

Exclusive π^0 muoproduction measurements at COMPASS

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on behalf of the COMPASS collaboration

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Generalised Parton Distributions

- Proton spin sum rule: $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$

Jaffe&Manohar Nucl. Phys. B337 (1990)

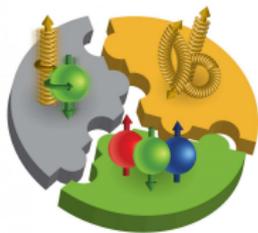
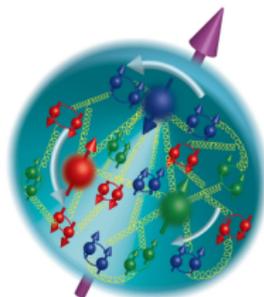
COMPASS experiment in μp DIS: $\Delta\Sigma = 0.32 \pm 0.03$

COMPASS Collaboration: Phys. Lett. B 693 (2010)

COMPASS, RHIC results: $\Delta G = 0.2^{+0.06}_{-0.07}$

de Florian et al. Phys.Rev.Lett. 113 (2014) no.1, 012001

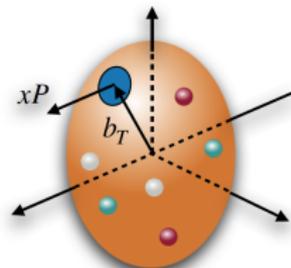
Missing component: $L_{q,g} = ? \rightarrow$ GPDs provides access



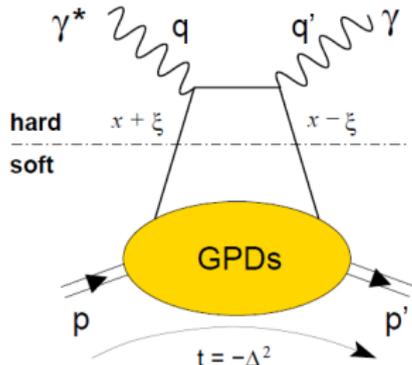
- Generalised Parton Distributions (GPD) give access to the 3D structure of a hadron
- GPDs encode the correlation between the longitudinal momentum of a parton and its position in the transverse plane

$$q^f(x, b_{\perp}) \xrightarrow{\int dx} \text{Form factors}$$

$$q^f(x, b_{\perp}) \xrightarrow{\int db_{\perp}} \text{PDFs}$$



Generalised Parton Distributions



Definition of variables:

$q \dots \gamma^*$ four-momentum
 $x \dots$ average longitudinal momentum fraction of initial and final parton (NOT accessible)

$\xi \dots$ difference of longitudinal-momentum fraction between initial and final parton $\approx x_B/(2 - x_B)$

$t \dots$ four-momentum transfer

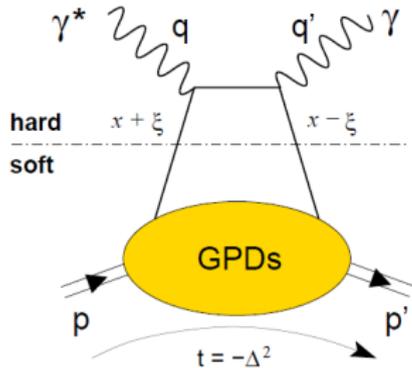
- 4 chiral-even GPDs (parton helicity conserved)
- 4 chiral-odd (or transversity) GPDs (parton helicity flipped)

		Quark Polarisation		
		Unpolarised (U)	Longitudinally polarised (L)	Transversely polarised (T)
Nucleon Polarisation	U	H		\bar{E}_T
	L		\tilde{H}	\tilde{E}_T
	T	E	\tilde{E}	H_T, \tilde{H}_T

GPDs enter the exclusive processes through **Compton Form Factors (CFF)**

$$\mathcal{H}(\xi, t) = \mathcal{P} \int_{-1}^1 dx \frac{H(x, \xi, t)}{x - \xi} + i\pi H$$

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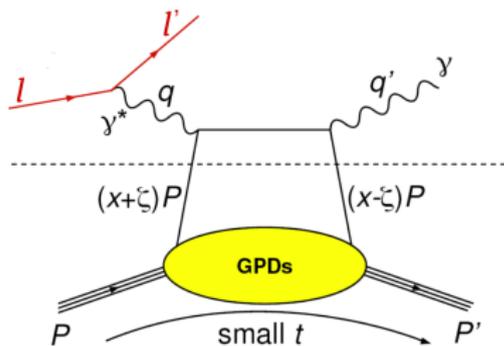
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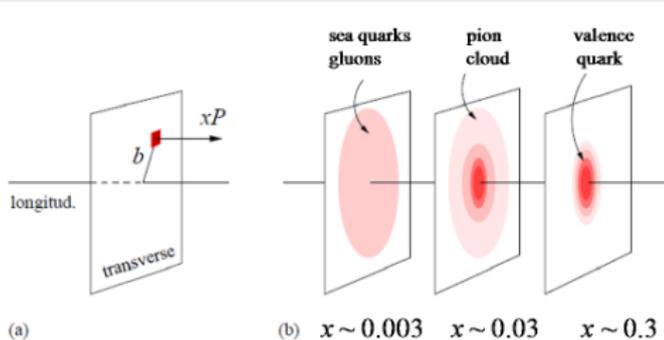
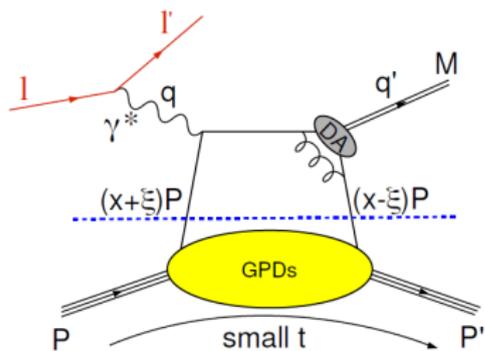
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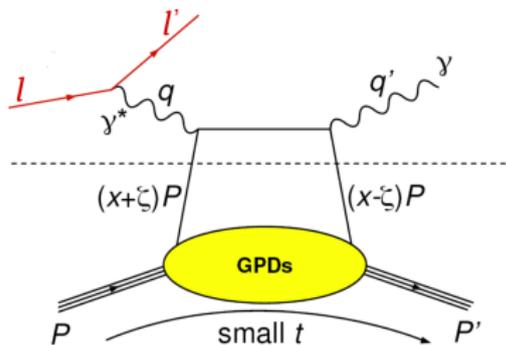


