Fit of the $a_1(1420)$ as a Triangle Singularity

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HISKP, Bonn University

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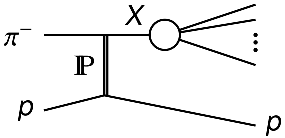
On behalf of the COMPASS collaboration

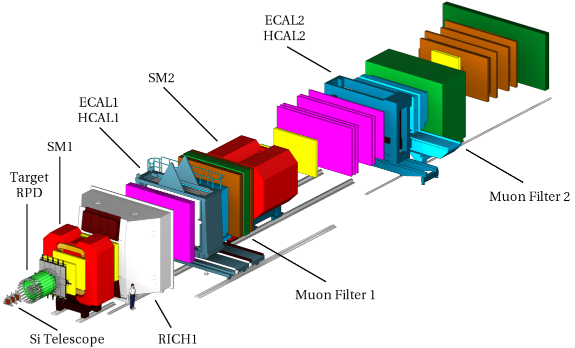
supported by BMBF

The COMPASS Experiment

The COMPASS Experiment

- Secondary hadron beam, mostly π^- ($\sim 97\,\%$)
  - $E_{\text{beam}} = 190\,\text{GeV}$
  - Liquid hydrogen target (40 cm)
  - $\pi^- + p \quad\rightarrow\quad \pi^- + \pi^- + \pi^+ + p$

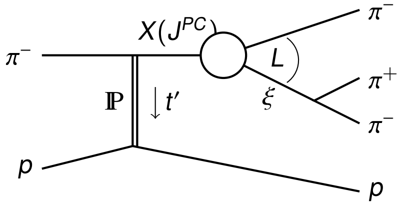




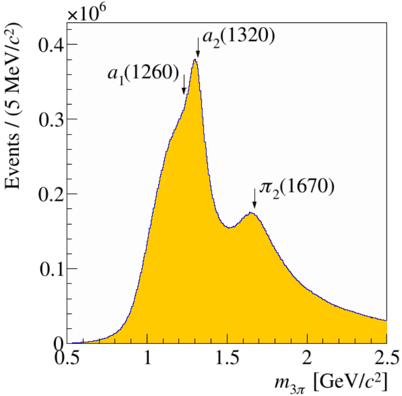
[COMPASS, NIM A**779**, 69-115 (2015)]

3π PWA

- Isobar model: $X^- \rightarrow \pi^- + \xi \rightarrow \pi^- + \pi^+ + \pi^-$
- Data binned in 100 $m_{3\pi}$ and 11 $t' = |t| - |t|_{\text{min}}$ slices
- PWA with 88 waves [COMPASS, PRD **95**, 032004 (2017)]



$\mathbb{P}$: Pomeron
X: Resonance with J^{PC}
ξ: Isobar



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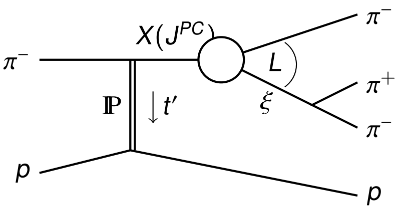

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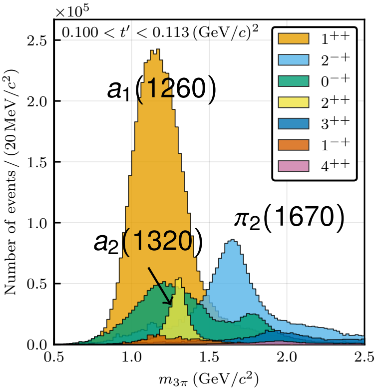

[Light-Meson Spectroscopy with COMPASS, arXiv:1909.06366v2]

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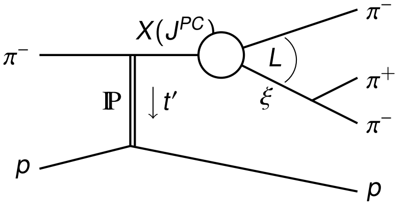

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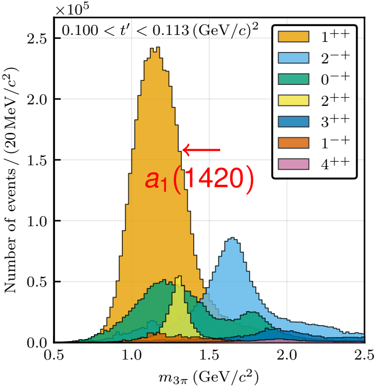

