Progress in the Partial-Wave Analysis Methods at COMPASS*

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Studies of Excited Light Mesons at COMPASS
Different Hadronic Final States

\[ \pi^- \rightarrow P \rightarrow X^- \rightarrow h_1, h_2, \ldots, h_n, P' \]

Where:
- \( \pi^- \) is a pion
- \( P \) is a particle
- \( X^- \) is a baryon
- \( h_1, h_2, \ldots, h_n \) are other particles
- \( P' \) is another particle
For this talk:

- COMPASS flagship channel:
  - > 100 Mio events
  - $\pi_j$ and $a_J$ resonances
  - $(J^{PC} = 0^{+-}, 1^{--}, 1^{++}, ...)$

  - Highly selective:
    - Final State: $J^{PC} = 1^{--}, 2^{++}, 3^{--}, ...$
    - Final State + dominant Pomeron exchange
    - $\rightarrow a_J$ for even $J$
    - $\rightarrow$ search for $a_6, a_4'$

  - $\rightarrow$ Probe for same resonances in different channels: Systematics!
For this talk:

- COMPASS flagship channel:
  - > 100 Mio events
  - → $\pi_j$ and $a_J$ resonances
  - ($J^{PC} = 0^{-+}, 1^{++}, 1^{++}, \ldots$)

  → Probe for same resonances in different channels: Systematics!

  **Final State:**
  - $J^{PC} = 1^{--}, 2^{++}, 3^{--}, \ldots$
  - + dominant Pomeron exchange
  - → $a_J$ for even $J$
  - → search for $a_6, a'_4$
Partial-Wave Analysis

Two Steps:

1) mass-independent fit
   model \( I(m_X, t'; \tau_n) \) in \((m_X, t')\) bins
   - factorization in \(T_a(m_X, t')\) and \(\psi_a(\tau; m_X)\)
   - parametrize \(T_a\) as step-wise functions
   - extract constant \(T_a\) in each bin

2) mass-dependent fit: model resonances
   1. results of first step: input
   2. \(\chi^2\) fit of resonant + background
      parameterization to subset of \(T_a(m_X, t')\)

→ resonance parameters = physics
Ambiguities in the Partial-Wave Decomposition of the $K_S^0 K^-$ Final State
Ambiguities in the Partial-Wave Decomposition

For any final state with **two spinless** particles ($\pi\pi$, $KK$, $\eta\pi$, ...):

Decomposition of intensity into $\{T_J\}$ is **not unique**

$\rightarrow$ Several sets of $\{T_J\}$ lead to the **same** $I(\theta, \phi)$ in each $(m_X, t')$ bin

\[
I(\theta, \phi) = \left| \sum_{JM} T^{(1)}_{JM} \psi_{JM}(\theta, \phi) \right|^2 = \left| \sum_{JM} T^{(2)}_{JM} \psi_{JM}(\theta, \phi) \right|^2
\]

Cannot distinguish between the **mathematically equivalent** solutions!
Ambiguities in the Partial-Wave Decomposition

\[ I(\theta, \phi) = \left| \sum_J T_J \psi_J(\theta, \phi) \right|^2 = \left| \sum_J T_J Y^1_J(\theta, 0) \right|^2 \sin^2 \phi \]

Polynomial in \( \tan^2 \theta \)

\[ a(\theta) = \sum_{j=0}^{J_{\text{max}}-1} c_j(\{T_j\}) \tan^{2j}(\theta) = \sum_{j=0}^{J_{\text{max}}-1} y_j \tan^{2j} \theta \]

Chung, PRD 56 7299–7316 (1997)

Barrelet, Nuov Cim A 8, 331–371 (1972)
Ambiguities in the Partial-Wave Decomposition

\[ I(\theta, \phi) = \left| \sum_J T_J Y^1_J(\theta, 0) \right|^2 |\sin \phi|^2 \]

\[ = \left| \sum_{j=0}^{J_{\text{max}}-1} c_j(\{T_J\}) \tan^2 j(\theta) \right|^2 |\sin \phi|^2 \]

\[ = c^2 \prod_{k=1}^{J_{\text{max}}-1} \left| \tan^2(\theta) - r_k^* \right|^2 |\sin \phi|^2 = c^2 \prod_{k=1}^{J_{\text{max}}-1} \left| \tan^2(\theta) - r_k^* \right|^2 |\sin \phi|^2 \]

