Partial-Wave Analyses at COMPASS

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for the COMPASS Collaboration

CERN

On leave of absence from
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PWA Tools in Hadron Spectroscopy
Mainz, 18. Feb 2013

The COMPASS Physics Program

COmmon **M**uon and **P**roton **A**pparatus for **S**tructure and **S**pectroscopy

Goal

- Study non-perturbative QCD
- Probe structure and dynamics of hadrons

Chiral dynamics

- $\pi\gamma$ reactions (Primakoff)
- π and K polarizabilities

Hadron spectroscopy

- Mass spectrum of hadrons
- Gluonic excitations

Nucleon structure

- Helicity, transversity PDFs
- $k_\perp$-dependent PDFs
- Generalized PDFs

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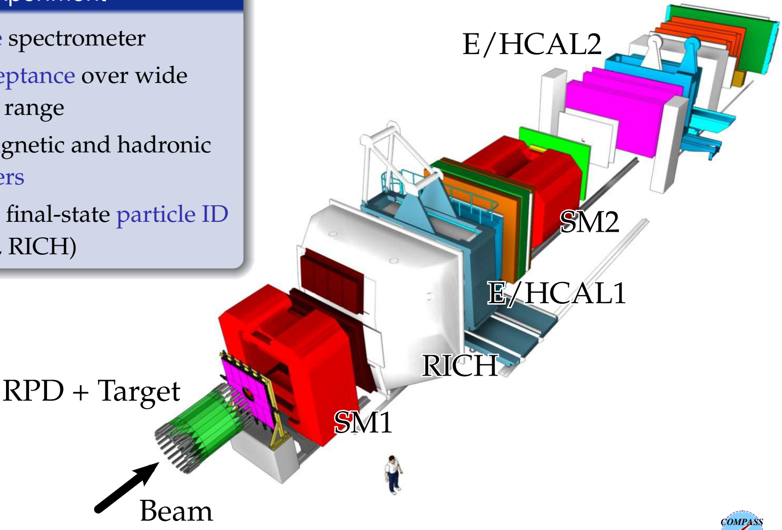
The COMPASS Experiment at the CERN SPS

Experimental Setup

NIM A **577**, 455 (2007)

Fixed-target experiment

- Two-stage spectrometer
- Large acceptance over wide kinematic range
- Electromagnetic and hadronic calorimeters
- Beam and final-state particle ID (CEDARs, RICH)



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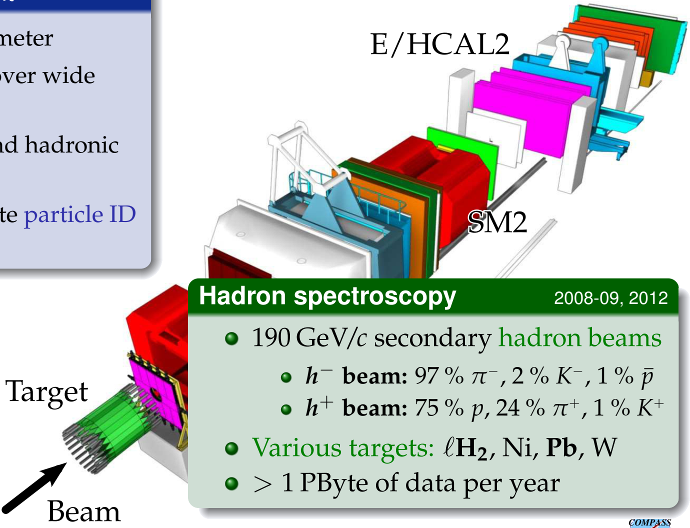


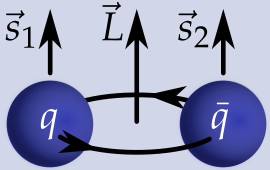
Hadron spectroscopy — 2008-09, 2012

- 190 GeV/c secondary hadron beams
  - h^- **beam:** 97 % π^-, 2 % K^-, 1 % $\bar{p}$
  - h^+ **beam:** 75 % p, 24 % π^+, 1 % K^+
- Various targets: $\ell\mathbf{H_2}$, Ni, **Pb**, W
- $>$ 1 PByte of data per year

Constituent Quark Model and Exotic Mesons

Spin-parity rules for bound $q\bar{q}$ system

- Quark spins couple to total intrinsic spin $S = 0$ (singlet) or 1 (triplet)
- Relative orbital angular Momentum $\vec{L}$ and total spin $\vec{S}$ couple to meson spin $\vec{J} = \vec{L} + \vec{S}$
- Parity $P = (-1)^{L+1}$
- Charge conjugation $C = (-1)^{L+S}$
- **Forbidden J^{PC}: $0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, \ldots$**
- Extension to charged mesons via G parity: $G = (-1)^{L+S+I}$



QCD allows for **states beyond the CQM**

- Hybrids $|q\bar{q}g\rangle$, glueballs $|gg\rangle$, multi-quark states $|q^2\bar{q}^2\rangle, \ldots$
- **Physical mesons:** superposition of all allowed basis states
- "Exotic" mesons have quantum numbers forbidden for $|q\bar{q}\rangle$
  - Particularly interesting: J^{PC}-exotic states

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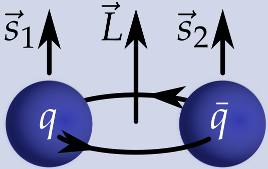

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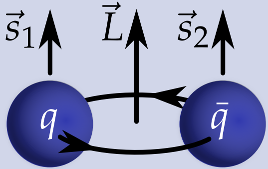

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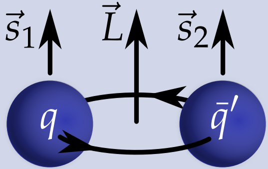

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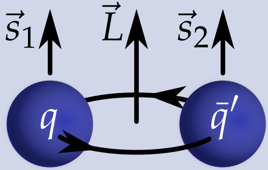

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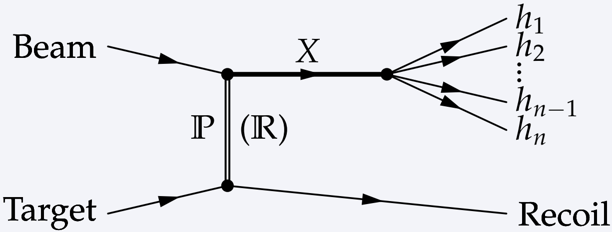
Production of Hadrons in Diffractive Dissociation



- Soft scattering of beam hadron off nuclear target (remains intact)
  - Beam particle is excited into intermediate state X
  - X decays into n-body final state
  - For p target recoil is measured (RPD)
  - For other nuclear targets elastic scattering at target vertex is assumed
- High $\sqrt{s}$ and low t': Pomeron exchange dominates strong interaction

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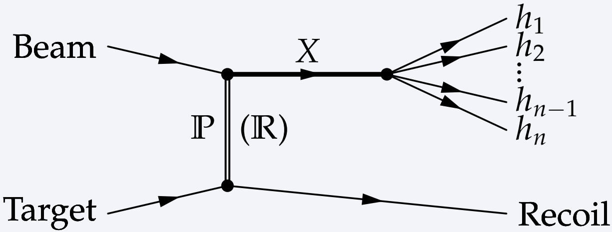

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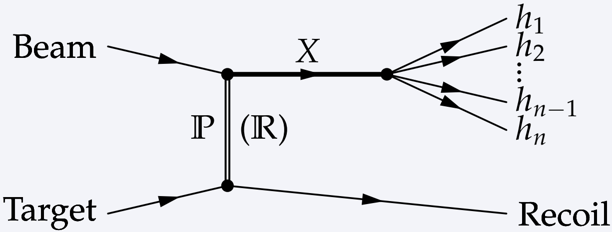

- Rich spectrum: large number of overlapping and interfering X
- **Goal:** use kinematic distribution of final-state particles to
  - Disentangle all resonances X
  - Determine their mass, width, and quantum numbers
- **Method:** partial-wave analysis (PWA)

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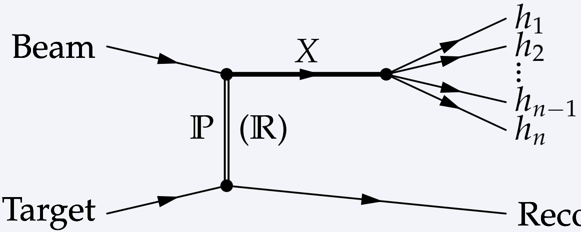

