Chiral Perturbation Theory: Recent Experimental Results and Perspectives

Norbert Kaiser
Stephan Paul
TU Munich

COVID Video contribution
Measuring $\chi$PT sensitive processes

- $\pi^0$ lifetime
- Chiral anomaly
  - $\pi^-\gamma^* \rightarrow \pi^-\pi^0$
- Polarizability
  - $\pi\gamma^* \rightarrow \pi\gamma$
  - $K\gamma^* \rightarrow K\gamma$
- $\pi\pi$ scattering
  - $K$ decays
  - $\pi^-\gamma^* \rightarrow \pi^-\pi^+\pi^-$, $\pi^-\pi^0\pi^0$
  - $\pi^-\pi^* \rightarrow \pi^-\pi$
- $K\pi$ scattering
  - $K^-\gamma^* \rightarrow K^-\pi^+\pi^-$, $K^-\pi^0\pi^0$
  - $K^-\pi^* \rightarrow K^-\pi$
- $\eta \rightarrow \pi\pi$, $\eta \rightarrow \pi\pi\pi$ (not covered here)
- Chiral symmetry restauration – parity doubling of mesons
Some highlights of ChPT

- **Pions** $\pi^{\pm 0}$: Goldstone bosons of spontaneous chiral symmetry breaking in QCD, $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$
- Their low-energy dynamics: systematically (and accurately) calculable in Chiral Perturbation Theory ($= \text{loop-expansion with effective Lagrangian}$)
- Leading-order pion-pion scattering amplitude in ChPT: involves as scale parameter the pion decay constant $f_\pi = 92.2 \text{ MeV}$

$$A(s, t, u)_{\text{LO}} = \frac{s - m_\pi^2}{f_\pi^2}$$

- 2-loop prediction for $I = 0$ $\pi\pi$-scattering length: $a_0 m_\pi = 0.220 \pm 0.005$
  confirmed by E865@BNL: $K^+ \rightarrow \pi^+\pi^- e^+\nu_e$ ($\pi^+\pi^- \text{ mass distribution}$)
- Cusp effect in $2\pi^0$ mass spectrum of $K^+ \rightarrow \pi^+\pi^0\pi^0$ at $\pi^+\pi^- \text{ threshold}$: $(a_0 - a_2)m_\pi = 0.257 \pm 0.006$ ($0.265 \pm 0.005)_{\text{ChPT}}$
- Conclusion: quark condensate $\langle 0|\bar{q}q|0 \rangle$ is large, linear term dominates quark mass expansion of $m_\pi^2$: $m_\pi^2 f_\pi^2 = -\langle 0|\bar{q}q|0 \rangle m_q + \mathcal{O}(m_q^2 \ln m_q)$
Beyond perturbation theory

- Moderately attractive isospin-0 S-wave $\pi\pi$-interaction can produce upon iteration to all orders (or unitarization) a complex resonance-pole

$$T_0^0(s) = A_0^0(s) + A_0^0(s)G(s)A_0^0(s) + A_0^0(s)G(s)A_0^0(s)G(s)A_0^0(s) + \cdots = \frac{A_0^0(s)}{1 - G(s)A_0^0(s)} ,$$

$$A_0^0(s) = \frac{2s - m_\pi^2}{f_\pi^2} , \quad G(s) = \frac{1}{16\pi^2} \left\{ \frac{1}{2} - \sqrt{1 - \frac{4m_\pi^2}{s}} \ln \frac{\sqrt{s} + \sqrt{4m_\pi^2 - s}}{2m_\pi} - \ln \frac{m_\pi}{\lambda} \right\}$$

- Inverse amplitude method in chiral limit $m_\pi = 0$ gives

$$T_0^0 = \frac{2s}{f_\pi^2} \left[ 1 - i \frac{s}{16\pi f_\pi^2} \right]^{-1} \text{ has complex pole at } \sqrt{s} = (1 - i)462 \text{ MeV}$$

- Scrutinized by dispersion relation analysis and Roy-Steiner equations
  [I. Caprini, G. Colangelo, J. Gasser, H. Leutwyler, B. Moussallam]

  $\sigma$-pole at $\sqrt{s} = m_\sigma - i\Gamma_\sigma/2 = [(450 \pm 20) - i(280 \pm 10)] \text{ MeV}$

- More to physics of $\pi\pi$ S-wave than occurrence of pole far from real axis

  - Further dynamically generated poles from mesonic final-state interactions:
    - $\kappa(700)$ in $\pi K$ S-wave with $I = 1/2$
    - $f_0(980)$ in $\pi\pi$-$K\bar{K}$ coupled channels
    - $a_0(980)$ in $\pi\eta$-$K\bar{K}$ coupled channels
    - $\Lambda(1405)$ in $\pi\Sigma$-$K\bar{N}$ coupled channels
Primakoff effect

