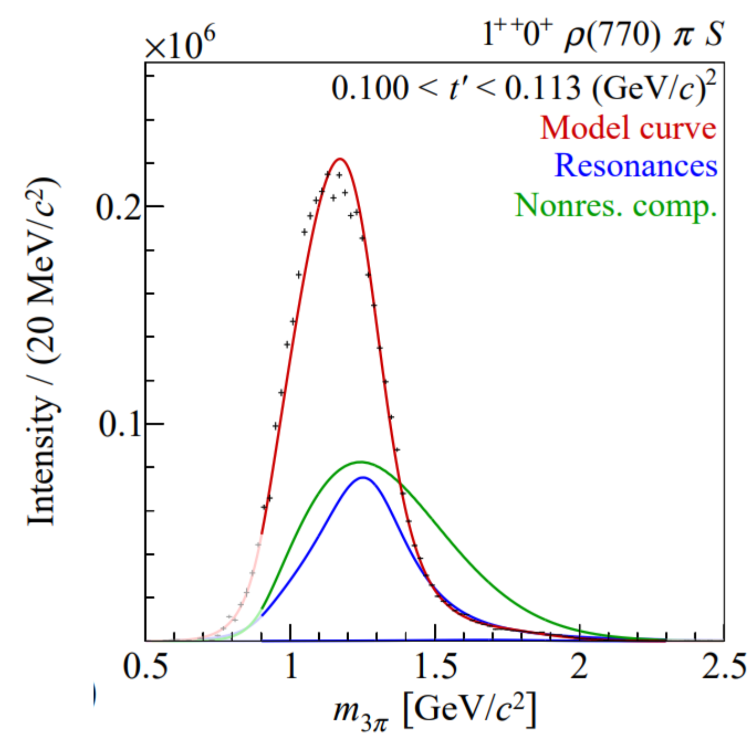
model $I(m_X, t'; \tau_n)$ in $(\boldsymbol{m_X}, \boldsymbol{t'})$ bins

- factorization in $T_a(m_X, t')$ and $\psi_a(\tau; m_X)$
- parametrize T_a as step-wise functions
- extract constant T_a in each bin

2) mass-dependent fit: model resonances

1. results of first step: input
2. χ^2 fit of resonant + background parameterization to subset of $T_a(m_X, t')$

$\rightarrow$ resonance parameters = physics



PRD 98 (2018) 092003

Details of the Partial-Wave Decomposition in: COMPASS Collaboration, Phys. Rev. D 95 (2017) 032004, arXiv:1509.00992 [hep-ex].

Ambiguities in the Partial-Wave Decomposition of the $K_S^0 K^-$ Final State

Ambiguities in the Partial-Wave Decomposition

For any final state with **two spinless** particles $(\pi\pi, KK, \eta\pi, \ldots)$:

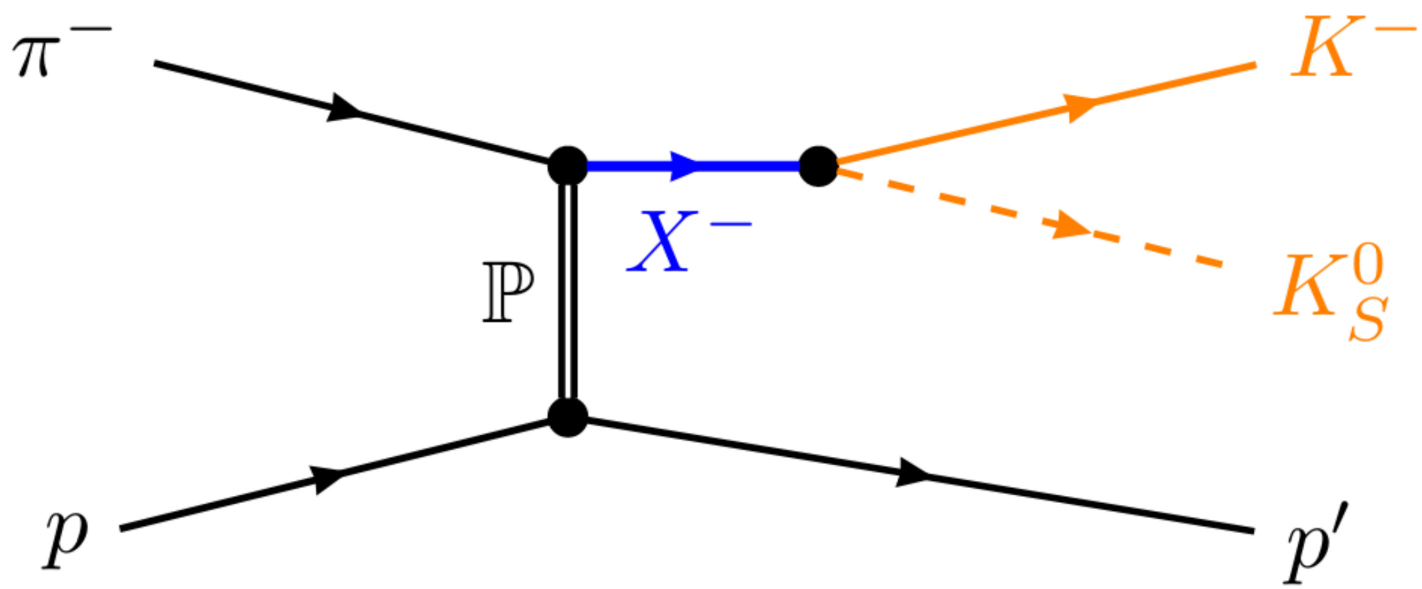
Decomposition of intensity into $\{T_J\}$ is **not unique**

→ Several sets of $\{T_J\}$ lead to the **same** $\boldsymbol{I}(\boldsymbol{\theta}, \boldsymbol{\phi})$ in each (m_X, t') bin

$$I(\theta, \phi) = \left| \sum_{JM} T_{JM}^{(1)} \psi_{JM}(\theta, \phi) \right|^2 = \left| \sum_{JM} T_{JM}^{(2)} \psi_{JM}(\theta, \phi) \right|^2$$

Cannot distinguish between the **mathematically equivalent** solutions!

Ambiguities in the Partial-Wave Decomposition



$$I(\theta,\phi) = \left| \sum_J T_J \, \psi_J(\theta,\phi) \right|^2 = \underbrace{\left| \sum_J T_J \, Y_J^1(\theta,0) \right|^2}_{a(\theta)} \left|\sin\phi\right|^2$$

$$Y_J^1(\theta,0) = \sum_{j=0}^{J-1} y_j \tan^{2j}\theta$$

Polynomial in $\tan^2\theta$

$$a(\theta) = \sum_{j=0}^{J_{\max}-1} c_j\left(\{T_J\}\right) \tan^{2j}(\theta) = c(\{T_J\}) \prod_{k=1}^{J_{\max-1}} \left(\tan^2(\theta) - r_k\left(\{T_J\}\right)\right)$$

root decomposition

$$a\left(\tan^2\theta = r_k\right) = 0$$

"Barrelet zeros"

Chung, PRD 56 7299–7316 (1997)

Barrelet, *Nuov Cim A* **8**, 331–371 (1972)

Ambiguities in the Partial-Wave Decomposition

$$I(\theta, \phi) = \left| \sum_J T_J \, Y_J^1(\theta, 0) \right|^2 \left| \sin\phi \right|^2$$

$$= \left| \sum_{j=0}^{J_{\max}-1} c_j\left(\{T_J\}\right) \tan^{2j}(\theta) \right|^2 \left| \sin\phi \right|^2$$

$$= c^2 \prod_{k=1}^{J_{\max-1}} \left| \tan^2(\theta) - r_k \right|^2 \left| \sin\phi \right|^2 \qquad = c^2 \prod_{k=1}^{J_{\max-1}} \left| \tan^2(\theta) - r_k^* \right|^2 \left| \sin\phi \right|^2$$

Conjugation of roots → different solution!

$$\{T_J{}'\} \neq \{T_J\}$$

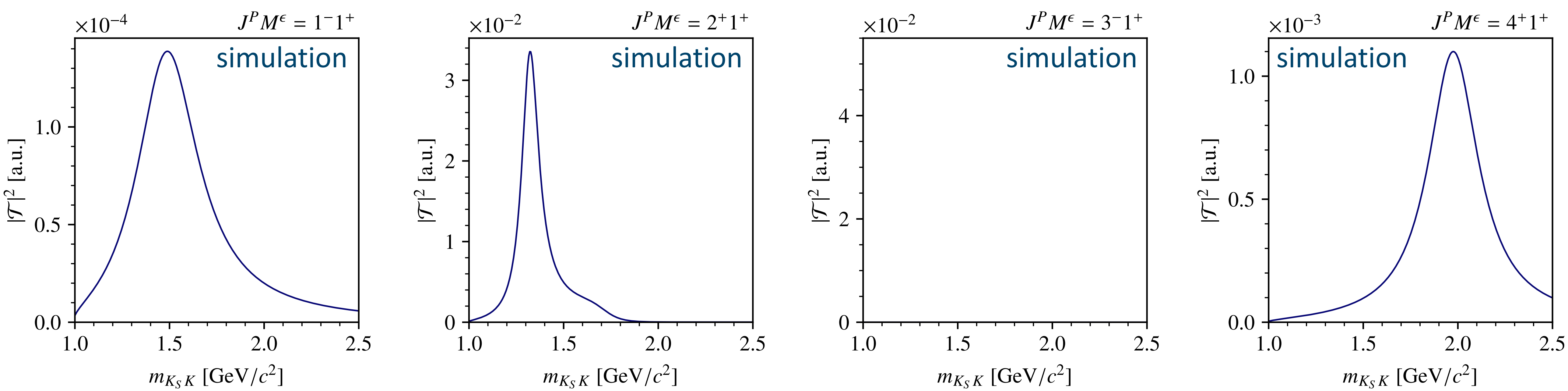
Study of Ambiguities

Study Continuous Intensity Model

Input:

- amplitude model for four selected partial waves
- m_X-dependence by Breit-Wigner amplitudes

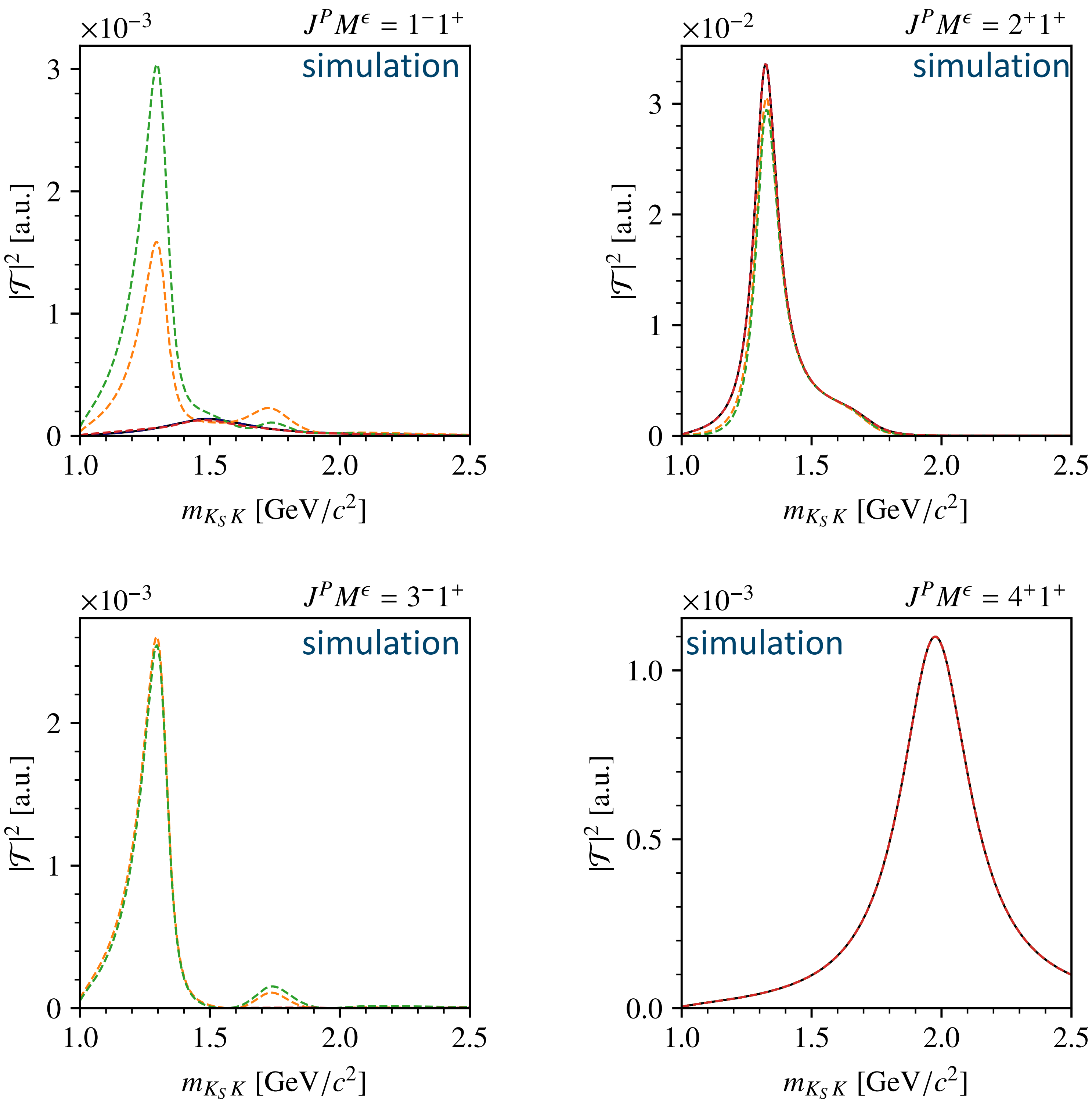
J^{PC}	Resonances
$\mathbf{1^{--}}$	$\rho(1450)$
$\mathbf{2^{++}}$	$a_2(1320), a_2'(1700)$
$\mathbf{3^{--}}$	None
$\mathbf{4^{++}}$	$a_4(1970)$



Study of Ambiguities

Calculate Ambiguous Solutions:

- Ambiguous intensities are also continuous in m_X
- Not all solutions are different from each other!
- Highest-spin (4^{++}) intensity is invariant!