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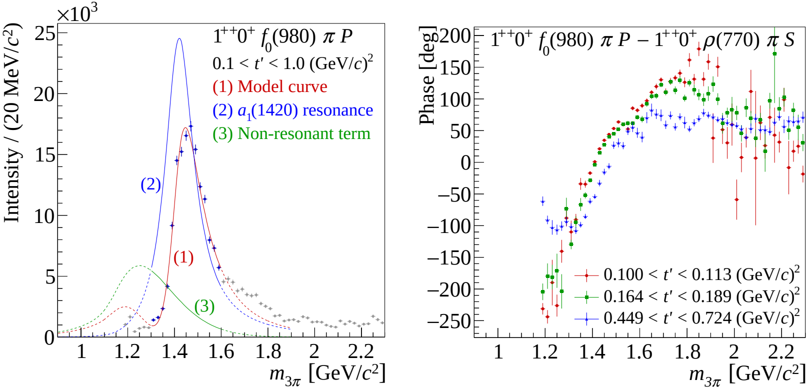
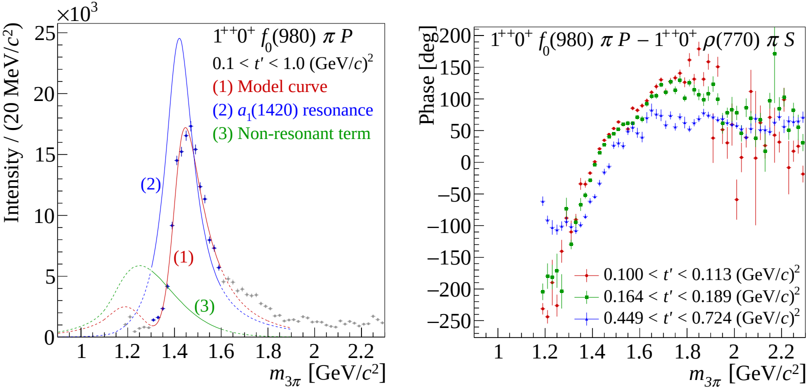
The $a_1(1420)$ signal



[COMPASS, PRL **115**, 082001 (2015)]

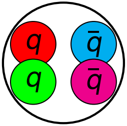


[COMPASS, PRL **115**, 082001 (2015)]

What makes this signal exotic?

- Only seen in the $J^{PC} = 1^{++}$ $f_0(980)\pi$ P-wave
- Very close to the ground state $a_1(1260)$
- Too narrow: 150 MeV (ground state has 250-600 MeV)

- 4-quark state
  [H.-X. Chen et al. (2015)],
  [T. Gutsche et al. (2017)]



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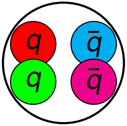

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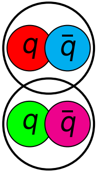

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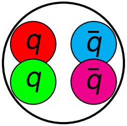

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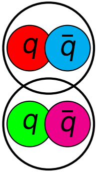
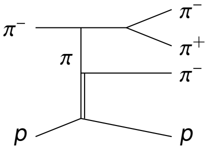


- Dynamic effect of interference with Deck-amplitude
  [Basdevant & Berger, PRL **114**, 192001 (2015)]



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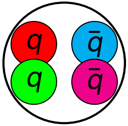

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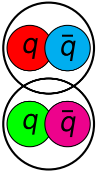

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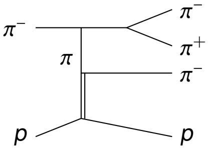
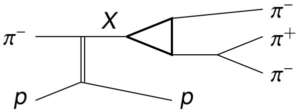


- **Triangle singularity**
  [Mikhasenko et al., PRD **91**, 094015 (2015)],
  [Aceti et al., PRD **94**, 096015 (2016)]

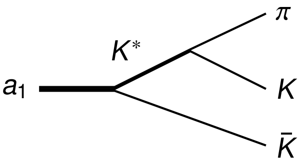


The $a_1(1420)$ signal

- The Triangle Diagram -

Include spin via partial-wave projection:

1. Look at the partial wave for $a_1(1260) \to K\bar{K}\pi$ with isobar K^*



(12)

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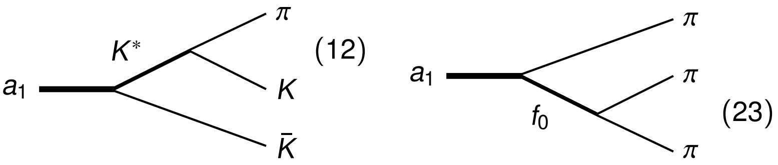

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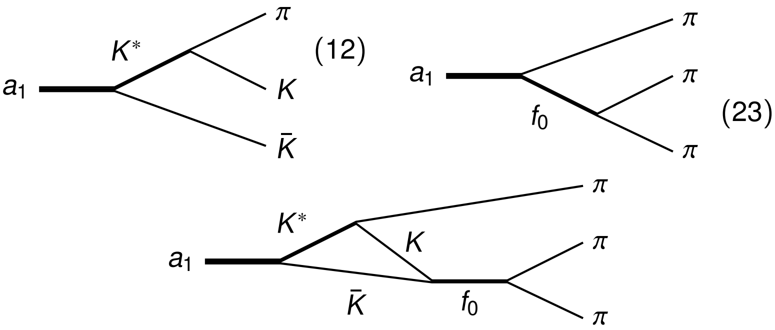
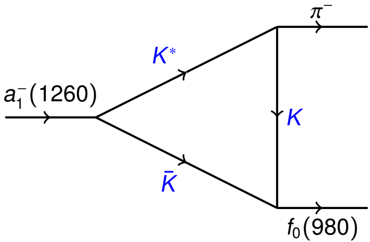
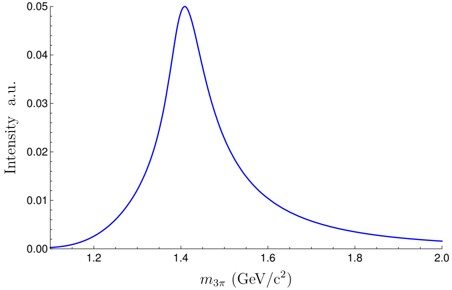
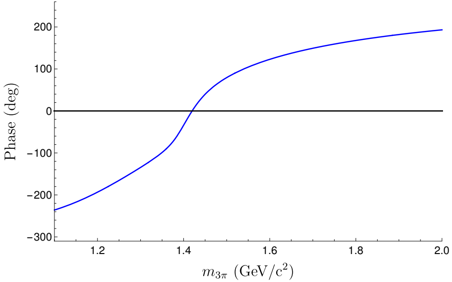
3. Obtain the first order approximation of the Khuri-Treiman approach