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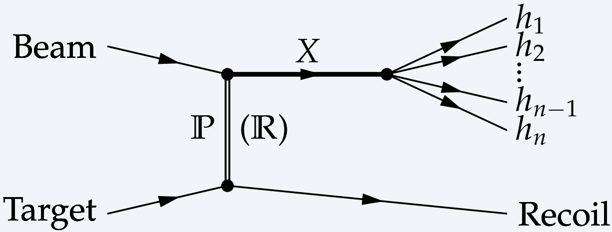

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Partial-Wave Analysis Formalism

Parity Conservation at Production Vertex

Reflectivity basis

S. U. Chung and T. L. Trueman, Phys. Rev. **D11** (1975) 633

- Reaction: beam + target $\to X$ + recoil
- Reflectivity operator Π_y: reflection through production plane
- Particles in production plane: Π_y acts like parity but leaves momenta unchanged
- Eigenstates to Π_y: $|J^P M^\epsilon\rangle \equiv c(M)\left[|J^P M\rangle - \epsilon\, P\,(-)^{J-M}|J^P\, {-M}\rangle\right]$

  where $M \geq 0$ and $c = \begin{cases} \frac{1}{2} & \text{for } M = 0, \\ \frac{1}{\sqrt{2}} & \text{for } M > 1 \end{cases}$
- Reflectivity $\epsilon = \pm 1$ (for bosons)
- **Parity conservation:** amplitudes with different ϵ do *not* interfere
- ϵ corresponds to naturality of exchanged Reggeon
  - Pomeron has positive naturality $\implies$ $\epsilon = +1$ amplitudes dominant

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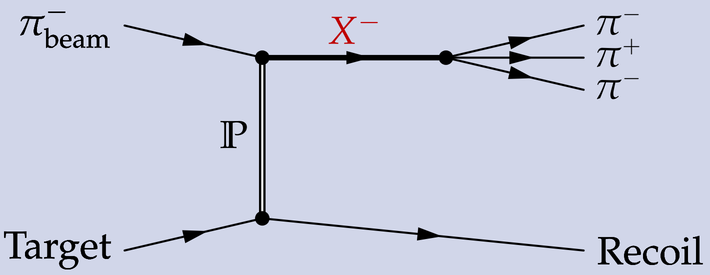
Partial-Wave Analysis of Multi-Body Final States

Isobar Model

X decay is chain of successive two-body decays

Example: Diffractive dissociation of π^- beam into $\pi^-\pi^+\pi^-$



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- Two-body decay amplitudes calculable

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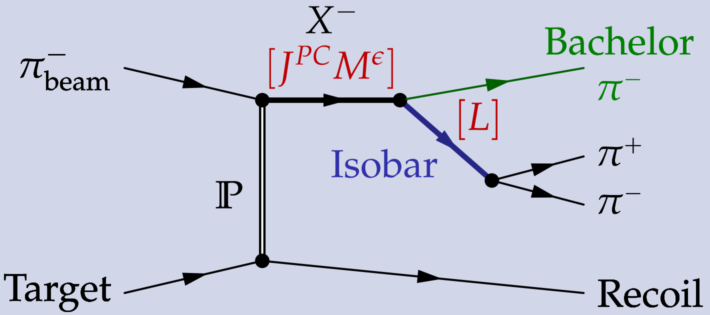

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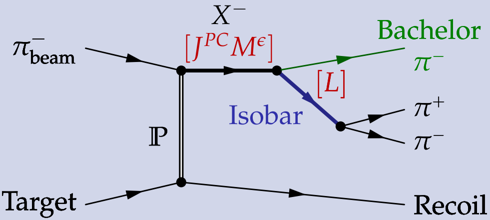

- Isobar has spin S and relative orbital angular momentum L w.r.t. bachelor π^-
  - L and S couple to spin J of X^-
- **"Wave"** = unique combination of isobar and quantum numbers
  - *Notation:* $J^{PC}M^\epsilon[\text{isobar}]L$
- 3-body kinematics fixed by m_X plus 5 phase-space variables
  - E.g. $\tau \equiv \{\theta_{GJ}, \phi_{GJ}, m_{\pi^+\pi^-}, \theta_H, \phi_H\}$

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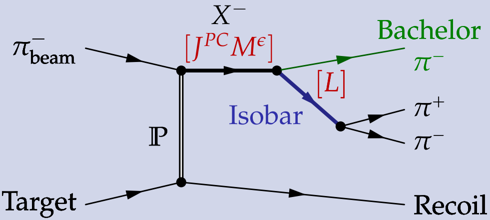

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Partial-Wave Analysis Formalism

Decay Amplitude in the Helicity Formalism

Two-body decay $a \to b + c$

- Kinematics defined by
  - Invariant mass m_a of a
  - Polar angles (θ, ϕ) of daughter b in rest frame of a
- Spin states of b and c are described in helicity basis
- J_b and J_c couple to total spin S
- Relative orbital angular momentum L between b and c
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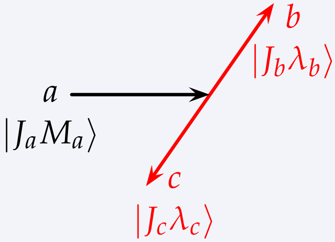

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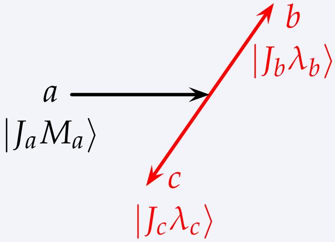

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$$A_a(m_a, \theta, \phi) = \sqrt{2L+1} \sum_{\lambda_b, \lambda_c} (J_b\, \lambda_b\; J_c\, {-\lambda_c} | S\, \delta)\, (L\, 0\; S\, \delta | J_a\, \delta)\; D^{J_a *}_{M_a \delta}(\theta, \phi, 0)\, F_L(q)\, \Delta(m_a)\, A_b\, A_c$$

Decay amplitude has no free parameters!

$\delta \equiv \lambda_b - \lambda_c$

$D^{J_a *}_{M_a \delta}(\theta, \phi, 0)$	D-function which describes rotation of helicity state
$F_L(q)$	Blatt-Weisskopf barrier factor for $a \to b\,[L]\,c$
q	Breakup momentum for $a \to b + c$
$\Delta(m_a)$	Amplitude that describes resonance shape of a
$A_{b,c}$	decay amplitudes of (unstable) daughters b and c

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- Recursive calculation of two-body decay amplitudes for each vertex in isobar decay tree
- E.g. 2 vertices in $\pi^-\pi^+\pi^-$ case
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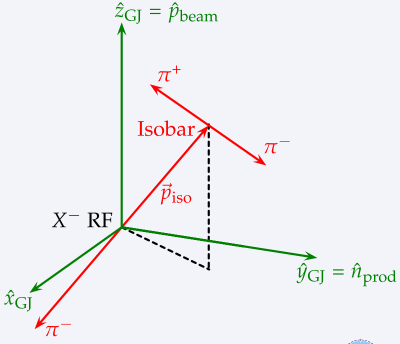

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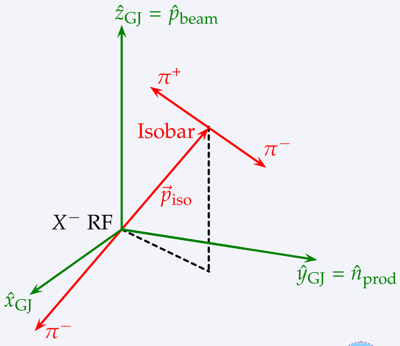

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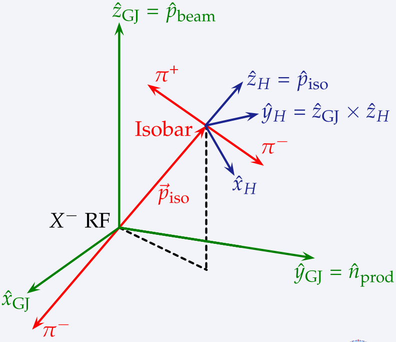

