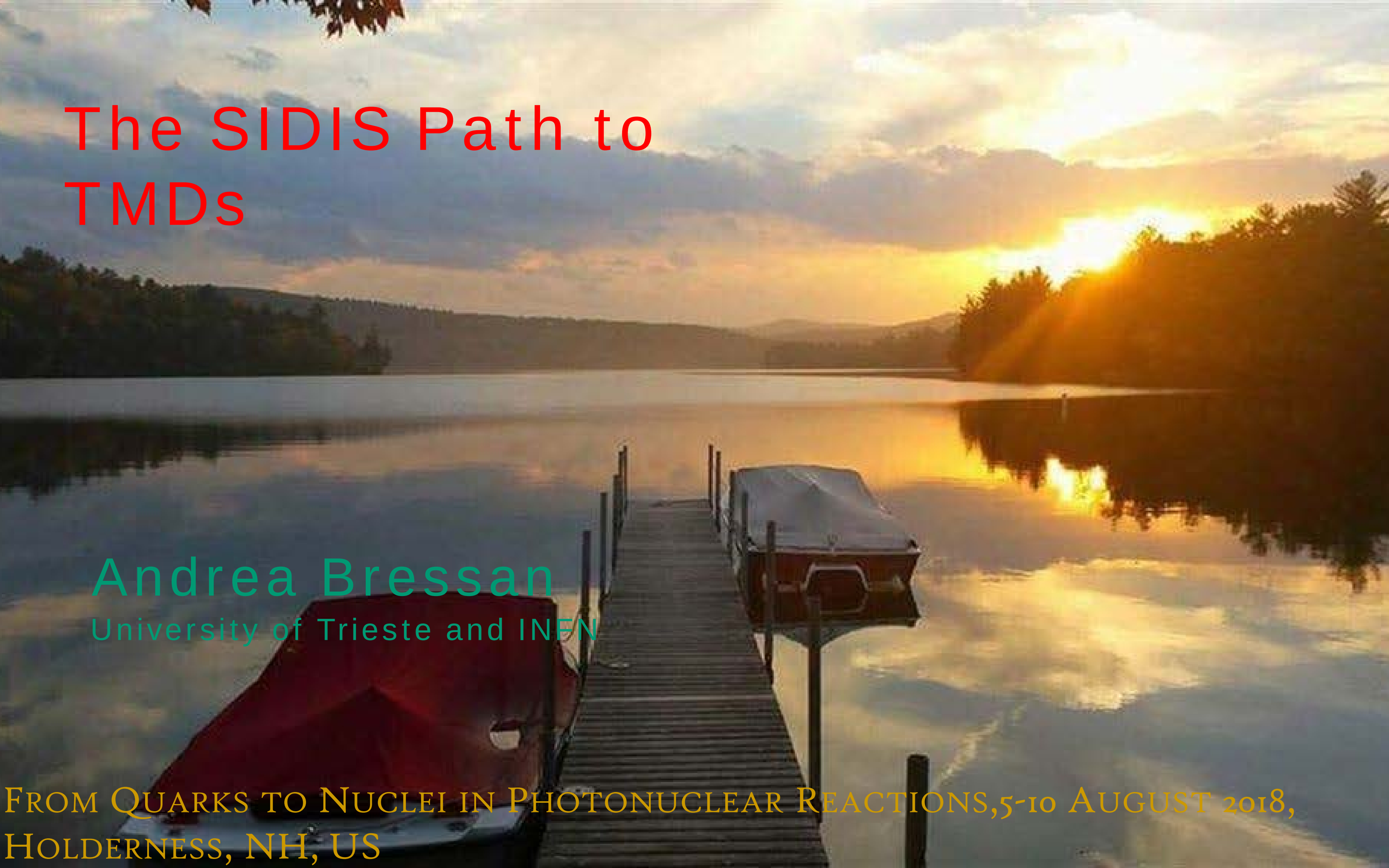
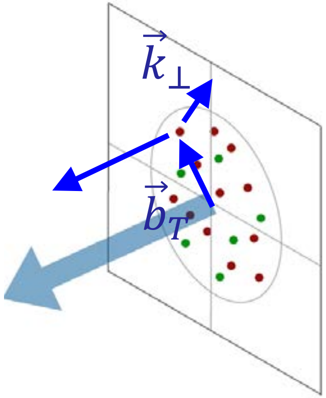


The SIDIS Path to TMDs

Andrea Bressan
University of Trieste and INFN

From Quarks to Nuclei in Photonuclear Reactions,5-10 August 2018, Holderness, NH, US

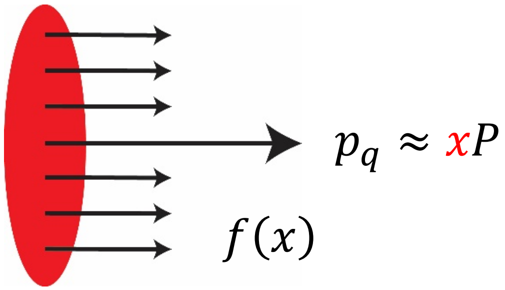
Confined parton motion in a hadron



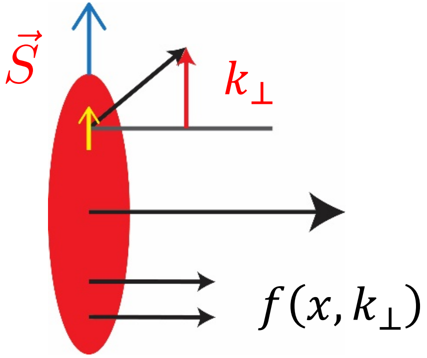
- Scattering with a large momentum transfer
    - Momentum scale of the hard probe $Q \gg {}^{1}/_{R} \sim \Lambda_{QCD} \sim 1\ \mathrm{fm}$
    - Combined motion $\sim {}^{1}/_{R}$ is too week to be sensitive to the hard probe
    - Collinear factorization – integrated into PDFs
- Scattering with multiple momentum scales observed
    - Two-scale observables (such as SIDIS, low p_T Drell-Yan) $Q \gg q_T \sim {}^{1}/_{R} \sim \Lambda_{QCD} \sim 1\ \mathrm{fm}$
    - “Hard” scale Q localizes the probe to see the quark or gluon d.o.f.
    - “Soft” scale q_T could be sensitive to the confined motion
    - TMD factorization: the confined motion is encoded into TMDs

Structure of proton

- Transverse Momentum Dependent parton distribution (TMDs)



Longitudinal motion only



Longitudinal + transverse motion

- Sivers function: an asymmetric parton distribution in a transversely polarized nucleon ($k_\perp$ correlated with the spin of the nucleon)

$$f_{q/h^\uparrow}(x, k_\perp, \vec{S}) = f_{q/h}(x, k_\perp) - \frac{1}{M} f_{1T}^{\perp q}(x, k_\perp) \vec{S} \cdot (\hat{p} \times \vec{k}_\perp)$$

- Boer-Mulders function: an asymmetric parton distribution in an unpolarised nucleon ($k_\perp$ correlated with the spin of the quark)

$$f_{q/h^\uparrow}(x, k_\perp, \vec{s}) = f_{q/h}(x, k_\perp) - \frac{1}{M} h_{1T}^{\perp q}(x, k_\perp) \vec{s} \cdot (\hat{p} \times \vec{k}_\perp)$$

Transverse-Momentum Dependent PDFs

- **Inclusive** processes → collinear factorisation: one or less hadrons detected
- **“More inclusive”** processes → TMD factorisation: one or more hadrons in the initial or final state detected

- **Collinear factorisation**: longitudinal momenta of the patrons are intrinsic, transverse momenta can be created by perturbative radiation effects (parton showers)
- **TMD factorisation**: both mechanisms of the transverse-momentum creation taken into account: intrinsic (essentially non-perturbative) and perturbative radiation

TMD PDFs

- The theoretical expression of TMDs has a more complicated structure of the gauge link, connecting two space-time points with a transverse separation

$$f_{q/N}(x,k_\perp) = \frac{1}{8\pi}\int dr^- \frac{dr_\perp^2}{(2\pi)^2} e^{-iMxr^-/2+ik_\perp\cdot r_\perp}$$

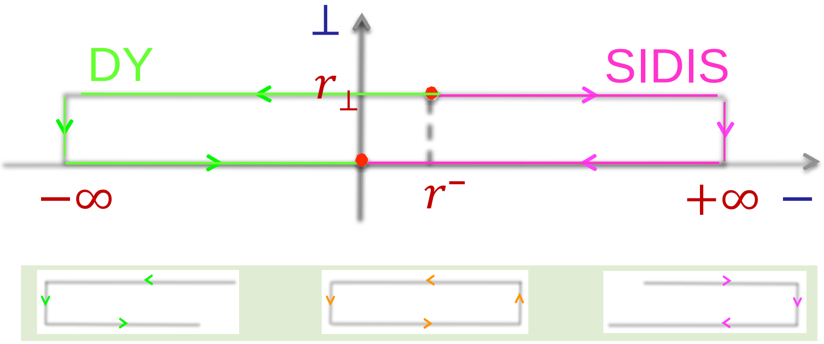
$$\langle N(P)|\bar{q}(r^-,r_\perp)\gamma^+ W[r^-,r_\perp;0]q(0)|N(P)\rangle|_{r^+\sim 1/\nu\to 0}$$

- The Wilson line W is no longer on the light-cone axis and may introduce a **process dependence**

Parity and Time reversal invariance ⇒

$$\left(f_{1Tq}^{\perp}\right)_{DY} = -\left(f_{1Tq}^{\perp}\right)_{SIDIS}$$

Most critical test to TMD approach to SSA



TMD evolution:

- QCD evolution of TMDs in Fourier space (solution of equation)

$$F(x,b;Q) \approx C \otimes F(x,c/b^*) \exp\left\{-\int_{c/b^*}^{Q_f} \frac{d\mu}{d}\left(A \ln \frac{Q_f^2}{\mu^2} + B\right)\right)\right\} \times \exp\left[-S_{\text{non-pert}}(b,Q)\right]$$

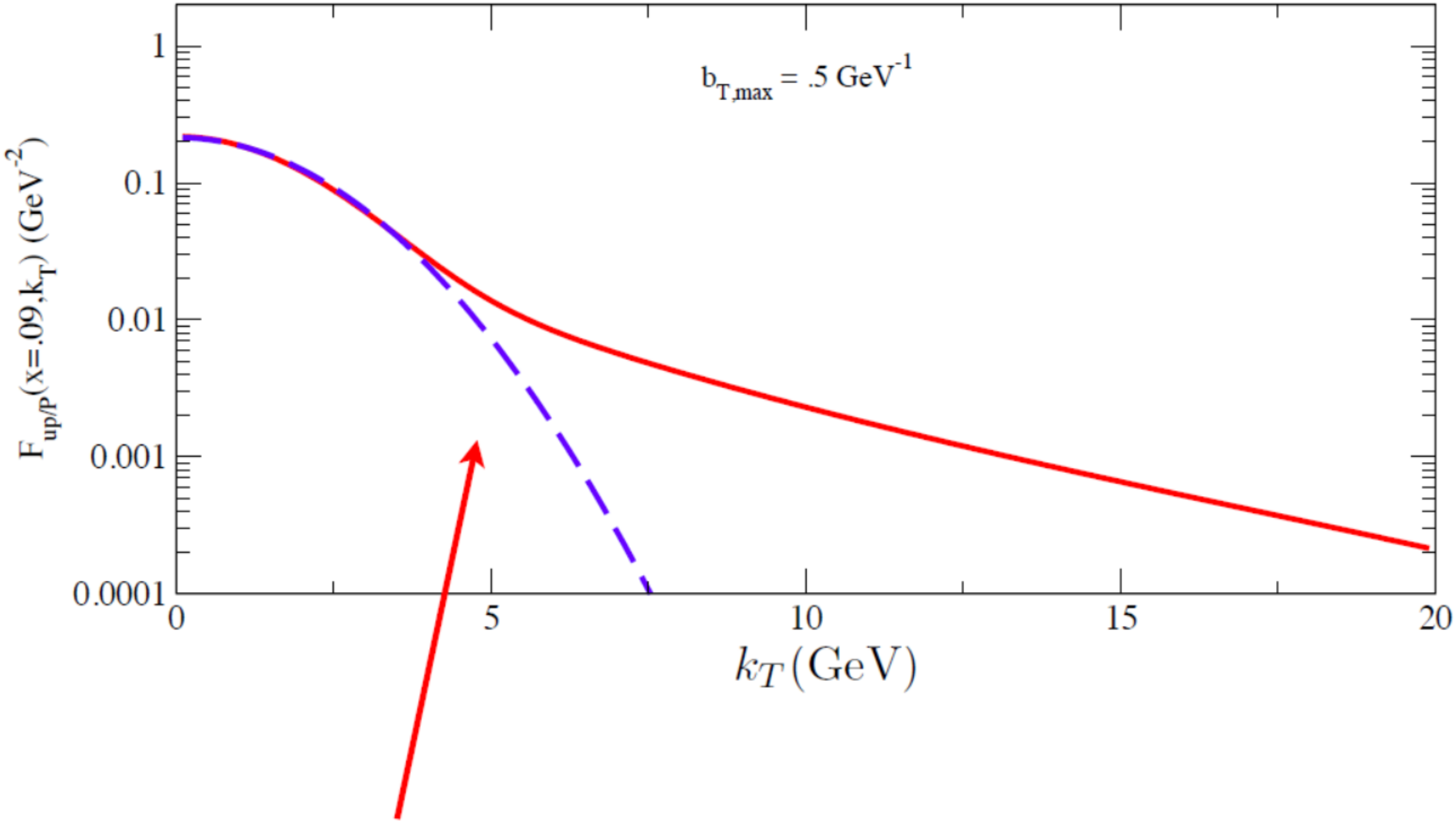
Evolution of longitudinal/ collinear part

Evolution of transverse part (Sudakov form factor)

Non-perturbative part has to be fitted to experimental data
The key ingredient is spin-independent

- Polarized scattering data comes as ratio: e.g. $A_{UT}^{\sin(\phi_h-\phi_s)} = F_{UT}^{\sin(\phi_h-\phi_s)}/F_{UU}$
- Unpolarized data is very important to constrain/extract the key ingredient for the non-perturbative part

Effect of QCD evolution

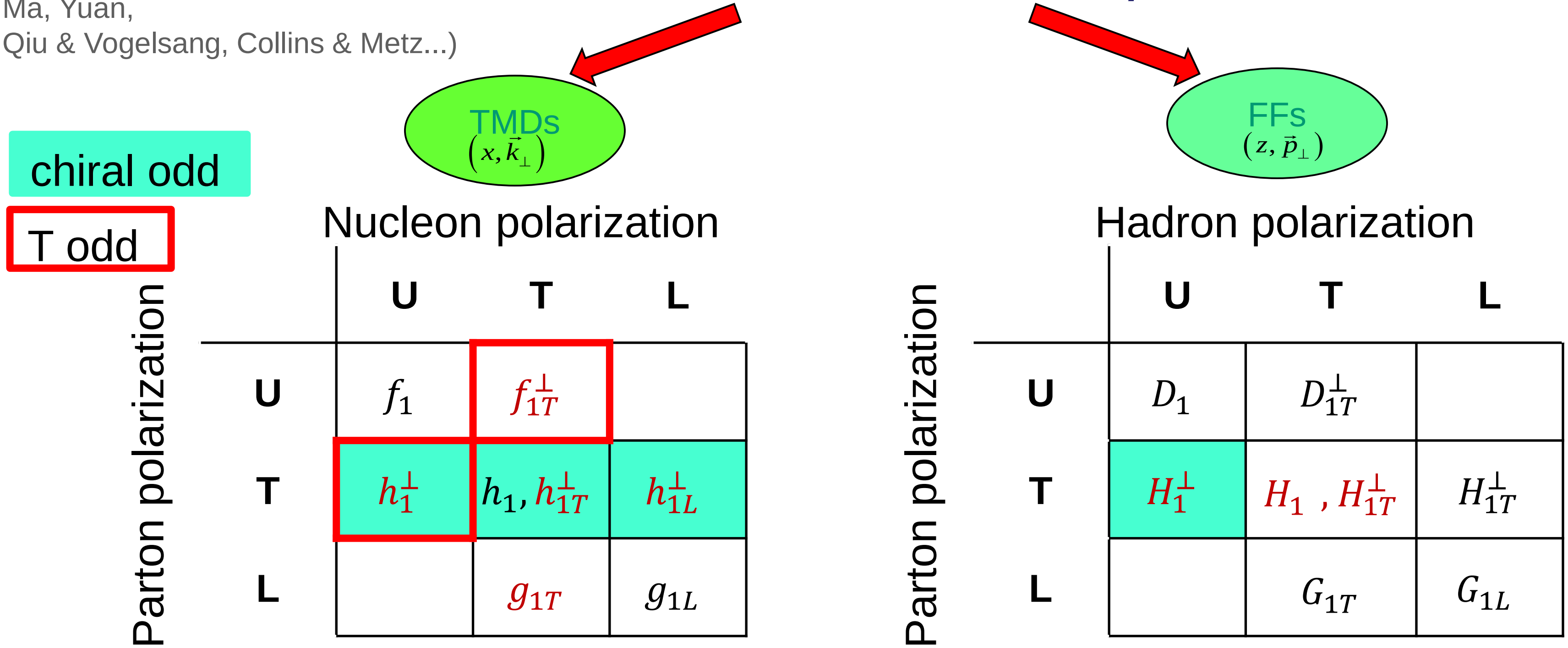


gaussian fit does not capture the effects of evolution quite well

SIDIS access to TMDs

Factorisation (Collins & Soper, Ji, Ma, Yuan,
Qiu & Vogelsang, Collins & Metz...)