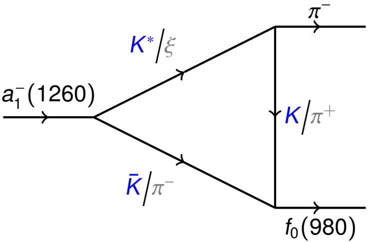




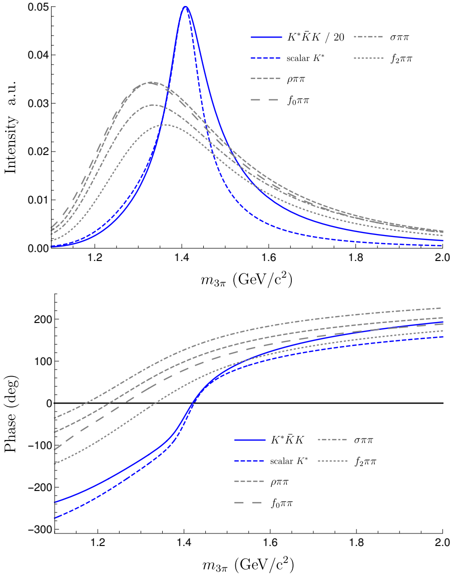
- Kaons in the loop produce peak and phase motion at 1.4 GeV

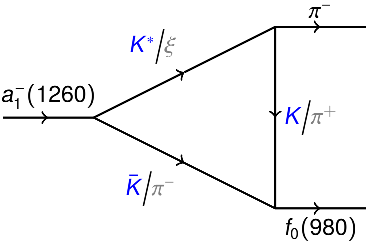
The Triangle Diagram

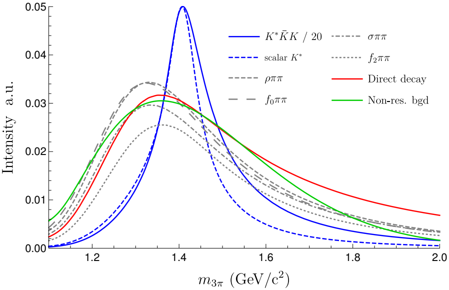
The $a_1(1420)$ signal

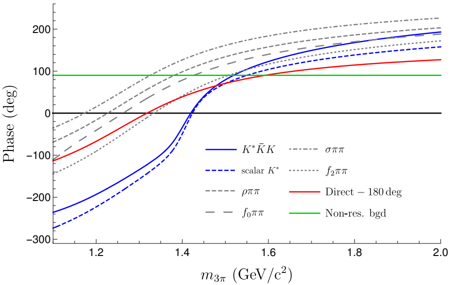


- Ground state rescatters through other intermediate isobar ξ
- Using Feynman calculation treating everything as scalars









- Other triangles similar to direct decay
- Phenomenological background is able to describe them

The Fit

- Model -

Wave Selection

Select interesting components from the pool of 88 partial waves:

1. $J^{PC} = 1^{++}$ $\rho\pi$ S-wave
   - Contains the ground state $a_1(1260)$

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   - Contains the signal of interest, the $a_1(1420)$
3. $J^{PC} = 2^{++}$ $\rho\pi$ D-wave
   - Contains the $a_2(1320)$
   - Clean signal, small background
   - Interferometer

- From PWA: #events per $m_{3\pi}$ and t' slices for a given partial wave
- Intensity: $\dfrac{\mathrm{d}^2 N}{\mathrm{d}m_{3\pi}\,\mathrm{d}t'} \propto \dfrac{\mathrm{d}^2 \sigma}{\mathrm{d}m_{3\pi}\,\mathrm{d}t'} \propto m_{3\pi} \left|\mathcal{M}_{\mathrm{tot}}\right|^2 \tilde{\Phi}_2,$

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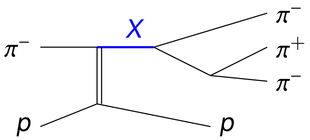
- Interferences between waves $\sim m_{3\pi}\mathcal{M}_{\text{tot}}^{(1)*}\mathcal{M}_{\text{tot}}^{(2)}\sqrt{\tilde{\Phi}_2^{(1)}\tilde{\Phi}_2^{(2)}}$
- $\mathcal{M}_{\text{tot}} = \mathcal{M}_{\text{signal}} + \mathcal{M}_{\text{bgd}}$

- $\mathcal{M}_{\text{signal}} \propto \dfrac{1}{M_X^2 - m_{3\pi}^2 - \mathrm{i}\, M_X \Gamma_X(m_{3\pi})}$
  - Propagator with energy-dependent width
  - Multiplied by triangle amplitude for $f_0\pi$ P-wave