Study of Ambiguities

Pseudo-Data Study

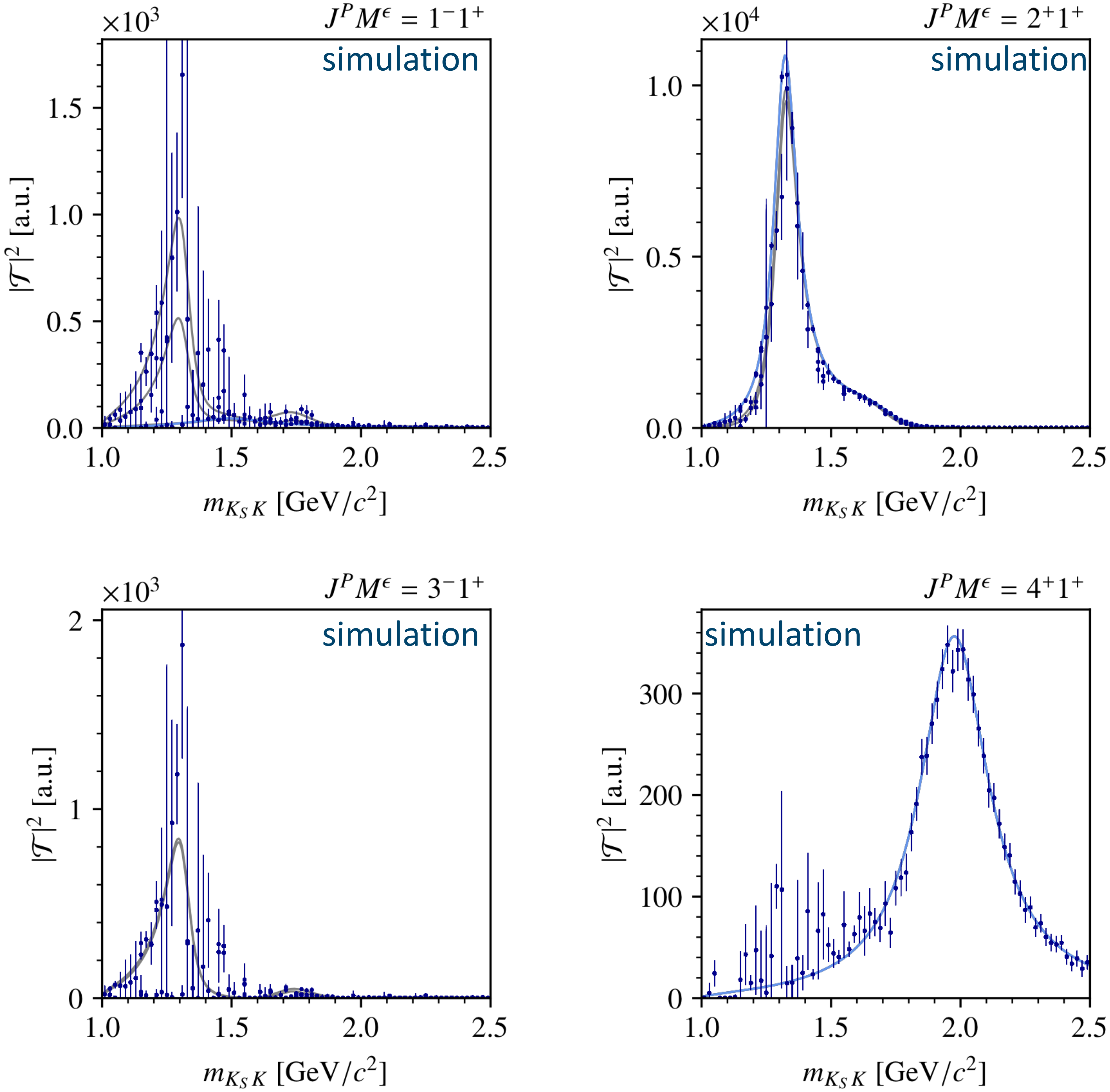
- generate pseudo-data according to model (10^5 events)
- perform a partial-wave decomposition fit

→ 3000 attempts with random start values

Ambiguous Solutions from Fit:

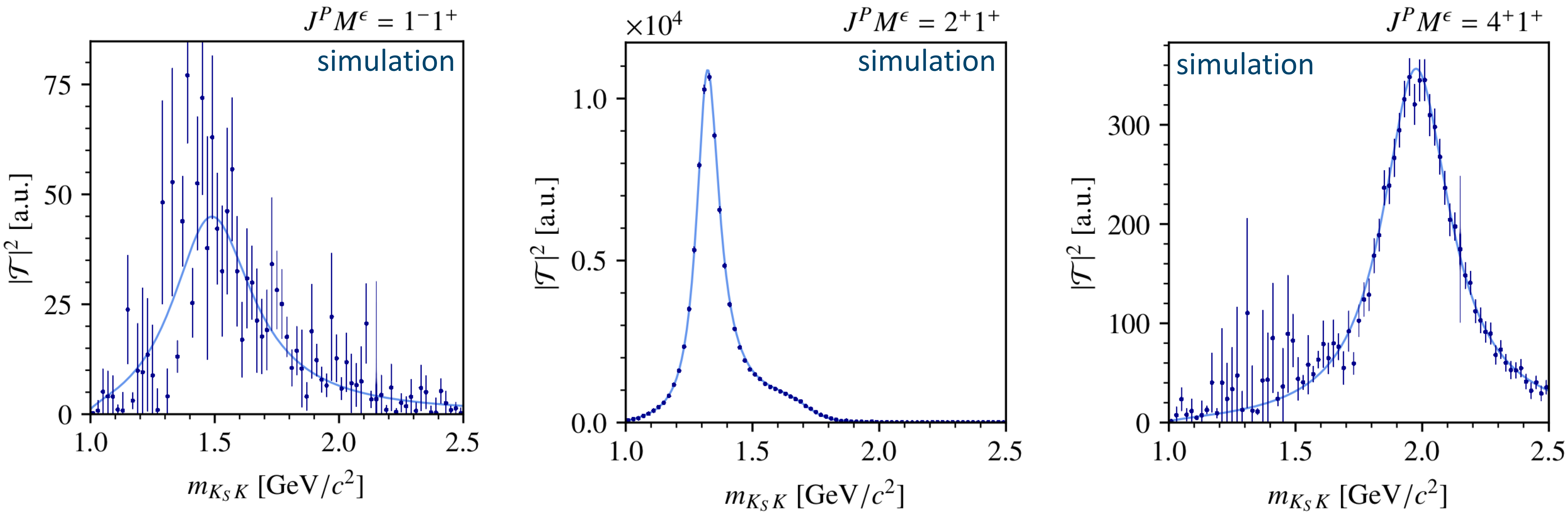
- 4^{++} intensity is still invariant!
- Overall, amplitude values found by the fit follow the calculated distributions
- Not all solutions are found in each m_X bin

→ PWD fit distorts the intensity distribution!



Resolving Ambiguities

- highest-spin wave is unaffected by ambiguities
- Including $M \geq 2$ $\rightarrow$ additional angular structure $\rightarrow$ **resolves ambiguities**
- Remove one wave with $J < J_{\text{max}}$ $\rightarrow$ **resolves ambiguities**



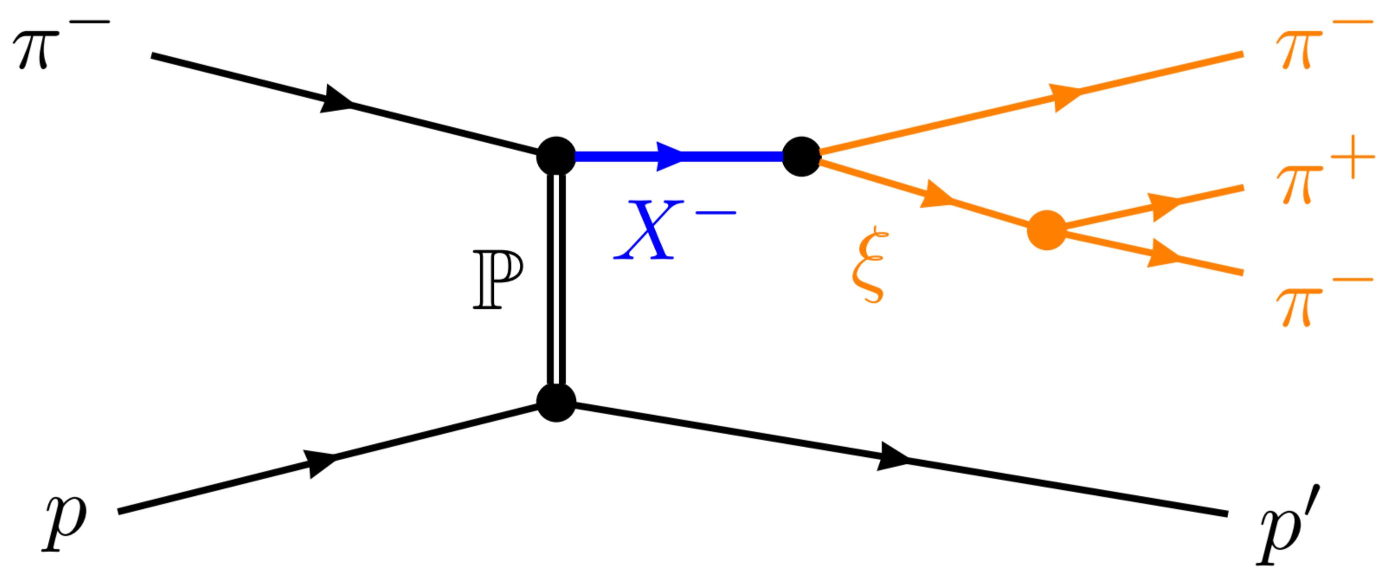
→ next: other possible solution

Continuity Constraints for Partial-Wave Analyses

Challenges of the $\pi^-\pi^-\pi^+$ Final State

$\pi^-\pi^-\pi^+$ Final State:

- no ambiguities
- large amount of data



Different Challenges:

- many contributing signals
- need to consider many partial-waves
- new signals are small / hidden among large ones
- selection of partial-wave model source of systematic uncertainty

Continuous Amplitude Models

Limitations of conventional PWA:

- Binned analysis limits statistics, especially for small signals
- We need to **select ("small") subset of partial waves** to include in the model

$\rightarrow$ important source of systematic uncertainty

More prior knowledge about $T(m_X, t')$:

- Physics should be (mostly) **continuous** in m_X and t'

$\rightarrow$ Solutions in close-by bins should be similar $\rightarrow$ correlations

- Amplitudes should follow **phase-space** and **production kinematics**

$\rightarrow$ use this information

Continuous Amplitude Models

Use of this information to stabilize partial-wave decomposition:

→ Replace discrete amplitudes with **smooth, non-parametric curves**

→ Incorporate **kinematic factors**

→ Include **regularization** for small amplitudes

Framework by group of Torsten Enßlin from the Max-Planck Institute for Astrophysics:

