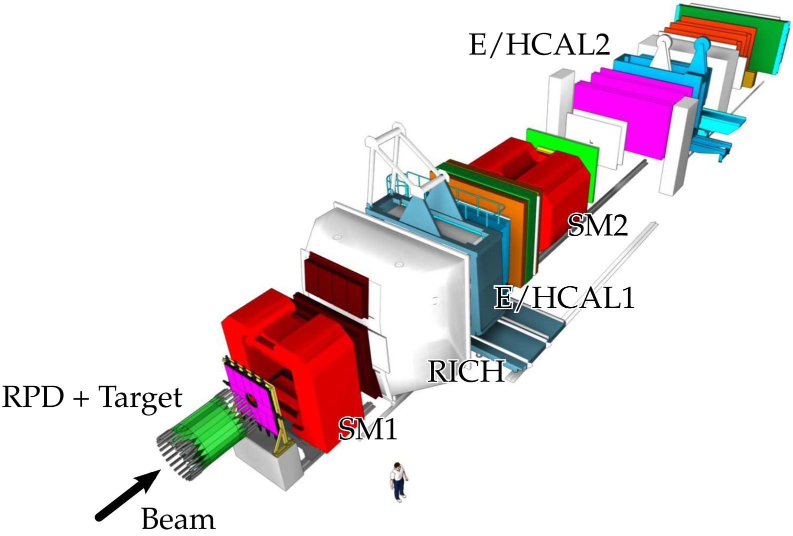
First Results from an extended Freed-Isobar Analysis of the 3π systems at COMPASS

Fabian Krinner
on behalf of the COMPASS Collaboration

Institute for Hadronic Structure
and Fundamental Symmetries
—
Technische Universität München

Salamanca
HADRON 2017
XVII International Conference on Hadron Spectroscopy and Structure

HADRON 2017 — Sep 27th 2017

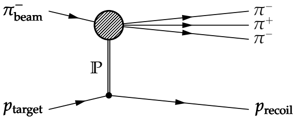


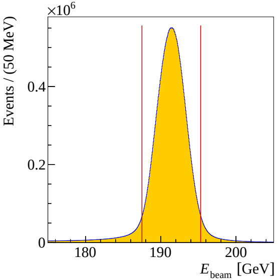
190 GeV$/c^2$ negative hadron (pion) beam on liquid hydrogen (proton) target.

Diffractive 3π production

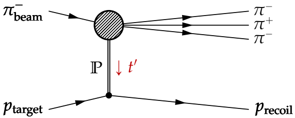
- COMPASS: World's largest data-set up to now for the diffractive process
  $\pi^-_{\text{beam}} + p \to \pi^-\pi^+\pi^- \quad + p$
  taken in 2008
  ($\sim 46 \cdot 10^6$ exclusive events)

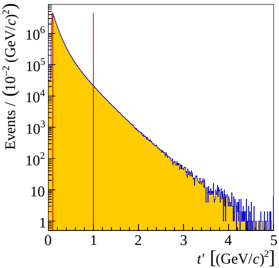
- Exclusive measurement



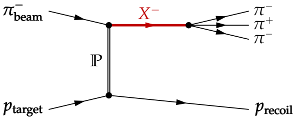


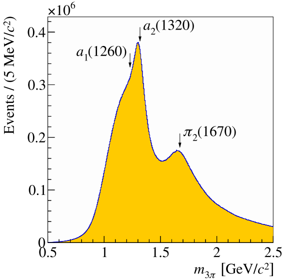
- COMPASS: World's largest data-set up to now for the diffractive process $\pi^-_{\text{beam}} + p \rightarrow \pi^-\pi^+\pi^- \quad + p$ taken in 2008 ($\sim 46 \cdot 10^6$ exclusive events)
- Exclusive measurement
- Squared four-momentum t' transferred by Pomeron $\mathbb{P}$





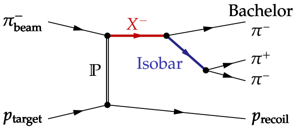
- COMPASS: World's largest data-set up to now for the diffractive process $\pi^-_{\text{beam}} + p \to \pi^-\pi^+\pi^- \quad + p$ taken in 2008 ($\sim 46 \cdot 10^6$ exclusive events)
- Exclusive measurement
- Squared four-momentum t' transferred by Pomeron $\mathbb{P}$
- Rich structure in $\pi^-\pi^+\pi^-$ mass spectrum: Intermediary states X^-

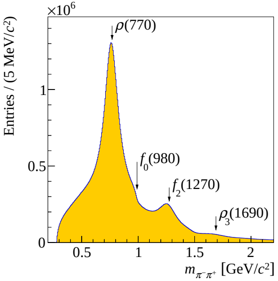




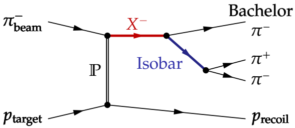
Diffractive 3π production

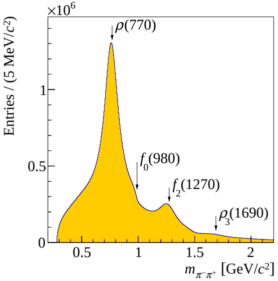
- COMPASS: World's largest data-set up to now for the diffractive process $\pi^-_{\text{beam}} + p \rightarrow \pi^-\pi^+\pi^-_{\text{bachelor}} + p$ taken in 2008 ($\sim 46 \cdot 10^6$ exclusive events)
- Exclusive measurement
- Squared four-momentum t' transferred by Pomeron $\mathbb{P}$
- Rich structure in $\pi^-\pi^+\pi^-$ mass spectrum: Intermediary states X^-
- Also structure in $\pi^+\pi^-$ subsystem: Intermediary states ξ (Isobar)





- COMPASS: World's largest data-set up to now for the diffractive process $\pi^-_{\text{beam}} + p \rightarrow \pi^-\pi^+\pi^-_{\text{bachelor}} + p$ taken in 2008 ($\sim 46 \cdot 10^6$ exclusive events)
- Exclusive measurement
- Squared four-momentum t' transferred by Pomeron $\mathbb{P}$
- Rich structure in $\pi^-\pi^+\pi^-$ mass spectrum: Intermediary states X^-
- Also structure in $\pi^+\pi^-$ subsystem: Intermediary states ξ (Isobar)
- Published in PRD 95 (2017) 032004

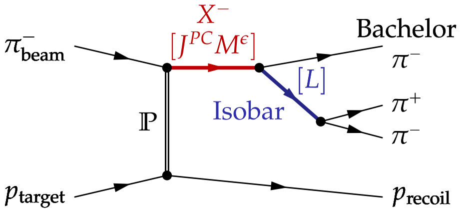




$$\mathcal{I}(\vec{\tau}) = \left| \sum_i \mathcal{T}_i \psi_i(\vec{\tau}) \Delta_i(m_{\pi^-\pi^+}) \right|^2$$

Waves defined by:

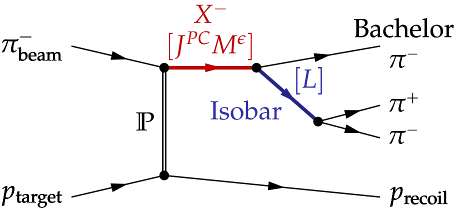
$$J^{PC} M^{\varepsilon} \xi \pi L$$



$$\mathcal{I}(\vec{\tau}) = \left| \sum_i \mathcal{T}_i \psi_i(\vec{\tau}) \Delta_i(m_{\pi^-\pi^+}) \right|^2$$

Waves defined by:

$$J^{PC} M^{\varepsilon} \xi \pi L$$

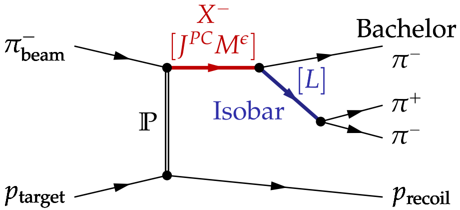


- J^{PC}: Spin and eigenvalues under parity and charge conjugation of X^-

$$\mathcal{I}(\vec{\tau}) = \left|\sum_i \mathcal{T}_i \psi_i(\vec{\tau})\, \Delta_i(m_{\pi^-\pi^+})\right|^2$$

Waves defined by:

$J^{PC} M^{\varepsilon} \xi \pi L$

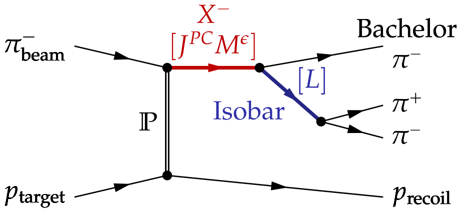


- J^{PC}: Spin and eigenvalues under parity and charge conjugation of X^-
- M^{ε}: Spin projection of X^- and naturality of the exchange particle

$$\mathcal{I}(\vec{\tau}) = \left| \sum_i \mathcal{T}_i \psi_i(\vec{\tau}) \Delta_i(m_{\pi^-\pi^+}) \right|^2$$

Waves defined by:

$$J^{PC} M^{\varepsilon} \xi \pi L$$

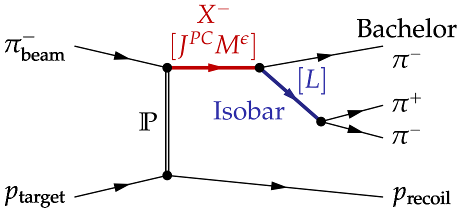


- J^{PC}: Spin and eigenvalues under parity and charge conjugation of X^-
- M^{ε}: Spin projection of X^- and naturality of the exchange particle
- ξ: Appearing fixed or freed isobar, e.g. $\rho(770)$ or $[\pi\pi]_{1^{--}}$

$$\mathcal{I}(\vec{\tau}) = \left| \sum_i \mathcal{T}_i \psi_i(\vec{\tau}) \Delta_i(m_{\pi^-\pi^+}) \right|^2$$

Waves defined by:

$$J^{PC} M^{\varepsilon} \xi \pi L$$



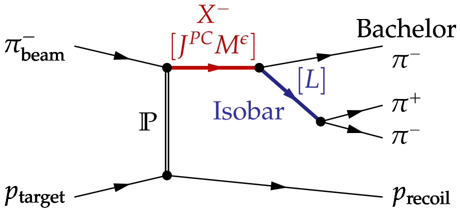
- J^{PC}: Spin and eigenvalues under parity and charge conjugation of X^-
- M^{ε}: Spin projection of X^- and naturality of the exchange particle
- ξ: Appearing fixed or freed isobar, e.g. $\rho(770)$ or $[\pi\pi]_{1^{--}}$
- π: Indicating the bachelor π^-. Always the same

Partial-Wave Analysis

$$\mathcal{I}(\vec{\tau}) = \left| \sum_i \mathcal{T}_i \psi_i(\vec{\tau}) \Delta_i(m_{\pi^-\pi^+}) \right|^2$$

Waves defined by:

$$J^{PC} M^{\varepsilon} \xi \pi L$$



- J^{PC}: Spin and eigenvalues under parity and charge conjugation of X^-
- M^{ε}: Spin projection of X^- and naturality of the exchange particle
- ξ: Appearing fixed or freed isobar, e.g. $\rho(770)$ or $[\pi\pi]_{1^{--}}$
- π: Indicating the bachelor π^-. Always the same
- L: Orbital angular momentum between isobar and bachelor pion

The isobar model

- Intermediary states: Dynamic amplitudes $\Delta(m)$:
  Complex-valued functions of the invariant mass of the state