- Most commonly used processes for GPDs parametrisation are Deeply Virtual Compton Scattering (DVCS) and Hard Exclusive Meson Production (HEMP)
- DVCS on an unpolarised proton target gives access to GPD $H \rightarrow$ 3D imaging of a hadron

$$H^q(x, \xi = 0, t) = \rho^q(x, b_\perp) \quad (\text{Burkardt 2000, 2003})$$



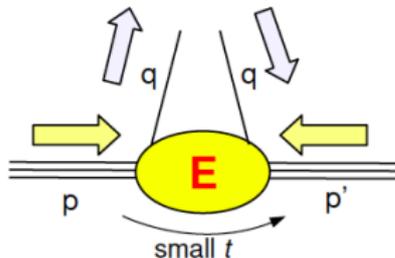
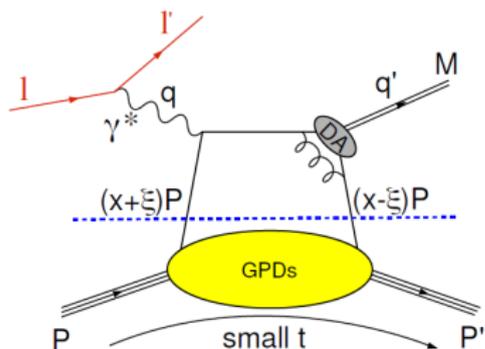
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- Most commonly used processes for GPDs parametrisation are Deeply Virtual Compton Scattering (DVCS) and Hard Exclusive Meson Production (HEMP)
- DVCS off neutrons, or DVCS or HEMP on transverse polarised proton target gives access to GPD $E \rightarrow$ helps constraining the total angular momentum of partons

$$J^f = \frac{1}{2} \lim_{t \rightarrow 0} \int_{-1}^1 dx x [H^f(x, \xi, t) + E^f(x, \xi, t)]$$

Phys. Rev. Lett. 78 (1997)



Road to HEMP cross-section

COMPASS measurement in 2012, and 2016/17 with μ^+ and μ^- beams of $E_\mu = 160$ GeV

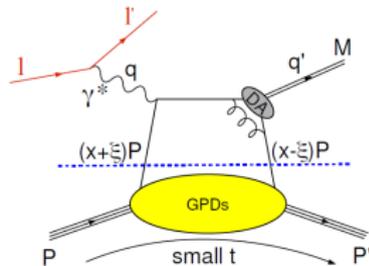
Collected events corrected for:

- Luminosity of μ^+ and μ^- beams
- Background subtraction
- Acceptance of the spectrometer
- Reduction of μp cross-section to $\gamma^* p$:

$$\frac{d^4\sigma_{\mu p}}{dQ^2 dt d\nu d\phi} = \Gamma \frac{d^2\sigma_{\gamma^* p}}{dt d\phi}$$

with the virtual photon flux

$$\Gamma = \Gamma(E_\mu, Q^2, \nu)$$



COMPASS 2012:

- 4 weeks \rightarrow **results published:**
PLB 805(2020) 135454

COMPASS 2016/17:

- 2 \times 6 months

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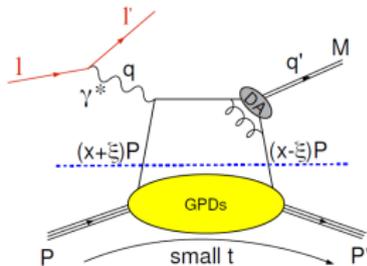
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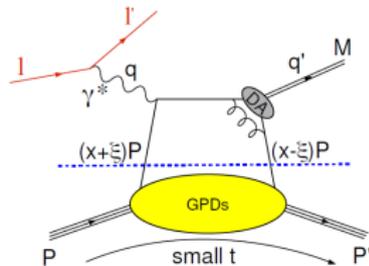
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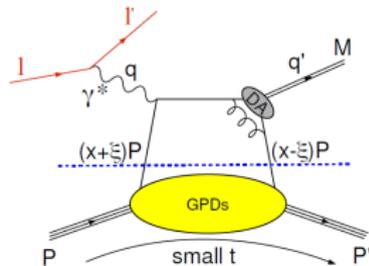
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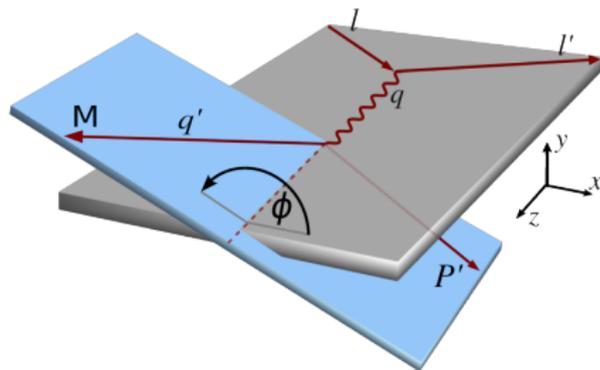
HEMP cross section

HEMP cross-section, reduced to γ^*p , for the **unpolarised target** and **polarised lepton beam** (relevant for COMPASS 2012, 2016/2017 measurements):

$$\frac{d^2\sigma_{\gamma^*p}^{\leftrightarrow}}{dtd\phi} = \frac{1}{2\pi} \left[\frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} + \epsilon \cos(2\phi) \frac{d\sigma_{TT}}{dt} + \sqrt{\epsilon(1+\epsilon)} \cos\phi \frac{d\sigma_{LT}}{dt} \right. \\ \left. \mp |P_l| \sqrt{\epsilon(1-\epsilon)} \sin\phi \frac{d\sigma'_{LT}}{dt} \right]$$

$$\epsilon = \frac{1-y - \frac{y^2\gamma^2}{4}}{1-y + \frac{y^2}{2} + \frac{y^2\gamma^2}{4}}$$

Factorization proven for σ_L ,
not for σ_T which is expected to
be suppressed by a factor $1/Q^2$
BUT large contributions are
observed at JLab



HEMP cross section

Spin independent HEMP cross-section after averaging the two spin-dependent cross-sections:

$$\frac{d^2\sigma_{\gamma^*p}}{dt d\phi} = \frac{1}{2} \left(\frac{d^2\sigma_{\gamma^*p}^{\leftarrow}}{dt d\phi} + \frac{d^2\sigma_{\gamma^*p}^{\rightarrow}}{dt d\phi} \right) =$$

$$\frac{1}{2\pi} \left[\frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} + \epsilon \cos(2\phi) \frac{d\sigma_{TT}}{dt} + \sqrt{\epsilon(1+\epsilon)} \cos\phi \frac{d\sigma_{LT}}{dt} \right]$$

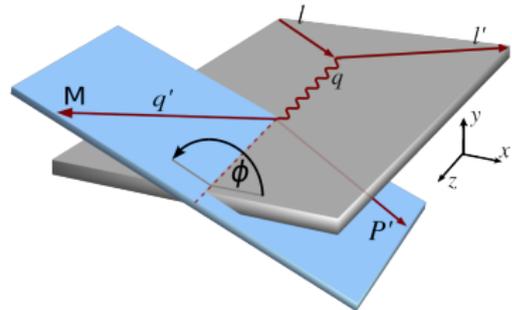
~~$$\mp |P_t| \sqrt{\epsilon(1-\epsilon)} \sin\phi \frac{d\sigma'_{LT}}{dt}$$~~

\Rightarrow study ϕ dependence

After integration in ϕ :

$$\frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt}$$

\Rightarrow study t dependence



HEMP cross section

$$\frac{d^2\sigma_{\gamma^*p}}{dt d\phi} = \frac{1}{2\pi} \left[\frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} + \epsilon \cos(2\phi) \frac{d\sigma_{TT}}{dt} + \sqrt{\epsilon(1+\epsilon)} \cos\phi \frac{d\sigma_{LT}}{dt} \right]$$

