Amplitude Analysis of the 5-Pion System

in Diffractive Pion Dissociation at COMPASS — Low t'

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on behalf of the COMPASS collaboration

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Cluster of Excellence: Origin and Structure of the Universe,

BMBF, EU

Diffractive Pion Dissociation

Partial Wave Decomposition in 5-Body-Mass Bins

Resonances Embedded in the 5π Continuum
- Formalism
- Exploring Resonant Contributions

Diffractive Dissociation into 5 Pions

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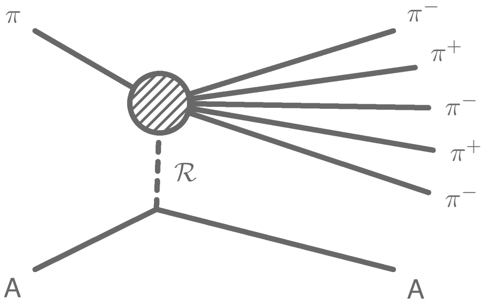

2004 COMPASS Hadron Run

- $190\,\mathrm{GeV}\,\pi^-$ beam
- Pb target
- Multiplicity Trigger
- NO Recoil Detector

Diffractive Dissociation into 5 Pions

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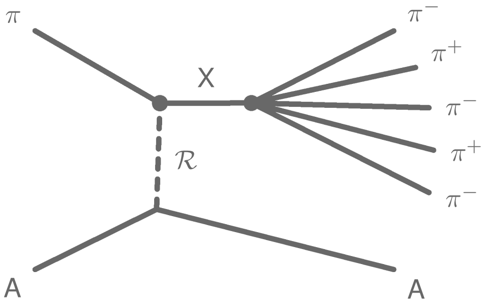

2004 COMPASS Hadron Run

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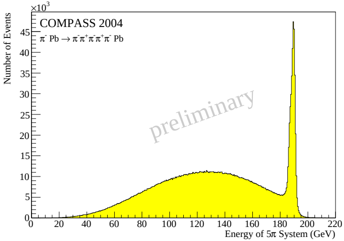
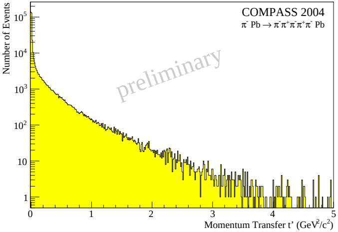
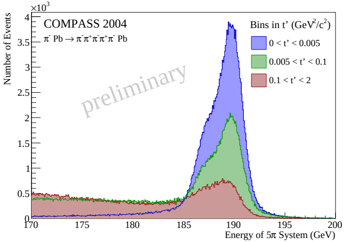
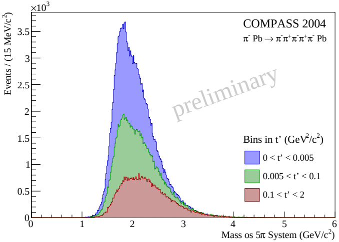
5π Data Sample

2004: $\pi + Pb \to 5\pi + Pb$









5π Data Sample

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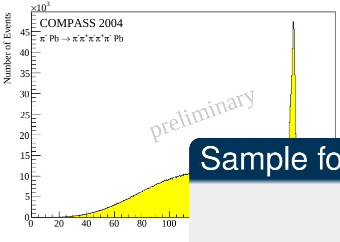
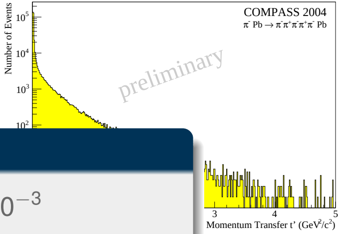
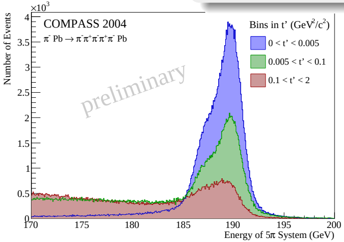