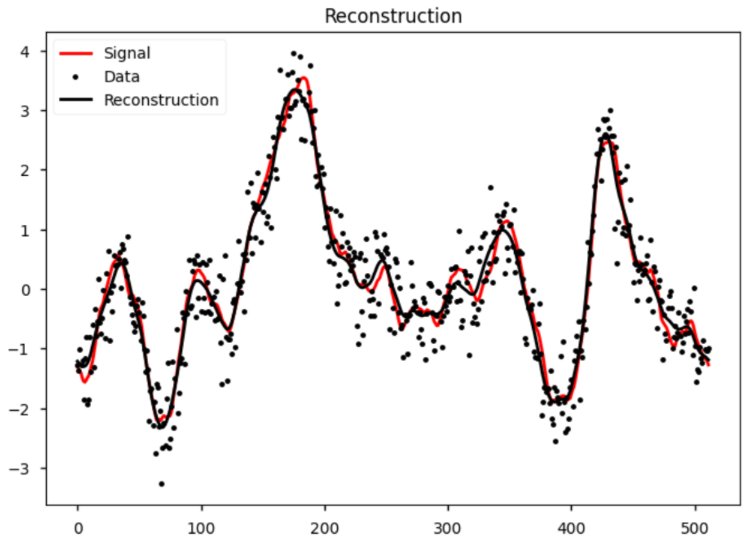
NIFTy: "**N**umerical **I**nformation **F**ield **T**heor**y**"

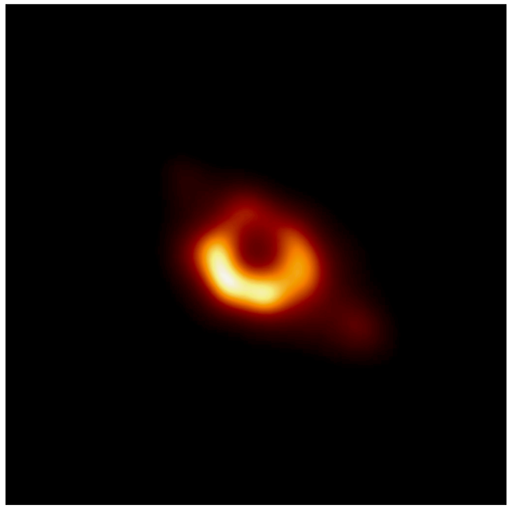
- Provides continuous non-parametric models
- Adapt to partial-wave analysis model
- **Learns smoothness and shape** of the amplitude curves

This work is done in collaboration with Jakob Knollmüller (TUM / ORIGINS Excellence Cluster)

A first attempt has been made together with Stefan Wallner and Philipp Frank

https://ift.pages.mpcdf.de/nifty/





M87* Black Hole: https://www.mpa-garching.mpg.de/1029092/hl202201

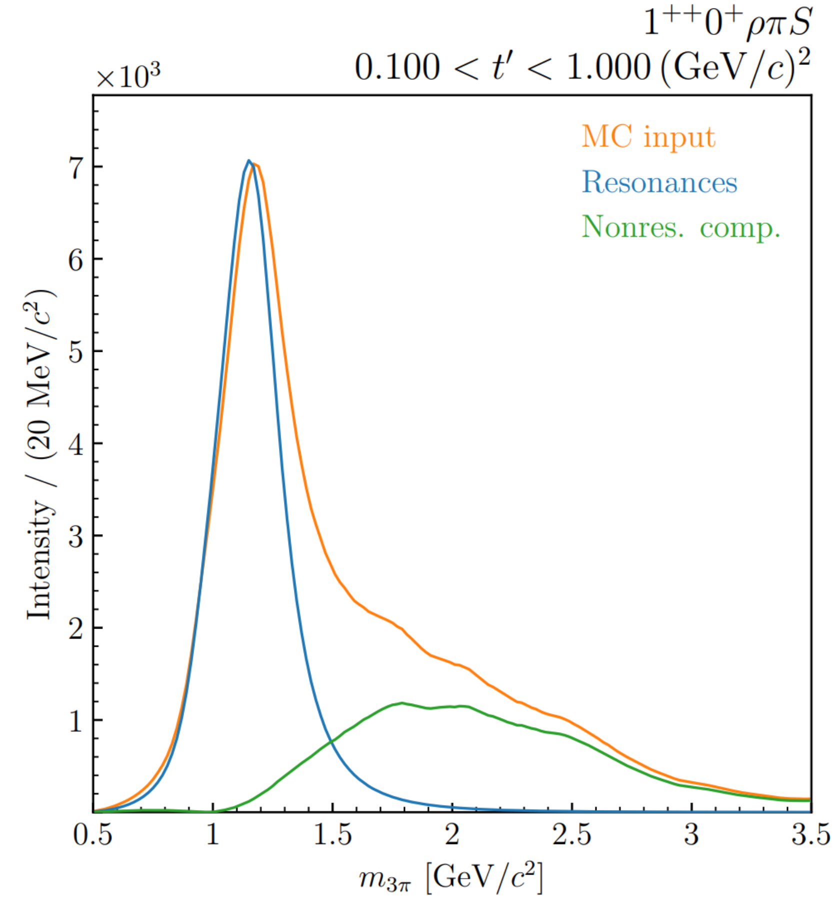
Verification on Simulated Data

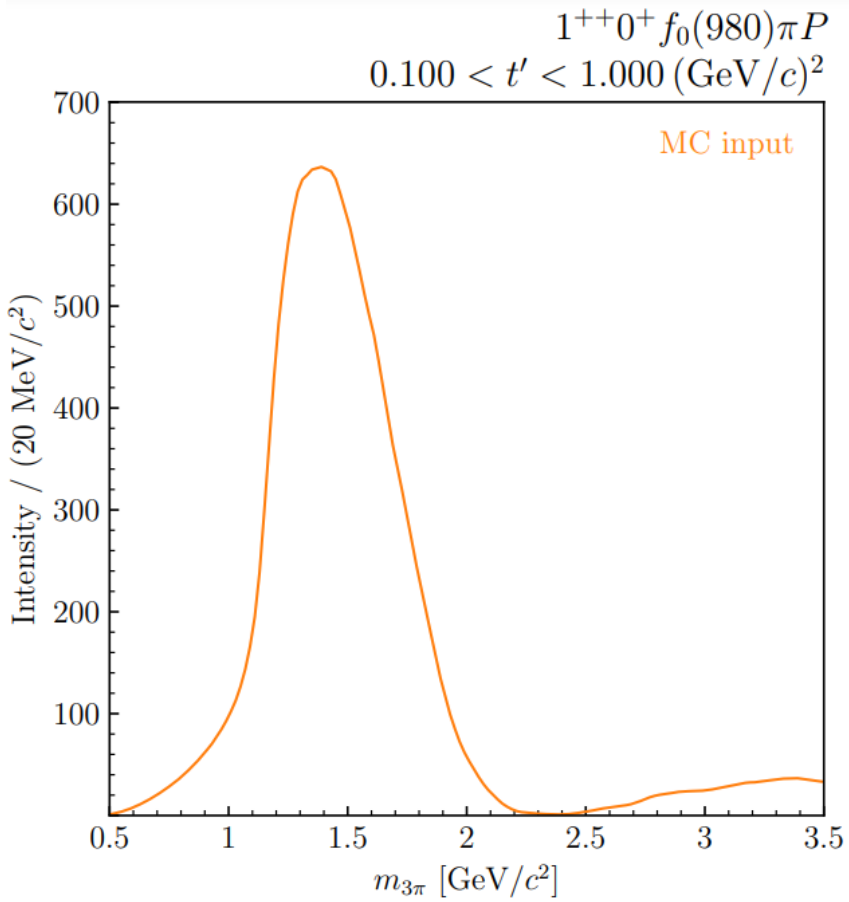
Create Pseudo-Data and try to recover!

Input-Output Study:

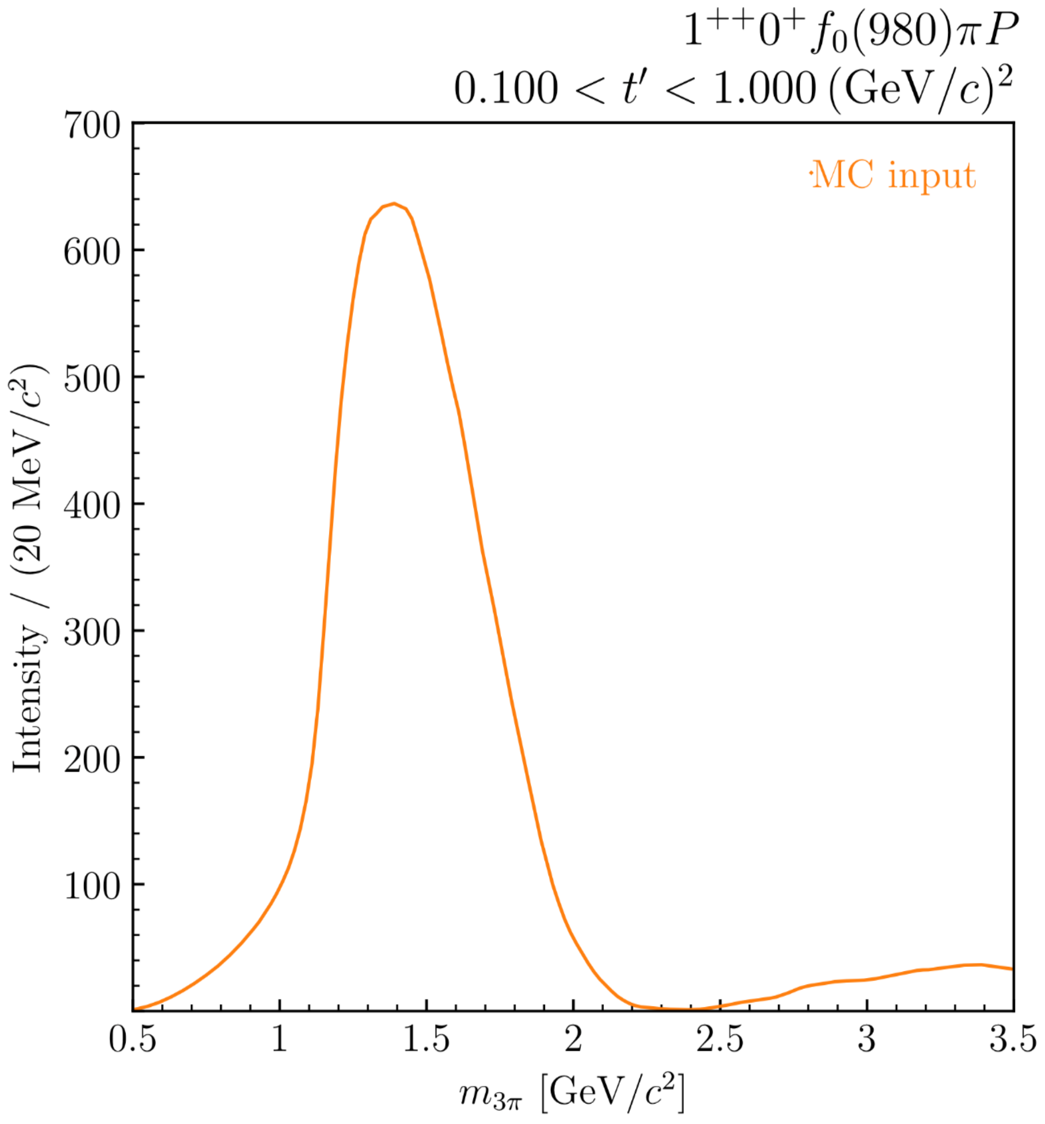
1. generate MC data according to:
   - smooth NIFTy model
   - 81 partial-waves
   - 5 resonances
2. try to recover input:

