Progress in the Partial-Wave Analysis Methods at COMPASS

Julien Beckers and Florian Kaspar for the COMPASS Collaboration

HADRON 2023: Analysis tools
June 8th, 2023

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funded by the DFG under Germany’s Excellence Strategy – EXC2094 –
390783311 and BMBF Verbundforschung 05P21WOCC1 COMPASS
Excited Light Mesons at COMPASS

- Inelastic scattering reactions of high-energetic meson beam

- Strong interaction (Pomeron exchange) between beam meson and target proton

- Intermediate hadronic resonances $X^-$ are created, then decay into $n$-body final state
  → wide range of allowed (spin) quantum numbers

- Final-state particles measured
The COMPASS Experiment

Large-acceptance magnetic spectrometer @ CERN-SPS

**Beam:**
- Secondary hadrons ($\pi^-, K^-$) at 190 GeV/c
- produced via primary proton beam from SPS

**Spectrometer:**
- Liquid-hydrogen target
- Two-stage spectrometer setup around two dipole magnets SM1/2

Excited Light Mesons at COMPASS

\[ \pi^- + p \rightarrow \pi^-\pi^-\pi^+ + p \]

\[ \pi^- + p \rightarrow K^0_S K^- + p \]

- Allowed quantum numbers:
  \[ J^{PC} = 0^{-+}, 1^{-+}, 1^{++}, \ldots \]
  → \( \pi_J \) and \( a_J \) resonances

- COMPASS flagship channel: \( 115 \times 10^6 \) evts

- Allowed quantum numbers:
  \[ J^{PC} = 1^{--}, 2^{++}, 3^{--}, \ldots \] from decay
  Dominated by Pomeron exchange
  → \( a_J \) for even \( J \)

- Highly selective → search for \( a'_4, a_6 \)
Excited Light Mesons at COMPASS

\( \pi^- + p \rightarrow \pi^-\pi^-\pi^+ + p \)

- Allowed decay
- \( J^P = 1^- \)
- \( \pi^- \rightarrow \pi^- + \pi^- \)
- COMPASS \( \pi^- \) data: \( \times 10^6 \) evts

\( \pi^- + p \rightarrow K_S^0K^- + p \)

- Allowed decay
- \( J^P = 1^- \)
- \( K_S^0 \rightarrow K^- + \pi^0 \)
- \( \pi^0 \rightarrow \gamma\gamma \)
- \( \gamma\gamma \) data: \( \times 10^4 \) evts

HADRON 2023 | June 8th, 2023
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Partial-Wave Decomposition

\[ I(m_X, t'; \tau_n) = |M_{fi}|^2 \]

- Separate process amplitude into **partial waves**
  - Spin \( J \) and spin-projection \( M \)
  - Parity \( P \), charge conjugation \( C \)
  - ...
  - \( \rightarrow \) Partial wave index \( \alpha \)

**Partial wave \( \alpha \):**
specific \( (J^{PC} M) \)
Partial-Wave Decomposition

\[ I(m_X, t'; \tau_n) = \left| \sum_a T_a(m_X, t') \psi_a(m_X, \tau_n) \right|^2 \]

- Separate process amplitude into **partial waves**
  - Partial wave index \(a\)
- Production, propagation of \(X^-\): \(T_a(m_X, t')\)

Partial wave \(a\):
- Specific \(J^{PC} M\)
Partial-Wave Decomposition

\[ I(m_X, t'; \tau_n) = \left| \sum_a T_a(m_X, t') \psi_a(m_X, \tau_n) \right|^2 \]

- Separate process amplitude into **partial waves**
  → Partial wave index \( a \)

- Production, propagation of \( X^- \): \( T_a(m_X, t') \)

- Decay of \( X^- \): \( \psi_a(m_X, \tau_n) \)

**Partial wave \( a \):**
specific \( (J^P C M) \)
Partial-Wave Decomposition

\[ I(m_X, t'; \tau_n) = \left| \sum_a T_a(m_X, t') \psi_a(m_X, \tau_n) \right|^2 \]