GDPs in exclusive π^0 production

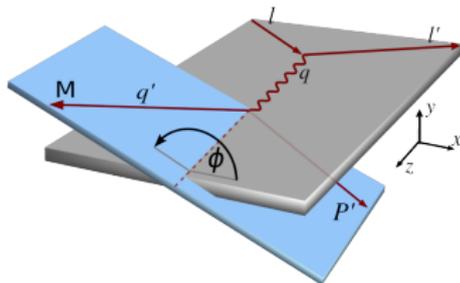
$$\frac{d\sigma_L}{dt} \propto \left[(1 - \xi^2) |\langle \tilde{\mathcal{H}} \rangle|^2 - 2\xi^2 \operatorname{Re}(\langle \tilde{\mathcal{H}} \rangle^* \langle \tilde{\mathcal{E}} \rangle) - \frac{t'}{4M^2} \xi^2 |\langle \tilde{\mathcal{E}} \rangle|^2 \right]$$

$$\frac{d\sigma_T}{dt} \propto \left[(1 - \xi^2) |\langle \mathcal{H}_T \rangle|^2 - \frac{t'}{8M^2} |\langle \bar{\mathcal{E}}_T \rangle|^2 \right]$$

$$\frac{d\sigma_{TT}}{dt} \propto t' |\langle \bar{\mathcal{E}}_T \rangle|^2$$

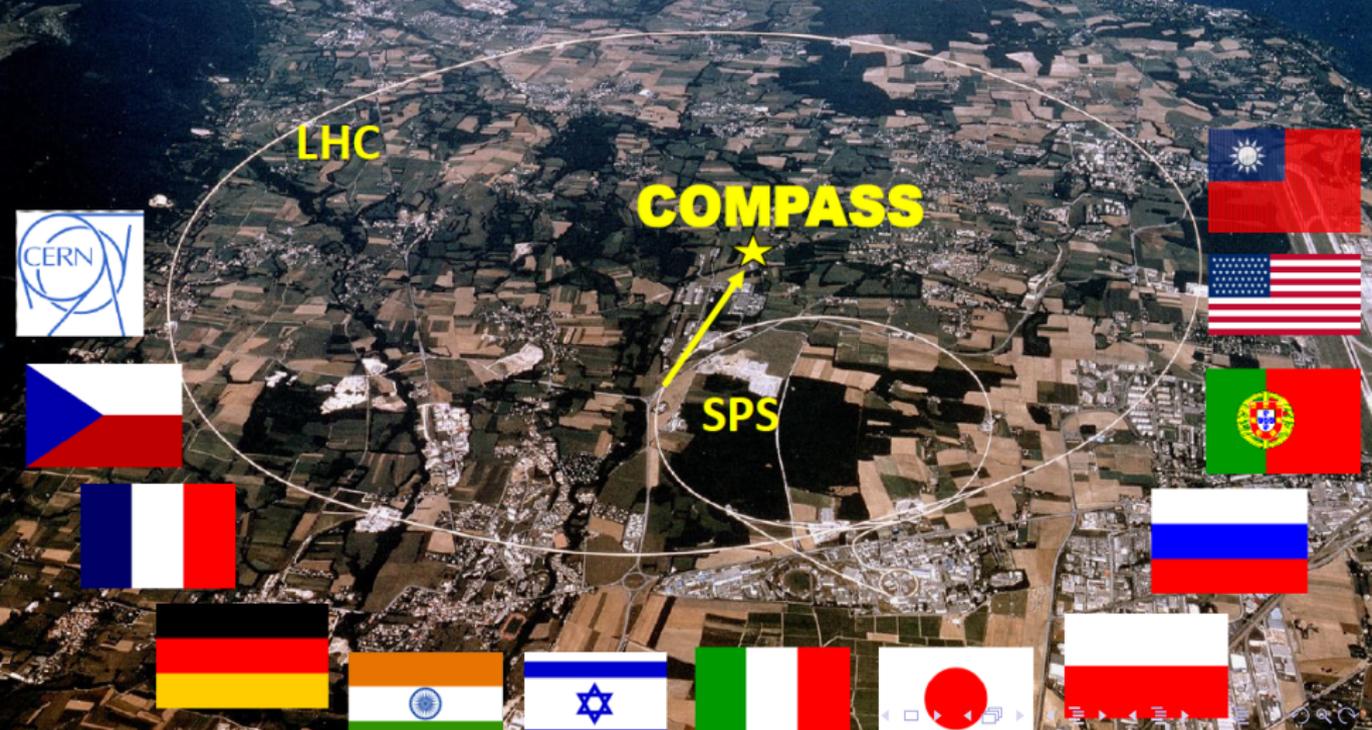
$$\frac{d\sigma_{LT}}{dt} \propto \xi \sqrt{1 - \xi^2} \sqrt{-t'} \operatorname{Re}(\langle \mathcal{H}_T \rangle^* \langle \tilde{\mathcal{E}} \rangle)$$

$$t' = t - t_{min}, t_{min} \text{ is the minimum value of } |t|$$



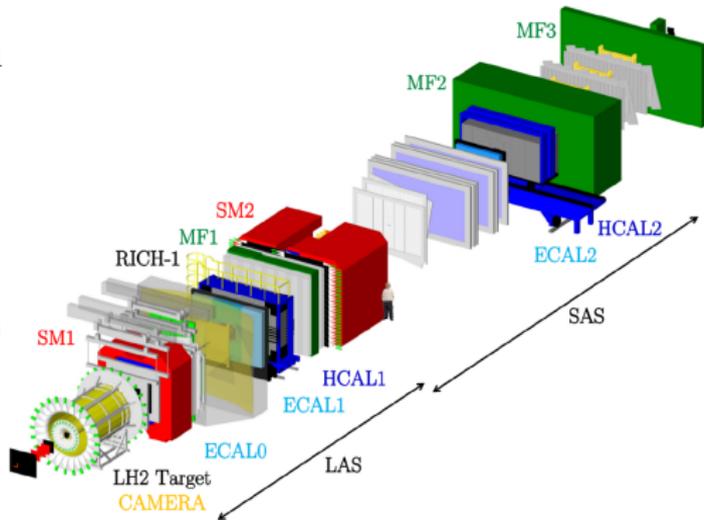
Impact of $\bar{\mathcal{E}}_T$ should be visible in $\frac{d\sigma_{TT}}{dt}$, and also a dip at small t of $\frac{d\sigma_T}{dt}$

COMPASS: Versatile facility to study QCD
with hadron (π^\pm , K^\pm , p ...) and lepton (polarized μ^\pm) beams
of ~ 200 GeV for hadron spectroscopy and
hadron structure studies using SIDIS, DY, DVCS, DVMP...



