The COMPASS Measurement of the Pion Polarizability

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COMPASS collaboration

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**CO**mmom **M**uon and **P**roton **A**pparatus for **S**tructure and **S**pectroscopy
CERN SPS: protons $\sim 400$ GeV (5 – 10 sec spills)

- secondary $\pi, K, (\bar{p})$: up to $2 \cdot 10^7$/s (typ. $5 \cdot 10^6$/s)
- tertiary muons: $4 \cdot 10^7$/s
  - 2002-04, 2006-07, 2010-11: spin structure of the nucleon
Fixed-target experiment

- two-stage magnetic spectrometer
- high-precision, high-rate tracking, PID, calorimetry
Fixed-target experiment

- Two-stage magnetic spectrometer
- High-precision, high-rate tracking, PID, calorimetry


- 190 GeV $\pi^-$ beam on $p$ and nuclear targets (C, Ni, W, Pb)
- Silicon microstrip detectors for “vertexing”
- Recoil and (digital) ECAL triggers
Electromagnetic Polarisabilities

- structure-dependent response to outer e.m. fields:
  \[ \Delta H = -\frac{1}{2} \left( \alpha \cdot \vec{E}^2 + \beta \cdot \vec{B}^2 \right) \]
- well-known for atoms and molecules
- measured on 10%-level for nucleons (also spin dependent)
pion polarisabilities $\alpha_\pi, \beta_\pi$ in units of $10^{-4} \, \text{fm}^3$

size of the pion $\sim 1 \, \text{fm}^3$ [cf. atoms: polarisability $\approx$ size $\approx 1 \, \text{Å}^3$]

Theory: ChPT (2-loop) prediction:

$$\alpha_\pi - \beta_\pi = 5.7 \pm 1.0$$
$$\alpha_\pi + \beta_\pi = 0.16 \pm 0.1$$

experiments for $\alpha_\pi - \beta_\pi$ lie in the range $4 \cdots 14$

($\alpha_\pi + \beta_\pi = 0$ assumed)

*ChPT: chiral perturbation theory: low-energy expansion of QCD*
Pion polarisability and ChPT

Pion polarisabilities $\alpha_\pi, \beta_\pi$ in units of $10^{-4}$ fm$^3$

size of the pion $\sim 1$ fm$^3$ [cf. atoms: polarisability $\approx$ size $\approx 1$ Å$^3$]

Theory: ChPT (2-loop) prediction:

\[
\begin{align*}
\alpha_\pi &= 2.93 \pm 0.5 \\
\beta_\pi &= -2.77 \pm 0.5
\end{align*}
\]

experiments for $\alpha_\pi$ lie in the range $2 \cdots 7$

($\alpha_\pi + \beta_\pi = 0$ assumed)

ChPT: chiral perturbation theory: low-energy expansion of QCD
Pion Compton Scattering

\[ \pi \gamma \rightarrow \pi \gamma \]

- Two kinematic variables, in CM: total energy \( \sqrt{s} \), scattering angle \( \theta_{cm} \)

\[
\frac{d\sigma_{\pi\gamma}}{d\Omega_{cm}} = \frac{\alpha^2 (s^2 z_+^2 + m_\pi^4 z_-^2)}{s (s z_+ + m_\pi^2 z_-)^2} - \frac{\alpha m_\pi^3 (s - m_\pi^2)^2}{4s^2 (s z_+ + m_\pi^2 z_-)} \cdot \mathcal{P}
\]

\[
\mathcal{P} = z_-^2 (\alpha_\pi - \beta_\pi) + \frac{s^2}{m_\pi^4} z_+^2 (\alpha_\pi + \beta_\pi) - \frac{(s - m_\pi^2)^2}{24s} z_-^3 (\alpha_2 - \beta_2)
\]

\[
z_\pm = 1 \pm \cos \theta_{cm}
\]
Pion Compton Scattering

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\[
z_\pm = 1 \pm \cos \theta_{cm}
\]
Pion Compton scattering: embedding the process