The isobar model

- Intermediary states: Dynamic amplitudes $\Delta(m)$:
  Complex-valued functions of the invariant mass of the state
- Simplest example: Breit-Wigner amplitude with known mass m_0 and width Γ_0 of a resonance:

$$\Delta_{\mathrm{BW}}(m) = \frac{m_0\Gamma_0}{m_0^2 - m^2 - im_0\Gamma_0}$$

The isobar model

- Intermediary states: Dynamic amplitudes $\Delta(m)$:
  Complex-valued functions of the invariant mass of the state
- Simplest example: Breit-Wigner amplitude with known mass m_0 and width Γ_0 of a resonance:

$$\Delta_{\mathrm{BW}}(m) = \frac{m_0\Gamma_0}{m_0^2 - m^2 - im_0\Gamma_0}$$

- Analysis performed in bins of $m_{X^-} = m_{3\pi}$. Dynamic amplitude of 3π state X^- inferred form the data (Previous two talks)

The isobar model

- Intermediary states: Dynamic amplitudes $\Delta(m)$:
  Complex-valued functions of the invariant mass of the state
- Simplest example: Breit-Wigner amplitude with known mass m_0 and width Γ_0 of a resonance:

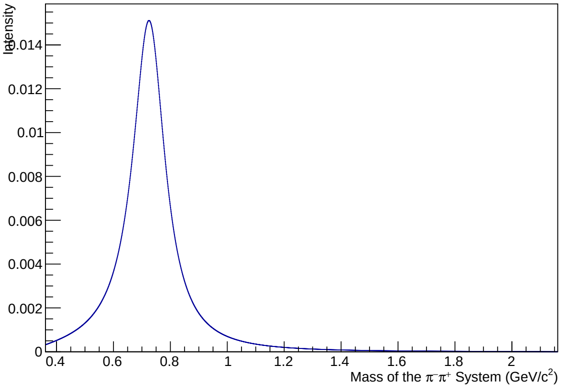
$$\Delta_{\mathrm{BW}}(m) = \frac{m_0 \Gamma_0}{m_0^2 - m^2 - i m_0 \Gamma_0}$$

- Analysis performed in bins of $m_{X^-} = m_{3\pi}$. Dynamic amplitude of 3π state X^- inferred form the data (Previous two talks)
- Dynamic amplitude of $\pi^-\pi^+$ state ξ: Model input in conventional PWA

The isobar model

- Inter Com
- Simp m_0 and widtl
- Anal e of 3π state X^- i
- Dyna onal PWA

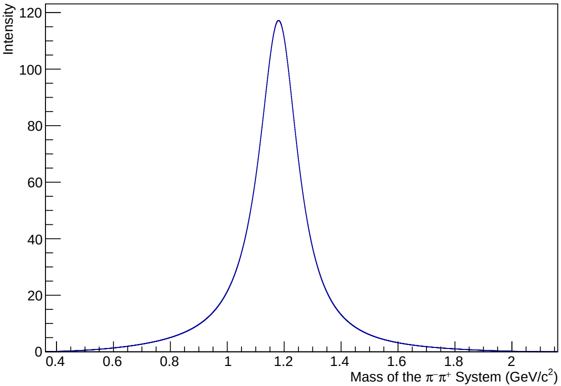
Dynamic isobar amplitude: $\rho(770)$, $J^{PC} = 1^{--}$



The isobar model

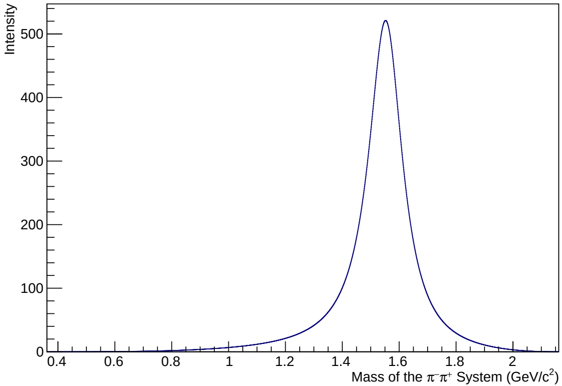
- Inter
  Com
- Simp … m_0 and
  widtl
- Anal … e of 3π state
  X^- i
- Dyn … onal PWA

Dynamic isobar amplitude: $f_2\,(1270)$, $J^{PC} = 2^{++}$



- Inter… Com…
- Simp… m_0 and widt…
- Anal… e of 3π state X^- i…
- Dyn… onal PWA

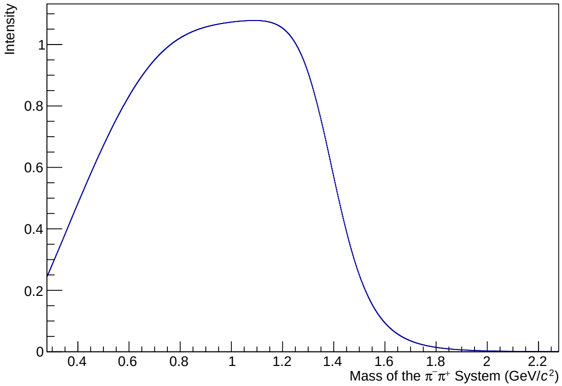
Dynamic isobar amplitude: ρ_3 (1690), $J^{PC} = 3^{--}$



The isobar model

- Inter
  Com
- Simp m_0 and
  widtl
- Anal e of 3π state
  X^- i
- Dyn onal PWA

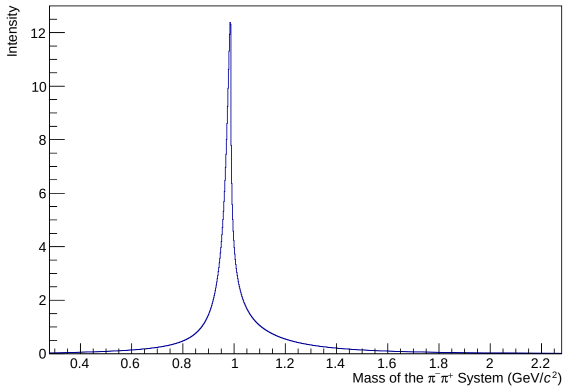
Dynamic isobar amplitude: $[\pi\pi]_S$ wave, $J^{PC} = 0^{++}$



The isobar model

- Inter
  Com
- Simp ... m_0 and
  widtl
- Anal ... e of 3π state
  X^- i
- Dyn ... onal PWA

Dynamic isobar amplitude: $f_0\,(980)$, $J^{PC} = 0^{++}$



The isobar model

- Intermediary states: Dynamic amplitudes $\Delta(m)$:
  Complex-valued functions of the invariant mass of the state
- Simplest example: Breit-Wigner amplitude with known mass m_0 and width Γ_0 of a resonance:

$$\Delta_{\mathrm{BW}}(m) = \frac{m_0\Gamma_0}{m_0^2 - m^2 - im_0\Gamma_0}$$

- Analysis performed in bins of $m_{X^-} = m_{3\pi}$. Dynamic amplitude of 3π state X^- inferred form the data (Previous two talks)
- Dynamic amplitude of $\pi^-\pi^+$ state ξ: Model input in conventional PWA
- Physical dynamic isobar amplitudes may differ from the model

The isobar model

- Intermediary states: Dynamic amplitudes $\Delta(m)$:
  Complex-valued functions of the invariant mass of the state
- Simplest example: Breit-Wigner amplitude with known mass m_0 and width Γ_0 of a resonance:

$$\Delta_{\mathrm{BW}}(m) = \frac{m_0\Gamma_0}{m_0^2 - m^2 - im_0\Gamma_0}$$

- Analysis performed in bins of $m_{X^-} = m_{3\pi}$. Dynamic amplitude of 3π state X^- inferred form the data (Previous two talks)
- Dynamic amplitude of $\pi^-\pi^+$ state ξ: Model input in conventional PWA
- Physical dynamic isobar amplitudes may differ from the model
- Free parameters in dynamic isobar amplitudes computationally unfeasible

- Total intensity in one $(m_{3\pi}, t')$-bin as function of phase-space variables $\vec{\tau}$:

$$\mathcal{I}(\vec{\tau}) = \left|\sum_i^{\text{waves}} \mathcal{T}_i \left[\psi_i(\vec{\tau})\,\Delta_i(m_{\pi^-\pi^+}) + \text{Bose Symm.}\right]\right|^2$$

Fit parameters: Production amplitudes $\mathcal{T}_i$

Fixed: Angular distributions $\psi_i(\vec{\tau})$, dynamic isobar amplitudes $\Delta_i(m_{\pi^-\pi^+})$

- Total intensity in one $(m_{3\pi}, t')$-bin as function of phase-space variables $\vec{\tau}$:

$$\mathcal{I}(\vec{\tau}) = \left| \sum_{i}^{\text{waves}} \mathcal{T}_i \left[\psi_i(\vec{\tau})\, \Delta_i(m_{\pi^-\pi^+}) + \text{Bose Symm.} \right] \right|^2$$

Fit parameters: Production amplitudes $\mathcal{T}_i$

Fixed: Angular distributions $\psi_i(\vec{\tau})$, dynamic isobar amplitudes $\Delta_i(m_{\pi^-\pi^+})$

- Replace fixed isobar amplitudes by piece-wise constant function:

$$\Delta_i(m_{\pi^-\pi^+}) \rightarrow \sum_{\text{bins}} \mathscr{T}_i^{\text{bin}} \Delta_i^{\text{bin}}(m_{\pi^-\pi^+}) \equiv [\pi\pi]_{J^{PC}}$$

$$\Delta_i^{\text{bin}}(m_{\pi^-\pi^+}) = \begin{cases} 1, & \text{if } m_{\pi^-\pi^+} \text{ in the bin.} \\ 0, & \text{otherwise.} \end{cases}$$

- Total intensity in one $(m_{3\pi}, t')$-bin as function of phase-space variables $\vec{\tau}$:

$$\mathcal{I}(\vec{\tau}) = \left| \sum_{i}^{\text{waves}} \mathcal{T}_i \left[\psi_i(\vec{\tau})\, \Delta_i(m_{\pi^-\pi^+}) + \text{Bose Symm.} \right] \right|^2$$

  Fit parameters: Production amplitudes $\mathcal{T}_i$

  Fixed: Angular distributions $\psi_i(\vec{\tau})$, dynamic isobar amplitudes $\Delta_i(m_{\pi^-\pi^+})$
- Replace fixed isobar amplitudes by piece-wise constant function:

$$\Delta_i(m_{\pi^-\pi^+}) \rightarrow \sum_{\text{bins}} \mathscr{T}_i^{\text{bin}} \Delta_i^{\text{bin}}(m_{\pi^-\pi^+}) \equiv [\pi\pi]_{J^{PC}}$$

$$\Delta_i^{\text{bin}}(m_{\pi^-\pi^+}) = \begin{cases} 1, & \text{if } m_{\pi^-\pi^+} \text{ in the bin.} \\ 0, & \text{otherwise.} \end{cases}$$

- Each $m_{\pi^-\pi^+}$ bin behaves like an independent partial wave $\mathcal{T}_i^{\text{bin}} = \mathcal{T}_i \mathscr{T}_i^{\text{bin}}$:

$$\mathcal{I}(\vec{\tau}) = \left| \sum_{i}^{\text{waves}} \sum_{\text{bin}}^{\text{bins}} \mathcal{T}_i^{\text{bin}} \left[\psi_i(\vec{\tau})\, \Delta_i^{\text{bin}}(m_{\pi^-\pi^+}) + \text{Bose Symm.} \right] \right|^2$$

- Total intensity in one $(m_{3\pi}, t')$-bin as function of phase-space variables $\vec{\tau}$:

$$\mathcal{I}(\vec{\tau}) = \left| \sum_i^{\text{waves}} \mathcal{T}_i \left[\psi_i(\vec{\tau})\, \Delta_i(m_{\pi^-\pi^+}) + \text{Bose Symm.} \right] \right|^2$$