$$\sigma(\ell p \to \ell' h X) \sim q(x) \otimes \hat{\sigma}^{\gamma q \to q} \otimes D_q^h(z)$$



Nucleon polarization

Parton polarization	U	T	L
U	f_1	$f_{1T}^\perp$	
T	$h_1^\perp$	$h_1, h_{1T}^\perp$	$h_{1L}^\perp$
L		g_{1T}	g_{1L}

Hadron polarization

Parton polarization	U	T	L
U	D_1	$D_{1T}^\perp$	
T	$H_1^\perp$	H_1 , $H_{1T}^\perp$	$H_{1T}^\perp$
L		G_{1T}	G_{1L}

- NOT directly accessible
- Their extractions require measurements of x-sections and asymmetries in **a large kinematic domain of** x, Q^2, z, P_{hT}

SIDIS Experiments

SIDIS Experiment must:

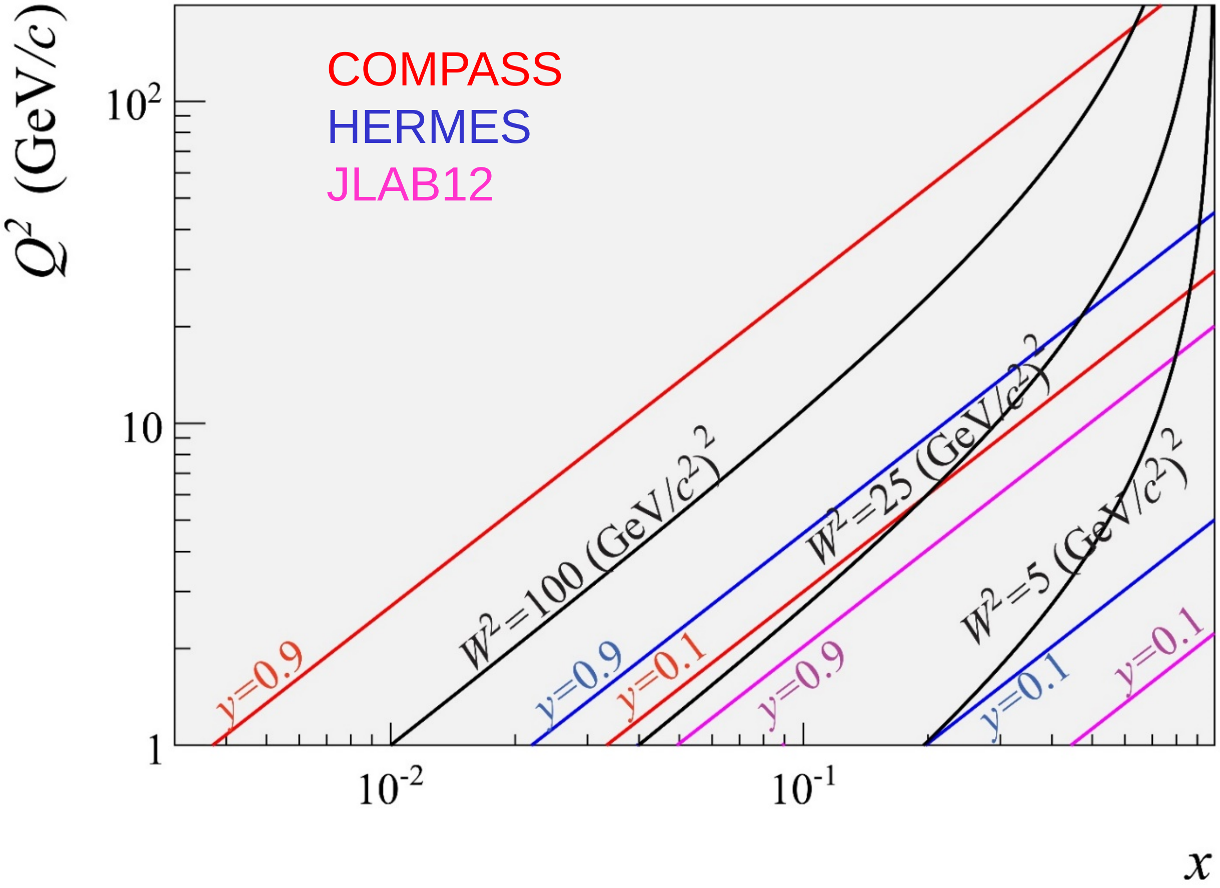
- Have large acceptances on all the relevant variables x, Q^2, z, P_{hT}, ϕ
- Use different targets (p, d, n) and identify hadrons to allow flavour separation
- Be ad different energies for to cover PDFs from the valence region down to small-x
- Large luminosity to allow multidimensional results needed by the complexity of TMDs

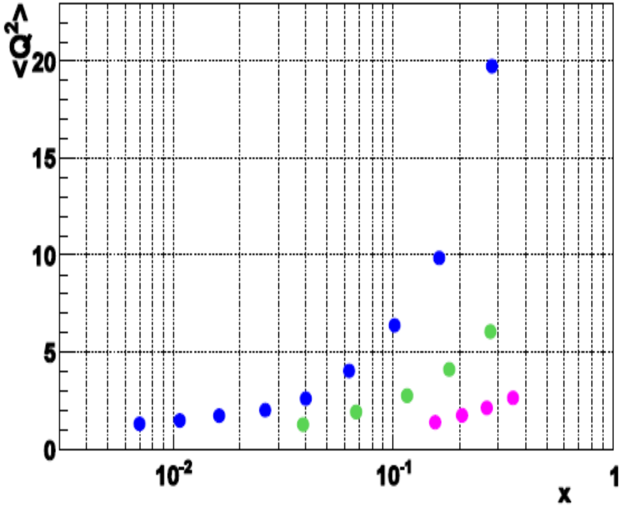
- **The polarized lepton-nucleon collider will be a mandatory tool to reach the level of ordinary PDF**

COMMENT ON TMD studies

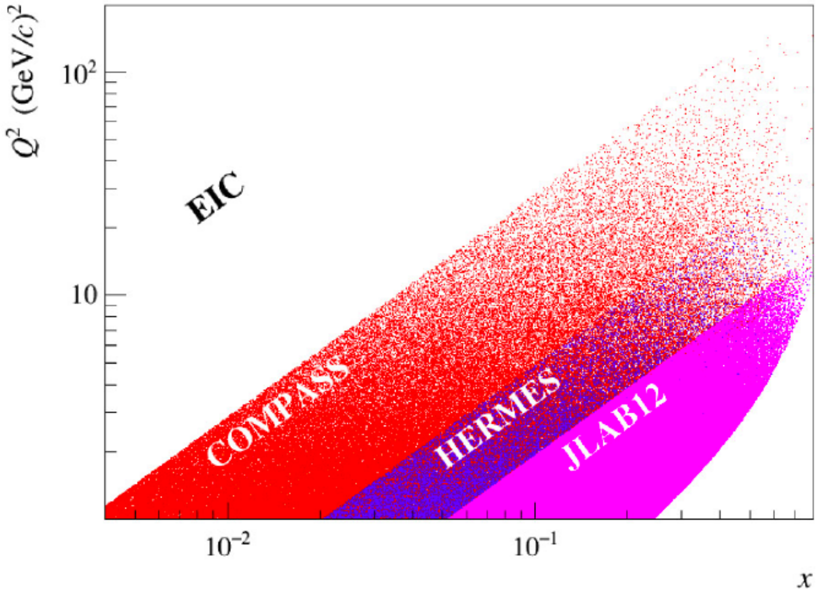
- SIDIS has opened the way to this field about 10 years ago:
    - Collins and DiHadron asymmetries on protons are sizeable
    - The Sivers asymmetry is also different from zero and we are now probing it’s pseudo universality
    - The other TMDs are small, compatible to zero in most of the cases, at present precision, with hints for the pretzelosity $g_{1T}^{\perp}$ in $\cos(\phi_h - \phi_S)$
    - P_{hT} multiplicities have been measured at HERMES, COMPASS and JLab
    - Sizeable $\cos\phi$ and $\cos 2\phi$ asymmetries have also bean measured but we don’t really know yet if the Boer-Mulders TMD PDF is different from zero.

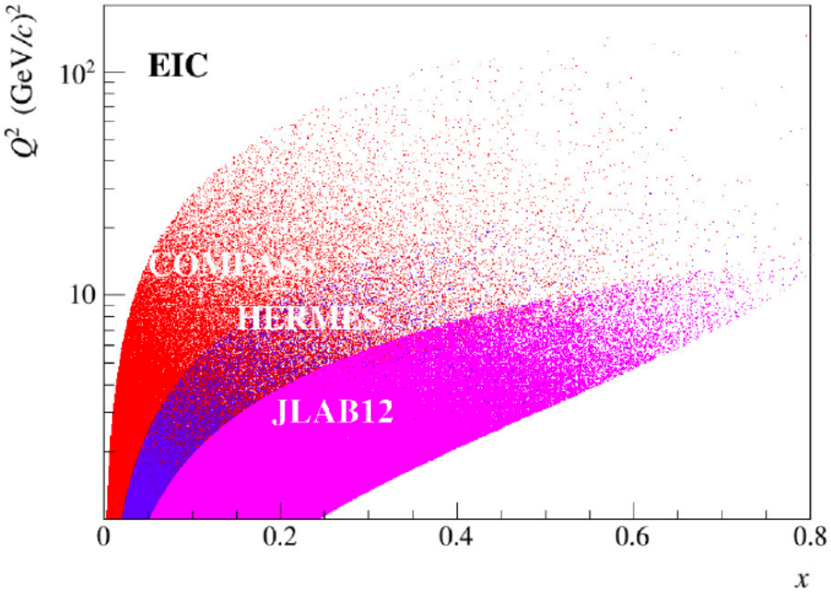
Kinematic coverage





Kinematic coverage





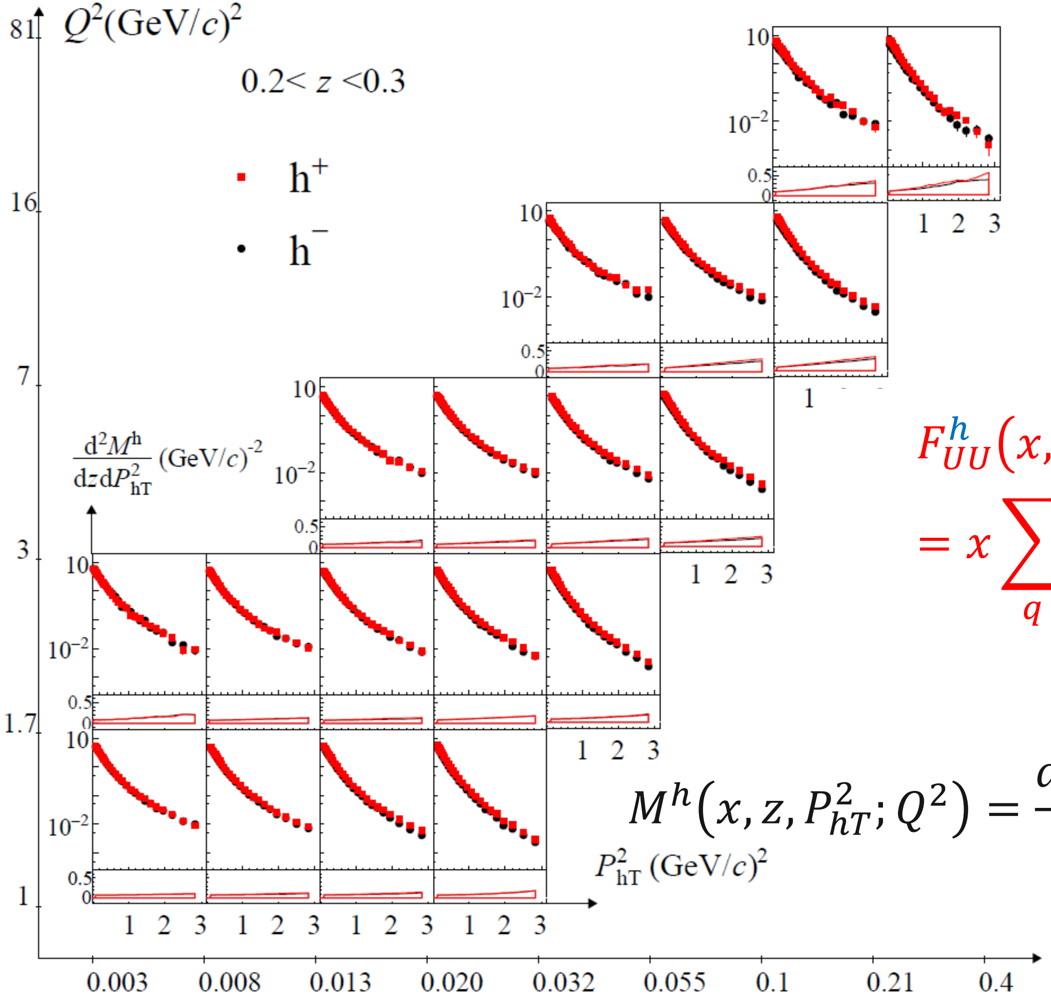
Multiplicity distributions

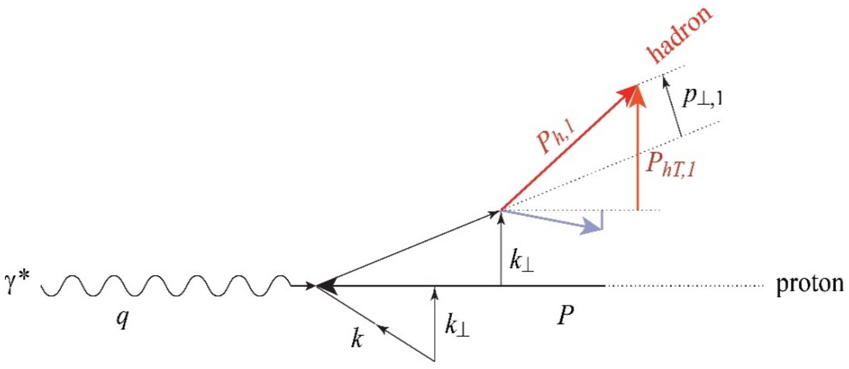
- Unpolarized hadron multiplicity distributions are the basic material for studying the mechanisms of P_{hT} generation and the applicability of TMD factorization.

