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# COMPASS measurement of $g_1$ and QCD fits

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On behalf of the COMPASS Collaboration

- **Measurement of  $g_1^p$  at 200 GeV**
- **NLO QCD fit of  $g_1$  world data**
- **Test of Bjorken sum rule**

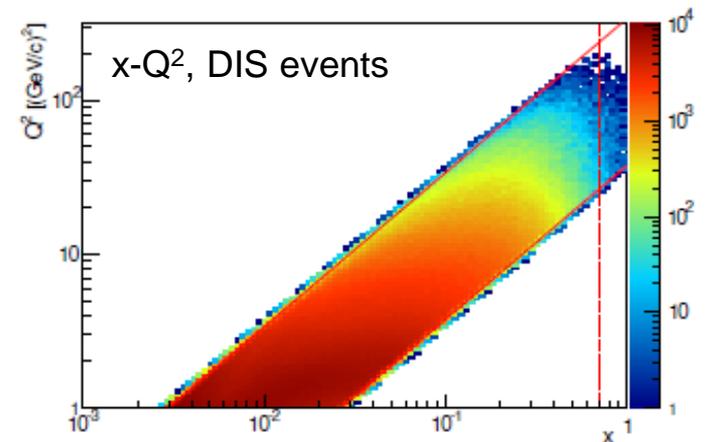


# Longitudinal spin asymmetry measurements at COMPASS

Year	Pol. Target	Pol. $\mu$ beam	Status
2002-2006	d ( $^6\text{LiD}$ )	160 GeV	PLB 612 (2005) 154 PLB 647 (2007) 8 PLB 647 (2007) 330 (low x) PLB 660 (2008) 458 (SIDIS) PLB 680 (2009) 171 (SIDIS)
2007	p ( $\text{NH}_3$ )	160 GeV	PLB 690 (2010) 240 PLB 693 (2010) 227 (SIDIS)
2011	p ( $\text{NH}_3$ )	200 GeV	prelim. DIS-2013

Goal of 200 GeV proton data:  
improve precision at low x, access slightly higher  $Q^2$ :

- Enlarge  $x$ - $Q^2$  coverage for QCD fits ( $\Delta G$ ,  $\Delta \Sigma$ )
- Improve precision of the integral of  $g_1^p$  ( $\Delta \Sigma$ )
- Improve precision on semi-inclusive data at low x ( $\Delta \bar{u}$ ,  $\Delta \bar{d}$ )
- Balance proton and deuteron statistics



# COMPASS at CERN

Fixed target

160-200 GeV muon and 190 GeV hadron beams from CERN SPS

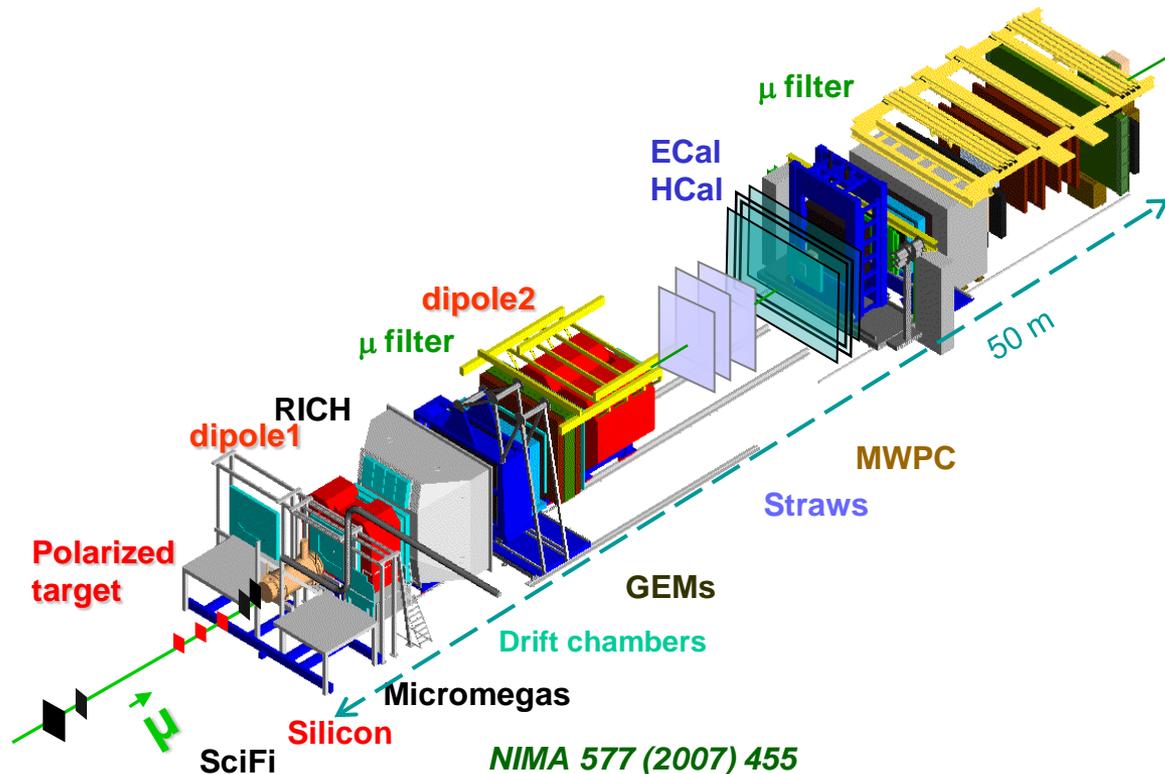
→ Multipurpose setup

Polarized muon beam  
& polarized target: d, p

Nucleon spin structure

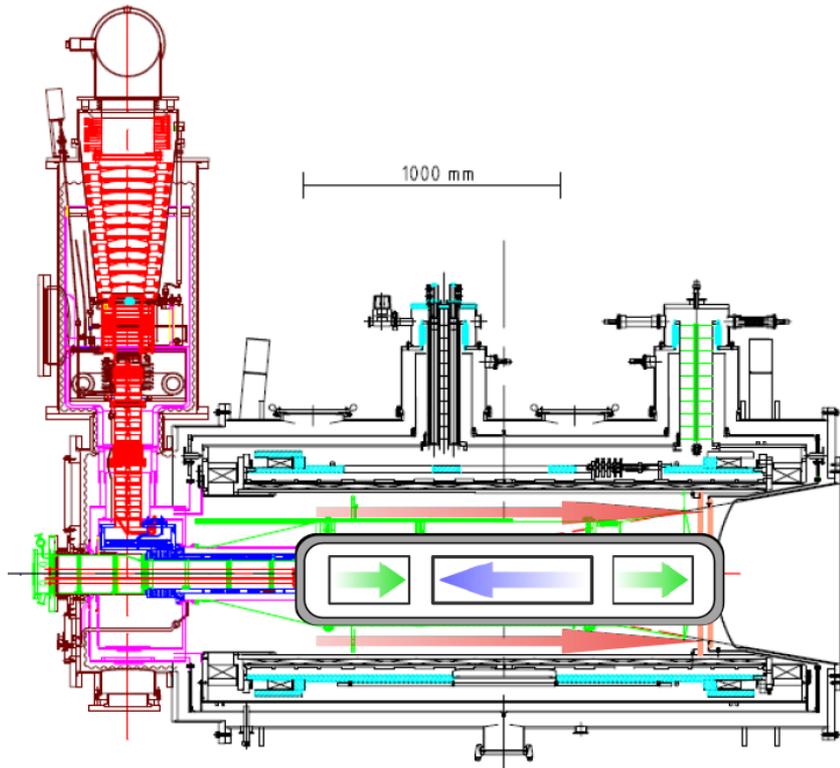
Hadron beam  $\pi / K / p$   
&  $LH_2$  or nuclei