- Separate process amplitude into **partial waves**
  - Partial wave index \( a \)

- Production, propagation of \( X^- \):
  \( T_a(m_X, t') \)

- Decay of \( X^- \):
  \( \psi_a(\tau_n) = Y_f^M(\theta, \phi) \)
  \( \tau_n = (\theta, \phi) \)

\( K_S^0 K^- \)
Partial-Wave Decomposition

\[ I(m_X, t'; \tau_n) = \left| \sum_a T_a(m_X, t') \psi_a(m_X, \tau_n) \right|^2 \]

- Separate process amplitude into **partial waves**
  \[ \rightarrow \text{Partial wave index } a \]

- Production, propagation of \( X^- \): \( T_a(m_X, t') \)

- Decay of \( X^- \): via **isobar model**

\[ \pi^- \pi^- \pi^+ \]

**Partial wave** \( a \): specific \( (J^{PC} M) \)
Partial-Wave Decomposition

\[ I(m_X, t'; \tau_n) = \left| \sum_a T_a(m_X, t') \psi_a(m_X, \tau_n) \right|^2 \]

- Separate process amplitude into **partial waves**
  - Partial wave index \( a \)

- Production, propagation of \( X^- \)

- Decay of \( X^- \): via isobar model

\[
\tau_n = (\theta_{GJ}, \phi_{GJ}, m_\xi, \theta_{HF}, \phi_{HF})
\]

\[
\psi_a = \psi_X(m_X, \theta_{GJ}, \phi_{GJ}) \cdot \psi_\xi(m_\xi, \theta_{HF}, \phi_{HF})
\]

\[ \pi^- \pi^- \pi^+ \]

**Partial wave \( a \):**

specific \( (J^{PC} + \text{decay}) \)
Partial-Wave Decomposition

\[ I(m_X, t'; \tau_n) = \left| \sum_a T_a(m_X, t') \psi_a(m_X, \tau_n) \right|^2 \]

- Separate process amplitude into partial waves
  → Partial wave index \( a \)

- Production, propagation of \( X^- \)

- Decay of \( X^- \): via isobar model

- Fit \( I(m_X, t'; \tau_n) \) to data in \( (m_X, t') \) bins
  → parametrize \( T_a \) as step-wise functions
  → extract constant \( T_a \) in each bin
Resonance-Model Fit

Second step: **extract resonance parameters**

- Build model for **mass dep. of partial-wave amplitudes**:
  - resonant (e.g. Breit-Wigner distribution)
  - + **non-resonant background** components

- \( \chi^2 \) fit to output of partial-wave decomposition

→ get **masses and widths** of parameterized resonances

COMPASS PRD 98 (2018) 092003
Understanding the Ambiguities in the Partial-Wave Decomposition of the $K_S^0 K^-$ Final State
Ambiguities in the Partial-Wave Decomposition

For any final state with two spinless particles ($\pi\pi$, $KK$, $\eta\pi$, ...):

- Decomposition of intensity into $\{T_J\}$ is not unique (see derivation later)

$\to$ Several sets of $\{T_J\}$ lead to the same $I(\theta, \phi)$ in each $(m_X, t')$ bin

$$I(\theta, \phi) = \left| \sum_{JM} T_{JM}^{(1)} \psi_{JM} (\theta, \phi) \right|^2 = \left| \sum_{JM} T_{JM}^{(2)} \psi_{JM} (\theta, \phi) \right|^2$$

- The fit cannot distinguish between the mathematically equivalent solutions!
Ambiguities in the Partial-Wave Decomposition

\[ I(\theta, \phi) = \left| \sum_{JM} T_{JM} \psi_{JM}(\theta, \phi) \right|^2 \]

Assume strong dominance of \(|M| = 1\) *

- Pomeron exchange dominant \( \rightarrow M \neq 0 \)
- Higher \(|M|\) suppressed