- It is important to have differential distributions in kinematic variables x, Q^2, z besides P_{hT}

- Not only low P_{hT}. Tails at $P_{hT} > 1$ GeV carries important perturbative & non-perturbative information

Importance of unpolarized SIDIS





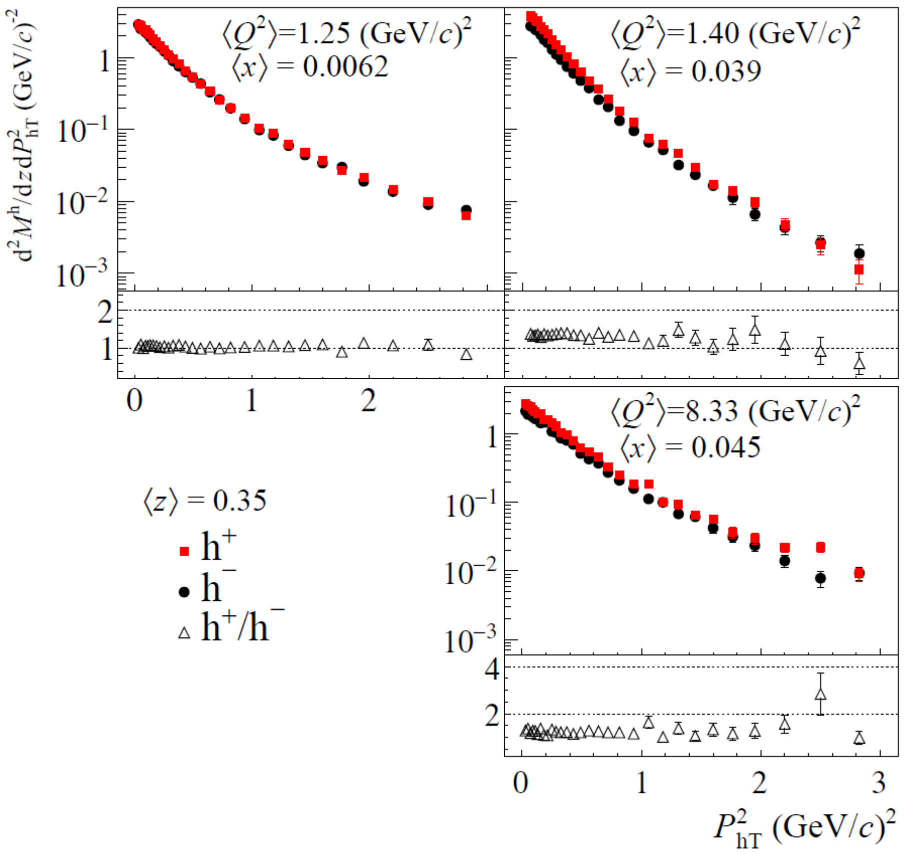
$$F_{UU}^{h}(x,z,P_{hT}^2;\ Q^2)$$

$$= x\sum_q e_q^2 \int d^2\vec{k}_\perp d^2\vec{p}_\perp \delta(\vec{p}_\perp + z\vec{k}_\perp)$$

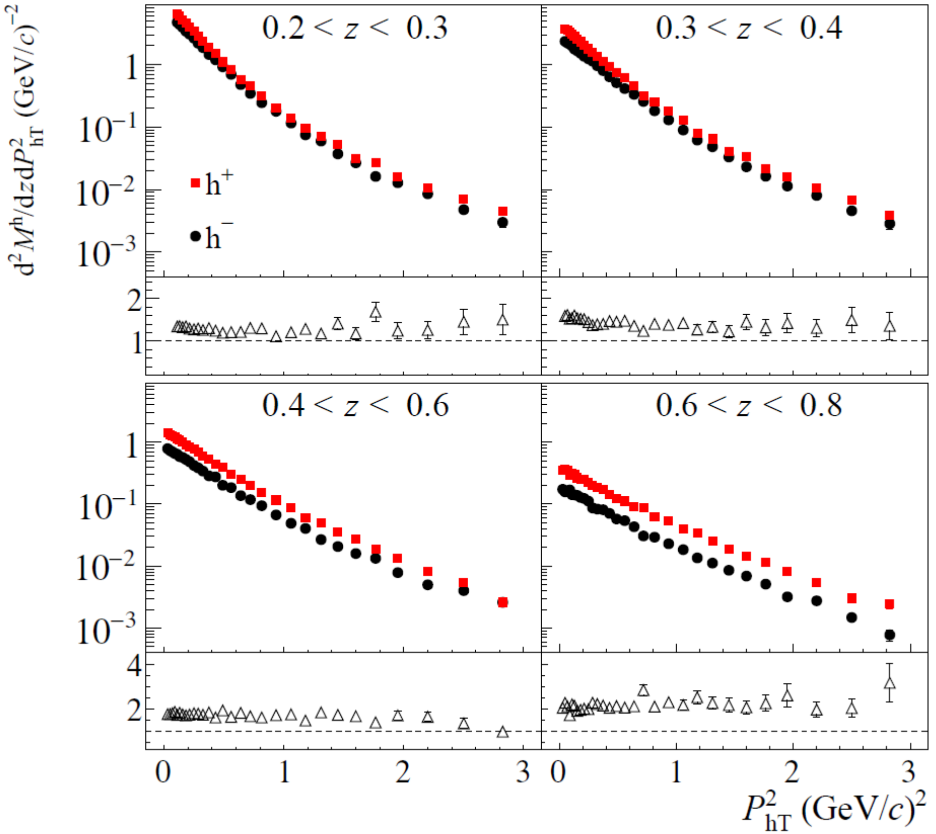
$$M^h(x,z,P_{hT}^2;Q^2) = \frac{d^5\sigma^h/dxdQ^2dzd^2\vec{p}_T}{d^2\sigma^{DIS}/dxdQ^2} \sim \frac{F_{UU}^h(x,z,P_{hT}^2;\ Q^2)}{F_{UU,T}+\varepsilon F_{UU,L}}$$

4918 data points

Positive vs Negative charged hadrons

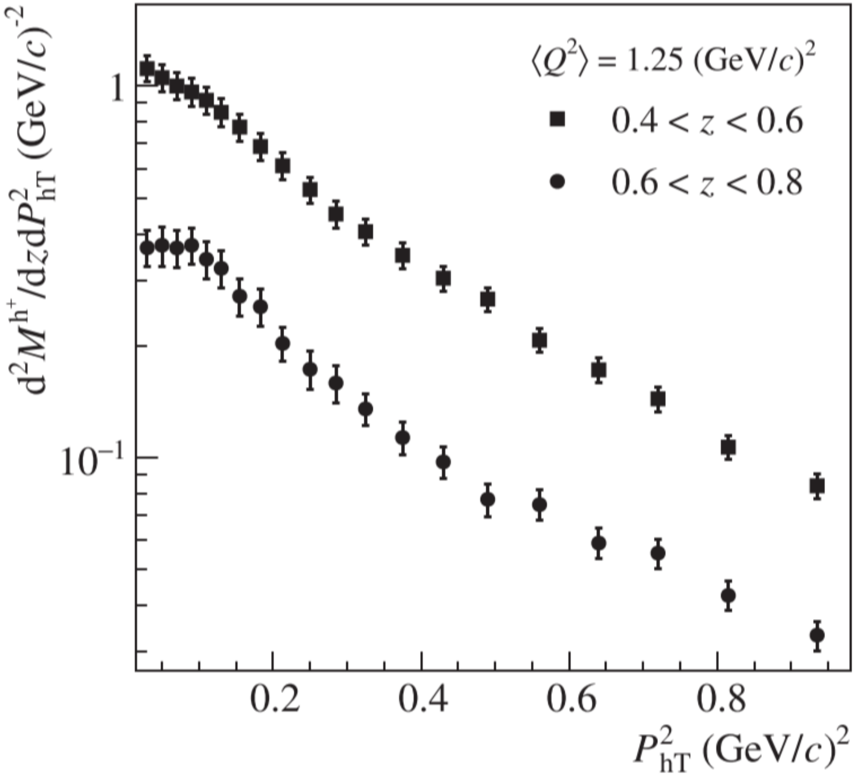


$\langle Q^2\rangle = 9.78\ (\mathrm{GeV}/c)^2$ and $\langle x\rangle = 0.149$

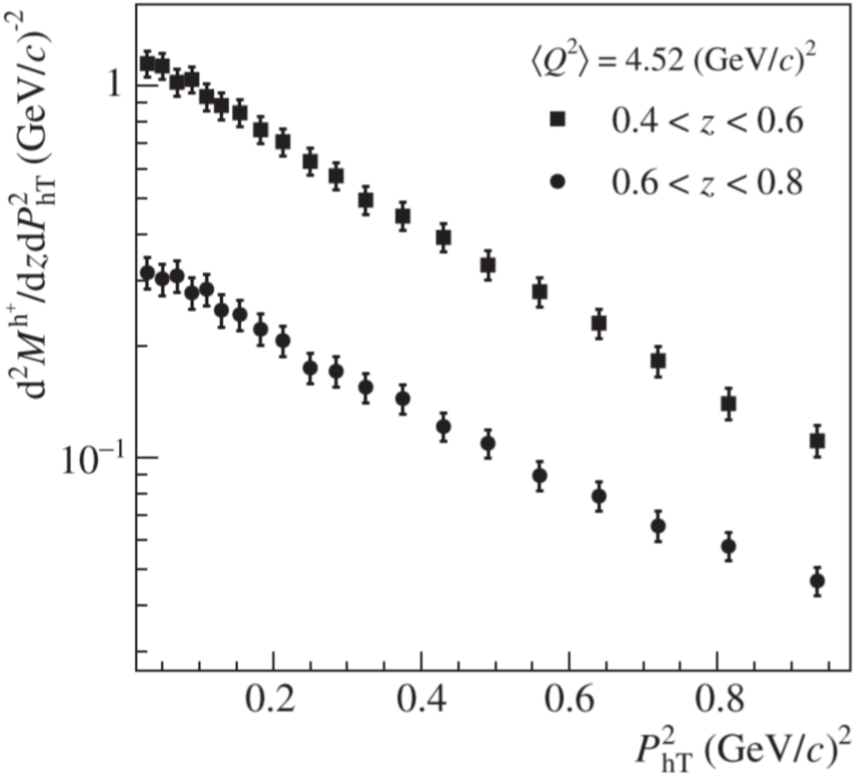


Low P_{hT} behavior

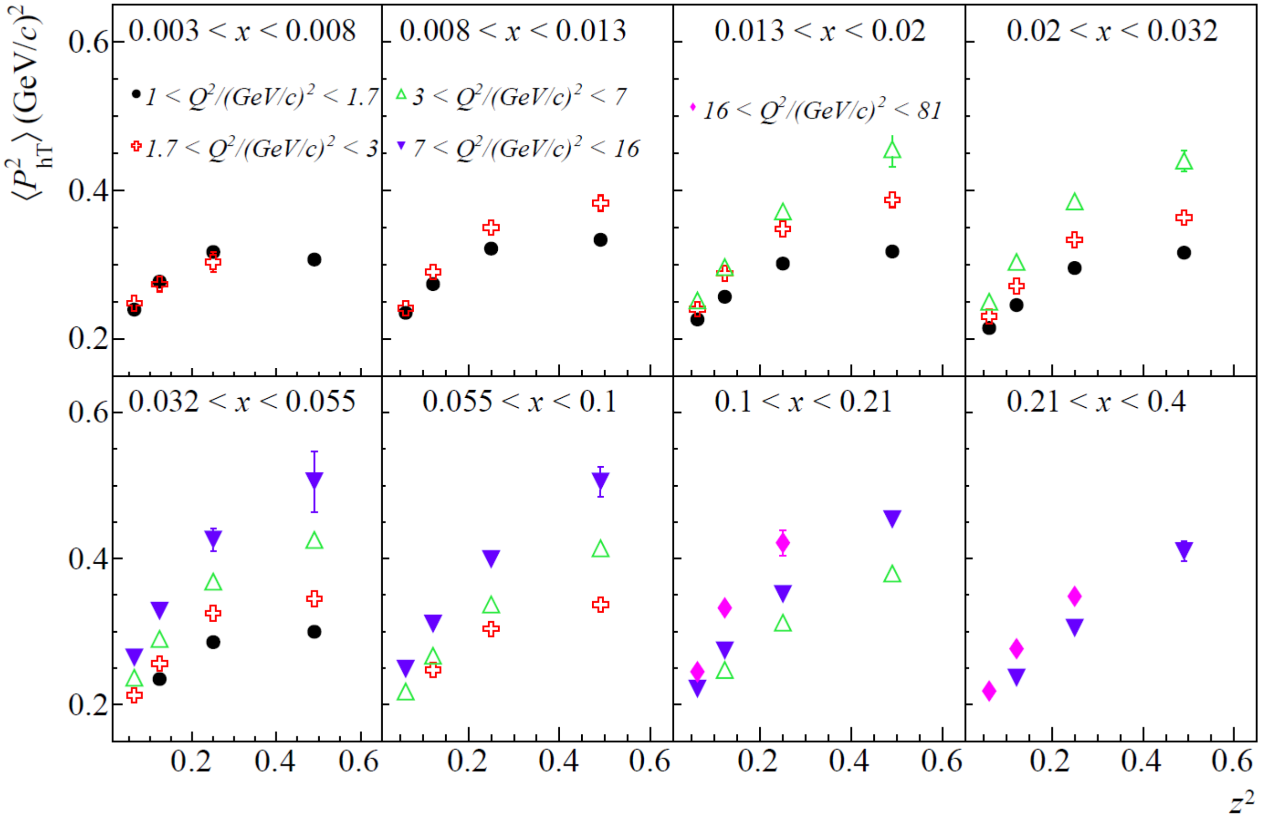
$\langle x \rangle = 0.006$

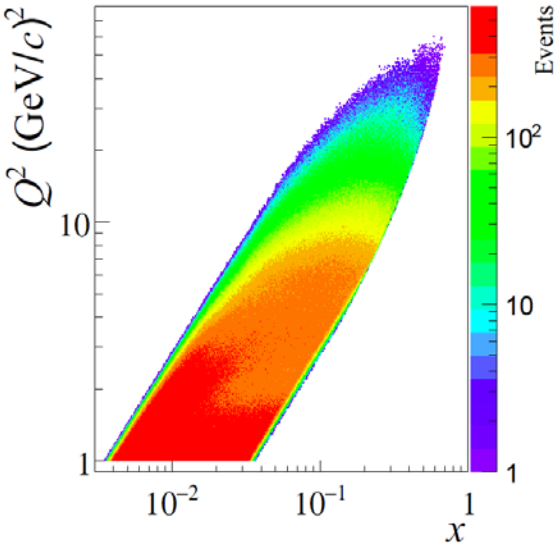


$\langle x \rangle = 0.043$



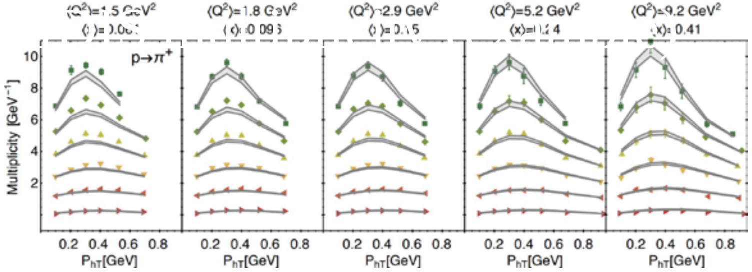
Mean values

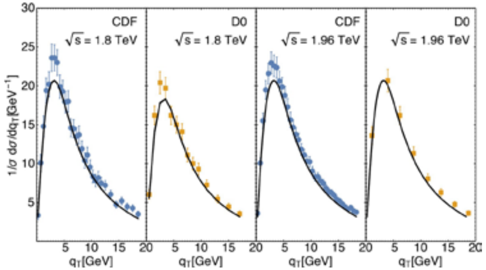

$\langle P^2_{hT} \rangle$ (GeV/c)^2
0.6
0.4
0.2
$0.003 < x < 0.008$
$1 < Q^2/(\mathrm{GeV}/c)^2 < 1.7$
$1.7 < Q^2/(\mathrm{GeV}/c)^2 < 3$
$0.008 < x < 0.013$
$3 < Q^2/(\mathrm{GeV}/c)^2 < 7$
$7 < Q^2/(\mathrm{GeV}/c)^2 < 16$
$0.013 < x < 0.02$
$16 < Q^2/(\mathrm{GeV}/c)^2 < 81$
$0.02 < x < 0.032$
$0.032 < x < 0.055$
$0.055 < x < 0.1$
$0.1 < x < 0.21$
$0.21 < x < 0.4$
0.2
0.4
0.6
z^2


Q^2 (GeV/c)^2
10
1
10^{-2}
10^{-1}
1
x
Events
10^2
10
1

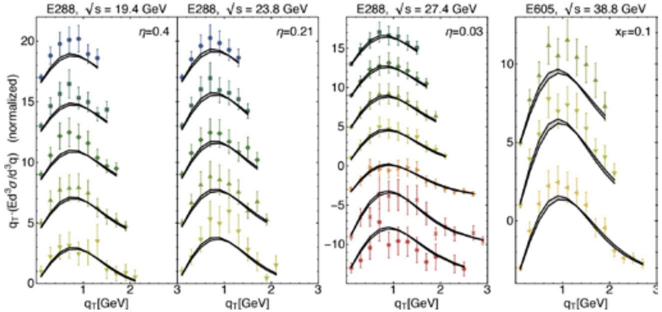
Global Analysis: Unpolarized TMD

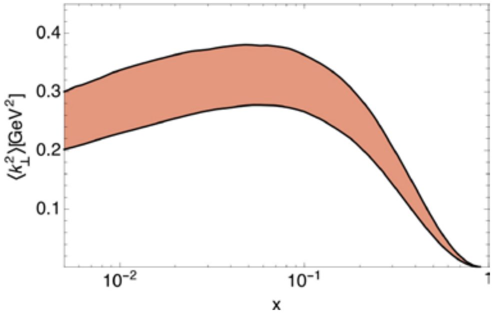
Global analysis of semi-inclusive DIS, Drell-Yan and Z production data with TMD evolution