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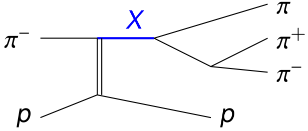
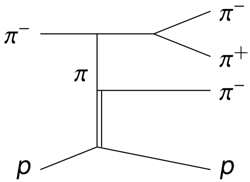
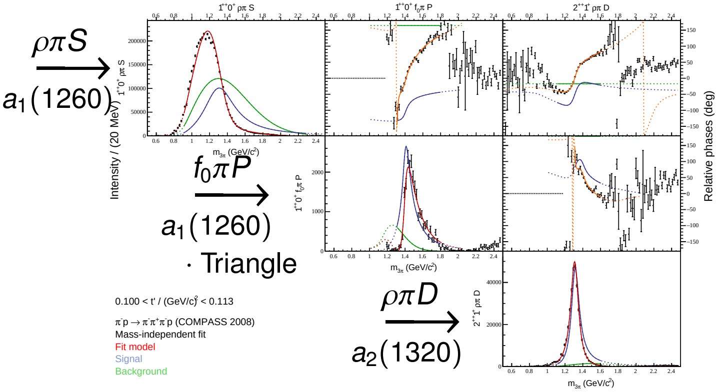


- $\mathcal{M}_{\text{bgd}} \propto \left(\dfrac{m_{3\pi} - m_{\text{thr}}}{m_{\text{thr}}}\right)^b e^{-c(t')\tilde{p}^2}$
    - Phenomenological parametrization for Deck background
    - $\tilde{p} = 4\pi m_{3\pi}\tilde{\Phi}_2$



Intensities

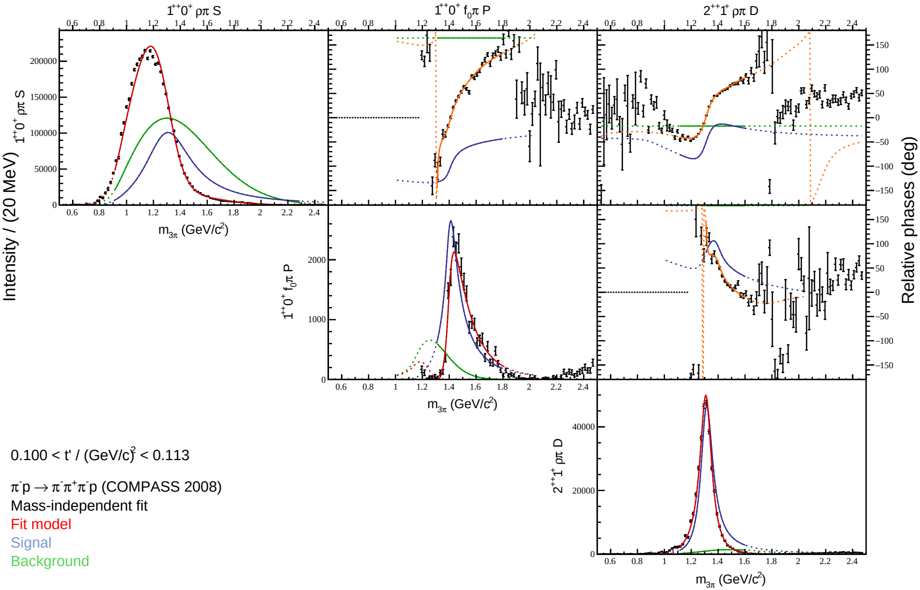
Interferences (fitted via real and imaginary part)



Fit model

Signal

Background



$0.100 < t' / (GeV/c^2) < 0.113$

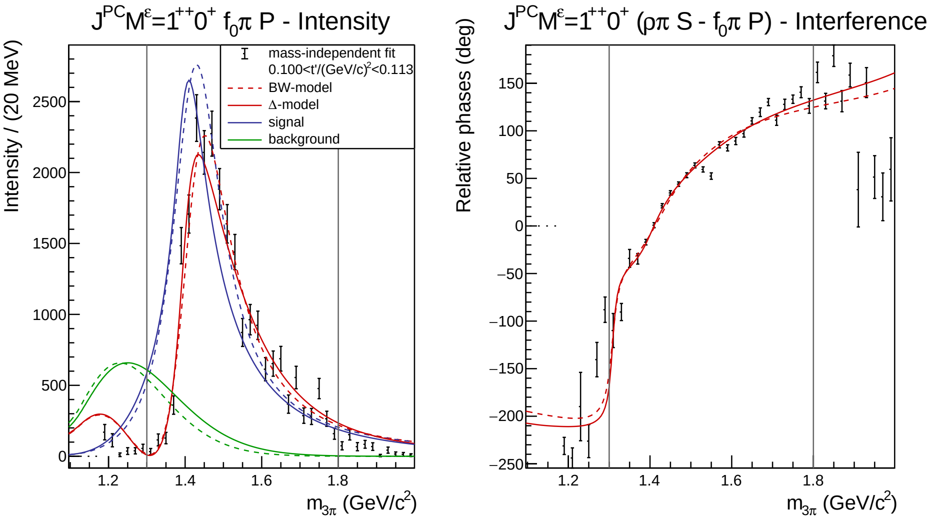
$\pi^- p \to \pi^-\pi^+\pi^- p$ (COMPASS 2008)