Fit parameters: Production amplitudes $\mathcal{T}_i$

Fixed: Angular distributions $\psi_i(\vec{\tau})$, dynamic isobar amplitudes $\Delta_i(m_{\pi^-\pi^+})$

- Replace fixed isobar amplitudes by piece-wise constant function:

$$\Delta_i(m_{\pi^-\pi^+}) \to \sum_{\text{bins}} \mathscr{T}_i^{\text{bin}} \Delta_i^{\text{bin}}(m_{\pi^-\pi^+}) \equiv [\pi\pi]_{J^{PC}}$$

$$\Delta_i^{\text{bin}}(m_{\pi^-\pi^+}) = \begin{cases} 1, & \text{if } m_{\pi^-\pi^+} \text{ in the bin.} \\ 0, & \text{otherwise.} \end{cases}$$

- Each $m_{\pi^-\pi^+}$ bin behaves like an independent partial wave $\mathcal{T}_i^{\text{bin}} = \mathcal{T}_i \mathscr{T}_i^{\text{bin}}$:

$$\mathcal{I}(\vec{\tau}) = \left| \sum_i^{\text{waves}} \sum_{\text{bin}}^{\text{bins}} \mathcal{T}_i^{\text{bin}} \left[\psi_i(\vec{\tau})\, \Delta_i^{\text{bin}}(m_{\pi^-\pi^+}) + \text{Bose Symm.} \right] \right|^2$$

- Approach similar to binning in $m_{3\pi}$

- Total intensity in one $(m_{3\pi}, t')$-bin as function of phase-space variables $\vec{\tau}$:

$$\mathcal{I}(\vec{\tau}) \quad \ldots \quad \text{m.}]\Big|^2$$

Fit parameters:

Fixed: Angular … udes $\Delta_i(m_{\pi^-\pi^+})$

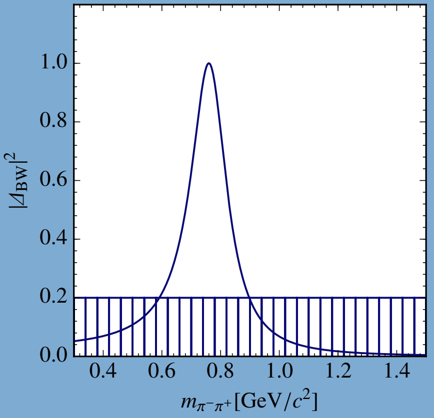
- Replace fixed is … function:

$$\Delta \quad \ldots \quad]_{J^{PC}}$$

- Each $m_{\pi^-\pi^+}$ bin … e $\mathcal{T}_i^{\text{bin}} = \mathcal{T}_i \mathscr{T}_i^{\text{bin}}$:

$$\mathcal{I}(\vec{\tau}) = \Big| \quad \ldots \quad \text{Symm.}]\Big|^2$$

Illustration



- Approach similar to binning in $m_{3\pi}$

- Total intensity in one $(m_{3\pi}, t')$-bin as function of phase-space variables $\vec{\tau}$:

$\mathcal{I}(\vec{\tau})$ ... m.]$|^2$

Fit parameters: ...

Fixed: Angular ... udes $\Delta_i(m_{\pi^-\pi^+})$

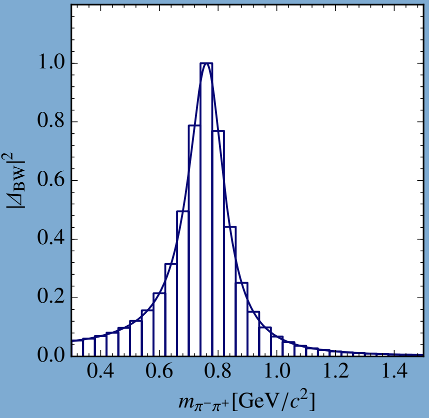
- Replace fixed is... function:

Δ ... $]_{J^{PC}}$

- Each $m_{\pi^-\pi^+}$ bin ... e $\mathcal{T}_i^{\text{bin}} = \mathcal{T}_i \mathscr{T}_i^{\text{bin}}$:

$\mathcal{I}(\vec{\tau}) = |$ ... Symm.$]|^2$

Illustration



- Approach similar to binning in $m_{3\pi}$

First freed-isobar analysis

- Fixed-isobar analysis
  - Wave set: 88 waves
  - 100 $m_{3\pi}$ bins, 11 t' bins
  - Introduced by S. Wallner

First freed-isobar analysis

- Fixed-isobar analysis
  - Wave set: 88 waves
  - 100 $m_{3\pi}$ bins, 11 t' bins
  - Introduced by S. Wallner
- Three waves with 0^{++} isobar freed:
  - $0^{-+}0^{+}[\pi\pi]_{0^{++}}\pi S$
  - $1^{++}0^{+}[\pi\pi]_{0^{++}}\pi P$
  - $2^{-+}0^{+}[\pi\pi]_{0^{++}}\pi D$

First freed-isobar analysis

- Fixed-isobar analysis
  - Wave set: 88 waves
  - 100 $m_{3\pi}$ bins, 11 t' bins
  - Introduced by S. Wallner
- Three waves with 0^{++} isobar freed:
  - $0^{-+}0^{+}[\pi\pi]_{0^{++}}\pi S$
  - $1^{++}0^{+}[\pi\pi]_{0^{++}}\pi P$
  - $2^{-+}0^{+}[\pi\pi]_{0^{++}}\pi D$
- Replace 7 fixed-isobar waves

- Fixed-isobar analysis
  - Wave set: 88 waves
- T
  fr
- F

Matching isobar quantum numbers

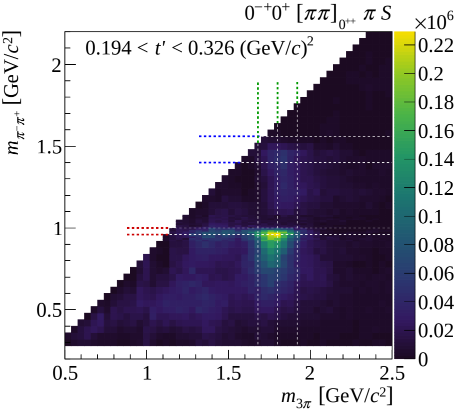
$$\left.\begin{array}{l} 0^{-+}0^{+}\,[\pi\pi]_S\,\pi S \\ 0^{-+}0^{+} f_0\,(980)\,\pi S \\ 0^{-+}0^{+} f_0\,(1500)\,\pi S \end{array}\right\}\quad 0^{-+}0^{+}[\pi\pi]_{0^{++}}\pi S$$

$$\left.\begin{array}{l} 1^{++}0^{+}\,[\pi\pi]_S\,\pi P \\ 1^{++}0^{+} f_0\,(980)\,\pi P \end{array}\right\}\quad 1^{++}0^{+}[\pi\pi]_{0^{++}}\pi P$$

$$\left.\begin{array}{l} 2^{-+}0^{+}\,[\pi\pi]_S\,\pi D \\ 2^{-+}0^{+} f_0\,(980)\,\pi D \end{array}\right\}\quad 2^{-+}0^{+}[\pi\pi]_{0^{++}}\pi D.$$

First freed-isobar analysis

- Fixed-isobar analysis
  - Wave set: 88 waves
  - 100 $m_{3\pi}$ bins, 11 t' bins
  - Introduced by S. Wallner
- Three waves with 0^{++} isobar freed:
  - $0^{-+}0^{+}[\pi\pi]_{0^{++}}\pi S$
  - $1^{++}0^{+}[\pi\pi]_{0^{++}}\pi P$
  - $2^{-+}0^{+}[\pi\pi]_{0^{++}}\pi D$
- Replace 7 fixed-isobar waves
- Published in PRD 95 (2017) 032004
- Promising results



- Extend freed-isobar wave set

- Extend freed-isobar wave set
- Free isobar dynamic amplitudes of 11 biggest waves:
  - Minimize potential leakage

- Extend freed-isobar wave set
- Free isobar dynamic amplitudes of 11 biggest waves:
  - Minimize potential leakage

Freed-isobar wave set

$0^{-+}0^{+}[\pi\pi]_{0^{++}}\pi S$	$1^{++}1^{+}[\pi\pi]_{1^{--}}\pi S$	$2^{-+}0^{+}[\pi\pi]_{2^{++}}\pi S$
$0^{-+}0^{+}[\pi\pi]_{1^{--}}\pi P$	$2^{-+}0^{+}[\pi\pi]_{0^{++}}\pi D$	$2^{-+}1^{+}[\pi\pi]_{1^{--}}\pi P$
$1^{++}0^{+}[\pi\pi]_{0^{++}}\pi P$	$2^{-+}0^{+}[\pi\pi]_{1^{--}}\pi P$	$2^{++}1^{+}[\pi\pi]_{1^{--}}\pi D$
$1^{++}0^{+}[\pi\pi]_{1^{--}}\pi S$	$2^{-+}0^{+}[\pi\pi]_{1^{--}}\pi F$	

- Extend freed-isobar wave set
- Free isobar dynamic amplitudes of 11 biggest waves:
    - Minimize potential leakage
- Add spin exotic $1^{-+}1^{+}[\pi\pi]_{1^{--}}\pi P$ wave
    - Wave of major interest

Freed-isobar wave set

Extended wave set

- Extend freed-isobar wave set
- Free isobar dynamic amplitudes of 11 biggest waves:
    - Minimize potential leakage
- Add spin exotic $1^{-+}1^{+}[\pi\pi]_{1^{--}}\pi P$ wave
    - Wave of major interest
- 12 freed-isobar waves replace 16 fixed-isobar waves

- Extend freed-isobar wave set
- Free isobar dynamic amplitudes of 11 biggest waves:
  - Minimize potential leakage
- Add spin exotic $1^{-+}1^{+}[\pi\pi]_{1^{--}}\pi P$ wave
  - Wave of major interest
- 12 freed-isobar waves replace 16 fixed-isobar waves
- In addition 72 fixed-isobar waves in the model

- Extend freed-isobar wave set
- Free isobar dynamic amplitudes of 11 biggest waves:
  - Minimize potential leakage
- Add spin exotic $1^{-+}1^{+}[\pi\pi]_{1^{--}}\pi P$ wave
  - Wave of major interest
- 12 freed-isobar waves replace 16 fixed-isobar waves
- In addition 72 fixed-isobar waves in the model
- 40 MeV wide $m_{3\pi}$ bins from 0.5 to 2.5 GeV
- 4 non-equidistant bins in t'

- Extend freed-isobar wave set
- Free isobar dynamic amplitudes of 11 biggest waves:
  - Minimize potential leakage
- Add spin exotic $1^{-+}1^{+}[\pi\pi]_{1^{--}}\pi P$ wave
  - Wave of major interest
- 12 freed-isobar waves replace 16 fixed-isobar waves
- In addition 72 fixed-isobar waves in the model
- 40 MeV wide $m_{3\pi}$ bins from 0.5 to 2.5 GeV
- 4 non-equidistant bins in t'
- 50 bins in $m_{3\pi}$, 4 bins in t': $4 \times 50 = 200$ independent bins

- Freed-isobar analysis: much more freedom than fixed-isobar analysis

- Freed-isobar analysis: much more freedom than fixed-isobar analysis
  - Causes continuous mathematical ambiguities in the model