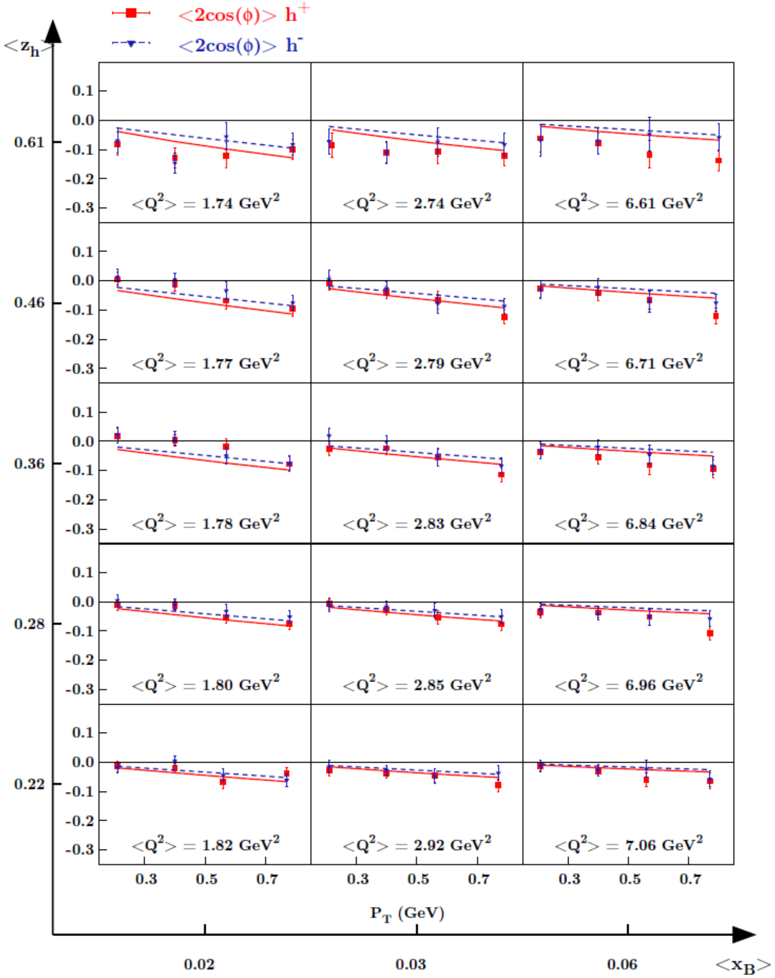
Transverse momentum distribution

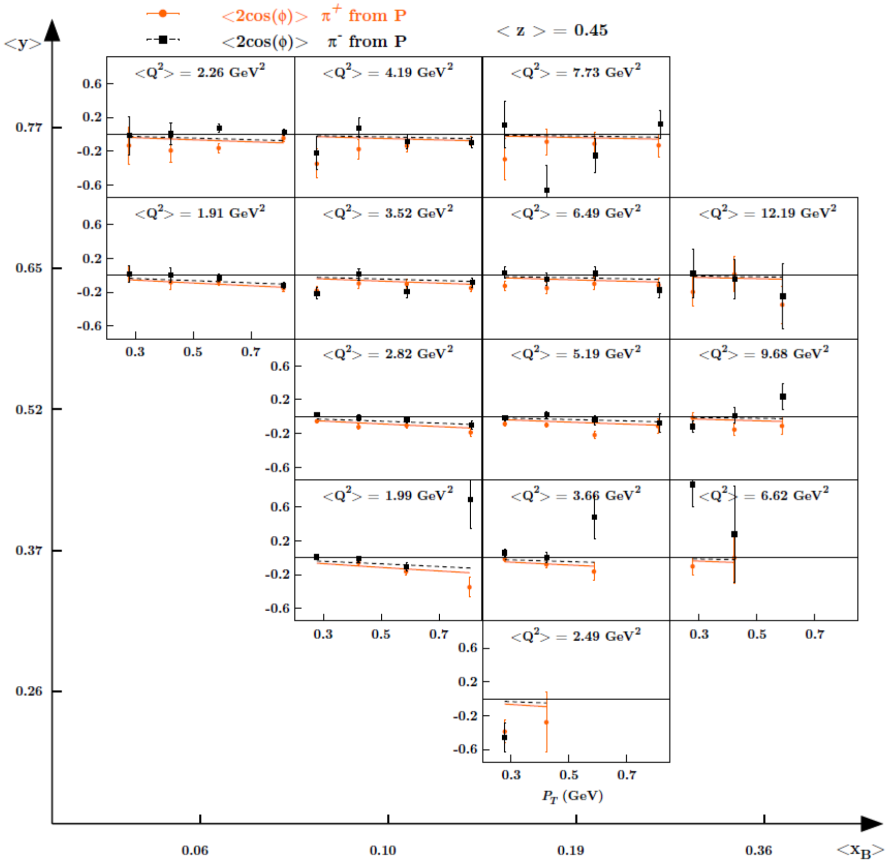




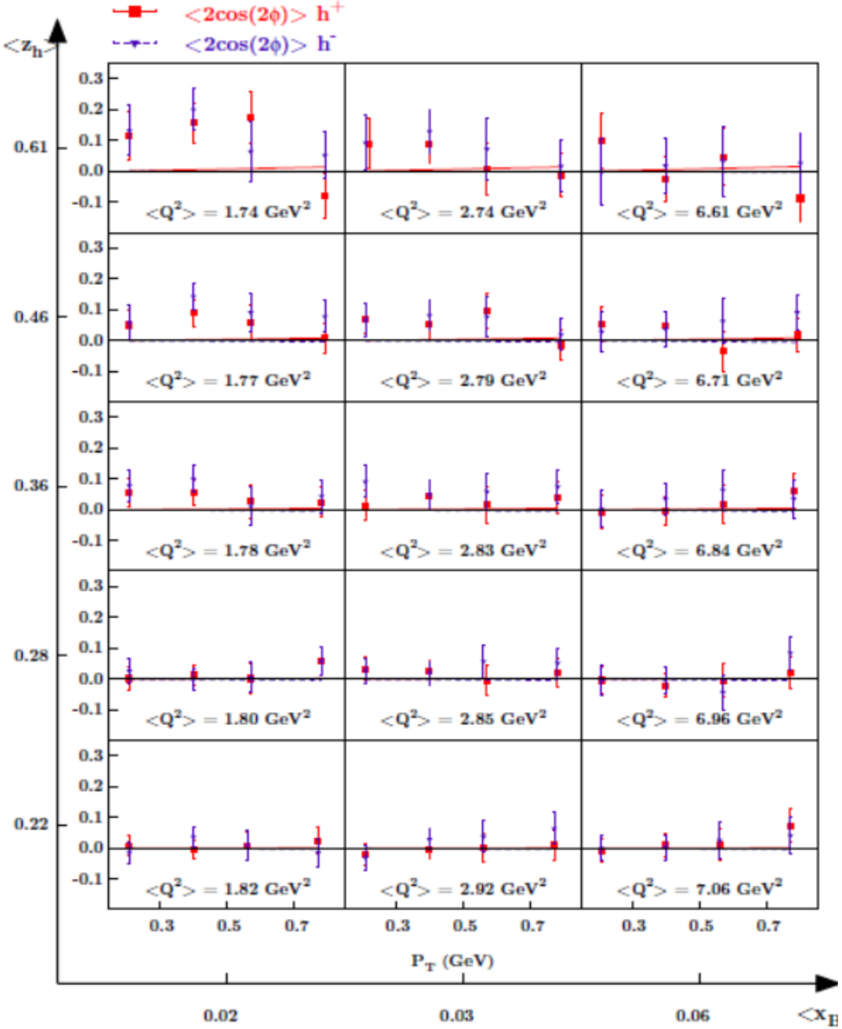
A. Bacchetta *et al.*, J. High Energy Phys. 06 (2017) 081.

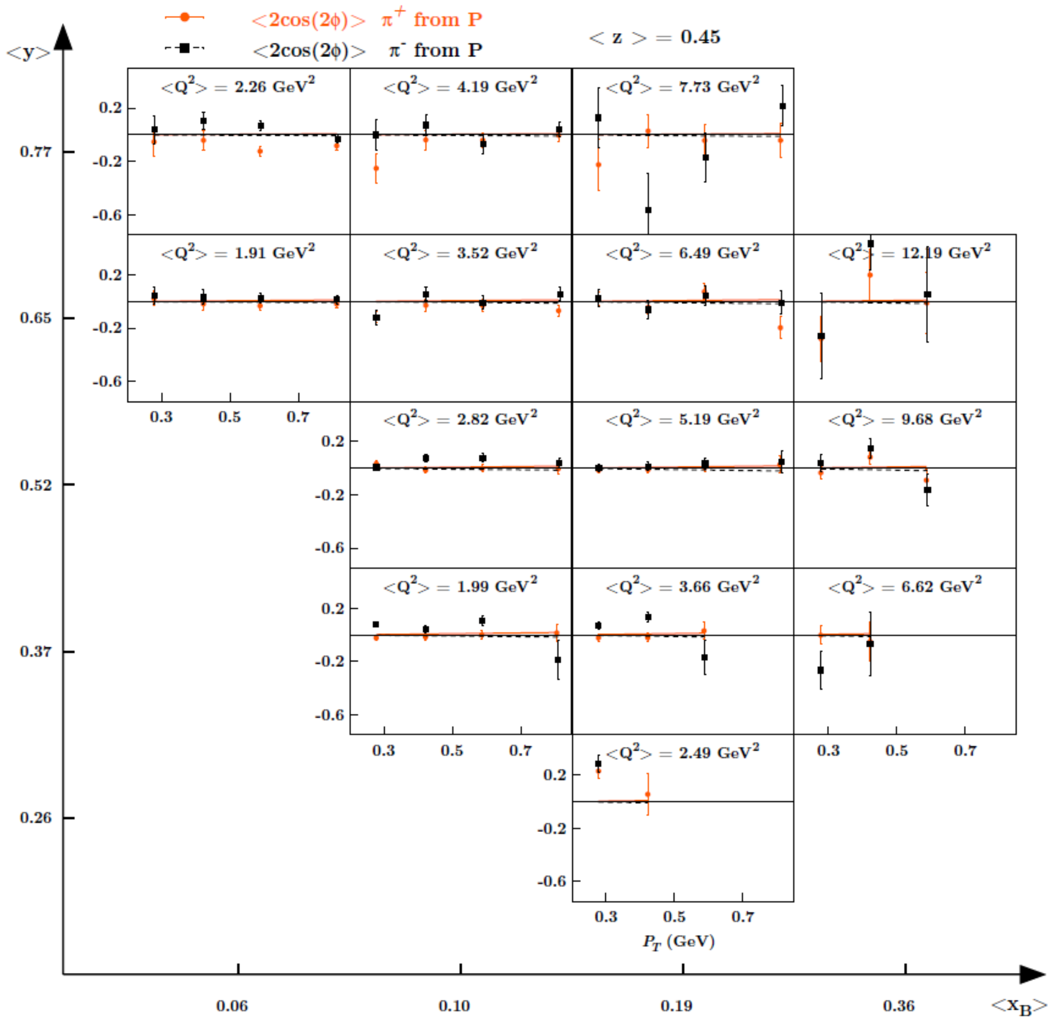
cos ϕ modulation





Boer-Mulders in $\cos 2\phi$ and in $\cos \phi$





The asymmetries

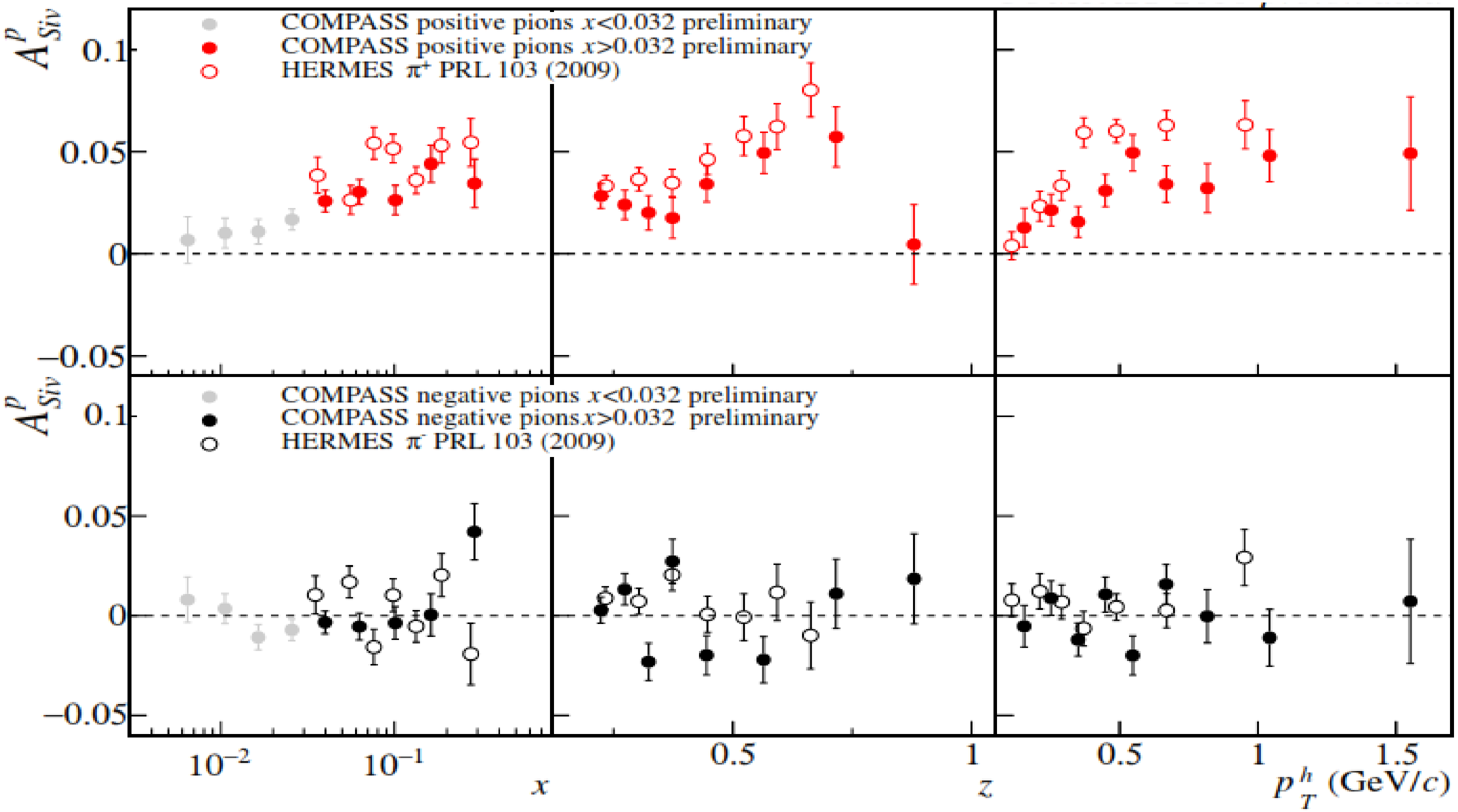
- The asymmetries are:
- $$A_{U(L),T}^{w(\phi_h,\phi_S)}(x,z,p_T;Q^2) = \frac{F_{U(L),T}^{w(\phi_h,\phi_S)}}{F_{UU,T}+\varepsilon F_{UU,L}}$$
- When we measure on 1D
- $$A_{U(L),T}^{w(\phi_h,\phi_S)}(x) = \frac{\int_{Q^2_{min}}^{Q^2_{max}} dQ^2 \int_{z_{min}}^{z_{max}} dz \int_{p_{T,min}}^{p_{T,max}} d^2\vec{p}_T F_{U(L),T}^{w(\phi_h,\phi_S)}}{\int_{Q^2_{min}}^{Q^2_{max}} dQ^2 \int_{z_{min}}^{z_{max}} dz \int_{p_{T,min}}^{p_{T,max}} d^2\vec{p}_T \left(F_{UU,T}+\varepsilon F_{UU,L}\right)}$$

Sivers Asymmetry

- Sivers: correlates nucleon spin & quark transverse momentum k_T /T-ODD: at LO for $\mu p^{\uparrow} \rightarrow \mu X h^{\pm}$
- $A_{Siv}(x,z) = \frac{F_{UT}^{\sin\Phi_{Siv}}(x,z)}{F_{UU}(x,z)} = \frac{\sum_q e_q^2 x f_{1T}^{\perp q}(x, k_\perp^2) \otimes D_{1q}^h(z, p_\perp^2)}{\sum_q e_q^2 x f_1^q(x, k_\perp^2) \otimes D_{1q}^h(z, p_\perp^2)}$
- To evaluate it we need to solve the convolutions (i.e. make hypothesis on the transverse momenta dependences of the TMDs)
- Gaussian ansatz: $f_{1T}^{\perp q}(x) \frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_S}}{\pi \langle k_\perp^2\rangle_S}$ $D_{1q}^h(z) \frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi \langle p_\perp^2\rangle}$
- Leading to: $A_{Siv,G}(x,z) = \frac{\sqrt{\pi} M}{\sqrt{z^2 \langle k_T^2\rangle_S + \langle p_T^2\rangle}} \frac{\sum_q e_q^2 \, x f_{1T}^{\perp(1)q}(x) z D_{1q}^h(z)}{\sum_q e_q^2 x f_1^q(x) D_{1q}^h(z)}$ with $f_{1T}^{\perp(1)q}(x) = \int d^2\vec{k}_T \frac{k_T^2}{2M^2} f_{1T}^{\perp q}(x, k_T^2)$