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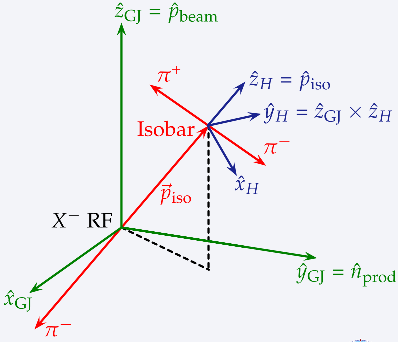
- E.g. 2 vertices in $\pi^-\pi^+\pi^-$ case
  - X^- decay: Gottfried-Jackson frame
    - $\Delta(m_X) \equiv 1$
  - Isobar decay: helicity frame
  - $A_{\pi^\pm} \equiv 1$



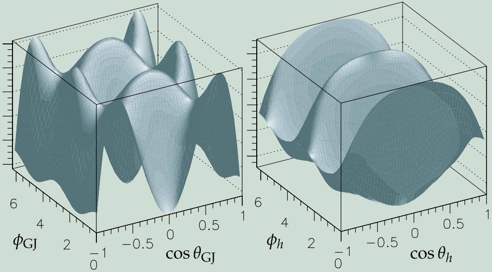
Partial-Wave Analysis Formalism

Decay Amplitude in the Helicity Formalism

Example: angular distribution for wave $2^{-+}1^+[f_2(1270)\pi]D$

Resonance decay $X^-(2^{-+}) \to f_2(1270)\pi^-$

Isobar decay $f_2(1270) \to \pi^+\pi^-$

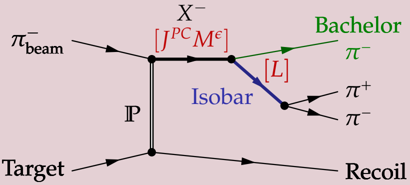


Dzierba *et al.*, PRD73 (2006) 072001

- 2D projections of a genuine 5D distribution (m_X = const.)
  - Orbital angular momentum between f_2 and π^- $L = 2$
  - $f_2(1270)$: $J^P = 2^+ \implies S = 2$

Partial-Wave Analysis Formalism

Isobar model: cross section parameterization



- Factorization of production and decay

$$\sigma(\tau; m_X) = \sigma_0 \sum_{\epsilon=\pm1} \sum_{r=1}^{\text{rank}} \left| \sum_{i}^{\text{waves}} T_{ir}^\epsilon(m_X)\, A_i^\epsilon(\tau; m_X) \right|^2$$

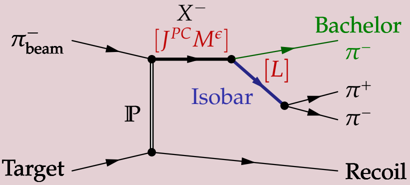
  - Transition amplitudes T_{ir}^ϵ form spin-density matrix ρ_{ij}^ϵ

    $$\sigma(\tau; m_X) = \sigma_0 \sum_{\epsilon=\pm1} \sum_{i,j}^{\text{waves}} \rho_{ij}^\epsilon(m_X)\, A_i^\epsilon(\tau; m_X)\, A_j^{\epsilon*}(\tau, m_X)$$

    where $\rho_{ij}^\epsilon(m_X) = \sum_{r=1}^{\text{rank}} T_{ir}^\epsilon(m_X)\, T_{jr}^{\epsilon*}(m_X)$
  - $\rho_{ij}^\epsilon(m_X)$ contains the interesting physics
    - Diagonal elements ρ_{ii}: wave intensities
    - Off-diagonal elements ρ_{ij}, $i \neq j$: interference terms

Partial-Wave Analysis Formalism

Isobar model: cross section parameterization



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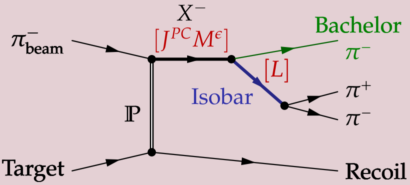

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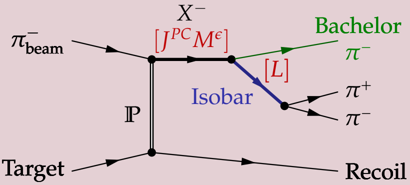

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- **Determination of $T_{ir}^\epsilon(m_X)$**
  1. Bin data in m_X; neglect m_X dependence within mass bin
  2. Calculate decay amplitudes $A_i^\epsilon(\tau; m_X)$ for every event
  3. Unbinned extended maximum likelihood fit of τ distribution in each m_X bin taking into account detector acceptance $\Longrightarrow T_{ir}^\epsilon(m_X)$
- **Method makes no assumptions about m_X dependence of T_{ir}^ϵ**

Partial-Wave Analysis Formalism

Unbinned extended maximum likelihood fit in mass bins

- Likelihood $\mathcal{L}$ to observe N events distributed according to model cross section $\sigma(\tau; m_X)$ and detector acceptance $\mathrm{Acc}(\tau; m_X)$

$$\mathcal{L} = \underbrace{\left[\frac{\overline{N}^N}{N!}\, e^{-\overline{N}}\right]}_{\text{Poisson likelihood to observe } N \text{ events}} \prod_{k=1}^{N} \underbrace{\left[\frac{\sigma(\tau_k; m_X)}{\int d\Phi_n(\tau; m_X)\, \sigma(\tau; m_X)\, \mathrm{Acc}(\tau; m_X)}\right]}_{\text{likelihood to observe event } k}$$

- Expected nmb. of events $\overline{N} \propto \int d\Phi_n(\tau; m_X)\, \sigma(\tau; m_X)\, \mathrm{Acc}(\tau; m_X)$
- n-body phase-space element $d\Phi_n(\tau; m_X)$

Partial-Wave Analysis Formalism

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- n-body phase-space element $\mathrm{d}\Phi_n(\tau; m_X)$

Partial-Wave Analysis Formalism

Unbinned extended maximum likelihood fit in mass bins

- Insert cross section parameterization

$$\sigma = \sigma_0 \sum_{\epsilon=\pm1} \sum_{r=1}^{\text{rank}} \left| \sum_i^{\text{waves}} T_{ir}^{\epsilon} A_i^{\epsilon} \right|^2$$

- Skip constant factors and take logarithm:

$$\ln \mathcal{L} = \sum_{k=1}^{N} \ln \left[\sum_{\epsilon=\pm1} \sum_{r=1}^{\text{rank}} \left| \sum_i^{\text{waves}} T_{ir}^{\epsilon} A_i^{\epsilon}(\tau_k) \right|^2 \right] - \sum_{\epsilon=\pm1} \sum_{r=1}^{\text{rank}} \sum_{i,j}^{\text{waves}} T_{ir}^{\epsilon} T_{jr}^{\epsilon *} \underbrace{\int \mathrm{d}\Phi_n(\tau)\, \mathrm{Acc}(\tau)\, A_i^{\epsilon}(\tau)\, A_j^{\epsilon *}(\tau)}_{\text{normalization integral } I_{ij}}$$

- Maximization of $\ln \mathcal{L}$ with $T_{ir}^{\epsilon}(m_X)$ as free parameters
- Decay amplitudes $A_i^{\epsilon}(\tau_k; m_X)$ are pre-calculated
- $I_{ij}(m_X)$ estimated using phase-space Monte Carlo

Partial-Wave Analysis Formalism

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$$- \sum_{\epsilon=\pm1} \sum_{r=1}^{\text{rank}} \sum_{i,j}^{\text{waves}} T_{ir}^{\epsilon} T_{jr}^{\epsilon *} \underbrace{\int \mathrm{d}\Phi_n(\tau)\, \text{Acc}(\tau)\, A_i^{\epsilon}(\tau)\, A_j^{\epsilon *}(\tau)}_{\text{normalization integral } I_{ij}}$$

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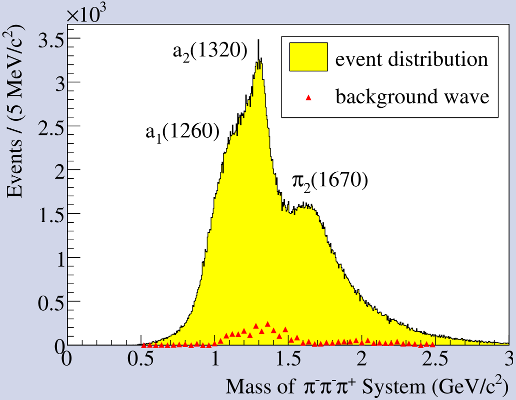
Partial-Wave Analysis of $\pi^-\pi^+\pi^-$ Final State