- Freed-isobar analysis: much more freedom than fixed-isobar analysis
  - Causes continuous mathematical ambiguities in the model
- “Zero mode” = dynamic isobar amplitude $\Omega\left(m_{\pi^-\pi^+}\right)$, that does not contribute to the **total** amplitude
- Spin-exotic wave:

$$\psi\left(\vec{\tau}\right)\Omega\left(m_{\pi^-\pi^+}\right) + \text{Bose Symm.} = 0$$

at **every point** $\vec{\tau}$ in phase space

- Process: $X^- \to \xi\pi_3^- \to \pi_1^-\pi_2^+\pi_3^-$.
- Condition for zero mode at all points $\vec{\tau}$ in phase-space:

$$\psi\left(\vec{\tau}_{123}\right)\Omega\left(m_{12}\right) + \mathrm{Bose\ Symm.} = 0 \tag{1}$$

- Tensor formalism with pion momenta defined in the X^- rest frame:

$$\psi\left(\vec{\tau}_{123}\right) \propto \vec{p}_1 \times \vec{p}_3$$

- Process: $X^- \to \xi\pi_3^- \to \pi_1^-\pi_2^+\pi_3^-$.
- Condition for zero mode at all points $\vec{\tau}$ in phase-space:

$$\psi\left(\vec{\tau}_{123}\right)\,\Omega\left(m_{12}\right) + \mathrm{Bose\ Symm.} = 0 \tag{1}$$

- Tensor formalism with pion momenta defined in the X^- rest frame:

$$\psi\left(\vec{\tau}_{123}\right) \propto \vec{p}_1 \times \vec{p}_3$$

- Bose symmetrization ($\pi_1^- \leftrightarrow \pi_3^-$):

$$\vec{p}_1 \times \vec{p}_3\,\Omega\left(m_{12}\right) + \vec{p}_3 \times \vec{p}_1\,\Omega\left(m_{23}\right) = \vec{p}_1 \times \vec{p}_3\left[\Omega\left(m_{12}\right) - \Omega\left(m_{23}\right)\right]$$

- Process: $X^{-} \rightarrow \xi\pi_3^{-} \rightarrow \pi_1^{-}\pi_2^{+}\pi_3^{-}$.
- Condition for zero mode at all points $\vec{\tau}$ in phase-space:

$$\psi\left(\vec{\tau}_{123}\right)\,\Omega\left(m_{12}\right) + \text{Bose Symm.} = 0 \tag{1}$$

- Tensor formalism with pion momenta defined in the X^{-} rest frame:

$$\psi\left(\vec{\tau}_{123}\right) \propto \vec{p}_1 \times \vec{p}_3$$

- Bose symmetrization ($\pi_1^{-} \leftrightarrow \pi_3^{-}$):

$$\vec{p}_1 \times \vec{p}_3\,\Omega\left(m_{12}\right) + \vec{p}_3 \times \vec{p}_1\,\Omega\left(m_{23}\right) = \vec{p}_1 \times \vec{p}_3\left[\Omega\left(m_{12}\right) - \Omega\left(m_{23}\right)\right]$$

  - Fulfill eq. (1) at every point in phase space $\Rightarrow \Omega\left(m_{\xi}\right) = \text{const.}$

- Process: $X^- \to \xi\pi_3^- \to \pi_1^-\pi_2^+\pi_3^-$.
- Condition for zero mode at all points $\vec{\tau}$ in phase-space:

$$\psi\left(\vec{\tau}_{123}\right)\,\Omega\left(m_{12}\right) + \mathrm{Bose\ Symm.} = 0 \qquad (1)$$

- Tensor formalism with pion momenta defined in the X^- rest frame:

$$\psi\left(\vec{\tau}_{123}\right) \propto \vec{p}_1 \times \vec{p}_3$$

- Bose symmetrization ($\pi_1^- \leftrightarrow \pi_3^-$):

$$\vec{p}_1 \times \vec{p}_3\,\Omega\left(m_{12}\right) + \vec{p}_3 \times \vec{p}_1\,\Omega\left(m_{23}\right) = \vec{p}_1 \times \vec{p}_3\left[\Omega\left(m_{12}\right) - \Omega\left(m_{23}\right)\right]$$

  - Fulfill eq. (1) at every point in phase space $\Rightarrow \Omega\left(m_\xi\right) = \mathrm{const.}$

- If $\Omega\left(m_\xi\right)$ is added to the physical dynamic isobar amplitude $\Delta^{\mathrm{phys}}\left(m_\xi\right)$, the total amplitude, and thus the intensity, is not altered:

$$\left|\psi\left(\vec{\tau}\right)\Delta^{\mathrm{phys}}\left(m_\xi\right) + \mathrm{B.\,S.}\right|^2 = \left|\psi\left(\vec{\tau}\right)\left[\Delta^{\mathrm{phys}}\left(m_\xi\right) + \mathcal{C}\Omega\left(m_\xi\right)\right] + \mathrm{B.\,S.}\right|^2$$

for any complex-valued zero-mode coefficient $\mathcal{C}$

Zero mode in the spin-exotic wave

Mathematical origin

- Process: $X^- \to \xi\pi_3^- \to \pi_1^-\pi_2^+\pi_3^-$.
- Condition for zero mode at all points $\vec{\tau}$ in phase-space:

$$\psi\left(\vec{\tau}_{123}\right)\Omega\left(m_{12}\right) + \text{Bose Symm.} = 0 \tag{1}$$

- Tensor formalism with pion momenta defined in the X^- rest frame:

$$\psi\left(\vec{\tau}_{123}\right) \propto \vec{p}_1 \times \vec{p}_3$$

- Bose symmetrization ($\pi_1^- \leftrightarrow \pi_3^-$):

$$\vec{p}_1 \times \vec{p}_3\,\Omega\left(m_{12}\right) + \vec{p}_3 \times \vec{p}_1\,\Omega\left(m_{23}\right) = \vec{p}_1 \times \vec{p}_3\left[\Omega\left(m_{12}\right) - \Omega\left(m_{23}\right)\right]$$

  - Fulfill eq. (1) at every point in phase space $\Rightarrow \Omega\left(m_\xi\right) = \text{const.}$

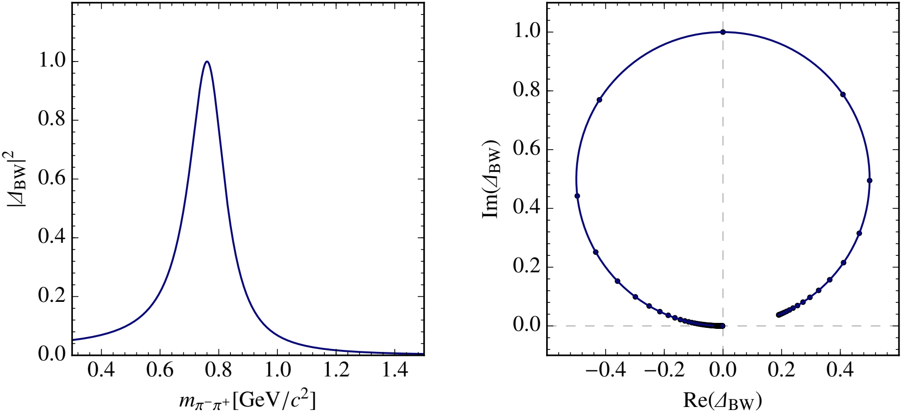
- If $\Omega\left(m_\xi\right)$ is added to the physical dynamic isobar amplitude $\Delta^{\text{phys}}\left(m_\xi\right)$, the total amplitude, and thus the intensity, is not altered:

$$\left|\psi\left(\vec{\tau}\right)\Delta^{\text{phys}}\left(m_\xi\right) + \text{B. S.}\right|^2 = \left|\psi\left(\vec{\tau}\right)\left[\Delta^{\text{phys}}\left(m_\xi\right) + \mathcal{C}\Omega\left(m_\xi\right)\right] + \text{B. S.}\right|^2$$

  for any complex-valued zero-mode coefficient $\mathcal{C}$
- $\mathcal{C}$: complex-valued ambiguity in the model

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \quad (2)$$

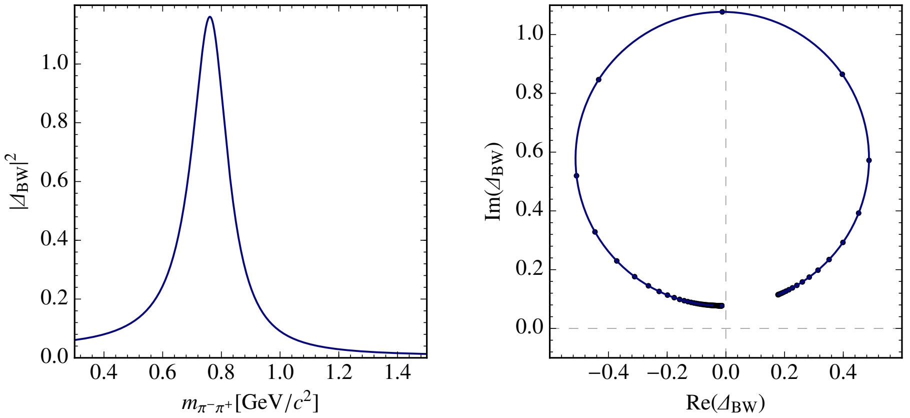
$$\mathcal{C} = 0.00 + 0.00i$$



All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \quad (2)$$

$\mathcal{C} = -0.01 + 0.08i$

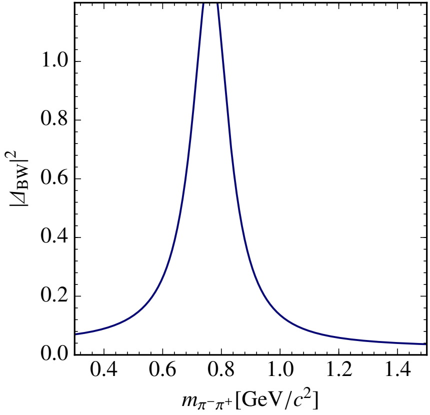


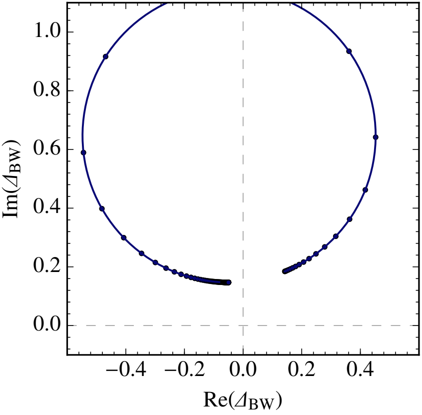
All amplitudes describe the same intensity