Primakoff processes

Radiative pion photoproduction

Photon-Photon fusion
Charged pions traverse the nuclear electric field
- typical field strength at $d = 5R_{Ni}$: $E \approx 300$ kV/fm

Bremsstrahlung process:
- particles scatter off equivalent photons
- tiny momentum transfer $Q^2 \approx 10^{-5}$ GeV$^2$/c$^2$
- pion/muon (quasi-)real Compton scattering

Polarisability contribution
- Compton cross-section typically diminished
- corresponding charge separation $\approx 10^{-5}$ fm $\cdot$ e
Polarisability effect in Primakoff technique

- Charged pions traverse the nuclear electric field
  - typical field strength at $d = 5R_{Ni}$:

  
  \[
  E \approx 300 \text{ kV/fm}
  \]

Bremsstrahlung process: particles scatter off equivalent photons tiny momentum transfer

\[
Q^2 \approx 10^{-5} \text{ GeV}^2/c^2
\]

Pion/muon (quasi-)real

Compton scattering

- Polarisability contribution to Compton cross-section typically diminished

- corresponding charge separation

\[
\approx 10^{-5} \text{ fm} \cdot e
\]

Typically diminished

photon exchange

strong interaction

details: see later

\[
\begin{array}{c|c|c|c|c|c|c}
q_t \text{ [GeV/c]} & 0 & 0.05 & 0.1 & 0.15 & 0.2 & 0.25 \\
\hline
Tq_0 & 0 & 0.05 & 0.1 & 0.15 & 0.2 & 0.25 \\
\end{array}
\]
Pion polarisability: world data before COMPASS

Primakoff processes

Radiative pion photoproduction

Photon-Photon fusion

GIS'06: ChPT prediction, Gasser, Ivanov, Sainio, NPB745 (2006), plots: T. Nagel, PhD
Fil'k'ov analysis objected by Pasquini, Drechsel, Scherer PRC81, 029802 (2010)
Measurement of the Charged-Pion Polarizability

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[213 authors]

(COMPASS Collaboration)

(Received 2 June 2014; revised manuscript received 24 December 2014; published 10 February 2015)

The COMPASS collaboration at CERN has investigated pion Compton scattering, $\pi^- \gamma \rightarrow \pi^- \gamma$, at center-of-mass energy below 3.5 pion masses. The process is embedded in the reaction $\pi^- \mathrm{Ni} \rightarrow \pi^- \gamma \mathrm{Ni}$, which is initiated by 190 GeV pions impinging on a nickel target. The exchange of quasireal photons is selected by isolating the sharp Coulomb peak observed at smallest momentum transfers, $Q^2 < 0.0015$ (GeV/c)$^2$. From a sample of 63,000 events, the pion electric polarizability is determined to be $\alpha_e = (2.0 \pm 0.6_{\text{stat}} \pm 0.7_{\text{syst}}) \times 10^{-4}$ fm$^3$ under the assumption $\alpha_e = -\beta_x$, which relates the electric and magnetic dipole polarizabilities. It is the most precise measurement of this fundamental low-energy parameter of strong
high-energetic pion beam on 4mm nickel disk
observe scattered pions in coincidence with produced hard photons
study of the cross-section shape
Principle of the measurement

- spatial resolution of tracks \( \sim 10 \mu m \)
- angular resolution of photons \( \sim 30 \mu rad \)
ECAL2: 3000 cells of different types
Figure 3.5: Profile of energy deviations shown for 1/4 of a shashlik block and for muon data photons within the range $133 \text{ GeV} < E_\gamma < 152 \text{ GeV}$.