Conjugation of roots \(\rightarrow\) different solution!

\[ \{T'_J\} \neq \{T_J\} \]
Study of Ambiguities

Study Continuous Intensity Model

Input:
- amplitude model for four selected partial waves
- \( m_\chi \)-dependence by Breit-Wigner amplitudes

<table>
<thead>
<tr>
<th>( J^{PC} )</th>
<th>Resonances</th>
</tr>
</thead>
<tbody>
<tr>
<td>1---</td>
<td>( \rho(1450) )</td>
</tr>
<tr>
<td>2++</td>
<td>( a_2(1320), a'_2(1700) )</td>
</tr>
<tr>
<td>3--</td>
<td>None</td>
</tr>
<tr>
<td>4++</td>
<td>( a_4(1970) )</td>
</tr>
</tbody>
</table>

![Graphs showing simulation data for different partial waves](graphica.png)
Study of Ambiguities

Calculate Ambiguous Solutions:

- Ambiguous intensities are also continuous in $m_X$
- Not all solutions are different from each other!
- Highest-spin ($4^{++}$) intensity is invariant!
**Study of Ambiguities**

**Pseudo-Data Study**
- generate pseudo-data according to model (10⁵ events)
- perform a partial-wave decomposition fit
  → **3000 attempts with random start values**

**Ambiguous Solutions from Fit:**
- 4++ intensity is still invariant!
- Overall, amplitude values found by the fit follow the calculated distributions
- Not all solutions are found in each $m_X$ bin

→ PWD fit distorts the intensity distribution!
Resolving Ambiguities

- highest-spin wave is unaffected by ambiguities
- Including $M \geq 2 \rightarrow$ additional angular structure $\rightarrow$ resolves ambiguities
- Remove one wave with $J < J_{\text{max}} \rightarrow$ resolves ambiguities

$\rightarrow$ next: other possible solution
Continuity Constraints for Partial-Wave Analyses
Challenges of the $\pi^-\pi^-\pi^+$ Final State

$\pi^-\pi^-\pi^+$ Final State:
- no ambiguities
- large amount of data

Different Challenges:
- many contributing signals
- need to consider many partial-waves
- new signals are small / hidden among large ones
- selection of partial-wave model source of systematic uncertainty
Continuous Amplitude Models

Limitations of conventional PWA:
• Binned analysis limits statistics, especially for small signals
• We need to select ("small") subset of partial waves to include in the model

→ important source of systematic uncertainty

More prior knowledge about $T(m_X, t')$:
• Physics should be (mostly) continuous in $m_X$ and $t'$
→ Solutions in close-by bins should be similar → correlations
• Amplitudes should follow phase-space and production kinematics

→ use this information
Continuous Amplitude Models

Use of this information to stabilize partial-wave decomposition:

→ Replace discrete amplitudes with smooth, non-parametric curves
→ Incorporate kinematic factors
→ Include regularization for small amplitudes

Framework by group of Torsten Enßlin from the Max-Planck Institute for Astrophysics:

NIFTy: “Numerical Information Field Theory”

• Provides continuous non-parametric models
• Adapt to partial-wave analysis model
• Learns smoothness and shape of the amplitude curves

This work is done in collaboration with Jakob Knollmüller (TUM / ORIGINS Excellence Cluster)

A first attempt has been made together with Stefan Wallner and Philipp Frank

M87* Black Hole: https://www.mpa-garching.mpg.de/1029092/hl202201
Verification on Simulated Data

Create Pseudo-Data and try to recover!

