Strange-Meson Spectroscopy – from COMPASS to AMBER

Stefan Wallner
for the COMPASS and AMBER collaborations
(swallner@mpp.mpg.de)

Max Planck Institute for Physics

XVth Quark Confinement and the Hadron Spectrum
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The Strange-Meson Spectrum

Understanding the light-meson spectrum

- Completing SU(3)$_{\text{flavor}}$ multiplets
- Identifying supernumerous states
  - Search for exotic strange mesons

Input to other fields of physics

- Strange mesons appear as resonances in multi-body hadronic final states with kaons
- Searches for CP violation
- Searches for physics beyond SM
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PDG lists 25 strange mesons

- 16 established states, 9 need further confirmation
- Missing states with respect to quark-model predictions
- Many measurements performed more than 30 years ago

(Ebert *et al.*, PRD 79 (2009) 114029)
- Diffractive scattering of high-energy kaon beam
- Strange mesons appear as intermediate resonances $X^-$
- Decay to multi-body hadronic final states
  - $K^-\pi^-\pi^+$ final state
    - Study in principle all strange mesons
    - Study a wide mass range
    - Study different decay modes
- Diffractive scattering of high-energy kaon beam
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Strange-Meson Spectroscopy at COMPASS
COMPASS Setup for Hadron Beams

CEDARs
* beam PID

H$_2$ Target
RPD

Beam
* 190 GeV
* 2.4 % K

30 m

RICH
* final-state PID

Strange-Meson Spectroscopy at COMPASS

The $K^−\pi^−\pi^+$ Data Sample

- World’s largest data set of about 720 k events
- Rich spectrum of overlapping and interfering $X^-$
  - Dominant well known states
  - States with lower intensity are “hidden”
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Partial wave: \( J^P M^\varepsilon \xi b^- L \)

- \( J^P \) spin and parity
- \( M^\varepsilon \) spin projection
- \( \xi \) isobar resonance
- \( b^- \) bachelor particle
- \( L \) orbital angular momentum
Strange-Meson Spectroscopy at COMPASS
Partial-Wave Analysis of $K^-\pi^-\pi^+$ Final State

Partial wave: $J^P M^\varepsilon \xi b^- L$

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Diagram:
- $K^-$
- $K^-$
- $\pi^-$
- $\pi^+$
- $\rho(770)$
- $L$ orbital angular momentum
Strange-Meson Spectroscopy at COMPASS

Partial Waves with $J^P = 2^+$

- Signal in $K^*_2(1430)$ mass region
- In different decays
  - $\rho(770) K D$
  - $K^*(892) \pi D$
- In agreement with previous measurements
- Cleaner signal in COMPASS data
Strange-Meson Spectroscopy at COMPASS
Partial Waves with $J^P = 2^+$

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![Graph showing the decay processes and signal in the $K^*(892)$ mass region with $0.10 \leq t' < 1.00 (GeV/c)^2$. The plot compares intensity in COMPASS data with previous measurements, showing a cleaner signal in COMPASS.
Strange-Meson Spectroscopy at COMPASS

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- In different decays
  - $\rho(770) \, K \, D$
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Strange-Meson Spectroscopy at COMPASS

Searching for Exotic Strange Mesons

- $K(1460)$ and $K(1830)$
- $K(1630)$
- Unexpectedly small width of only 16 MeV/$c^2$
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PDG (2021)
COMPASS $K^-\pi^-\pi^+$ data

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Indications for 3 excited $K$ from a single analysis

Quark-model predicts only two excited states: potentially $K(1460)$ and $K(1830)$

- $K(1630)$ supernumerary state
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Final-state particle identification does not cover full momentum range
- Loss of distinguishing power for some partial waves
- Analysis artifacts in these partial waves

Artifacts can be identified
- Mainly affects only a sub-set of partial waves
- the range $m_{K\pi\pi} \lesssim 1.6 \text{ GeV}/c^2$

Limits access to certain decay modes
- Induces non-negligible systematic uncertainties
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$m_{K\pi\pi}$ [GeV/$c^2$]
Intensity / (1.0 GeV/$c^2$) $\times 10^7$
$0 - 0^+ \bar{K}^*(892)\pi P$

