

Spin-orbit correlations in Monte Carlo simulations

Focus on recent work on the polarized quark fragmentation process

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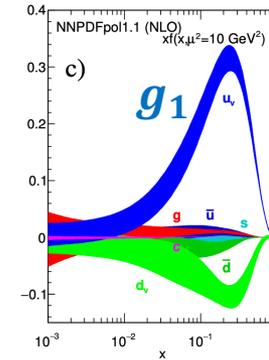
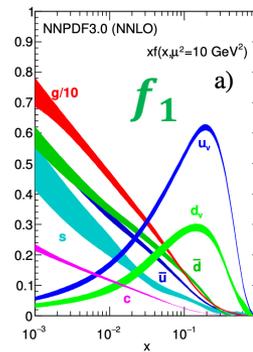
in collaboration with X. Artru and A. Martin

nucleon structure at leading twist

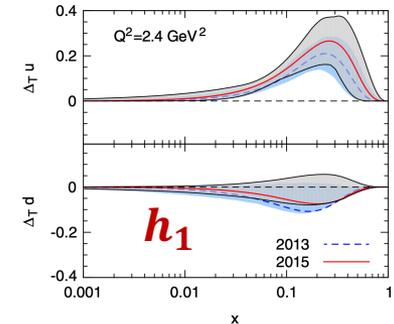
The collinear nucleon structure at leading twist is described by unpolarised f_1 , helicity g_1 , transversity h_1

	nucleon		
quark	U	L	T
U	$f_1(x, k_T^2)$ (unpolarized)		$f_{1T}(x, k_T^2)$ (Sivers)
L		$g_1(x, k_T^2)$ (helicity)	$g_{1T}(x, k_T^2)$ (worm-gear)
T	$h_{1T}(x, k_T^2)$ (Boer-Mulders)	$h_{1L}(x, k_T^2)$ (worm-gear)	$h_1(x, k_T^2)$ (transversity) $h_{1T}(x, k_T^2)$ (pretzelosity)

from PDG (2019)



Anselmino et al., PRD 92, 114023 (2015)



transversity is the less known

nucleon structure at leading twist

The collinear nucleon structure at leading twist is described by unpolarised f_1 , helicity g_1 , transversity h_1

	nucleon		
quark	U	L	T
U	$f_1(x, k_T^2)$ (unpolarized)		$f_{1T}(x, k_T^2)$ (Sivers)
L		$g_1(x, k_T^2)$ (helicity)	$g_{1T}(x, k_T^2)$ (worm-gear)
T	$h_1^\perp(x, k_T^2)$ (Boer-Mulders)	$h_{1L}^\perp(x, k_T^2)$ (worm-gear)	$h_1(x, k_T^2)$ (transversity) $h_{1T}^\perp(x, k_T^2)$ (pretzelosity)

+ 5 other TMD PDFs when the intrinsic k_T is taken into account
 → correlations between the nucleon spin and the spin and k_T of partons
 ~ basically unknown, but Sivers

In SIDIS some TMDs are coupled to the **unpolarized FF** D_1

$$A_{UU}^{\cos\phi_h} \sim f_1 \otimes D_1 \quad \sim \text{Cahn effect}$$

$$A_{UT}^{\sin(\phi_h - \phi_S)} \sim f_{1T} \otimes D_1 \quad \text{Sivers asymmetry}$$

$$A_{LT}^{\cos(\phi_h - \phi_S)} \sim g_{1T} \otimes D_1 \quad \text{Kotzinian-Mulders}$$

others coupled to the **Collins FF** H_1^\perp

$$A_{UT}^{\sin(\phi_h + \phi_S - \pi)} \sim h_1 \otimes H_1^\perp \quad \text{Collins asymmetry}$$

$$A_{UU}^{\cos 2\phi_h} \sim h_1^\perp \otimes H_1^\perp \quad \sim \text{Boer-Mulders}$$

$$A_{UL}^{\sin(2\phi_h)} \sim h_{1L}^\perp \otimes H_1^\perp \quad \text{Kotzinian-Mulders}$$

$$A_{UT}^{\sin(3\phi_h - \phi_S)} \sim h_{1T}^\perp \otimes H_1^\perp \quad \text{pretzelosity}$$

what about MC simulations of these effects?

MC implementation of spin-orbit effects

Monte Carlo event generators (MCEGs) are important tools
event generation in the full phase space, access to correlations,
multi-dimensional studies, phenomenology

A systematic implementation of all leading order effects related to the intrinsic transverse momentum of quarks and to their spin degree of freedom in complete MCEGs is still missing

Different effects considered separately

- TMD PDFs

k_T dependence in $f_1 \rightarrow$ «primordial k_T » in PYTHIA? (but PDFs in PYTHIA do not depend on k_T)

Cahn and Sivers effects implemented in LEPTO *Kotzinian '05*

- TMD FFs

unpolarized quarks, $D_1 \rightarrow$ Lund String Model, Cluster Model .. (is relation with theory OK?)