Meson spectroscopy  
 $\pi$ , K polarisabilities



Future:  
GPDs from DVCS  
TMDs from Polarized  
Drell-Yan

# Polarized target

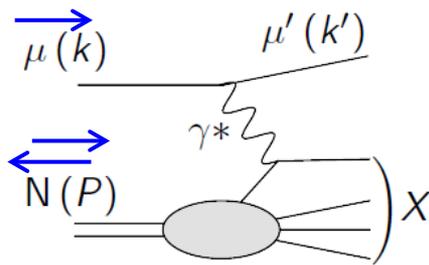


- NH<sub>3</sub> material
- Dilution ( $15 \pm 0.3$ ) %
- Polarisation ( $80 \pm 3$ )%
- Dynamic Nuclear Polarization
- Superconducting solenoid 2.5T

- 3 cells with opposite polarizations  
Nice balance of acceptance from the 2 spin states
- Reversal of polarization by:
  - Adiabatic rotation of solenoid field
  - Different microwave settings→ 4 measurements

→ Minimize systematics

# Results for $A_1^p$

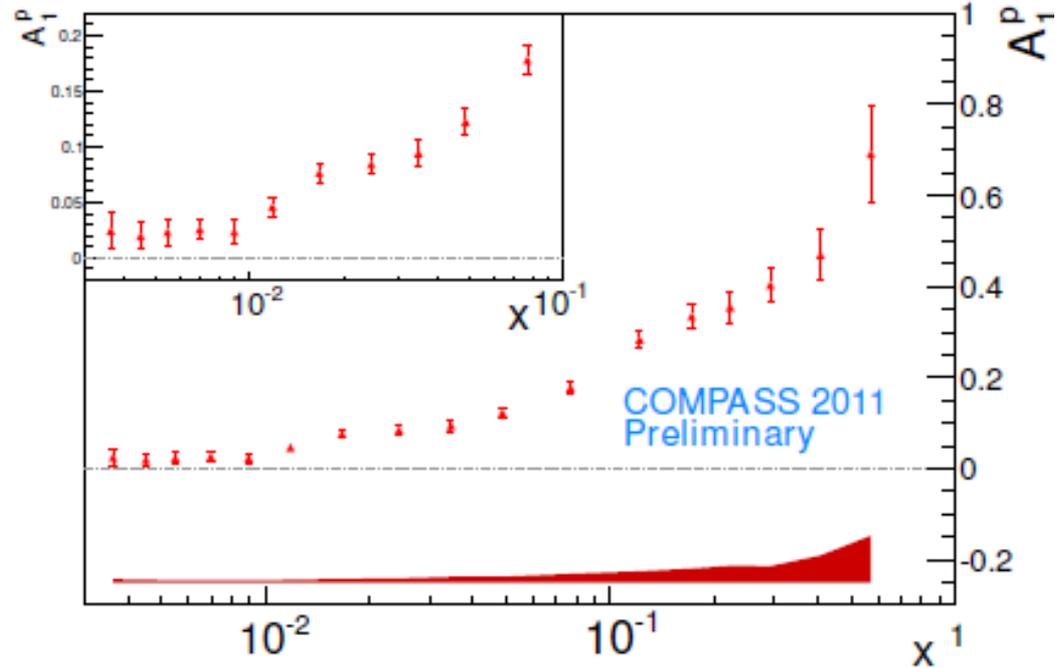


DIS events,  $Q^2 > 1$

$$A_{\parallel} = \frac{d\sigma^{\rightarrow\rightarrow} - d\sigma^{\rightarrow\leftarrow}}{d\sigma^{\rightarrow\rightarrow} + d\sigma^{\rightarrow\leftarrow}} = D(A_1 + \eta A_2)$$

$$\frac{A_{\parallel}}{D} = \left\langle \frac{1}{2} \frac{1}{|P_B P_T| f D} \left( \frac{N^{\rightarrow\rightarrow} - N^{\rightarrow\leftarrow}}{N^{\rightarrow\rightarrow} + N^{\rightarrow\leftarrow}} + \frac{N^{\leftarrow\rightarrow} - N^{\leftarrow\leftarrow}}{N^{\leftarrow\rightarrow} + N^{\leftarrow\leftarrow}} \right) \right\rangle$$

f: target dilution      D: depolarization factor



$$A_{\parallel} \approx D \cdot \frac{g_1}{F_1}$$

# Systematic errors on $A_1$

- Multiplicative
- Additive

$$A_1^{1\gamma} = \frac{1}{fDP_B P_T} A^{raw} - \left( A_1^{RC} + \mathcal{O}\left(\frac{x}{Q} A_2\right) + \mathcal{O}(A_{False}) \right)$$

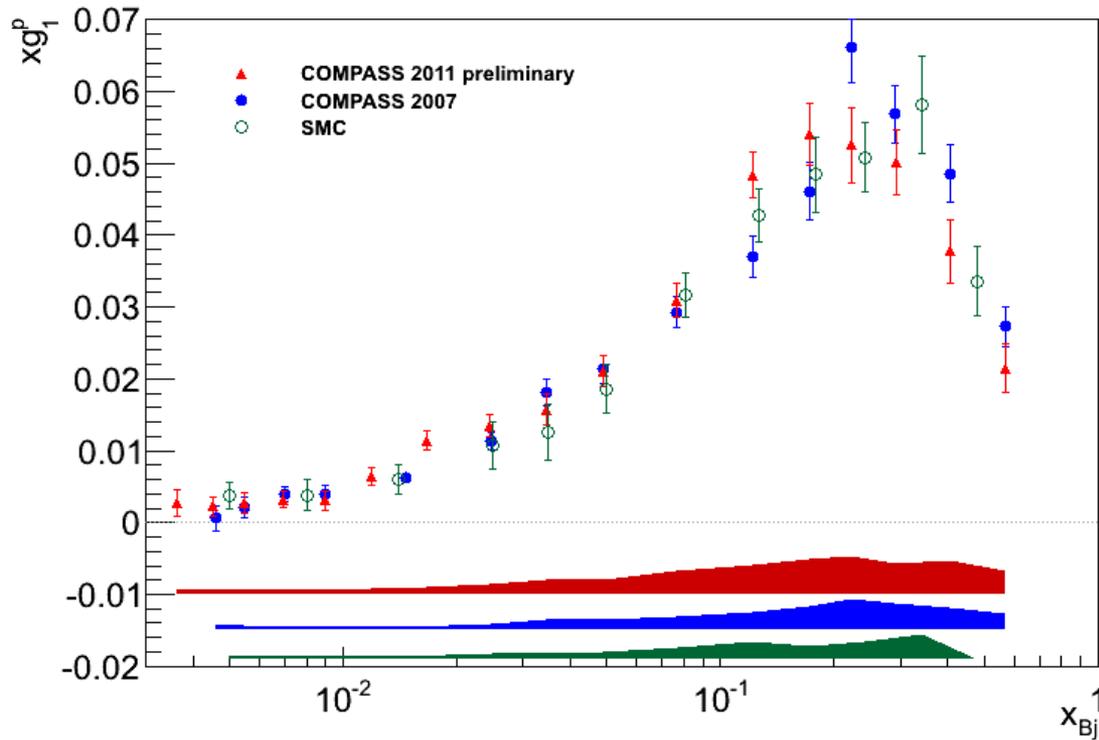
Total of systematics always smaller than statistical error.

