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Hard exclusive π^0 production at COMPASS IWHSS 2023, Prague

Karolína Lavičková* on behalf of the COMPASS collaboration

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2023 June 27

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Outline

- Introduction
- Measurement at COMPASS
- Data selection and background estimation
- Cross section determination
- Results



Introduction

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Hard exclusive π^{0} production at COMPASS

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Generalized Parton Distributions (GPDs)

- Tool for the investigation of the nucleon structure
- Provide a 3D picture of how quarks and gluons build up the nucleon
- Access to GPDs via
 - Deeply Virtual Compton Scattering (DVCS)
 - Hard Exclusive Meson production (HEMP)
 - Hard exclusive π^0 muoproduction
 - Cross section measurement of $\mu p \rightarrow \mu' \pi^0 p'$



Hard exclusive π^0 muoproduction



Sensitive to the GPDs

- $\tilde{H}(x, \xi, t)$ and $\tilde{E}(x, \xi, t)$ chiral-even (conserving the parton helicity)
- $H_T(x, \xi, t)$ and $\overline{E}_T(x, \xi, t)$ chiral-odd (parton helicity flip)

Reduced cross section for hard exclusive π^0 production

$$\frac{\mathrm{d}^2 \sigma^{\gamma^* p}}{\mathrm{d}t \mathrm{d}\phi_{\pi^0}} = \frac{1}{2\pi} \bigg[\epsilon \frac{\mathrm{d}\sigma_L}{\mathrm{d}t} + \frac{\mathrm{d}\sigma_T}{\mathrm{d}t} + \epsilon \cos(2\phi_{\pi^0}) \frac{\mathrm{d}\sigma_{TT}}{\mathrm{d}t} + \sqrt{2\epsilon(1+\epsilon)} \cos(2\phi_{\pi^0}) \frac{\mathrm{d}\sigma_{LT}}{\mathrm{d}t} \bigg]$$

$$\begin{split} \frac{\mathrm{d}\sigma_L}{\mathrm{d}t} &\propto \left[(1-\xi^2) |\langle \tilde{H} \rangle|^2 - 2\xi^2 \mathrm{Re} \left[\langle \tilde{H} \rangle * \langle \tilde{E} \rangle \right] - \frac{t'}{4M^2} \xi^2 |\langle \tilde{E} \rangle|^2 \right] \\ \frac{\mathrm{d}\sigma_T}{\mathrm{d}t} &\propto \left[(1-\xi^2) |\langle H_T \rangle|^2 - \frac{t'}{8M^2} |\langle \bar{E}_T \rangle|^2 \right] \\ \frac{\mathrm{d}\sigma_{TT}}{\mathrm{d}t} &\propto t' |\langle \bar{E}_T \rangle|^2 \\ \frac{\mathrm{d}\sigma_{LT}}{\mathrm{d}t} &\propto \xi \sqrt{1-\xi^2} \sqrt{-t'} \mathrm{Re} \left[\langle H_T \rangle * \langle \tilde{E} \rangle \right] \end{split}$$

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Measurement at COMPASS

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Collected data

- 2012 pilot run (results published in PLB 805 135454)
- **2016** current analysis (2.3 times larger statistics than 2012)
- \blacksquare 2017 to be included soon (2016+2017 \sim 9 times larger statistics than 2012)

Exclusive π^0 measurement at COMPASS



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Data selection and background estimation

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Hard exclusive π° production at COMPASS

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- Vertex candidate
 - μ and μ' associated to the vertex
 - \blacksquare requirements on μ and μ'

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 - μ and μ' associated to the vertex
 - \blacksquare requirements on μ and μ'
- π^0 candidate from 2 photons
 - pair of neutral clusters in ECALs
 - energy thresholds
 - timing requirement

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Exclusive variables measured either by the spectrometer or by CAMERA

$$P_{spec}' = P + k - k' - p_{\pi^0}$$

- $\Delta \varphi = \varphi_{CAM} \varphi_{spec}$
- $\Delta p_T = p_{T_{CAM}} p_{T_{spec}}$ • $M_x^2 = (P + k - k' - p_{\pi^0} - P'_{CAM})^2$



- $|\Delta \varphi| < 0.4 \text{ rad}$
- $|\Delta p_T| < 0.3 \text{ GeV/c}$
- $|M_x^2| < 0.3 \, (\text{GeV}/\text{c}^2)^2$
- $|\Delta z_A| < 16 \text{ cm}$