polarized quarks \rightarrow (T) Collins effect H_1^\perp , (L) jet handedness

recursive string+3P0 model \rightarrow this talk

stand alone MC

AK et al., PRD 100 (2019) 1, 014003, PRD 97 (2018) 7, 074010

inclusion in PYTHIA 8.2

AK and L. Lönnblad, PoS DIS2019 (2019) 179

extended NJL-jet model

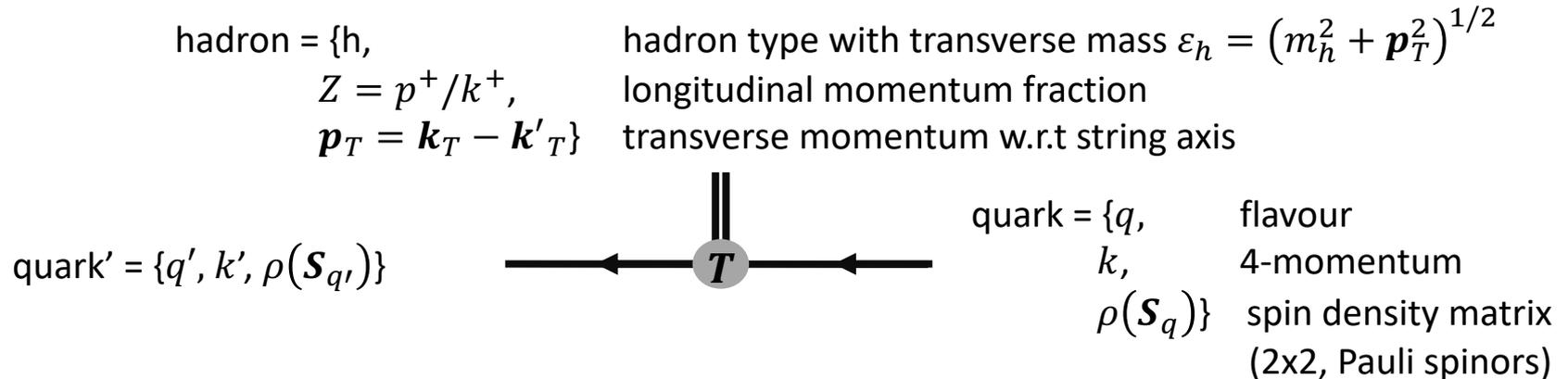
stand alone MC

Matevosyan et al., PRD 95 (2017) 1, 014021

elementary splitting: emission of a PS

string decay = recursive repetition of the elementary splitting $q \rightarrow h + q'$

X. Artru, Z. Belghobsi DSPIN-2011, 2013



Splitting in flavour \otimes momentum \otimes spin space

→ Transition Amplitude $T_{q',h,q}(Z, \mathbf{p}_T | \mathbf{k}_T)$

Splitting Probability

→ Splitting Function $F_{q'hq}(Z, \mathbf{p}_T | \mathbf{k}_T, \mathcal{S}_q) = \text{tr} T_{q',h,q} \rho(\mathcal{S}_q) T_{q',h,q}^\dagger$

Spin transfer to q'

→ spin density matrix of q' $\rho(\mathcal{S}_{q'}) \propto T_{q',h,q} \rho(\mathcal{S}_q) T_{q',h,q}^\dagger$

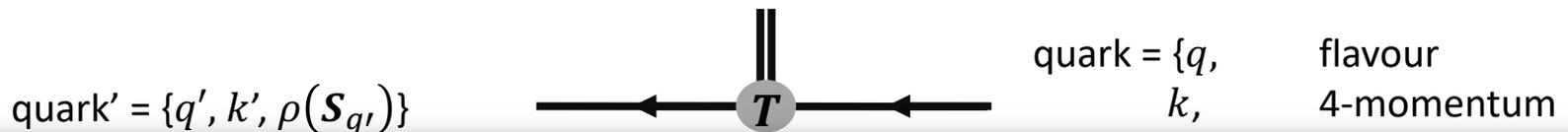
the very basic ingredients for the MC simulations

elementary splitting: emission of a PS

string decay = recursive repetition of the elementary splitting $q \rightarrow h + q'$

X. Artru, Z. Belghobsi DSPIN-2011, 2013

hadron = {h, $Z = p^+ / k^+$, $\mathbf{p}_T = \mathbf{k}_T - \mathbf{k}'_T$ } hadron type with transverse mass $\varepsilon_h = (m_h^2 + \mathbf{p}_T^2)^{1/2}$
 longitudinal momentum fraction
 transverse momentum w.r.t string axis



- Expression for the Transition Amplitude

$$T_{q',h,q} = C_{q',h,q} \times \left(\frac{1-Z}{\varepsilon_h^2} \right)^{a/2} e^{-\frac{b_L \varepsilon_h^2}{2Z}} \times e^{-\frac{b_T \mathbf{k}'_T{}^2}{2}} \times \check{g}(\varepsilon_h^2) \times [\mu + \sigma_z \sigma \cdot \mathbf{k}'_T] \times \sigma_z \times \hat{u}^{-1/2}(\mathbf{k}_T)$$

$$T_{q',h,q} = \text{Lund String Fragmentation Model} \times {}^3P_0 \text{ operator} \times \text{PS coupling} \times \dots$$

- Few free parameters

$a, b_L, b_T \rightarrow$ string fragmentation dynamics (Lund Model, e.g. PYTHIA, LEPTO)

μ **complex mass** from 3P_0 mechanism \rightarrow responsible for spin effects ($\text{Im}(\mu) \rightarrow$ transverse)

- Input function $\check{g} \rightarrow$ governs spin-independent $\mathbf{k}_T - \mathbf{k}'_T$ correlations

correlations \rightarrow Model **M18**

PRD 97 (2018) 7, 074010

NO correlations \rightarrow Model **M19** (much simpler)

PRD100 (2019) no.1, 014003

Stand alone simulations (M19)

M18 and M19 have been implemented in stand alone MC programs
give very similar results, in spite for M19 being much simpler