Effects on dynamic isobar amplitudes

$$\Delta_{\mathrm{BW}}(m_{\pi^-\pi^+}) + \mathcal{C}\Omega(m_{\pi^-\pi^+}) \quad (2)$$

$$\mathcal{C} = -0.05 + 0.15i$$

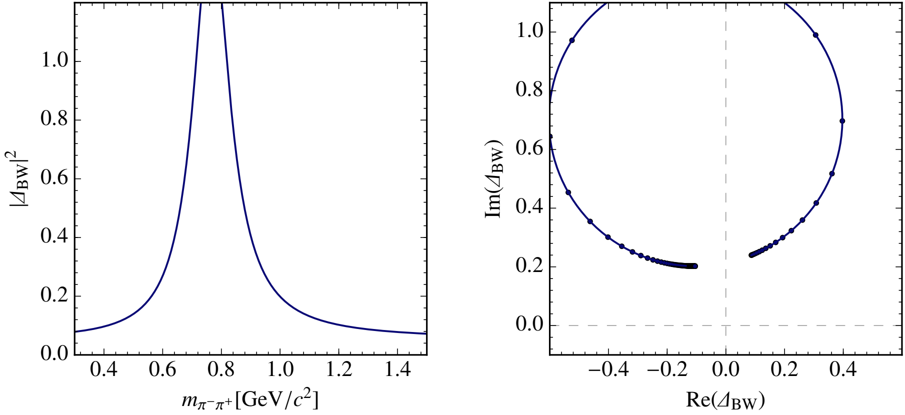




All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \quad (2)$$

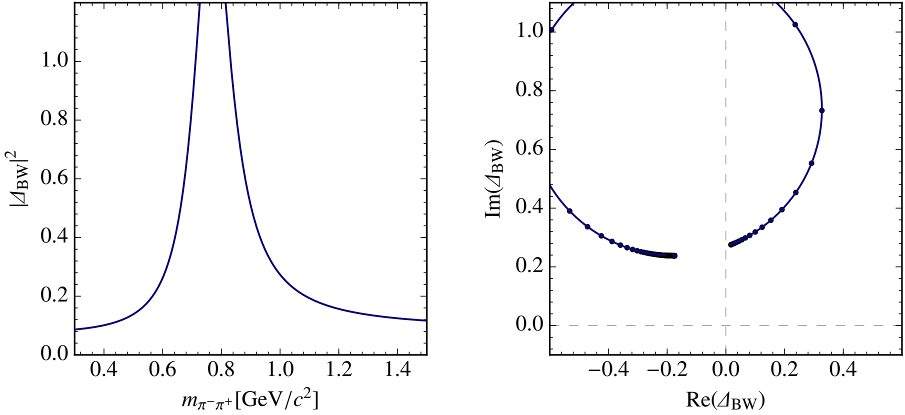
$$\mathcal{C} = -0.10 + 0.20i$$



All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \quad (2)$$

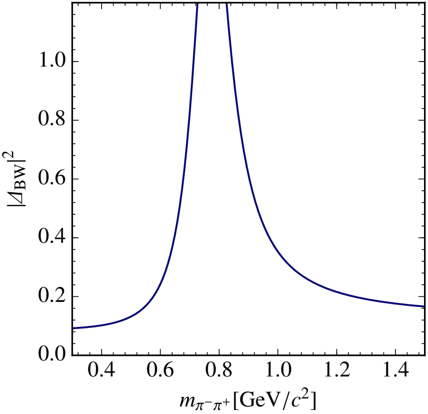
$$\mathcal{C} = -0.17 + 0.24i$$

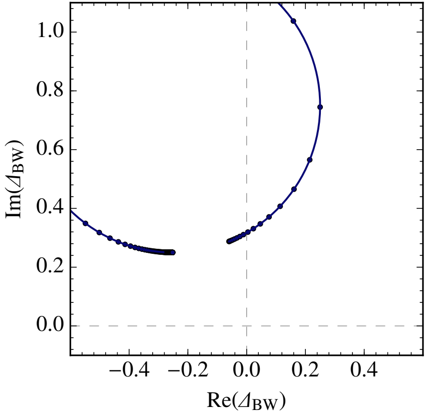


All amplitudes describe the same intensity

$$\Delta_{\text{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \tag{2}$$

$\mathcal{C} = -0.25 + 0.25i$





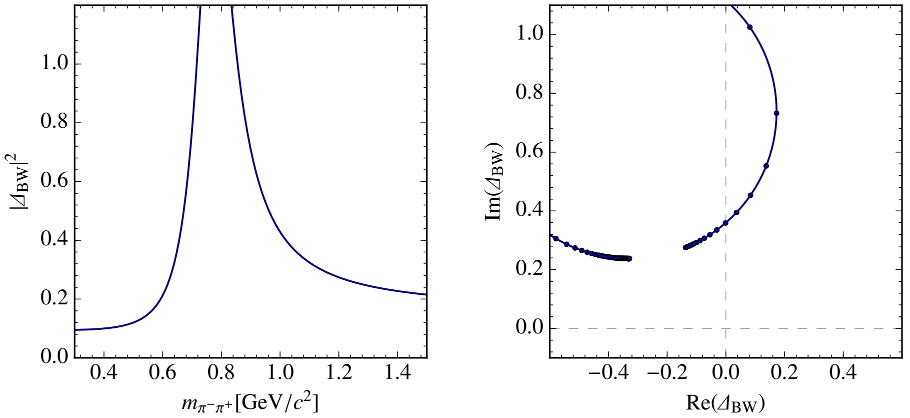
All amplitudes describe the same intensity

Zero mode in the spin-exotic wave

Effects on dynamic isobar amplitudes

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \quad (2)$$

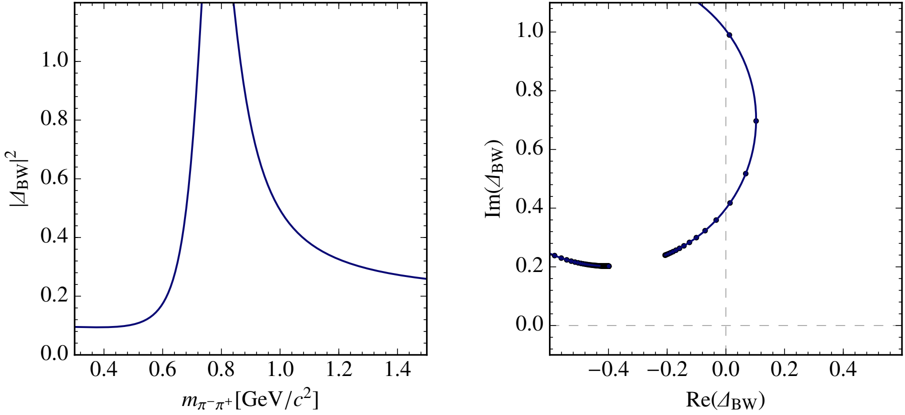
$$\mathcal{C} = -0.33 + 0.24i$$



All amplitudes describe the same intensity

$$\Delta_{\text{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \qquad (2)$$

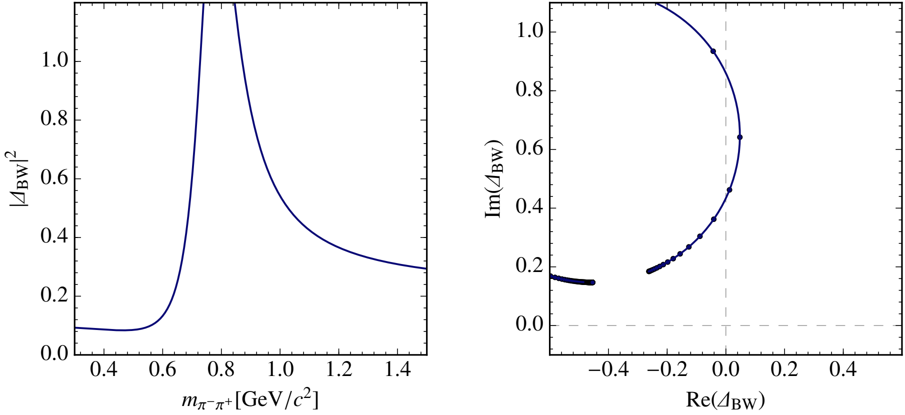
$$\mathcal{C} = -0.40 + 0.20i$$



All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \quad (2)$$

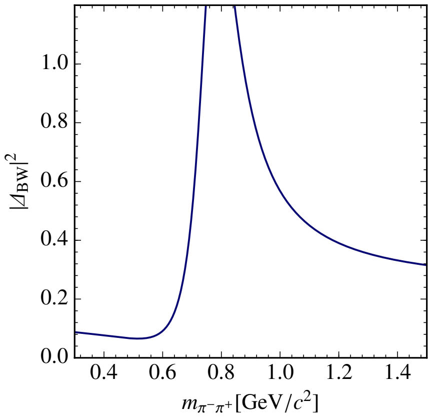
$\mathcal{C} = -0.45 + 0.15i$

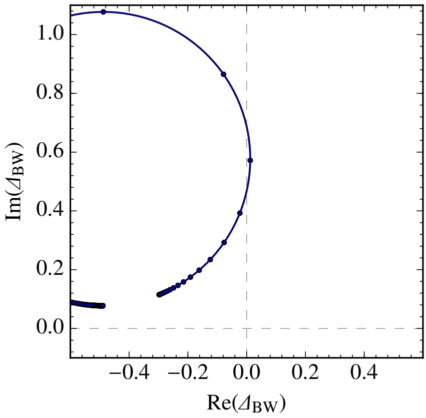


All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \quad (2)$$

$\mathcal{C} = -0.49 + 0.08i$

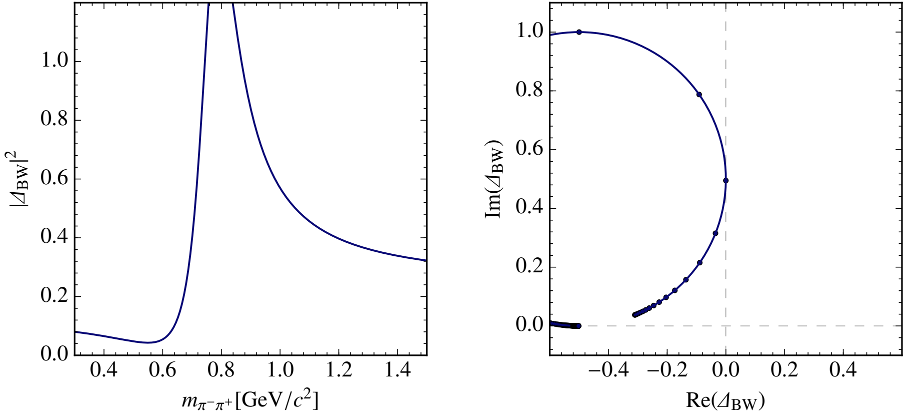




All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \qquad (2)$$

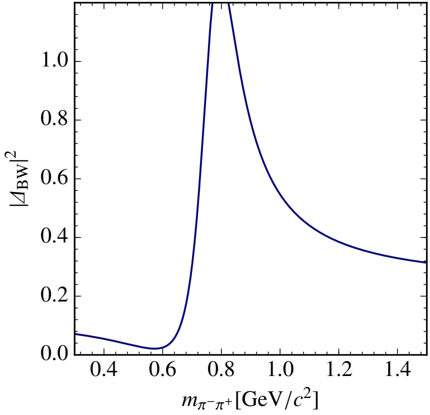
$\mathcal{C} = -0.50 + 0.00i$

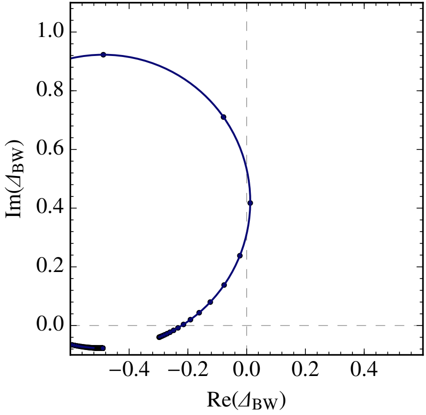


All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \quad (2)$$

$$\mathcal{C} = -0.49 - 0.08i$$





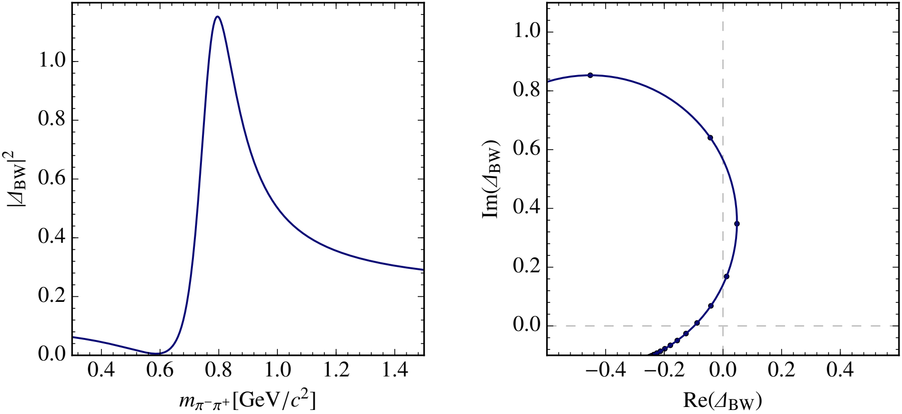
All amplitudes describe the same intensity