Sivers asymmetry on p

charged pions (and kaons), HERMES and COMPASS



π+

●

○

π-

●

○

The weighted Sivers asymmetry

- If we weight the spin dependent part of the cross-section

$$F_{UT}^{sin\Phi_{Siv}}(x,z) = \Sigma_q\ e_q^2 \int d^2\vec{P}_T P_T F_q(x,z,P_T^2)$$

- with $w = P_T/zM$, i.e.

$$F_{UT}^{sin\Phi_{Siv},w}(x,z) = \Sigma_q\ e_q^2 \int d^2\vec{P}_T \frac{P_T^2}{zM} F_q(x,z,P_T^2) = 2\ \Sigma_q\ e_q^2 x f_{1T}^{\perp(1)q}(x) D_{1q}^h(z)$$

and $F_q(x,z,P_T^2) = \int d^2\vec{k}_T \int d^2\vec{p}_T\ \delta^2\big(\vec{P}_T - z\vec{k}_T - \vec{p}_T\big) \frac{\vec{P}_T\cdot\vec{k}_T}{MP_T^2} x f_{1T}^{\perp q}(x,k_T^2)\ D_{1q}(z,p_T^2)$

- we have no longer a convolution but a product of two integrals and we can write

$$A_{Siv}^{w}(x,z) = \frac{F_{UT}^{sin\Phi_{Siv},w}(x,z)}{F_{UU}(x,z)} = 2\,\frac{\Sigma_q\, e_q^2 x f_{1T}^{\perp(1)q}(x) D_{1q}^h\,(z)}{\Sigma_q\, e_q^2 x f_1^q(x) D_{1q}^h(z)}$$

- with $f_{1T}^{\perp(1)q}(x) = \int d^2\vec{k}_T \frac{k_T^2}{2M^2} f_{1T}^{\perp q}(x,k_T^2)$

The weighted Sivers asymmetry

- In one dimension: for x

$$A^{w}_{Siv}(x) = 2\frac{\sum_q e_q^2 x f_{1T}^{\perp(1)q}(x) \int D_{1q}^{h}(z)dz}{\sum_q e_q^2 x f_1^{q}(x) \int D_{1q}^{h}(z)dz}$$

and for z

$$A^{w}_{Siv}(z) = 2\frac{\sum_q e_q^2 D_{1q}^{h}(z) \int C(x) x f_{1T}^{\perp(1)q}(x)dx}{\sum_q e_q^2 D_{1q}^{h}(z) \int C(x) x f_1^{q}(x)dx}$$

with $C(x) = \int_{\Omega_y} dy \frac{1-y+y^2/2}{x^2y^2}$

- Note that assuming u-dominance at large x for positive hadrons:

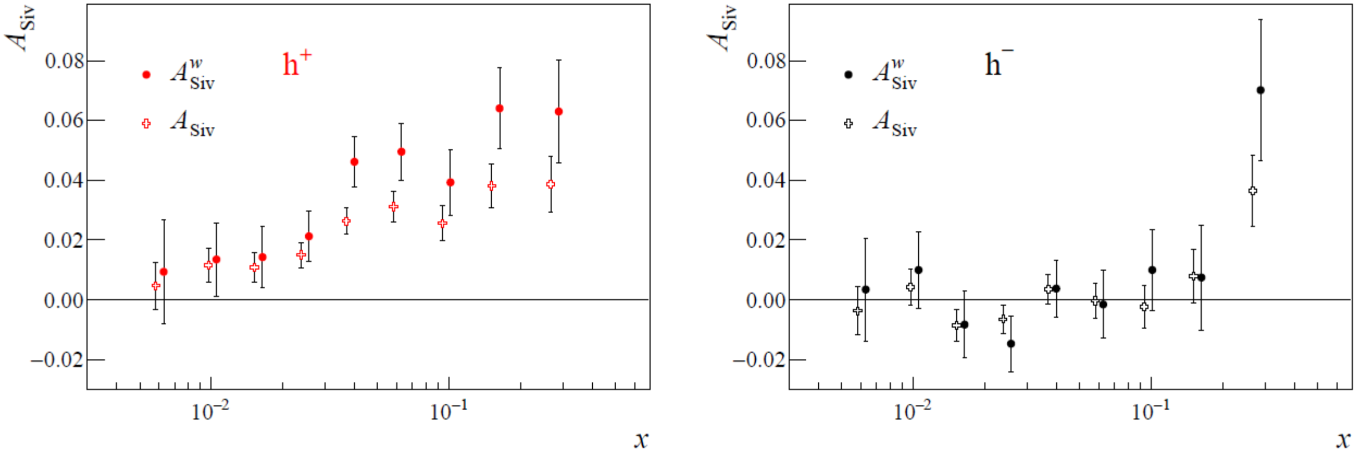
$$A^{w,h^+}_{Siv}(x) \cong 2\frac{f_{1T}^{\perp(1)u}(x)}{f_1^{u}(x)} \quad \text{and} \quad A^{w,h^+}_{Siv}(z) = 2\frac{\int C(x) x f_{1T}^{\perp(1)u}(x)dx}{\int C(x) x f_1^{u}(x)dx}$$

The weighted Sivers asymmetry

$$A^{w}_{Siv}(x) = 2\frac{\sum_q e_q^2 x f_{1T}^{\perp(1)q}(x) \int D_{1q}^h(z)dz}{\sum_q e_q^2 x f_1^q(x) \int D_{1q}^h(z)dz}$$

$w = P_T/zM$

standard cuts
z>0.2



$\sim 2\frac{f_{1T}^{\perp(1)u}(x)}{f_1^u(x)}$

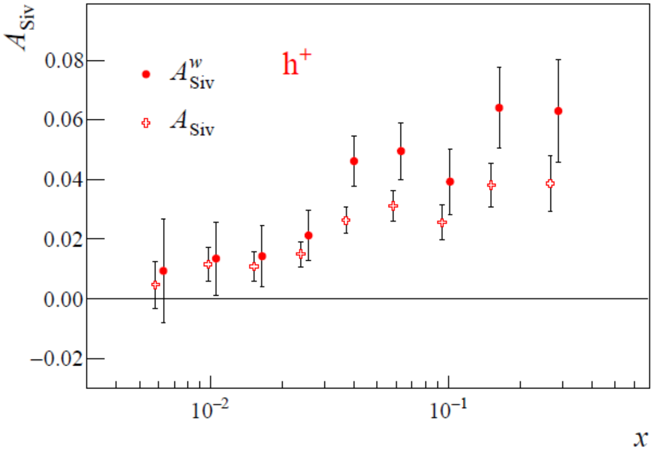
both $f_{1T}^{\perp(1)u}$ and $f_{1T}^{\perp(1)d}$
contribute

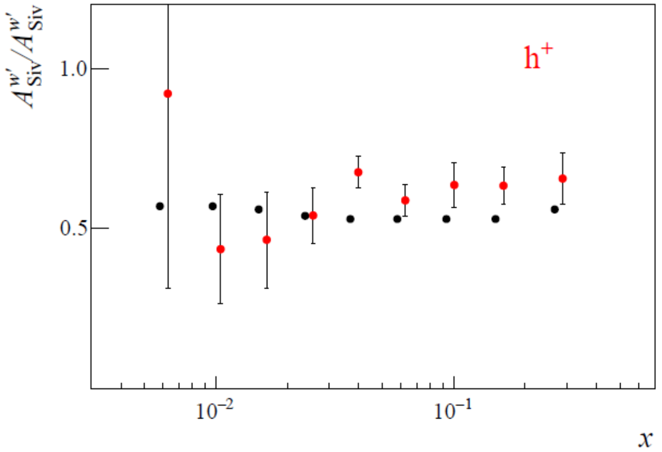
C. Adolph *et al.* [COMPASS Collaboration], Phys. Lett.
B **717** (2012) 383.

The weighted Sivers asymmetry

$$A^{w}_{Siv}(x) = 2\,\frac{\sum_q e_q^2 x f_{1T}^{\perp(1)q}(x)\int D_{1q}^h(z)dz}{\sum_q e_q^2 x f_1^q(x)\int D_{1q}^h(z)dz} \qquad w = P_T/zM$$

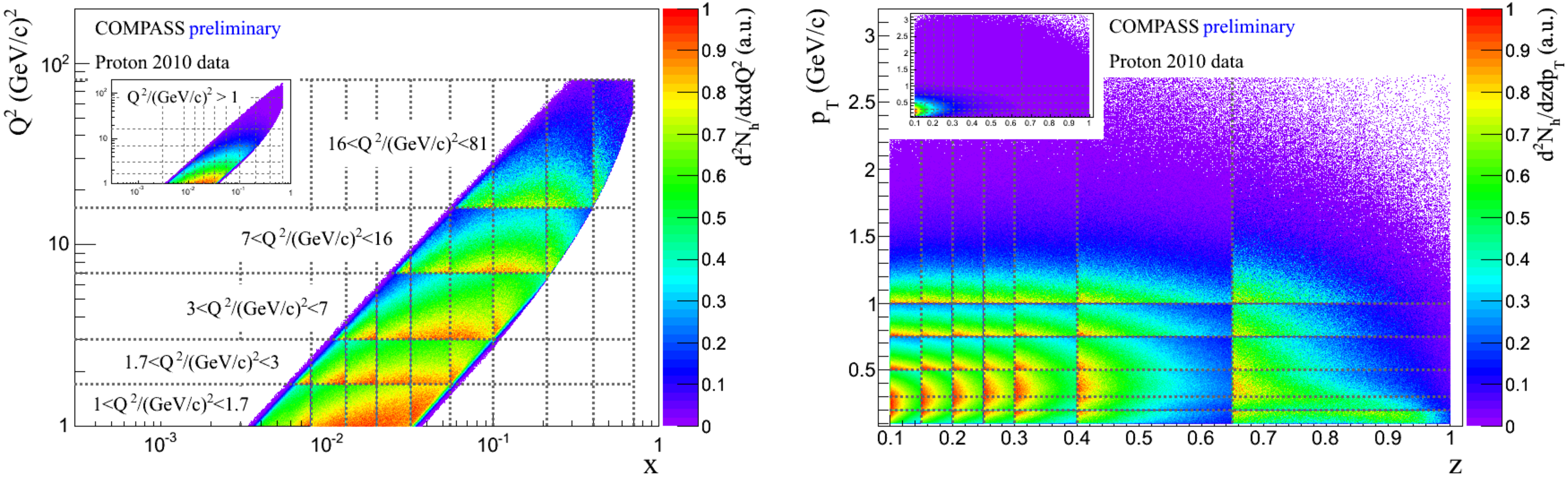
standard cuts
z>0.2





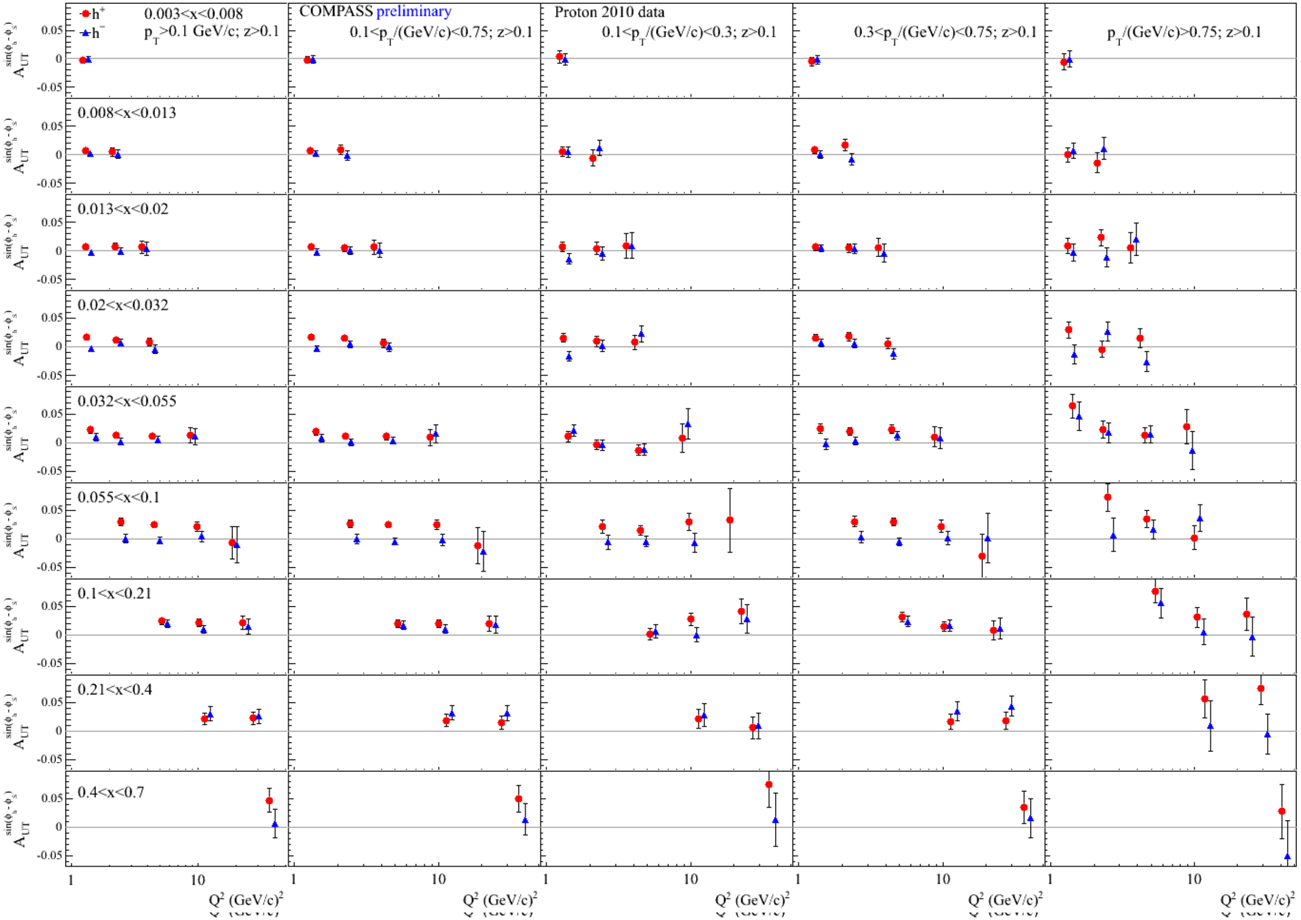
The ratio between weighted and unweighted Sivers asymmetries follows the average of $\left\langle \frac{P_{hT}}{zM} \right\rangle$ of the unpolarised sample

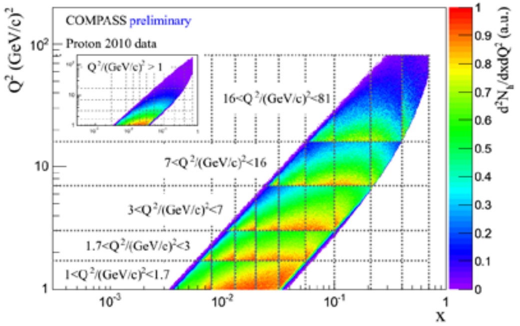
Multi D

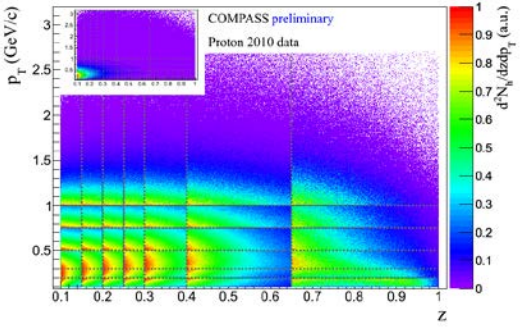

COMPASS preliminary
Proton 2010 data
$Q^2/(GeV/c)^2 > 1$
$16<Q^2/(GeV/c)^2<81$
$7<Q^2/(GeV/c)^2<16$
$3<Q^2/(GeV/c)^2<7$
$1.7<Q^2/(GeV/c)^2<3$
$1<Q^2/(GeV/c)^2<1.7$
Q^2 (GeV/c)^2
10^2
10
1
10^{-3}
10^{-2}
10^{-1}
1
x
$d^2N_h/dxdQ^2$ (a.u.)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
COMPASS preliminary
Proton 2010 data
p_T (GeV/c)
0.5
1
1.5
2
2.5
3
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
z
$d^2N_h/dzdp_T$ (a.u.)