Next slide, simulated Collins analysing power $a^{u\uparrow\rightarrow h+X}$
with initial conditions

u quarks fully transversely polarized along \hat{y}

Energy calculated from a $\{x_B, Q^2\}$ sample of SIDIS events

no primordial KT

Values of the free parameters

$$a = 0.9$$

$$b_L = 0.5 (\text{GeV}/c^2)^{-2}$$

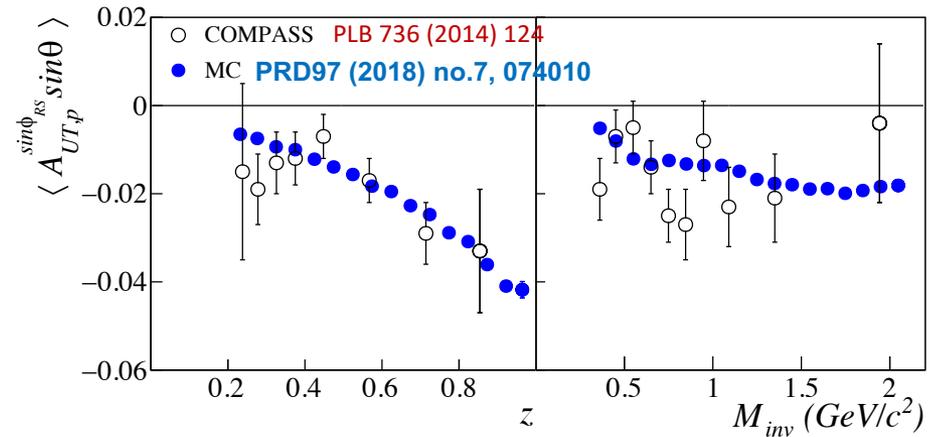
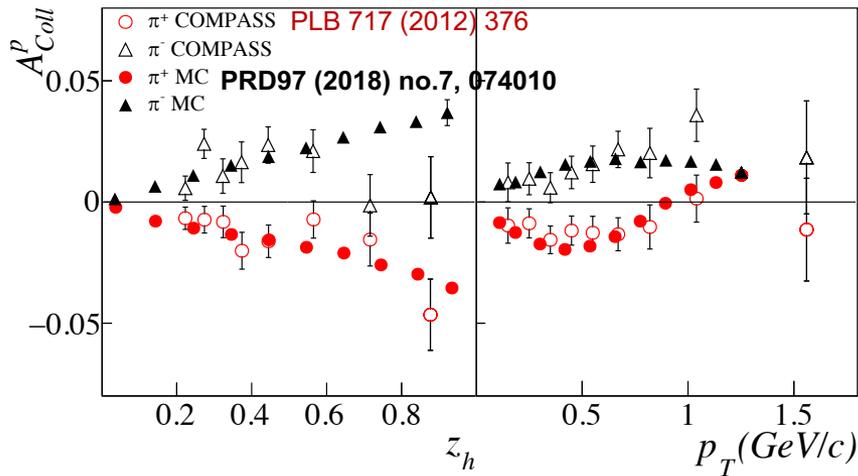
$$b_T = 5.17 (\text{GeV}/c)^{-2}$$

$$\mu = (0.42 + i0.76) \text{ GeV}/c^2$$

see AK, X. Artru, Z. Belghobsi, F. Bradamante, A. Martin PRD 100 (2019) 1, 014003

Azimuthal spectrum of hadrons
 $dN_h \propto 1 + a^{u\uparrow\rightarrow h+X} \sin(\phi_h - \phi_{S_u})$
Collins analysing power
 $a^{u\uparrow\rightarrow h+X} = 2 \langle \sin(\phi_h - \phi_{S_u}) \rangle$

Stand alone simulations: comparison with SIDIS data (M19)



Collins asymmetry on protons $A_{Coll}^p \simeq \frac{h_1^u}{f_1^u} a^{u \uparrow \rightarrow \pi+X}$

MC \rightarrow Collins analysing power $a^{u \uparrow \rightarrow \pi+X}$

scaled by $\lambda \sim \langle h_1^u / f_1^u \rangle = 0.055 \pm 0.010$

only u quarks fully transversely polarized

λ is estimated by comparison with A_{Coll}^p for π^-

Di-hadron asymmetry for h^+h^- pairs

MC \rightarrow same scale factor λ

same mechanism as for Collins

the model describes the main properties of data

Improving the model

The model gives already a good description of the main properties of data
few parameters, same mechanism for Collins and dihadron asymmetries,
jet handedness (not shown here)

We have improved it further following two directions ..

a) Exploit the true predictive power of the model via a more complete simulation of the event

- interface M19 with PYTHIA 8.2 for SIDIS
- introduction of transversity PDF

in collaboration with L. Lönnblad

the first step towards a systematic implementation of spin effects in PYTHIA!

b) Improve the description of the polarized fragmentation process

- extend M19 by introducing vector mesons → **the NEW model M20**

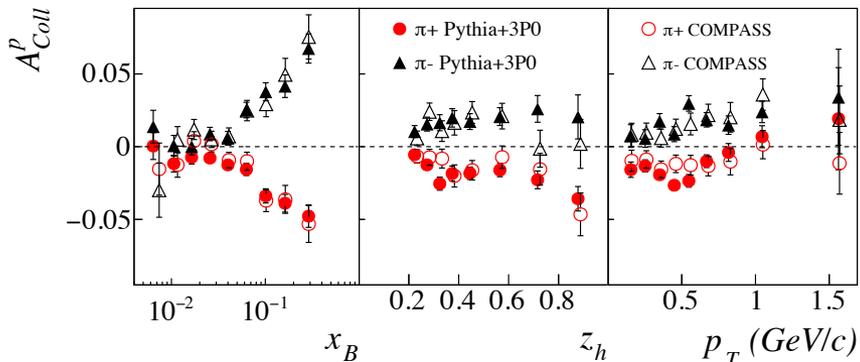
PYTHIA+3P0

- M19 is interfaced with PYTHIA 8.2 for the simulation of polarized SIDIS
- spin effects introduced for the first time in the hadronization part of a complete MCEG
- parameterizations for u^v and d^v transversity PDFs implemented
- **PYTHIA+3P0 allows to simulate the Collins and dihadron asymmetries**