Figure 3.6: Technical drawing of a full shashlik cell to be compared with the figure to the left.

from: Th. Nagel, PhD thesis TUM 2012
Extraction of the pion polarisability

- Identify exclusive reactions

\[ \pi \gamma \{ \text{Ni} \rightarrow \text{Ni'} \} \rightarrow \pi \gamma \]

at smallest momentum transfer \(< 0.001 \text{ GeV}^2/c^2\)

- Assuming \(\alpha_\pi + \beta_\pi = 0\), from the cross-section

\[
R = \frac{\sigma(x_\gamma)}{\sigma_{\alpha_\pi=0}(x_\gamma)} = \frac{N_{\text{meas}}(x_\gamma)}{N_{\text{sim}}(x_\gamma)} = 1 - \frac{3}{2} \cdot \frac{m_\pi^3}{\alpha} \cdot \frac{x_\gamma^2}{1 - x_\gamma} \cdot \alpha_\pi
\]

is derived, depending on \(x_\gamma = E_\gamma(\text{lab})/E_{\text{Beam}}\).
Measuring \(R\) the polarisability \(\alpha_\pi\) can be concluded.

- Control systematics by

\[ \mu \gamma \{ \text{Ni} \rightarrow \text{Ni'} \} \rightarrow \mu \gamma \]

and

\[ K^- \rightarrow \pi^- \pi^0 \rightarrow \pi \gamma \gamma \]
Extraction of the pion polarisability

- Identify exclusive reactions
  \[ \pi \gamma \{ \text{Ni} \rightarrow \text{Ni}' \} \rightarrow \pi \gamma \]
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Measuring \( R \) the polarisability \( \alpha_\pi \) can be concluded.

- Control systematics by
  \[ \mu \gamma \{ \text{Ni} \rightarrow \text{Ni}' \} \rightarrow \mu \gamma \]
  and
  \[ K^- \rightarrow \pi^- \pi^0 \rightarrow \pi \gamma \gamma \]
Identifying the $\pi \gamma \rightarrow \pi \gamma$ reaction


- **Energy balance** $\Delta E = E_\pi + E_\gamma - E_{\text{Beam}}$
- **Exclusivity peak** $\sigma \approx 2.6$ GeV (1.4%)
- $\sim 63,000$ exclusive events ($x_\gamma > 0.4$) (Serpukhov $\sim 7000$ for $x_\gamma > 0.5$)
Primakoff peak


- $\Delta Q_T \approx 12$ MeV/c (190 GeV/c beam → requires few-$\mu$rad angular resolution)
- first diffractive minimum on Ni nucleus at $Q \approx 190$ MeV/c
- data a little more narrow than simulation → negative interference?
Primakoff peak: muon data


- **muon control measurement**: pure electromagnetic interaction
- e.m. nuclear effects well understood
Photon energy spectra for muon and pion beam

Pion polarisability: COMPASS result

\[ \alpha_\pi = (2.0 \pm 0.6_{\text{stat}}) \times 10^{-4} \text{ fm}^3 \]

(assuming \( \alpha_\pi = -\beta_\pi \))

"false polarisability" from muon data:

\[ (0.5 \pm 0.5_{\text{stat}}) \times 10^{-4} \text{ fm}^3 \]

Radiative corrections (Compton scattering part)

muon Compton scattering: $\mu^- + \gamma \rightarrow \mu^- + \gamma$

$pion$ Compton scattering: $\pi^- + \gamma \rightarrow \pi^- + \gamma$

\[ \lambda = 3.8 \text{ MeV} \]

\[ \lambda = 5 \text{ MeV} \]

<table>
<thead>
<tr>
<th>source of systematic uncertainty</th>
<th>estimated magnitude</th>
</tr>
</thead>
<tbody>
<tr>
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<td><strong>quadratic sum</strong></td>
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$\text{COMPASS result for the pion polarisability:} \quad \alpha_\pi = \left( 2.0 \pm 0.6 \text{ stat} \pm 0.7 \text{ syst} \right) \times 10^{-4} \text{ fm}^3$

$\alpha_\pi = -\beta_\pi$ assumed
### source of systematic uncertainty

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COMPASS result for the pion polarisability:

$$\alpha_\pi = (2.0 \pm 0.6_{\text{stat}} \pm 0.7_{\text{syst}}) \times 10^{-4} \text{ fm}^3$$

with $\alpha_\pi = -\beta_\pi$ assumed
The new COMPASS result is in significant tension with the earlier measurements of the pion polarisability.