**COMPASS**: Measurement of low-energy pion-photon reactions

- Scattering high-energy pions in nuclear Coulomb field (charge $Z$) allows to extract cross sections for $\pi^-\gamma$ reactions (equivalent-photon method)

\[
\frac{d\sigma}{ds\,dQ^2} = \frac{Z^2\alpha}{\pi(s - m^2_\pi)} \frac{Q^2 - Q^2_{\text{min}}}{Q^4} \sigma_{\pi^-\gamma}(s), \quad Q_{\text{min}} = \frac{s - m^2_\pi}{2E_{\text{beam}}}
\]

- $s = (\pi^-\gamma$ invariant mass$)^2$, $Q \to 0$ momentum transfer by virtual photon
- Isolate Coulomb peak from strong interaction background
- Different final-states $\pi^-\gamma$, $\pi^-\pi^0$, $\pi^-\pi^0\pi^0$, $\pi^+\pi^-\pi^-$ allow to test different aspects of chiral dynamics (low-energy QCD)
- Polarizabilities, chiral anomaly, $\pi\pi$-scattering in el-magn. environment

S. Paul, N. Kaiser (TUM)  
Chiral Perturbation Theory for $\pi\gamma$ reactions
Pion electromagnetic polarizabilities

- Two-loop prediction of ChPT: $\bar{\ell}_6 - \bar{\ell}_5 = 3.0 \pm 0.3$ from radiative pion decay $\pi^+ \rightarrow e^+ \nu_e \gamma$, PIBETA@PSI: axial-to-vector ratio $F_A/F_V = 0.44$

$$\alpha_\pi - \beta_\pi = \frac{\alpha(\bar{\ell}_6 - \bar{\ell}_5)}{24\pi^2 f_\pi^2 m_\pi} + \frac{\alpha m_\pi}{(4\pi f_\pi)^4} \left\{ c' + \frac{8}{3} \left( \frac{1}{2} - 1 + \frac{1}{5} - 6 \right) \ln \frac{m_\pi}{m_\rho} + \frac{4}{9} \left( \frac{1}{3} - 2 \right) - \frac{3}{8} \left( \frac{53\pi^2}{48} - \frac{41}{324} \right) \right\}$$

$$\alpha_\pi - \beta_\pi = (5.7 \pm 1.0) \cdot 10^{-4} \text{ fm}^3, \quad \alpha_\pi + \beta_\pi = 0.16 \cdot 10^{-4} \text{ fm}^3$$

- COMPASS result: $\alpha_\pi - \beta_\pi = (4.0 \pm 1.8) \cdot 10^{-4} \text{ fm}^3$ [PRL 114, 062002 (’15)]

$$x_\gamma = E_\gamma/E_\pi \text{ in lab, } \cos \theta_{cm} = 1 - 2x_\gamma s/(s - m_\pi^2)$$

Analysis of data includes:

- chiral pion-loop corrections $A(s, t) \sim \ln^2(t)$.
- radiative corrections [NPA 812, 186 (’08)]
- isospin-breaking correction $\sim (m_\pi^2 - m_\pi^0) \ln^2(t)$.
- previous results from Mainz and Serpukhov: $\alpha_\pi - \beta_\pi = (12 - 16) \cdot 10^{-4} \text{ fm}^3$
Extracting the chiral anomaly

- $\pi^0 \rightarrow 2\gamma$ and $\gamma \rightarrow 3\pi$ couplings determined by chiral anomaly of QCD
- Amplitude and cross section for $\pi^- (p_1) + \gamma (k, \epsilon) \rightarrow \pi^- (p_2) + \pi^0 (p_0)$:

$$T_{\gamma 3\pi} = \frac{e}{4\pi^2 f_\pi^3} \epsilon_{\mu\nu\kappa\lambda} \epsilon^{\mu} p_1^\nu p_2^\kappa p_0^\lambda M(s, t), \quad F_{3\pi} = 9.8 \text{ GeV}^{-3}$$

$$\sigma_{\text{tot}}(s) = \frac{\alpha (s - m_{\pi}^2) (s - 4m_{\pi}^2)^{3/2}}{(4f_{\pi})^6 \pi^4 \sqrt{s}} \int_{-1}^{1} dz \left(1 - z^2\right) |M(s, t)|^2$$

