Testing Predictions of the Chiral Anomaly in Primakoff Reactions at COMPASS

Dominik Ecker, Andrii Maltsev on behalf of the COMPASS collaboration
The chiral anomaly

- Lagrange density of QCD:

\[ \mathcal{L}_{QCD} = \sum_{f=u,d,s, c,b,t} \bar{q}_f (i \gamma^\mu \not{\psi} - m_f) q_f - \frac{1}{4} G_{\mu \nu}^a G_a^{\mu \nu} \]
The chiral anomaly

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\[ \mathcal{L}_{QCD} = \sum_{f=u,d,s,} \bar{q}_f (i\slashed{\partial} - m_f) q_f - \frac{1}{4} G^{\alpha}_{\mu\nu} G^{\mu\nu}_{\alpha} \]

Symmetry breaking term: \( m_f = \begin{pmatrix} m_u & m_d & m_s \end{pmatrix} \)

Chiral limit: \( m_u, m_d, m_s = 0 \)
The chiral anomaly

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- Features axial $U(1)$-symmetry in chiral limit:
  \[ q(x) \rightarrow e^{i\theta \gamma_5} q(x) \]
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The chiral anomaly

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- Anomaly: Symmetry of classical Lagrangian violated at quantum level (by renormalization choice)

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- Features \textit{axial} \(U(1)\)-symmetry in chiral limit:
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- \textbf{Anomaly}: Symmetry of classical Lagrangian violated at quantum level (by renormalization choice)

- Adler, Bell, Jackiw 1969: \( \tau_{\text{anom}}(\pi^0) = (9.5 \pm 1.5) \cdot 10^{-17} \text{s} \neq \tau_{\text{theory}}(\pi^0) \approx 10^{-13} \text{s} \)
The chiral anomaly

- Lagrange density of QCD:
  \[ \mathcal{L}_{QCD} = \sum_{f=u,d,s} c_{b,t} \bar{q}_f (i\not{\partial} - m_f) q_f - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} \]

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well tested in \( \pi^0 \) decay
Anomalous processes

• Chiral anomaly governs couplings of odd number of Goldstone bosons:

<table>
<thead>
<tr>
<th>$SU(2)$ flavor</th>
<th>$SU(3)$ flavor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$π^0 \rightarrow γγ$</td>
<td>$K^+K^- \rightarrow π^+π^-π^0$</td>
</tr>
<tr>
<td>$γπ^- \rightarrow π^-π^0$</td>
<td>$η \rightarrow π^+π^-γ$</td>
</tr>
<tr>
<td>$π^+ \rightarrow e^+ν_eγ$</td>
<td>$K^+ \rightarrow π^+π^-e^+ν_e$</td>
</tr>
<tr>
<td>etc.</td>
<td>etc.</td>
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• On tree-level: low-energy theorems with few parameters, e.g. pion decay constant $F_π$ measured from leptonic decays of the charged pion ($π^± \rightarrow μ^± + ν$)

• Higher order corrections via Chiral Perturbation Theory (ChPT)

\[ F_{πγγ} = \frac{e^2N_C}{12π^2F_π} = 2.52 \cdot 10^{-2}\text{GeV}^{-1} \]

\[ F_{3π} = \frac{eN_C}{12π^2F_π^3} = (9.78 ± 0.05)\text{GeV}^{-3} \]
Testing the chiral anomaly - $F_{3\pi}$

- $F_{3\pi}$: Direct coupling of $\gamma$ to $3\pi$ - process proceeds primarily via the chiral anomaly

- Accessible in Primakoff reactions via: $\pi^- \gamma^* \rightarrow \pi^-\pi^0$ ultra-relativistic pion scatters in e.m. field of nucleus (characterized by very low momentum transfer)
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- Problem of explicit chiral symmetry breaking:

  \[
  F_{3\pi} = \frac{eN_C}{12\pi^2 F_{\pi}^3} = (9.78 \pm 0.05) \text{GeV}^{-3} = F(s = t = u = 0)
  \]

  We measure at $s > (2m_\pi)^2$: use ChPT to bridge "gap"

  \[
  F_{3\pi}(s, t, u) = F_{3\pi} \left( f^{(0)}(s, t, u) + f^{(1)}(s, t, u) + f^{(2)}(s, t, u) + \ldots \right)
  \]

Dominik Ecker | HADRON | 08/06/2023
Radiative width of $\rho$-meson

- Cross section of $\pi^-\pi^0$ final state result of two coherent processes:

- At kinematic threshold: dominated by chiral anomaly
- Interference between Chiral Anomaly and $\rho$ gives additional information

$\Rightarrow$ possibility of extraction of radiative width of $\rho$-meson: $\Gamma_{(\rho\rightarrow\pi\gamma)}/\Gamma_{\text{tot}} \approx 4.5 \cdot 10^{-4}$

Low-mass tail: mainly driven by $F_{3\pi}$
Radiative width of $\rho$-meson

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Previous measurements – $F_{3\pi}$

Antipov, Y. et al. PRD 36 (1987) 101103
and reanalyzed by
Ametller, L. et al. PRD 64 (2001) 094009

$$F_{3\pi} = (10.7 \pm 1.2) \text{ GeV}^{-3}$$

- Neglecting $s$-channel production of $\rho$ meson
- No proper consideration of systematics

from cross-section data of
Amendolia, S.R. et al., PLB 155, 457 (1985)

$$F_{3\pi} = (9.6 \pm 1.1) \text{ GeV}^{-3}$$

- Neglecting $s$-channel production of $\rho$ meson
- No proper consideration of systematics
- Dominant background of elastically scattered pions
Previous measurements – $\Gamma_{\rho \rightarrow \pi \gamma}$

Radiative width of $\rho$-meson:


- From fit to cross section (BW shape):
  $\Gamma(\rho \rightarrow \pi \gamma) = (81 \pm 4 \pm 4)$ keV
Cross section:

\[
\sigma(s) = \frac{(s - 4m_{\pi}^2)^\frac{3}{2}(s - m_{\pi}^2)}{1024\pi\sqrt{s}} \int_{-1}^{+1} dz \ (1 - z^2)|F_{3\pi}(s, t, u)|^2
\]
\[ \pi \gamma \to \pi \pi \text{ from dispersion relations} \]

- Cross section:
  \[
  \sigma(s) = \frac{(s - 4m^2_{\pi})^3(s - m^2_{\pi})}{1024\pi\sqrt{s}} \int_{-1}^{+1} dz \left(1 - z^2\right)|F_{3\pi}(s, t, u)|^2
  \]

- Dispersive framework to deduce \( F_{3\pi} \) from a fit to the full data set up to 1.0 GeV including the \( \rho(770) \)-resonance:
  \[
  F_{3\pi}^{DR}(s) = \frac{1}{3} \left( C_2^{(1)} + C_2^{(2)}s \right) + \frac{1}{\pi} \int \frac{ds'}{4m^2_{\pi}} \frac{s^2}{s'^2} \frac{C_2^{(1)}\text{Im} \mathcal{F}_2^{(1)}(s') + C_2^{(2)}\text{Im} \mathcal{F}_2^{(2)}(s')}{s' - s}
  \]
\[ \pi \gamma \rightarrow \pi \pi \text{ from dispersion relations} \]

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\sigma(s) = \frac{(s - 4m_{\pi}^2)^{\frac{3}{2}}(s - m_{\pi}^2)}{1024\pi\sqrt{s}} \int_{-1}^{+1} dz \frac{(1 - z^2)}{2} |F_{3\pi}(s, t, u)|^2
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\]

- Basis functions provided in:

M. Hoferichter, B. Kubis, and D. Sakkas, *PRD 86 (2012) 116009*

M. Hoferichter, B. Kubis, and M. Zanke, *PRD 96 (2017) 114016*
\( \pi \gamma \rightarrow \pi \pi \) from dispersion relations

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Fit parameters

Basis functions provided in:

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M. Hoferichter, B. Kubis, and M. Zanke, *PRD* 96 (2017) 114016
COmmom Muon and Proton Apparatus for Structure and Spectroscopy
COMPASS spectrometer

- 190 GeV negative hadron beam: 96.8% $\pi^-$, 2.4% $K^-$, 0.8% $\bar{p}$
- Beam PID by Cherenkov detectors
- Two stage magnetic spectrometer
- 4mm Ni target disk ($\approx 25\%$ $X/X_0$)
- Calorimetric trigger on photons

- Measure scattered $\pi^-$ and photons of $\pi^0$ decay
- Select exclusive events at very low $Q^2$
Principle of measurement

- Measure scattered $\pi^-$ and photons of $\pi^0$ decay
- Select exclusive events at very low $Q^2$
- For absolute cross-section measurements: Luminosity