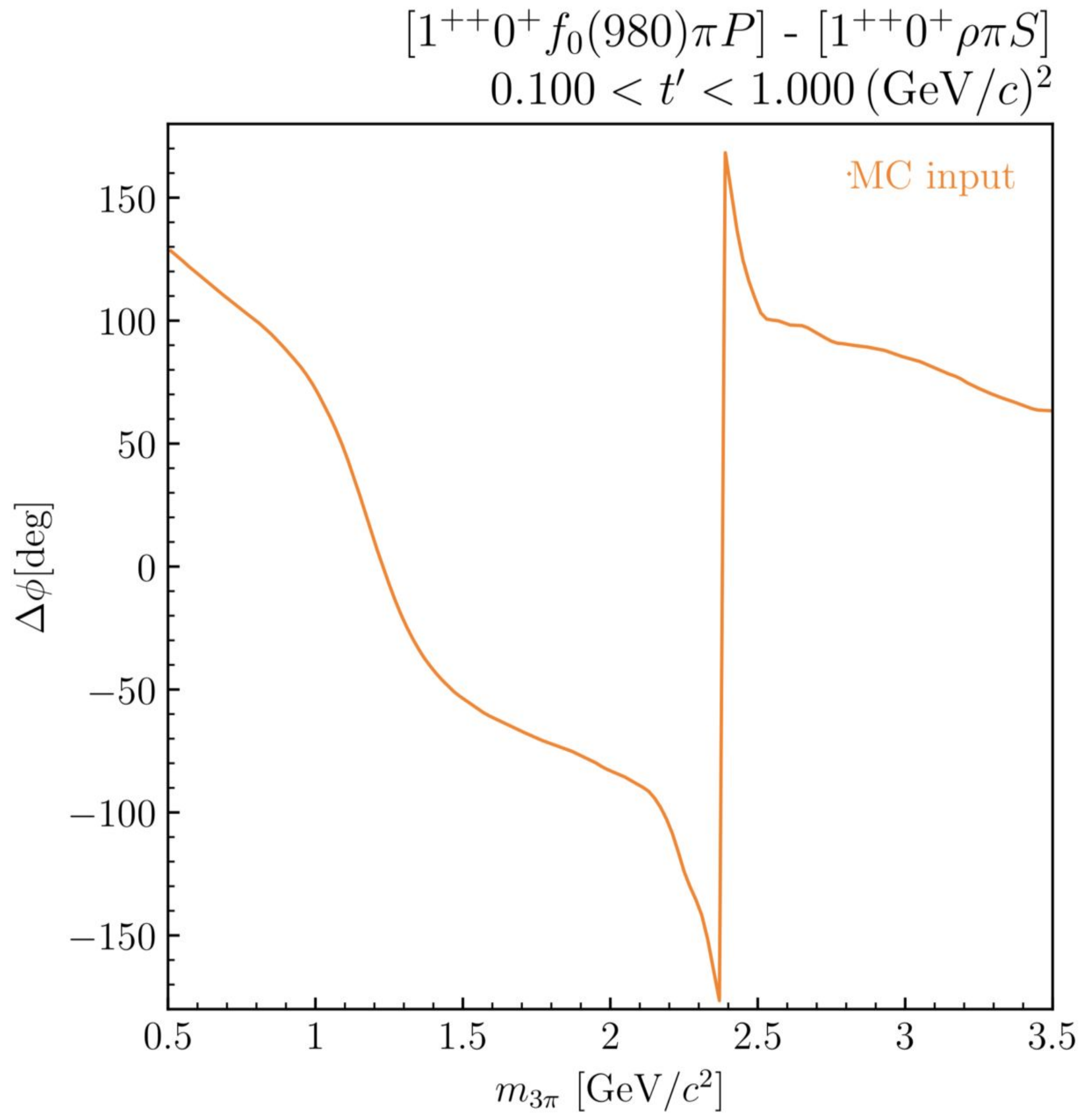
   - resonance(s) (Breit-Wigner)
   - nonres. component (broad curve)
   - Combined signal → **input model**