- $\rho(770)$-resonance must be included:

$$M(s, t) = 1 + 0.46 \left\{ \frac{s}{m_{\rho}^2 - s - i\sqrt{s} \Gamma_{\rho}(s)} + \frac{t}{m_{\rho}^2 - t} + \frac{u}{m_{\rho}^2 - u} \right\}$$
Dispersive representation of $\pi \gamma \rightarrow \pi \pi$ with p-wave phase shifts as input

\[ \frac{e}{4\pi^2 f^3_\pi} M(s, t) = F(s) + F(t) + F(u), \quad u = 3m^2_\pi - s - t, \]

\[ F(s) = a + b s + \frac{s^2}{\pi} \int ds' \frac{\text{Im} F(s')}{s'^2(s' - s)}, \quad \text{Im} F(s) = [F(s) + \hat{F}(s)] \sin \delta^1_1(s) e^{-i\delta^1_1(s)} \]

Relevant subtraction constant $C = 3(a + b m^2_\pi)$ is fitted to data and matched via the chiral representation to $F_{3\pi}$

\[ C = \left\{ 1 + \frac{m^2_\pi}{(4\pi f_\pi)^2} \left( 2.9 - \ln \frac{m_\pi}{m_\rho} \right) \right\} F_{3\pi} = 1.067 F_{3\pi} \]

- solid line: $C = 9.78$ GeV$^{-3}$
- dashed line: $C = 12.9$ GeV$^{-3}$
- close to threshold, one-photon exchange an important correction: $1 \rightarrow 1 - 2e^2 f^2_\pi / t$
- Good theory waiting for good data
- Extension to $K^- \gamma \rightarrow \pi K$ [B. Kubis et al.]
Tree level cross sections for $\pi^- \gamma \rightarrow 3\pi$

- Coulomb gauge $\epsilon \cdot p_1 = \epsilon \cdot k = 0$, photon does not couple to incoming $\pi^-$
- No $\gamma 4\pi$ vertex at leading order

**Example:**

Total cross section for $\pi^- (p_1) + \gamma (k, \epsilon) \rightarrow \pi^- \pi^0 \pi^0$

\[
\sigma_{\text{tot}}(s) = \frac{\alpha}{16\pi^2(s - m_{\pi}^2)^3} \int_{2m_{\pi}} \sqrt{s} - m_{\pi} d\mu \sqrt{\mu^2 - 4m_{\pi}^2} \left[ \frac{\mu^2 - m_{\pi}^2}{f_{\pi}^2} \right]^2 
\times \left\{ (s + m_{\pi}^2 - \mu^2) \ln \frac{s + m_{\pi}^2 - \mu^2 + \lambda^{1/2}(s, \mu^2, m_{\pi}^2)}{2m_{\pi}\sqrt{s}} - \lambda^{1/2}(s, \mu^2, m_{\pi}^2) \right\}
\]

- $(\mu^2 - m_{\pi}^2)/f_{\pi}^2$ is LO chiral $\pi\pi$-interaction, rest from 3-body phase space
- How large are next-to-leading order corrections from chiral loops + cts?
Charged pion-pair production

- Total cross section for $\pi^- \gamma \rightarrow \pi^+ \pi^- \pi^-$

\[
\sigma_{\text{tot}}(s) \text{ for } \sqrt{s} < 6m_\pi \text{ almost unchanged in comparison to tree approx.}
\]

- Suggestive explanation: $\pi^- \pi^- \rightarrow \pi^- \pi^- \text{ final state interaction } (1 - 0.02)^2$

\[
a_2 = -\frac{m_\pi}{16\pi f_\pi^2} \left[ 1 - \frac{m_\pi^2}{12\pi^2 f_\pi^2} \left( \bar{l}_1 + 2\bar{l}_2 - \frac{3\bar{l}_3}{8} - \frac{3\bar{l}_4}{2} + \frac{3}{8} \right) \right]
\]

- Analysis of COMPASS data for $\sqrt{s} \leq 5m_\pi \text{ agrees with ChPT prediction}$
  
  First measurement of chiral dynamics in $\pi^- \gamma \rightarrow \pi^- \pi^- \pi^+$, PRL108, 192001 ('12)

- Agreement on level of full 5-dimensional phase space distribution

- For $\pi^- \gamma \rightarrow \pi^- \pi^0 \pi^0$ chiral corrections are substantially larger $\sim (1+0.2)^2$
Coupling of Mesons to Photons
\[ \pi^- \gamma^* \rightarrow \pi^- \pi^0 \]

\[ \pi^0 \rightarrow \gamma \gamma \text{ and } \gamma \rightarrow 3\pi \ (\pi^- \gamma \rightarrow \pi^- \pi^0) \] determined by chiral anomaly of QCD
\[ \pi^- \gamma^* \rightarrow \pi^- \pi^0 \]