COMPASS GPD program

- Two stage magnetic spectrometer with large angular and momentum acceptance
- Versatile usage: hadron and muon beams
- Particle identification:
 - Ring Imaging Cherenkov (RICH) detector
 - Electromagnetic calorimeters (ECAL0, ECAL1, ECAL2)
 - Hadronic calorimeters (HCAL1, HCAL2)
 - 2 muon walls
- GPD program: 2012 pilot run, 2016/17 main measurement

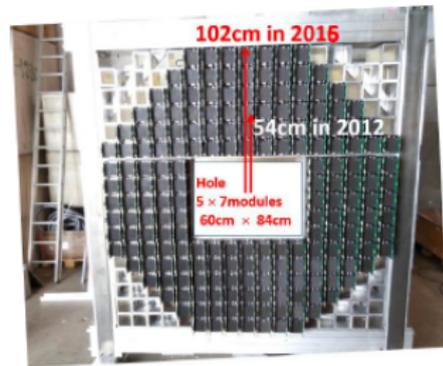
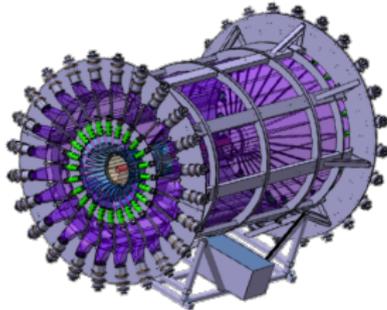
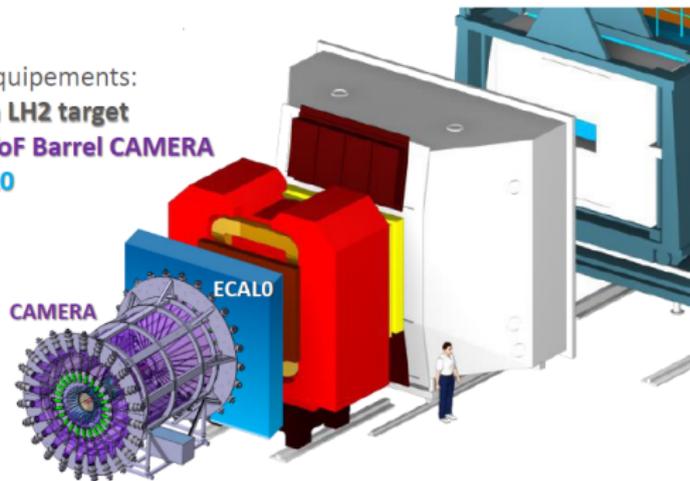


COMPASS GPD program

- Target ToF system:
 - 24 inner and outer scintillators
 - 1 GHz readout
 - 310 ps ToF resolution
- ECAL0 calorimeter:
 - shaslyk modules
 - 2×2 m, 2200 channels

New equipments:

- 2.5m LH2 target
- 4m ToF Barrel CAMERA
- ECAL0

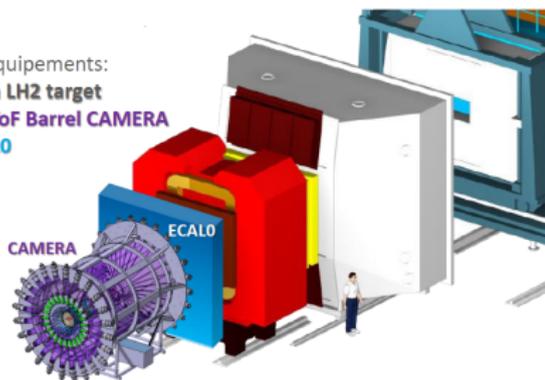


Exclusive π^0 production: Selection

- Incoming and outgoing μ connected to primary vertex
- Two photons in ECALs from π^0 decay, attached to the vertex
- Recoil proton candidate
- $1 < Q^2 < 5 \text{ (GeV/c)}^2$,
 $8.5 < \nu < 28 \text{ GeV}$,
 $0.08 < |t| < 0.64 \text{ (GeV/c)}^2$

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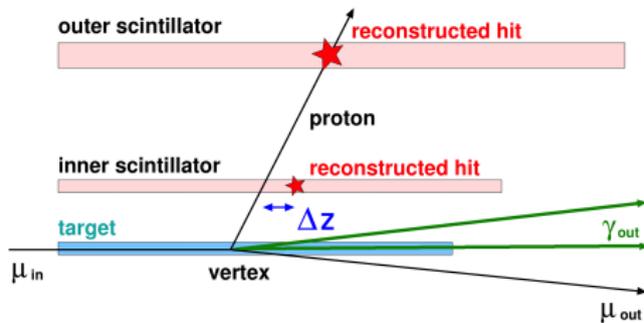


Selections for exclusive π^0 events:

- Transverse momentum constraint:
$$\Delta p_T = p_{T,spect}^p - p_{T,recoil}^p$$
- $$\Delta \varphi = \varphi_{spect}^p - \varphi_{recoil}^p$$
- Z coordinate of inner CAMERA ring:
$$\Delta z = z_{spect}^p - z_{recoil}^p$$
- Energy-momentum conservation:
$$M_X^2 = (p_\mu + p_p - p_{\mu'} - p_{p'} - p_{\pi^0})^2$$
- Invariant mass $M_{\gamma\gamma}$ cut
- Kinematic fit

