The $2\pi$ subsystem in diffractively produced $\pi^-\pi^+\pi^-$ at COMPASS

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Hadron 2015
Newport News, VA, USA
The COMPASS experiment

COMPASS hadron setup

The $2\pi$ subsystem

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COMPASS: World’s largest data-set up to now for 
\[ \pi^- p \rightarrow \pi^- \pi^+ \pi^- p \]
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Results of “conventional” PWA shown by Boris Grube yesterday

![Graph showing events vs. mass for the \( m_{3\pi} \) subsystem. Peaks at \( a_1(1260) \), \( a_2(1320) \), and \( \pi_2(1670) \).]
COMPASS: World’s largest data-set up to now for $\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$

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Detailed PWA with 88 partial waves
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- Detailed PWA with 88 partial waves
  - Reliable extraction of waves contributing less than 1% to the intensity
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Detailed PWA with 88 partial waves

- Reliable extraction of waves contributing less than 1% to the intensity

Good agreement with 
\[ \pi^- p \rightarrow \pi^- \pi^0 \pi^0 p \]
3π spectroscopy at COMPASS

Overview over results

Fabian Krinner (TUM E18)
Conventional PWA method
The isobar model

The intermediate state $X^-$ undergoes subsequent two-particle decays $N^3\pi\to\pi^+\pi^-$.

Narrow bins in $m_{\pi^+\pi^-}$ allow for fixed amplitudes of the isobars $\xi$.

Direct fit of resonance parameters of the isobars computationally very expensive.

How good is the isobar model?
How good are the parametrizations used?

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Fixed amplitudes of the *isobars* $\xi$
**Conventional PWA method**

The isobar model

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- Narrow bins in $m_{3\pi} \to$ no assumptions on $X^-$
- Fixed amplitudes of the isobars $\xi$
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Conventional PWA method
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Novel approach
Steplike isobar amplitudes

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- $0^{++}$: $f_0(500)$ (or $\sigma$, $[\pi\pi]$$_S$)
- $0^{++}$: $f_0(980)$

![Graph showing intensity of $f_0(980)$ vs mass of $\pi^-\pi^-$ system]
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- $0^{++}$: $f_0(500)$ (or $\sigma$, $\pi\pi_s$)
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Example: Shape of $0^{++}$ resulting from interference of $f_0(500)$ and $f_0(980)$
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![Plot showing decay widths and mass spectrum of $\pi^+\pi^-$ system](image.png)
Novel approach
Steplike isobar amplitudes

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- Fit of isobar resonance parameters not practical → “binned isobars”

- Extract binned amplitudes

Example: Shape of $0^{++}$ resulting from interference of $f_0(500)$ and $f_0(980)$
Total intensity in conventional PWA

\[ I(m_{3\pi}, m_{\pi^+\pi^-}, \tau) = \left| \sum_{i}^{\text{waves}} T_i(m_{3\pi}) \psi_i(\tau) \Delta_i(m_{\pi^+\pi^-}) \right|^2 \]

Production amplitudes \( T_i(m_{3\pi}) \), angular distributions \( \psi(\tau) \) and isobar amplitudes \( \Delta_i(m_{\pi^+\pi^-}) \) (e.g. Breit-Wigner)
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Replace fixed isobar amplitudes by bins:

\[ \Delta_i(m_{\pi^+\pi^-}) \rightarrow \sum_{\text{bins}} \Delta_i^{\text{bin}}(m_{\pi^+\pi^-}) = [\pi\pi]_{J^PC} \]

\[ \Delta_i^{\text{bin}}(m_{\pi^+\pi^-}) = \begin{cases} 1, & \text{if } m_{\pi^+\pi^-} \text{ in corresponding bin. } \\ 0, & \text{otherwise.} \end{cases} \]
Freed isobar fit
Formulation

- Total intensity in conventional PWA

\[ I(m_{3\pi}, m_{\pi^+\pi^-}, \tau) = \left| \sum_{i}^{\text{waves}} T_i(m_{3\pi}) \psi_i(\tau) \Delta_i(m_{\pi^+\pi^-}) \right|^2 \]

Production amplitudes \( T_i(m_{3\pi}) \), angular distributions \( \psi(\tau) \) and isobar amplitudes \( \Delta_i(m_{\pi^+\pi^-}) \) (e.g. Breit-Wigner)