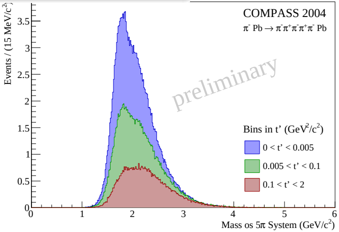


Sample for PWA

$$t'_{\mathrm{reco}} < 5 \cdot 10^{-3}$$

$\rightarrow$ Nonexclusive background small

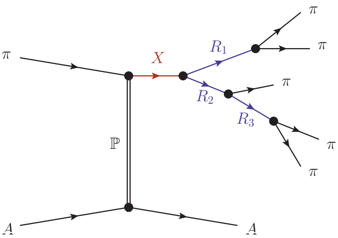


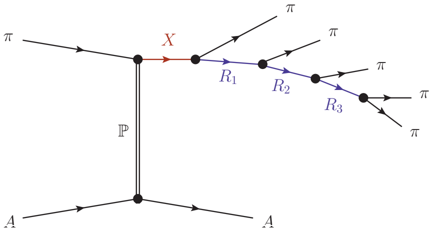


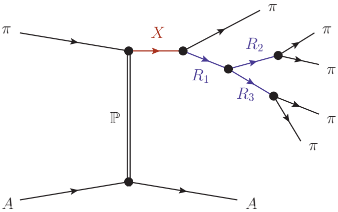
Isobar Model for 5π Final State

Challenges and Approaches

5-body isobar model







Isobar Decay Tree

- 11 indepependent variables τ:
  4 vertices $\times$ 2 angles $+$ 3 isobar masses
- Decay amplitudes $\psi(\tau)$
  in Helicity formalism
- Non-relativistic model

Isobar Model for 5π Final State

Challenges and Approaches

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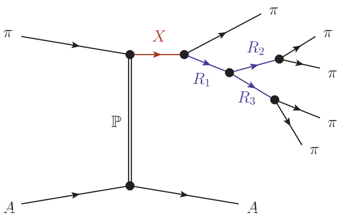
5-body isobar model

5-Body PWA Specials

- Decay topologies
- Many possible partial waves
- Assembly of waveset not possible by hand
- $\Rightarrow$ Waveset evolution
- 284 waves tested



Isobar Decay Tree

- 11 indepependent variables τ:
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- Decay amplitudes $\psi(\tau)$ in Helicity formalism
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Evolutionary Waveset Exploration

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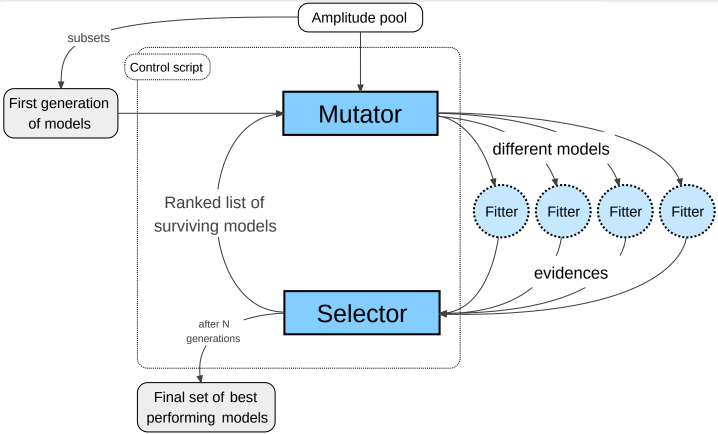
Genetic Algorithm — 284 Waves in Pool



Evidence = Goodness of fit

- Bayesian Statistics → regularized Log-Likelihood
- Takes into account model complexity