Example: π^- Pb $\to \pi^-\pi^+\pi^-$ Pb (2004 pilot run data) PRL **104** (2010) 241803

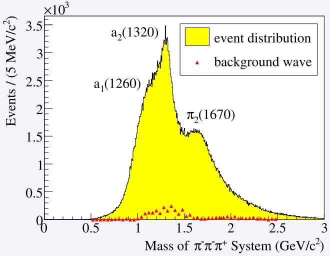
- 190 GeV/c π^- beam on Pb target
- 420 000 exclusive $\pi^-\pi^+\pi^-$ events
- Kinematic range $0.1 < t' < 1.0$ (GeV/c)^2



PWA of π^- Pb $\to \pi^-\pi^+\pi^-$ Pb (2004 Data)

PRL 104 (2010) 241803

$\pi^-\pi^+\pi^-$ mass spectrum



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PRL 104 (2010) 241803

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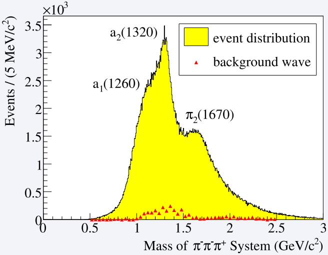
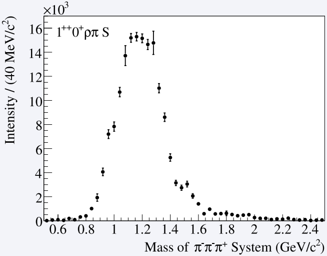


$1^{++}\,0^+\,[\rho\pi]S$: $a_1(1260)$



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PRL 104 (2010) 241803

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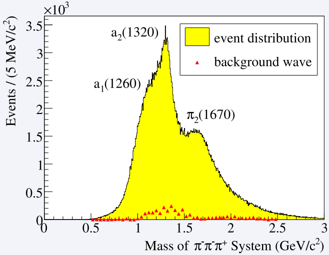

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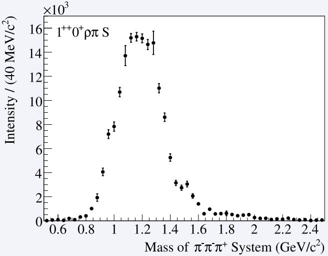
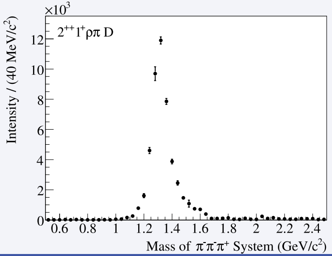


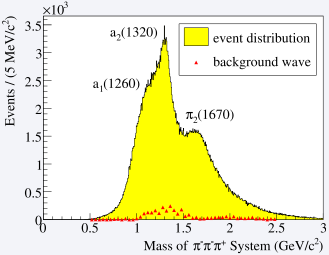
$2^{++}\,1^+\,[\rho\pi]D$: $a_2(1320)$



PWA of π^- Pb $\rightarrow \pi^-\pi^+\pi^-$ Pb (2004 Data)

PRL 104 (2010) 241803

$\pi^-\pi^+\pi^-$ mass spectrum



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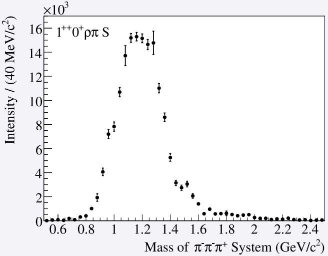
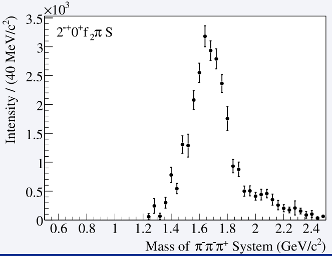


$2^{-+}\,0^+\,[f_2\pi]S$: $\pi_2(1670)$



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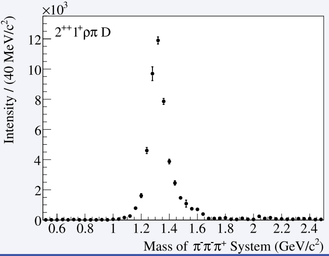

PWA of π^- Pb $\rightarrow \pi^-\pi^+\pi^-$ Pb (2004 Data)

PRL 104 (2010) 241803

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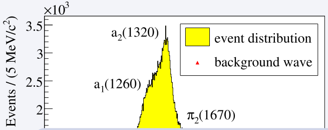

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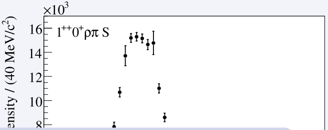
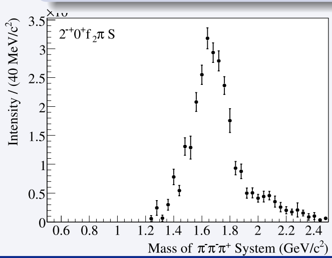
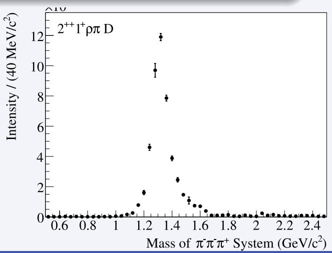


- Data described by model consisting of 41 waves + incoherent isotropic background wave
  - Isobars: $(\pi\pi)_S$, $f_0(980)$, $\rho(770)$, $f_2(1270)$, and $\rho_3(1690)$
- Rank-2 spin-density matrix



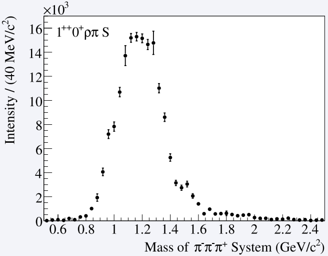


PWA of π^- Pb $\to \pi^-\pi^+\pi^-$ Pb (2004 Data)

PRL 104 (2010) 241803

- Relative phases from off-diagonal elements of spin-density matrix

$1^{++}\,0^+\,[\rho\pi]S$: $a_1(1260)$



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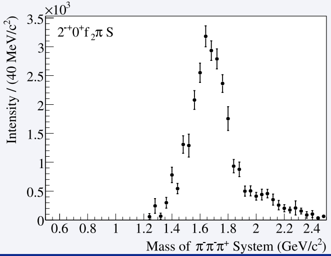
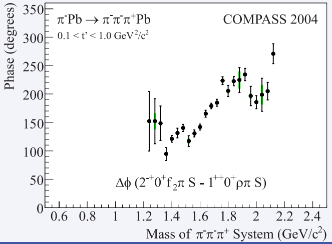


$2^{-+}\,0^+\,[f_2\pi]S - 1^{++}\,0^+\,[\rho\pi]S$



Partial-Wave Analysis Formalism

Parametrization of Mass-Dependence of Spin-Density Matrix

Ansatz for parametrization of $\rho_{ij}^{\epsilon}(m_X) = \sum_{r=1}^{\text{rank}} T_{ir}^{\epsilon}(m_X)\, T_{jr}^{\epsilon *}(m_X)$

$$T_{ir}^{\epsilon}(m_X) = \sum_{k}^{\text{resonances}} C_{irk}^{\epsilon}\, \mathcal{A}_k(m_X) \sqrt{\underbrace{\int \mathrm{d}\Phi_n(\tau)\, |A_i^{\epsilon}(\tau; m_X)|^2}_{\text{phase space for wave } i}}$$

Dynamic amplitudes $\mathcal{A}_k(m_X)$ describe

- Resonance line shapes
  - Typically relativistic Breit-Wigner with mass-dependent width

$$\mathcal{A}_k^{\text{BW}}(m_X) = \frac{m_0\, \Gamma_0}{m_0^2 - m_X^2 - i\, m_0\, \Gamma_{\text{tot}}(m_X)}$$

$$\Gamma_{\text{tot}}(m_X) = \sum_{v}^{\text{decays}} \Gamma_v(m_X) = \sum_{v}^{\text{decays}} \Gamma_{0,v}\, \frac{m_0}{m_X}\, \frac{q_v}{q_{0,v}}\, \frac{F_{L_v}(q_v)}{F_{L_v}(q_{0,v})}$$