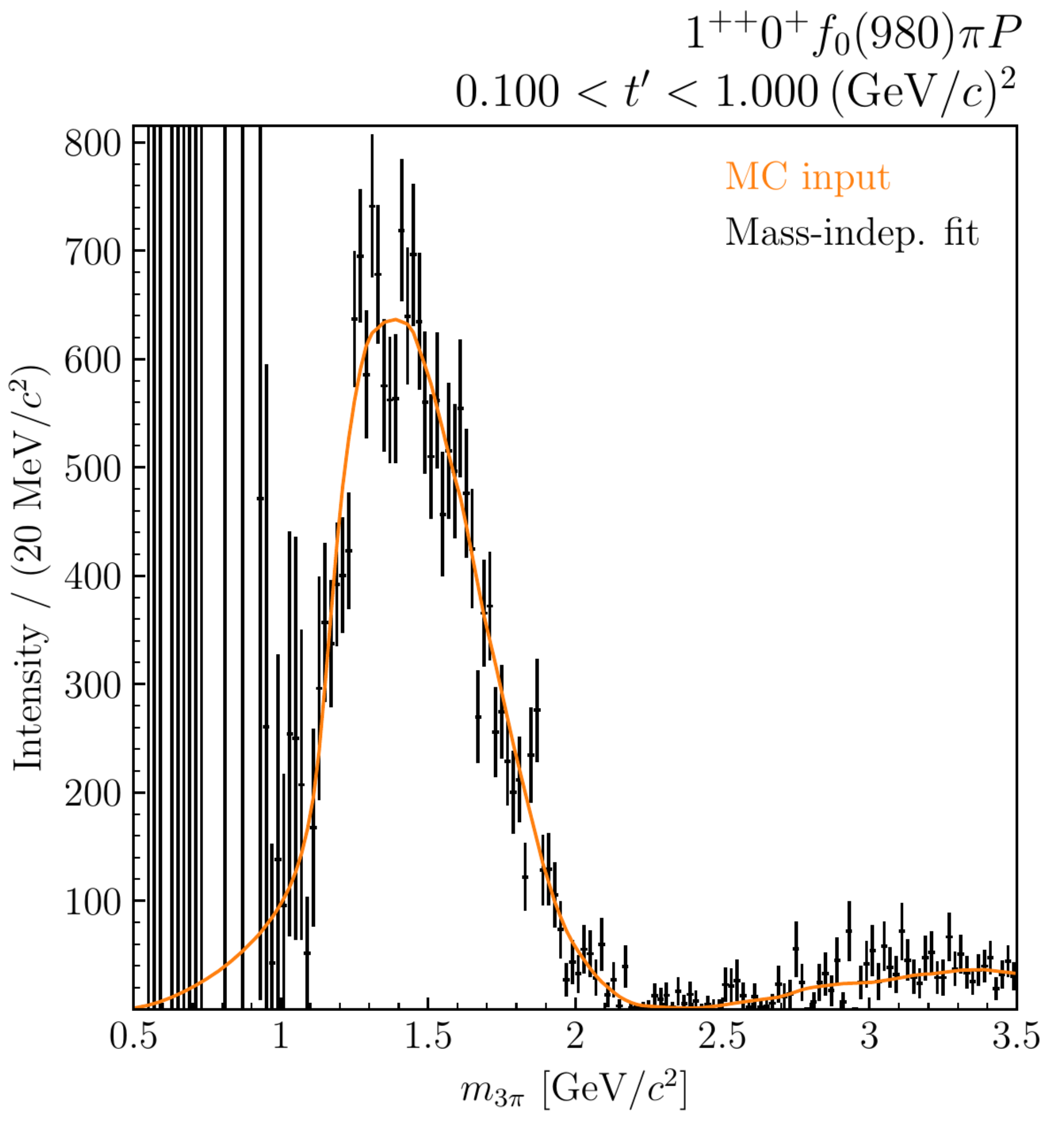


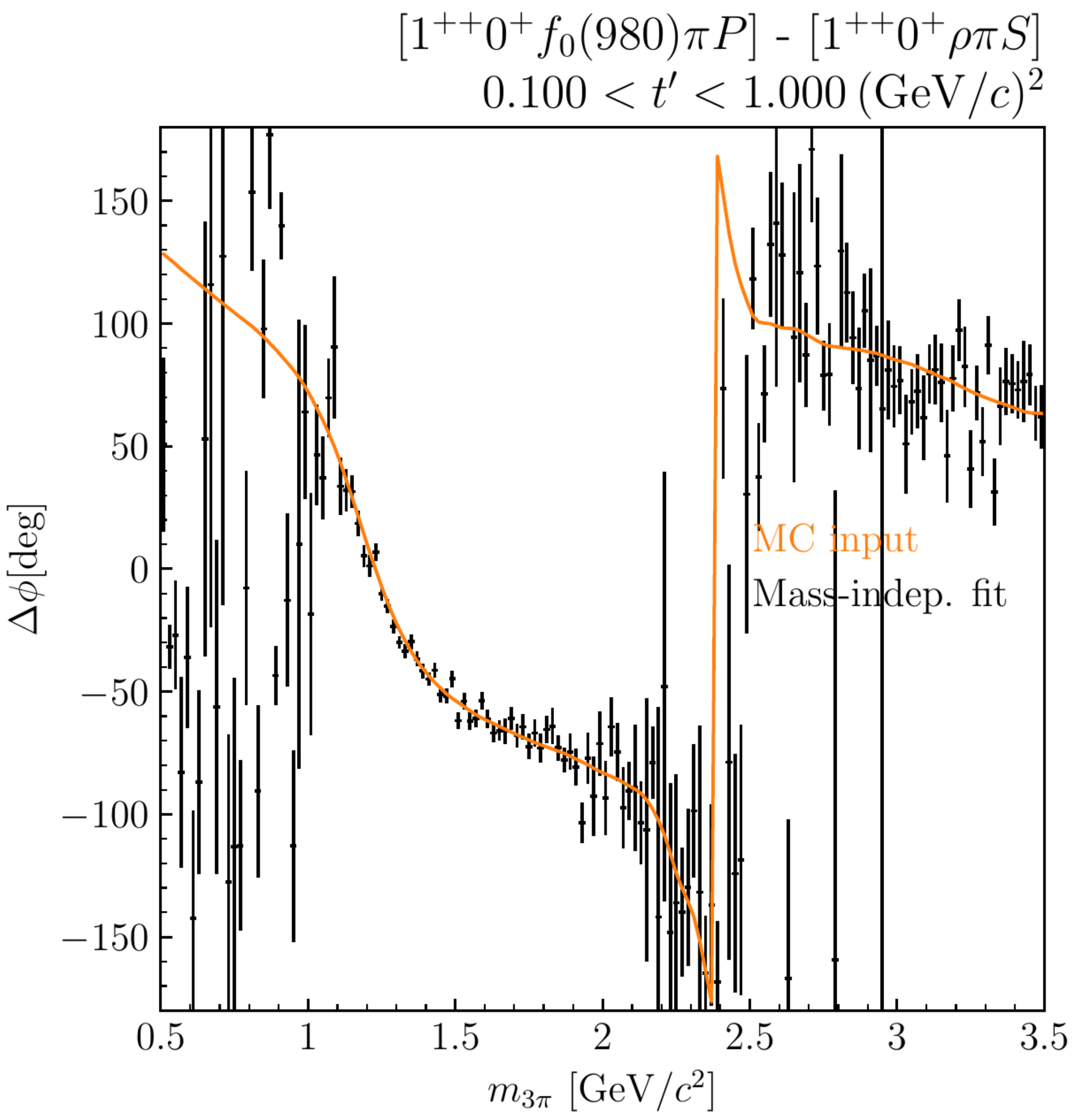
Input-Output Study



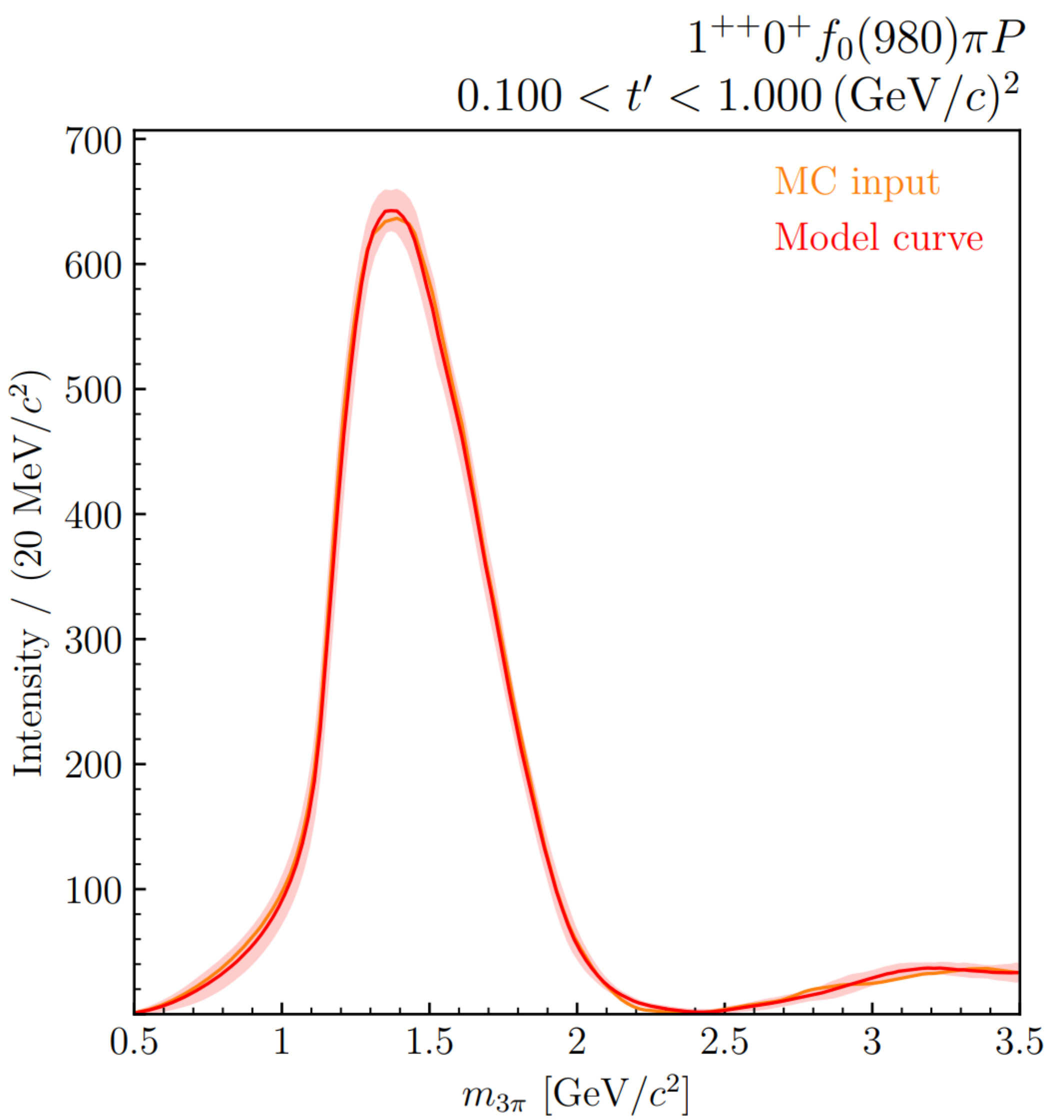


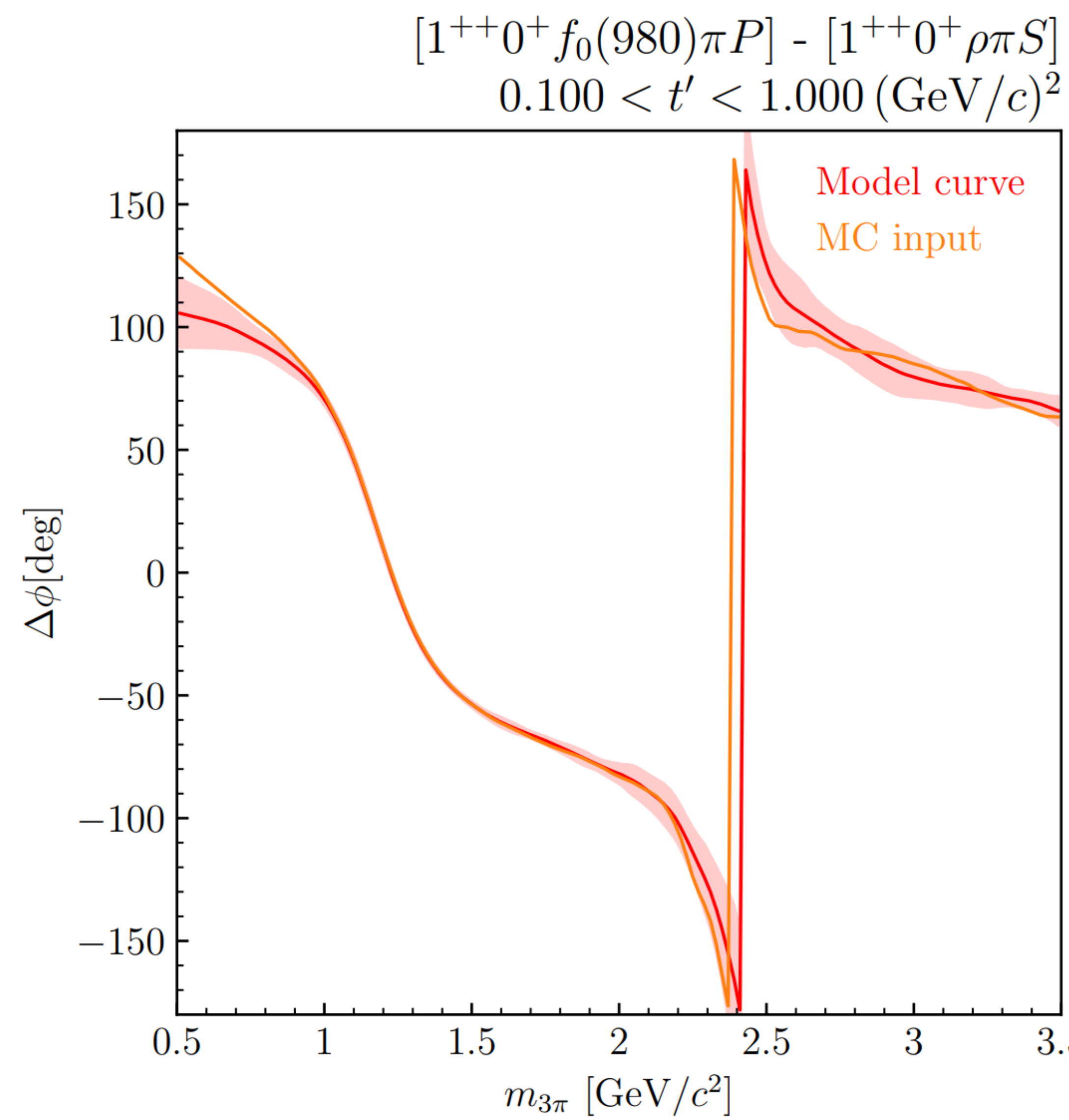
Input-Output Study



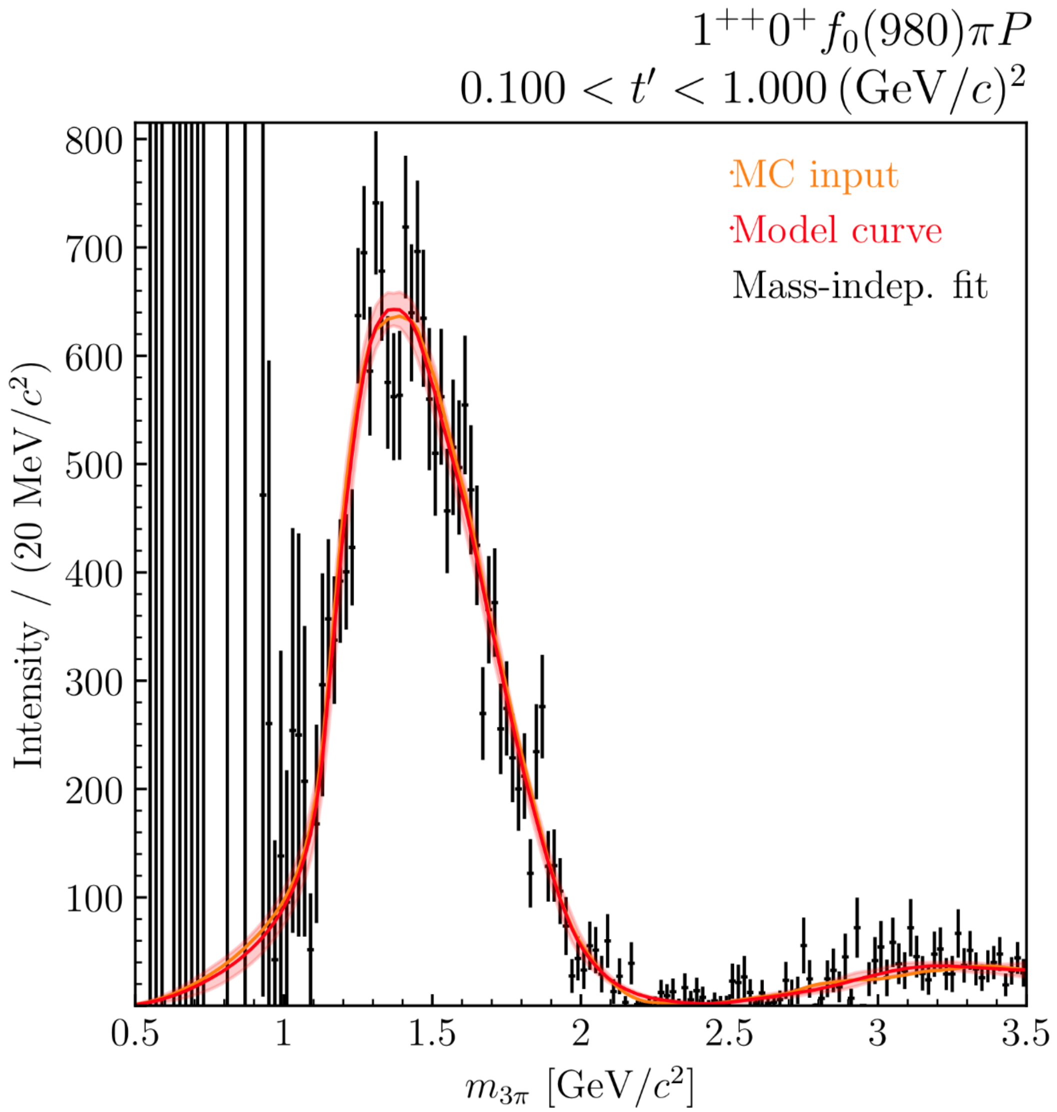


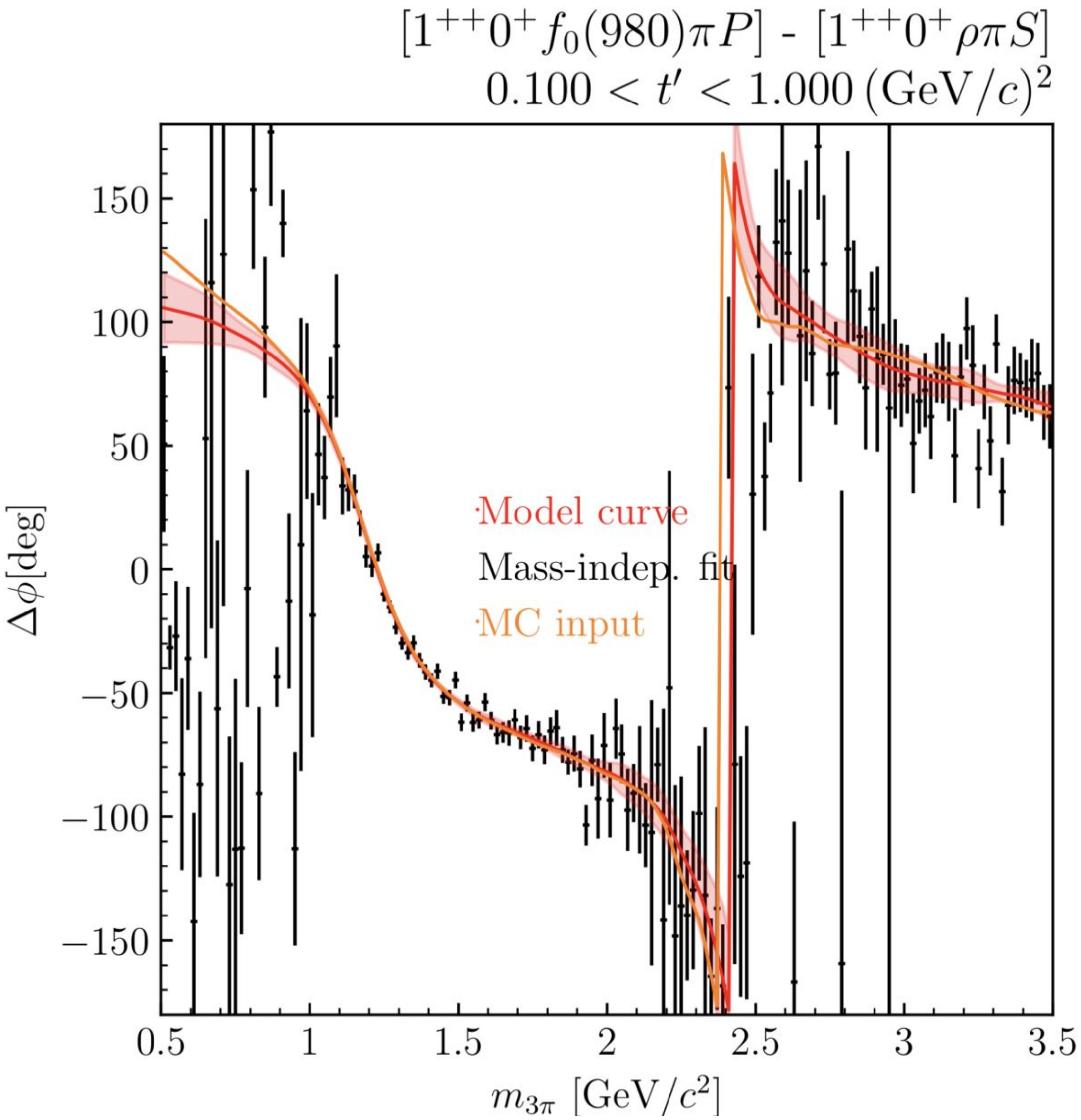
Input-Output Study





Input-Output Study





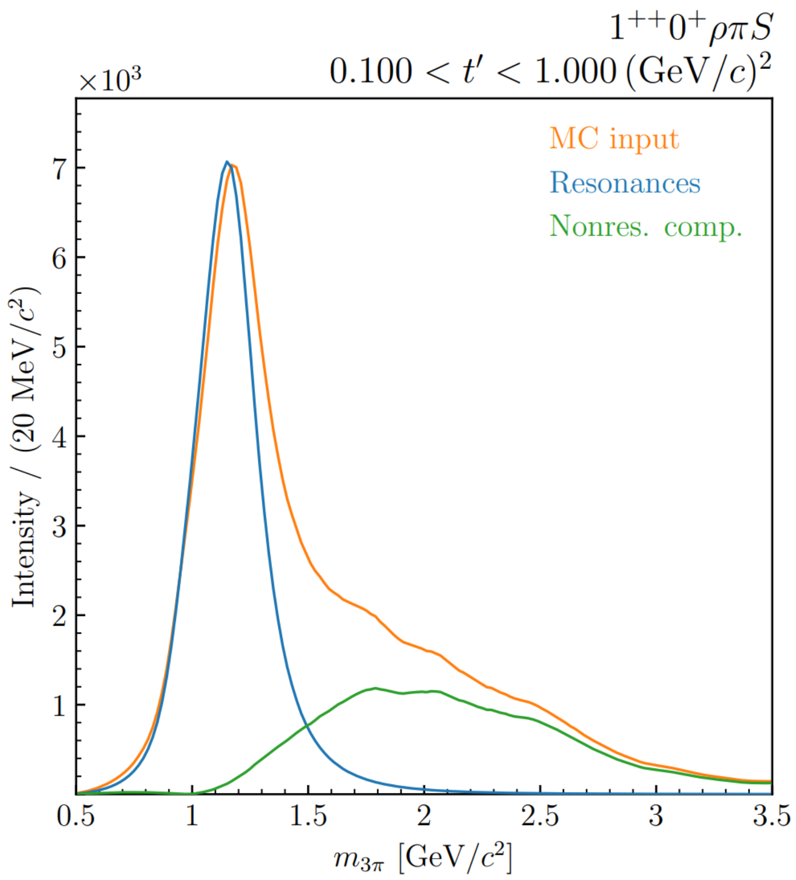
Single-Step Resonance Model Fit

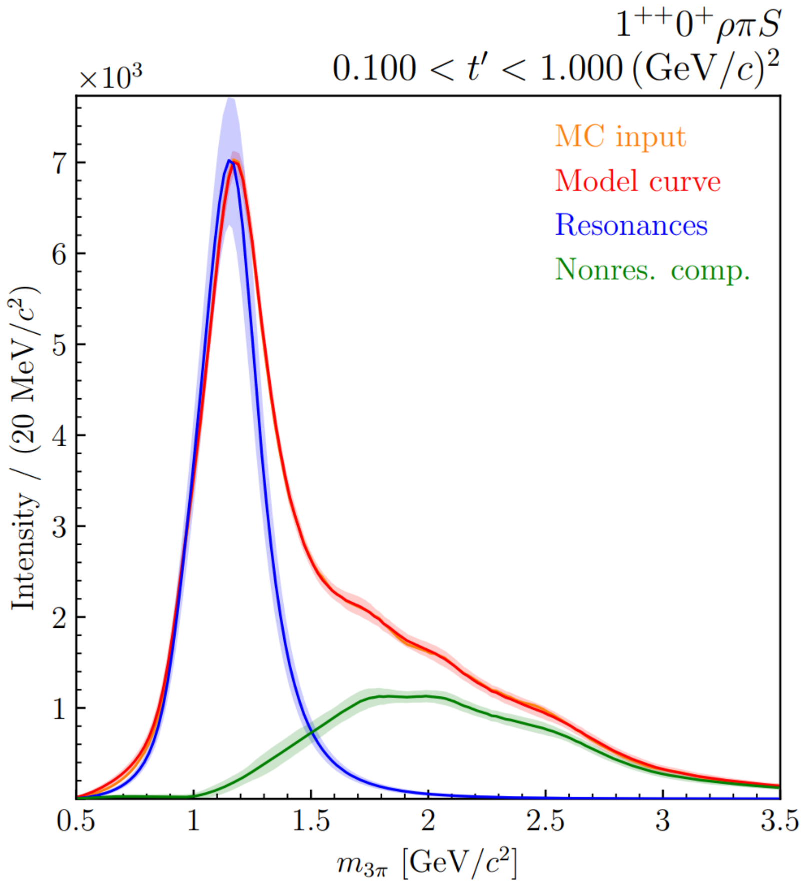
We can go one step further:

for selected waves add resonant part

- from NIFTy: flexible non-res. background
- resonant signal sum of Breit-Wigners
- coherent sum describes $T_i(m_{3\pi}, t')$

Goal: overcome limitations of the conventional approach



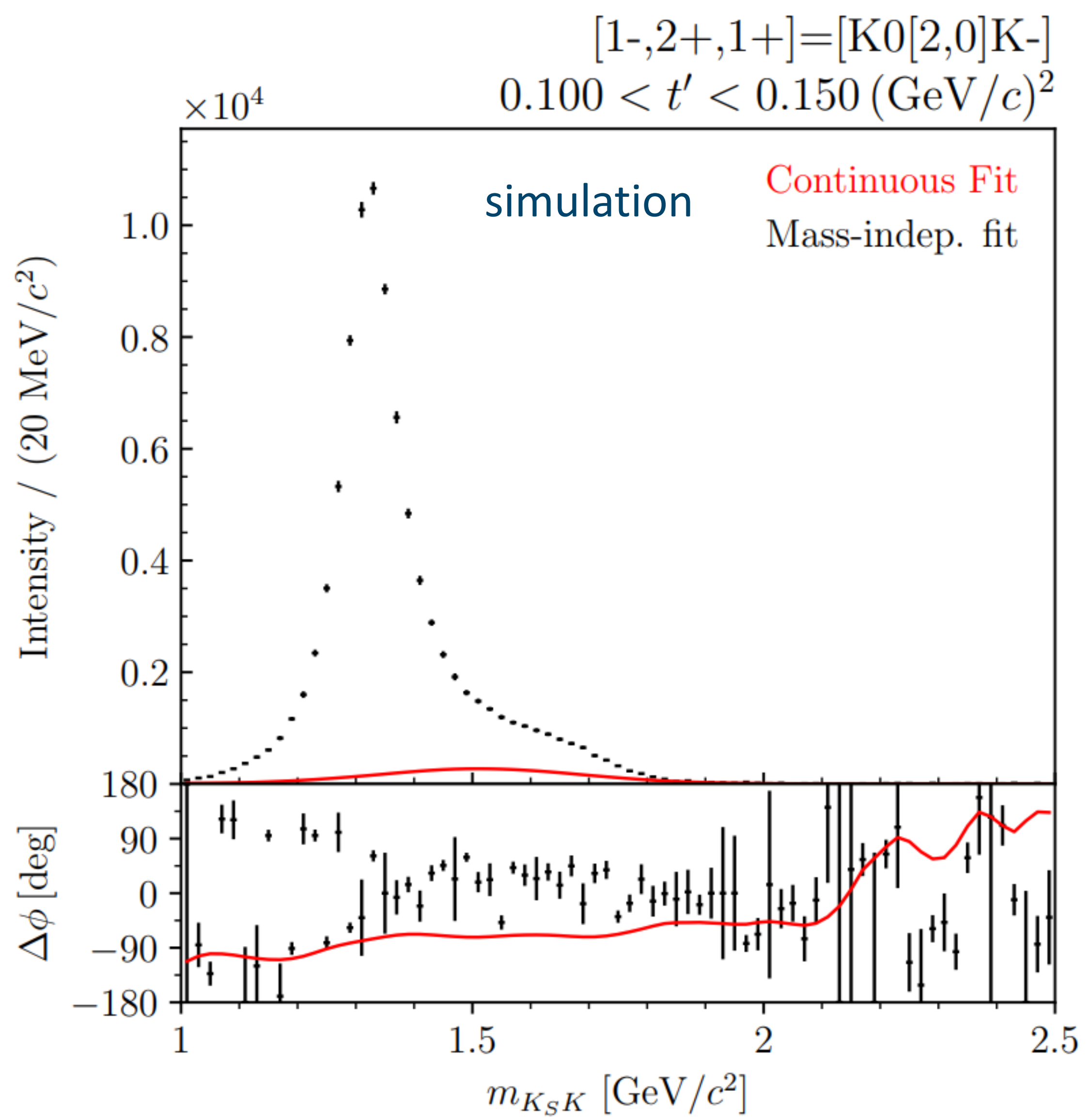


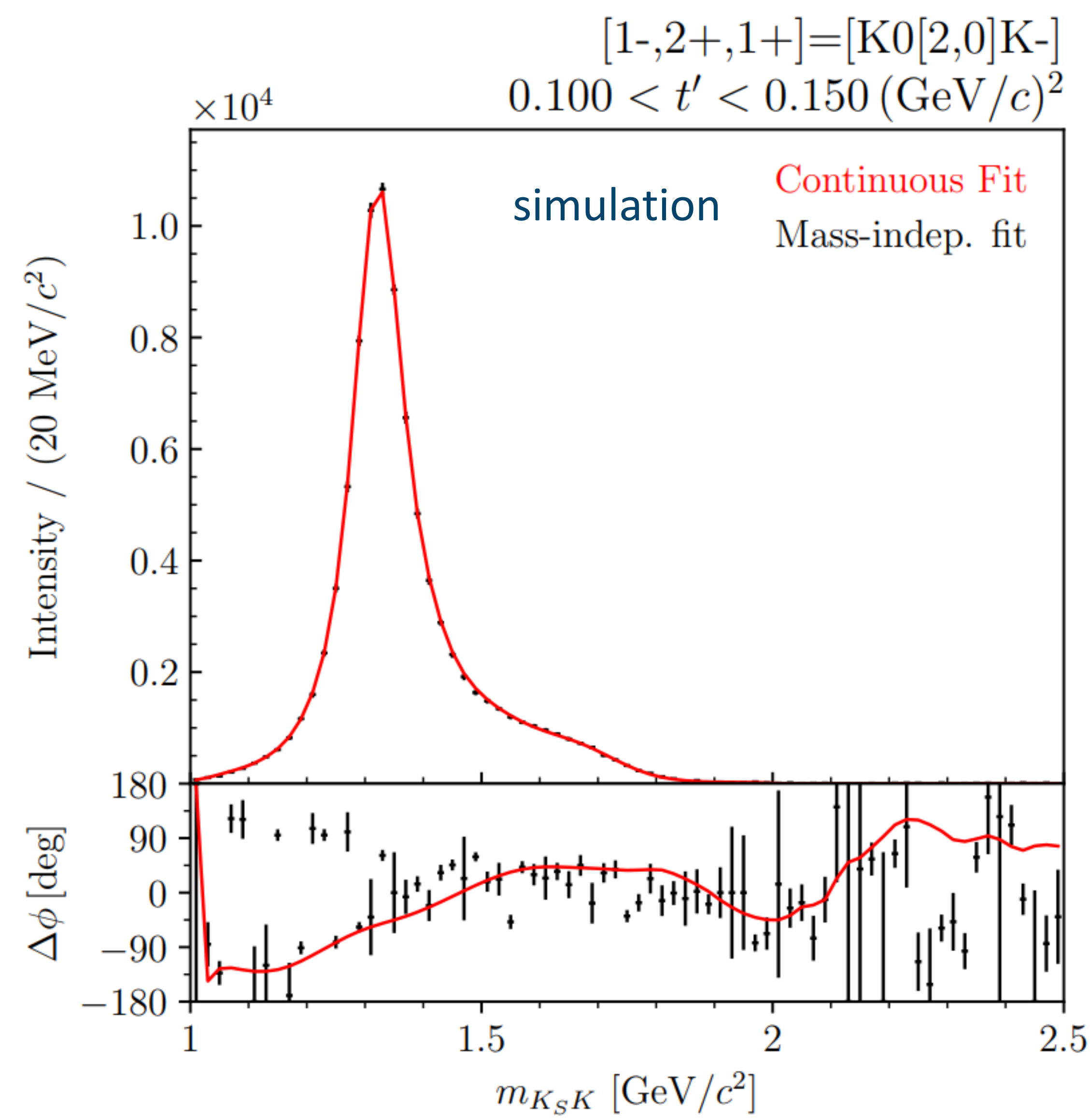
Application to $K_S^0K^-$ Final State

First attempts on simulated data: NIFTy seems to separate ambiguous solutions!

→ Apply NIFTy method on ambiguity problem in $K_S^0K^-$

- try separate ambiguous solutions over entire mass range
- improve fit quality





Conclusions & Outlook

Conclusions and Outlook

Ambiguities of two-body states

- ambiguous amplitudes are **continuous** and can be calculated
- PWD fit/finite data has an effect on ambiguous solutions
- several approaches to treat them

NIFTy + Partial-Wave Analysis:

- new approach to PWA
- **continuity, kinematics and regularization**
- combined with resonance-model fit

Currently:

- NIFTy method for $K_S^0 K^-$
- NIFTy method successfully applied to real data

Thank you for your attention!

Acknowledgements

Thank you for your attention!

I would like to thank Jakob Knollmüller who helped me develop the NIFTy model

I would also like to thank Stefan Wallner and Philipp Frank of the Max-Planck for Astrophysics with whom I worked on a first version of the NIFTy fit. The current work is partially based on this.