Final Waveset

From evolutionary exploration

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$J^{PC}M^{\epsilon}$	ℓs	Isobar1 Isobar2	Decay Isobar2	
$0^{-+}0^{+}$	$S\,0$	$\pi^{-} f_0(1500)$	$\rho(770) \binom{0}{0} \rho(770)$	●
$0^{-+}0^{+}$	$S\,0$	$\pi^{-} f_0(1500)$	$(\pi\pi)_S \binom{0}{0} (\pi\pi)_S$	
$0^{-+}0^{+}$	$S\,0$	$\rho(770) a_1(1260)$	$\pi^{-} \binom{0}{1} \rho(770)$	●
$0^{-+}0^{+}$	$D\,2$	$\rho(770) a_1(1260)$	$\pi^{-} \binom{0}{1} \rho(770)$	
$0^{-+}0^{+}$	$S\,0$	$(\pi\pi)_S \pi(1300)$	$\pi^{-} (\pi\pi)_S$	
$0^{-+}0^{+}$	$P\,1$	$\rho(770) \pi(1300)$	$\pi^{-} \binom{0}{0} (\pi\pi)_S$	
$1^{++}0^{+}$	$S\,1$	$\pi^{-} \rho(1600)$	$\rho(770) \binom{0}{1} (\pi\pi)_S$	
$1^{++}0^{+}$	$P\,0$	$\pi^{-} f_0(1370)$	$\rho(770) \binom{0}{0} \rho(770)$	●
$1^{++}0^{+}$	$P\,0$	$\pi^{-} (4\pi)_{0^{++}}$	$(\pi\pi)_S (\pi\pi)_S$	
$1^{++}0^{+}$	$P\,1$	$\pi^{-} f_1(1285)$	$\pi^{\mp} \binom{1}{1} a_1(1260)$	●
$1^{++}0^{+}$	$S\,1$	$\rho(770) \pi(1300)$	$\pi^{-} \binom{1}{1} \rho(770)$	●
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$1^{++}0^{+}$	$D\,1$	$\rho(770) \pi(1300)$	$\pi^{-} \binom{1}{1} \rho(770)$	
$1^{++}0^{+}$	$S\,1$	$(\pi\pi)_S a_1(1260)$	$\pi^{-} \binom{0}{1} \rho(770)$	
$1^{++}0^{+}$	$S\,1$	$(\pi\pi)_S a_1(1260)$	$\pi^{-} \binom{1}{0} (\pi\pi)_S$	
$1^{++}0^{+}$	$D\,1$	$(\pi\pi)_S a_1(1260)$	$\pi^{-} \binom{0}{1} \rho(770)$	●
$1^{++}0^{+}$	$D\,2$	$(\pi\pi)_S a_2(1320)$	$\pi^{-} \binom{2}{1} \rho(770)$	
$1^{++}0^{+}$	$P\,0$	$(\pi\pi)_S \pi(1300)$	$\pi^{-} \binom{0}{0} (\pi\pi)_S$	
$1^{++}0^{+}$	$S\,1$	$\pi^{-} \eta_1(1600)$	$\pi^{\mp} \binom{0}{1} a_1(1260)$	
$1^{++}0^{+}$	$S\,1$	$\pi^{-} \rho(1700)$	$\pi^{\mp} \binom{1}{0} \pi(1300)$	
$1^{++}0^{+}$	$P\,2$	$\rho(770) a_1(1260)$	$\pi^{-} \binom{0}{1} \rho(770)$	

$2^{-+}0^{+}$	$S\,2$	$\pi^{-} f_2(1270)$	$\pi^{\mp} \binom{1}{1} a_1(1260)$	●
$2^{-+}0^{+}$	$S\,2$	$\rho(770) a_1(1260)$	$\pi^{-} \binom{0}{1} \rho(770)$	●
$2^{-+}0^{+}$	$S\,2$	$\rho(770) a_2(1320)$	$\pi^{-} \binom{2}{1} \rho(770)$	●
$2^{-+}0^{+}$	$D\,0$	$\rho(770) a_1(1260)$	$\pi^{-} \binom{0}{1} \rho(770)$	●
$2^{-+}0^{+}$	$P\,0$	$(\pi\pi)_S \pi(1800)$	$\pi^{-} \binom{0}{0} (\pi\pi)_S$	
$2^{-+}0^{+}$	$D\,2$	$\pi^{-} f_2(1270)$	$\pi^{\mp} \binom{1}{1} a_1(1260)$	
$2^{-+}0^{+}$	$S\,2$	$f_2(1270) \pi_2(1670)$	$\pi^{-} \binom{0}{2} f_2(1270)$	
$2^{-+}0^{+}$	$P\,1$	$\pi^{-} \rho(1600)$	$\rho(770) \binom{0}{1} (\pi\pi)_S$	
$3^{++}0^{+}$	$D\,1$	$(\pi\pi)_S a_1(1260)$	$\pi^{-} \binom{0}{1} \rho(770)$	
$1^{-+}0^{-}$	$D\,1$	$\rho(770) a_1(1260)$	$\pi^{-} \binom{0}{1} \rho(770)$	
FLAT				

● waves used in mass independent fit.