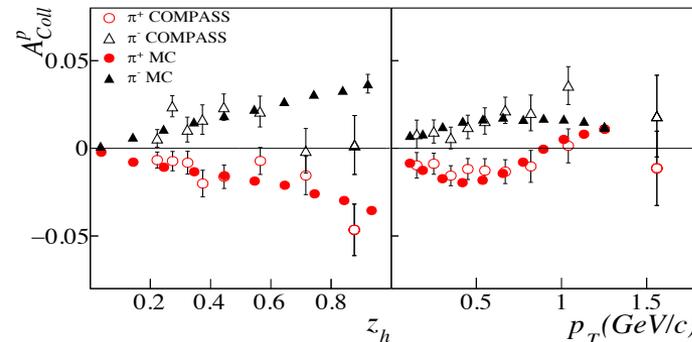
NEW tool!

Simulation of SIDIS off protons @ COMPASS kinematics
ISR/FSR switched OFF, no intrinsic k_{\perp}
complex mass retuned to $\mu = (0.78 + i0.38) \text{ GeV}/c^2$

Collins asymmetry PYTHIA+3P0



Collins asymmetry M19



COMPASS PLB 717 (2012) 376
MC PRD97 (2018) no.7, 074010

Nice description of data!

trend vs z_h is modified in PYTHIA
also good description of di-hadron asymmetries

PYTHIA+3P0 will be available soon!!
AK and L. Lönnblad, in preparation

Improving the model

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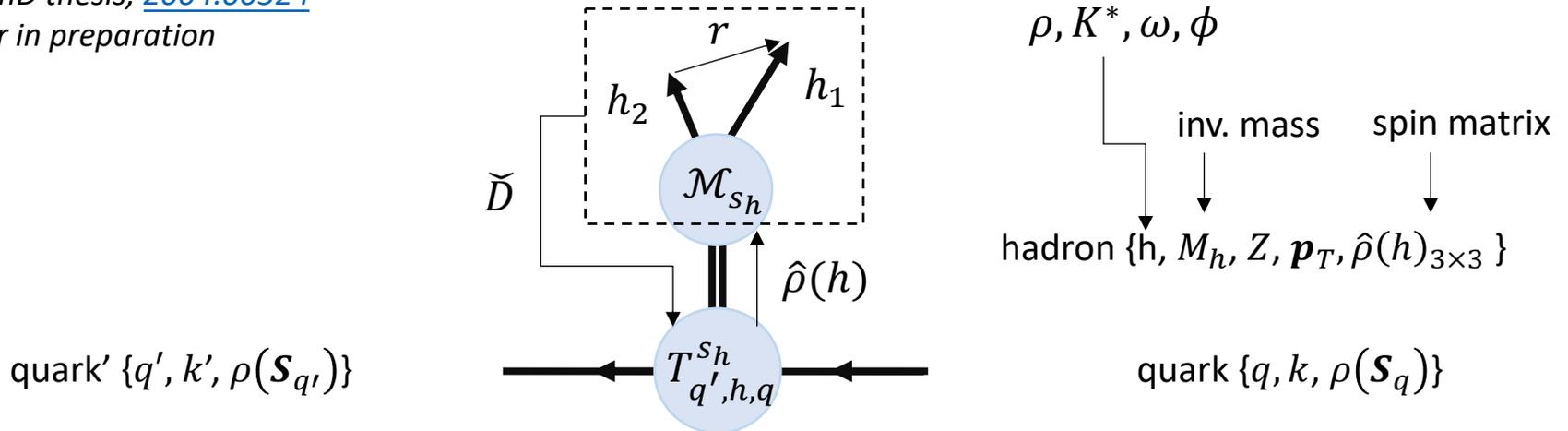
b) Improve the description of the polarized fragmentation process

→ extend M19 by introducing vector mesons → **the NEW model M20**

spin effects with PS and VM production are treated systematically, for the first time

elementary splitting: emission of a VM

AK, PhD thesis, [2004.00524](#)
paper in preparation



NEW Transition Amplitude

Splitting Probability

→ **NEW Splitting Function**

Spin transfer to h

→ spin density matrix of h

Decay (e.g. $VM \rightarrow h_1 h_2$)

→ angular distribution

Bring decay information back to q'

→ decay matrix

Transfer spin information to q'

→ spin density matrix of q'

$$T_{q',h,q}^{sh}(M, Z, \mathbf{p}_T, s_h | \mathbf{k}_T)$$

$$F_{q'hq}(M, Z, \mathbf{p}_T | \mathbf{k}_T, \mathbf{S}_q) = \text{tr} T_{q',h,q}^{sh} \rho(\mathbf{S}_q) T_{q',h,q}^{sh \dagger}$$

$$\hat{\rho}_{s_h s_{h'}}(h) \propto \text{tr} T_{q',h,q}^{sh} \rho(\mathbf{S}_q) T_{q',h,q}^{sh \dagger}$$