- Non-resonant coherent background contributions
  - Typically exponentially damped phase space: $\mathcal{A}_k^{\text{BG}}(m_X) = e^{-\alpha_k q_k^2}$

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Model parameters determined by χ^2 fit to $\rho_{ij}^{\epsilon}(m_X)$

Free parameters:

- Complex amplitudes C_{irk}^{ϵ}
- Resonance or background parameters in $\mathcal{A}_k(m_X)$

Partial-Wave Analysis — $\pi^-\pi^+\pi^-$ Final State

Example: π^- Pb $\rightarrow \pi^-\pi^+\pi^-$ Pb (2004 pilot run data) PRL **104** (2010) 241803

- Choose submatrix of ρ_{ij}
  - Here: 6 significant waves (out of 41) with clear phase motion
- Fit model: 6 resonances + backgrounds
- Mass dependence of intensities *and* interferences of *all* waves in subset is fit simultaneously
- E.g. $\pi_2(1670)$: $m = 1\,658 \pm 3^{+24}_{-8}$ MeV/c^2, $\Gamma = 271 \pm 9^{+22}_{-24}$

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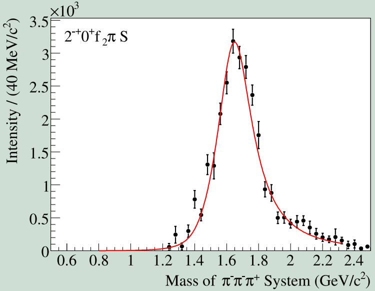
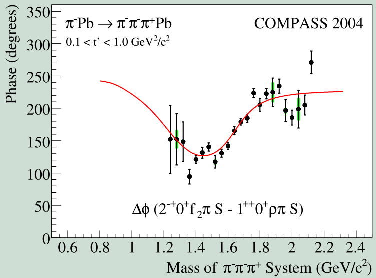
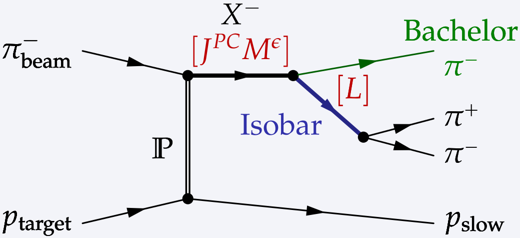




Outline

PWA of $\pi^- p \to \pi^-\pi^+\pi^- p_{\text{slow}}$



- 190 GeV/c negative hadron beam: 97 % π^-, 2 % K^-, 1 % $\bar{p}$
- Liquid hydrogen target
- Recoil proton p_{slow} measured by RPD
- Kinematic range $0.1 < t' < 1.0\ (\text{GeV}/c)^2$

PWA of $\pi^- p \to \pi^-\pi^+\pi^- p_{\text{slow}}$

World's largest 3π data set: ≈**50 M exclusive events**

- Challenging analysis
  - Needs precise understanding of apparatus
  - Model deficiencies become visible

PWA of $\pi^- p \to \pi^-\pi^+\pi^- p_{\text{slow}}$

World's largest 3π data set: $\approx$50 M exclusive events

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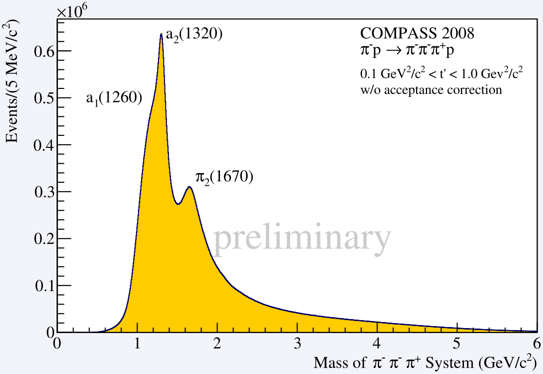
  - Needs precise understanding of apparatus
  - Model deficiencies become visible

$\pi^-\pi^+\pi^-$ invariant mass distribution

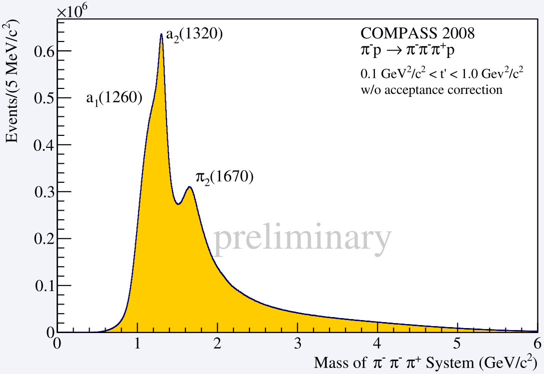


PWA of $\pi^- p \to \pi^-\pi^+\pi^- p_{\text{slow}}$

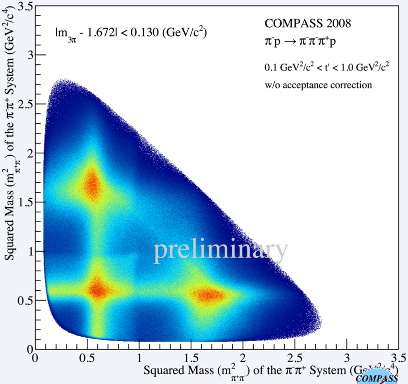
World's largest 3π data set: $\approx$**50 M exclusive events**

- Challenging analysis
  - Needs precise understanding of apparatus
  - Model deficiencies become visible

$\pi^-\pi^+\pi^-$ invariant mass distribution

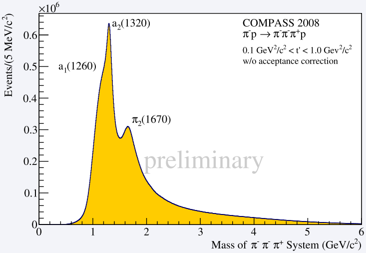


Dalitz plot for $\pi_2(1670)$ region



PWA of $\pi^- p \to \pi^-\pi^+\pi^- p_{\text{slow}}$

$\pi^-\pi^+\pi^-$ invariant mass spectrum



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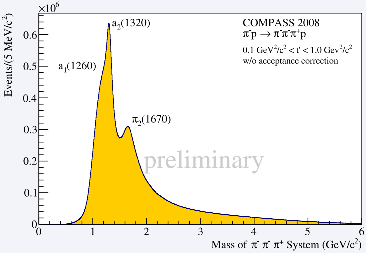
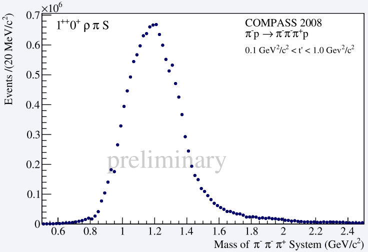


$1^{++}\,0^+\,[\rho\pi]S$: $a_1(1260)$



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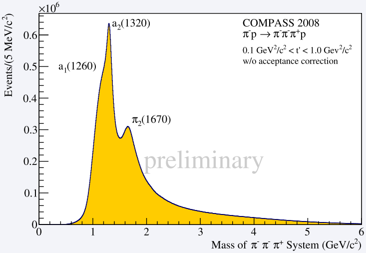

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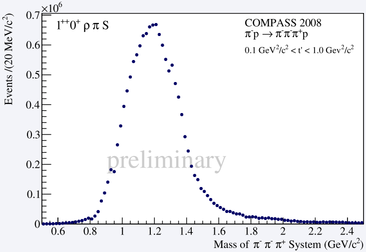
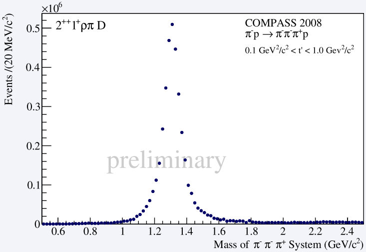


$2^{++}\,1^+\,[\rho\pi]D$: $a_2(1320)$



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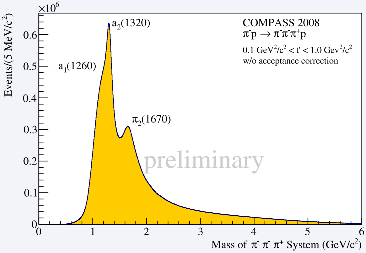
$\pi^-\pi^+\pi^-$ invariant mass spectrum