$0.10 \leq t' < 1.00$ (GeV/$c$)$^2$
10.5%
Main limiting factors

- Final-state particle identification
- Size of the data samples
  - Low kaon fraction in the beam ($\approx 2\%$)
  - Sample for strange-mesons about 150-times smaller than sample for non-strange mesons
    - $720 \text{k } K^- + p \rightarrow K^-\pi^-\pi^+ + p$ events
    - $115 \text{M } \pi^- + p \rightarrow \pi^-\pi^-\pi^+ + p$ events
Phase I: After long shutdown 2 of LHC
[CERN-SPSC-2019-022]
- Proton charge-radius measurement
- Drell-Yan and charmonium production
- $p$-induced $\bar{p}$ production cross section

Phase II: After long shutdown 3 of LHC
[arXiv:1808.00848]
- Physics with kaon beams
  - Strange-meson spectroscopy
    - goal: 10$\times$ larger data sample
  - Kaon-induced charmonium production
  - ...
  - ...

Key Requirements for the Experimental Setup

- Upgrade of final-state particle identification
  - Cover wide momentum range
  - Large and uniform acceptance
- Efficient beam-particle identification for high-purity sample
- High-resolution track reconstruction
- Efficient photon detection for access to final states with neutral particles

- Eliminate artifacts caused by limited final-state particle identification
- Increase size of the data sample by increasing acceptance
Increase size of the data sample by increasing kaon fraction in beam

Radio-frequency separation

- Particle species discrimination by time-of-flight
  - Same momentum
  - But different velocity
- Transverse kick by RF cavities
- Kick by RF1 compensated or amplified by RF2, depending on phase (velocity)
- Feasibility studies ongoing
Increase size of the data sample by increasing kaon fraction in beam

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High-Precision Strange-Meson Spectroscopy at AMBER

The virtue of larger data samples

- Improved precision
- Study also small signals in data
- Access to novel analysis methods

Freed-isobar partial-wave analysis

- $K_0^*$ mesons ($J^P = 0^+$) cannot be directly produced in diffractive scattering
- $K_0^*$ mesons appear in $K^-\pi^+$ sub-system of the $K^-\pi^-\pi^+$ final state
- Freed-isobar method allows us to study mesons in sub-systems
  - Developed and successfully applied to COMPASS $\pi^-\pi^-\pi^+$ sample
  - Requires large data samples
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The Strange-Meson Spectrum

- Many strange mesons require further confirmation
- Search for strange partners of exotic non-strange light mesons

COMPASS

- World’s largest data sample on $K^- + p \rightarrow K^-\pi^-\pi^+ + p$
  - Most detailed and comprehensive analysis of the $K^-\pi^-\pi^+$ final state so far
  - Limited by final-state particle identification and small kaon fraction in beam

AMBER: High-Precision Strange-Meson Spectroscopy

- Goal: Collect $10\times$ larger sample using high-intensity and high-energy kaon beam
- Rewrite the PDG for strange mesons, with a single and self-consistent measurement
- Requires experimental setup with uniform acceptance over wide kinematic range including particle identification and measurement of neutral particles
- AMBER is open for interested collaborators to join
Summary

The Strange-Meson Spectrum

- Many strange mesons require further confirmation
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Backup
Outline

6 Kinematic Distribution of $K^−\pi^−\pi^+$ Events
   - Subsystem
   - $m_{K^−\pi^-}$
   - $t'$ Spectrum
   - Exclusivity

7 Partial-Wave Decomposition of $K^−\pi^−\pi^+$
   - Partial Waves with $J^P = 2^+$
   - Partial Waves with $J^P = 0^-$

8 Partial Waves with $J^P = 0^-$

9 Leakage Effect

10 Incoherent Background

11 Freed-Isobar Method
   - Freed-Isobar Method: $0^{-+} 0^{++} [\pi\pi]_{0^{++}} \pi S$

12 Freed-Isobar Analysis
   - Zero Modes and $1^{--}$
Kinematic Distribution of $K^-$π$^-$π$^+$ Events

Subsystem

Also structure in $\pi^-\pi^+$ and $K^-\pi^+$ subsystems
  - Successive 2-body decay via $\pi^-\pi^+$ / $K^-\pi^+$ resonance called isobar

Also structure in angular distributions
Kinematic Distribution of $K^-\pi^-\pi^+$ Events

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Subsystem

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  ➡ Successive 2-body decay via $\pi^-\pi^+$ / $K^-\pi^+$ resonance called isobar

Also structure in angular distributions
No dominant resonant structures
Kinematic Distribution of $K^-\pi^-\pi^+$ Events

$t'$ Spectrum

- Exponential shape
- Shallower for larger $t'$
Kinematic Distribution of $K^-\pi^-\pi^+$ Events