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- Kinematics
 Kinematic fit
 6.4 < ν < 40 GeV
 1 < Q² < 8 (GeV/c)²
 0.08 < |t| < 0.64 (GeV/c)²

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- Kinematics
 Kinematic fit $6.4 < \nu < 40 \text{ GeV}$ $1 < Q^2 < 8 \text{ (GeV/c)}^2$ $0.08 < |t| < 0.64 \text{ (GeV/c)}^2$

 Events with only one combination: vertex candidate + π⁰ candidate + proton candidate pass the selection





Background estimation

- π^0 from deep inelastic scattering
- LEPTO MC non exclusive
- HEPGEN++ MC excl. π^0
- scaling of HEPGEN and "LEPTO background"
- background fraction $r_{\rm LEPTO}$
- $r_{\rm LEPTO}$ =17 ± 5 %



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- $\blacksquare \ {\rm 6.4} < \nu < {\rm 40 \ GeV}$
- $\blacksquare~1 < \mathsf{Q}^2 < 8~(\text{GeV/c})^2$
- \bullet 0.08 < |t| < 0.64 (GeV/c)²
- $\blacksquare \ -\pi < \phi < \pi \ \mathrm{rad}$

$$\frac{\mathrm{d}^2 \sigma^{\gamma^* p \to \pi^0 p'}}{\mathrm{d} t \mathrm{d} \phi_{\pi^0}} = \frac{1}{\Gamma(Q^2, \nu)} \frac{\mathrm{d}^4 \sigma^{\mu p \to \mu' \pi^0 p'}}{\mathrm{d} Q^2 \mathrm{d} \nu \mathrm{d} t \mathrm{d} \phi_{\pi^0}}$$

• $\Gamma(Q^2, \nu)$ - virtual photon flux



$$\left\langle \frac{\mathrm{d}^2 \sigma^{\gamma^* p}}{\mathrm{d} t \mathrm{d} \phi} \right\rangle_{\Delta \Omega}^{\pm} = \left(\sum_{i=1}^{N_{data^{\pm}}^{\Delta \Omega}} \frac{1}{\Gamma(\Omega_i) a(\Omega_i) \mathcal{L}^{\pm} \Delta \Omega} - f^{\pm} \sum_{i=1}^{N_{LEPTO^{\pm}}^{\Delta \Omega}} \frac{1}{\Gamma(\Omega_i) a(\Omega_i) \mathcal{L}^{\pm} \Delta \Omega} \right)$$

- phase space element $\Delta \Omega = \Delta |t| \Delta \Phi \Delta Q^2 \Delta \nu$
- acceptance $a(\Omega_i)$
- $\blacksquare \text{ luminosity } \mathcal{L}$
- background normalization

$$f^{\pm} = r^{\pm}_{LEPTO} \cdot rac{N_{data^{\pm}}}{N_{LEPTO^{\pm}}}$$

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spin-independent cross section

$$\left\langle \frac{\mathrm{d}^2 \sigma^{\gamma^* p}}{\mathrm{d}t \mathrm{d}\phi} \right\rangle_{\Delta\Omega} = \frac{1}{2} \left(\left\langle \frac{\mathrm{d}^2 \sigma^{\gamma^* p}}{\mathrm{d}t \mathrm{d}\phi} \right\rangle_{\Delta\Omega}^+ + \left\langle \frac{\mathrm{d}^2 \sigma^{\gamma^* p}}{\mathrm{d}t \mathrm{d}\phi} \right\rangle_{\Delta\Omega}^- \right)$$

Acceptance averaged over ϕ



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Results

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ϕ -dependent cross section for $\nu \in$ (6.4,40) GeV and Q² \in (1,8) GeV²/c²



- Only statistical uncertainties
- The systematic uncertainties are preliminary evaluated using the main contributor $r_{\rm LEPTO}$
 - $\blacksquare~\sim$ 10 % for the largest values around $\pm\pi/2$
 - $\blacksquare~\sim$ 20 % for the smallest values around $-\pi, 0, \pi$

$\phi\text{-dependent}$ cross section for $\nu \in$ (6.4,40) GeV and $\mathsf{Q}^2 \in$ (1,8) $\mathsf{GeV}^2/\mathsf{c}^2$