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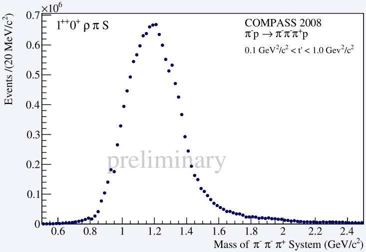
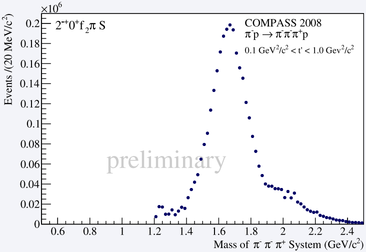
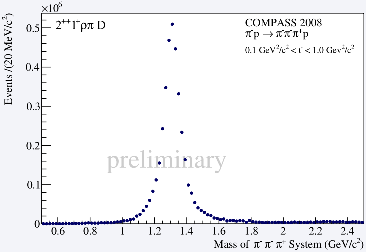


$2^{-+}0^+[f_2\pi]S$: $\pi_2(1670)$



$2^{++}1^+[\rho\pi]D$: $a_2(1320)$



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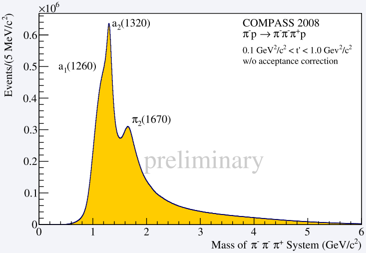

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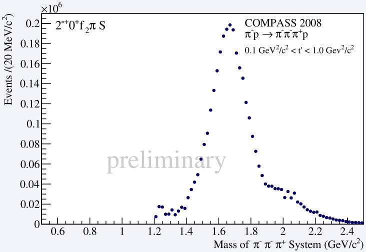
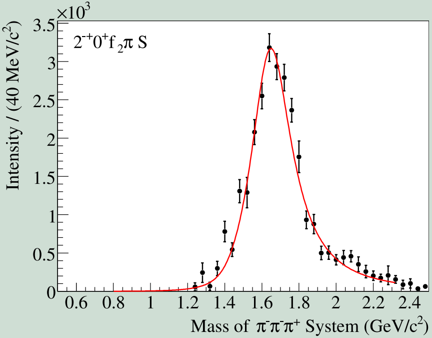


Cf. data from 2004 pilot-run

PRL **104** (2010) 241803

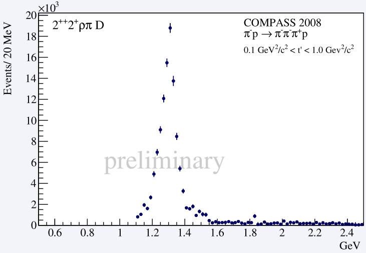
$2^{-+}\,0^+\,[f_2\pi]S$: $\pi_2(1670)$



- 420 000 events
- Pb target

PWA of $\pi^- p \to \pi^-\pi^+\pi^- p_{\text{slow}}$

$2^{++}\,2^+\,[\rho\pi]D$: $a_2(1320)$



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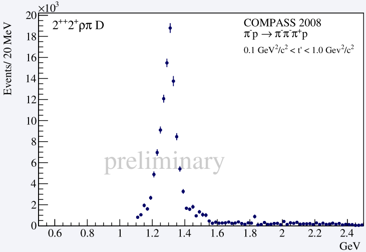
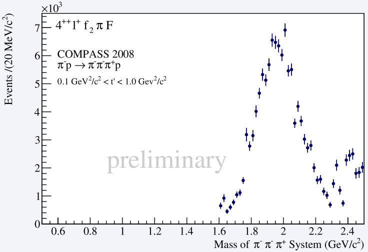


$4^{++}\,1^+\,[f_2\pi]F$: $a_4(2040)$



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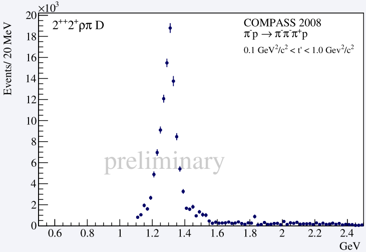

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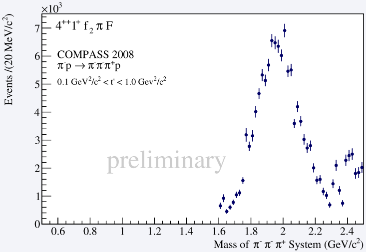
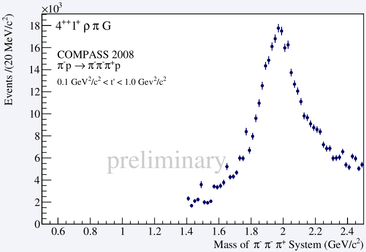


$4^{++}\,1^{+}\,[\rho\pi]G$: $a_4(2040)$



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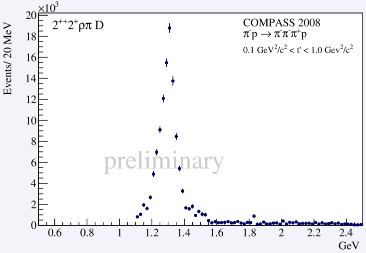

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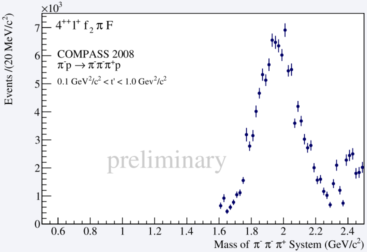
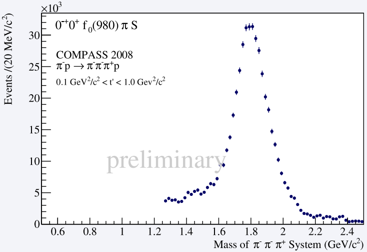


$0^{-+}\,0^+\,[f_0(980)\pi]S$: $\pi(1800)$



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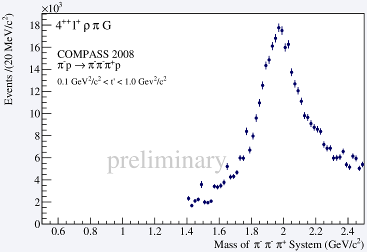
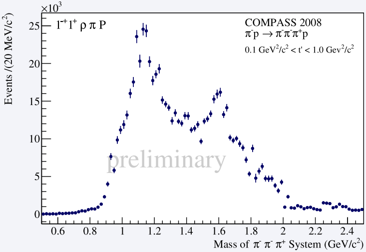


PWA of $\pi^- p \to \pi^-\pi^+\pi^- p_{\text{slow}}$

Spin-exotic $1^{-+}1^+ [\rho\pi]P$



- Structure around 1.1 GeV/c^2 unstable w.r.t. fit model
- Enhancement around 1.6 GeV/c^2 depends on t'
- Stable phase motion w.r.t. to tail of $a_1(1260)$
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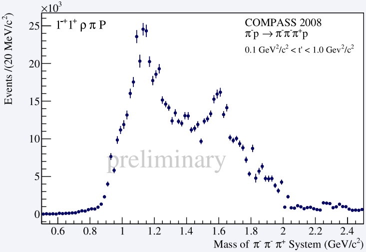

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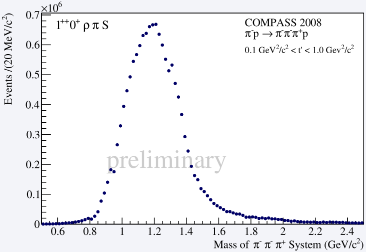
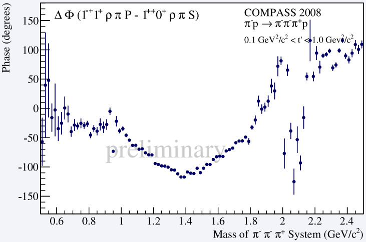


$1^{-+}\,1^+\,[\rho\pi]P - 1^{++}\,0^+\,[\rho\pi]S$



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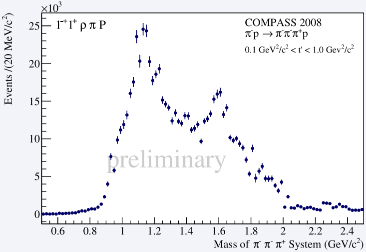
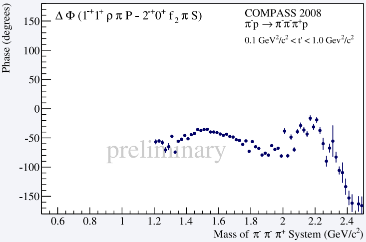