Structures of the Spin Density Matrix

32 Waves

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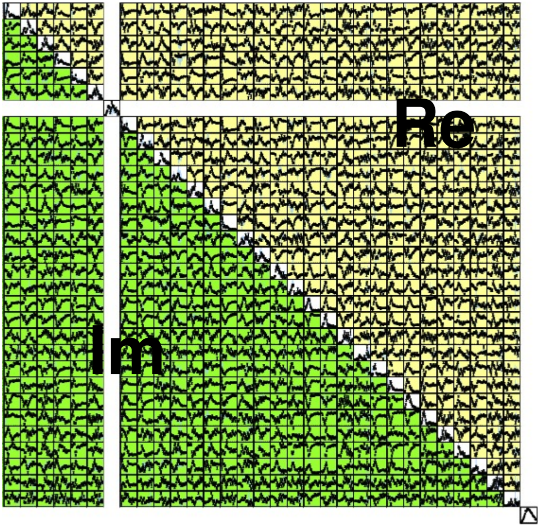

Mass Dependent Parameterization

of Spin Density Matrix

$$T^{\epsilon}_{\alpha} T^{\epsilon *}_{\beta} = \rho^{\epsilon}_{\alpha\beta}(m) = \left(\sum_{k} C^{\epsilon}_{\alpha k} \mathcal{A}_{\alpha k}(m) \sqrt{\rho_{\alpha}(m)} \right) \left(\sum_{l} C^{\epsilon}_{\beta l} \mathcal{A}_{\beta l}(m) \sqrt{\rho_{\beta}(m)} \right)^{*} \cdot \rho_{5\pi}(m) F(m) \qquad (1)$$

[1] [N. A. Törnqvist Z. Phys. **C68**(1995)647]

Mass Dependent Parameterization
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with Breit-Wigner amplitudes:

$$\mathcal{A}_{\alpha k}(m, M_0, \Gamma_0) = \frac{M_0 \Gamma_0}{m^2 - M_0^2 + i\Gamma_0 M_0} \qquad k = \mathrm{resonance} \qquad (2)$$

and fixed width, including meson “formfactor” F(m)[1] In each fitted wave a coherent, constant background term is allowed, such that

$$\mathcal{A}_{\alpha k}(m) = c_{\alpha \mathrm{bkg}} \qquad k = \mathrm{bkg.} \qquad (3)$$