Dominant contributions:

- conservative limit put on possible false asymmetries (not seen)
- beam polarization at medium x

Multiplicative variables error, $\Delta A_1^{mult}$	Beam polarisation	$\sigma_{P_B}/P_B$	5 %
	Target polarisation	$\sigma_{P_T}/P_T$	3.5 %
	Depolarisation factor	$\sigma_D/D$	2 – 3 %
	Dilution factor	$\sigma_f/f$	2 %
Additive variables error, $\Delta A_1^{add}$	Transverse asymmetry	$\mathcal{O}(x/Q) \cdot \sigma_{A_2}$	$10^{-3} - 10^{-2}$
	Rad. corrections	$\sigma_{A_1^{RC}}$	$10^{-5} - 10^{-3}$
	False asymmetry	$\sigma_{A_{false}}$	$< 0.34 - 0.84 \cdot \sigma_{A_1^{stat}}$

# Results for $g_1^p$ at 200 GeV



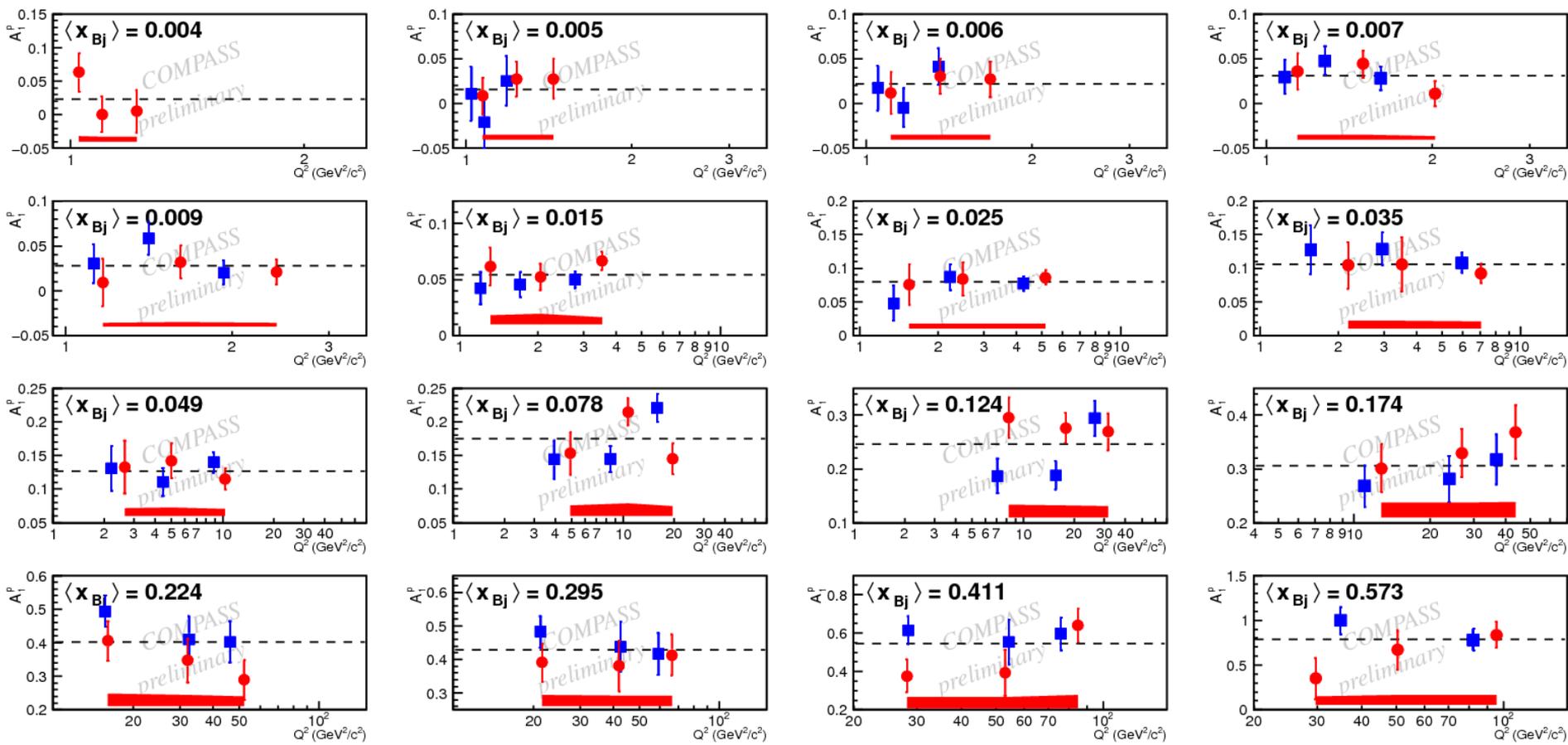
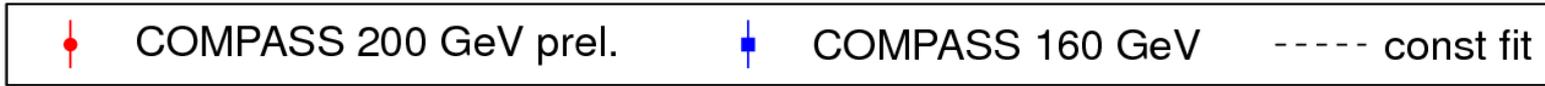
- COMPASS 200 GeV
- COMPASS 160 GeV
- SMC 190 GeV