Exclusivity

$E_{\text{beam}}$ [GeV]

Events / (0.1 GeV) $\times 10^4$

$\Delta\phi_{\text{recoil}}$ [deg]

Events / (0.11 deg) $\times 10^4$
Kinematic Distribution of $K^-\pi^-\pi^+$ Events

Exclusivity

- $E_{\text{beam}}$ [GeV]
- $\Delta\phi_{\text{recoil}}$ [deg]
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S. Wallner Strange-Meson Spectroscopy – from COMPASS to AMBER
Partial-Wave Decomposition of $K^-\pi^-\pi^+$

Partial Waves with $J^P = 2^+$

- Signal in $K_2^*(1430)$ mass region
- In Different decays
  - $\rho(770) K D$
  - $K^*(892) \pi D$
- Clear phase motion in $K_2^*(1430)$ region
  - Characteristic of narrow isolated resonances
- In agreement with previous measurement

Graph:
- $2^+\rho^0(770)KD$
- $0.10 \leq t' < 1.00 \text{ (GeV/c)}^2$
- Preliminary

Graph data:
- $m_{K\pi\pi} \text{ [GeV/c}^2\text{]}$
- Intensity / $(1.0 \text{ GeV/c}^2) \times 10^5$

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![Graph with data points and axis labels](image-url)
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Partial Waves with $J^P = 0^-$

$K(1460)$ and $K(1830)$ potentially quark-model states

$K(1630)$ candidate for supernumerary state

- Unexpectedly small width: $16\text{ MeV}/c^2$
- $J^P$ of $K(1630)$ unclear
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$0^- \ 0^+ \ \rho(770) \ K P$ partial wave

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$K^-\pi^-\pi^+$ from ACCMOR

- Potential $K(1630)$ signal already in ACCMOR analysis

$K^-\pi^-\pi^+$ from LHCb

- Measurement of $D^0 \rightarrow K^+\pi^-\pi^\pm\pi^\mp$ at LHCb
- Study strange mesons in $K\pi\pi$ subsystem
- MIPWA of $J^P = 0^-$ amplitude
- Potential signal above 1.6 GeV/$c^2$
- Limited by kinematic range

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Partial Waves with $J^P = 0^-$

\[ 0^{-0^+} \rho^0(770) K P \]

\[ 0.10 \leq t' < 1.00 (\text{GeV}/c)^2 \]

\[ 0.7\% \]

Intensity / $(1.0 \text{ GeV}/c^2)$

$\times 10^5$

$m_{K\pi\pi}$ [GeV/c$^2$]
- Unexpected low-mass enhancement in $3^+ 1^+ K^*(892)\pi D$ wave
- Similar to dominant $1^+$ wave
- Sensitive to systematic effects
- Decay amplitudes of different $J^P$ are orthogonal
- Loss of orthogonality taking acceptance into account
- Limited acceptance due to limited kinematic range of final-state PID
- Only a small sub-set of partial waves affected
Leakage Effect

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![Graph showing leakage effect](image)
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![Graph showing intensity vs. $m_{K\pi\pi}$](image)
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\[ I_{a,b} = \int d\varphi_3(\tau) \Psi_a(\tau)\Psi^*_b(\tau) \]
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\[ \bar{I}_{a,b} = \int d\varphi_3(\tau) \eta(\tau) \Psi_a(\tau) \Psi^*_b(\tau) \]

Accepted phase space

| Preliminary | 0.00 | 0.07 | 0.14 |
Leakage Effect

- Unexpected low-mass enhancement in $3^{+}1^{+} K^*(892) \pi D$ wave
- Similar to dominant $1^{+}$ wave
- Sensitive to systematic effects
- Decay amplitudes of different $J^P$ are orthogonal
- Loss of orthogonality taking acceptance into account
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Incoherent Background

- $K^-\pi^-\pi^+$ and $\pi^-\pi^-\pi^+$ similar experimental footprint
- Distinguishable only by
  - Beam particle identification
  - Final-state particle identification
- Excellent beam PID:
  - Expect small contamination from beam $\pi^-$
- Final-state PID does not suppress $\pi^-\pi^-\pi^+$ background
  - Non-negligible $\pi^-\pi^-\pi^+$ background in $K^-\pi^-\pi^+$ sample of about 7%
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  - Dominant background in $K^-\pi^-\pi^+$ sample
Incoherent Background