Mass-independent fit

Fit model

Signal

Background



$J^{PC}M^{\varepsilon}=1^{++}0^{+}$ $f_0\pi$ P - Intensity
Intensity / (20 MeV)
mass-independent fit
$0.100<t'/(GeV/c)^2<0.113$
BW-model
Δ-model
signal
background
$m_{3\pi}$ (GeV/c^2)
$J^{PC}M^{\varepsilon}=1^{++}0^{+}$ ($\rho\pi$ S - $f_0\pi$ P) - Interference
Relative phases (deg)
$m_{3\pi}$ (GeV/c^2)

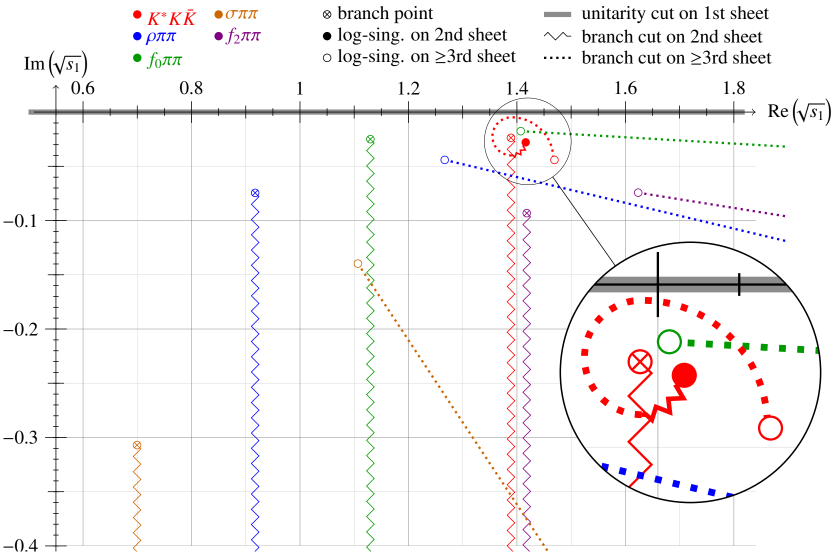
Conclusion

- It is possible to fit $a_1(1420)$ with the Triangle-Model.
- The inclusion of spins is done via the partial-wave-projection method.
- Spin only affects the shape, not the position of peak.
- The Scalar-Triangle-Model is sufficient for first studies.
- Many systematic studies have been performed.
  - Very stable w.r.t. manipulations of the data and the fit model.
- Comparison of Triangle-Model and BW-Model:
  - Competitive fit quality between both models.
  - In the Triangle-Model: No free fit parameters are present to describe the peak position and width of the signal.
  - Rescattering has to be present!

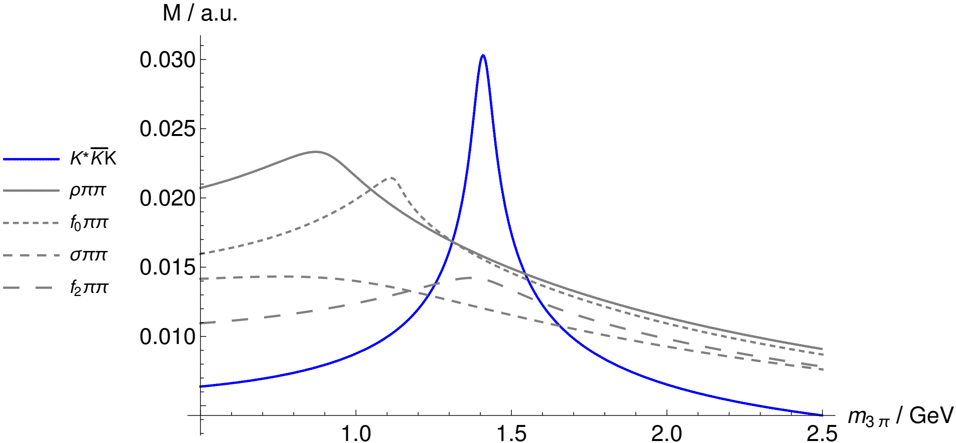
Thank you for your attention!