The phase space factors

$$\rho_\alpha(m) = \int |\psi^{\epsilon}_{\alpha}|^2 d\tau \qquad (4)$$

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Final Waveset

From evolutionary exploration

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$0^{-+}0^{+}$	$S\,0$	$(\pi\pi)_S\, \pi(1300)$	$\pi^{-}\,(\pi\pi)_S$	
$0^{-+}0^{+}$	$P\,1$	$\rho(770)\pi(1300)$	$\pi^{-}\binom{0}{1}(\pi\pi)_S$	
$1^{++}0^{+}$	$S\,1$	$\pi^{-}\,\rho(1600)$	$\rho(770)\binom{0}{1}(\pi\pi)_S$	
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$1^{++}0^{+}$	$P\,0$	$\pi^{-}\,(4\pi)_{0^{++}}$	$(\pi\pi)_S(\pi\pi)_S$	
$1^{++}0^{+}$	$P\,1$	$\pi^{-}\, f_1(1285)$	$\pi^{\mp}\binom{1}{1}a_1(1260)$	●
$1^{++}0^{+}$	$S\,1$	$\rho(770)\pi(1300)$	$\pi^{-}\binom{1}{1}\rho(770)$	●
$1^{++}0^{+}$	$S\,1$	$\rho(770)\pi(1300)$	$\pi^{-}\binom{0}{0}(\pi\pi)_S$	
$1^{++}0^{+}$	$D\,1$	$\rho(770)\pi(1300)$	$\pi^{-}\binom{1}{1}\rho(770)$	
$1^{++}0^{+}$	$S\,1$	$(\pi\pi)_S\, a_1(1260)$	$\pi^{-}\binom{0}{1}\rho(770)$	
$1^{++}0^{+}$	$S\,1$	$(\pi\pi)_S\, a_1(1260)$	$\pi^{-}\binom{1}{0}(\pi\pi)_S$	
$1^{++}0^{+}$	$D\,1$	$(\pi\pi)_S\, a_1(1260)$	$\pi^{-}\binom{0}{1}\rho(770)$	●
$1^{++}0^{+}$	$D\,2$	$(\pi\pi)_S\, a_2(1320)$	$\pi^{-}\binom{2}{1}\rho(770)$	
$1^{++}0^{+}$	$P\,0$	$(\pi\pi)_S\, \pi(1300)$	$\pi^{-}\binom{0}{0}(\pi\pi)_S$	
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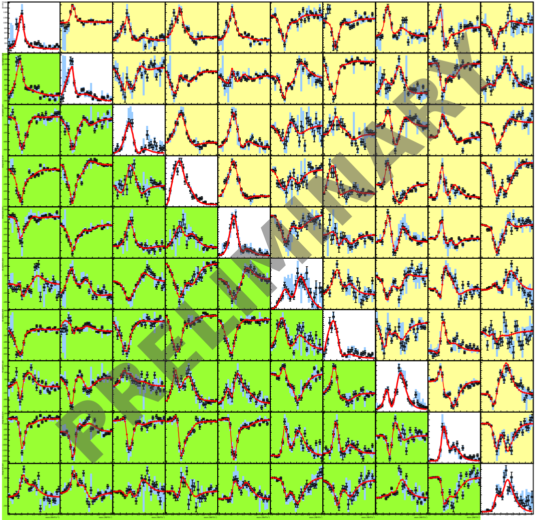
$1^{++}0^{+}$	$S\,1$	$\pi^{-}\,\rho(1700)$	$\pi^{\mp}\binom{1}{0}\pi(1300)$	
$1^{++}0^{+}$	$P\,2$	$\rho(770) a_1(1260)$	$\pi^{-}\binom{0}{1}\rho(770)$	
$2^{-+}0^{+}$	$S\,2$	$\pi^{-}\, f_2(1270)$	$\pi^{\mp}\binom{1}{1}a_1(1260)$	●
$2^{-+}0^{+}$	$S\,2$	$\rho(770) a_1(1260)$	$\pi^{-}\binom{0}{1}\rho(770)$	●
$2^{-+}0^{+}$	$S\,2$	$\rho(770) a_2(1320)$	$\pi^{-}\binom{2}{1}\rho(770)$	●
$2^{-+}0^{+}$	$D\,0$	$\rho(770) a_1(1260)$	$\pi^{-}\binom{0}{1}\rho(770)$	●
$2^{-+}0^{+}$	$P\,0$	$(\pi\pi)_S\, \pi(1800)$	$\pi^{-}\binom{0}{0}(\pi\pi)_S$	
$2^{-+}0^{+}$	$D\,2$	$\pi^{-}\, f_2(1270)$	$\pi^{\mp}\binom{1}{1}a_1(1260)$	
$2^{-+}0^{+}$	$S\,2$	$f_2(1270)\pi_2(1670)$	$\pi^{-}\binom{0}{2}f_2(1270)$	
$2^{-+}0^{+}$	$P\,1$	$\pi^{-}\,\rho(1600)$	$\rho(770)\binom{0}{1}(\pi\pi)_S$	
$3^{++}0^{+}$	$D\,1$	$(\pi\pi)_S\, a_1(1260)$	$\pi^{-}\binom{0}{1}\rho(770)$	
$1^{-+}0^{-}$	$D\,1$	$\rho(770) a_1(1260)$	$\pi^{-}\binom{0}{1}\rho(770)$	
FLAT				

● waves used in mass independent fit.

7-Resonance Fit

Final Fit Results Overview



The 0^{-+} Sector

0^{-+} Waves

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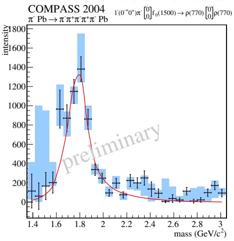
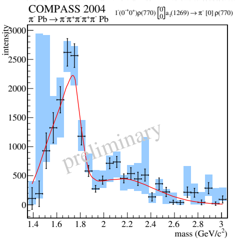
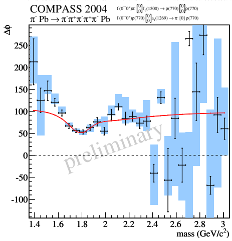


(a) $\pi f_0(1500)$



(b) $\rho(770) a_1(1269)$



(c) Phase difference.

The 1^{++} Sector

How Many a_1 States Do We Need?