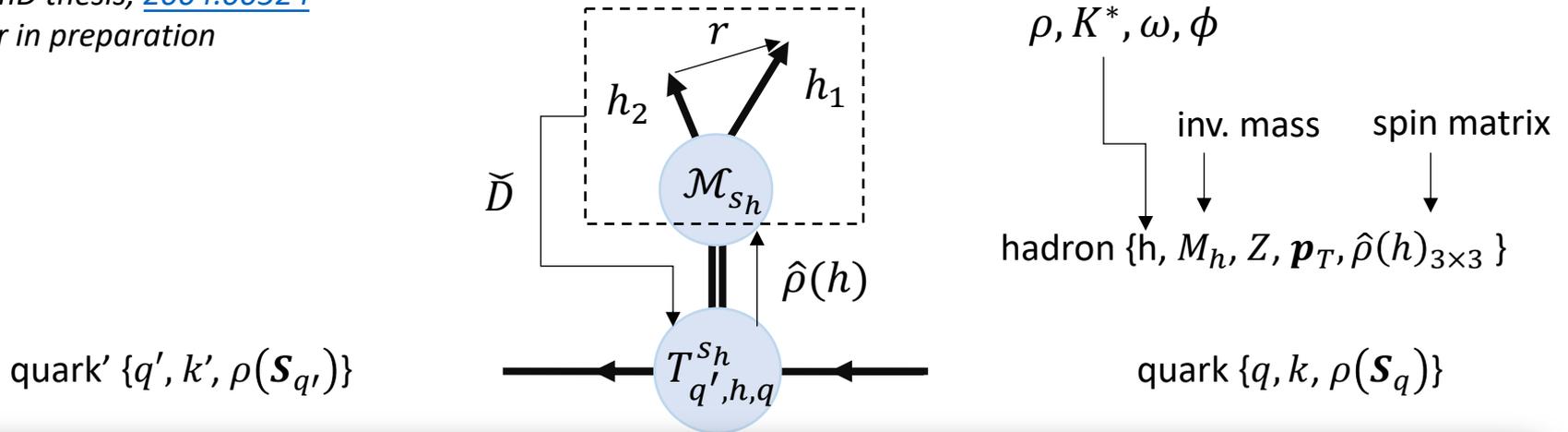
$$dN/d\Omega \propto \mathcal{M}_{s_h} \hat{\rho}_{s_h s_{h'}} \mathcal{M}_{s_{h'}}^\dagger$$

$$\check{D}_{s_h' s_h} = \mathcal{M}_{s_h'}^\dagger \mathcal{M}_{s_h}$$

$$\rho(\mathbf{S}_{q'}) = \check{D}_{s_h' s_h} T_{q',h,q}^{sh'} \rho(\mathbf{S}_q) T_{q',h,q}^{sh \dagger}$$

elementary splitting: emission of a VM

AK, PhD thesis, [2004.00524](#)
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Complicated recipe!

respects entanglement $q' \leftrightarrow$ momenta of decay hadrons (Collins '88, Knowles '88)

Form of the NEW splitting amplitude

$T_{q',h,q}^{Sh}$ = relativistic BW \times Lund String Fragmentation Model \times 3P_0 operator \times VM coupling \times ..

VM coupling \rightarrow complex free parameters

$G_L \rightarrow$ coupling of q to VM with linear L polarisation along the string axis

$G_T \rightarrow$ coupling of q to VM with linear T polarisation w.r.t the string axis

Only two new parameters in the model

$|G_L|/|G_T| \rightarrow$ global Collins effect of the VM (depends on VM polarisation!)

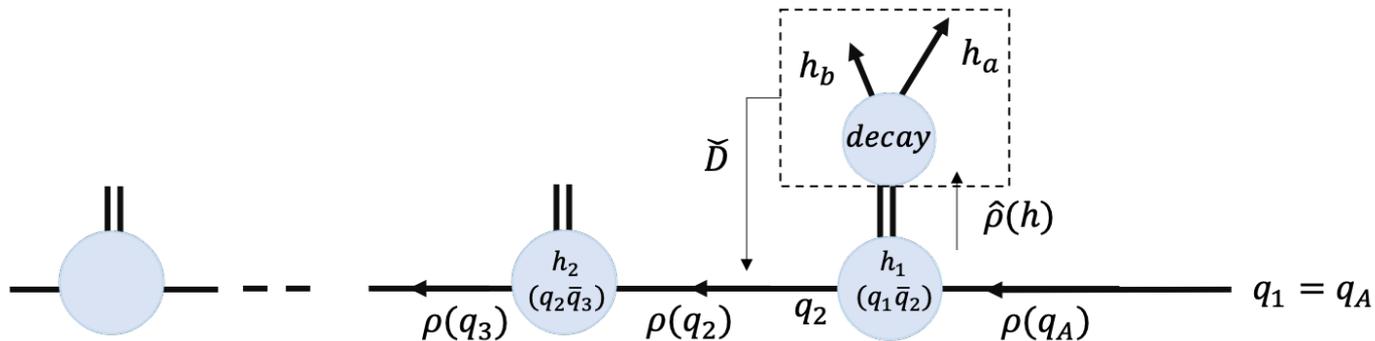
$\theta_{LT} = \arg(G_L/G_T) \rightarrow$ oblique polarisation (LT)

Stand alone MC implementation of M20

For each event define initial quark $q_A \equiv q_1$, i.e. flavour (u, d, s), momentum, density matrix $\rho(q_A)$

1. Generate a $q_2 \bar{q}_2$ pair and form the hadron $h_1(q_A \bar{q}_2)$, VM with prob. $\frac{f_{VM}}{f_{VM} + f_{PS}}$
2. Generate M_{h_1} (if VM), \mathbf{k}_{2T} , Z_1 , using $F_{q_2 h_1 q_A} \rightarrow$ construct p_1
3. If $h_1 = PS$ go to 4.
If $h_1 = VM \rightarrow$ calculate $\hat{\rho}(h)$
 - a) generate decay hadrons in VM rest frame and boost to string frame
 - b) construct the decay matrix \check{D}
4. Calculate the spin density matrix of q_2

- Iterate points 1-4 until the exit condition (enough remaining mass to produce at least one baryonic resonance)