The expectation from ChPT is confirmed within the uncertainties.
Polarisability and Loop Contributions $z=-1.0$

\[ \sigma / \sigma_{\text{Born}} \]

- LEX $\alpha=-\beta=2.00$
- LEX $\alpha=-\beta=2.85$
- DR [B. Pasquini]
- LEX($\alpha=0$)+chiral loops
- LEX($\alpha=2$)+chiral loops

\[ \sqrt{s}/m_\pi \]

**Dispersion relations and ChPT**

**J. M. Friedrich — Pion Polarizability at COMPASS**
Pion polarisability on the lattice

\[ \alpha_\pi + [10^{-4} \text{ fm}^3] \]

**FIGURE 3.** Left: electric polarizability for the charged pions as a function of the valence quark mass. The data for \( m_\pi = 390 \text{ MeV} \) is taken from [5]. Right: effective mass for a charged pion correlator together with the scalar particle correlator determined from the fit. The fitting range is indicated by the vertical bars.

Alexandru *et al.*, Pion electric polarizability from lattice QCD, arXiv:1501.06516
Pion polarisability at COMPASS: further efforts

~63k events
0.3X₀ Ni
~3 weeks
Primakoff run 2009

~1 week
~10k events
0.5X₀ Pb
Primakoff
pilot run 2004

~3 months
~200–400k events
0.3X₀ Ni
Primakoff run 2012

\[ \mathcal{P} = z_{-}^{2}(\alpha_{\pi} - \beta_{\pi}) + \frac{s^{2}}{m_{\pi}^{4}}z_{+}^{2}(\alpha_{\pi} + \beta_{\pi}) - \frac{(s - m_{\pi}^{2})^{2}}{24s}z_{-}^{3}(\alpha_{2} - \beta_{2}) \]

\[ z_{\pm} = 1 \pm \cos \theta_{cm} \]
Transition to GEANT4

- COMGEANT: original Monte Carlo Simulation for COMPASS based on GEANT3
- Since 2012 efforts to upgrade to C++ based on GEANT4
Transition to GEANT4

![Graph showing counts vs. Q with TGEANT and COMGEANT lines]
Access to $\pi + \gamma$ reactions via the Primakoff effect

**Primakoff reactions accessible at COMPASS**

\[
\pi^- + \gamma \rightarrow \begin{cases} 
\pi^- + \gamma \\
\pi^- + \pi^0 / \eta \\
\pi^- + \pi^0 + \pi^0 \\
\pi^- + \pi^- + \pi^+ \\
\pi^- + \pi^- + \pi^+ + \pi^- + \pi^+ \\
\pi^- + ... 
\end{cases}
\]

**analogously: Kaon-induced reactions** $K^- + \gamma \rightarrow \cdots$
explorative study with 2009 data on “double tungsten” target

ongoing study with 2012 data on “(side-)segmented tungsten” target

best future opportunity: measure $\pi^0$ decays

$$\pi^0 \rightarrow \gamma \gamma \quad \text{and} \quad \pi^0 \rightarrow \gamma \, e^+ e^-$$

from the (exclusive) $\pi^- \pi^0$ final state

on a double tungsten target with varying distance

inspired by and extending the

1980’s “CERN direct measurement of the $\pi^0$ lifetime”

complementary to the PRIMEX measurement
Measurement of the pion polarisability at COMPASS

Via the Primakoff reaction, COMPASS has determined

\[ \alpha_\pi = (2.0 \pm 0.6_{\text{stat}} \pm 0.7_{\text{syst}}) \times 10^{-4} \text{fm}^3 \]

most direct access to the $\pi\gamma \to \pi\gamma$ process
Most precise experimental determination
Systematic control: $\mu\gamma \to \mu\gamma$, $K^- \to \pi^-\pi^0$

Related topics at COMPASS: radiative widths and chiral dynamics in $\pi^-\gamma \to \pi^-\pi^0$ and $\pi\gamma \to \pi\pi\pi$ reactions