Fit With TWO Resonances

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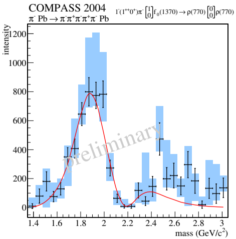
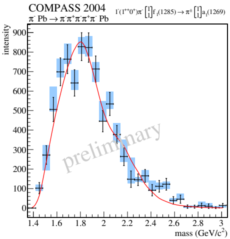
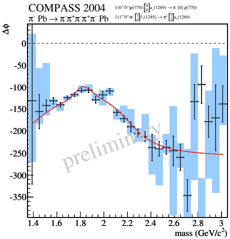


(d) $\pi f_0(1370)$



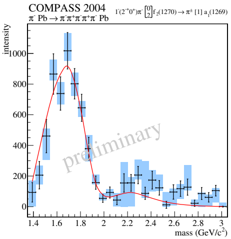
(e) $\pi f_1(1285)$



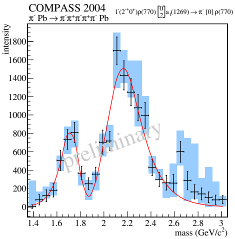
(f) Phase difference.

The 2^{-+} Sector

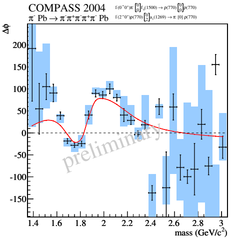
2^{-+} Waves



(g) $\pi^- f_2(1270)$ S-wave



(h) $\rho a_1(1269)$ S-wave



(i) Phase difference

How Many π_2 States Do We Need?

Fit With ONE Resonance

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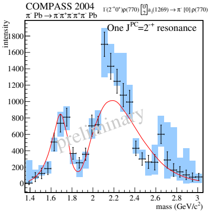
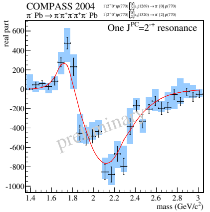
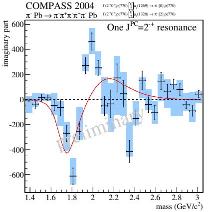


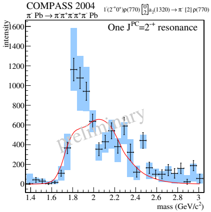
(j)



(k)



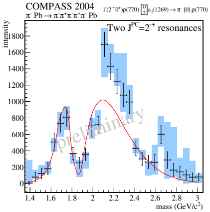
(l)



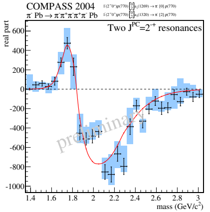
(m)

How Many π_2 States Do We Need?

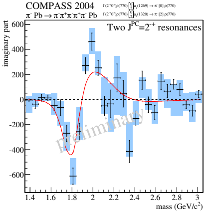
Fit With TWO Resonances



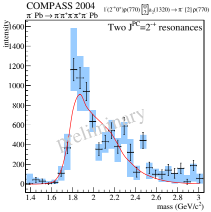
(a)



(b)



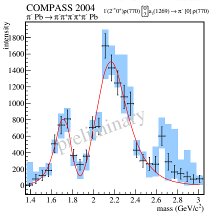
(c)



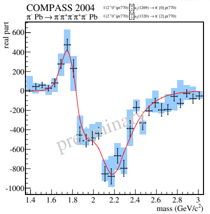
(d)

How Many π_2 States Do We Need?

Fit With THREE Resonances —



(a)



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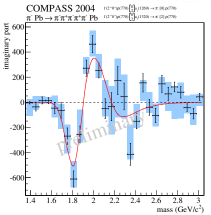

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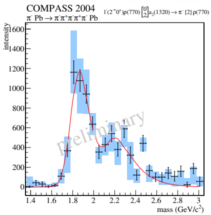
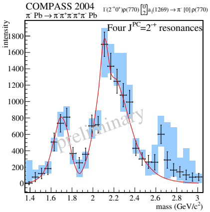


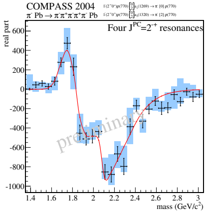
(d)

How Many π_2 States Do We Need?