shown at measured  $Q^2$

*V. Andrieux, DIS 2013*

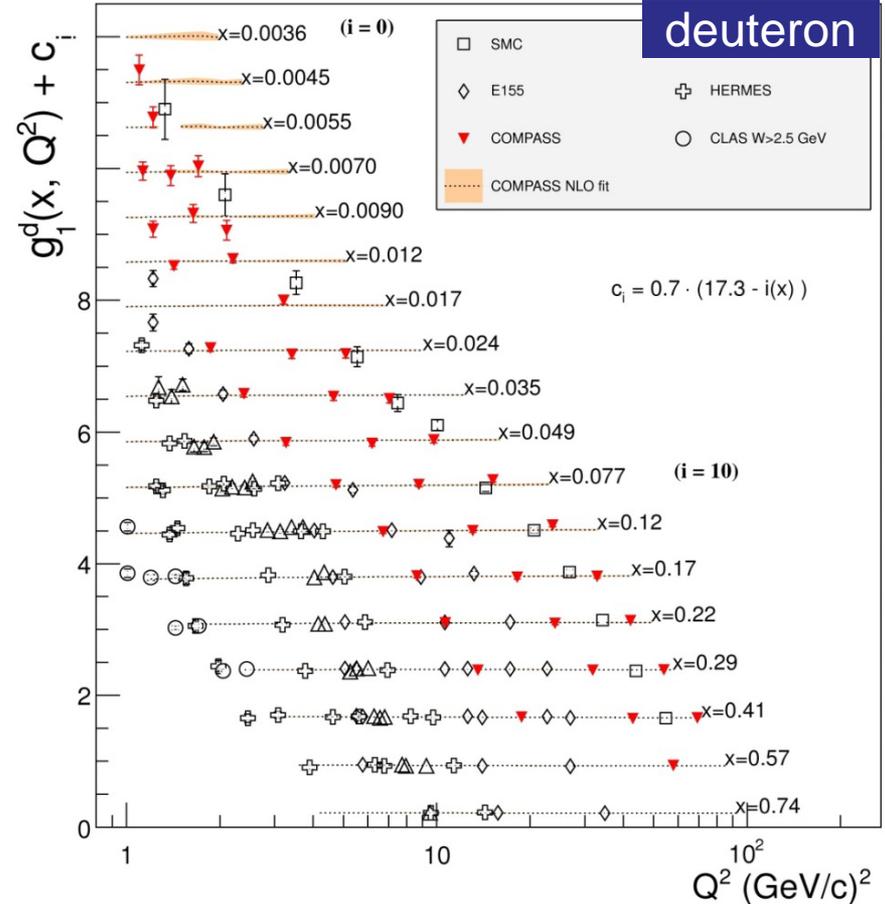
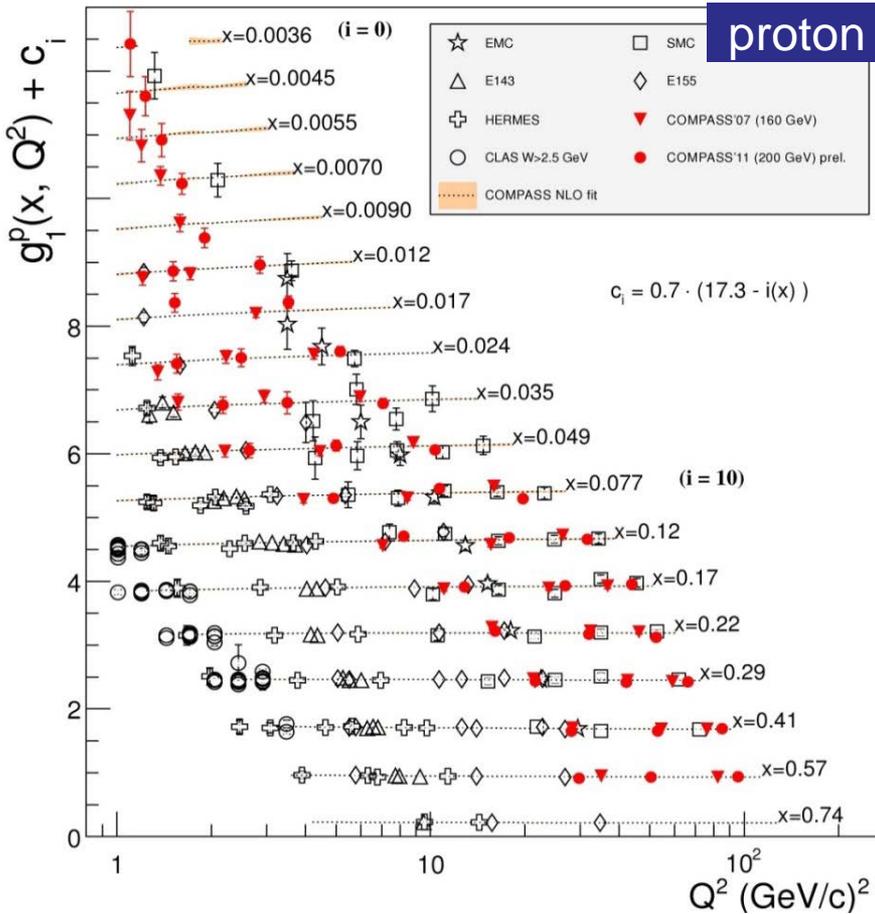
- All 3 sets of data compatible
- Lowest x value reached  $x=0.0036$
- Statistical uncertainties (160 and 200 GeV) 3 times smaller than SMC in similar beam time

# A1 (Q<sup>2</sup>) at various x



160 and 200 Gev data: all Q<sup>2</sup> dependences well fitted by a constant

# World data on $g_1^p$ and $g_1^d$



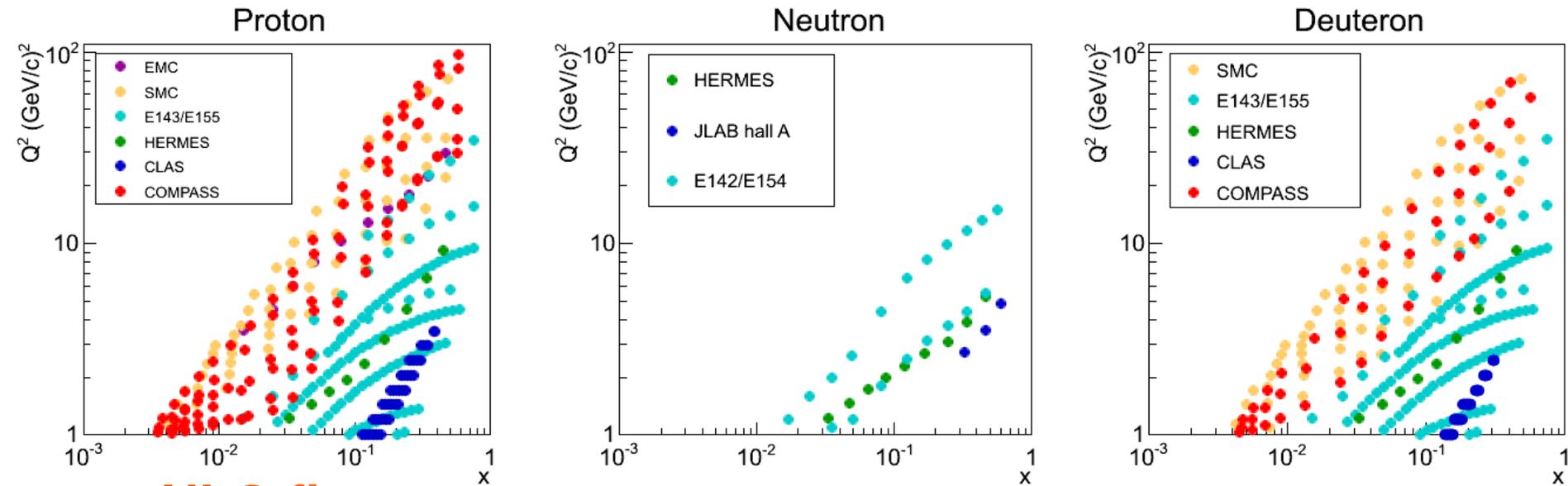
- New **proton** data from COMPASS 200 GeV : low  $x$  and high  $Q^2$