Stand alone simulations with M20

Values of the free parameters used in simulations
all mesons

$$a = 0.9$$

$$b_L = 0.5 (\text{GeV}/c^2)^{-2}$$

$$b_T = 8.43 (\text{GeV}/c)^{-2}$$

$$\mu = (0.42 + i0.76) \text{ GeV}/c^2$$

as in model M19

VM production

$$f_{VM}/f_{PS} = 0.62 (0.725) \text{ for } u, d (s)$$

as in PYTHIA 8

$$\begin{aligned} |G_L|/|G_T| &= 1 \\ \theta_{LT} &= 0 \end{aligned}$$



following the NR quark model

Czyzewski '96

sensitivity to parameters values also explored

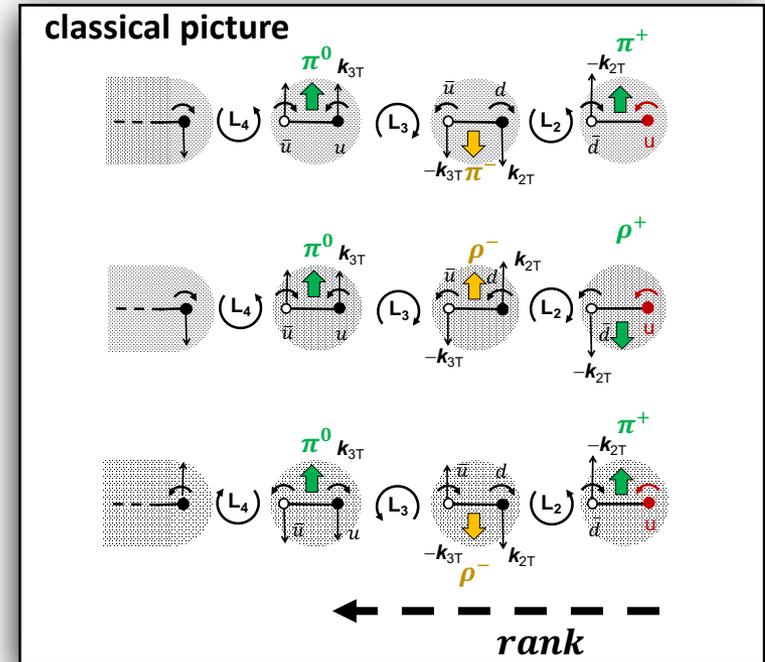
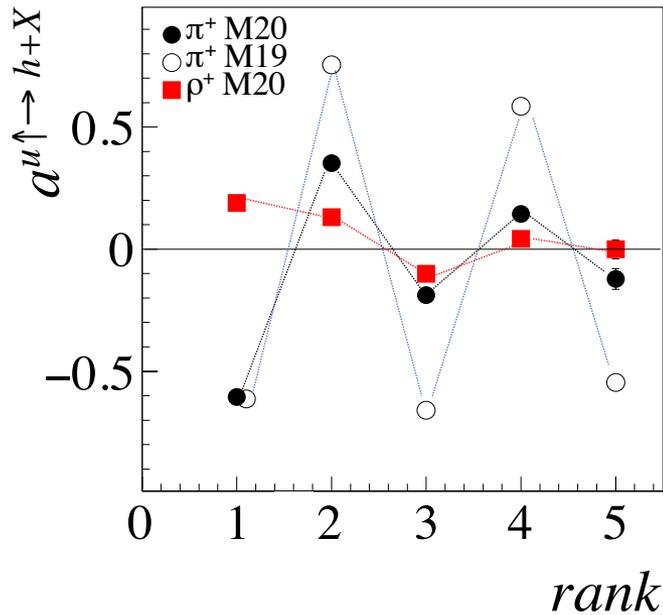
Initial conditions

u quarks fully transversely polarized along \hat{y}

Energy calculated from a $\{x_B, Q^2\}$ sample of SIDIS events

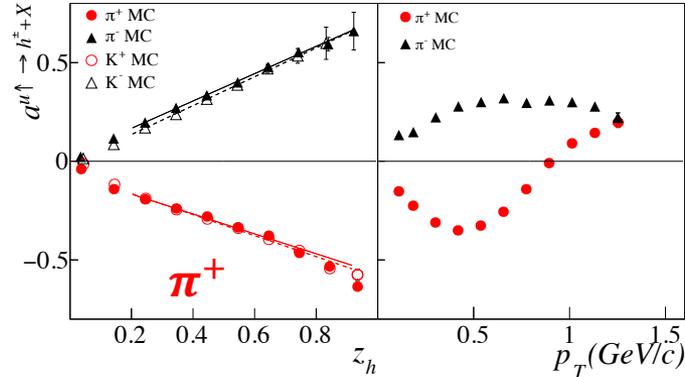
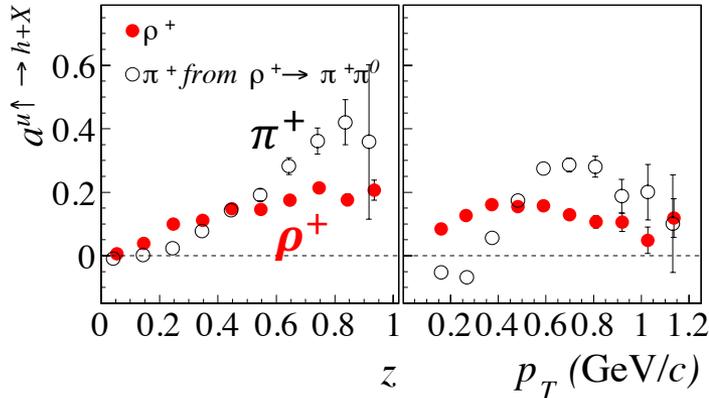
no primordial KT