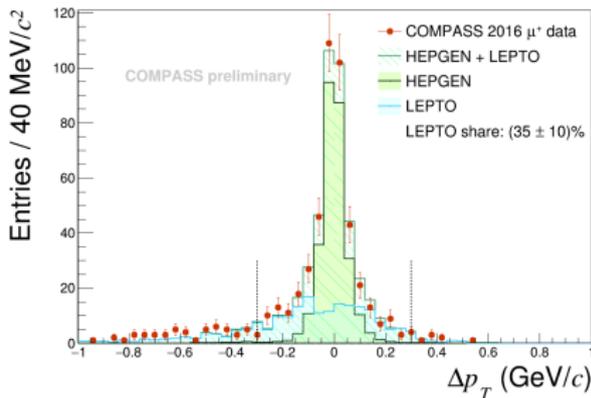
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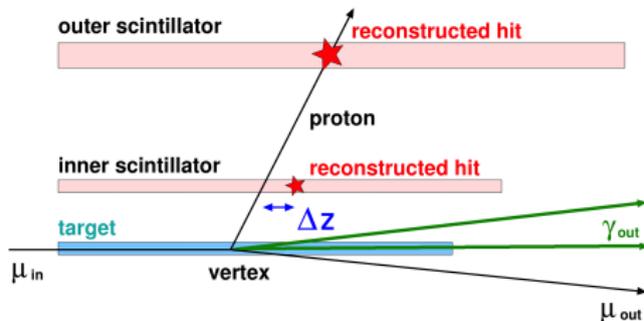
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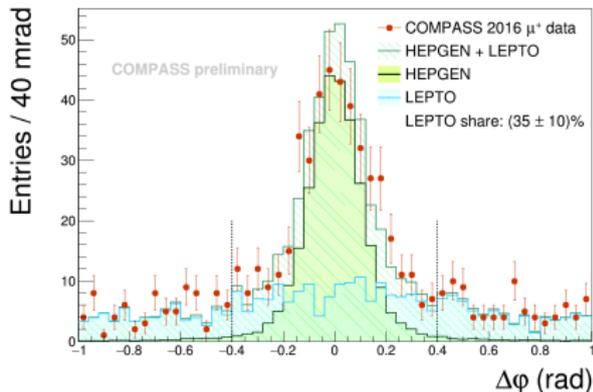
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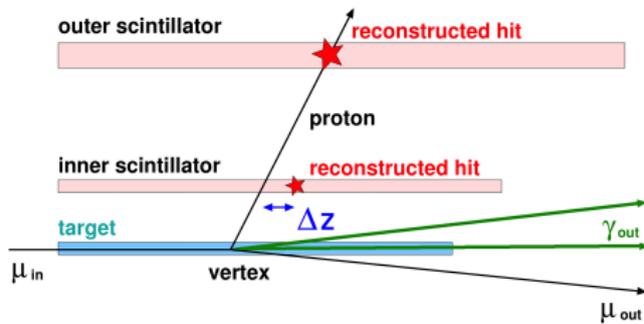
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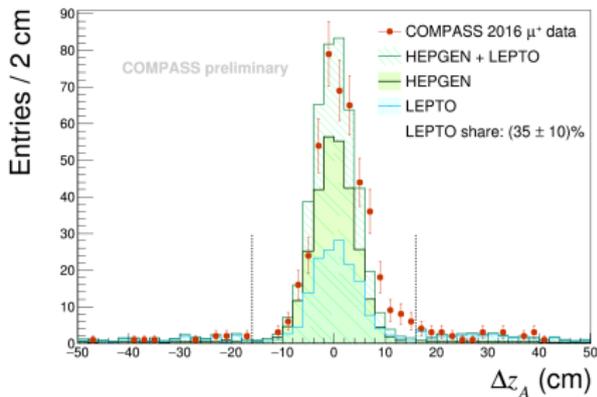
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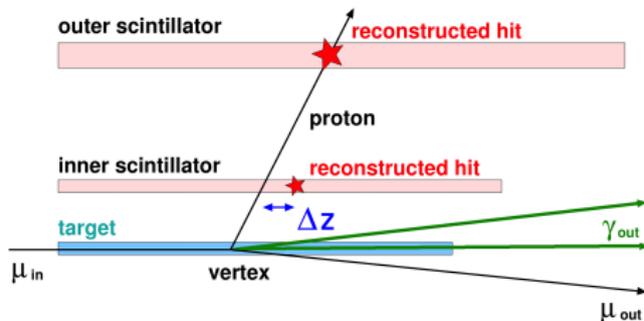
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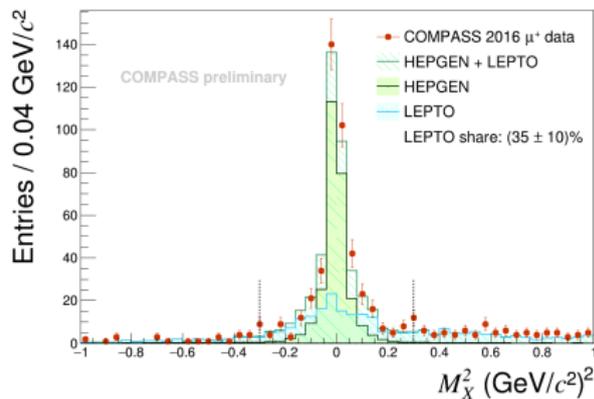
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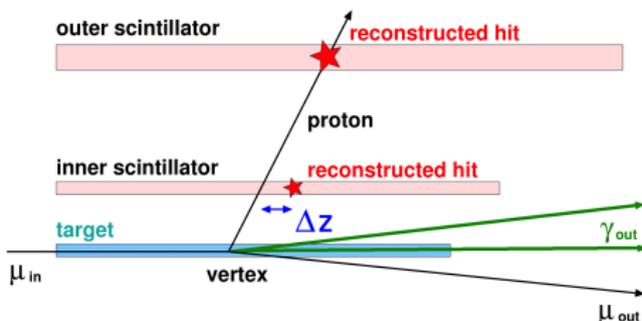
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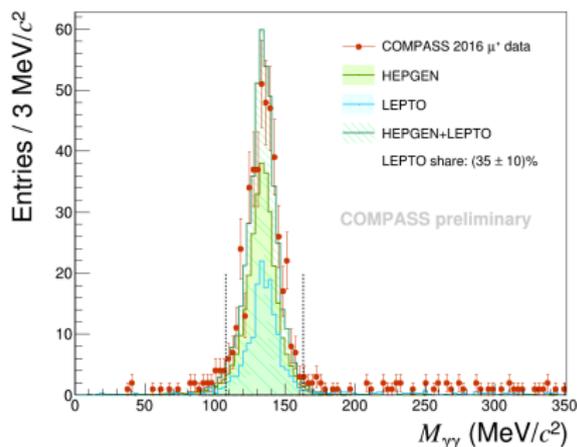
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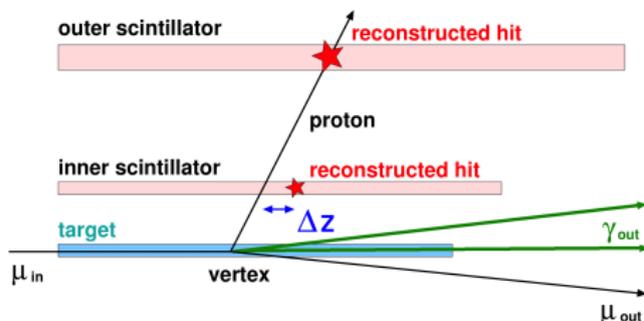
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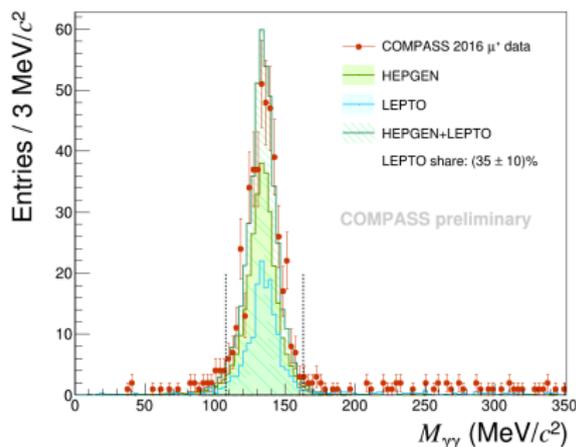
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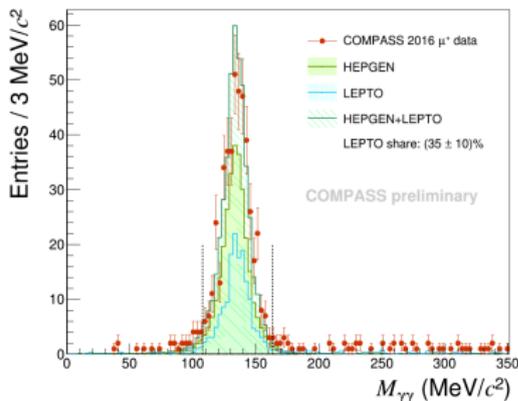
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 $\Delta z = z_{spect}^p - z_{recoil}^p$
- Energy-momentum conservation:
 $M_X^2 = (p_\mu + p_p - p_{\mu'} - p_{p'} - p_{\pi^0})^2$
- Invariant mass $M_{\gamma\gamma}$ cut
- Kinematic fit



Exclusive π^0 production: SIDIS background estimation

- Main background of π^0 production \Rightarrow non-exclusive DIS processes
- 2 Monte Carlo simulations with the same π^0 selection criteria:
 - LEPTO for the non-exclusive background
 - HEPGEN++ shape of distributions of exclusive π^0 production (signal contribution)
- Search for best description of data fitting by mixture of both MC
- Both MC samples normalised to the experimental $M_{\gamma\gamma}$ distribution
- The ratio of background events r_{LEPTO} is determined by a fit on the exclusivity distributions