- Well established model for $\pi^- + p \rightarrow \pi^- \pi^- \pi^+ + p$
  - From very same data set
  - Measured with high precision
  - Acceptance corrected
- Generate $\pi^- \pi^- \pi^+$ Monte Carlo sample
- Mis-interpret $\pi^- \pi^- \pi^+$ Monte Carlo events as $K^- \pi^- \pi^+$
  - Apply wrong mass assumption
  - Same event reconstruction and selection as for $K^- \pi^- \pi^+$
- Perform partial-wave decomposition of mis-interpreted $\pi^- \pi^- \pi^+$ Monte Carlo sample
  - Using the same PWA model as for measured $K^- \pi^- \pi^+$ sample
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Study $\pi^- \pi^- \pi^+$ background in individual $K^- \pi^- \pi^+$ partial waves
Freed-Isobar Method

\[ [J^{PC} M^\varepsilon] \]

\[ \pi^- \rightarrow X^- \rightarrow \pi^- \pi^- \pi^- + \xi \]

\[ J = J^pC \]

\[ \pi^+ \rightarrow \pi^- \]

\[ P \rightarrow P_{\text{recoil}} \]

Challenge

Need knowledge of isobar amplitude to calculate decay amplitudes \( \Psi_\alpha(\tau) \)

- How good are the parameterizations?
- Single isobar may not be approximated well by a Breit-Wigner amplitude
- Effects of rescattering may distort the isobar shape
Freed-Isobar Method

**Challenge**

Need knowledge of isobar amplitude to calculate decay amplitudes $\Psi_\Delta(\tau)$

- How good are the parameterizations?
  - Single isobar may not be approximated well by a Breit-Wigner amplitude
  - Effects of rescattering may distort the isobar shape
Freed-Isobar Method

Extract isobar amplitudes from data

- Replace model for isobar amplitude with step-like amplitude
- Extract binned shape from data
- Computationally more expensive
  - Up to 100 additional parameters per wave with freed isobar
- Needs large data sets

\[ [\pi\pi]_S \text{ isobar amplitude} \]
Freed-Isobar Method

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\[
[S\pi\pi]_S \text{ isobar amplitude}
\]

\[
m_{\pi^-\pi^+} \text{ [GeV/c}^2]\]

Intensity

0.5 1.0 1.5 2.0

0 1 2 3 4 5 6

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Freed-Isobar Method

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$[\pi\pi]_S$ isobar amplitude

$m_{\pi^-\pi^+}$ [GeV/c$^2$]

Intensity

0 0.5 1.0 1.5 2.0

$0 \leq x < 500$

$500 \leq x < 1000$

$1000 \leq x < 1500$

$1500 \leq x < 2000$

$2000 \leq x < 2500$

$2500 \leq x < 3000$

$3000 \leq x < 3500$

$3500 \leq x < 4000$

$4000 \leq x < 4500$

$4500 \leq x < 5000$

$5000 \leq x < 5500$

$5500 \leq x < 6000$

$6000 \leq x < 6500$

$6500 \leq x < 7000$

$7000 \leq x < 7500$

$7500 \leq x < 8000$

$8000 \leq x < 8500$

$8500 \leq x < 9000$

$9000 \leq x < 9500$

$9500 \leq x < 10000$

$10000 \leq x < 10500$

$10500 \leq x < 11000$

$11000 \leq x < 11500$

$11500 \leq x < 12000$

$12000 \leq x < 12500$

$12500 \leq x < 13000$

$13000 \leq x < 13500$

$13500 \leq x < 14000$

$14000 \leq x < 14500$

$14500 \leq x < 15000$

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$15500 \leq x < 16000$

$16000 \leq x < 16500$

$16500 \leq x < 17000$

$17000 \leq x < 17500$

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$18500 \leq x < 19000$

$19000 \leq x < 19500$

$19500 \leq x < 20000$
Example: $0^+ - 0^+ [\pi \pi \ S\text{-wave}] \pi \ S \text{ wave}$

- Comparison of $0^+ - 0^+ [\pi \pi \ S\text{-wave}] \pi \ S \text{ wave}$ intensity between
  - sum of all conventional isobar waves
  - freed-isobar method

- Compatible shapes
- $\pi(1800)$ peak prominent
Freed-Isobar Method

Example: $0^{-+} 0^{+} [\pi \pi \text{ S-wave}] \pi S$ wave

- Comparison of $0^{-+} 0^{+} [\pi \pi \text{ S-wave}] \pi S$ wave intensity between
  - sum of all conventional isobar waves
  - freed-isobar method
- Compatible shapes
- $\pi(1800)$ peak prominent