Collins analysing power as function of rank



- classical picture reproduced
- ρ^+ have opposite effect w.r.t π^+
- quark spin information decays along the chain
- faster decay in M20

Collins analysing power for ρ and decay π



AK et al., PRD 100 (2019) 1, 014003,

model M19 \rightarrow only PS

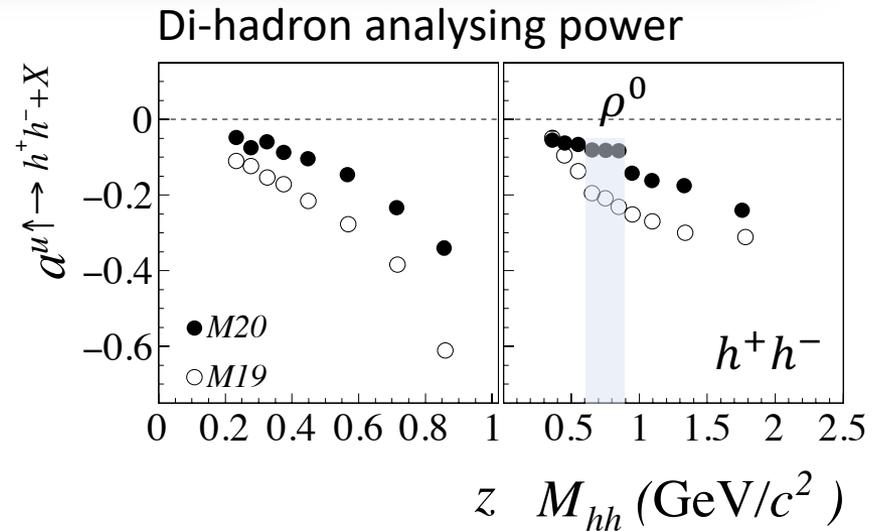
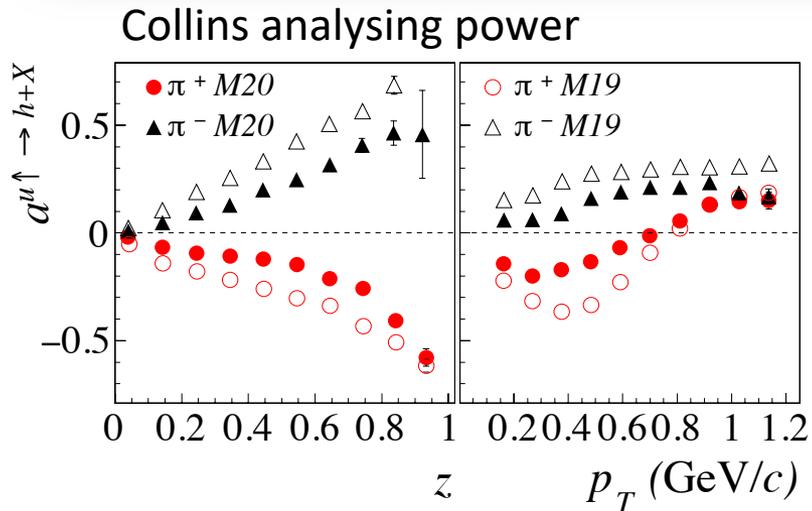
ρ^+ analysing power
opposite to π^+
 ~ 3 times smaller than π^+
as expected from the M20 prediction

$$\frac{a^{u\uparrow\rightarrow\rho+X}}{a^{u\uparrow\rightarrow\pi+X}} \Big|_{rank=1} = - \frac{|G_L|^2}{2|G_T|^2 + |G_L|^2}$$

$\rho^-, \rho^0 \sim$ to ρ^+ for $|G_L|/|G_T| = 1$

decay $\pi^+ \rightarrow$ larger analysing power at large z_h and large p_T w.r.t ρ^+
large $z_h \rightarrow \pi^+$ emitted along \hat{z} from longitudinally polarized ρ^+
large $p_T \rightarrow \pi^+$ emitted along \vec{p}_T^ρ from rank 1 ρ^+
emitted along $\hat{z} \times \vec{p}_T^\rho$ from rank 2 ρ^+
small $p_T \rightarrow \pi^+$ emitted along $-\vec{p}_T^\rho$ from rank 1 ρ^+

Effect of VM decays on transverse spin asymmetries



- Large effect on Collins analysing power w.r.t M19
- different trends
 - average analysing power diluted by 50%

di-hadron analysing power calculated using the relative transverse momentum

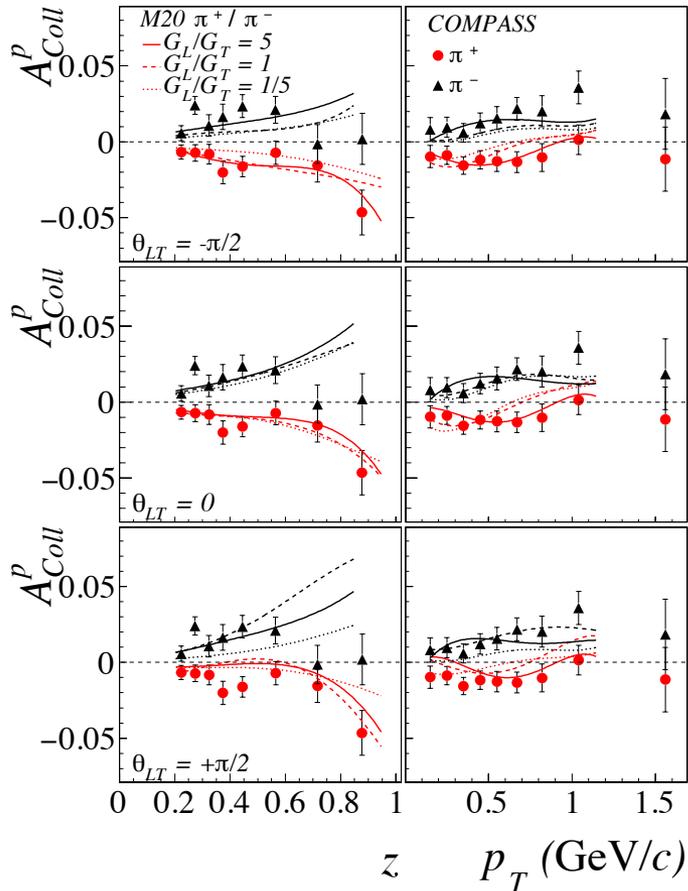
$$\mathbf{R}_T = z_2 \mathbf{p}_{1T}/z - z_1 \mathbf{p}_{2T}/z$$