Indirect determination of luminosity via free Kaon decays

\[ (K^- \rightarrow \pi^-\pi^0 \text{ or } K^- \rightarrow \pi^-\pi^0\pi^0) \]

\[ \int L \, dt = \left( 5.21 \pm 0.04_{\text{stat}} \pm 0.48_{\text{syst}} \right) \text{nb}^{-1} \]
Potential background processes to $\pi \gamma \rightarrow \pi \pi$

$\pi^- \pi^0$ via strong interaction

- Pomeran exchange: forbidden by $G$-parity conservation
- $\pi$ and $\omega$ exchange: 
  low cross section at COMPASS beam energies

$\pi^- \pi^0 \pi^0$ via Pomeran exchange

- Large cross section
- Main background: loss of one (soft) $\pi^0$
- Approach:
  - Using the model from COMPASS $\pi^- \pi^0 \pi^0$ data
  - Apply $\pi^- \pi^0$ event selection -> realistic distributions of leakage in $\pi^- \pi^0$
Subtraction of $3\pi$ background

Model from COMPASS $\pi^-\pi^0\pi^0$ data:
- Realistic shapes for signal and background contributions
- Fit yields (signal vs background) to match observed momentum transfer distribution

$0.75 \text{ GeV}/c^2 < M_{\pi^-\pi^0} < 0.80 \text{ GeV}/c^2$
Subtraction of $3\pi$ background

COMPASS preliminary

- $0.30 \, \text{GeV}^2 < M_{3\pi} < 0.35 \, \text{GeV}^2$
- $0.35 \, \text{GeV}^2 < M_{3\pi} < 0.40 \, \text{GeV}^2$
- $0.40 \, \text{GeV}^2 < M_{3\pi} < 0.45 \, \text{GeV}^2$
- $0.45 \, \text{GeV}^2 < M_{3\pi} < 0.50 \, \text{GeV}^2$
- $0.50 \, \text{GeV}^2 < M_{3\pi} < 0.55 \, \text{GeV}^2$
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- $0.60 \, \text{GeV}^2 < M_{3\pi} < 0.65 \, \text{GeV}^2$
- $0.65 \, \text{GeV}^2 < M_{3\pi} < 0.70 \, \text{GeV}^2$
- $0.70 \, \text{GeV}^2 < M_{3\pi} < 0.75 \, \text{GeV}^2$
- $0.75 \, \text{GeV}^2 < M_{3\pi} < 0.80 \, \text{GeV}^2$
- $0.80 \, \text{GeV}^2 < M_{3\pi} < 0.85 \, \text{GeV}^2$
- $0.85 \, \text{GeV}^2 < M_{3\pi} < 0.90 \, \text{GeV}^2$
- $0.90 \, \text{GeV}^2 < M_{3\pi} < 0.95 \, \text{GeV}^2$
- $0.95 \, \text{GeV}^2 < M_{3\pi} < 1.00 \, \text{GeV}^2$

- Data
- Primakoff MC
- $3\pi$ MC
- Sum of MC contributions
Results of dispersive fits

- Determine subtraction constants from fit
  - Use data up to 1 GeV/$c^2$
  - Exclude data around 500 MeV/$c^2$ due to background of free kaon decay

\[
C_2^{(1)} = \left( 10.5 \pm 0.1_{\text{stat}} \pm 0.6_{\text{syst}} \right) \text{GeV}^{-3}
\]
\[
C_2^{(2)} = \left( 24.5 \pm 0.1_{\text{stat}}^{+1.6}_{-1.4_{\text{syst}}} \right) \text{GeV}^{-5}
\]

- Use ChPT expansion (NLO) to determine $F_{3\pi}(0,0,0)$:

\[
F_{3\pi} = \left( 10.3 \pm 0.1_{\text{stat}} \pm 0.6_{\text{syst}} \right) \text{GeV}^{-3}
\]
\[
\Gamma_{\rho \to \pi \gamma} = \left( 76 \pm 1_{\text{stat}}^{+10}_{-8_{\text{syst}}} \right) \text{keV}
\]
Comparison to previous measurements

- **COMPASS**: First combined measurement of $F_{3\pi}$ and $\Gamma_{\rho\to\pi\gamma}$

  \[ F_{3\pi} = (10.3 \pm 0.1_{\text{stat}} \pm 0.6_{\text{syst}}) \text{GeV}^{-3} \]
  \[ \Gamma_{\rho\to\pi\gamma} = (76 \pm 1_{\text{stat}}^{+10}_{-8_{\text{syst}}}) \text{keV} \]