![Graph showing comparison of isobar waves](image-url)
This is not a Dalitz-plot
Freed-Isobar Method

Investigate the $\pi\pi$ subsystem with $J^{PC} = 0^{-+}$

- No constrains on $\pi\pi$ resonances
- Extract $\pi\pi$ amplitude (intensity & phase)
  - Extract $\pi\pi$ resonances
- Investigate effects of rescattering

This is not a Dalitz-plot
Freed-Isobar Method

Freed-Isobar Method: $0^{-+} 0^{++} [\pi\pi]_{0^{++}} \pi S$

This is not a Dalitz-plot

[Adolph et al., PRD 95, 032004 (2017)]

S. Wallner
Strange-Meson Spectroscopy – from COMPASS to AMBER
Freed-Isobar Method

Freed-Isobar Method: $0^{-+} 0^{++} [\pi \pi]_{0^{++}} S$

[Adolph et al., PRD 95, 032004 (2017)]

This is not a Dalitz-plot
Freed-Isobar Method

Freed-Isobar Method: $0^{-+} 0^{++} [\pi \pi]_{0^{++}} \pi S$

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Freed-Isobar Method: $0^{-+} 0^{++} [\pi\pi]_{0^{++}} \pi S$

This is not a Dalitz-plot
Freed-Isobar Method

Freed-Isobar Method: \( 0^- + 0^+ [\pi\pi]_{0^+ + \pi} \)

[Adolph et al., PRD 95, 032004 (2017)]
Freed-Isobar Method

Freed-Isobar Method: $0^{-+} 0^{+} [\pi\pi]_{0^{++} \pi S}$

$[\text{Adolph et al., PRD 95, 032004 (2017)}]$
Freed-Isobar Method

Freed-Isobar Method: $0^{-} + 0^{+} \pi \pi_0^+ \pi S$

$0^{-} 0^{+} [\pi \pi]_{\rho^*} \pi S$

$0.194 < \tau' < 0.326 (\text{GeV}/c)^2$

$1.66 < m_{3\pi} < 1.70 \text{ GeV}/c^2$

$0^{-} 0^{+} [\pi \pi]_{\rho'} \pi S$

$0.194 < \tau' < 0.326 (\text{GeV}/c)^2$

$1.78 < m_{3\pi} < 1.82 \text{ GeV}/c^2$

$0^{-} 0^{+} [\pi \pi]_{\omega^*} \pi S$

$0.194 < \tau' < 0.326 (\text{GeV}/c)^2$

$1.90 < m_{3\pi} < 1.94 \text{ GeV}/c^2$

[Adolph et al., PRD 95, 032004 (2017)]
Freed-isobar method
Step-like isobar amplitudes

- Total intensity in one \((m_{3\pi}, t')\)-bin as function of phase-space variables \(\vec{\tau}\):

\[
I(\vec{\tau}) = \left| \sum_i \text{waves} \mathcal{T}_i \left[ \psi_i(\vec{\tau}) \Delta_i(m_{\pi^-\pi^+}) + \text{Bose Symm.} \right] \right|^2
\]

- Fit parameters: Production amplitudes \(\mathcal{T}_i\)
  - Fixed: Angular distributions \(\psi_i(\vec{\tau})\), dynamic isobar amplitudes \(\Delta_i(m_{\pi^-\pi^+})\)
  - Replace fixed isobar amplitudes by piece-wise constant function:

\[
\Delta_i(m_{\pi^-\pi^+}) \to \sum_{\text{bins}} \mathcal{T}_i^{\text{bin}} \Delta_i^{\text{bin}}(m_{\pi^-\pi^+}) \equiv [\pi\pi]_{jPC}
\]

\[
\Delta_i^{\text{bin}}(m_{\pi^-\pi^+}) = \begin{cases} 
1, & \text{if } m_{\pi^-\pi^+} \text{ in the bin.} \\
0, & \text{otherwise.}
\end{cases}
\]

- Each \(m_{\pi^-\pi^+}\) bin behaves like an independent partial wave \(\mathcal{T}_i^{\text{bin}} = \mathcal{T}_i \mathcal{T}_i^{\text{bin}}\):

\[
I(\vec{\tau}) = \left| \sum_i \sum_{\text{bins}} \mathcal{T}_i^{\text{bin}} \left[ \psi_i(\vec{\tau}) \Delta_i^{\text{bin}}(m_{\pi^-\pi^+}) + \text{Bose Symm.} \right] \right|^2
\]