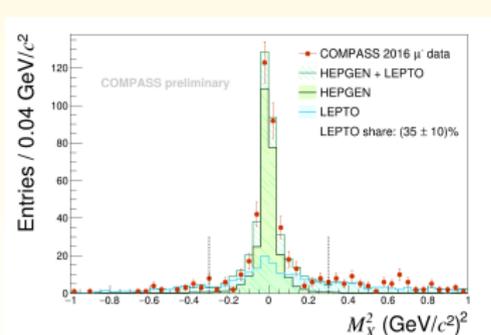
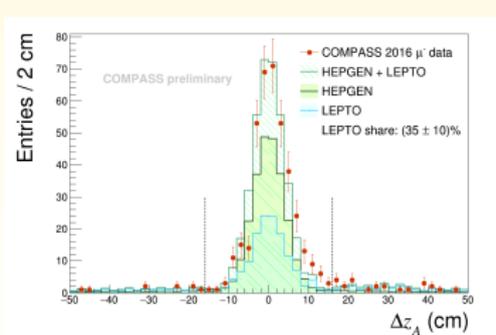
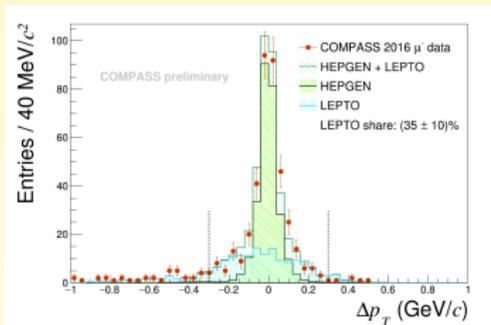
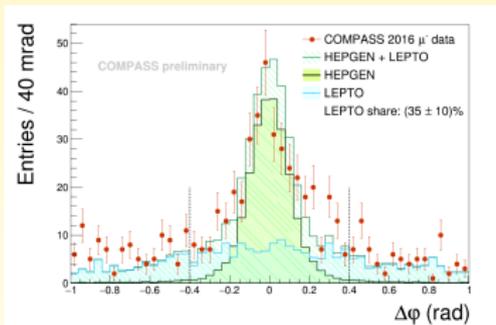


- Resulting fraction of non-exclusive background in data \Rightarrow **(35 \pm 10) %**
- Background fit method is currently the main source of systematic uncertainty

Exclusive π^0 production: SIDIS background estimation

- Main background of π^0 production \Rightarrow non-exclusive DIS processes

Exclusive variables

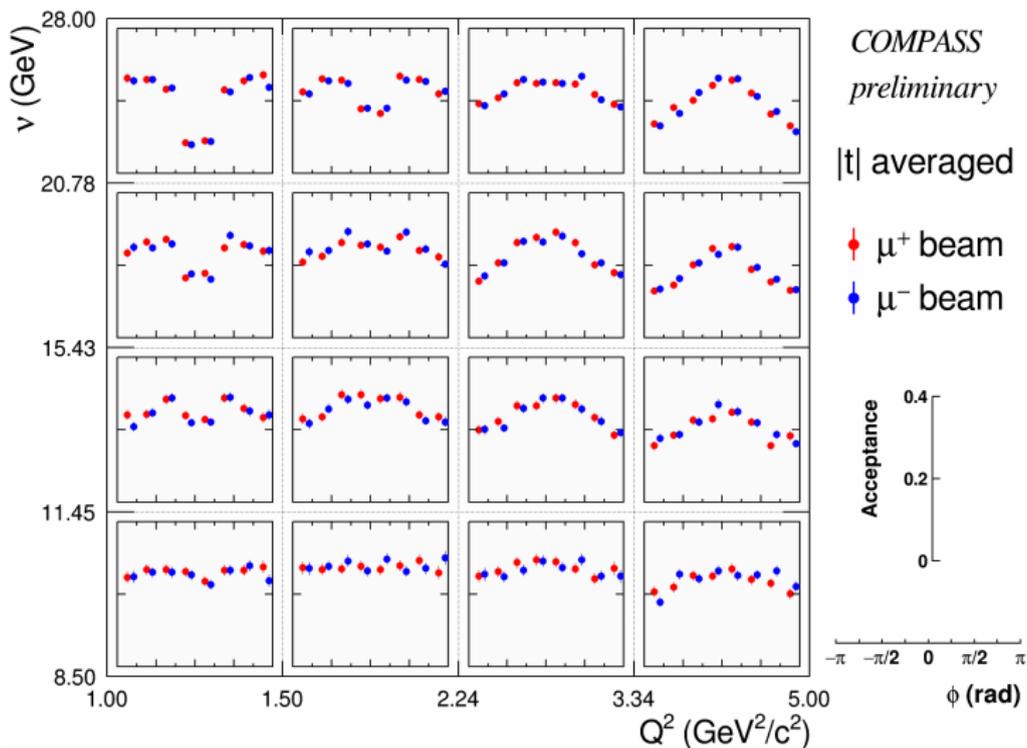


Entries / 3 MeV/c²

) %

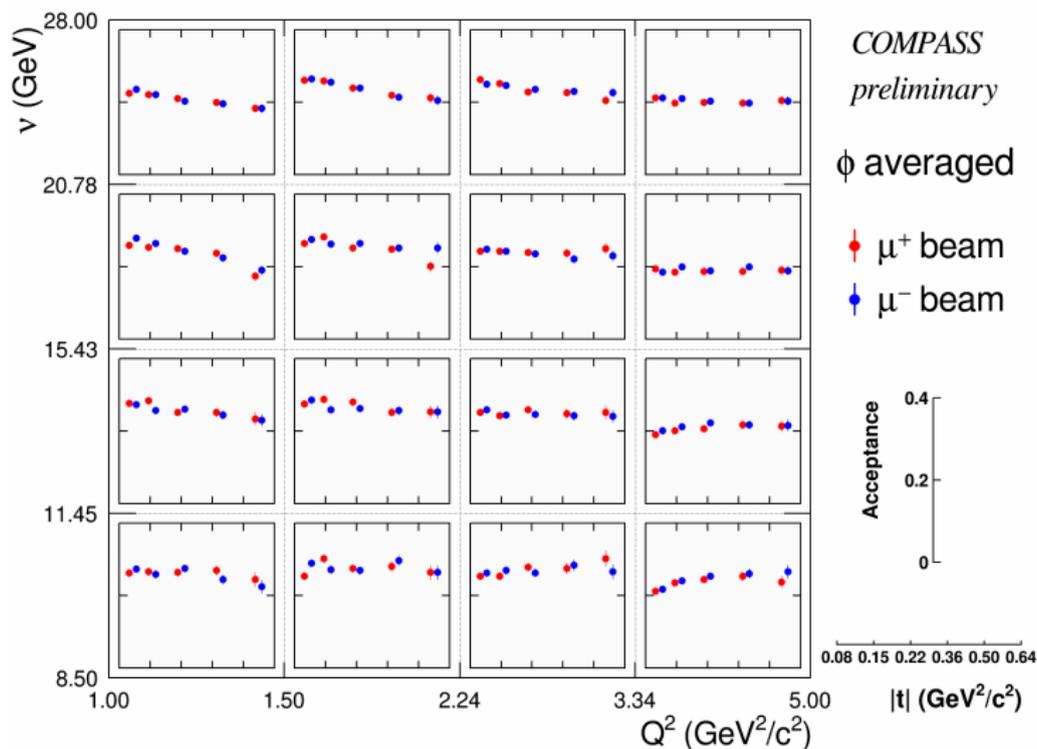
Exclusive π^0 production: COMPASS acceptance

- 4D acceptance in bins of ϕ_{π^0} , ν , $|t|$, Q^2
- figure shows 3D projection, as a function of ϕ_{π^0}