$g_1$  as input to global QCD fits for extraction of  $\Delta q_f(x)$  and  $\Delta G(x)$

$$\frac{d g_1}{d \text{Log}(Q^2)} \propto -\Delta g(x, Q^2)$$

# Fit to proton, neutron and deuteron world data

$x$ - $Q^2$  coverage of world data



## NLO fit:

- Assume functional forms for  $\Delta\Sigma$ ,  $\Delta G$  and  $\Delta q^{\text{NS}}$  at a reference  $Q_0^2 = 1(\text{GeV}/c)^2$

e.g.: 
$$\Delta q_{si}(x|Q_0^2) = \eta_s x^{\alpha_s} (1-x)^{\beta_s} (1 + \gamma_s x) / N_s$$

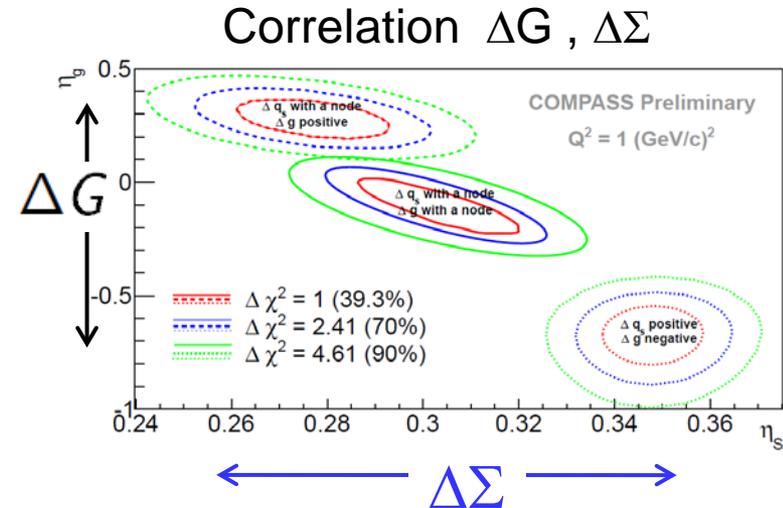
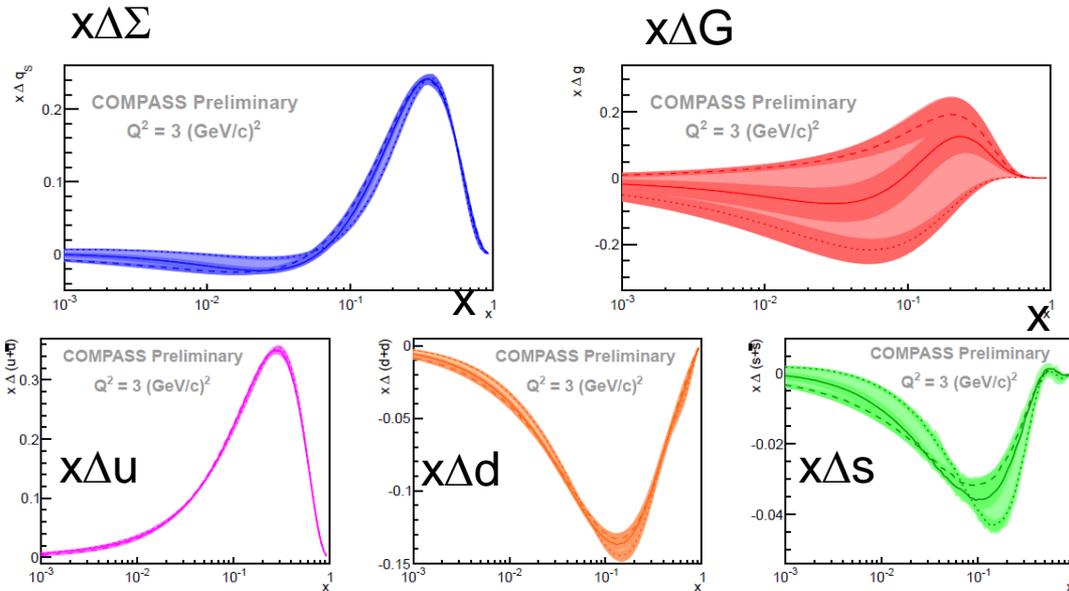
- Assume  $\text{SU}_3$
- Use DGLAP equations
- Fit world data

679 points, 28 free parameters.

# COMPASS NLO pQCD fit of $g_1$ world data

→ 3 classes of solutions,  $\Delta G > 0$ ,  $\Delta G$  with a node, and  $\Delta G < 0$