- Approach similar to binning in \(m_{3\pi}\)
- Extend freed-isobar wave set
- Free isobar dynamic amplitudes of 11 biggest waves:
  - Minimize potential leakage

### Freed-isobar wave set

<table>
<thead>
<tr>
<th>Wave Set</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>0−+0+[ππ]0++πS</td>
<td>1++1+[ππ]1−−πS</td>
</tr>
<tr>
<td>0−+0+[ππ]1−−πP</td>
<td>2−+0+[ππ]0++πD</td>
</tr>
<tr>
<td>1++0+[ππ]0++πP</td>
<td>2−+0+[ππ]1−−πP</td>
</tr>
<tr>
<td>1++0+[ππ]1−−πS</td>
<td>2−+0+[ππ]1−−πF</td>
</tr>
<tr>
<td>2−+0+[ππ]2++πS</td>
<td>2−+1+[ππ]1−−πP</td>
</tr>
<tr>
<td>2−+1+[ππ]1−−πP</td>
<td>2++1+[ππ]1−−πD</td>
</tr>
</tbody>
</table>
Extend freed-isobar wave set

Free isobar dynamic amplitudes of 11 biggest waves:
  ▶ Minimize potential leakage

Add spin exotic $1^{-+}1^{+} [\pi \pi]_{1-} \pi P$ wave
  ▶ Wave of major interest

12 freed-isobar waves replace 16 fixed-isobar waves

In addition 72 fixed-isobar waves in the model

40 MeV wide $m_{3\pi}$ bins from 0.5 to 2.5 GeV

4 non-equidistant bins in $t'$

50 bins in $m_{3\pi}$, 4 bins in $t'$: $4 \times 50 = 200$ independent bins
Zero mode in the spin-exotic wave

What is a “zero mode”?

- Freed-isobar analysis: much more freedom than fixed-isobar analysis
  - Causes continuous mathematical ambiguities in the model
- “Zero mode” = dynamic isobar amplitude $\Omega (m_{\pi-\pi^+})$, that does not contribute to the total amplitude
- Spin-exotic wave:

  $$\psi (\vec{\tau}) \Omega (m_{\pi-\pi^+}) + \text{Bose Symm.} = 0$$

  at every point $\vec{\tau}$ in phase space
Zero mode in the spin-exotic wave

Mathematical origin

- Process: \( X^- \rightarrow \xi \pi^- \rightarrow \pi_1^- \pi_2^+ \pi_3^- \).
- Condition for zero mode at all points \( \vec{\tau} \) in phase-space:
  \[
  \psi (\vec{\tau}_{123}) \Omega (m_{12}) + \text{Bose Symm.} = 0 \tag{1}
  \]
- Tensor formalism with pion momenta defined in the \( X^- \) rest frame:
  \[
  \psi (\vec{\tau}_{123}) \propto \vec{p}_1 \times \vec{p}_3
  \]
- Bose symmetrization (\( \pi_1^- \leftrightarrow \pi_3^- \)):
  \[
  \vec{p}_1 \times \vec{p}_3 \Omega (m_{12}) + \vec{p}_3 \times \vec{p}_1 \Omega (m_{23}) = \vec{p}_1 \times \vec{p}_3 [\Omega (m_{12}) - \Omega (m_{23})]
  \]
  ▶ Fulfill eq. (1) at every point in phase space \( \Rightarrow \Omega (m_\xi) = \text{const.} \)
- If \( \Omega (m_\xi) \) is added to the physical dynamic isobar amplitude \( \Delta^{\text{phys}} (m_\xi) \), the total amplitude, and thus the intensity, is not altered:
  \[
  |\psi (\vec{\tau}) \Delta^{\text{phys}} (m_\xi) + \text{B. S.}|^2 = |\psi (\vec{\tau}) [\Delta^{\text{phys}} (m_\xi) + C \Omega (m_\xi)] + \text{B. S.}|^2
  \]
  for any complex-valued zero-mode coefficient \( C \)
- \( C \): complex-valued ambiguity in the model
$\Delta_{BW} \left( m_{\pi^-\pi^+} \right) + C \Omega \left( m_{\pi^-\pi^+} \right) \quad \quad (2)$

$C = 0.00 + 0.00i$

All amplitudes describe the same intensity
\[ \Delta_{BW}(m_{\pi^-\pi^+}) + C\Omega(m_{\pi^-\pi^+}) \]
\[ C = -0.01 + 0.08i \]