\( \pi^0 \rightarrow \gamma \gamma \) and \( \gamma \rightarrow 3\pi \) (\( \pi^- \gamma \rightarrow \pi^- \pi^0 \)) determined by chiral anomaly of QCD

\[ T_{\gamma 3\pi} \propto \frac{e}{4\pi^2 f^2_{\pi}} = F^3_{3\pi} \]

Antipov et al in Serpukhov:

\[ F^3_{3\pi} = 12.9 \pm 0.9 \pm 0.5 \text{ GeV}^{-3} \]

- strong final state interaction:

\[ \pi^- \pi^0 \rightarrow \rho \rightarrow \pi^- \pi^0 \]

Tree:

Loop:

\[ \begin{array}{c}
\gamma \\
\pi
\end{array} \quad \begin{array}{c}
\gamma \\
\pi
\end{array} \quad \begin{array}{c}
\gamma \\
\pi
\end{array} \quad \begin{array}{c}
\gamma \\
\pi
\end{array} \quad \begin{array}{c}
\gamma \\
\pi
\end{array} \]

\[ \begin{array}{c}
\gamma \\
\pi
\end{array} \quad \begin{array}{c}
\gamma \\
\pi
\end{array} \quad \begin{array}{c}
\gamma \\
\pi
\end{array} \quad \begin{array}{c}
\gamma \\
\pi
\end{array} \quad \begin{array}{c}
\gamma \\
\pi
\end{array} \]

\[ \begin{array}{c}
\gamma \\
\pi
\end{array} \quad \begin{array}{c}
\gamma \\
\pi
\end{array} \quad \begin{array}{c}
\gamma \\
\pi
\end{array} \quad \begin{array}{c}
\gamma \\
\pi
\end{array} \quad \begin{array}{c}
\gamma \\
\pi
\end{array} \]

\[ \begin{array}{c}
\gamma \\
\pi
\end{array} \quad \begin{array}{c}
\gamma \\
\pi
\end{array} \quad \begin{array}{c}
\gamma \\
\pi
\end{array} \quad \begin{array}{c}
\gamma \\
\pi
\end{array} \quad \begin{array}{c}
\gamma \\
\pi
\end{array} \]
• Use (quasireal) photons as target
• Scattering off Coulomb field in heavy nucleus → Primakoff cross-section: $\alpha Z^2 \Rightarrow$ large $Z$
• Very high $Z$ (e.g. Pb): large corrections from 2$\gamma$ processes and from screening, conversions
  
  Optimum choice: medium heavy Ni target

Artistic depiction of $\pi^- + \text{Ni} \rightarrow \pi^- + \gamma + \text{Ni}$ via Primakoff process
• 190 GeV negative hadron beam: $96.8\% \pi^-\), $2.4\% K^-$, $0.8\% p^-$
• beam particle identification by Cherenkov detectors
• 4 mm Ni target disk ($\approx 25\% X_0$)
• Measure scattered $\pi^-$ and produced photons (number depends on final state)
• Select exclusive events at lowest momentum transfers
• Small scattering angles require high resolution
  • spatial resolution of tracking $\approx 10$ $\mu$m
  • angular resolution of ECAL $\approx 30$ $\mu$rad
Intermediate COMPASS Results

Theory: use $\chi$PT and dispersion theory to accommodate for FSI

- allows to address higher masses
- extract $F_{3\pi}$ and radiative width of $\rho$ meson (only 2 free parameters)
- possibly access radiative width of excited $\rho$ mesons
- Experimental issues: subtraction of diffractive production, luminosity

Cross section [arbitrary units]

$p^+ p \rightarrow p^+ p$

$\pi^- \gamma^* \rightarrow \pi^- \pi^0$

Very Preliminary

$K^- \rightarrow \pi\pi$

Remaining diffractive processes
Experimental Challenges

- **Measure absolute cross sections**
  - Target thickness (geometry)
  - Beam flux including PID
    - Known ratio of $\pi/K$ in beam (previous measurements at CERN)
    - Count kaons
    - $K^- \rightarrow \pi^- \pi^0$ (20.7%), $K^- \rightarrow \pi^- \pi^+ \pi^-$ (5.6%)
    - Consider backgrounds for these decays
      - $K^- \rightarrow \pi^0 e^- \bar{\nu}_e$ (5%) identification of electrons not unambiguous owing to Bremsstrahlung before detection
      - $K^- \rightarrow \pi^0 \mu^- \bar{\nu}_\mu$ (3.4%)
  - **In principle**: all decays can be used — X-check of normalization
  - **In fact**: requires excellent understanding of efficiencies and cross talk