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- Step-like functions behave like independent Partial Waves:

\[ I = \left| \sum_{i}^{\text{waves}} \sum_{\text{bins}} T_i^{\text{bin}}(m_{3\pi}) \psi_i(\tau) \Delta_i^{\text{bin}}(m_{\pi^+\pi^-}) \right|^2 \]
Two dimensional results

- Conventional analysis: Binned in $m_{3\pi}$: $T(m_{3\pi})$

- Now: Two-dimensional binning: $T(m_{3\pi}, m_{\pi^+\pi^-})$

→ Two-dimensional picture

Here: Three waves with freed isobars:

- $0^{-+} 0^{++} [\pi\pi]_{0^{++}} \pi S$
- $1^{++} 0^{++} [\pi\pi]_{0^{++}} \pi P$
- $2^{-+} 0^{++} [\pi\pi]_{0^{++}} \pi D$

- All other waves still with fixed isobar amplitudes

- In principle also possible for $1^{--}$, $2^{++}$, ... isobars
Two-dimensional intensities for waves with freed isobars

These plots should not be mistaken as Dalitz plots
Two dimensional results

Two-dimensional intensities for waves with freed isobars

MASS OF THE $\pi^-\pi^+\pi^-$ SYSTEM

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Two-dimensional results

Results

Two-dimensional intensities for waves with freed isobars

\[ 0^{-}0^{+} [\pi \pi]_{0^{++}} \pi \ S \]

\[ 1^{++}0^{+} [\pi \pi]_{0^{++}} \pi \ P \]

These plots should not be mistaken as Dalitz plots

MASS OF THE \( \pi^- \pi^+ \pi^- \) SYSTEM
Two-dimensional intensities for waves with freed isobars

Two-dimensional results

These plots should not be mistaken as Dalitz plots
Different regions of the four-momentum transfer $t'$

0.10 < $t' < 0.14 \, (GeV/c)^2$

0−+0+[ππ]0++ πS

0.19 < $t' < 0.32 \, (GeV/c)^2$

0−+0+[ππ]0++ πS
$0^{-+0^{+} [\pi \pi]_{0^{++}} \pi S}$

Slices at constant $m_{3\pi}$

1.70 < $m_{3\pi}$ < 1.72 GeV/c²

1.80 < $m_{3\pi}$ < 1.82 GeV/c²

1.90 < $m_{3\pi}$ < 1.92 GeV/c²

High 0.19 < $t'$ < 0.32(GeV/c)²
Comparison with conventional analysis

- Sum up all bins in $m_{\pi^+\pi^-}$

![Diagram showing intensity distribution with bins]
Sum up all bins in $m_{\pi^+\pi^-}$

Compare with sum of conventional $0^{-+}f_0$ and $0^{-+}\sigma$ waves
Comparison with conventional analysis

- Sum up all bins in $m_{\pi^+\pi^-}$
- Compare with sum of conventional $0^{-+}f_0$ and $0^{-+}\sigma$ waves
- Compatible shapes
Comparison with conventional analysis

- Sum up all bins in $m_{\pi^+\pi^-}$
- Compare with sum of conventional $0^{-+} f_0$ and $0^{-+} \sigma$ waves
- Compatible shapes
- $\pi(1800)$ peak visible
Sum up all bins in $m_{\pi^+\pi^-}$

Compare with sum of conventional $0^{-+} f_0$ and $0^{-+} \sigma$ waves

Compatible shapes

$\pi(1800)$ peak visible

Novel method reproduces shape in $m_{3\pi}$
Different regions of the four-momentum transfer $t'$

$0.10 < t' < 0.14 \ (\text{GeV/c})^2$

$1^{++} 0^+ [\pi\pi]_{0^{++}} \pi P$

$0.19 < t' < 0.32 \ (\text{GeV/c})^2$

$1^{++} 0^+ [\pi\pi]_{0^{++}} \pi P$
Slices at constant $m_{3\pi}$

High $t' = 0.19 - 0.32(GeV/c)^2$
Sum all bins in $m_{\pi^+\pi^-}$
Comparison with conventional analysis

- Sum all bins in $m_{\pi^+\pi^-}$
- Compare with sum of corresponding waves in conventional PWA
Comparison with conventional analysis

- Sum all bins in $m_{\pi^+\pi^-}$
- Compare with sum of corresponding waves in conventional PWA
- Shapes are compatible

![Graph showing comparison between conventional analysis and proposed method](image-url)
Comparison with conventional analysis