$1^{-+}\, 1^+\, [\rho\pi] P - 2^{-+}\, 0^+\, [f_2\pi] S$



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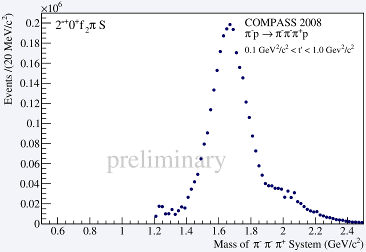

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PWA of $\pi^- p \to \pi^-\pi^+\pi^- p_{\text{slow}}$

Summary

- Data described by model consisting of 52 waves
  + incoherent isotropic background
  - Isobars: $(\pi\pi)_{S-\text{wave}}$, $f_0(980)$, $\rho(770)$, $f_2(1270)$, $f_0(1500)$, $\rho_3(1690)$

Understanding of small waves is work in progress

- Intensity in spin-exotic $1^{-+}1^+ [\rho\pi]P$ wave
  - Interpretation in terms of resonances still unclear
- Improvements of wave set and isobar parameterization

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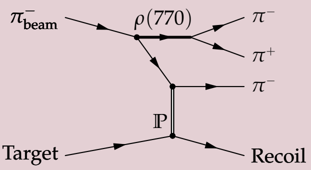
- Intensity in spin-exotic $1^{-+}1^+[\rho\pi]P$ wave
  - Interpretation in terms of resonances still unclear
- Significant contributions from non-resonant Deck-like processes
  - Inclusion into fit model
- Take into account t'-dependence of partial-wave amplitudes
  - Two-dimensional PWA in narrow $m_{\pi^-\pi^+\pi^-}$ and t' bins



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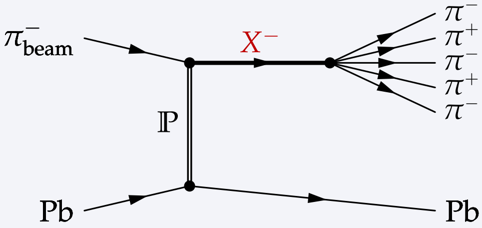
Outline

1. Introduction
2. Search for spin-exotic mesons produced in π^- diffraction
   - *Introduction:* Partial-wave analysis of multi-body final states
   - Partial-wave analysis of the $\pi^-\pi^+\pi^-$ system
   - Partial-wave analysis of the $\pi^-\pi^+\pi^-\pi^+\pi^-$ channel
3. Conclusions

PWA of π^- Pb $\to \pi^-\pi^+\pi^-\pi^+\pi^-$ Pb



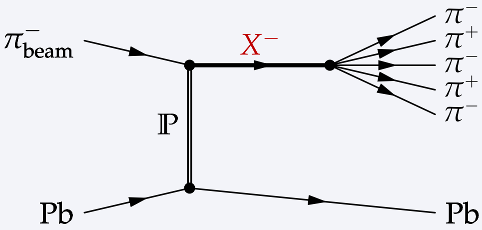
First mass-dependent PWA of this reaction

- **Light-meson frontier:** access to mesonic states in 2 GeV/c^2 region
- Little information from previous experiments

Data from pilot run

- Pb target
- Recoil not measured
- Kinematic range $t' < 5 \cdot 10^{-3}$ (GeV/c)^2

PWA of π^- Pb $\to \pi^-\pi^+\pi^-\pi^+\pi^-$ Pb



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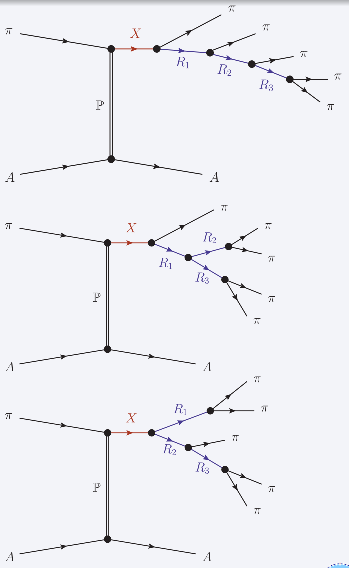
PWA model

- Complicated isobar structure
  - Large number of possible waves
  - Data exhibit no dominant waves
  - $(2\pi)^0$ isobars: $(\pi\pi)_{S-\text{wave}}$, $\rho(770)$
  - $(3\pi)^\pm$ isobars: $a_1(1260)$, $a_2(1320)$
  - $(4\pi)^0$ isobars: $f_2(1270)$, $f_1(1285)$, $f_0(1370, 1500)$, and $\rho'(1450, 1700)$
    - Only few information available for $(4\pi)^0$ isobars
- Exploration of model space using evolutionary algorithm
  - Bayesian goodness-of-fit criterion that takes into account model complexity $\implies$ "evidence"
  - Pool of 284 allowed waves



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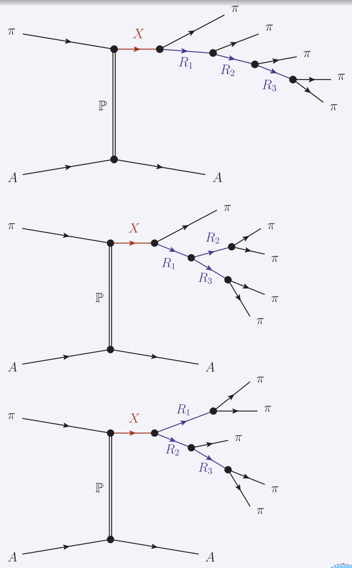
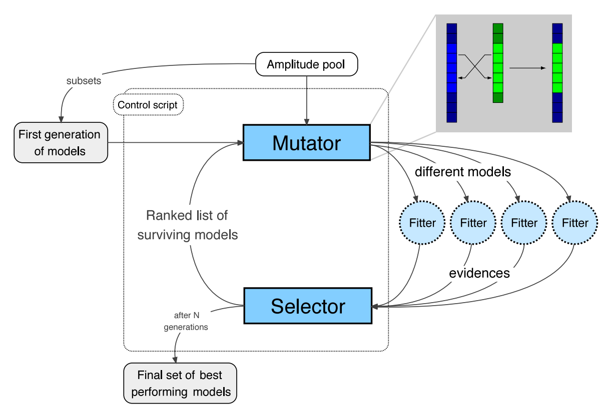


PWA of $\pi^- \text{Pb} \to \pi^-\pi^+\pi^-\pi^+\pi^- \text{Pb}$

Evolutionary Algorithm for Wave Set Selection



- Best model found: 31 waves + incoherent isotropic background
- Also provides estimate for systematic uncertainty from fit model

PWA of π^- Pb $\to \pi^-\pi^+\pi^-\pi^+\pi^-$ Pb

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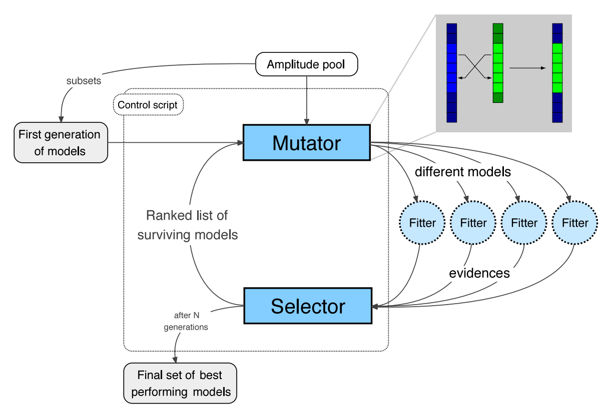