Back-Up

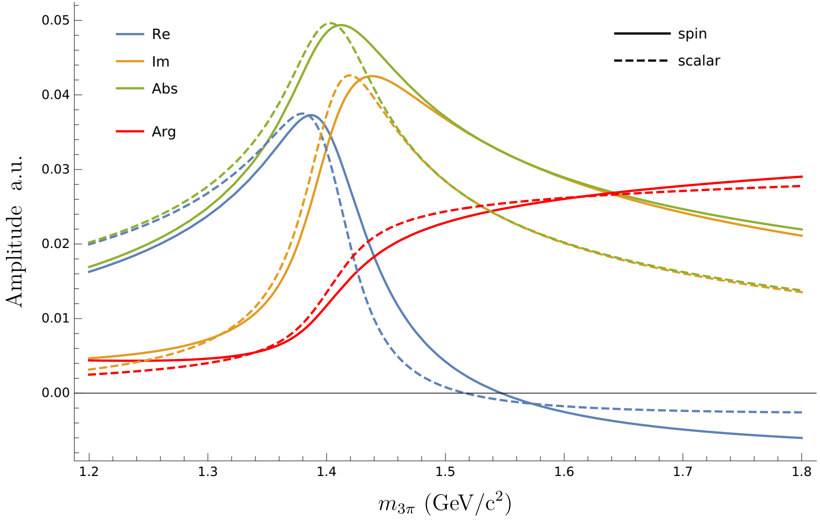
Structure of Complex Plane



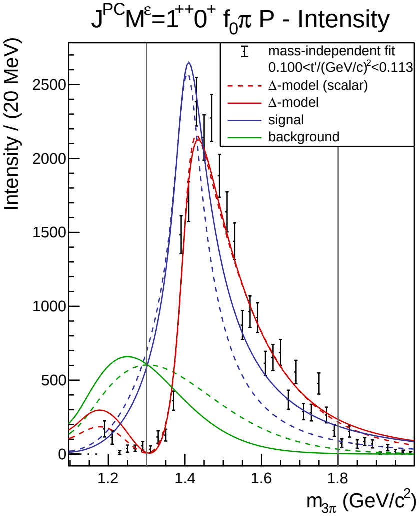
All Scalar Amplitudes

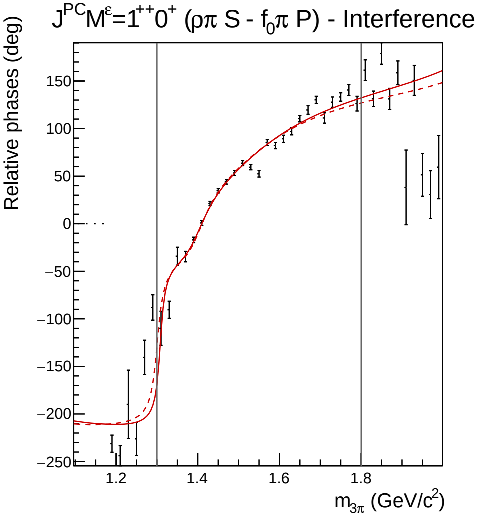


Complex Amplitude

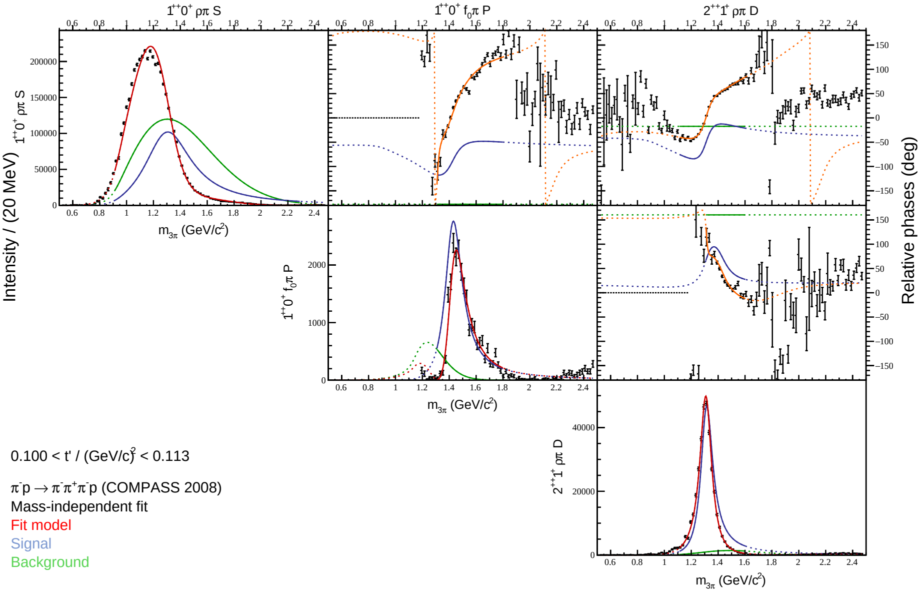


Comparison with Scalar-Triangle





Stamp Plot - Breit-Wigner



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Mass-independent fit
Fit model
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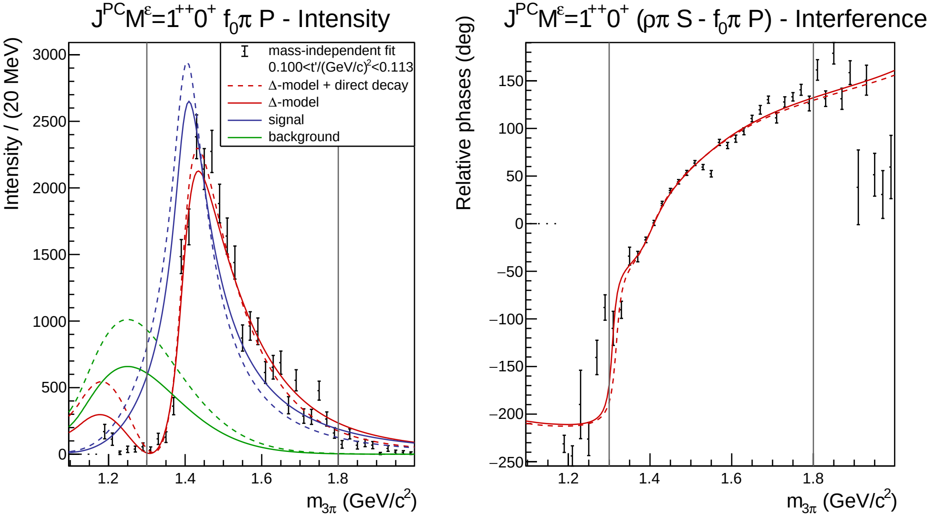
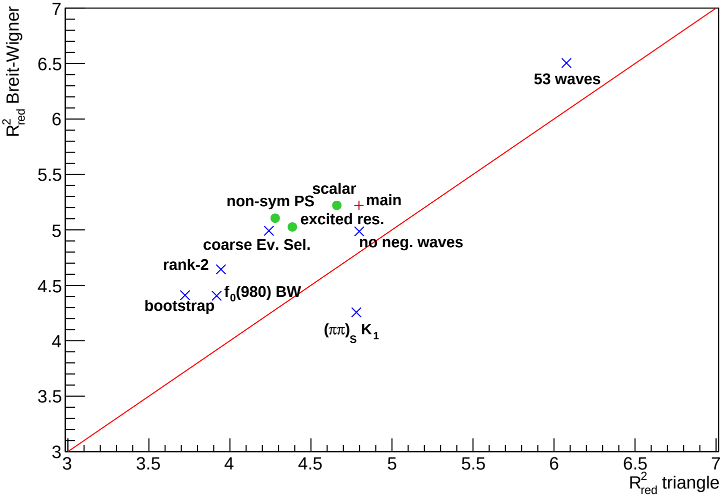
Background

Comparison with Direct Decay

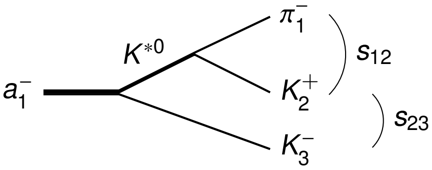


Systematic Studies



PWP - Calculations

$$A(\tau) = \sum_{w=(JMLS)} \left[F_w(s_{12}) Z_w^*(\Omega_{3,12}) + F_w(s_{23}) Z_w^*(\Omega_{1,23}) \right]$$



Simple model: $F_w(s_{12}) = C_{a_1} \cdot t_{K^*}(s_{12})$

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Projection to channel (23):

$$\begin{aligned} A_w(s_{23}) &= \int \mathrm{d}\Omega_{1,23} Z_w(\Omega_{1,23}) A(\tau) \\ &= F_w(s_{23}) + \hat{F}_w(s_{23}) \end{aligned}$$