Zero mode in the spin-exotic wave

Effects on dynamic isobar amplitudes

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \tag{2}$$

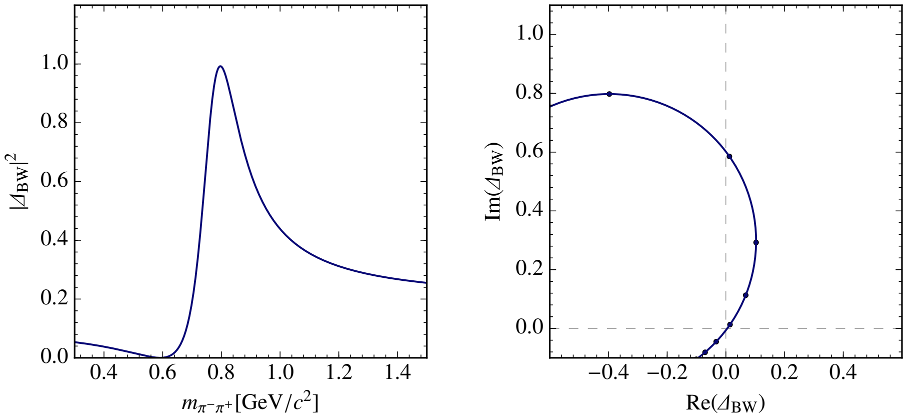
$$\mathcal{C} = -0.45 - 0.15i$$



All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \tag{2}$$

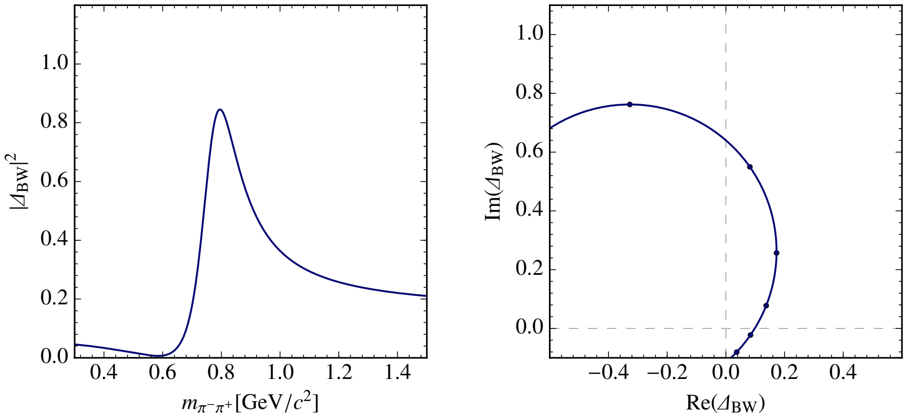
$\mathcal{C} = -0.40 - 0.20i$



All amplitudes describe the same intensity

$$\Delta_{\text{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \quad (2)$$

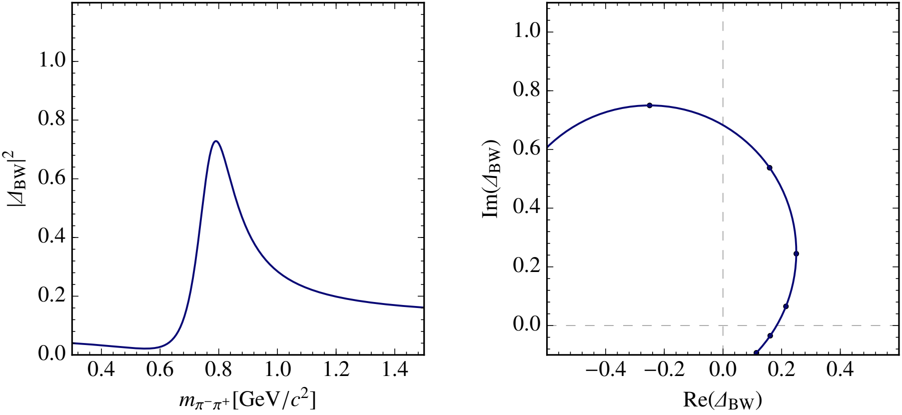
$$\mathcal{C} = -0.33 - 0.24i$$



All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \quad (2)$$

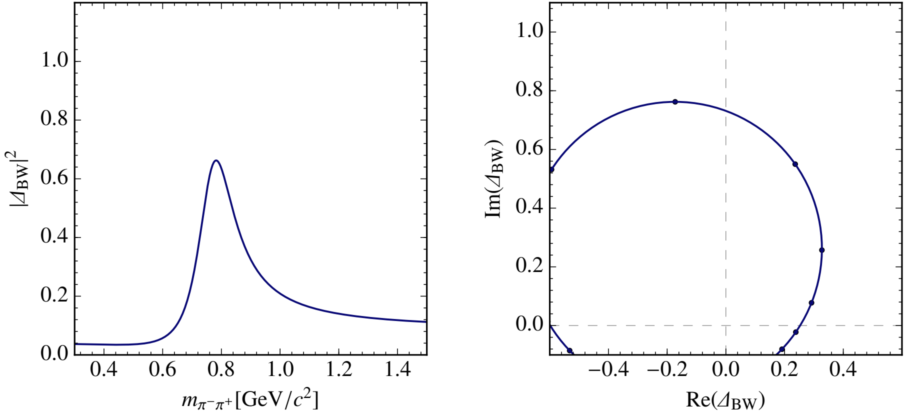
$\mathcal{C} = -0.25 - 0.25i$



All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \tag{2}$$

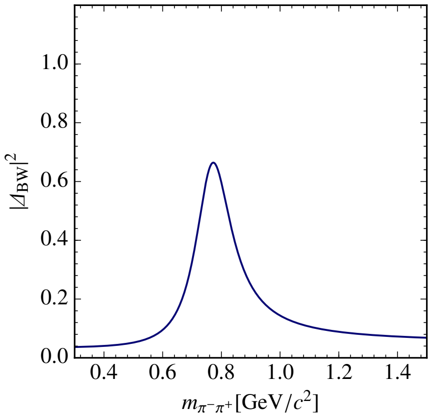
$\mathcal{C} = -0.17 - 0.24i$

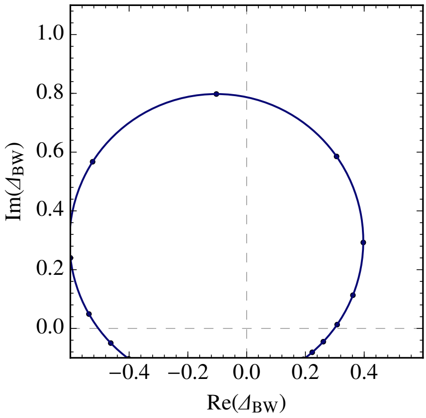


All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \quad (2)$$

$\mathcal{C} = -0.10 - 0.20i$

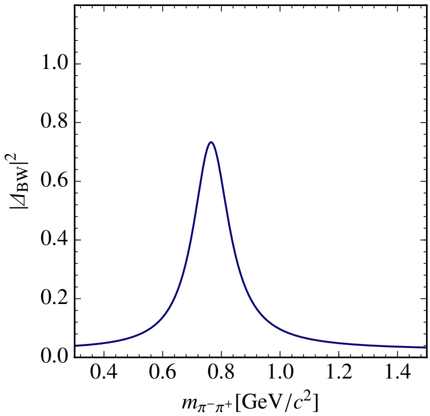


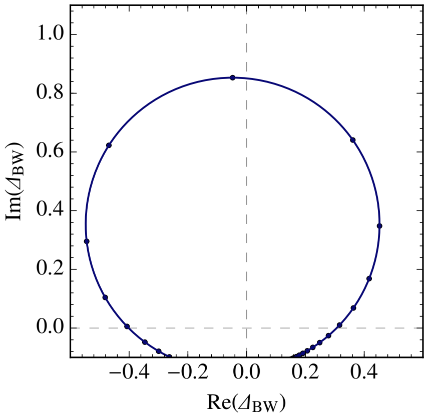


All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \quad (2)$$

$\mathcal{C} = -0.05 - 0.15i$



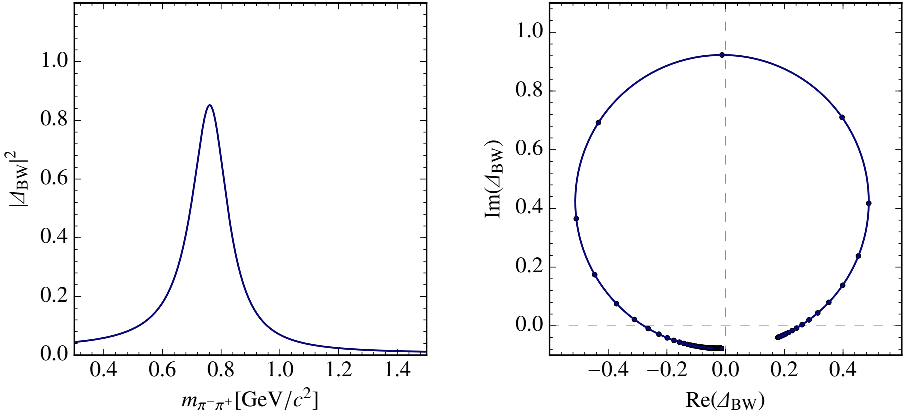


All amplitudes describe the same intensity

Zero mode in the spin-exotic wave
Effects on dynamic isobar amplitudes

$$\Delta_{\mathrm{BW}}(m_{\pi^-\pi^+}) + \mathcal{C}\Omega(m_{\pi^-\pi^+}) \quad (2)$$

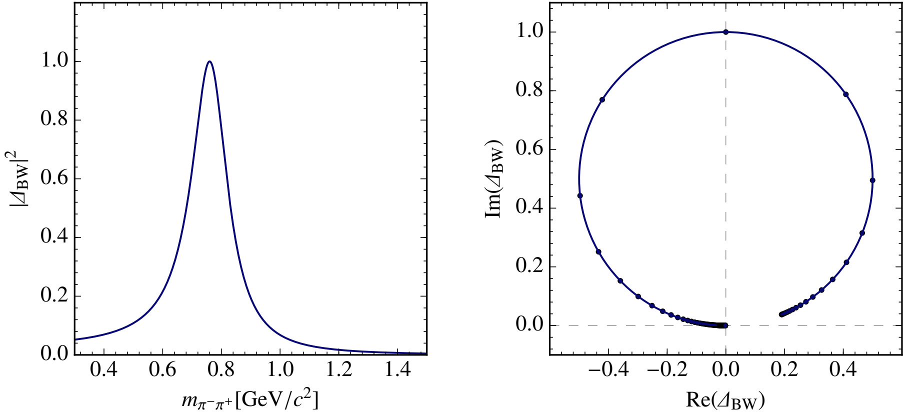
$$\mathcal{C} = -0.01 - 0.08i$$



All amplitudes describe the same intensity

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Omega\left(m_{\pi^-\pi^+}\right) \tag{2}$$