Sivers asymmetry on proton. Multidimensional

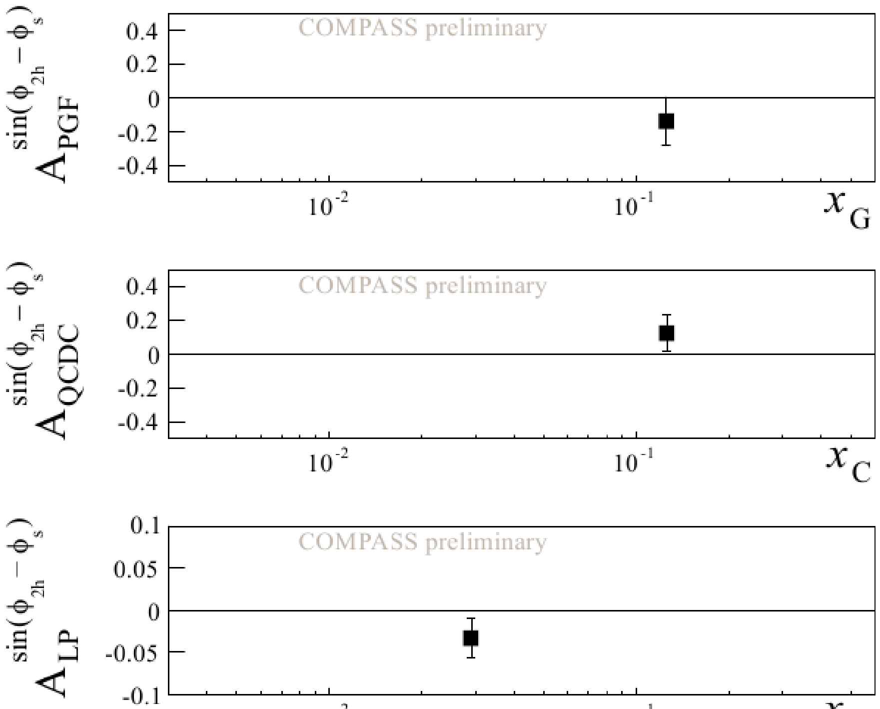
Extraction of TSAs with a Multi-D $\left(x{:}\,Q^2{:}\,z{:}\,p_T\right)$ approach

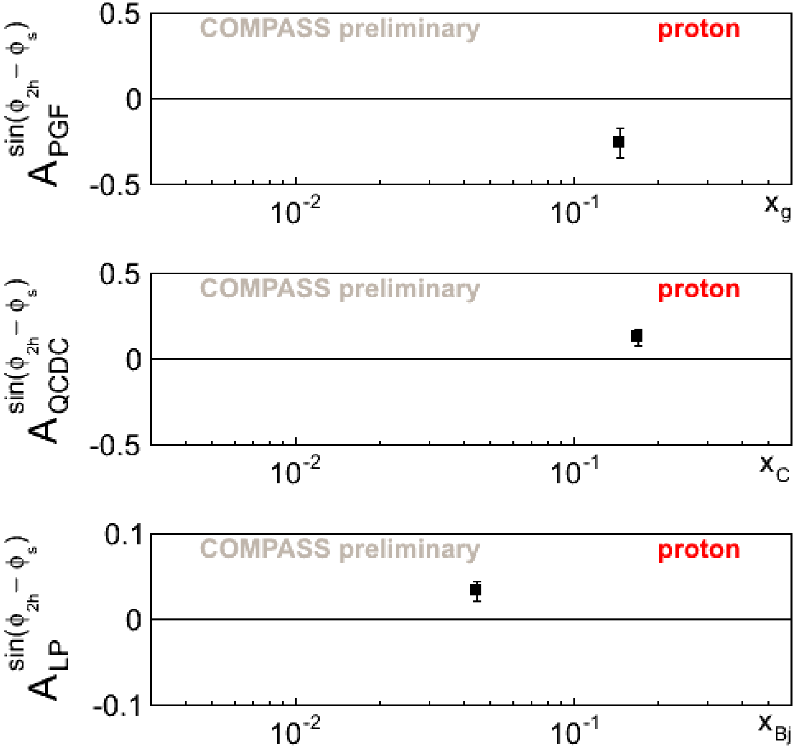






Sivers asymmetry on deuteron and proton for Gluons





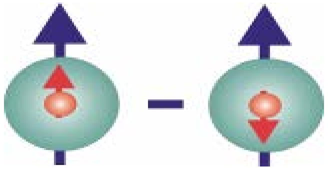
Transversity PDF

h_1^q (x) = $q^{\uparrow\uparrow}$(x) - $q^{\uparrow\downarrow}$(x)

$\Delta_T q(x)$,

$\delta q(x)$,

$\delta_T q(x)$



q=u_v, d_v, q_{sea}

quark with **spin** parallel to the nucleon spin in a transversely polarised nucleon

- probes the relativistic nature of quark dynamics
- no contribution from the gluons → simple Q^2 evolution
- Positivity: Soffer bound…………….. $2|h_1| \leq q + \Delta q$ *Soffer, PRL 74 (1995)*
- first moments: tensor charge………. $\delta q \equiv \int dx \left[h_1^q(x) - h_1^{\bar{q}}(x) \right]$
- sum rule for transverse spin in PM… $\frac{1}{2} = \frac{1}{2} \sum h_1^q + L_q + L_g$ *Bakker, Leader, Trueman, PRD 70 (04)*
- is chiral-odd: decouples from inclusive DIS

Collins asymmetry on proton

x > 0.032 region

charged pions

COMPASS and HERMES results

π+

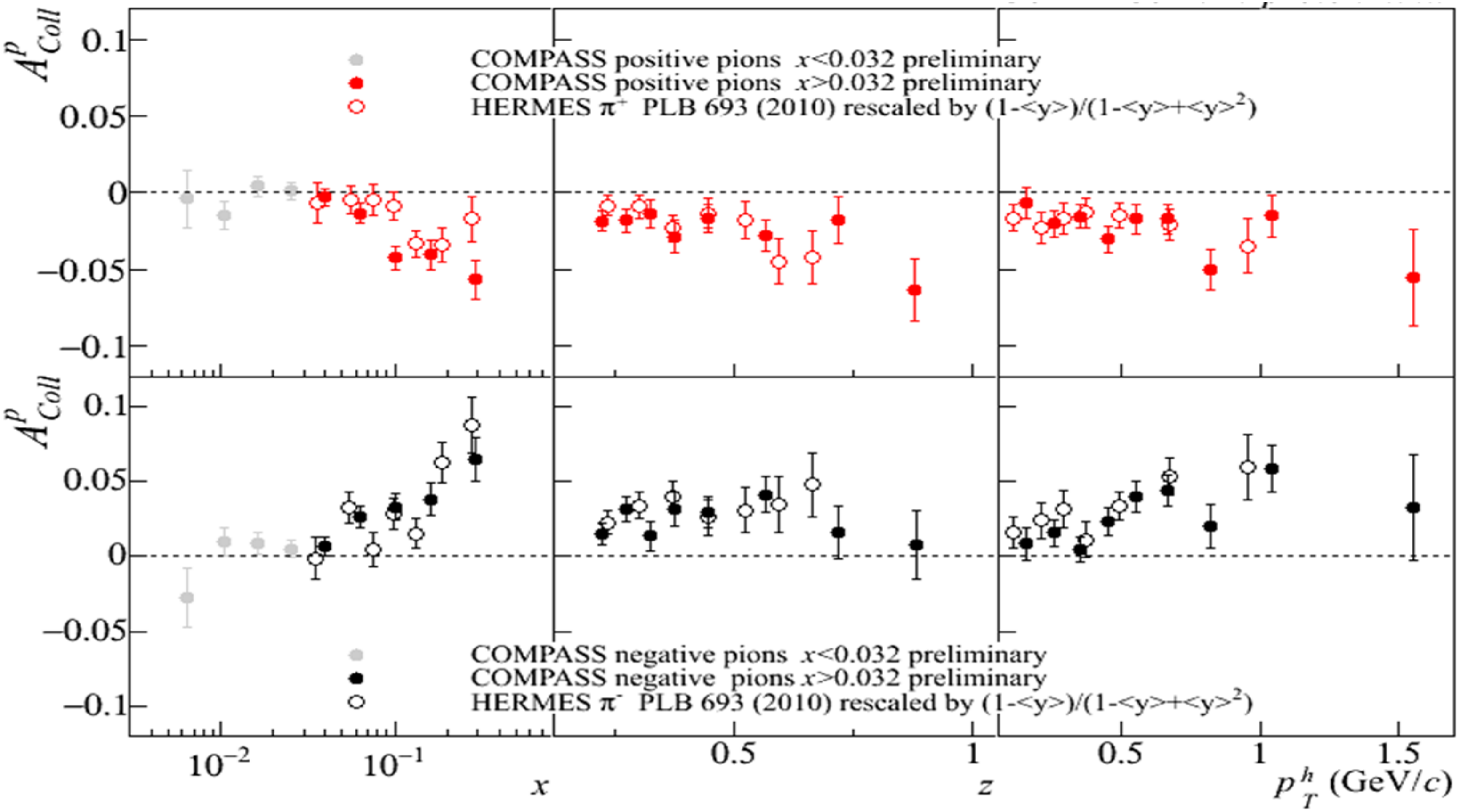
●

○

π-

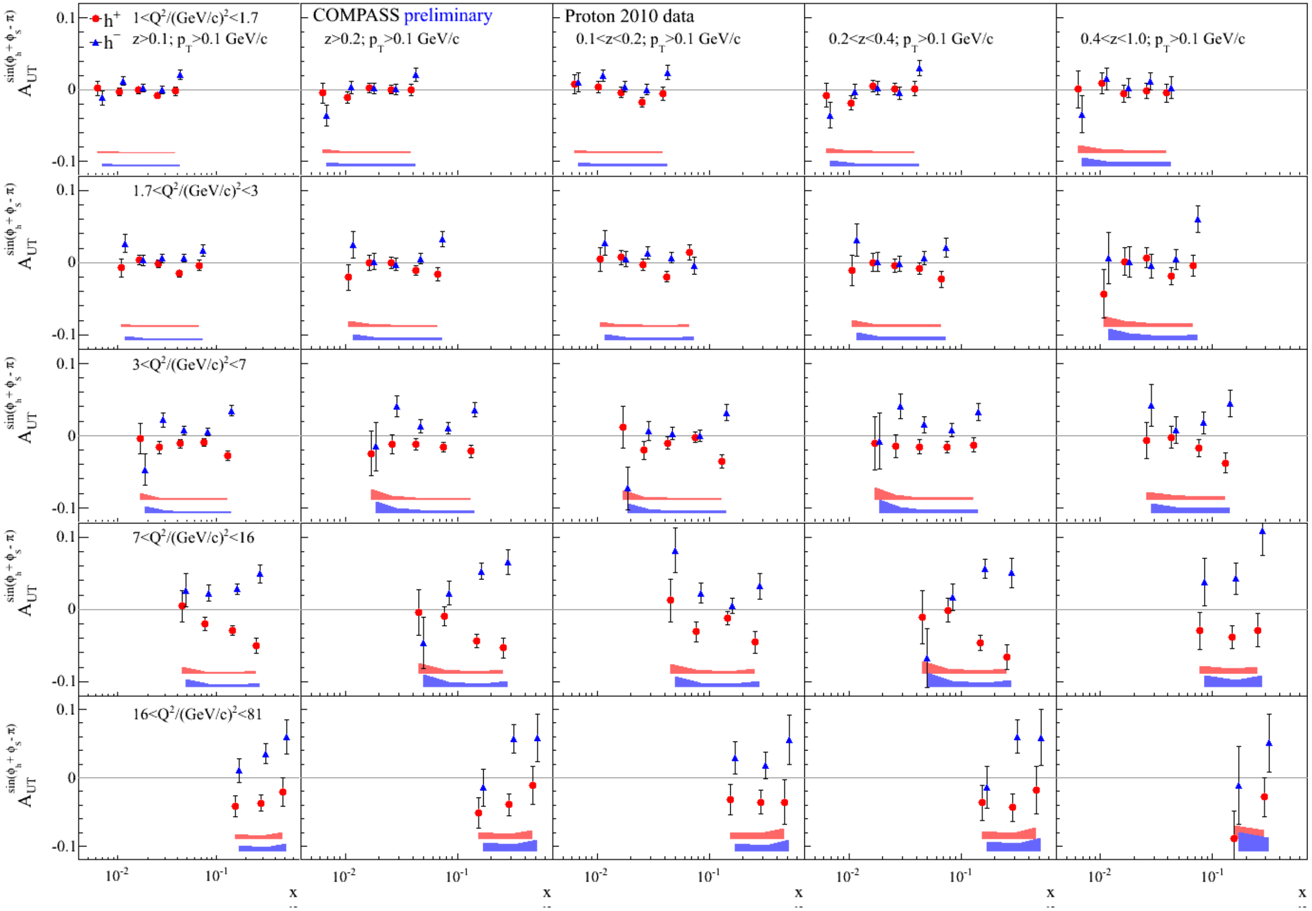
●

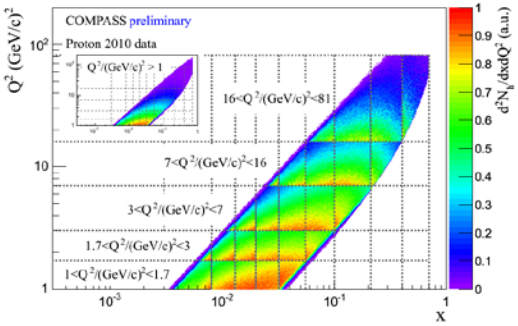
○



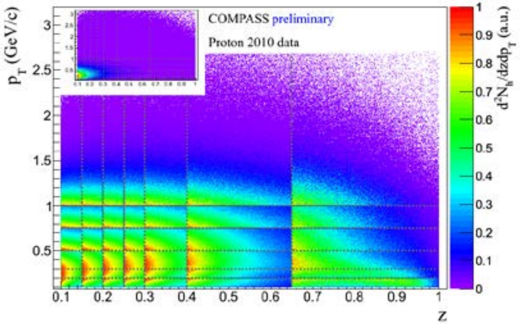
Collins asymmetry on proton. Multidimensional

Extraction of TSAs with a Multi-D $(x: Q^2: z: p_T)$ approach

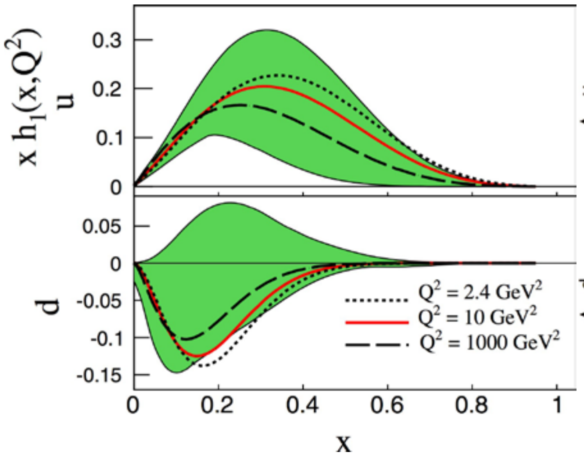




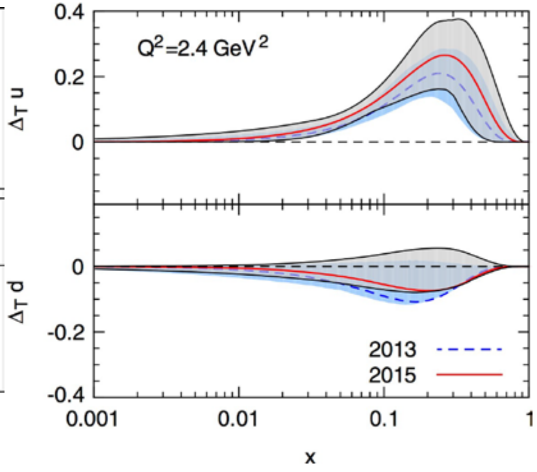
One dense plot out of many



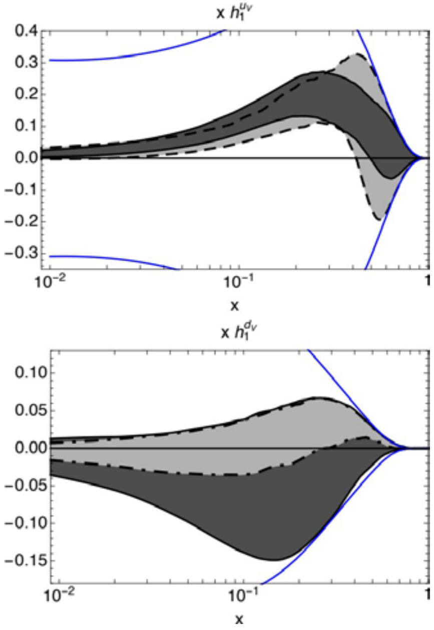
Global Analysis: Transversity



Z.-B. Kang et al., Phys. Rev. D 93, 014009 (2016).