with $\quad \hat{F}_w(s_{23}) := \int \mathrm{d}Z_w(s_{23}) \sum_{w'} F_{w'}(s_{12}) Z_{w'}^*(\Omega_{3,12})$

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unitarity for PW amplitude A_w:

$$\Rightarrow \quad F_w(s_{23}) = t_\xi(s_{23}) \left[C_w + \frac{1}{2\pi} \int_{s_{\mathrm{th}}}^{\infty} \frac{\rho(\tilde{s}_{23}) \hat{F}_w(\tilde{s}_{23})}{\tilde{s}_{23} - s_{23}} \mathrm{d}\tilde{s}_{23} \right]$$

PWP - Calculations

$$A(\tau) = \sum_{w=(JMLS)} \left[F_w(s_{12}) Z_w^*(\Omega_{3,12}) + F_w(s_{23}) Z_w^*(\Omega_{1,23}) \right]$$

Projection to channel (23):

$$\begin{aligned} A_w(s_{23}) &= \int \mathrm{d}\Omega_{1,23} Z_w(\Omega_{1,23}) A(\tau) \\ &= F_w(s_{23}) + \hat{F}_w(s_{23}) \end{aligned}$$

with $\quad \hat{F}_w(s_{23}) := \int \mathrm{d}Z_w(s_{23}) \sum_{w'} F_{w'}(s_{12}) Z_{w'}^*(\Omega_{3,12})$

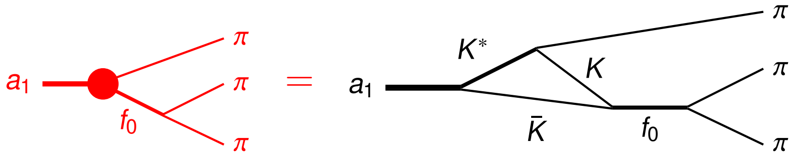
unitarity for PW amplitude A_w:

$$\Rightarrow \quad F_w(s_{23}) = t_\xi(s_{23}) \left[C_w + \frac{1}{2\pi} \int_{s_{\mathrm{th}}}^{\infty} \frac{\rho(\tilde{s}_{23}) \hat{F}_w(\tilde{s}_{23})}{\tilde{s}_{23} - s_{23}} \mathrm{d}\tilde{s}_{23} \right]$$