Questions?

Backup Slides

Partial-Wave Analysis: Limitations

mass-independent fit:

- select set of partial-waves $\{i\}$ → partial-wave model
- in principle: infinitely many waves
- in practice: finite data → select relevant waves
  - truncate high spins: large wavepool (several hundred waves)
  - select subset (otherwise unstable inference)

→ partial-wave model is a large systematic uncertainty

mass-dependent fit:

- fit to mass-independent result
- approximate uncertainties as Gaussian

→ source of systematic uncertainty

→ How can we improve the extraction?

Likelihood & Thresholds

Likelihood

$$\mathscr{L} = \frac{\bar{n}^n}{n!} e^{-\bar{n}} \prod_j^n P(\tau^j; m^j_{3pi}, t'^j) = \frac{1}{n!} e^{-\bar{n}} \prod_j^n I(\tau^j; m^j_{3pi}, t'^j)$$

with expected number of events $\bar{n} = \int_\Omega I(\tau; m_{3pi}, t')$d LIPS$(\tau) \approx \vec{T}^\dagger M \vec{T}$ within one bin

$\rightarrow$ maximize $\log(\mathscr{L}) \rightarrow$ transition amplitudes in bin $\vec{T} \in \mathbb{C}^n$

Integral Matrix $\tilde{M}_{ij} = \int_\Omega \psi(\tau)_i \psi(\tau)^*_j$d LIPS$(\tau)$ and $M_{ij} = \frac{\tilde{M}_{ij}}{\sqrt{\tilde{M}_{ii}\tilde{M}_{jj}}}$

This way:

- within one bin the phase-space information is moved to the transition amplitudes $\vec{T} \in \mathbb{C}^n$ or in other words: the fit chooses the value
- $|T_i|^2$ normalized to nmb. events
- $\tilde{M}_{ii}$ contains information of the wave opening with phase-space
- M_{ii} = 1
- M_{ij} are overlaps of decay amplitudes

Generative Model

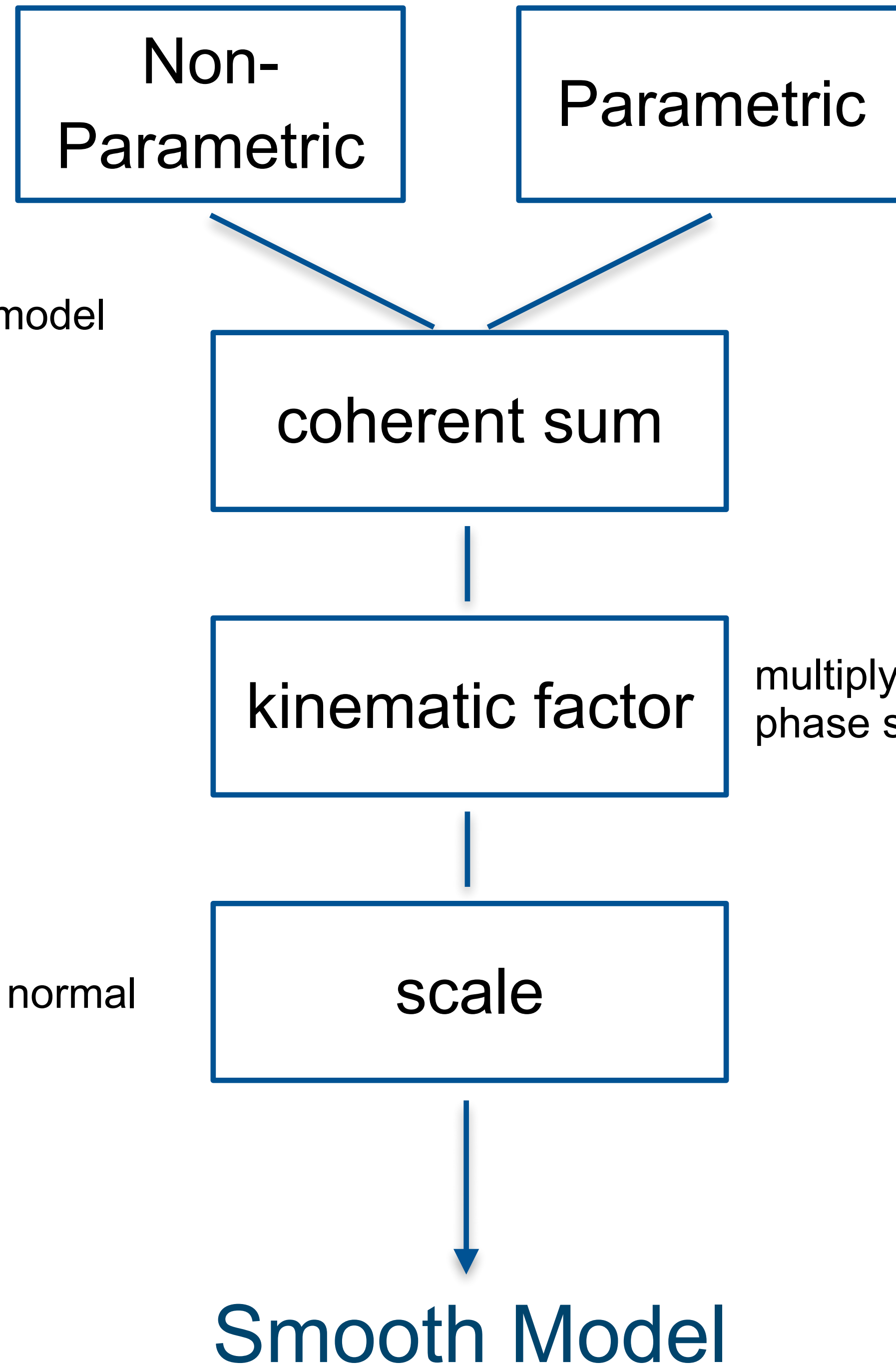
Generative Model (per wave):

Modified NIFTy correlated field maker:
→ fixed fluctuations to 1
→ loglog average slope -4
→ flexibility
→ offset

for real and imag part indiv.

functions as:
→ coh. background if there is a parametric model
→ description of transition amplitude

e.g. nothing or sum of Breit-Wigners:
Priors in masses and width
→ Lognormal
Prior on complex-valued scale:
→ 2d-normal
(scale to set relative prior strength)



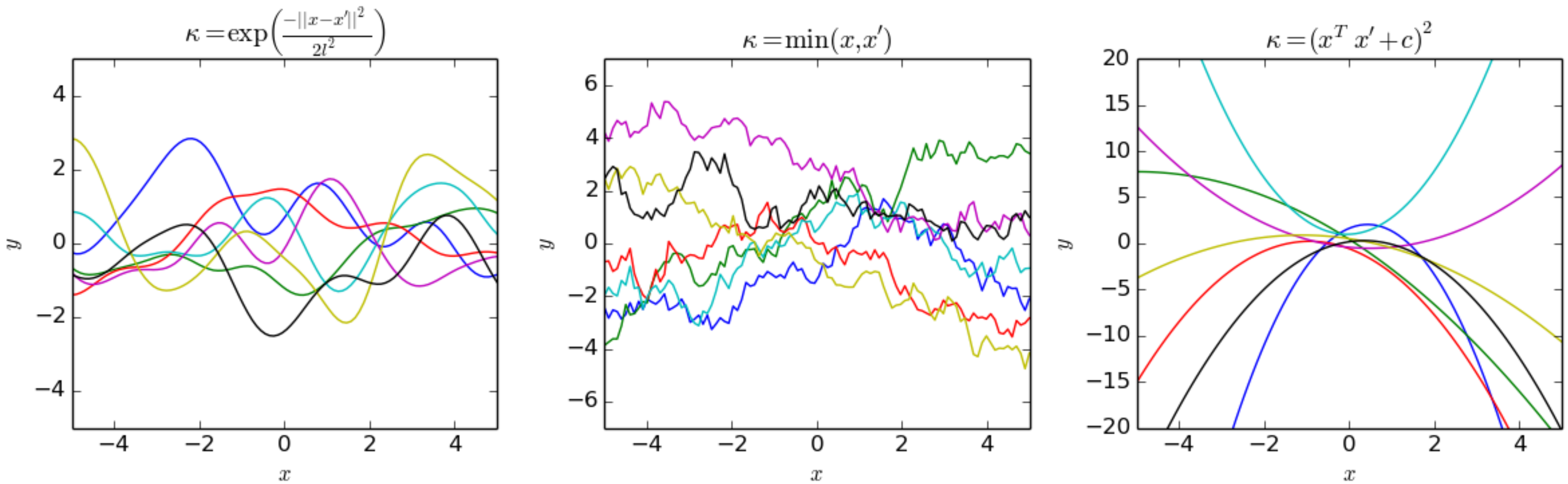
multiply with kinematic factor:
phase space of wave times production factor

scale for combined signal: 1d normal

Gaussian Processes

Formalize continuity:

- Gaussian Process: Infinite dimensional multivariate normal distribution
- Continuity given by covariance function: $\kappa(x, x')$
- encode our prior knowledge within choice of $\kappa(x, x')$



https://upload.wikimedia.org/wikipedia/commons/b/b4/Gaussian_process_draws_from_prior_distribution.png

How to chose $\kappa(x, x')$? → learn from data → NIFTy software framework

Model & Fit:

Bayes Theorem:

$$P(\{\theta_i\}\,|\,D) = \frac{P(D\,|\,\{\theta_i\})P(\{\theta_i\})}{P(D)}$$

- Prior: NIFTy: Generative Model → encodes:
  - smoothness
  - kinematic factor
  - prior on resonance parameters

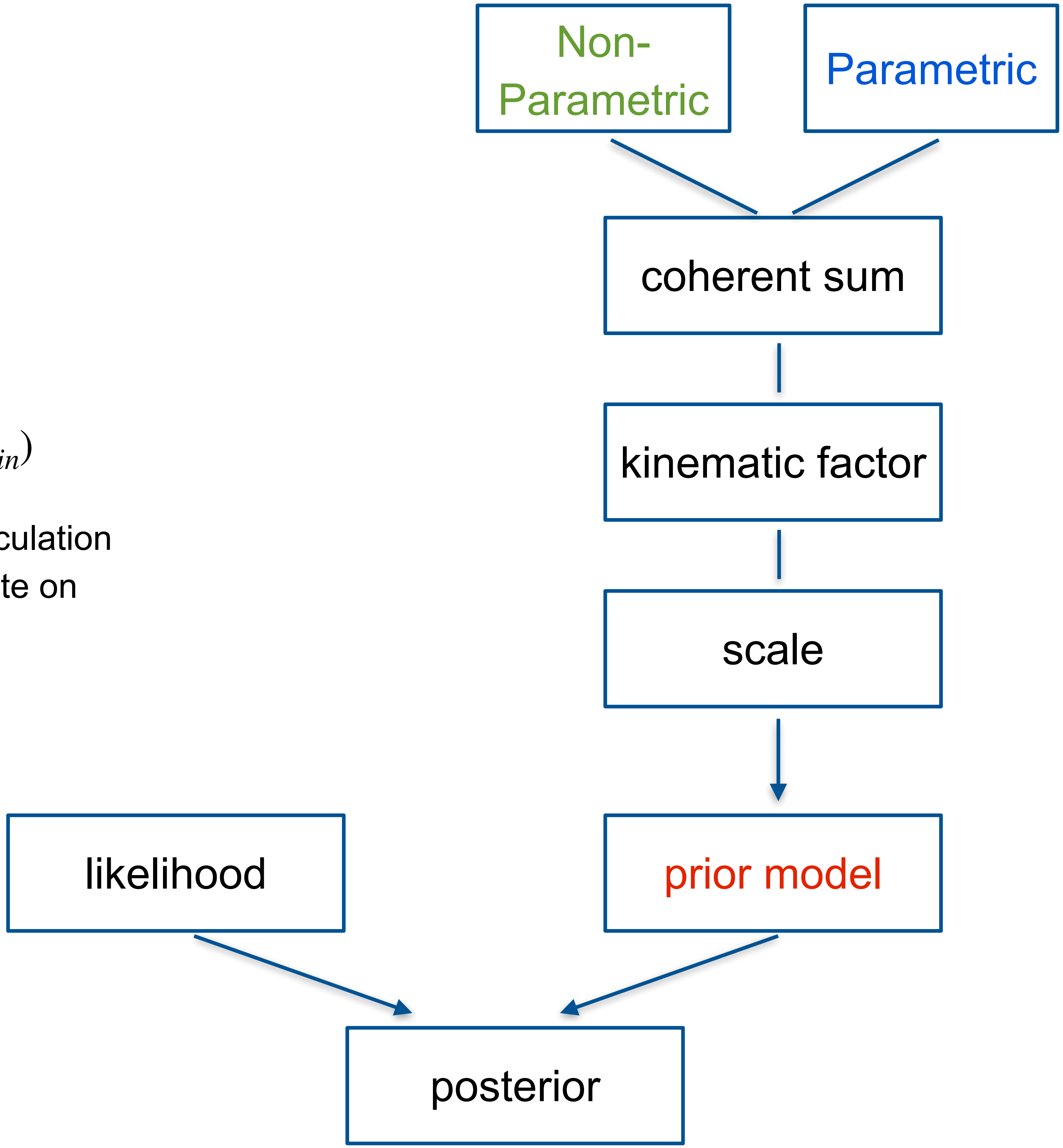
- Likelihood: From PWA framework:

$$\log \mathscr{L}(T_i\,|\,D) = \sum_{iBin} \log \mathscr{L}(T_i\,|\,D_{iBin})$$

  - cannot fit bins individually → likelihood calculation needs all bins at the same time! → distribute on multiple CPUs / machines with MPI
  - needs tens to hundreds of GB of memory