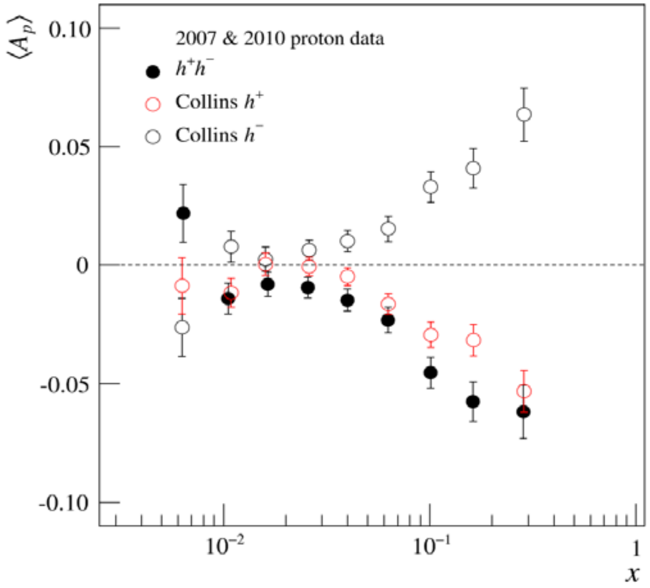
M. Anselmino et al., Phys. Rev. D 92, 114023 (2015).



M. Radici and A. Bacchetta, arXiv: 1802.05212[hep-ph]

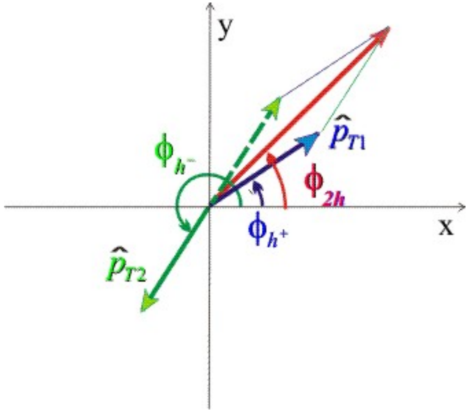
Interplay among dihadron and single hadron asymmetries

- Collins asymmetry for h+ and for h-
  "mirror symmetry"
- dihadron asymmetry
  only somewhat larger than h+ Collins



hints for a common origin
of the Collins FF and DiFF

Como 2013, DSpin2013, PLB736 (2014) 124



look at
the $\Delta\phi = \phi_1 - \phi_2$
dependence of the asymmetries

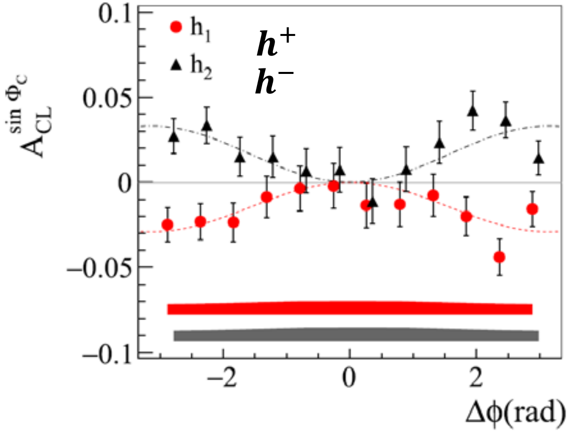
Interplay among dihadron and single hadron asymmetries

PLB 753 (2016) 406

analitically

$$A_{CL1}^{\sin \Phi_C} = a_1 + a_2 \cos \Delta\phi$$
$$A_{CL2}^{\sin \Phi_C} = a_2 + a_1 \cos \Delta\phi$$

mirror symmetry



agreement with data if $a_1 = -a_2 = a$

$$A_{CL\,2h}^{\sin \Phi_{2h,S}} = a\sqrt{2(1 - \cos \Delta\phi)}$$

ratio of the $\Delta\phi$ integrated 2h and 1h asymmetries: $4/\pi$

slightly larger than h^+

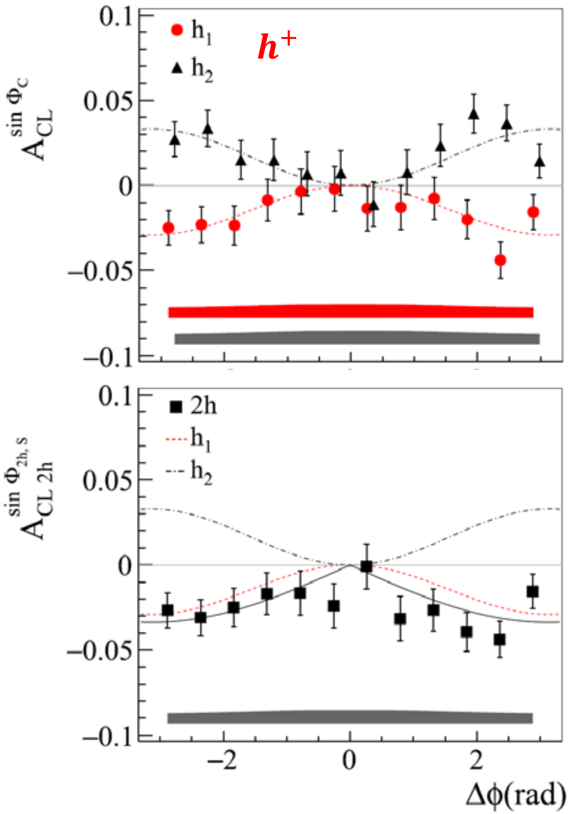
Interplay among dihadron and single hadron asymmetries

PLB 753 (2016) 406

analytically

$$A_{CL1}^{\sin\Phi_C} = a_1 + a_2 \cos\Delta\phi$$
$$A_{CL2}^{\sin\Phi_C} = a_2 + a_1 \cos\Delta\phi$$

mirror symmetry



agreement with data if

⇩

$$A_{CL\ 2h}^{\sin\Phi_{2h,S}} = a\sqrt{2(1-\cos\Delta\phi)}$$

agreement with data

a very simple relationships among the asymmetries in the “2h sample”

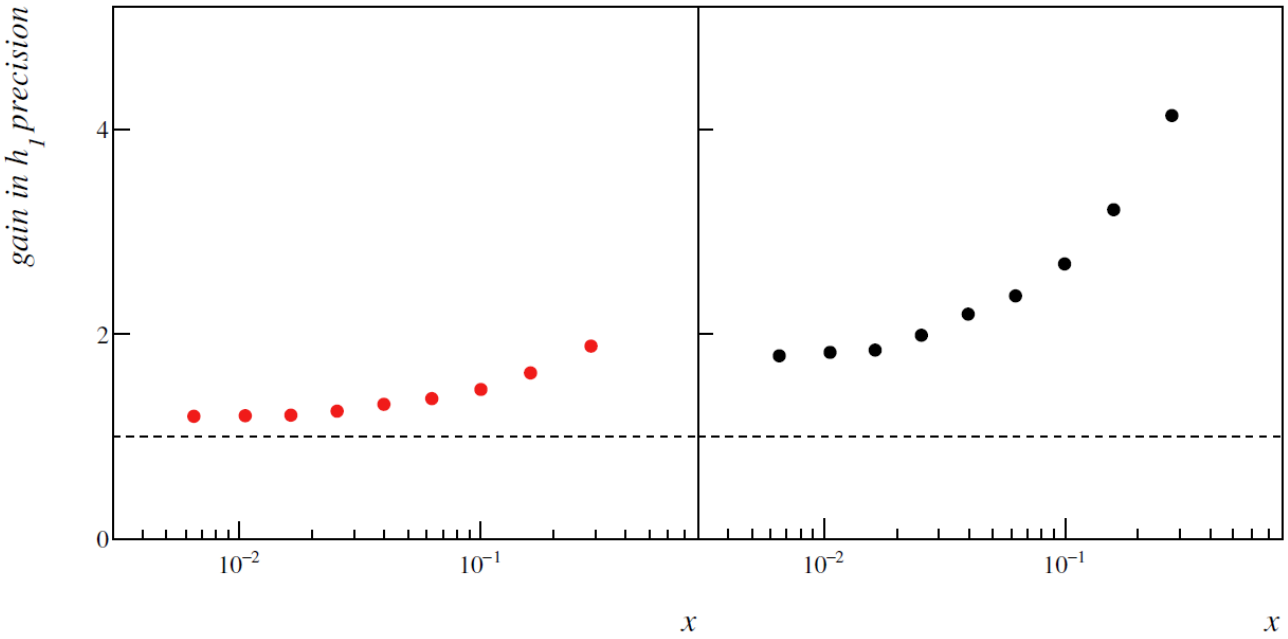
they are driven by the same elementary mechanism.

ratio of the $\Delta\phi$ integrated 2h and 1h asymmetries: 4/π

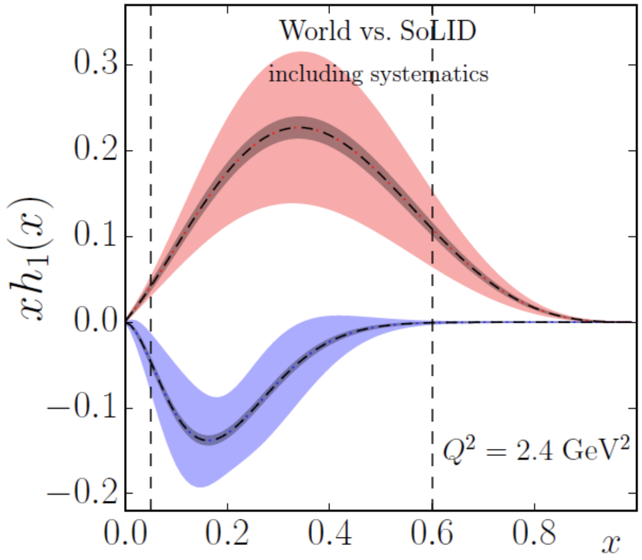
slightly larger than h^+

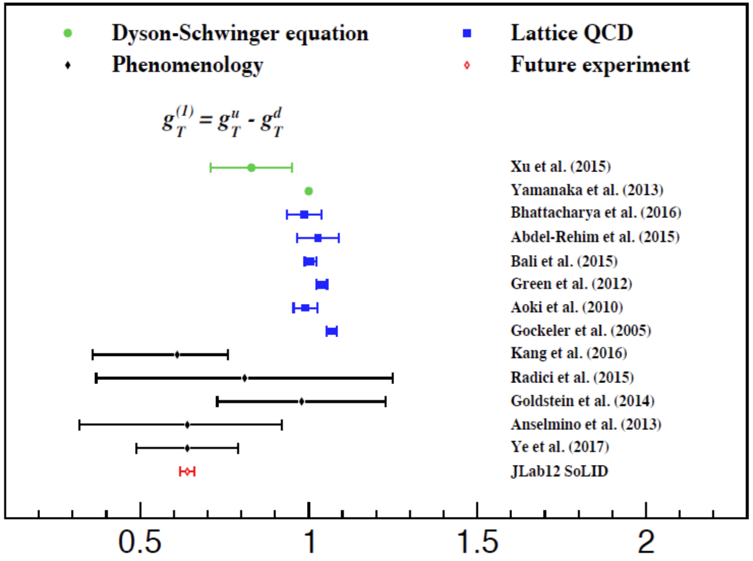
COMPASS deuteron data in 2021

- Expected gain in precision on u- and d-quark transversity

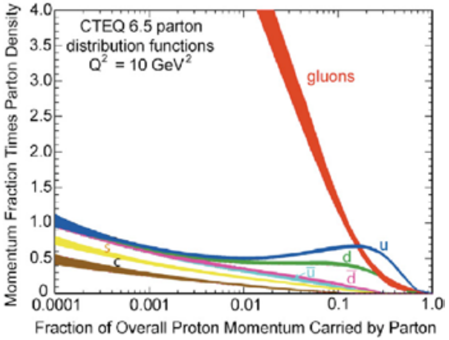


SoLID Impact on h_1 and Tensor Charges

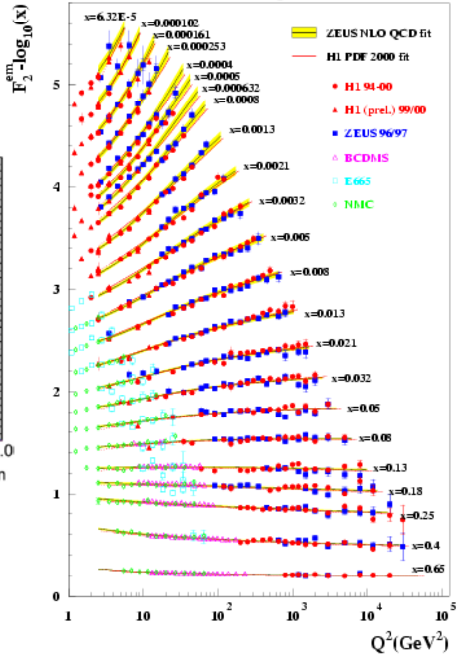




Far Future perspective

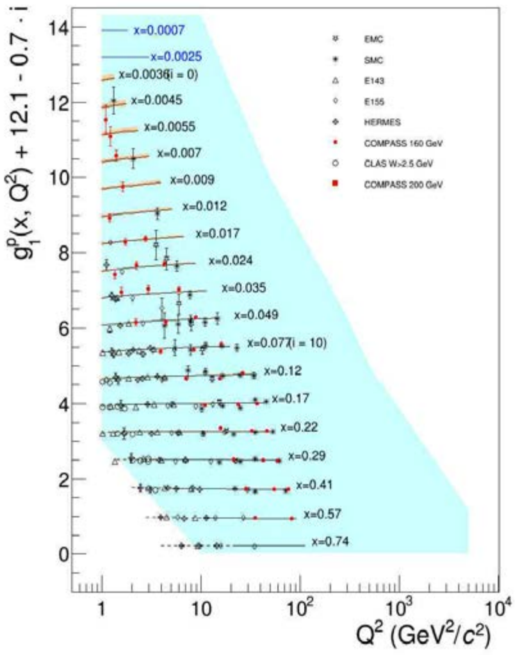


World Data on F_2^p



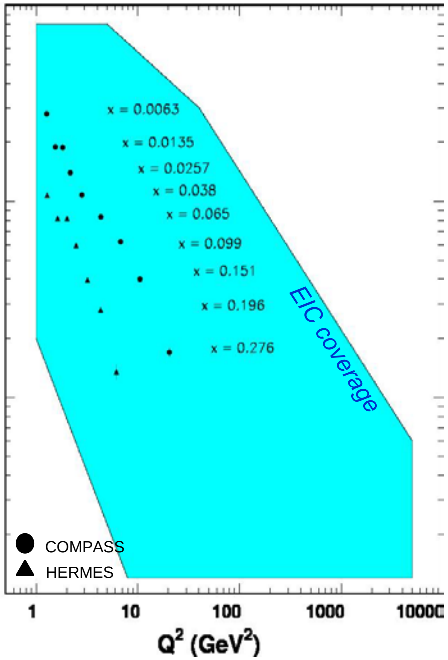
momentum

World Data on g_1^p



spin

World Data on h_1^p



transverse spin ~ angular momentum

Thank you

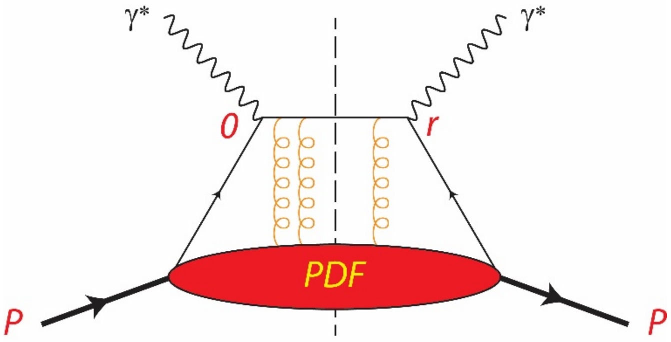
Ordinary PDFs

- The Bj limit ($Q^2, \nu \to \infty$ with $x = Q^2/2M\nu$ fixed) selects scattering on a single parton in the target. $\sigma_{DIS}(\ell N \to \ell' X)$ determines the ordinary PDF