Problem: $\hat{F}$ depends on F as well! $\rightsquigarrow$ solve iteratively

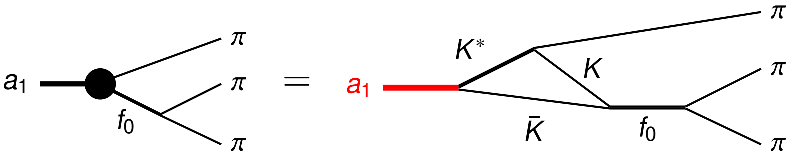
PWP - Iterative Procedure

$$F(s_{23}) = t_{f_0}(s_{23}) \frac{1}{2\pi} \int_{4m_K^2}^{\infty} d\tilde{s}_{23} \frac{\rho(\tilde{s}_{23}) \int dZ_{f_0}(\tilde{s}_{23})\ C_{a_1} t_{K^*}(s_{12}) Z^*_{K^*}(\Omega_{3,12})}{\tilde{s}_{23} - s_{23}}$$



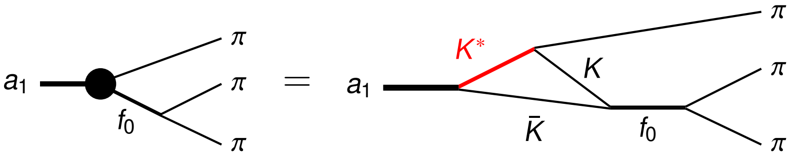
PWP - Iterative Procedure

$$F(s_{23}) = t_{f_0}(s_{23}) \frac{1}{2\pi} \int_{4m_K^2}^{\infty} d\tilde{s}_{23} \frac{\rho(\tilde{s}_{23}) \int dZ_{f_0}(\tilde{s}_{23})\ C_{a_1} t_{K^*}(s_{12}) Z^*_{K^*}(\Omega_{3,12})}{\tilde{s}_{23} - s_{23}}$$



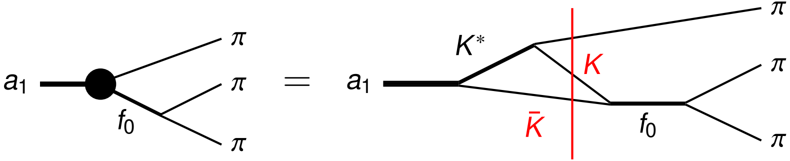
PWP - Iterative Procedure

$$F(s_{23}) = t_{f_0}(s_{23}) \frac{1}{2\pi} \int_{4m_K^2}^{\infty} d\tilde{s}_{23} \frac{\rho(\tilde{s}_{23}) \int dZ_{f_0}(\tilde{s}_{23})\, C_{a_1} {\color{red} t_{K^*}(s_{12})} Z^*_{K^*}(\Omega_{3,12})}{\tilde{s}_{23} - s_{23}}$$



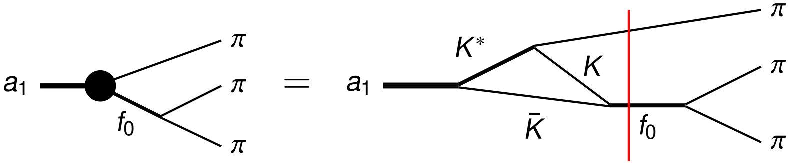
PWP - Iterative Procedure

$$F(s_{23}) = t_{f_0}(s_{23})\frac{1}{2\pi}\int_{4m_K^2}^{\infty} \mathrm{d}\tilde{s}_{23} \frac{\rho(\tilde{s}_{23}) \int \mathrm{d}Z_{f_0}(\tilde{s}_{23})\; C_{a_1} t_{K^*}(s_{12}) Z_{K^*}^{*}(\Omega_{3,12})}{\tilde{s}_{23} - s_{23}}$$



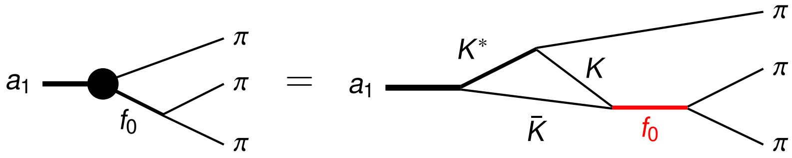
PWP - Iterative Procedure

$$F(s_{23}) = t_{f_0}(s_{23}) \frac{1}{2\pi} \int_{4m_K^2}^{\infty} d\tilde{s}_{23} \frac{\rho(\tilde{s}_{23}) \int dZ_{f_0}(\tilde{s}_{23})\ C_{a_1} t_{K^*}(s_{12}) Z_{K^*}^*(\Omega_{3,12})}{\tilde{s}_{23} - s_{23}}$$



PWP - Iterative Procedure

$$F(s_{23}) = t_{f_0}(s_{23}) \frac{1}{2\pi} \int_{4m_K^2}^{\infty} \mathrm{d}\tilde{s}_{23} \frac{\rho(\tilde{s}_{23}) \int \mathrm{d}Z_{f_0}(\tilde{s}_{23})\, C_{a_1} t_{K^*}(s_{12}) Z^*_{K^*}(\Omega_{3,12})}{\tilde{s}_{23} - s_{23}}$$



PWP - Iterative Procedure

$$F(s_{23}) = C_{a_1} t_{f_0}(s_{23}) \frac{1}{2\pi} \int_{4m_K^2}^{\infty} d\tilde{s}_{23} \frac{\rho(\tilde{s}_{23}) \int dZ_{f_0}(\tilde{s}_{23}) \; t_{K^*}(s_{12}) Z_{K^*}^*(\Omega_{3,12})}{\tilde{s}_{23} - s_{23}}$$