*using reflectivity basis for \(\psi_{JM} :\)
doi.org/10.1103/PhysRevD.11.633
Ambiguities in the Partial-Wave Decomposition

\[ I(\theta, \phi) = \left| \sum_J T_J \psi_J(\theta, \phi) \right|^2 \]

Assume strong dominance of \(|M| = 1^*\):

- Pomeron exchange dominant \( \rightarrow M \neq 0 \)
- Higher \(|M|\) suppressed

*using reflectivity basis for \(\psi_{JM}\):
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Ambiguities in the Partial-Wave Decomposition

\[ I(\theta, \phi) = \left| \sum_j T_j \psi_j(\theta, \phi) \right|^2 = \left| \sum_j T_j Y_j^1(\theta, 0) \right|^2 |\sin \phi|^2 \]

\[ Y_j^1(\theta, 0) = \sum_{j=0}^{j-1} y_j \tan^{2j} \theta \]

Polynomial in \( \tan^2 \theta \)

\[ a(\theta) = \sum_{j=0}^{J_{\text{max}}-1} c_j(\{T_j\}) \tan^{2j}(\theta) = c(\{T_j\}) \prod_{k=1}^{J_{\text{max}}-1} \left( \tan^2(\theta) - r_k(\{T_j\}) \right) \]

root decomposition

\[ a(\tan^2 \theta = r_k) = 0 \]

"Barrelet zeros"

Chung, PRD 56 7299–7316 (1997)

Barrelet, Nuov Cim A 8, 331–371 (1972)
Ambiguities in the Partial-Wave Decomposition

\[ a(\theta) = c(\{T_j\}) \prod_{k=1}^{J_{\text{max}}-1} \left( \tan^2(\theta) - r_k(\{T_j\}) \right) \]

\[ I(\theta, \phi) = \left| \sum_j T_j Y_j^1(\theta, 0) \right|^2 |\sin \phi|^2 \]

\[ = \left| \sum_{j=0}^{J_{\text{max}}-1} c_j(\{T_j\}) \tan^{2j}(\theta) \right|^2 |\sin \phi|^2 \]

\[ = c^2 \prod_{k=1}^{J_{\text{max}}-1} \left| \tan^2(\theta) - r_k^* \right|^2 |\sin \phi|^2 = c^2 \prod_{k=1}^{J_{\text{max}}-1} \left| \tan^2(\theta) - r_k^* \right|^2 |\sin \phi|^2 \]
Study of the Ambiguities

- How do the ambiguous solutions look like (continuity, signals, ...)?
- What are the effects of the partial-wave decomposition fit on finite data on the ambiguities?

I. Continuous intensity model

- create an amplitude model for selected partial waves
- calculate exact ambiguities

II. Finite pseudo-data

- generate pseudo-data according to model
- perform partial-wave decomposition
I. Continuous intensity model

- create an amplitude model for four selected partial waves
- In $1.0 < m_X < 2.5\text{ GeV/c}^2$
- $m_X$-dependence by Breit-Wigner amplitudes (PDG parameters)

$$T(m_X) = \sqrt{m_X} \sqrt{\rho_2(m_X)} \cdot C e^{i\phi} \cdot D_{BW}(m_X; M_0, \Gamma_0)$$

- Phase-space factor
- Complex scale
- $\frac{M_0 \Gamma_0}{M_0^2 - m_X^2 - iM_0\Gamma_0}$

<table>
<thead>
<tr>
<th>$J^{PC}$</th>
<th>Resonances</th>
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<tbody>
<tr>
<td>1⁻⁻</td>
<td>$\rho(1450)$</td>
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<tr>
<td>2⁺⁺</td>
<td>$a_2(1320), a_2'(1700)$</td>
</tr>
<tr>
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</tr>
<tr>
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Continuous Amplitude Model

I. Continuous intensity model

- create an amplitude model for four selected partial waves
- $1.0 < m_X < 2.5 \text{ GeV}/c^2$
- $m_X$-dependence by Breit-Wigner amplitudes (PDG parameters)

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</table>
I. Continuous intensity model $N_a = 3$

- Sample points in $m_X$ and calculate ambiguous solutions

- Ambiguous intensities are also continuous

- Not all solutions are different from each other!