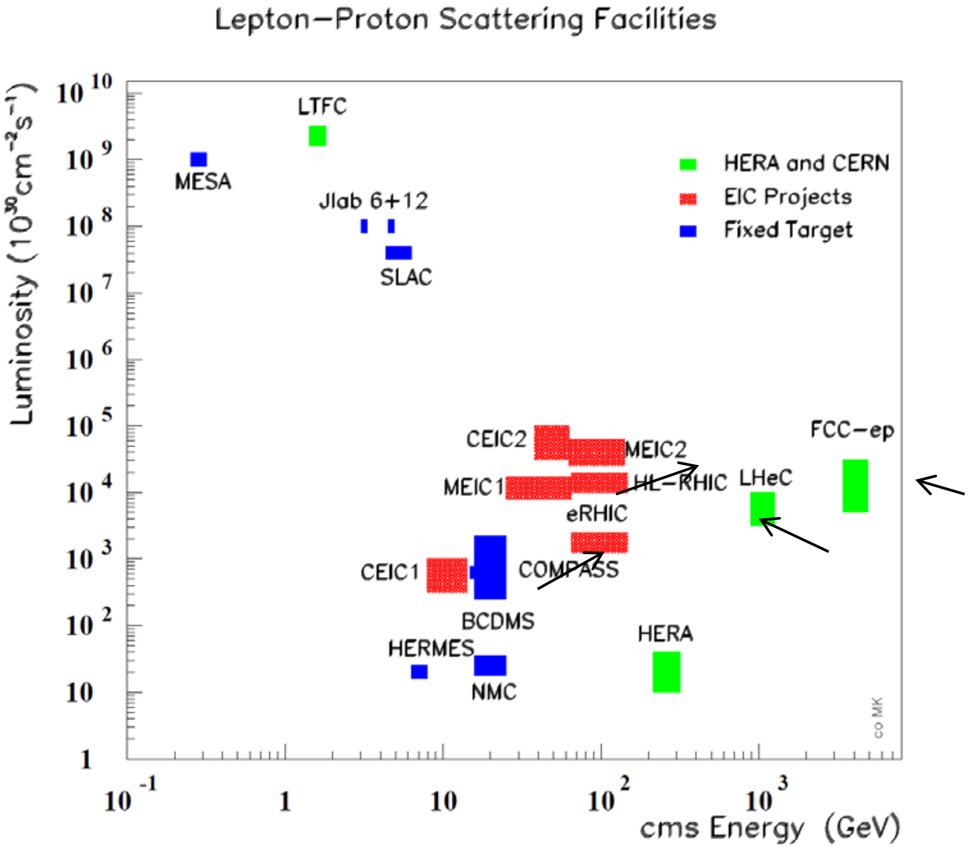
$$f_{q/N}(x) = \frac{1}{8\pi} \int dr^- e^{-iMxr^-/2} \langle N(P)|\bar{q}(r^-)\gamma^+ W[r^-,0] q(q)|N(P)\rangle \Big|_{\substack{r^+\sim 1/\nu\to 0 \\ r_\perp \sim 1/Q \to 0}}$$

- The Wilson line W arises from scattering of the struck quark in the target. It does not cancel between the amplitude and (amplitude)* since the photon vertices are separated by r^-… In the light-cone gauge, $A^+ = 0$, the Wilson link reduces to unity and can be omitted

- $W[r^-,0] \equiv P \exp\left[\frac{ig}{2}\int_0^{r^-} dx^- A^+(x^-)\right]$



The CM Energy vs Luminosity Landscape



CEIC1 = Chinese version of Electron-Ion Collider

("A dilution-free mini-COMPASS")

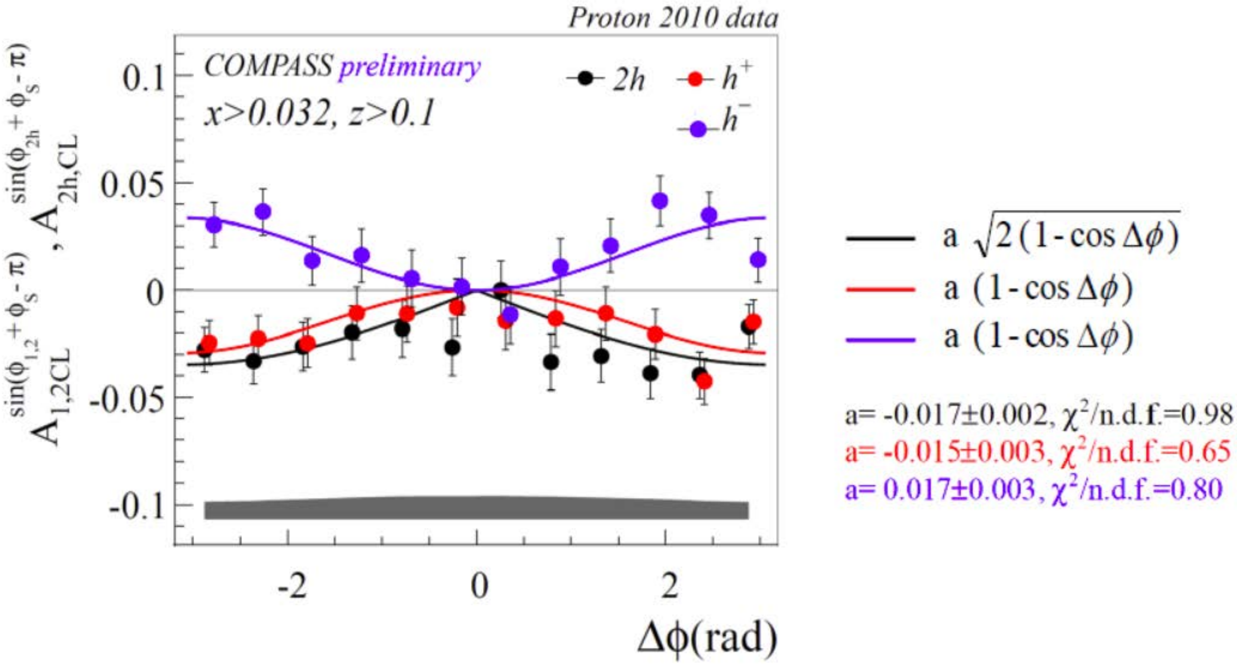
MEIC1 = EIC@Jlab

eRHIC = EIC@BNL

LHeC = ep/eA collider @ CERN

CEIC2
MEIC2
HL-eRHIC
FCC-he

Asymmetries for $x > 0.032$ vs $\Delta\phi = \phi_{h^+} - \phi_{h^-}$

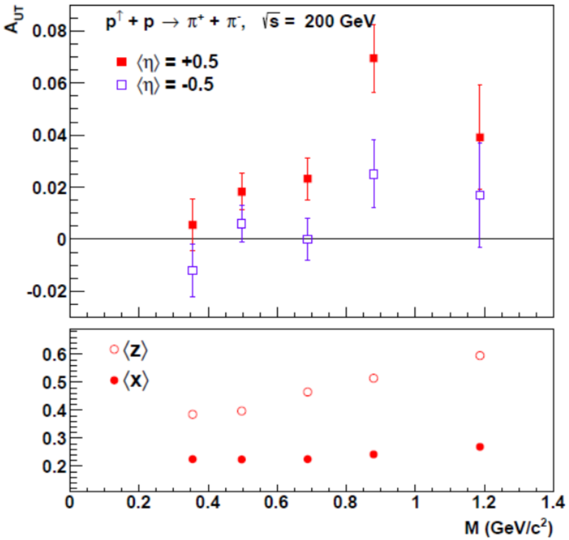


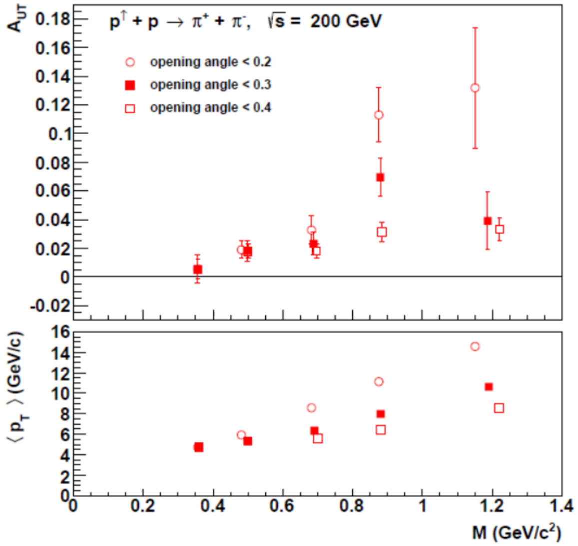
$$a = \frac{\sigma_{1C}^{h^+h^-}(\Delta\phi)}{\sigma_U(\Delta\phi)}$$

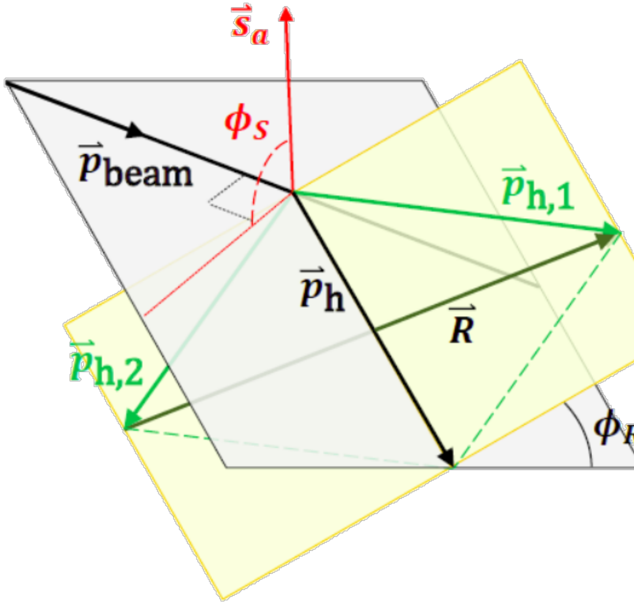
$$= -\frac{\sigma_{2C}^{h^+h^-}(\Delta\phi)}{\sigma_U(\Delta\phi)}$$

ratio of the integrals compatible with $4/\pi$

2h asymmetries in $p^{\uparrow}p \longrightarrow \pi\pi X$





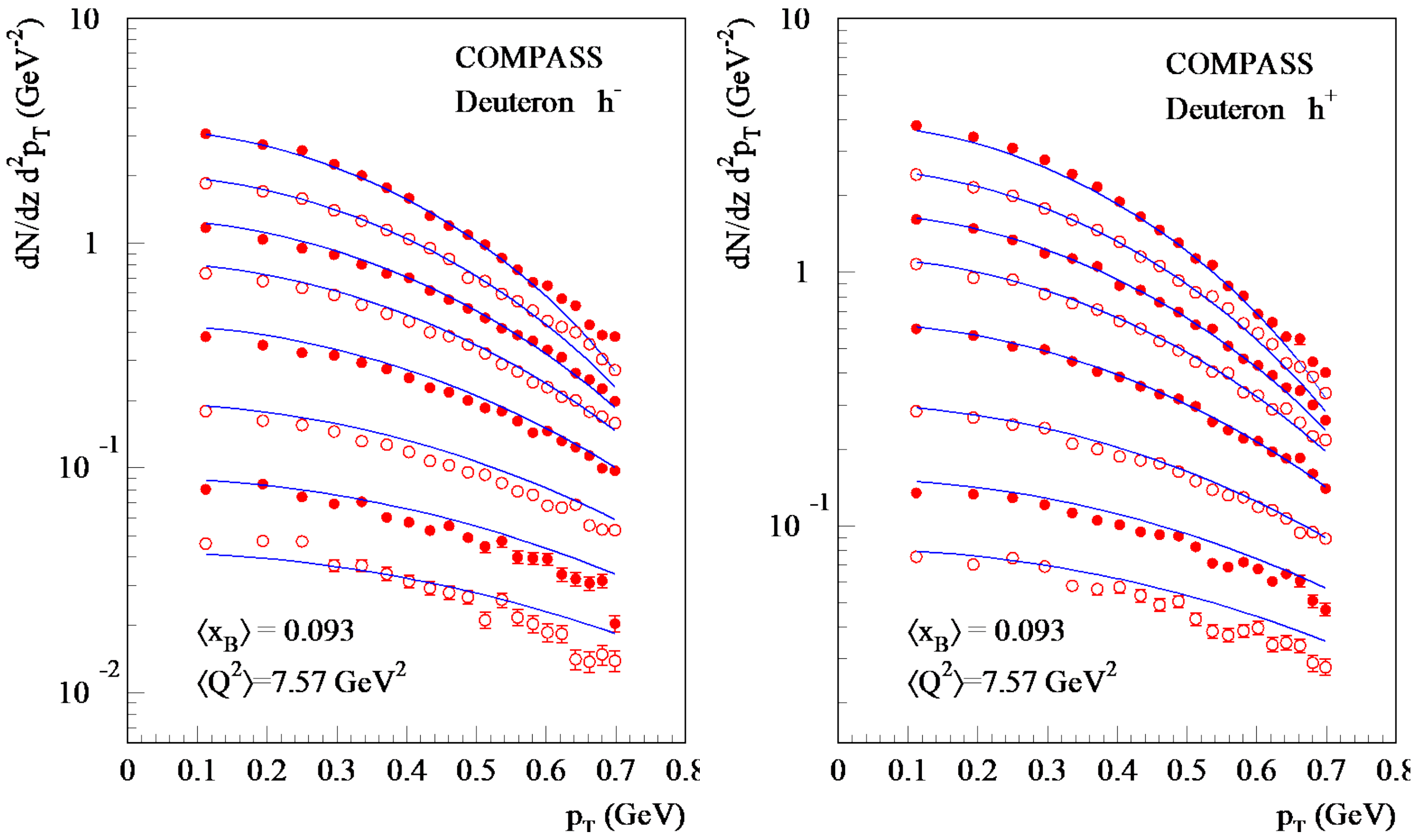


$$d\sigma_{UT} \propto \sin\phi_{RS}\, f_1 \otimes h_1 \otimes \hat{\sigma}^{qq\to qq} \otimes H_{1,q}^{\sphericalangle}(z,M)$$

TMD evolution works: multiplicity distribution in SIDIS

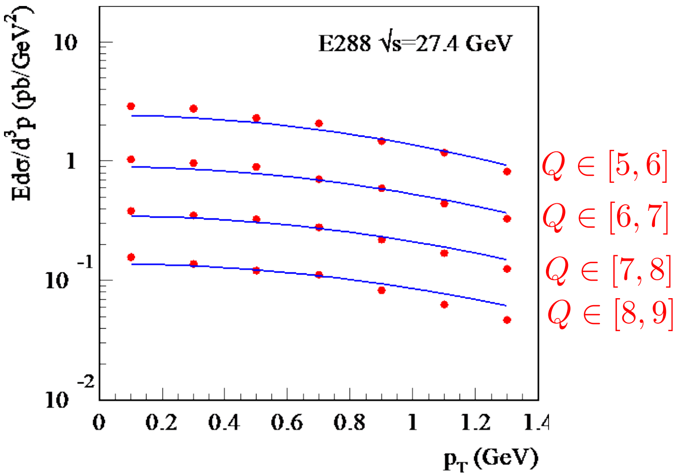
- Comparison to COMPASS data

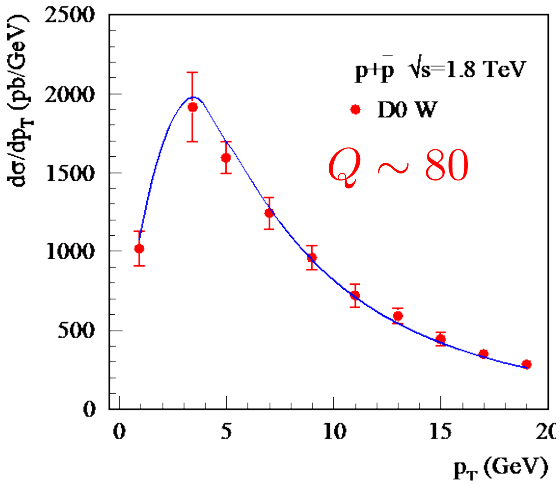
Echevarria, Idilbi, Kang, Vitev

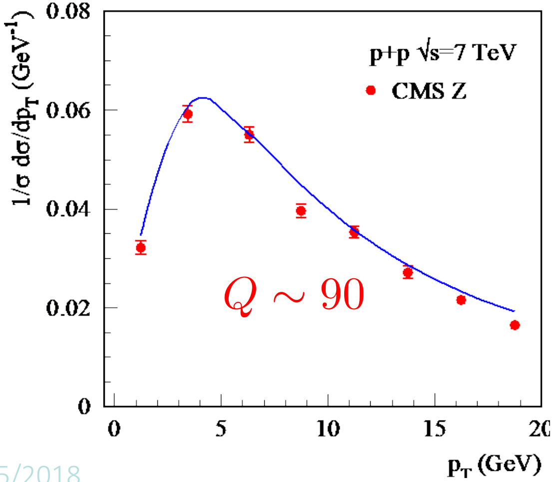


TMD evolution works: Drell-Yan and W/Z production

- Comparison with DY, W/Z pt distribution







- Works for SIDIS, DY, and W/Z in all the energy ranges
- Make predictions for future JLab 12, COMPASS, Fermilab, RHIC experiments