High-statistics run 2012

- separate determination of $\alpha_\pi$ and $\beta_\pi$
- $s$–dependent quadrupole polarisabilities
- First measurement of the kaon polarisability
Thank you for your attention!
Photon-photon fusion process \(\gamma\gamma \rightarrow \pi^+\pi^-\)

Planned measurements at ALICE and JLab

\[
\sigma_{\text{tot}}(s) = \frac{2\pi\alpha^2}{\hat{s}^3 m_{\pi}^2} \left\{ \left[ 4 + \hat{s} + \hat{s}|C(\hat{s})|^2 \right] \sqrt{\hat{s}(\hat{s} - 4)} \right. \\
\left. + 8 \left[ 2 - \hat{s} + \hat{s} \Re C(\hat{s}) \right] \ln \frac{\sqrt{\hat{s}} + \sqrt{\hat{s} - 4}}{2} \right\}
\]

\[
C(\hat{s}) = -\beta_\pi \frac{m_{\pi}^3}{2\alpha} \hat{s} - \frac{m_{\pi}^2}{(4\pi f_{\pi})^2} \left\{ \hat{s} + 2 \left[ \ln \frac{\sqrt{\hat{s}} + \sqrt{\hat{s} - 4}}{2} - \frac{i\pi}{2} \right]^2 \right\}
\]

courtesy Norbert Kaiser (TUM)

limited sensitivity to the polarisability contribution
Cross Sections for $\pi^- \gamma \to \ldots$
Pair production and $\pi^0$ lifetime

Compass projected pair probability
$t_1 = 25\mu$, $t_2 = 50\mu$ W targets

probability per pion $p_{\pi0}(\text{GeV})$

J. M. Friedrich — Pion Polarizability at COMPASS
2004 Primakoff results

\[ \pi^- \text{Pb} \rightarrow \text{Pb} \pi^- \pi^- \pi^+ \]

**COMPASS 2004**

- "Low \( t' \)": \( 10^{-3} \text{(GeV}/\text{c})^2 < t' < 10^{-2} \text{(GeV}/\text{c})^2 \) \( \sim 2 \times 10^6 \) events
- "Primakoff region": \( t' < 10^{-3} \text{(GeV}/\text{c})^2 \) \( \sim 1 \times 10^6 \) events
Chiral dynamics in $\pi \gamma \rightarrow 3\pi$

Published in PRL 108 (2012) 192001

Normalization: analysis ongoing
Chiral dynamics in $\pi \gamma \rightarrow 3\pi$
Radiative Coupling of $a_2(1320)$ and $\pi_2(1670)$

\[ \Gamma_0(a_2(1320) \rightarrow \pi\gamma) \, M2 \]

\[ \Gamma_0(\pi_2(1670) \rightarrow \pi\gamma) \, E2 \]

\[ \leftrightarrow \text{meson w.f.'s: } \, \Gamma_{i \rightarrow f} \propto |\langle \Psi_f | e^{-i \vec{q} \cdot \hat{r}} \hat{e} \cdot \vec{p} | \Psi_i \rangle |^2, \, \text{VMD} \]

- normalization via beam kaon decays
- large Coulomb correction

*published in EPJ A50 (2014) 79*
does (charged-)pion exchange play a role at COMPASS collision energies?

search for $Z_c$ photo-production:

\[ \gamma \rightarrow J/\psi Z_c^+ \rightarrow J/\psi \pi^+ \text{ or } J/\psi \text{ pomeron} \]

\[ p \rightarrow n \]

\[ \gamma \rightarrow J/\psi a_2^+ \rightarrow J/\psi \pi^+ \pi^- \text{ or } J/\psi \text{ pomeron} \]

\[ p \rightarrow n \]

constrain contribution from pionic trajectory
calculation following G. Fäl dt (Phys. Rev. C79, 014607)
eikonal approximation: pions traverse Coulomb and strong-interaction potentials
About crossing

Physical regions in Mandelstam plane

- red hatched: physical regions
  \( \gamma + \gamma \rightarrow \pi + \pi \)
  \( \gamma + \pi \rightarrow \gamma + \pi \)