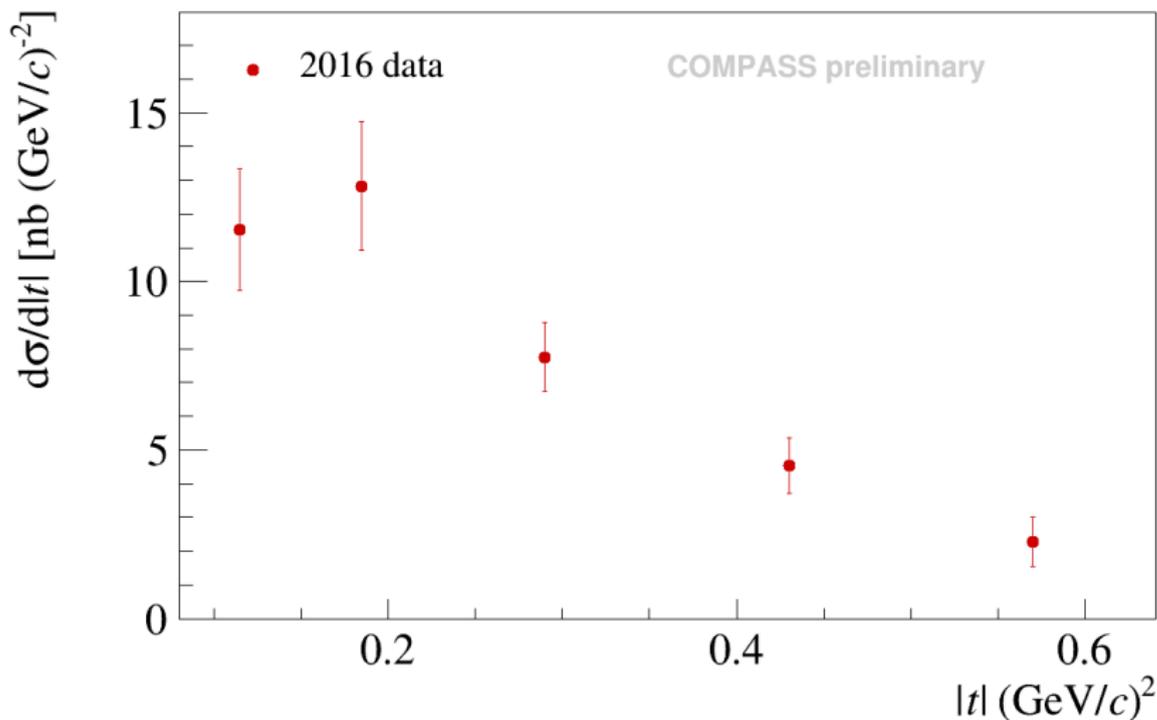


Exclusive π^0 production: COMPASS acceptance

- 4D acceptance in bins of ϕ_{π^0} , ν , $|t|$, Q^2
- figure shows 3D projection, as a function of $|t|$



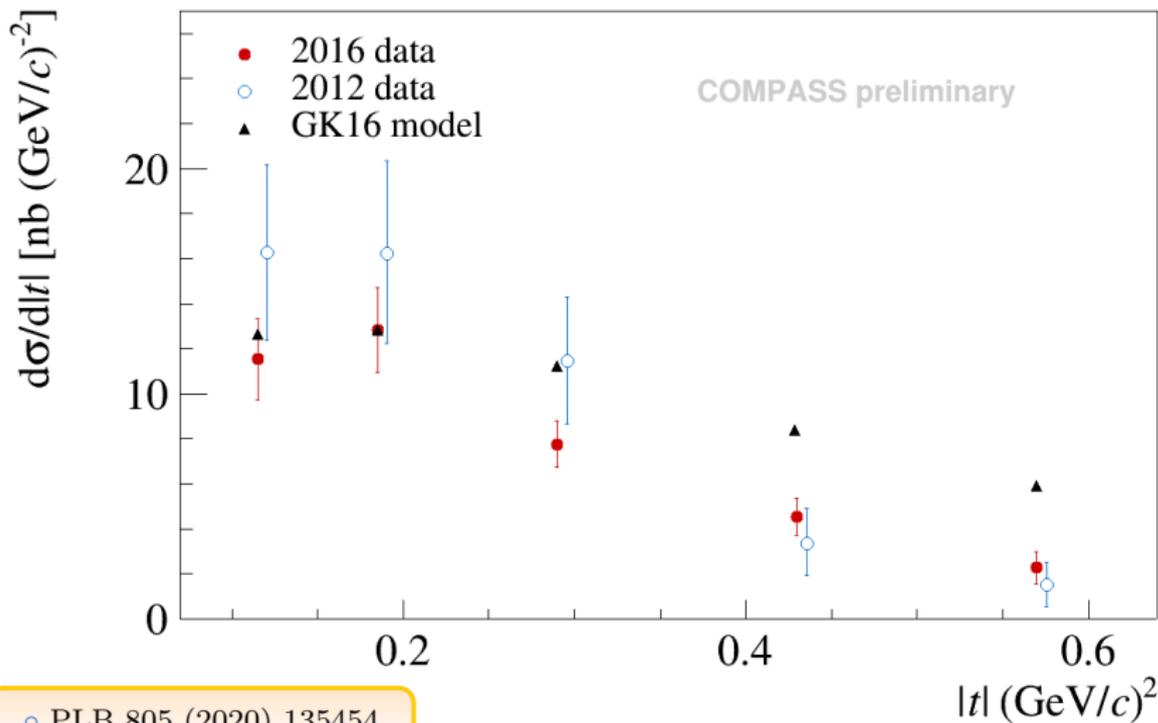
Exclusive π^0 cross-section as a function of $|t|$



Systematic error is of the order of the statistical one

Exclusive π^0 cross-section as a function of $|t|$

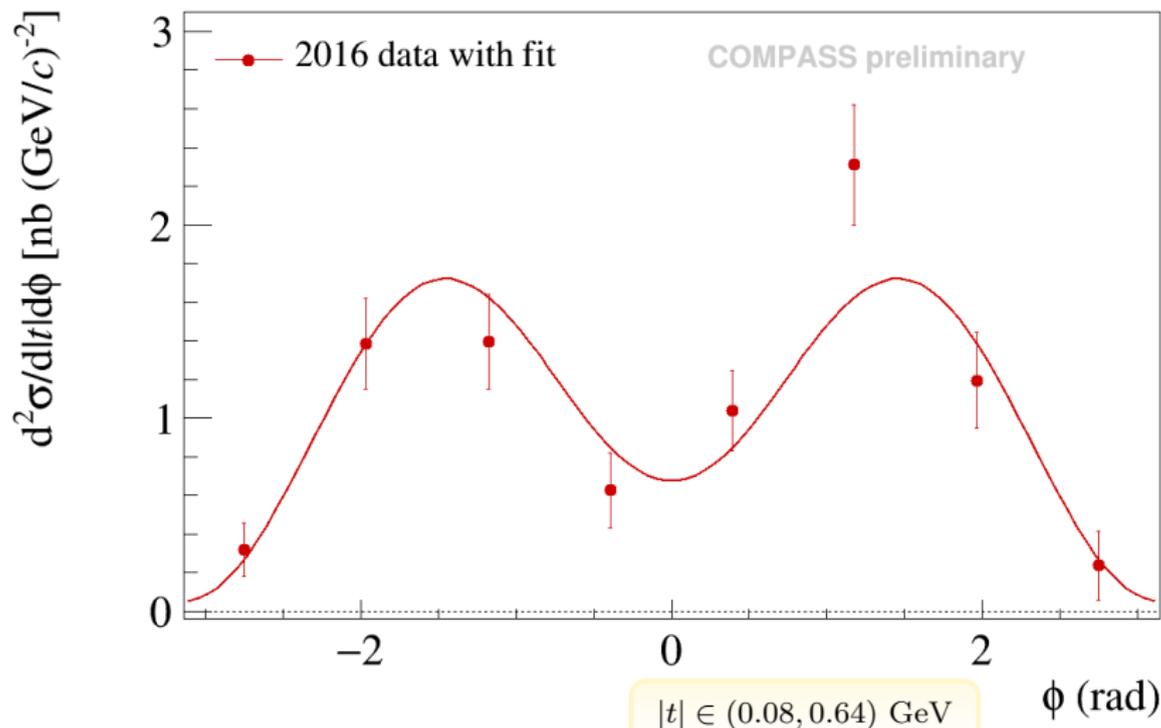
Goloskokov&Kroll model EPJ A47 (2011) 112