- Intensive test of systematics (dominant contributions):
  - Luminosity
  - Radiative corrections
  - Background of $\omega$, $\pi$ exchange
  - Background from $\pi\gamma$ final state

- Accompanied with intensive analysis of $\pi^-\text{Ni} \to \pi^-\pi^0\pi^0\text{Ni}$ for background estimation
Conclusion and outlook

- Measurement of $F_{3\pi}$ - fundamental test of low-energy QCD

- COMPASS did first combined measurement of $F_{3\pi}$ and $\Gamma_{\rho \rightarrow \pi \gamma}$

- Result for $F_{3\pi}$ is in agreement with prediction from ChPT

- Results dominated by systematic uncertainties -> improvement expected
  - Background prediction
  - Luminosity determination

- On the future program of successor experiment AMBER: similar program on kaon sector
  (see talk by Oleg Denisov, “From COMPASS to AMBER”, Fri 14:00)

Thank you for your attention
Backup
Cross sections for Primakoff effect
Chiral tree, chiral loop

- Direct (point-like) coupling of photon to 4 pions
- Prediction from ChPT at tree- and loop-level available

Grabmüller S. (2012). Cryogenic Silicon Detectors and Analysis of Primakoff Contributions to the Reaction $\pi^- Pb \rightarrow$

Krämer M. (2016) Evaluation and Optimization of a digital calorimetric trigger and analysis of $\pi^- Ni \rightarrow$
Pion-photon reactions through the Primakoff technique

- Photon is provided by the strong Coulomb field of a nucleus (typical field strength at \( d = 5R_N \): \( E \approx 300 \text{ kV/fm} \))

- Coulomb field of nucleus is a source of quasi-real \((P_\gamma \ll m_{\pi}^2)\) photons

- Large impact parameters (ultra-peripheral scattering)

\[
\frac{d\sigma}{ds dQ^2 d\Phi_n} = \frac{Z^2 \alpha}{\pi (s - m_{\pi}^2)} F^2(Q^2) \frac{Q^2 - Q_{\text{min}}^2}{Q^4} \cdot \frac{d\sigma_{\pi\gamma \rightarrow X}}{d\Phi_n}
\]

- Flux of quasi-real photons
- \(\pi\gamma\) scattering cross section
Integrated luminosity

- Needed for absolute cross section measurement: effective integrated luminosity

\[ L_{\text{eff}} = L \cdot (1 - \varepsilon_{\text{DAQ}}) \]

- Can be determined via free kaon decays:
  - Use CEDAR detectors for beam PID
  - Free decays where no material
  - Exclusive events with no momentum transfer

\[ N_K = \Phi_\pi \frac{n_K}{n_\pi} BR(K \to X) (1 - e^{-\frac{L}{\gamma_{\text{tr}}}}) \varepsilon_{K\to X} \]
Previous measurements – $F_{3\pi}$

Antipov, Y. et al. PRD 36 (1987) 101103

$F_{3\pi} = (12.9 \pm 0.9) \text{ GeV}^{-3}$

- Assuming $F_{3\pi} = \bar{F}_{3\pi}(s, t, u)$
- Neglecting $s$-channel production of $\rho$ meson
- No proper consideration of systematics
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**Antipov, Y. et al.** PRD 36 (1987) 101103

and reanalyzed by

**Ametller, L. et al.** PRD 64 (2001) 094009

$F_{3\pi} = (11.4 \pm 1.3) \text{ GeV}^{-3}$

- Neglecting $s$-channel production of $\rho$ meson
- No proper consideration of systematics
- Using ChPT to extrapolate to chiral limit (NNLO)

$$F_{3\pi} = \frac{eN_C}{12\pi^2 F_{\pi}^3} = (9.78 \pm 0.05) \text{ GeV}^{-3}$$
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$$F_{3\pi} = (10.7 \pm 1.2) \text{ GeV}^{-3}$$

- Neglecting s-channel production of $\rho$ meson
- No proper consideration of systematics
- Using ChPT to extrapolate to chiral limit (NNLO)
- Considering dominant correction

$$F_{3\pi} = \frac{e N_C}{12 \pi^2 F_{\pi}^3} = (9.78 \pm 0.05) \text{ GeV}^{-3}$$