V. Andrieux, QCD@Work -2014

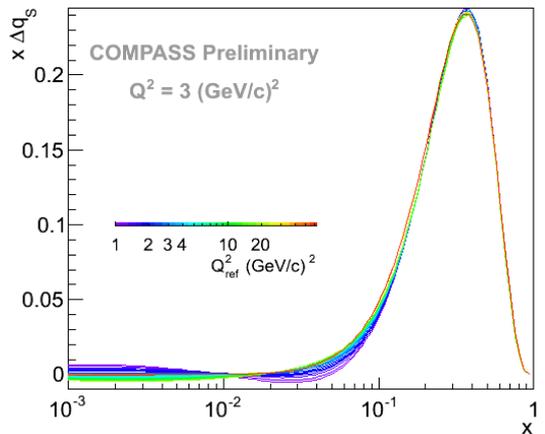
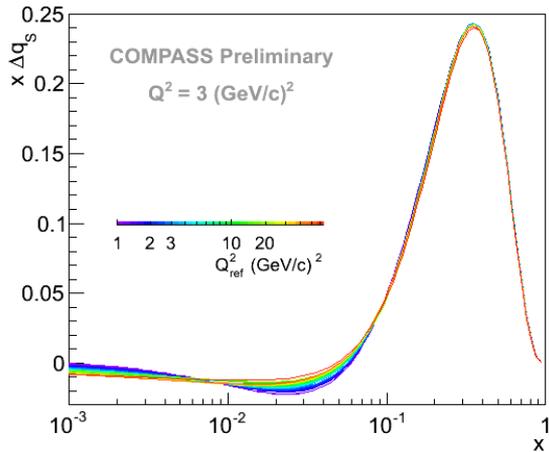
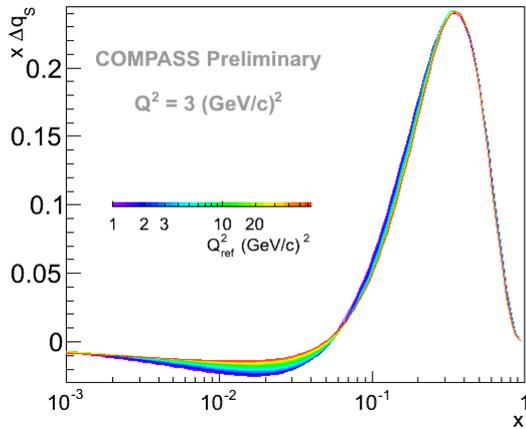
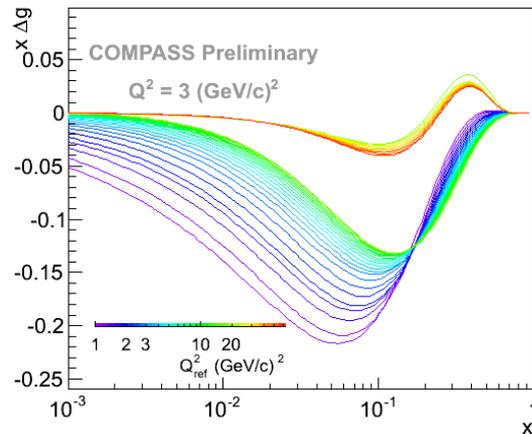
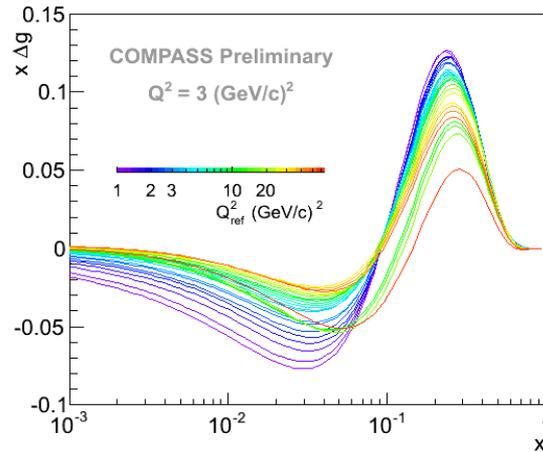
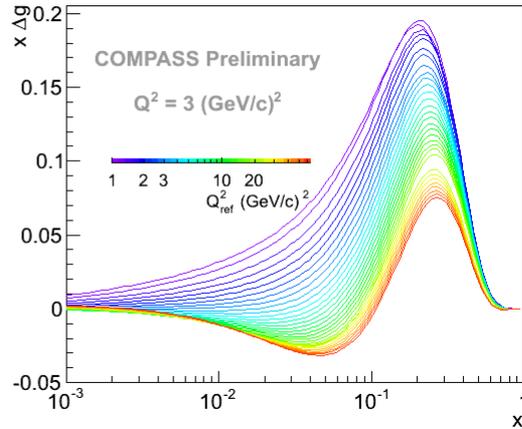


- Quark spin contribution :  $0.26 < \Delta\Sigma < 0.34$  at  $Q^2=3 \text{ (GeV/c)}^2$

Largest uncertainty comes from the bad knowledge of functional forms.

Result in fair agreement with other global fits, and with Lattice QCD.

- Gluon spin contribution:  $\Delta G$  not well constrained, even the sign, using DIS only  
Solution with node agrees with result from DSSV++ using RHIC pp data

$\Delta\Sigma$  $\Delta G$ 

Varying  $Q_{0\text{ref}}$   
 from 1 to 60  $\text{GeV}^2$

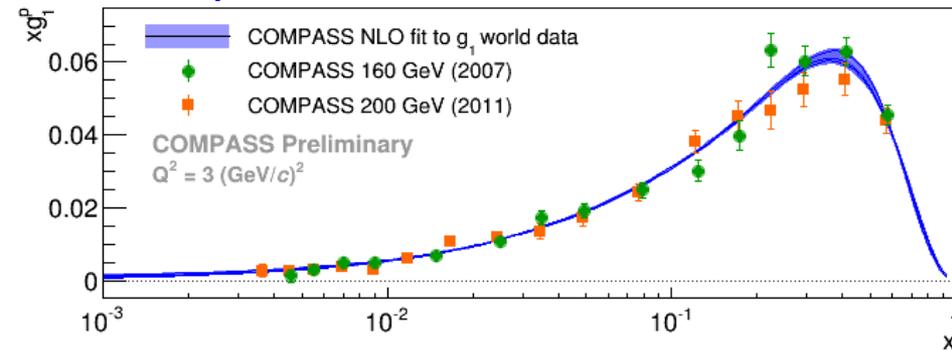
→ equivalent to  
 changing the  
 functional form

→ well visible on  $\Delta G$

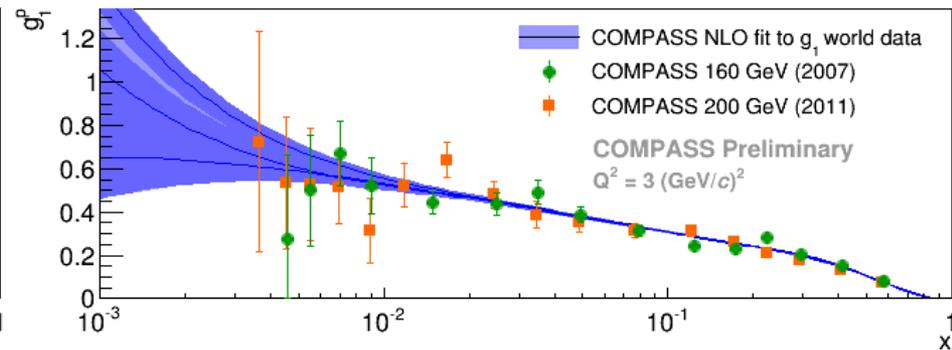
# $g_1^p$ and $g_1^d$

COMPASS data and NLO QCD fit

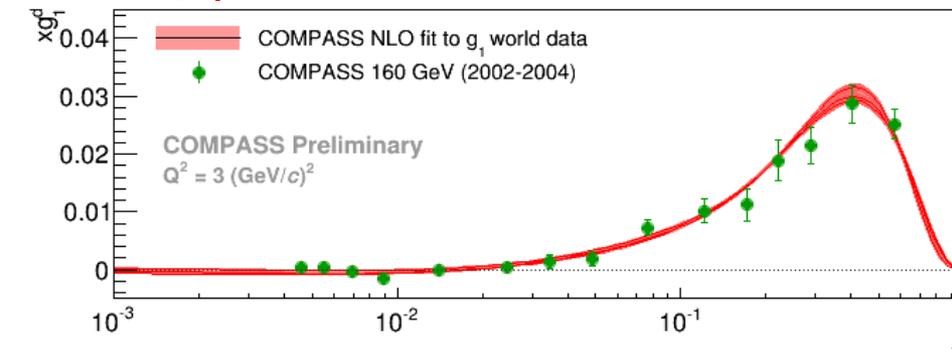
$xg_1^p$



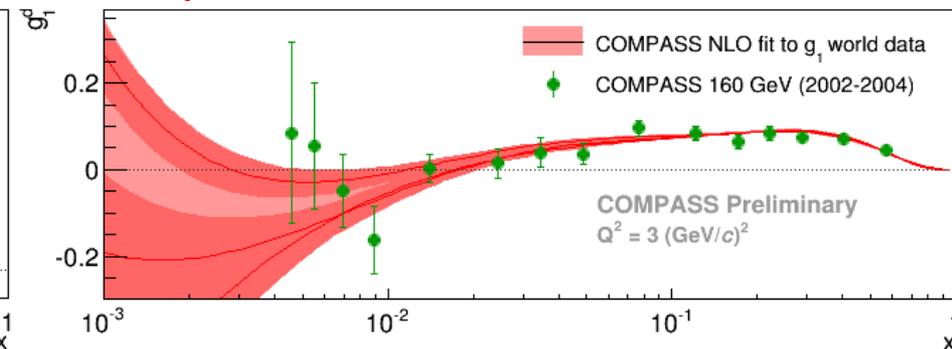
$g_1^p$



$xg_1^d$



$g_1^d$



- $g_1^p$  positive at low  $x$
- Lower  $x$  data needed for sensitivity to  $\Delta G$