Input-Output Study:
1. generate MC data according to:
   - smooth NIFTy model
   - 81 partial-waves
   - 5 resonances
2. try to recover input:
   - resonance(s) (Breit-Wigner)
   - nonres. component (broad curve)
   - Combined signal → input model
Input-Output Study

\[1^{++}0^+ f_0(980)\pi P\]
\[0.100 < t' < 1.000 \text{ (GeV/c)}^2\]

\[\text{Intensity} / (20 \text{ MeV/c}^2)\]

\[m_{3\pi} \text{ [GeV/c}^2]\]

\[\Delta\phi [\text{deg}]\]

\[m_{3\pi} \text{ [GeV/c}^2]\]
Input-Output Study

\[ 1^{++}0^{+} f_{0}(980) \pi P \]
\[ 0.100 < t' < 1.000 \text{ (GeV/c)}^2 \]

MC input
Mass-indep. fit

\[ [1^{++}0^{+} f_{0}(980) \pi P] - [1^{++}0^{+} \rho \pi S] \]
\[ 0.100 < t' < 1.000 \text{ (GeV/c)}^2 \]

MC input
Mass-indep. fit
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Input-Output Study

$1^{++}0^+ f_0(980)\pi P$

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Intensity / (20 MeV/c$^2$)

$m_{3\pi} [GeV/c^2]$

MC input
Model curve
Mass-indep. fit

$[1^{++}0^+ f_0(980)\pi P] - [1^{++}0^+ \rho\pi S]$

$0.100 < t' < 1.000 (GeV/c)^2$

$\Delta\phi [\text{deg}]$

$m_{3\pi} [GeV/c^2]$

Model curve
Mass-indep. fit
MC input
Single-Step Resonance Model Fit

We can go one step further:

for selected waves add resonant part
- from NIFTy: flexible non-res. background
- resonant signal sum of Breit-Wigners
- coherent sum describes $T_i(m_{3\pi}, t')$

Goal: overcome limitations of the conventional approach
Application to $K_S^0K^-$ Final State

First attempts on simulated data: NIFTy seems to separate ambiguous solutions!

→ Apply NIFTy method on ambiguity problem in $K_S^0K^-$
  • try separate ambiguous solutions over entire mass range
  • improve fit quality
Conclusions & Outlook
Conclusions and Outlook

**Ambiguities** of two-body states

- ambiguous amplitudes are **continuous** and can be calculated
- PWD fit/finite data has an effect on ambiguous solutions
- several approaches to treat them

**NIFTy + Partial-Wave Analysis:**

- new approach to PWA
- **continuity, kinematics and regularization**
- combined with resonance-model fit

**Currently:**

- NIFTy method for $K_S^0 K^-$
- NIFTy method successfully applied to real data
Thank you for your attention!
Thank you for your attention!

I would like to thank Jakob Knollmüller who helped me develop the NIFTy model.

I would also like to thank Stefan Wallner and Philipp Frank of the Max-Planck for Astrophysics with whom I worked on a first version of the NIFTy fit. The current work is partially based on this.
Questions?
Backup Slides
Partial-Wave Analysis: Limitations

mass-independent fit:
- select set of partial-waves \( \{i\} \rightarrow \) partial-wave model
- in principle: infinitely many waves
- in practice: finite data \( \rightarrow \) select relevant waves
  - truncate high spins: large wavepool (several hundred waves)
  - select subset (otherwise unstable inference)

  \[ \rightarrow \text{partial-wave model is a large systematic uncertainty} \]

mass-dependent fit:
- fit to mass-independent result
- approximate uncertainties as Gaussian

  \[ \rightarrow \text{source of systematic uncertainty} \]

  \[ \rightarrow \text{How can we improve the extraction?} \]
Likelihood & Thresholds
Likelihood

$$\mathcal{L} = \frac{\tilde{n}^n}{n!} e^{-\tilde{n}} \prod_j P(\tau^j; m_{3pi}^j, t^j) = \frac{1}{n!} e^{-\tilde{n}} \prod_j I(\tau^j; m_{3pi}^j, t^j)$$

with expected number of events \(\tilde{n} = \int_{\Omega} I(\tau; m_{3pi}, t') \, d\text{LIPS}(\tau) \approx \vec{T}^\dagger M \vec{T}\) within one bin

\[\rightarrow\text{ maximize } \log(\mathcal{L}) \rightarrow \text{ transition amplitudes in bin } \vec{T} \in \mathbb{C}^n\]