$$a_1^{u\uparrow \rightarrow h^+h^-+X} = 2 \langle \sin(\phi_R - \phi_{SA}) \rangle$$

50% dilution w.r.t M19

- effect at ρ^0 peak due to $\rho^0 \rightarrow \pi^+\pi^-$ symmetric w.r.t $\mathbf{R}_T \leftrightarrow -\mathbf{R}_T$

Comparison with SIDIS data



MC scaled by a factor λ depending on $|G_L|/|G_T|$ and θ_{LT}

Large variations for π^+ due to different values of $|G_L|/|G_T|$ and θ_{LT}

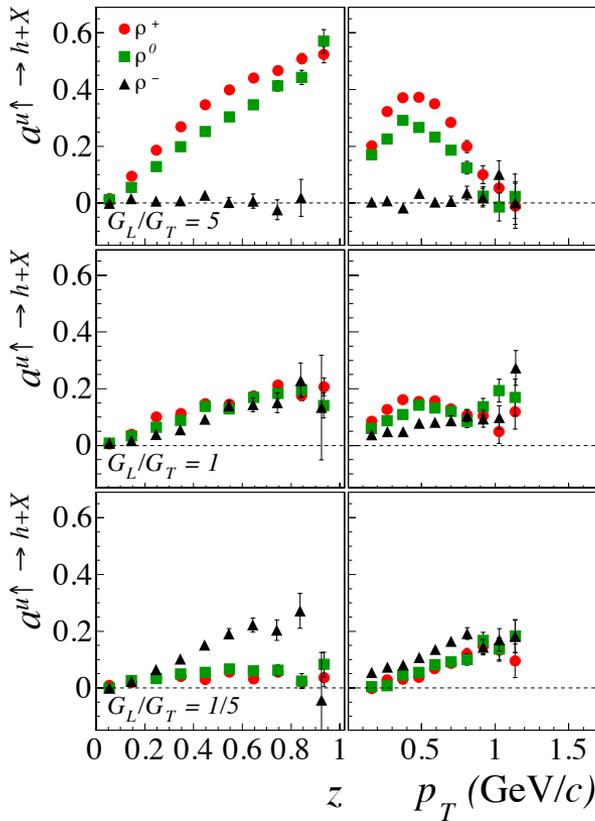
somewhat smaller for π^-

→ both parameters are important

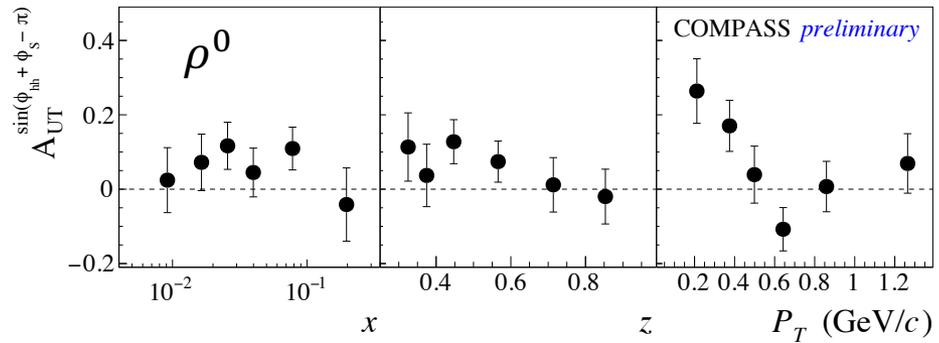
hint for $\frac{|G_L|}{|G_T|} > 1, \theta_{LT} < 0$?

more precise data would help to fix the free parameters

Sensitivity to free parameters: Collins effect for ρ mesons



from AK talk at DIS-2021



COMPASS results: NEW! Measurements feasible

they could be used to fix the parameters

Hint for $\frac{|G_L|}{|G_T|} > 1$ (in particular from p_T)

Strong dependence on $|G_L|/|G_T|$
both size and shapes change

Conclusions

The string+3P0 model with PS meson emission (M18, M19) implemented in a stand alone MC
→ describes the main features of Collins and di-hadron asymmetries!

M19 has been interfaced to PYTHIA 8

→ parameterisations for the transversity PDF implemented

→ more complete description of TSA

the code will available very soon

(AK and L. Lönnblad, in preparation)

For the first time implementation of the string+3P0 model with PS and VM production (M20) in a stand alone MC
(paper in preparation)

→ detailed study of Collins effect for VM

→ only 4 free parameters for spin effects, to be fixed from comparison with data

μ OK, hints for $|G_L|/|G_T| > 1$ and $\theta_{LT} < 0$

more precise data would help (COMPASS 2021-2022 d run, JLab12 ..)

ongoing work, promising results ...