Fit With FOUR Resonances



(a)



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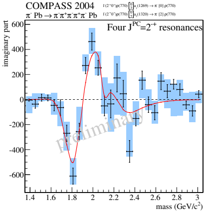
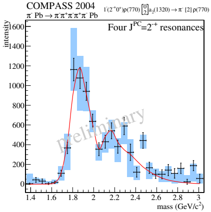


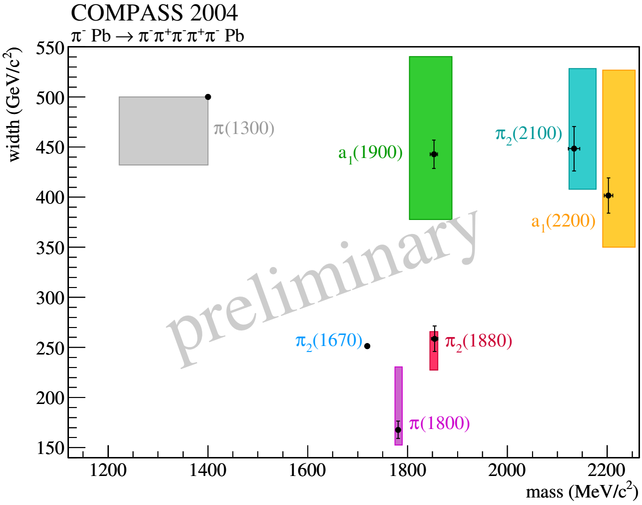
(c)



(d)

Final Result

Summary of Resonance Parameters



Summary

Summary

- Diffractive dissociation of π^- into 5π on lead (COMPASS 2004)
- First full 5-body PWA in 5π mass bins
- Semi-automatic model selection with genetic optimization
  - $\rightarrow$ handle on systematic uncertainties
- First successful mass-dependent fits
  - Known states: $\pi_2(1670), \pi(1800)$ observed
  - Elusive $\pi_2(1880)$ fitted in $a_1\rho$ and $a_2\rho$
  - Fit with two 1^{++} resonances
  - Possible $\pi_2(2200)$ signal

Outlook

- Large data-set $\pi^- + p \rightarrow 5\pi + p$ at high t' on tape
- Analysis of 4π subsystem

Resonance Parameters

Cmparison to PDG

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Parameter				Fit	PDG
	Resonance	J^{PC}		(MeV/c^2)	
	$\pi(1300)$	0^{-+}	M	1400^*	1300 ± 100
			Γ	$500^\dagger$	200…600
	$\pi(1800)$	0^{-+}	M	$1781 \pm 5^{+1(+8)}_{-6(-6)}$	1816 ± 14
			Γ	$168 \pm 9^{+5(+62)}_{-14(-15)}$	208 ± 12
○	$a_1(1900)$	1^{++}	M	$1853 \pm 7^{+36(+36)}_{-6(-49)}$	1930^{+30}_{-70}
			Γ	$443 \pm 14^{+12(+98)}_{-45(-65)}$	155 ± 45
○	$a_1(2200)$	1^{++}	M	$2202 \pm 8^{+15(+53)}_{-8(-11)}$	$2096 \pm 17 \pm 121$
			Γ	$402 \pm 17^{+41(+125)}_{-52(-51)}$	$451 \pm 41 \pm 81$
	$\pi_2(1670)$	2^{-+}	M	$1719.0^\dagger$	1672.4 ± 3.2
			Γ	$251.4^\dagger$	259 ± 9
	$\pi_2(1880)$	2^{-+}	M	$1854 \pm 6^{+6(+6)}_{-4(-9)}$	1895 ± 16
			Γ	$259 \pm 13^{+7(+7)}_{-17(-31)}$	235 ± 34
○	$\pi_2(2100)$	2^{-+}	M	$2133 \pm 12^{+7(+43)}_{-18(-18)}$	2090 ± 29
			Γ	$448 \pm 22^{+60(+80)}_{-40(-40)}$	625 ± 50
			M		2245 ± 60
			Γ		320^{+100}_{-40}