$\mathcal{C} = 0.00 + 0.00i$



All amplitudes describe the same intensity

- Now for $m_{\pi^-\pi^+}$ bins: $\vec{\mathcal{T}}^0 = \{\Omega\left(m_{\text{bin}}\right)\}$ for all $m_{\pi^-\pi^+}$bins

- Now for $m_{\pi^-\pi^+}$ bins: $\vec{\mathcal{T}}^0 = \{\Omega\,(m_{\text{bin}})\}$ for all $m_{\pi^-\pi^+}$ bins
- The fitting algorithm might find a solution, shifted away from the physical solution $\vec{\mathcal{T}}^{\text{phys}}$:

$$\vec{\mathcal{T}}^{\text{phys}} = \vec{\mathcal{T}}^{\text{fit}} + \mathcal{C}\vec{\mathcal{T}}^0$$

- Now for $m_{\pi^-\pi^+}$ bins: $\vec{\mathcal{T}}^0 = \{\Omega\left(m_{\mathrm{bin}}\right)\}$ for all $m_{\pi^-\pi^+}$bins
- The fitting algorithm might find a solution, shifted away from the physical solution $\vec{\mathcal{T}}^{\mathrm{phys}}$:

$$\vec{\mathcal{T}}^{\mathrm{phys}} = \vec{\mathcal{T}}^{\mathrm{fit}} + \mathcal{C}\vec{\mathcal{T}}^0$$

- Obtain physical solution: constrain $\mathcal{C}$ by conditions on the resulting dynamic amplitudes $\vec{\mathcal{T}}^{\mathrm{fit}}$

- Now for $m_{\pi^-\pi^+}$ bins: $\vec{\mathcal{T}}^0 = \{\Omega\,(m_{\text{bin}})\}$ for all $m_{\pi^-\pi^+}$bins
- The fitting algorithm might find a solution, shifted away from the physical solution $\vec{\mathcal{T}}^{\text{phys}}$:

$$\vec{\mathcal{T}}^{\text{phys}} = \vec{\mathcal{T}}^{\text{fit}} + \mathcal{C}\vec{\mathcal{T}}^0$$

- Obtain physical solution: constrain $\mathcal{C}$ by conditions on the resulting dynamic amplitudes $\vec{\mathcal{T}}^{\text{fit}}$
- In the case of the $1^{-+}1^{+}[\pi\pi]_{1^{--}}\pi P$ wave:
  - use the Breit-Wigner for the $\rho\,(770)$ resonance with fixed resonance parameters as in the fixed-isobar analysis
  - use a Breit-Wigner for the $\rho\,(770)$ resonance with floating resonance parameters

- Now for $m_{\pi^-\pi^+}$ bins: $\vec{\mathcal{T}}^0 = \{\Omega(m_{\mathrm{bin}})\}$ for all $m_{\pi^-\pi^+}$ bins
- The fitting algorithm might find a solution, shifted away from the physical solution $\vec{\mathcal{T}}^{\mathrm{phys}}$:

$$\vec{\mathcal{T}}^{\mathrm{phys}} = \vec{\mathcal{T}}^{\mathrm{fit}} + \mathcal{C}\vec{\mathcal{T}}^0$$

- Obtain physical solution: constrain $\mathcal{C}$ by conditions on the resulting dynamic amplitudes $\vec{\mathcal{T}}^{\mathrm{fit}}$
- In the case of the $1^{-+}1^+[\pi\pi]_{1^{--}}\pi P$ wave:
  - use the Breit-Wigner for the $\rho(770)$ resonance with fixed resonance parameters as in the fixed-isobar analysis
  - use a Breit-Wigner for the $\rho(770)$ resonance with floating resonance parameters
- Final results: weighted average of these two methods

- Now for $m_{\pi^-\pi^+}$ bins: $\vec{\mathcal{T}}^0 = \{\Omega\,(m_{\text{bin}})\}$ for all $m_{\pi^-\pi^+}$bins
- The fitting algorithm might find a solution, shifted away from the physical solution $\vec{\mathcal{T}}^{\text{phys}}$:

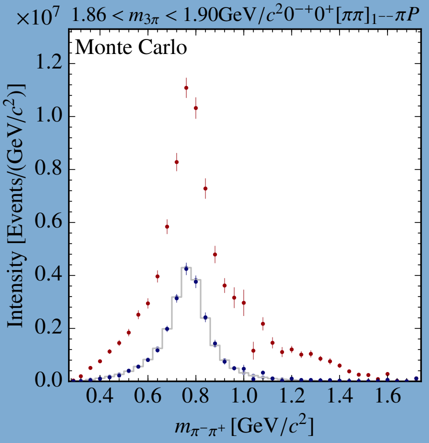
$$\vec{\mathcal{T}}^{\text{phys}} = \vec{\mathcal{T}}^{\text{fit}} + \mathcal{C}\vec{\mathcal{T}}^0$$

- Obtain physical solution: constrain $\mathcal{C}$ by conditions on the resulting dynamic amplitudes $\vec{\mathcal{T}}^{\text{fit}}$
- In the case of the $1^{-+}1^+[\pi\pi]_{1^{--}}\pi P$ wave:
  - use the Breit-Wigner for the $\rho\,(770)$ resonance with fixed resonance parameters as in the fixed-isobar analysis
  - use a Breit-Wigner for the $\rho\,(770)$ resonance with floating resonance parameters
- Final results: weighted average of these two methods
- **Note:** Resolving the ambiguity fixes only a single complex-valued degree of freedom. $n_{\text{bins}} - 1$ complex-valued degrees of freedom remain free.

Zero mode in the spin-exotic wave
Resolving the ambiguity

- Now for $m_{\pi^-\pi^+}$
- The fitting algor... om the physical solution $\vec{\mathcal{T}}^{\text{phys}}$:
- Obtain physical ... e resulting dynamic amplitu...
- In the case of th...
  - use the Breit... resonance parameters ...
  - use a Breit-W... resonance parameters
- Final results: we...
- **Note:** Resolving ... ex-valued degree of freedom. $n_{\text{bins}} - 1$ complex-valued degrees of freedom remain free.

Constrain $\rho(770)$:
$m_0 = 770\,\text{MeV}/c^2$; $\Gamma_0 = 160\,\text{MeV}/c^2$

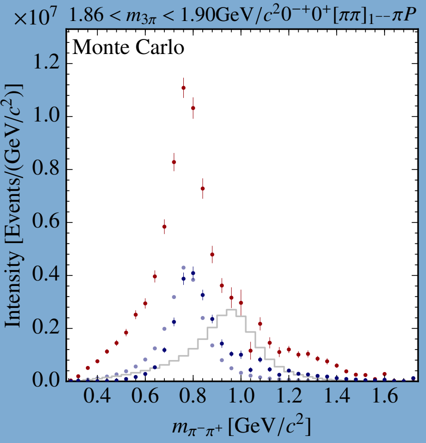


Zero mode in the spin-exotic wave

Resolving the ambiguity

- Now for $m_{\pi^-\pi^+}$ ...
- The fitting algor... ...om the physical solution $\vec{\mathcal{T}}^{\text{phys}}$:
- Obtain physical ... ...e resulting dynamic amplitu...
- In the case of th...
  - use the Breit... ...resonance parameters ...
  - use a Breit-W... ...resonance parameters
- Final results: we...
- **Note:** Resolving ... ...ex-valued degree of freedom. $n_{\text{bins}} - 1$ complex-valued degrees of freedom remain free.

Constrain $\rho(770)$:
$m_0 = 970\,\text{MeV}/c^2$; $\Gamma_0 = 260\,\text{MeV}/c^2$

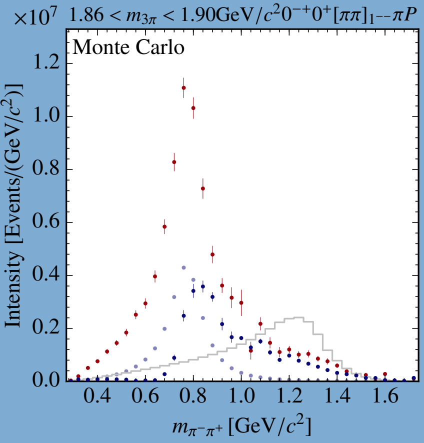


Zero mode in the spin-exotic wave

Resolving the ambiguity

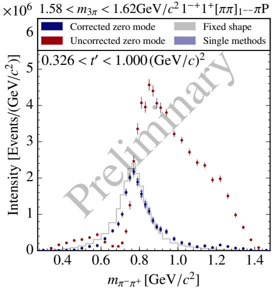
- Now for $m_{\pi^-\pi^+}$
- The fitting algor ... om the physical solution $\vec{\mathcal{T}}^{\text{phys}}$:
- Obtain physical ... e resulting dynamic amplitu
- In the case of th
  - use the Breit ... resonance parameters
  - use a Breit-W ... resonance parameters
- Final results: we
- **Note:** Resolving ... ex-valued degree of freedom. $n_{\text{bins}} - 1$ complex-valued degrees of freedom remain free.

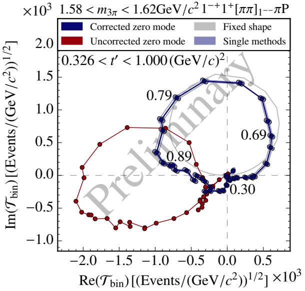
Constrain $\rho(770)$:
$m_0 = 1270\,\text{MeV}/c^2$; $\Gamma_0 = 360\,\text{MeV}/c^2$



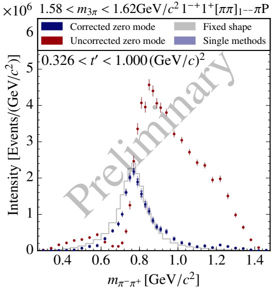
The spin-exotic wave

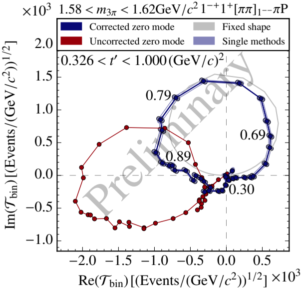
- Example: Single $(m_{3\pi}, t')$ bin
  - $1.58 < m_{3\pi} < 1.62\,\mathrm{GeV}/c^2$
  - $0.326 < t' < 1.000\ (\mathrm{GeV}/c)^2$





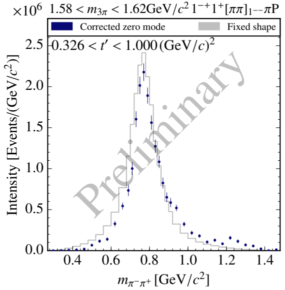
- Example: Single $(m_{3\pi}, t')$ bin
  - $1.58 < m_{3\pi} < 1.62\,\text{GeV}/c^2$
  - $0.326 < t' < 1.000\ (\text{GeV}/c)^2$
- Zero-mode ambiguity resolved with $\rho(770)$ used as constraint

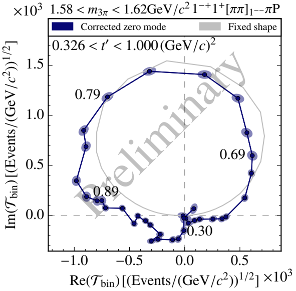




The spin-exotic wave

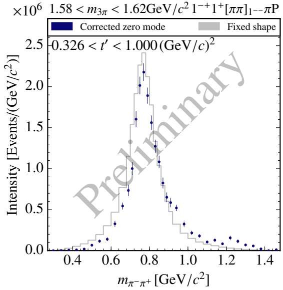
- Example: Single $(m_{3\pi}, t')$ bin
  - $1.58 < m_{3\pi} < 1.62\,\text{GeV}/c^2$
  - $0.326 < t' < 1.000\ (\text{GeV}/c)^2$
- Zero-mode ambiguity resolved with $\rho(770)$ used as constraint