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|t|-dependent cross section for $u \in$ (6.4,40) GeV and $\mathsf{Q}^2 \in$ (1,8) GeV $^2/\mathsf{c}^2$



Only statistical uncertainties

• The systematic uncertainties are preliminary evaluated using the main contributor $r_{\rm LEPTO}$

 $\blacksquare ~\sim 10\%$ in each $|t|{
m -bin}$

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$\phi\text{-dependent}$ cross section for $\nu \in$ (8.5,28) GeV and $\mathsf{Q}^2 \in$ (1,5) $\mathsf{GeV}^2/\mathsf{c}^2$



2012 results (PLB 805 135454):

$$\begin{split} \left\langle \frac{\mathrm{d}\sigma_{T}}{\mathrm{d}t} + \epsilon \frac{\mathrm{d}\sigma_{L}}{\mathrm{d}t} \right\rangle &= (8.1 \pm 0.9_{\mathsf{stat}} \stackrel{+}{}_{-1.0}^{-1.1} \big|_{\mathsf{sys}}) \frac{\mathsf{nb}}{(\mathsf{GeV}/c)^{2}} \\ \left\langle \frac{\mathrm{d}\sigma_{TT}}{\mathrm{d}t} \right\rangle &= (-6.0 \pm 1.3_{\mathsf{stat}} \stackrel{+}{}_{-0.7}^{-0.7} \big|_{\mathsf{sys}}) \frac{\mathsf{nb}}{(\mathsf{GeV}/c)^{2}} \\ \left\langle \frac{\mathrm{d}\sigma_{LT}}{\mathrm{d}t} \right\rangle &= (1.4 \pm 0.5_{\mathsf{stat}} \stackrel{+}{}_{-0.2}^{-0.3} \big|_{\mathsf{sys}}) \frac{\mathsf{nb}}{(\mathsf{GeV}/c)^{2}} \\ \epsilon &= 0.996 \end{split}$$

- 2012 statistical and systematic uncertainties are displayed
- 2016 only statistical uncertainties

$\phi\text{-dependent}$ cross section for $\nu \in$ (8.5,28) GeV and $\mathsf{Q}^2 \in$ (1,5) $\mathsf{GeV}^2/\mathsf{c}^2$



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$$\left\langle \frac{\mathrm{d}\sigma_{TT}}{\mathrm{d}t} \right\rangle = (-6.0 \pm 1.3_{\mathrm{stat}} + \frac{0.7}{0.7} \big|_{\mathrm{sys}}) \frac{\mathrm{nb}}{(\mathrm{GeV}/c)^{2}}$$

$$\left\langle \frac{\mathrm{d}\sigma_{LT}}{\mathrm{d}t} \right\rangle = (1.4 \pm 0.5_{\mathrm{stat}} + \frac{0.3}{-0.2} \big|_{\mathrm{sys}}) \frac{\mathrm{nb}}{(\mathrm{GeV}/c)^{2}}$$

$$\epsilon = 0.996$$

S. Goloskokov, P. Kroll EPJC47 (2011) + private communication

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|t|-dependent cross section for $u \in (8.5,28)$ GeV and $\mathsf{Q}^2 \in (1,5)$ GeV $^2/\mathsf{c}^2$



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|t|-dependent cross section for $u \in (8.5,28)$ GeV and $\mathsf{Q}^2 \in (1,5)$ GeV $^2/\mathsf{c}^2$



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Conclusion

- New preliminary results in a larger kinematic domain $\nu \in (6.4,40)$ GeV and Q² $\in (1,8)$ GeV²/c²
- Comparison with published results (PLB 805 135454) in a smaller kinematic domain $\nu \in$ (8.5,28) GeV and Q² \in (1,5) GeV²/c²
- All theoreticians are invited to present their predictions, namely S. Goloskokov, P. Kroll, K. Passek-Kumericki and S. Liuti
- Prospects
 - Complete systematic studies
 - 2017 data
 - Evolution in 3 ν bins