Integral Matrix \(\tilde{M}_{ij} = \int_{\Omega} \psi(\tau)_i \bar{\psi(\tau)}_j \, d\text{LIPS}(\tau)\) and \(M_{ij} = \frac{\tilde{M}_{ij}}{\sqrt{\tilde{M}_{ii} \tilde{M}_{jj}}}\)

This way:

- within one bin the phase-space information is moved to the transition amplitudes \(\vec{T} \in \mathbb{C}^n\) or in other words: the fit chooses the value
- \(|T_i|^2\) normalized to nmb. events
- \(\tilde{M}_{ii}\) contains information of the wave opening with phase-space
- \(M_{ii} = 1\)
- \(M_{ij}\) are overlaps of decay amplitudes
Generative Model
Generative Model (per wave):

Modified NIFTy correlated field maker:
→ fixed fluctuations to 1
→ loglog average slope -4
→ flexibility
→ offset

for real and imag part indiv.

functions as:
→ coh. background if there is a parametric model
→ description of transition amplitude

e.g. nothing or sum of Breit-Wigners:
Priors in masses and width
→ Lognormal
Prior on complex-valued scale:
→ 2d-normal
(scale to set relative prior strength)

multiply with kinematic factor:
phase space of wave times production factor

scale for combined signal: 1d normal
Gaussian Processes

Formalize continuity:

- Gaussian Process: Infinite dimensional multivariate normal distribution
- Continuity given by covariance function: $\kappa(x, x')$
- encode our prior knowledge within choice of $\kappa(x, x')$

$$\kappa = \exp\left(\frac{-||x-x'||^2}{2l^2}\right)$$

- $\kappa = \min(x, x')$
- $\kappa = (x^T x' + c)^2$

How to chose $\kappa(x, x')$? → learn from data → NIFTy software framework

https://upload.wikimedia.org/wikipedia/commons/b/b4/Gaussian_process_draws_from_prior_distribution.png
Model & Fit:

Bayes Theorem:

\[
P(\{\theta_i\} | D) = \frac{P(D | \{\theta_i\})P(\{\theta_i\})}{P(D)}
\]

- **Prior**: NIFTy: Generative Model \(\rightarrow\) encodes:
  - smoothness
  - kinematic factor
  - prior on resonance parameters

- **Likelihood**: From PWA framework:
  \[
  \log \mathcal{L}(T_i | D) = \sum_{iBin} \log \mathcal{L}(T_i | D_{iBin})
  \]
  - cannot fit bins individually \(\rightarrow\) likelihood calculation
    needs all bins at the same time! \(\rightarrow\) distribute on multiple CPUs / machines with MPI
  - needs tens to hundreds of GB of memory

- **Posterior**: NIFTy Model & Likelihood

\(\rightarrow\) Fit to posterior
Regularized Fit
MC Model: Larger Fit Model

Non-Parametric (NIFTy) + Breit-Wigner resonance = model curve

Hybrid and mass-indep. fit reconstruction (NOW: 330 waves):

Mass-Indep. Fit with regularization:
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\[ 0.100 < t' < 1.000 \ (\text{GeV}/c)^2 \]
Verification on MC: Extended Model

More realistic: consider 332 waves for fit
- mass-indep. fit: signs of overfitting bias
- single-stage fit: prior informations stabilizes fit
- still able to recover input & to separate non-res. and resonant components

\[ 1^{++}0^{+}\rho\pi S \]
\[ 0.100 < t' < 1.000 \text{ (GeV/c)}^2 \]

![Graph showing intensity over mass 3π](image)

- MC input
- Model curve
- Mass-indep. fit
- Resonances
- Nonres. comp.

![Graph showing FLAT over mass 3π](image)

- MC input
- Model curve
- Mass-indep. fit