- **Understand underlying strong interaction background**
Reward: Enlarged Physics Program

- Radiative width of excited mesons
  - $\rho(770)$, $\rho_3(1670)$

- Perform angular analysis for quantum numbers and yield (radiative width)
\( \pi^0 \) lifetime

- Use Primakoff reaction to produce \( \pi^0 \)
- Coherent photoproduction
- Measure \( \gamma \gamma \) width through differential production cross section

Theory:  \[ \Gamma(\pi^0 \to \gamma \gamma) = \frac{m_{\pi^0}^3 N_c^2 \alpha^2}{576 \pi^3 F_{\pi^0}^2} = 7.750 \pm 0.016 \text{ eV} \]
PrimEx @ JLAB

- Use real photons as beam

\[ \gamma \gamma^* \rightarrow \pi^0 \rightarrow \gamma \gamma \]
\[ \Gamma(\pi^0 \rightarrow \gamma\gamma) = 7.798 \pm 0.056(\text{stat}) \pm 0.109(\text{sys}) \text{ eV} \]

- World average: \[ \Gamma(\pi^0 \rightarrow \gamma\gamma) = 7.802 \pm 0.052(\text{stat}) \pm 0.105(\text{sys}) \text{ eV} \]
χPT Physics with Kaons

- Polarizability
  - $K\gamma^* \rightarrow K\gamma$
- $K\pi$ scattering
  - $K^-\gamma^* \rightarrow K^-\pi^+\pi^-, \ K^-\pi^0\pi^0$
  - $K^-\pi^* \rightarrow K^-\pi$

Goals:
1. Determine $K\pi$ scattering length
2. Access $K\pi$ resonances and their radiative width
3. Chiral anomaly for kaons - SU(3) symmetry breaking (dispersive analysis prepared)
4. Vector formfactor for kaon $K^- \rightarrow \ell\nu\gamma$ (analogon to $\pi^0 \rightarrow \gamma\gamma$)
5. Corrections larger than for $\pi\pi$ („large“ strange quark mass)
Special case: $\kappa - K_0(700)$

$K\pi$ S-wave

• perform **scattering** experiment: use „pion cloud“ as virtual $\pi$ target
  • $\pi$ exchange dominates at pion pole (in scattering experiments: $t < 0$)
    • extrapolate to pion pole
    • enhance $\pi$ exchange through $\Delta$ production (recoil particle detection)
    • examples: SLAC experiments, $K_L$ beam at JLAB (planned)
  • theoretically complex

• extract $K\pi$ S-wave from decays (Watson theorem)
  • weak: $D \to K\pi\pi$, $\tau \to K\pi\nu_\tau$
  • strong: $K^* \to K\pi\pi$, $J/\psi \to K^*(892)K\pi$
Model Free Analysis of Hadronic Decays

- Example: CLEO
  \[ D \rightarrow K\pi\pi \]

\[ \tau \rightarrow K\pi\nu_\tau \]

Existing analyses suffer from unrecognized ambiguities (zero modes)

- \[ \tau \rightarrow K\pi\nu_\tau \] difficult to perform PWA (missing \( \nu_\tau \)) - new technique developed
  analyze: \[ \tau^+\tau^- \rightarrow K\pi\nu_\tau + \pi\pi\pi \]
Example Hadronic Decay (Kππ) System

- Analyse Kπ subsystem in Kππ final states
- Preliminary: Kπ P-Wave
- To come: Kπ S-Wave

Freed of ambiguities !!

Kπ S-Wave:
from threshold into resonance region
Chiral Symmetry Restauration (parity doubling)

Nucleon mass: 1% from quark mass (Higgs mechanism) $\rightarrow$ 99% from the strong interaction (QCD)

Spontaneously broken: chiral symmetry

- Hadrons with different parity do not have same mass

Unique test of fundamental QCD property:

- Change order parameter $\rightarrow$ change temperature
- Symmetry restoration at high temperatures
  - Measure hadron properties (spectral functions)
• Low energy meson dynamics described by $\chi$PT
• Experiments address key observables (also sensitive to quark masses)
• Precision experiments allow to determine low energy constants of effective theory $\rightarrow$ universal description of QCD at low energies
• Extension into strangeness sector ongoing (see AMBER proposal)
• PWA important tool: extract S-wave scattering and single out Primakoff processes from high backgrounds
• Important test: Parity doubling in hot medium with restored chiral symmetry
Kπ elastic scattering

- LASS data and projected JLAB data
- Measure t-dependence and extrapolate to $t = m_\pi^2$