All amplitudes describe the same intensity
Zero mode in the spin-exotic wave
Effects on dynamic isobar amplitudes

\[
\Delta_{BW} (m_{\pi^-\pi^+}) + C \Omega (m_{\pi^-\pi^+})
\]

\[
C = -0.05 + 0.15i
\]

All amplitudes describe the same intensity.
Zero mode in the spin-exotic wave
Effects on dynamic isobar amplitudes

\[
\Delta_{BW} (m_{\pi^-\pi^+}) + C \Omega (m_{\pi^-\pi^+})
\]

\[
C = -0.10 + 0.20i
\]
$\Delta_{BW} (m_{\pi^{-}\pi^{+}}) + C\Omega (m_{\pi^{-}\pi^{+}})$

$C = -0.17 + 0.24i$

All amplitudes describe the same intensity
Zero mode in the spin-exotic wave
Effects on dynamic isobar amplitudes

\[ \Delta_{BW} (m_{\pi^-}\pi^+) + C \Omega (m_{\pi^-}\pi^+) \]

\[ C = -0.25 + 0.25i \]

All amplitudes describe the same intensity
Zero mode in the spin-exotic wave
Effects on dynamic isobar amplitudes

\[ \Delta_{BW} (m_{\pi^-\pi^+}) + C\Omega (m_{\pi^-\pi^+}) \]

\[ C = -0.33 + 0.24i \] (2)

All amplitudes describe the same intensity
Zero mode in the spin-exotic wave

Effects on dynamic isobar amplitudes

\[ \Delta_{BW} (m_{\pi^-\pi^+}) + C \Omega (m_{\pi^-\pi^+}) \]  

\[ C = -0.40 + 0.20i \]  

All amplitudes describe the same intensity
Zero mode in the spin-exotic wave
Effects on dynamic isobar amplitudes

\[ \Delta_{BW} (m_{\pi^- \pi^+}) + C \Omega (m_{\pi^- \pi^+}) \]

\[ C = -0.45 + 0.15i \]

All amplitudes describe the same intensity
Zero mode in the spin-exotic wave
Effects on dynamic isobar amplitudes

\[ \Delta_{BW} (m_{\pi^-\pi^+}) + C \Omega (m_{\pi^-\pi^+}) \]

\[ C = -0.49 + 0.08i \]

All amplitudes describe the same intensity
Zero mode in the spin-exotic wave
Effects on dynamic isobar amplitudes

\[ \Delta_{BW} (m_{\pi^-\pi^+}) + C\Omega (m_{\pi^-\pi^+}) \]

\[ C = -0.50 + 0.00i \]
Zero mode in the spin-exotic wave

Effects on dynamic isobar amplitudes

\[ \Delta_{BW} (m_{\pi^-\pi^+}) + C \Omega (m_{\pi^-\pi^+}) \]

\[ C = -0.49 - 0.08i \]

All amplitudes describe the same intensity
Zero mode in the spin-exotic wave
Effects on dynamic isobar amplitudes

\[ \Delta_{BW} (m_{\pi^-\pi^+}) + C\Omega (m_{\pi^-\pi^+}) \]

\[ C = -0.45 - 0.15i \]  

All amplitudes describe the same intensity
Zero mode in the spin-exotic wave
Effects on dynamic isobar amplitudes

\[ \Delta_{BW} (m_{\pi^-\pi^+}) + C \Omega (m_{\pi^-\pi^+}) \]

\[ C = -0.40 - 0.20i \]

All amplitudes describe the same intensity
Zero mode in the spin-exotic wave
Effects on dynamic isobar amplitudes

\[ \Delta_{\text{BW}} (m_{\pi^- \pi^+}) + C \Omega (m_{\pi^- \pi^+}) \]

\[ C = -0.33 - 0.24i \]  \hspace{1cm} (2)

All amplitudes describe the same intensity
Zero mode in the spin-exotic wave
Effects on dynamic isobar amplitudes

\[ \Delta_{BW} (m_{\pi^-\pi^+}) + C \Omega (m_{\pi^-\pi^+}) \]

\[ C = -0.25 - 0.25i \]

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Zero mode in the spin-exotic wave
Effects on dynamic isobar amplitudes

\[ \Delta_{BW} (m_{\pi^-\pi^+}) + C \Omega (m_{\pi^-\pi^+}) \] 
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Zero mode in the spin-exotic wave
Effects on dynamic isobar amplitudes

\[ \Delta_{BW} (m_{\pi^-\pi^+}) + C \Omega (m_{\pi^-\pi^+}) \]  
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Effects on dynamic isobar amplitudes