- Highest-spin ($4^{++}$) intensity is invariant!
I. Continuous intensity model

- Sample points in $m_X$ and calculate ambiguous solutions
- Ambiguous intensities are also continuous
- Not all solutions are different from each other!
- Highest-spin ($4^{++}$) intensity is invariant!
II. Finite pseudo-data

- reality: **finite data** and **amplitudes unknown**
- generate pseudo-data according to model ($10^5$ events)
- perform a partial-wave decomposition fit
  → **3000 attempts with random start values**

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II. Finite pseudo-data

- $4^{++}$ intensity is still invariant!

- Overall, amplitude values found by the fit follow the calculated distributions

- Not all solutions are found in each $m_X$ bin
  → PWD fit distorts the intensity distribution!
II. Finite pseudo-data

- $4^{++}$ intensity is still invariant!

- Overall, amplitude values found by the fit follow the calculated distributions

- Not all solutions are found in each $m_\chi$ bin
  $\rightarrow$ PWD fit distorts the intensity distribution!
II. Finite pseudo-data

- $4^{++}$ intensity is still invariant!

- Overall, amplitude values found by the fit follow the calculated distributions

- Not all solutions are found in each $m_X$ bin
  → PWD fit distorts the intensity distribution!
Reducing the Ambiguities

- Intensity of highest-spin wave is unaffected by ambiguities

- Including $M \geq 2$ → allows for additional angular structure → resolves ambiguities

- Remove one wave with $J < J_{\text{max}}$ → resolves ambiguities
Continuity Constraints for Partial-Wave Analyses
Conventional Partial-Wave Analysis

We have some knowledge about the partial-wave amplitudes $T(m_X, t')$:

- Physics should be (mostly) **continuous** in $m_X$ and $t'$
  → Solutions in neighboring bins should be similar (→ correlations between bins)
- Amplitudes should follow **phase-space** and **production kinematics**

Limitations of conventional PWA:

\[
I(m_X, t'; \tau_n) = \left| \sum_{\text{waves}} T_i(m_X, t') \psi_i(m_X, \tau_n) \right|^2
\]

- Binned analysis limits statistics, especially for small signals
- Continuity information is not imposed in the model
- We need to **select ("small") subset of partial waves** to include in the model
  → important source of systematic uncertainty
Constraints for Partial-Wave Analyses

Make use of this information to stabilize partial-wave decomposition fit:
- Replace discrete amplitudes with smooth, non-parametric curves
- Incorporate kinematic factors
- Include regularization for small amplitudes

Framework by team from the Max-Planck Institute for Astrophysics:
**NIFTY**: “Numerical Information Field Theory”

- Provides continuous non-parametric models
- Adapt to partial-wave analysis model
- **Learns smoothness and shape** of the amplitude curves

This work is done in collaboration with Jakob Knollmueller (TUM / ORIGINS Excellence Cluster)
Verification on pseudodata:

• Generate pseudodata according to:
  – smooth model in mass
  – 81 partial waves
  – 5 resonances in selected waves

• resonance(s) (Breit-Wigner)
• nonres. component (broad curve)
• Combined signal → **input model**

• Perform PWA fit with NIFTy model on generated dataset
  – Same set of partial waves
Input-Output Study

$1^{++} 0^+ f_0(980) \pi P$

$0.100 < t' < 1.000 \text{(GeV/c)}^2$

$[1^{++} 0^+ f_0(980) \pi P] - [1^{++} 0^+ \rho \pi S]$

$0.100 < t' < 1.000 \text{(GeV/c)}^2$

---

**MC input**

---

Intensity / (20 MeV/c^2)