# Results for Bjorken sum rule from $g_1$ COMPASS data

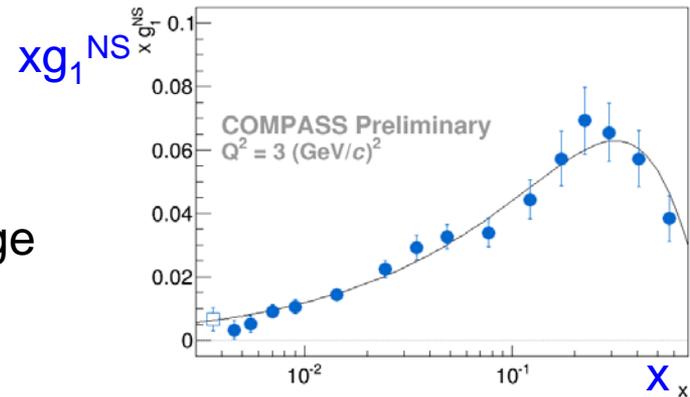
Fundamental QCD sum rule, which relates proton and neutron spin structure functions  $g_1$ .

$$\int_0^1 (g_1^p(x, Q^2) - g_1^n(x, Q^2)) dx = \frac{1}{6} \left| \frac{g_A}{g_V} \right| C_1^{NS}(Q^2)$$

Using COMPASS data alone:

- Non-singlet fit: independent from  $\Delta G$
- Reduce systematics

94% of the sum is from the measured range



COMPASS  $(g_A/g_V)_{\text{NLO}} = 1.219 \pm 0.052(\text{stat.}) \pm 0.095(\text{syst.})$

*M. Wilfert, DIS-2014*

to be compared to:  $\left| \frac{g_A}{g_V} \right| = 1.269 \pm 0.002$  obtained from neutron  $\beta$ -decay.

**→ Bjorken sum rule verified to 8%**

**Better statistics and extended systematics studies compared to past**

Note that experimental value increases from 1.22 to 1.25 when  $C_1^{NS}$  at NNLO

# Summary

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$g_1^p$  : Measurement down to  $x=0.0036$

Statistical precision improved by factor of  $\sim 3$  compared to SMC

## NLO QCD fit of $g_1$ world data:

Gluon and quark contributions to nucleon spin

Quarks : Sum  $0.26 < \Delta\Sigma < 0.34$

Uncertainty dominated by initial functional forms

Gluon : Shape not constrained enough by DIS data alone

**Bjorken sum rule from COMPASS p and d data:** Verified to 8%

## In progress

Semi-inclusive data  $A_1^{\pi, k}$  for flavour separation  $\Delta q_f(x, Q^2)$

## COMPASS future

2015 : TMDs via polarized Drell-Yan  $\pi p \uparrow \rightarrow \gamma \gamma$

2016-2017 : GPDs via Deep Virtual Compton Scattering  $\mu p \rightarrow \mu p \gamma$

In parallel, SIDIS  $\mu p \rightarrow \mu h X$ : Fragmentation functions and PDFs

# Spares

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# NLO QCD fit – $Q^2$ evolution equations

- Decomposition of spin structure functions at NLO ( $\overline{MS}$ ):

$$g_1 = \frac{1}{2} \langle e^2 \rangle \left( C^s(\alpha_s) \otimes \underbrace{\Delta q_S}_{\text{singlet}} + C^{NS}(\alpha_s) \otimes \underbrace{\Delta q_{NS}}_{\text{non-singlet}} + C^g(\alpha_s) \otimes \Delta g \right)$$

with  $\Delta q_S = \Delta u + \Delta d + \Delta s$  and

$\Delta q_{NS}$ : a combination of  $\Delta q_3 = \Delta u - \Delta d$  and  $\Delta q_8 = \Delta u + \Delta d - 2\Delta s$

- DGLAP equations:

$$\frac{d}{d \ln Q^2} \Delta q_{NS} = \frac{\alpha_s(Q^2)}{2\pi} \Delta P_{qq} \otimes \Delta q_{NS}$$

$$\frac{d}{d \ln Q^2} \begin{pmatrix} \Delta q_S \\ \Delta g \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi} \begin{pmatrix} \Delta P_{qq} & 2 n_f \Delta P_{qg} \\ \Delta P_{gq} & \Delta P_{gg} \end{pmatrix} \otimes \begin{pmatrix} \Delta q_S \\ \Delta g \end{pmatrix}$$

with  $\Delta P_{ij}$ : splitting functions

Probability for a parton  $j$  to radiate a parton  $i$

# NLO QCD fit – functional forms

Functional forms postulated at a given “reference” scale  $Q_{ref}^2$ :

$$\Delta q_S(x|Q_{ref}^2) = \eta_S x^{\alpha_S} (1-x)^{\beta_S} (1 + \gamma_S x + \rho_S \sqrt{x}) / N_S$$

$$\Delta g(x|Q_{ref}^2) = \eta_g x^{\alpha_g} (1-x)^{\beta_g} (1 + \gamma_g x + \rho_g \sqrt{x}) / N_g$$

$$\Delta q_3(x|Q_{ref}^2) = \eta_3 x^{\alpha_3} (1-x)^{\beta_3} / N_3$$

$$\Delta q_8(x|Q_{ref}^2) = \eta_8 x^{\alpha_8} (1-x)^{\beta_8} / N_8$$

$N_q$  defined such as  $\eta_q$  represent the first moment of the distributions at  $Q_{ref}^2$

↓

DGLAP equations to evolve to any  $Q^2$

↓

give

$$\Delta q_S(x, Q^2), \Delta g(x, Q^2), \Delta q_3(x, Q^2), \Delta q_8(x, Q^2)$$

for all  $Q^2$  values

# NLO QCD fit – Constrains

- Additional inputs:

- $\eta_3 = F + D = g_A/g_V$  with  $SU(2)_{\text{flavour}}$
- $\eta_8 = 3F - D$  with  $SU(3)_{\text{flavour}}$  (PRD 87 (2013) 016002)
- Positivity :  $|\Delta g| < g(x)$  and  $|\Delta(s + \bar{s})| < s(x) + \bar{s}(x)$  (MSTW2008)
- $\beta_g$  fixed to 7.5 (preferential value)