- two-pion thresholds at \( s = 4m^2_{\pi}, u = 4m^2_{\pi}, t = 4m^2_{\pi} \)

- DR integration paths
  \( t = 0 \) (forward),
  \( \theta = 180^\circ \) (backward)
  \( u = m^2_{\pi}, s = m^2_{\pi}, \ldots \)

from: D. Drechsel, talk at IWHSS 2011 Paris
Polarisability effect (LO ChPT values)

\[ \alpha_\pi = 3.00, \beta_\pi = -3.00 \]

loop effects not shown

\[ \frac{d\sigma}{d\Omega_{\text{cm}}} \] [\mu b]

\[ s=3m_\pi^2 \]
\[ s=5m_\pi^2 \]
\[ s=8m_\pi^2 \]
\[ s=15m_\pi^2 \]

Primakoff \[ E_\gamma < 20 \text{ GeV} \]
Polarisability effect (NLO ChPT values)

loop effects not shown

\[ \alpha_\pi = 3.00, \beta_\pi = -2.86 \]
Polarisability effect with “wrong-sign” \( \alpha_\pi + \beta_\pi < 0 \)

\[ \text{loop effects not shown} \]

\[ \frac{d\sigma}{d\Omega_{\text{cm}}} \] vs. \( \cos \theta_{\text{cm}} \)

- - - \( \alpha_\pi = 3.00, \beta_\pi = -3.14 \)
Polarisability effect (Serpukhov values)

\[ \theta \cos^{-1} \]

\[ \begin{array}{cccccc}
-0.8 & -0.6 & -0.4 & -0.2 & 0 & 0.2 \\
0.2 & 0.4 & 0.6 & 0.8 & 1
\end{array} \]

\[ d\sigma / d\Omega_{\text{cm}} [\mu b] \]

\[ \begin{array}{cccc}
0.02 & 0.1 & 0.2 & 0.3 & 0.4
\end{array} \]

\[ s=3m^2, 5m^2, 8m^2, 15m^2 \]

\[ E_\gamma < 20 \text{ GeV} \]

\[ \alpha_\pi = 6.10, \beta_\pi = -6.10 \]

loop effects not shown
- Radiative $\pi^+$ production on the proton:
  \[ \gamma \pi^* \rightarrow \pi \gamma \quad [\text{via } \gamma p \rightarrow n \pi^+ \gamma] \]
  Mainz (2005) measurement: $\alpha_\pi - \beta_\pi = 11.6 \pm 1.5 \pm 3.0 \pm 0.5$
  “$\pm 0.5$”: model error only within the used ansatz, full systematics not under control

- Primakoff Compton reaction:
  \[ \gamma^* \pi \rightarrow \pi \gamma \quad [\text{via } \pi Z \rightarrow Z \pi \gamma] \]
  tiny extrapolation $\gamma^* \rightarrow \gamma \quad O(10^{-3} m^2_\pi)$
  fully under theoretical control
Minimum transverse momentum of the charged particle

\[ \pi^- \text{Ni} \rightarrow \pi^- \gamma \text{Ni} \]

Counts / 2.5 MeV/c

- Data
- Simulation (normalised)
CM energy in $\pi \gamma \rightarrow \pi \gamma$

$\rho$ contribution from $\pi \gamma \rightarrow \pi \pi^0$
Exclusivity vs. $\sqrt{s}$

\begin{itemize}
  \item $\rho$ contribution from $\pi\gamma \rightarrow \pi\pi^0$
\end{itemize}
Mandelstam \( \{s, t\} \leftrightarrow \) Laboratory \( \{E_\gamma, \theta_\gamma\} \)

for \( \pi \gamma \rightarrow \pi \gamma \)
Cross section

J. M. Friedrich — Pion Polarizability at COMPASS
M.R. Pennington in the 2\textsuperscript{nd} DAΦNE Physics Handbook, “What we learn by measuring $\gamma\gamma \rightarrow \pi\pi$ at DAΦNE”:

All this means that the only way to measure the pion polarisabilities is in the Compton scattering process near threshold and not in $\gamma\gamma \rightarrow \pi\pi$. Though the low energy $\gamma\gamma \rightarrow \pi\pi$ scattering is seemingly close to the Compton threshold (...) and so the \textit{extrapolation} not very far, the dominance of the pion pole (...) means that the energy scale for this continuation is $m_\pi$. Thus the polarisabilities cannot be determined accurately from $\gamma\gamma$ experiments in a model-independent way and must be measured in the Compton scattering region.
2004 Primakoff results

\[ \pi^- \text{Pb} \rightarrow \text{Pb} \pi^- \pi^- \pi^+ \]

- "Low \( t' \)": \( 10^{-3} \text{(GeV/c)}^2 < t' < 10^{-2} \text{(GeV/c)}^2 \) \( \sim 2 \, 000 \, 000 \) events
- "Primakoff region": \( t' < 10^{-3} \text{(GeV/c)}^2 \) \( \sim 1 \, 000 \, 000 \) events
2004 Primakoff results

\[ \pi^- \text{Pb} \rightarrow \text{Pb} \pi^- \pi^- \pi^+ \]

COMPASS 2004
\[ \pi \text{Pb} \rightarrow \pi \pi^+ \text{Pb} \]

PWA of \(a_1(1260), a_2(1320)\) contributions in \(t\) slices

- "Low \(t'\)": \(10^{-3} \text{ (GeV/c)}^2 < t' < 10^{-2} \text{ (GeV/c)}^2\) \(\sim 2\,000\,000\) events
- "Primakoff region": \(t' < 10^{-3} \text{ (GeV/c)}^2\) \(\sim 1\,000\,000\) events
PWA: $a_1, a_2$ and $\Delta \Phi$ in separated $t'$ regions

COMPASS 2004
$\pi \text{Pb} \rightarrow \pi \pi \pi^+\text{Pb}$
$0.0015 < t' < 0.01 \text{ GeV}^2/c^2$
$t' < 0.0005 \text{ GeV}^2/c^2$

$1^{++}0^+ \rho \pi S$

$2^{++}1^+ \rho \pi D$

$\Delta \Phi ( 2^{++}1^+ \rho \pi D - 1^{++}0^+ \rho \pi S )$
Phase $a_2 - a_1$ in detail: $t'$ dependence

- transition of $\pi \gamma$ to $\pi IP \rightarrow a_2$ production
- work in progress
- interference can be used to map details of resonances and production mechanisms
No evidence for $a_1(1260) \to \pi\gamma$
Mass-independent PWA (narrow mass bins):

$$\sigma_{\text{indep}}(\tau, m, t') = \sum_{\epsilon=\pm 1} \sum_{r=1}^{N_r} \left| \sum_i T^c_{ir} f^c_i(t') \psi^c_i(\tau, m) \right|^2 \sqrt{\int |f^c_i(t')|^2 dt'} \sqrt{\int |\psi^c_i(t', m)|^2 d\tau'}$$

- Production strength assumed constant in single bins
- Decay amplitudes $\psi^c_i(\tau, m)$, with $t'$ dependence $f^c_i(t')$
- Production amplitudes $T^c_{ir} \rightarrow$ Extended log-likelihood fit
- Acceptance corrections included

Spin-density matrix: $\rho_{ij}^c = \sum_r T^c_{ir} T^c_{jr}^*$

$\rightarrow$ Physical parameters:

- Intens$^c_i = \rho^c_{ii}$
- relative phase $\Phi^c_{ij}$

$$\text{Coh}^c_{i,j} = \sqrt{(\text{Re} \rho^c_{ij})^2 + (\text{Im} \rho^c_{ij})^2} / \sqrt{\rho^c_{ii} \rho^c_{jj}}$$

Mass-dependent $\chi^2$-fit (not presented here):

- $X$ parameterized by Breit-Wigner (BW) functions
- Background can be added
Mass dependence of the diffractive slope

COMPASS 2004

$\pi^- \mathrm{Pb} \rightarrow \pi^- \pi^+ \pi^+ \mathrm{Pb}$

Preliminary
Partial Wave Analysis Formalism
Isobar Model

\[ J^P_C M^\epsilon \]