BACKUP

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Prospects

Determine $\sin \phi$ contribution (for μ^+ and μ^- beam)

$$\frac{\mathrm{d}^2 \sigma^{\gamma^* p}}{\mathrm{d}t \mathrm{d}\phi_{\pi^0}} = \frac{1}{2\pi} \left[\frac{\mathrm{d}\sigma_T}{\mathrm{d}t} + \epsilon \frac{\mathrm{d}\sigma_L}{\mathrm{d}t} + \epsilon \cos(2\phi_{\pi^0}) \frac{\mathrm{d}\sigma_{TT}}{\mathrm{d}t} + \sqrt{2\epsilon(1+\epsilon)} \cos(2\phi_{\pi^0}) \frac{\mathrm{d}\sigma_{LT}}{\mathrm{d}t} \right]$$
$$\mp |P_l| \sqrt{2\epsilon(1-\epsilon)} \sin(2\phi_{\pi^0}) \frac{\mathrm{d}\sigma'_{LT}}{\mathrm{d}t} \right]$$

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Photon flux

$$\Gamma(Q^2,\nu) = \frac{\alpha_{em}(1-x_{Bj})}{2\pi Q^2 y E_{\mu}} \left[y^2 \left(1 - \frac{2m_{\mu}^2}{Q^2} \right) + \frac{2}{1+Q^2/\nu^2} \left(1 - y - \frac{Q^2}{4E_{\mu}^2} \right) \right]$$

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Virtual photon polarisation parameter

$$\epsilon = \frac{1 - y - \frac{1}{4}y^2 Q^2/\nu^2}{1 - y + \frac{1}{2}y^2 + \frac{1}{4}y^2 Q^2/\nu^2}$$

• $\epsilon = 0.997$ (it was 0.996 in 2012 analysis)

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Kinematic fit

- Measurements of exclusive processes at COMPASS are over-constrained
- This can be used to improve the resolution on the measured kinematic quantities by the usage of a kinematically constrained fit
- In case of exclusive π^0 analysis, the fit can be used to improve signal to background ratio
- The goal is to minimize the least square function $\chi^2(\vec{k}) := (\vec{k}_{fit} \vec{k})^T \widehat{C}^{-1} (\vec{k}_{fit} \vec{k})$
- k is the vector of measured quantities (e.g. variables such as momentum and transverse position corresponding to the incoming/scattered muon, initial/final proton and lower/higher energetic photon)
- \widehat{C} is the covariance matrix corresponding to the measured quantities
- The Lagrange multiplier method with constraints g_i is used for the minimization:

$$L(\vec{k},\vec{\alpha}) = \chi^2(\vec{k}) + 2\sum_{i=1}^N \alpha_i g_i.$$
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Kinematic fit - introduction - kinematic constraints

Momentum and energy conservation

$$g_{i} = p_{\mu,i}^{fit} - p_{\mu',i}^{fit} - p_{p',i}^{fit} - p_{\gamma_{h},i}^{fit} - p_{\gamma_{h},i}^{fit} = 0, \ i = 1, 2, 3$$
$$g_{4} = E_{\mu}^{fit} + m_{p}c^{2} - E_{\mu'}^{fit} - E_{\rho'}^{fit} - E_{\gamma_{h}}^{fit} - E_{\gamma_{l}}^{fit} = 0$$

• Common vertex for all tracks (except initial and final state proton)

$$g_{5+i} = p_{j,3}^{fit} \left(x_{\nu} - x_{j}^{fit} \right) - p_{j,1}^{fit} \left(z_{\nu} - z_{j}^{fit} \right) = 0 \quad (i,j) \in \{(0,\mu), (2,\mu\prime), (4,\gamma_h), (6,\gamma_l)\}$$

$$g_{6+i} = p_{j,3}^{fit} \left(y_{\nu} - y_{j}^{fit} \right) - p_{j,2}^{fit} \left(z_{\nu} - z_{j}^{fit} \right) = 0, \quad (i,j) \in \{(0,\mu), (2,\mu\prime), (4,\gamma_h), (6,\gamma_l)\}$$

Constraints for final state proton

$$g_{13+i} = p_{\rho',3}^{fit} \left(x_j^{fit} - x_v \right) - p_{\rho',1} \left(z_j^{fit} - z_v \right) = 0, \quad (i,j) \in \{(0,A), (2,B)\}$$

$$g_{14+i} = p_{\rho',3}^{fit} \left(y_j^{fit} - y_v \right) - p_{\rho',2} \left(z_j^{fit} - z_v \right) = 0, \quad (i,j) \in \{(0,A), (2,B)\}$$
constraint $g_{17} = (E_{\gamma_b}^{fit} + E_{\gamma_l}^{fit})^2 - (p_{\gamma_b}^{fit} + p_{\gamma_l}^{fit})^2 - m_{\pi^0}^2 = 0$

Mass