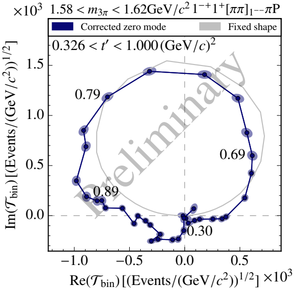




The spin-exotic wave

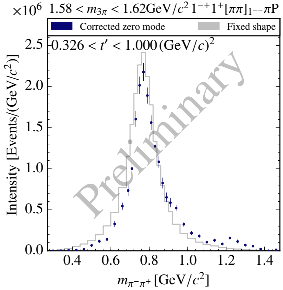
- Example: Single $(m_{3\pi}, t')$ bin
  - $1.58 < m_{3\pi} < 1.62\,\text{GeV}/c^2$
  - $0.326 < t' < 1.000\ (\text{GeV}/c)^2$
- Zero-mode ambiguity resolved with $\rho(770)$ used as constraint
- Dynamic isobar amplitude dominated by $\rho(770)$

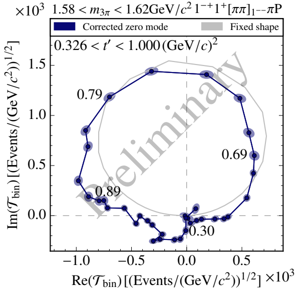




The spin-exotic wave

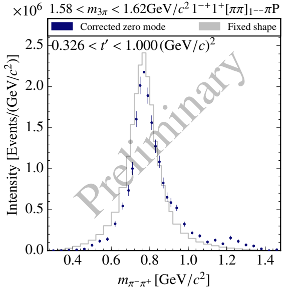
- Example: Single $(m_{3\pi}, t')$ bin
  - $1.58 < m_{3\pi} < 1.62\,\mathrm{GeV}/c^2$
  - $0.326 < t' < 1.000\ (\mathrm{GeV}/c)^2$
- Zero-mode ambiguity resolved with $\rho(770)$ used as constraint
- Dynamic isobar amplitude dominated by $\rho(770)$
- Significant deviations from a pure Breit-Wigner shape

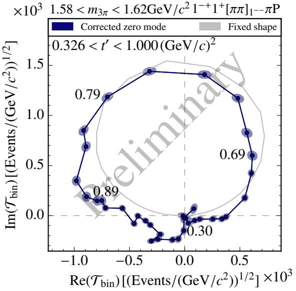




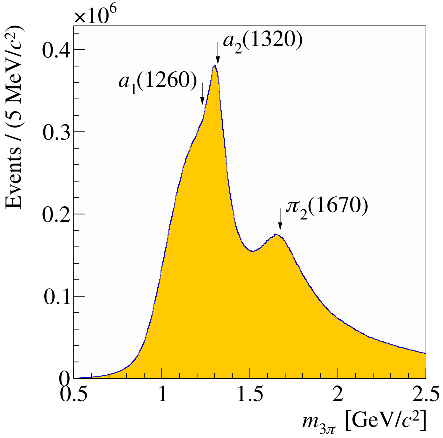
The spin-exotic wave

- Example: Single ($m_{3\pi}, t'$) bin
  - $1.58 < m_{3\pi} < 1.62\,\mathrm{GeV}/c^2$
  - $0.326 < t' < 1.000\ (\mathrm{GeV}/c)^2$
- Zero-mode ambiguity resolved with $\rho(770)$ used as constraint
- Dynamic isobar amplitude dominated by $\rho(770)$
- Significant deviations from a pure Breit-Wigner shape
  - Non-resonant contributions (Deck effect)
  - Rescattering



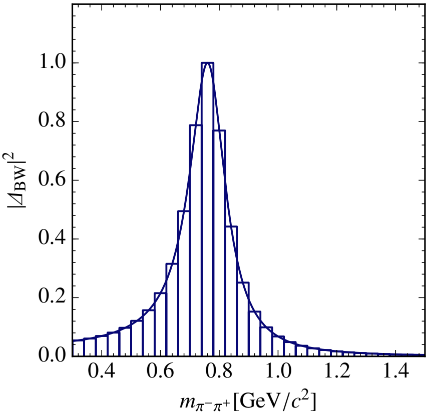


- Starting point: Conventional PWA



Conclusions

- Starting point: Conventional PWA
- Freed-isobar PWA: Replace fixed dynamic isobar amplitudes by step-like functions



Conclusions

- Starting point: Conventional PWA
- Freed-isobar PWA: Replace fixed dynamic isobar amplitudes by step-like functions
- Freed-isobar PWA of COMPASS data with an extended wave set

$0^{-+}\,0^{+} f_0(980)\,\pi\,S$

$0^{-+}\,0^{+} \rho(770)\,\pi\,P$

$1^{-+}\,1^{+} \rho(770)\,\pi\,P$

$1^{++}\,0^{+} f_0(980)\,\pi\,P$

$1^{++}\,0^{+} \rho(770)\,\pi\,S$

$1^{++}\,1^{+} \rho(770)\,\pi\,S$

$2^{-+}\,0^{+} f_0(980)\,\pi\,D$

$2^{-+}\,0^{+} \rho(770)\,\pi\,P$

$2^{-+}\,0^{+} \rho(770)\,\pi\,F$

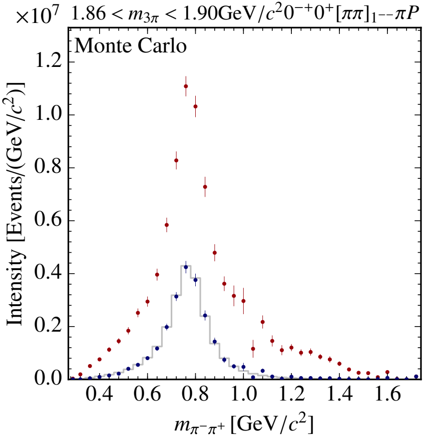
$2^{-+}\,0^{+} \rho(770)\,\pi\,P$

$2^{-+}\,1^{+} f_2(1270)\,\pi\,S$

$2^{++}\,1^{+} \rho(770)\,\pi\,S$

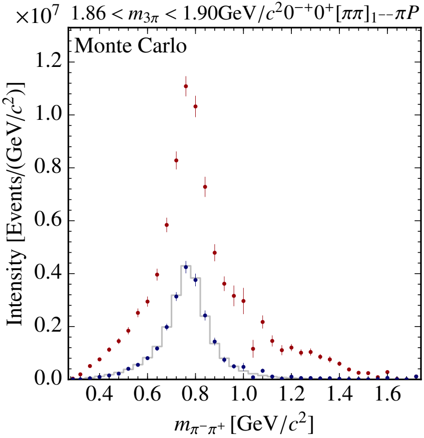
Conclusions

- Starting point: Conventional PWA
- Freed-isobar PWA: Replace fixed dynamic isobar amplitudes by step-like functions
- Freed-isobar PWA of COMPASS data with an extended wave set
- Zero mode: Continuous ambiguity in the $1^{-+}1^{+}\rho\pi P$ wave



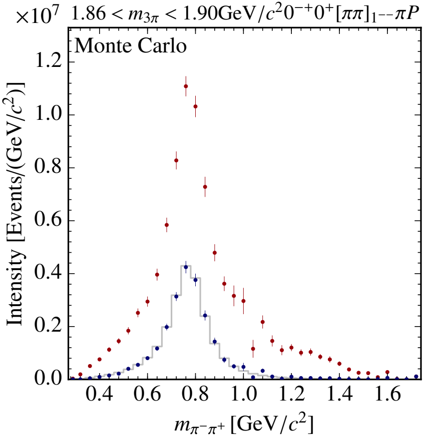
Conclusions

- Starting point: Conventional PWA
- Freed-isobar PWA: Replace fixed dynamic isobar amplitudes by step-like functions
- Freed-isobar PWA of COMPASS data with an extended wave set
- Zero mode: Continuous ambiguity in the $1^{-+}1^{+}\rho\pi P$ wave
  - Similar modes in other waves

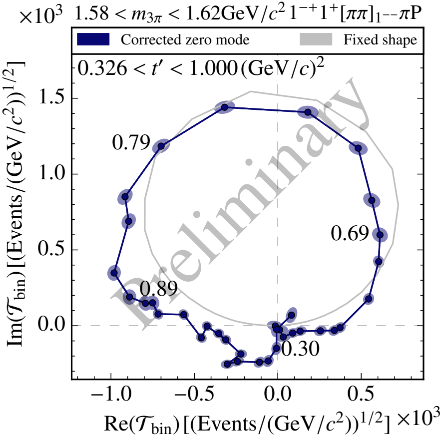


Conclusions

- Starting point: Conventional PWA
- Freed-isobar PWA: Replace fixed dynamic isobar amplitudes by step-like functions
- Freed-isobar PWA of COMPASS data with an extended wave set
- Zero mode: Continuous ambiguity in the $1^{-+}1^{+}\rho\pi P$ wave
  - Similar modes in other waves
- Resolved with physics constraints

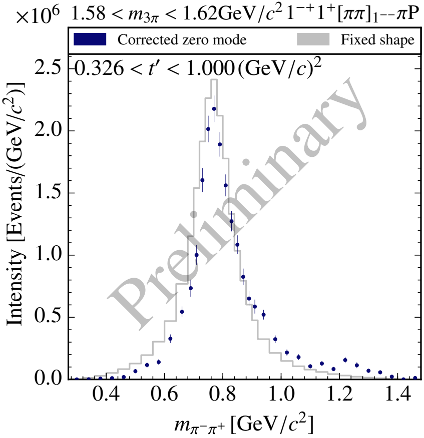


- Starting point: Conventional PWA
- Freed-isobar PWA: Replace fixed dynamic isobar amplitudes by step-like functions
- Freed-isobar PWA of COMPASS data with an extended wave set
- Zero mode: Continuous ambiguity in the $1^{-+}1^{+}\rho\pi P$ wave
  - Similar modes in other waves
- Resolved with physics constraints
- Dynamic isobar amplitude for this wave extracted



Conclusions

- Starting point: Conventional PWA
- Freed-isobar PWA: Replace fixed dynamic isobar amplitudes by step-like functions
- Freed-isobar PWA of COMPASS data with an extended wave set
- Zero mode: Continuous ambiguity in the $1^{-+}1^{+}\rho\pi P$ wave
  - Similar modes in other waves
- Resolved with physics constraints
- Dynamic isobar amplitude for this wave extracted
- Dominated by $\rho(770)$, no pure Breit-Wigner



Outlook

- Only one example shown

Outlook

- Only one example shown
- Results for:
  - 50 $m_{3\pi}$ bins
  - 4 t' bins
  - 12+ waves

Outlook

- Only one example shown
- Results for:
  - 50 $m_{3\pi}$ bins
  - 4 t' bins
  - 12+ waves
- Extract resonance parameters of isobar resonance:
  - Excited resonances, small signals: ρ'

Outlook

- Only one example shown
- Results for:
  - 50 $m_{3\pi}$ bins
  - 4 t' bins
  - 12+ waves
- Extract resonance parameters of isobar resonance:
  - Excited resonances, small signals: ρ'
- Study non-resonant effects:
  - Rescattering

Outlook

- Only one example shown
- Results for:
  - 50 $m_{3\pi}$ bins
  - 4 t' bins
  - 12+ waves
- Extract resonance parameters of isobar resonance:
  - Excited resonances, small signals: ρ'
- Study non-resonant effects:
  - Rescattering
- Validate/improve model in fixed-isobar PWA