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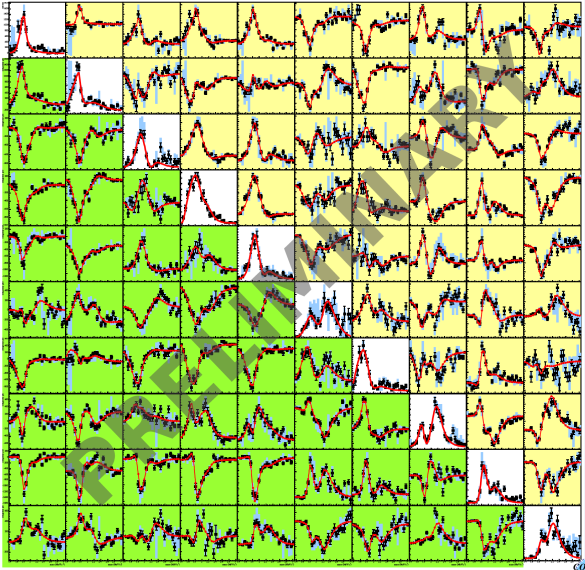
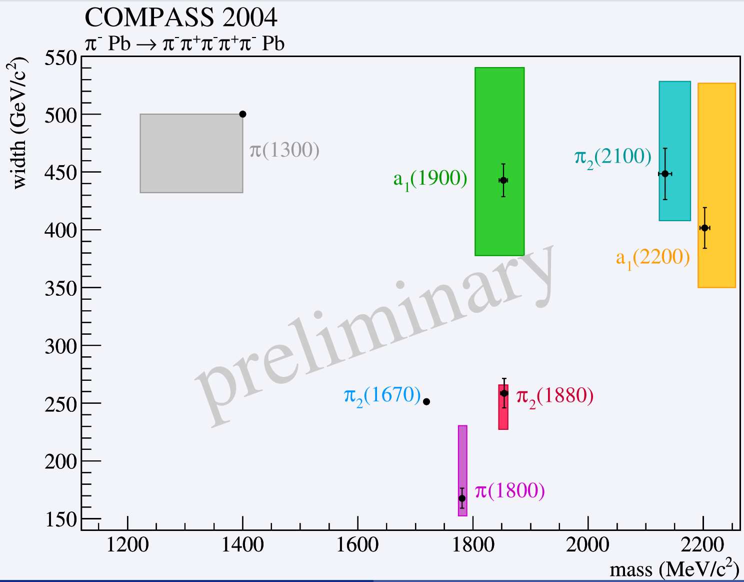
PWA of $\pi^- \mathrm{Pb} \to \pi^-\pi^+\pi^-\pi^+\pi^- \mathrm{Pb}$

- $0^{-+}\pi^- f_0(1500)\ S$
- $0^{-+}\rho a_1(1260)\ S$
- $1^{++}\pi^- f_0(1370)\ P$
- $1^{++}\pi^- f_1(1285)\ P$
- $1^{++}\rho\pi(1300)\ S$
- $1^{++}(\pi\pi)_S a_1\ D$
- $2^{-+}\pi^- f_2(1270)\ S$
- $2^{-+}\rho a_1(1260)\ S$
- $2^{-+}\rho a_2(1320)\ S$
- $2^{-+}\rho a_1(1260)\ D$



PWA of π^- Pb $\to \pi^-\pi^+\pi^-\pi^+\pi^-$ Pb



PWA of π^- Pb $\rightarrow \pi^-\pi^+\pi^-\pi^+\pi^-$ Pb

Proof of Principle: First mass-dependent five-body PWA

- Spin-density sub-matrix of 10 waves described using 7 resonances + background terms
- Rather simplistic fit model
    - Parameterization by sum of relativistic constant-width Breit-Wigners
    - Mixing and coupled-channel effects neglected
    - Multi-peripheral processes (Deck-effect) not taken into account
- Good description of data

Work in progress

- Much more data on tape
    - Proton target, kinematic range $0.1 < t' < 1\,(\text{GeV}/c)^2$
- Improvement of fit models
- Analysis of $(4\pi)^0$ subsystem

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Conclusions

- **COMPASS has large data sets for many final states**
  - Diffractive π^-, K^-, and p dissociation on various targets
  - Central production with π^- and p beams
  - $\pi^-\gamma$ and $K^-\gamma$ Primakoff reaction
- Main focus on J^{PC}-exotic mesons and glueballs
  - *Pilot run:* significant $J^{PC} = 1^{-+}$ signal consistent with $\pi_1(1600)$ seen in $\pi^-\pi^+\pi^-$ data on Pb target PRL **104** (2010) 241803
  - Detailed study of $\pi^-\pi^+\pi^-$ final state on p target
  - First mass-dependent $\pi^-\pi^+\pi^-\pi^+\pi^-$ PWA in diffractive production
  - Significant 1^{-+} signal also in $\eta\pi^-$ and $\eta'\pi^-$
  - Search for scalar glueballs in central production of $\pi\pi$ and KK
  - Further channels under analyses
    - K^- diffraction into $K^-\pi^+\pi^-$
    - π^- diffraction into $\pi^-\pi^0\pi^0$, $\pi^-\eta\eta$, $(\pi\pi K\bar{K})^-$, …

Software framework ROOTPWA available at
`http://sourceforge.net/projects/rootpwa/`

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  - Further channels under analyses
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Conclusions

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Baryon Spectroscopy at COMPASS

Large data sets with positive hadron beam (**75 % p**, 24 % π^+, 1 % K^+)

- Identification of beam protons via CEDARs
- Final-state particle ID using RICH
- *Main goal:* study of central-production reactions
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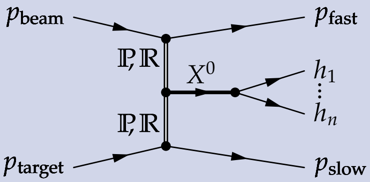

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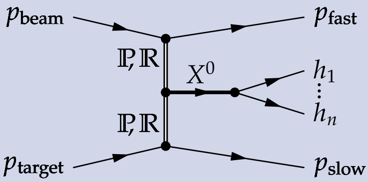
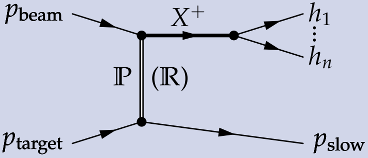
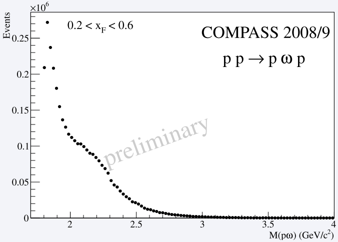




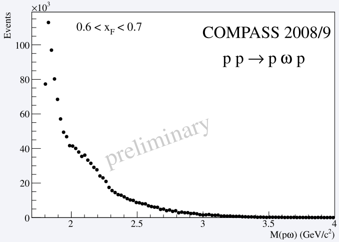
Baryon Spectroscopy at COMPASS

$pp \to p\omega\, p_{\text{slow}}$

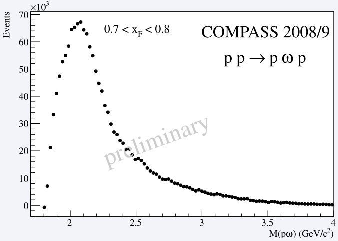
$m_{p\omega}$ for $0.2 < x_F(p) < 0.6$



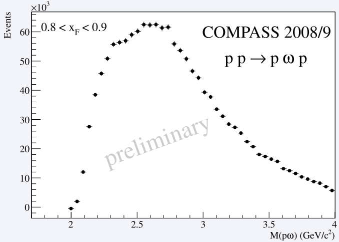
$m_{p\omega}$ for $0.6 < x_F(p) < 0.7$



$m_{p\omega}$ for $0.7 < x_F(p) < 0.8$



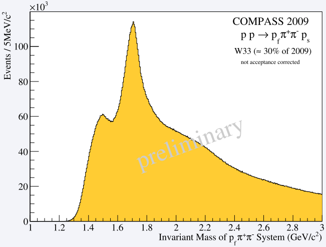
$m_{p\omega}$ for $0.8 < x_F(p) < 0.9$



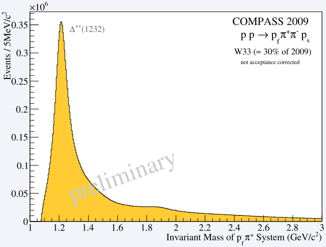
Baryon Spectroscopy at COMPASS

$pp \to p\pi^+\pi^- \, p_{\text{slow}}$

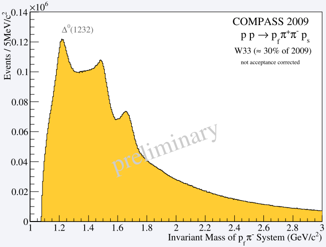
$p\pi^+\pi^-$ invariant mass



$p\pi^+$ subsystem



$p\pi^-$ subsystem



$\pi^+\pi^-$ subsystem