- Sum all bins in $m_{\pi^+\pi^-}$
- Compare with sum of corresponding waves in conventional PWA
- Shapes are compatible
- New resonance, $a_1(1420)$, visible as peak

![Graph showing $1^{++}0^+[\pi\pi S\text{-wave}]\,\pi P$](image-url)

- Intensity / (40 MeV/c²)
- $1^{++}0^+[\pi\pi S\text{-wave}]\,\pi P$
- Fixed isobars
- Freed isobars
- $0.194 < t' < 0.326 (\text{GeV/c})^2$
Different regions of the four-momentum transfer $t'$

\[ 0.10 < t' < 0.14 \ (\text{GeV/c})^2 \]
\[ 2^{-+}0^+ [\pi\pi]_{0^+} \pi D \]

\[ 0.19 < t' < 0.32 \ (\text{GeV/c})^2 \]
\[ 2^{-+}0^+ [\pi\pi]_{0^+} \pi D \]

Origin of broad structures not clear at the moment
(Shadows of fixed-shape waves?)
Conclusions

- Isobar amplitudes are replaced by sets of step-like functions $[\pi\pi]_{JPC}$
- Novel method allows to extract the amplitudes of isobars

![Graph showing intensity vs mass of the $\pi\pi$ system](image)
Conclusions

- Isobar amplitudes are replaced by sets of step-like functions \( [\pi\pi]_{JPC} \)
- Novel method allows to extract the amplitudes of isobars
- Study resonance production in three dimensions: \( m_{3\pi}, m_{\pi^+\pi^-} \) and \( t' \)

![Graph showing 0^-0^+ [\pi\pi]_0^+ \pi S with mass constraints 0.194 < t' < 0.326 (GeV/c)^2]
Conclusions

- Isobar amplitudes are replaced by sets of step-like functions $[\pi\pi]_{J^P}$
- Novel method allows to extract the amplitudes of isobars
- Study resonance production in three dimensions: $m_{3\pi}$, $m_{\pi^+\pi^-}$ and $t'$
- Known waves and decay modes reproduced, especially the new $a_1(1420) \rightarrow f_0(980)\pi^-$

![Graph](image)
Conclusions

- Isobar amplitudes are replaced by sets of step-like functions $[\pi\pi]_{J^P}$
- Novel method allows to extract the amplitudes of isobars
- Study resonance production in three dimensions: $m_{3\pi}$, $m_{\pi^+\pi^-}$ and $t'$
- Known waves and decay modes reproduced, especially the new $a_1(1420) \rightarrow f_0(980)\pi^-$
- $t'$ dependent, broad structures at small $m_{3\pi}$, $m_{\pi^+\pi^-}$ → Possible non-resonant processes

![Graph showing $m_{3\pi}$ vs $m_{\pi^+\pi^-}$ with $0.194 < t' < 0.326 (GeV/c)^2$]
Outlook

- Reduce effects from imperfect parameterizations in other waves

→ Free isobar-amplitudes for all large waves
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  → Free isobar-amplitudes for all large waves

Goal at the moment: Free 11 waves

\[
\begin{align*}
0^{-+} f_0(980)\pi S \\
0^{-+} \rho(770)\pi P \\
1^{++} f_0(980)\pi P \\
1^{++} \rho(770)\pi S \\
1^{++} 1^+ \rho(770)\pi S \\
2^{--} f_0(980)\pi D \\
2^{--} \rho(770)\pi P \\
2^{--} \rho(770)\pi F \\
2^{--} \rho(770)\pi P \\
2^{--} f_2(1270)\pi S \\
2^{++} f_2(1270)\pi S \\
2^{++} 1^+ \rho(770)\pi S
\end{align*}
\]
Outlook

- Reduce effects from imperfect parameterizations in other waves
  → Free isobar-amplitudes for all large waves

- Goal at the moment: Free 11 waves
  - 75% of the total intensity
  - All waves above 1% of the intensity

Some challenges:
- Freeing isobars heavily increases the number of parameters
- Some problems with non-orthogonality of Partial Waves

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2^{-+} f_0(980)\pi D \\
2^{-+} \rho(770)\pi P \\
2^{-+} \rho(770)\pi F \\
2^{-+} \rho(770)\pi P \\
2^{-+} f_2(1270)\pi S \\
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\[0^{-+} 0^{+} f_0(980)_{\pi S}\]
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\[1^{++} 0^{+} f_0(980)_{\pi P}\]
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