○ PLB 805 (2020) 135454

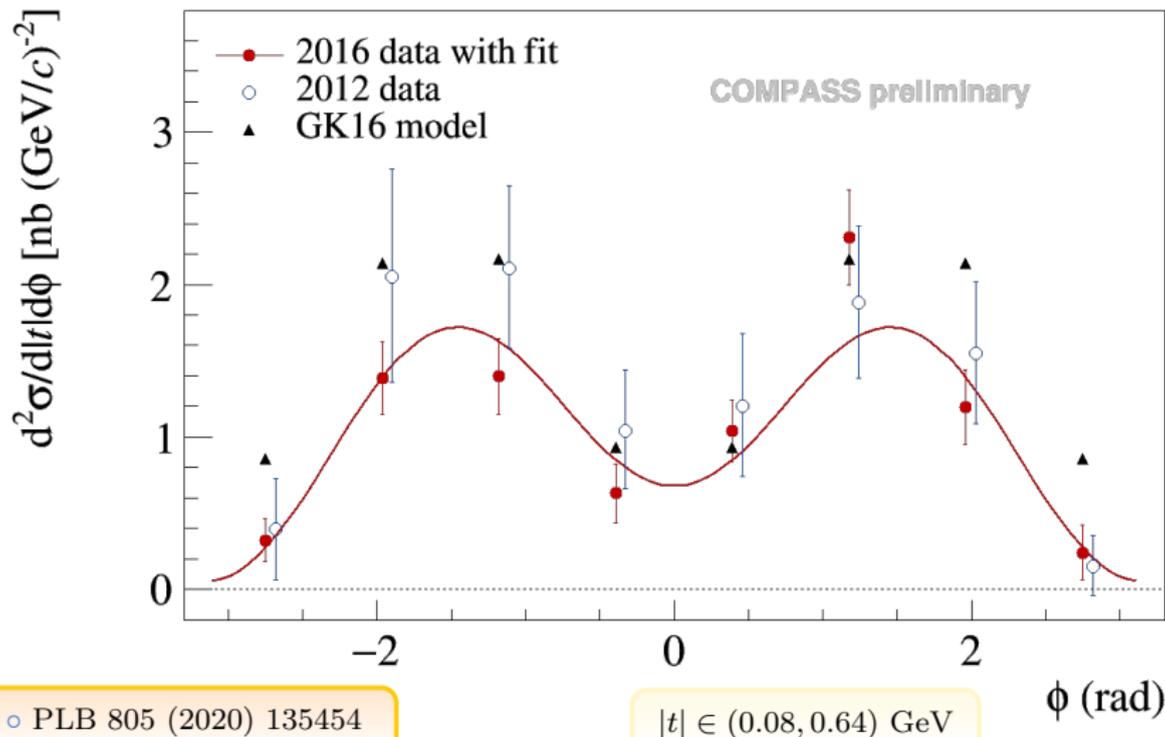
Exclusive π^0 cross-section as a function of ϕ

$$\frac{d^2\sigma_{\gamma^*p}}{dt d\phi} = \frac{1}{2\pi} \left[\frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} + \epsilon \cos(2\phi) \frac{d\sigma_{TT}}{dt} + \sqrt{\epsilon(1+\epsilon)} \cos\phi \frac{d\sigma_{LT}}{dt} \right]$$



Exclusive π^0 cross-section as a function of ϕ

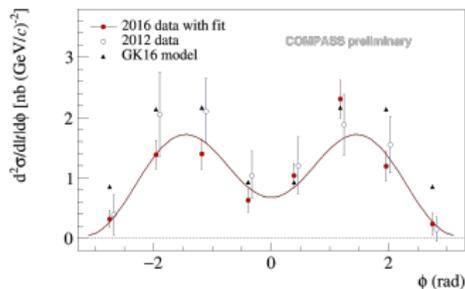
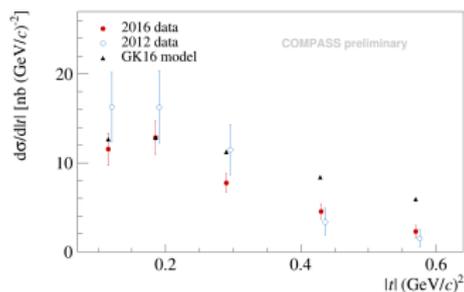
$$\frac{d^2\sigma_{\gamma^*p}}{dt d\phi} = \frac{1}{2\pi} \left[\frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt} + \epsilon \cos(2\phi) \frac{d\sigma_{TT}}{dt} + \sqrt{\epsilon(1+\epsilon)} \cos\phi \frac{d\sigma_{LT}}{dt} \right]$$



Summary

$|t|$ -dependence and ϕ -dependence of exclusive π^0 cross-section on unpolarised proton target:

- ➡ New, preliminary results of 2016 COMPASS measurement at low ξ (or $\langle x_B \rangle = 0.096$), input for constraining phenomenological models (e.g. Goloskokov&Kroll, Goldstein&Luiti, etc.)



- ➡ Statistics of 2016 shown here is about $2.3 \times$ larger than of published results from 2012 pilot run (PLB 805 (2020) 135454)

- ➡ The whole collected 2016/2017 statistics $\sim 9 \times$ larger than 2012 \rightarrow plan to process all available data

- ➡ Heading towards publication of 2016/2017 results soon



Thank you for your attention!

SPARES

- Measurement of exclusive processes at COMPASS is overconstrained \rightarrow can be used to improve precision of kinematic quantities using kinematically constrained fit
- Kinematic fit improves the resolution of the signal and lowers the background
- It works in a principle of minimisation of least square function $\chi^2(\vec{k}) = (\vec{k}_{fit} - \vec{k})^T \hat{C}^{-1} (\vec{k}_{fit} - \vec{k})$, where \vec{k} is a vector of measured quantities and \hat{C} is their covariance matrix

- Method used for the minimisation is Lagrange multipliers with constraints g_i :

$$L(\vec{k}, \vec{\alpha}) = \chi^2(\vec{k}) + 2 \sum_{i=1}^N \alpha_i g_i$$

- Constraints include momentum and energy conservation, common vertex for all tracks (except proton), constraints for final proton, and mass constraint

Past and future GPD measurements

Q^2 [GeV²]

Start
2001

After
2016