\( \pi^- \) (beam)
\( \pi^- \) (bachelor)
\( \pi^- \)
\( \pi^+ \)

\( \epsilon = +: \) natural parity exchange
\( \epsilon = -: \) unnatural parity exchange

- Isobar model: Intermediate 2-particle decays
- Partial wave in reflectivity basis: \( J^P_C M^\epsilon [isobar] L \)

- Mass-independent PWA (40 MeV/\( c^2 \) mass bins): 38 waves
  Fit of angular dependence of partial waves, interferences
- Mass-dependent \( \chi^2 \)-fit (Not presented here)
Major intensities in m(3π)-bins (acceptance corrected)

COMPASS 2004
\(\pi Pb \rightarrow \pi \pi \pi^+ Pb\)
\(t' < 0.001 \text{ GeV}^2/c^2\)

- M=0 Spin Total
- M=1 Spin Total

**a_1(1260)**
\(1^{++} \rho \pi S\)

**a_2(1320)**

COMPASS 2004
\(\pi Pb \rightarrow \pi \pi \pi^+ Pb\)
\(t' < 0.001 \text{ GeV}^2/c^2\)

- 2^{++} 1 Spin Total

J. M. Friedrich — Pion Polarizability at COMPASS
PWA of data with low $t'$

Intensity of selected waves: $0^{-+} 0^+ f_0(980) \pi S$, $1^{++} 0^+ \rho \pi S$, $2^{++} 1^+ \rho \pi D$, $2^{-+} 0^+ f_2(1270) \pi S$
"Spin Totals": Sum of all contributions for given M (i.e. z-projection of J)

\[ t' \]-dependent amplitudes:

Primakoff production: \( M=1: \sigma(t') \propto e^{-b_{\text{Prim}}t'} \rightarrow \text{arises at } t' \approx 0 \text{ (resolved shape!)} \)

Diffractive production: \( M=0: \sigma(t') \propto e^{-b_{\text{diff}}(m)t'} \)
\( M=1: \sigma(t') \propto t'e^{-b_{\text{diff}}(m)t'} \rightarrow \text{vanishes for } t' \approx 0 \)
Theory: Phase $a_2$(strong+Coulomb) - $a_1$(strong)

Glauber modell


Plot: N. Kaiser (TU München)

⇒ indicates confirmation of interference Coulomb-interaction - strong interaction
⇒ detailed studies of the nature of resonances
Primakoff contribution at \( t' < 10^{-3} \text{ (GeV/c)}^2 \)

Primakoff: \( \sigma(t') \propto e^{-b_{\text{Prim}} t'} \), \( b_{\text{Prim}} \approx 2000 \text{ (GeV/c)}^{-2} \) (mainly resolution)

Diffractive: \( \sigma(t') \propto e^{-b_{\text{diff}} t'} \), \( b_{\text{diff}} \approx 400 \text{ (GeV/c)}^{-2} \) for lead target

(Mass) spectrum of this Primakoff contribution?
\( \Rightarrow \) Statistical subtraction of diffractive background (for bins of \( m_{3\pi} \))
Higher-order effects

Chiral loops, e.g.
(N. Kaiser, NPA848 (2010) 198)

ρ terms:
First Measurement of $\pi\gamma \rightarrow 3\pi$ Absolute Cross-Section

Measured absolute cross-section of $\pi^-\gamma \rightarrow \pi^-\pi^-\pi^+$

COMPASS 2004

$\pi^-\gamma \rightarrow \pi^-\pi^-\pi^+$
from $\pi^-\text{Pb} \rightarrow \pi^-\pi^-\pi^+\text{Pb}$

- Fitted ChPT Intensity
- Leading Order ChPT Prediction

Full Systematic Error
Luminosity Uncertainty

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Partial Wave Analysis

Isobaric Model – Chiral Wave

\[ \pi^- \gamma \rightarrow \pi^- \pi^0\pi^0 \]

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Partial Wave Analysis

*Chiral Model - Amplitudes*