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All amplitudes describe the same intensity
Zero mode in the spin-exotic wave
Effects on dynamic isobar amplitudes

\[ \Delta_{BW} (m_{\pi^-\pi^+}) + C \Omega (m_{\pi^-\pi^+}) \]

where

\[ C = -0.01 - 0.08i \]

All amplitudes describe the same intensity.
\[ \Delta_{BW} (m_{\pi^-\pi^+}) + C \Omega (m_{\pi^-\pi^+}) \]
\[ C = 0.00 + 0.00i \]

All amplitudes describe the same intensity
Zero mode in the spin-exotic wave

Resolving the ambiguity

Now for $m_{\pi^-\pi^+}$ bins: $\vec{T}^0 = \{ \Omega(m_{\text{bin}}) \}$ for all $m_{\pi^-\pi^+}$ bins

The fitting algorithm might find a solution, shifted away from the physical solution $\vec{T}^{\text{phys}}$:

$$\vec{T}^{\text{phys}} = \vec{T}^{\text{fit}} + C \vec{T}^0$$

Obtain physical solution: constrain $C$ by conditions on the resulting dynamic amplitudes $\vec{T}^{\text{fit}}$

In the case of the $1^{-+}1^{++}[\pi\pi]_{1--\pi} P$ wave:

- use the Breit-Wigner for the $\rho (770)$ resonance with fixed resonance parameters as in the fixed-isobar analysis
- use a Breit-Wigner for the $\rho (770)$ resonance with floating resonance parameters

Final results: weighted average of these two methods

**Note:** Resolving the ambiguity fixes only a single complex-valued degree of freedom. $n_{\text{bins}} - 1$ complex-valued degrees of freedom remain free.
The spin-exotic wave

- Example: Single \((m_{3\pi}, t')\) bin
  - \(1.58 < m_{3\pi} < 1.62 \text{ GeV}/c^2\)
  - \(0.326 < t' < 1.000 \text{ (GeV}/c)^2\)

- Zero-mode ambiguity resolved with \(\rho(770)\) used as constraint
Freed-Isobar Analysis

$J^{PC} = 1^{--}$ Wave with freed $j^{pc} = 1^{--}$ Isobar Amplitude

\[ m_{3\pi} \text{ [GeV/c}^2\text{]} \]

$J^{PC} = 1^{++}$

Preliminary

$0.326 < t' < 1.000 \text{ (GeV/c)}^2$
Freed-Isobar Analysis

\( J^{PC} = 1^{--} \) Wave with freed \( J^{PC} = 1^{--} \) Isobar Amplitude

\[
J^{PC} = 1^{--} \quad \text{Wave with freed } J^{PC} = 1^{--} \quad \text{Isobar Amplitude}
\]
Freed-Isobar Analysis

\[ j^{PC} = 1^{--} \text{ Wave with freed } j^{PC} = 1^{--} \text{ Isobar Amplitude} \]

- Study \( \pi^- \pi^+ \) amplitude as a function of \( m_{3\pi} \)
- \( m_{\pi^- \pi^+} \) spectrum shows good agreement with \( \rho(770) \) Breit-Wigner
- Extract \( m_{\pi^- \pi^+} \) dependence of complex-valued amplitude
- Shape of \( m_{3\pi} \) spectrum is in fair agreement with fixed-isobar analysis
  - \( \pi_1(1600) \) signal at about 1.6 GeV/c\(^2\) robust
Freed-Isobar Analysis

$J^{PC} = 1^{--}$ Wave with freed $J^{PC} = 1^{--}$ Isobar Amplitude

- Study $\pi^- \pi^+$ amplitude as a function of $m_{3\pi}$
- $m_{\pi^- \pi^+}$ spectrum shows good agreement with $\rho(770)$ Breit-Wigner
- Extract $m_{\pi^- \pi^+}$ dependence of complex-valued amplitude
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\( j^{PC} = 1^{--} \) Wave with freed \( j^{PC} = 1^{--} \) Isobar Amplitude

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\( m_{\pi^-\pi^+} \) spectrum shows good agreement with \( \rho(770) \) Breit-Wigner

Extract \( m_{\pi^-\pi^+} \) dependence of complex-valued amplitude

Shape of \( m_{3\pi} \) spectrum is in fair agreement with fixed-isobar analysis

\( \pi_1(1600) \) signal at about 1.6 GeV/c\(^2\) robust