Meson Charge Radii
at AMBER

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CERN / zoom

RF-separated beams for Amber- Kick Off Meeting
How small is small?

Depicting microcosm, we amplify the scales and simplify a lot – it’s all quantum fields!

There is a rigorous way to define the extension of particles via form factors

Specific interest in hadronic systems as their extension is a consequence of quantum chromodynamics

Experimental data for many baryons and mesons still scarce and of limited precision
Hadron radius measurements

From: EPJC 8 (1999) 59, The WA89 Collaboration (measurement of $\Sigma^-$ charge radius)

Measured $\langle r_{ch}^2 \rangle$ in $fm^2$ of various hadrons

<table>
<thead>
<tr>
<th></th>
<th>Experiment</th>
<th>Soliton</th>
<th>Skyrme</th>
<th>non-relat. quark</th>
<th>Skyrme</th>
<th>Cloudy Bag</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>0.74 ± 0.02 [1]</td>
<td>0.78</td>
<td>1.20</td>
<td>0.67</td>
<td>0.775</td>
<td>0.714</td>
</tr>
<tr>
<td></td>
<td>0.67 ± 0.02 [2]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.79 ± 0.03 [3]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>-0.11 ± 0.03 [4]</td>
<td>-0.09</td>
<td>-0.15</td>
<td>-0.308</td>
<td>-0.121</td>
<td></td>
</tr>
<tr>
<td>$\Sigma^-$</td>
<td>0.91 ± 0.32 ±0.4</td>
<td>0.75</td>
<td>1.21</td>
<td>0.55</td>
<td>0.751</td>
<td>0.582</td>
</tr>
</tbody>
</table>

comparatively good accuracies (pion radius ~1%) stem from assuming a theoretical shape of the form factor
Hadron charge radii through elastic lepton scattering at low $Q^2$

Protons in hydrogen target (or other stable nuclei):
Measurement via elastic electron or muon scattering
Cross section:

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} R \left( e G_E^2 + \tau G_M^2 \right)$$

Charge radius from the slope of $G_E$

$$\langle r_E^2 \rangle = -6\hbar^2 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2 \to 0}$$

For unstable particles, electron scattering can be realised in *inverse kinematics*
S.R. Amendolia et al. / Pion electromagnetic form factor

\[ |F_R|^2 \]

\[ q^2 \text{ (GeV/c)}^2 \]

Fig. 17. The square of the pion form factor, \( |F_R|^2 \) versus \( q^2 \), with statistical error bars only. The line corresponds to the pole fit with \( \langle r^2 \rangle = 0.34 \text{ fm}^2 \).

\~380,000 pion-electron scattering events

\~400,000 kaon triggers

(\~30,000 kaon-electron scatterings?)


Fig. 3. The measured kaon form factor squared. The line corresponds to the pole fit with \( \langle r^2 \rangle = 0.34 \text{ fm}^2 \).
Summary

- **Meson radii** are of **key interest** in understanding their inner structure and the emergence of hadron mass

- For **pions**, some deeper investigations would be needed to see whether and how the data of previous experiments can be challenged

- For **kaons**, a significant increase of the form factor knowledge in the range $0.001 < Q^2 < 0.07$ appears in reach with an **80 GeV rf-separated kaon beam**
Kinematics

\[ K^- e^{-}_{\text{target}} \rightarrow K^- e^- \]

\[ s = 2E_b m_e + m_b^2 + m_e^2 \]

\[ Q_{\text{max}}^2 = \frac{4p_b^2 m_e^2}{s} \]

<table>
<thead>
<tr>
<th>Beam</th>
<th>( E_b ) [GeV]</th>
<th>( Q_{\text{max}}^2 ) [GeV²]</th>
<th>( E_{b,\text{min}}' ) [GeV]</th>
<th>Relative charge-radius effect on c.s. at ( Q_{\text{max}}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi )</td>
<td>190</td>
<td>0.176</td>
<td>17.3</td>
<td>~40%</td>
</tr>
<tr>
<td>( K )</td>
<td>190</td>
<td>0.086</td>
<td>105.7</td>
<td>~20%</td>
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<tr>
<td>80</td>
<td>0.066</td>
<td>59.9</td>
<td>~15%</td>
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</tr>
<tr>
<td>50</td>
<td>0.037</td>
<td>41.3</td>
<td>~8%</td>
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</table>
Principle of the measurement
Extrapolation from COMPASS analysis: Count rate estimate for AMBER

By-product of COMPASS 2009 “Primakoff” analysis (constrained by cuts)

- Electrons identified in ECAL2, trigger on $E_e > 40$ GeV

Plot prepared by Dominik Steffen (TUM)

- 190 GeV pion beam
- 9 days of beam time
- 1,500 identified elastic pion-electron scattering events in cut range $50 < E'_{\pi} < 140$
- i.e. $0.05 < Q^2 < 0.14$
- naïve estimate: for reproducing the NA7 result, roughly a factor 30 larger data sample would be needed

yet a similar measurement with separated kaon beam may become competitive
Backup
Determination of the rms radius from a form factor measurement

- the rms radius of a charge distribution seen in lepton scattering is defined as the slope of the electric form factor at vanishing momentum transfer $Q^2$

\[ \langle r_E^2 \rangle = -6\hbar^2 \frac{dG_E(Q^2)}{dQ^2} \bigg|_{Q^2 \to 0} \]

- elastic scattering experiments provide data for $G_E$ at non-vanishing $Q^2$ and thus require an extrapolation procedure towards zero → mathematical ansatz may take more or less bounds into account (physics/theory/whatever motivated)

- Any approach (Padé, CF, DI, CM,...) must boil down to a series expansion

\[ G_E(Q^2) = 1 + c_2 Q^2 + c_4 Q^4 + ... \]

introducing possibly very different assumptions on the coefficients $c_i$

- recipe for experimenters: measure a sufficiently large range of $Q^2$ down to values as small as possible and as precise as possible
Radiative corrections for electron and muon scattering

QED radiative corrections

- for soft bremsstrahlung photon energies ($E_\gamma/E_{beam} \sim 0.01$), QED radiative corrections amount to $\sim 15$-20% for electrons, and to $\sim 1.5\%$ for muons
- important contribution to the uncertainty of elastic scattering intensities: *change* of this correction over the kinematic range of interest
- check: impact of exponentiation procedure (strictly valid only for vanishing photon energies): $e^-$: $2 - 4\%$, $\mu^-$: $0.1\%$
- integrating the radiative tail out to large fraction of beam energy: shifts the correction to smaller values, but only *increases* the uncertainty