- Posterior: NIFTy Model & Likelihood

→ Fit to posterior



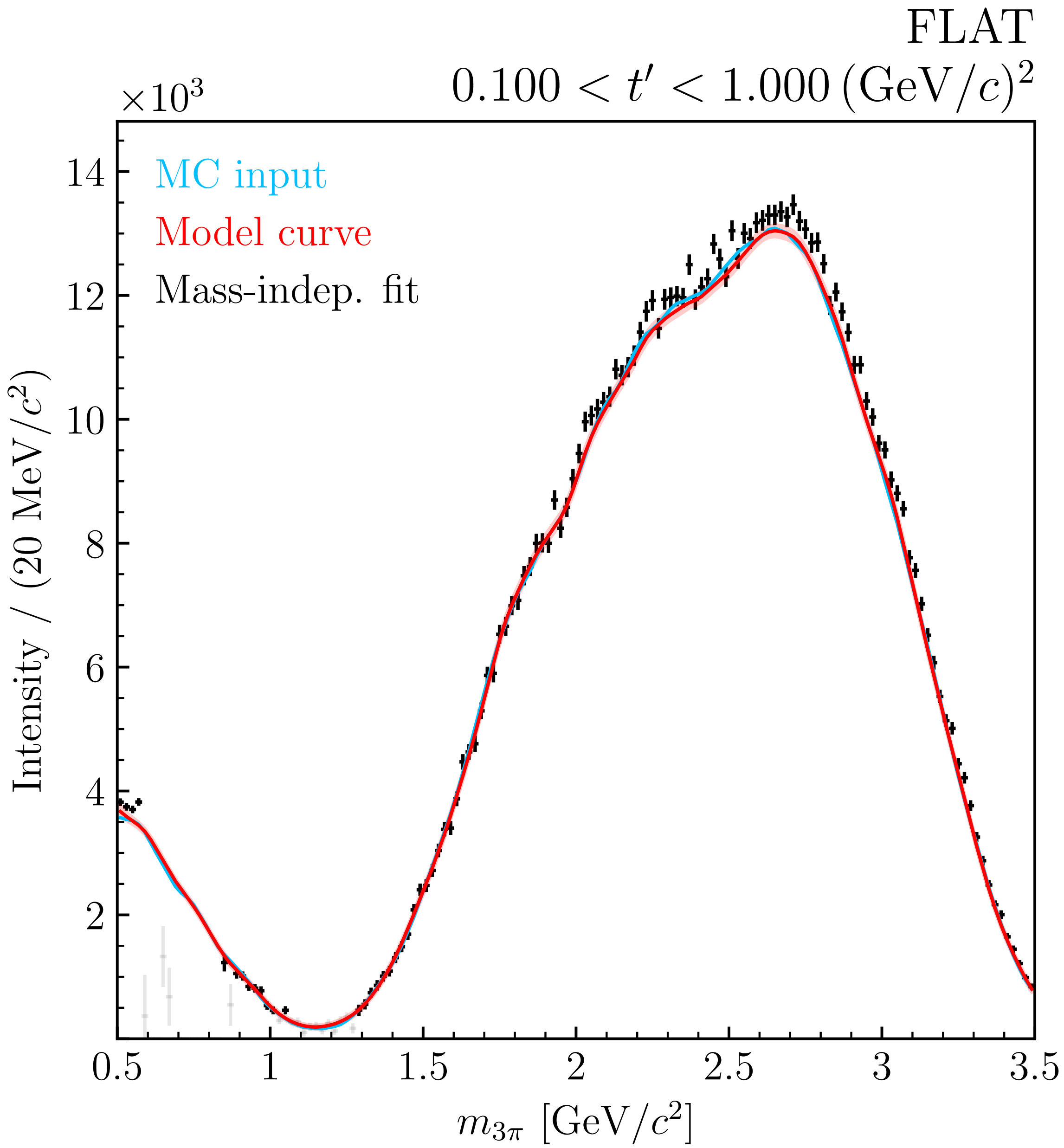
Regularized Fit

MC Model: Larger Fit Model

Non-Parametric (NIFTy) + Breit-Wigner resonance = model curve

Hybrid and mass-indep. fit reconstruction (NOW: 330 waves):

Mass-Indep. Fit with regularization:

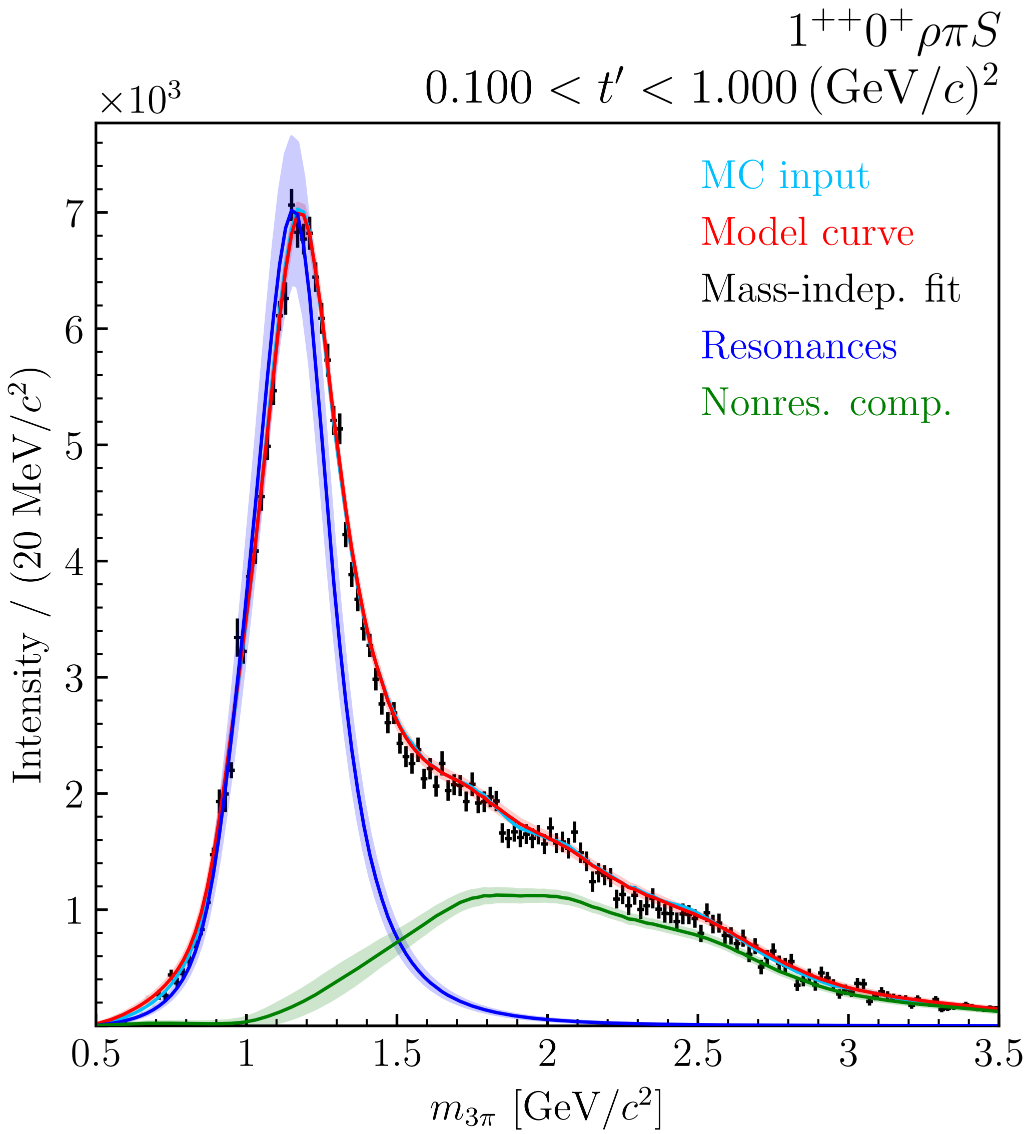


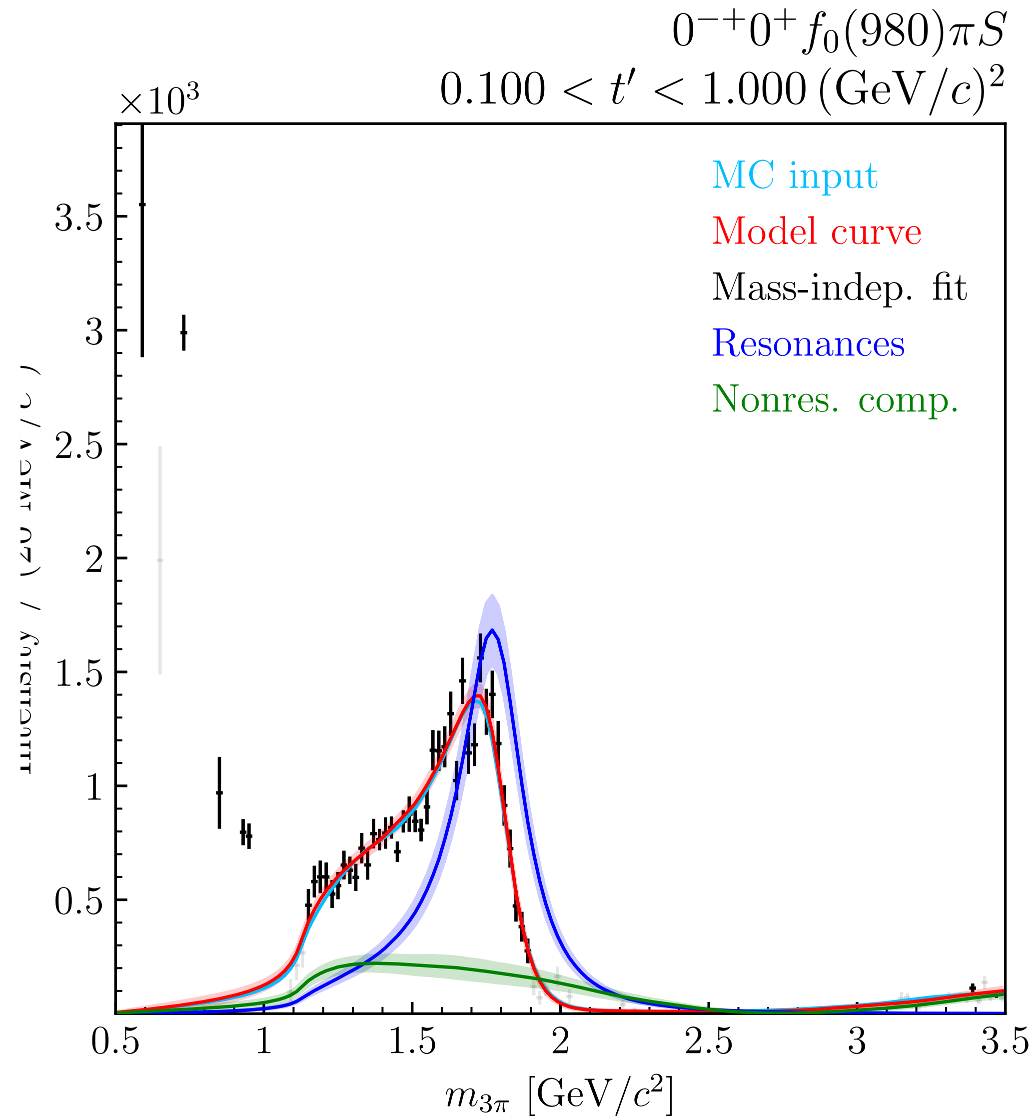
MC Model: Larger Fit Model

Non-Parametric (NIFTy) + Breit-Wigner resonance = model curve

Hybrid and mass-indep. fit reconstruction (NOW: 330 waves):

Mass-Indep. Fit with regularization:





Verification on MC: Extended Model

More realistic: consider 332 waves for fit

- mass-indep. fit: signs of overfitting bias
- single-stage fit: prior informations stabilizes fit
- still able to recover input & to separate non-res. and resonant components