$m_{3\pi} \text{[GeV/c}^2\text{]}$

---

$\Delta\phi \text{[deg]}$

$m_{3\pi} \text{[GeV/c}^2\text{]}$
Input-Output Study

$1^{++}0^+ f_0(980)\pi P$
$0.100 < t' < 1.000 \text{ (GeV/c)}^2$

MC input
Mass-indep. fit

$[1^{++}0^+ f_0(980)\pi P] - [1^{++}0^+ \rho\pi S]$
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MC input
Mass-indep. fit
Input-Output Study

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$0.100 < t' < 1.000 \text{ (GeV/c)}^2$

MC input
Continuous fit
Input-Output Study

\[ 1^{++}0^+ f_0(980) \pi P \]
\[ 0.100 < t' < 1.000 \text{ (GeV/c)}^2 \]

**MC input**
**Continuous fit**
**Mass-indep. fit**

\[ [1^{++}0^+ f_0(980) \pi P] - [1^{++}0^+ \rho \pi S] \]
\[ 0.100 < t' < 1.000 \text{ (GeV/c)}^2 \]

**Continuous fit**
**Mass-indep. fit**
**MC input**
Single-Step Resonance Model Fit

We can go one step further!

In selected waves:

• add resonant part
  (e.g. as sum of Breit-Wigner distributions)
• use NIFTy as flexible non-res. background
• amplitude described by coherent sum

→ extract resonance parameters in a single fit
Single-Step Resonance Model Fit

\[ \frac{1^{++} 0^+ \rho \pi S}{\nu \chi^2 (GeV/c)^2} \quad 0.100 < t' < 1.000 \]

- **Input model**
- **MC input**
- **Continuous fit**
- **Resonances**
- **Nonres. comp.**
High-precision data from COMPASS in $\pi^-\pi^-\pi^+$ and $K_S^0K^-$ allow in-depth study of $a_J$ and $\pi_J$ states

**Ambiguities** appear in the partial-wave decomposition of two-body states

- Ambiguous amplitudes are **continuous** and can be calculated
- PWD fit/finite data has an effect on ambiguous solutions
- **Choice of included partial waves** may suppress the ambiguities

**NIFTy:** new approach to partial-wave analysis

- **Includes continuity, kinematics and regularization**
- Overcomes limitations of conventional approach
- Can include resonance-model fit
- Demonstrated in Monte Carlo pseudodata studies
Outlook

We can combine both presented topics

→ Apply NIFTy method on ambiguity problem in $K_S^0 K^-$

• Use continuity constraints to separate ambiguous solutions over entire mass range
• Improve fit quality

Partial-wave analysis using NIFTy model is being successfully applied on real data

Thank you for your attention!
QCD in the Resonance Region

• At low energies (hadron regime): **QCD not solvable perturbatively**

• Theory: rely on models and effective theories, e.g. **quark model** (hadrons as bound states of **valence quarks**)

• Experimentally: **precision measurements** of hadronic states and search for so-called **exotic states** (forbidden in the quark model)
Partial-Wave Decomposition

\[ I(m_X, t'; \theta, \phi) = \frac{dN}{dm_X dt' d\theta d\phi} = |M_{fi}|^2 \]

- Separate process amplitude into **partial waves**
  - Spin \( J \) and spin-projection \( M \)
  \[ J = L, M = M_L \]
  \[ P = C = (-1)^J \]

**Partial wave:** specific \( (J^{PC} M) \)
Partial-Wave Decomposition

\[ I(m_X, t'; \theta, \phi) = \frac{dN}{dm_X dt' d\theta d\phi} = \left| \sum_j T_j(m_X, t') \psi_j(\theta, \phi) \right|^2 \]

- Separate process amplitude into **partial waves**
  - Spin \(J\) and spin-projection \(M\)

- Production, propagation and decay of \(X^-\)

\[ T_{JM}(m_X, t') \]
\[ \psi_J(\theta, \phi) = Y_J^1(\theta, \phi) \]
\[ M = 1 \]