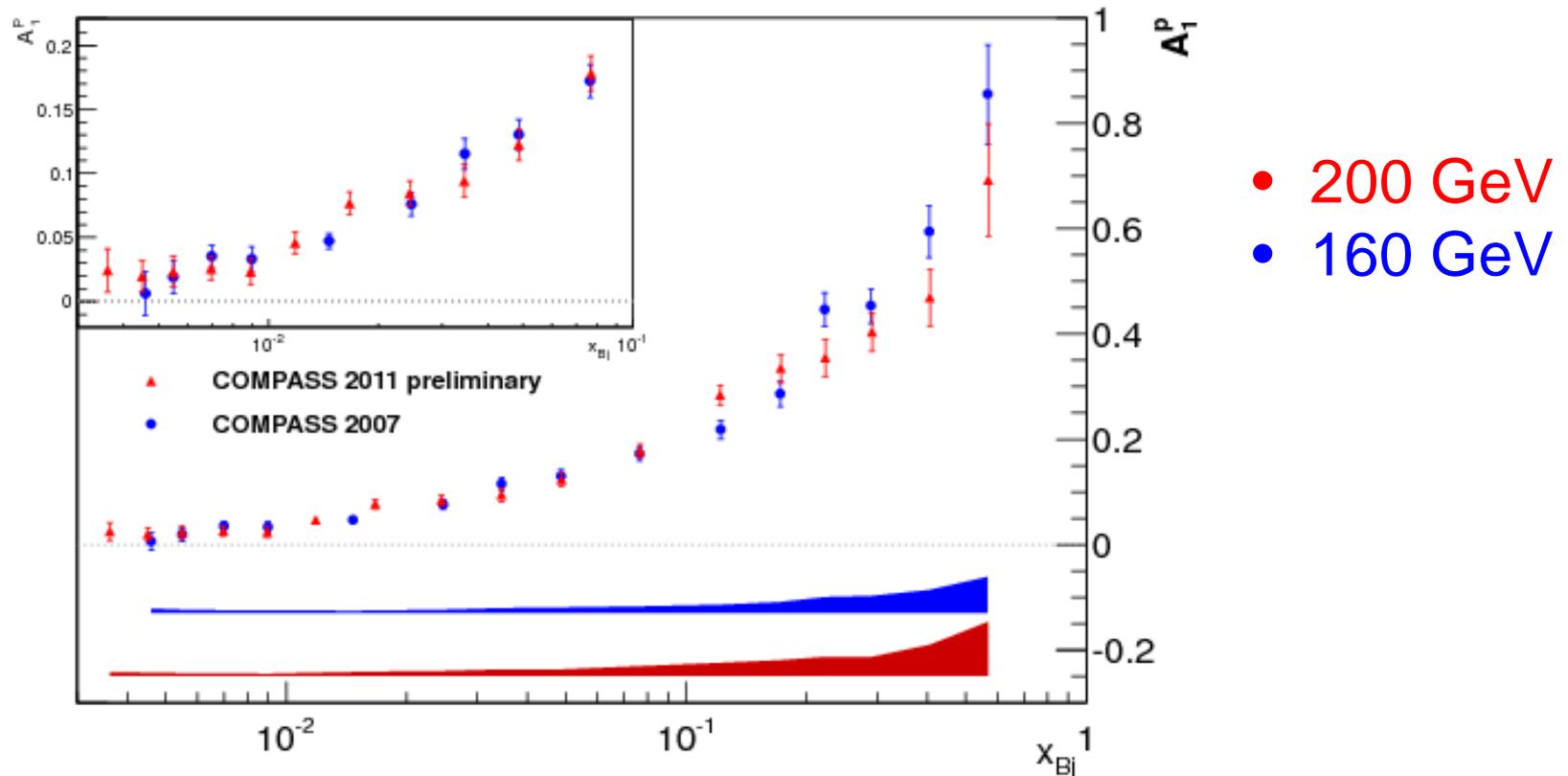
- Minimisation of  $\chi^2 =$

$$\sum_{data} \left( \frac{g_1^{fit}(x_{data}, Q_{data}^2) - \mathcal{N}_{exp} \cdot g_1^{data}}{\mathcal{N}_{exp} \cdot \sigma_{data}^{stat}} \right)^2 + \sum_{exp} \left( \frac{1 - \mathcal{N}_{exp}}{\delta \mathcal{N}_{exp}} \right)^2$$

$\mathcal{N}_{exp}$  parameters accounting for experimental normalisation uncertainties

Overall:  $\sim 28$  free parameters for 679 data points

# Results for $A_1^p$ at 200 GeV



- Compatible results for the different beam energies at their measured  $Q^2$
- Statistical uncertainties (160 GeV and 200 GeV) 3 times smaller than SMC for a similar beam time.
- Lowest  $x$  value reached:  $x = 0.0036$

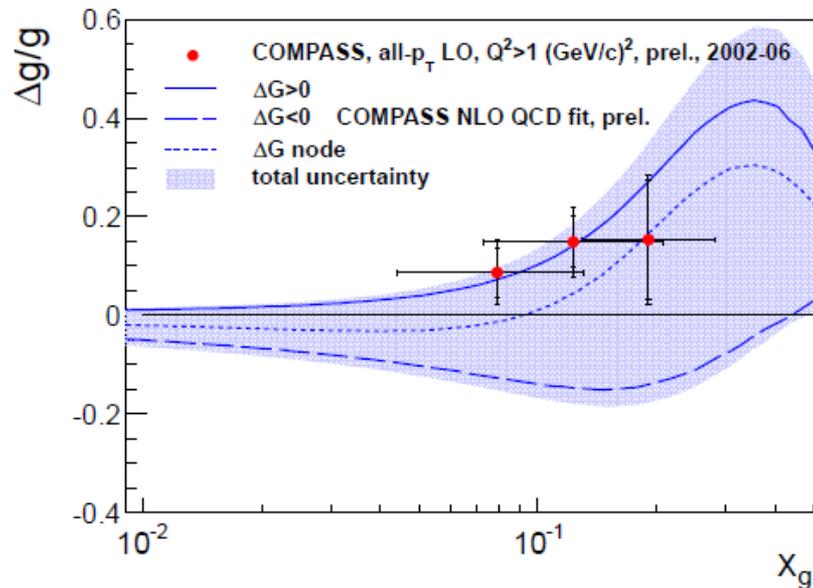
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$$\begin{aligned}\Delta\Sigma &\in [0.264, 0.356] \text{ at } Q^2 = 1 \text{ (GeV/c)}^2 \\ \Delta\Sigma &\in [0.256, 0.335] \text{ at } Q^2 = 3 \text{ (GeV/c)}^2 \\ \Delta\Sigma &\in [0.258, 0.299] \text{ at } Q^2 = 10 \text{ (GeV/c)}^2\end{aligned}$$

# $\Delta G/G$ from hadron prod. in DIS (all- $p_T$ )

New COMPASS results (better precision)

$\Delta G/G$  extracted at LO, in 3 x-bins



3 solutions from  
COMPASS NLO QCD fit of  $g_1$   
world data

*M. Stolarski, DIS 2014*

Uncertainty on  $\Delta G/G$  could be reduced if these results could be included in NLO fits

# Statistical error on fit

Estimated from multiple fits on replicas of pseudo data generated within the error on  $g_1$ . Example for the 'positive' solution.