(reflectivity basis, \(\varepsilon = -1\) suppressed \(\rightarrow M \neq 0\) )
Partial-Wave Decomposition

\[ I(m_X, t'; \theta, \phi) = \frac{dN}{dm_X dt' d\theta d\phi} = \left| \sum_j T_j(m_X, t') \psi_j(\theta, \phi) \right|^2 \]

- Separate process amplitude into **partial waves**
  - Spin \( J \) and spin-projection \( M \)

- Production, propagation and decay of \( X^- \)
  \[ T_j(m_X, t') = P(m_X, t') D(m_X) \]
  \[ \psi_j(\theta, \phi) = Y_j^M(\theta, \phi) \]
  \( M = 1 \)

- Fit \( I(m_X, t'; \theta, \phi) \) to data in \( (m_X, t') \) bins:
  - Choose finite set of \( \{ J^{PC} \} \)

From B. Ketzer et al., *Prog.Part.Nucl.Phys. 113* (2020) 103755, Light-Meson Spectroscopy with COMPASS
Ambiguities in Incoherent Sectors

\[ \varepsilon = \pm 1: \quad I(m_X, t'; \theta, \phi) = \left| \sum_{JM} T_{JM}^+(m_X, t') \psi_{JM}^+(\theta, \phi) \right|^2 + \left| \sum_{JM} T_{JM}^-(m_X, t') \psi_{JM}^-(\theta, \phi) \right|^2 \]

\[ a_0^\pm = \sum_{J=0}^{J_{\text{max}}} T_{J0}^\pm Y_J^0(\theta, 0) \quad \varepsilon = -1, M = 0 \]

\[ a_1^- = \sum_{J=1}^{J_{\text{max}}} T_{J1}^- Y_J^1(\theta, 0) \quad \varepsilon = -1, M = 1 \]

\[ a_1^+ = \sum_{J=1}^{J_{\text{max}}} T_{J1}^+ Y_J^1(\theta, 0) \quad \varepsilon = +1, M = 1 \]

- \[ a_s^- = a_0^- + a_1^- \], then same procedure as for a single sector

- New amplitudes for \( \varepsilon = +1: |a_1^+|^2 = |a_1^-|^2 - \text{const.} \rightarrow \text{positivity requirement!} \]
I. Continuous intensity model

- create a model for the amplitudes in four waves
- $m_X$-dependence by Breit-Wigner amplitudes
<table>
<thead>
<tr>
<th>$J^P$</th>
<th>resonance content</th>
<th>$m_0$ [GeV/$c^2$]</th>
<th>$\Gamma_0$ [GeV/$c^2$]</th>
<th>$c$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1$^-$</td>
<td>$\rho(1450)$</td>
<td>1.465</td>
<td>0.400</td>
<td>0.0564</td>
<td>1.8023</td>
</tr>
<tr>
<td>2$^+$</td>
<td>$a_2(1320)$</td>
<td>1.3181</td>
<td>0.1098</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$a_2(1700)$</td>
<td>1.698</td>
<td>0.265</td>
<td>0.1480</td>
<td>$\pi$</td>
</tr>
<tr>
<td>3$^-$</td>
<td>None</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>4$^+$</td>
<td>$a_4(1970)$</td>
<td>1.967</td>
<td>0.324</td>
<td>0.1274</td>
<td>6.0072</td>
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Implementation: NIFTy Framework

Framework by team from the Max-Planck Institute for Astrophysics:

**NIFTY**: “Numerical Information Field Theory”

- Provides continuous non-parametric models

This work is done in collaboration with Jakob Knollmueller (TUM / ORIGINS Excellence Cluster)
Input-Output Study

Mc input
Continuous fit
Resonances
Nonres. comp.

2^{++} 1^+ \rho \pi D
0.100 < t' < 1.000 \text{ (GeV/c)^2}

Intensity / (20 \text{ MeV/c^2})

m_{3\pi} \text{ [GeV/c^2]}

Outgoing analysis
julien.beckers@tum.de
Outlook: NIFTy on KsK- pseudodata