○ not established   * at limit: † fixed in fit

Isobars that have been used

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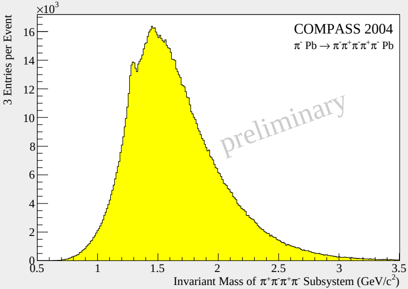
4π Isobars ($G = +$)			3π Isobars ($G = -$)
Name	Mass / GeV	$I^G J^{PC}$	
f_0	1370 / 1500 / 1700	$0^+(0^{++})$	
η	1405	$0^+(0^{-+})$	
ρ'	1450 / 1700	$1^+(1^{--})$	
b_1	1235 / 1800	$1^+(1^{+-})$	
f_1	1285 / 1420	$0^+(1^{++})$	
f_2	1270 / 1565	$0^+(2^{++})$	
η_2'	1645	$0^+(2^{-+})$	
ρ_3	1690	$1^+(3^{--})$	
η_1	1600	$0^+(1^{-+})$	
b_0	1800	$1^+(0^{+-})$	
b_2	1800	$2^+(2^{+-})$	

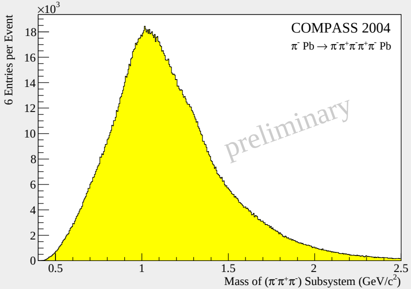
4π subsystem



Isobars that have been used

4π Isobars ($G = +$)			3π Isobars ($G = -$)		
			Name	Mass / GeV	$I^G J^{PC}$
			a_1	1270	$1^-(1^{++})$
			a_2	1320	$1^-(2^{++})$
			π'	1300	$1^-(0^{-+})$
			π_2	1670	$1^-(2^{-+})$
			π_1	1600	$1^-(1^{-+})$
b_2	1800	$2^+(2^{+-})$			

3π subsystem



Isobars that have been used

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4π Isobars ($G=+$)			3π Isobars ($G=-$)		
Name	Mass / GeV	$I^G J^{PC}$	Name	Mass / GeV	$I^G J^{PC}$
f_0	1370 / 1500 / 1700	$0^+(0^{++})$			
η	1405	$0^+(0^{-+})$	a_1	1270	$1^-(1^{++})$
ρ'	1450 / 1700	$1^+(1^{--})$	a_2	1320	$1^-(2^{++})$
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ρ_3	1690	$1^+(3^{--})$			
η_1	1600	$0^+(1^{-+})$			
b_0	1800	$1^+(0^{+-})$	π_1	1600	$1^-(1^{-+})$
b_2	1800	$2^+(2^{+-})$			

2π subsystem: σ, $\rho(770)$, $f_2(1270)$

Acceptance Correction

Accepted Phase-Space MC $m_{5\pi} \in [1840, 2080]\ \mathrm{MeV}/c^2$

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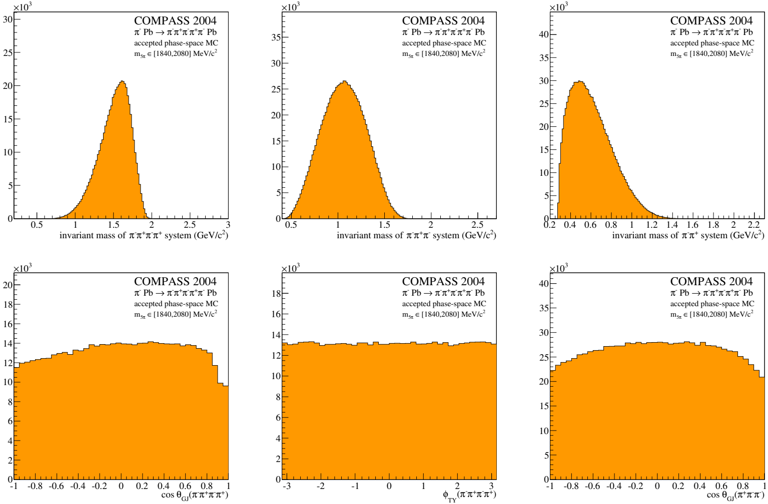

Acceptance Correction II

Accepted Phase-Space MC $m_{5\pi} \in [1840, 2080]\,\mathrm{MeV}/c^2$

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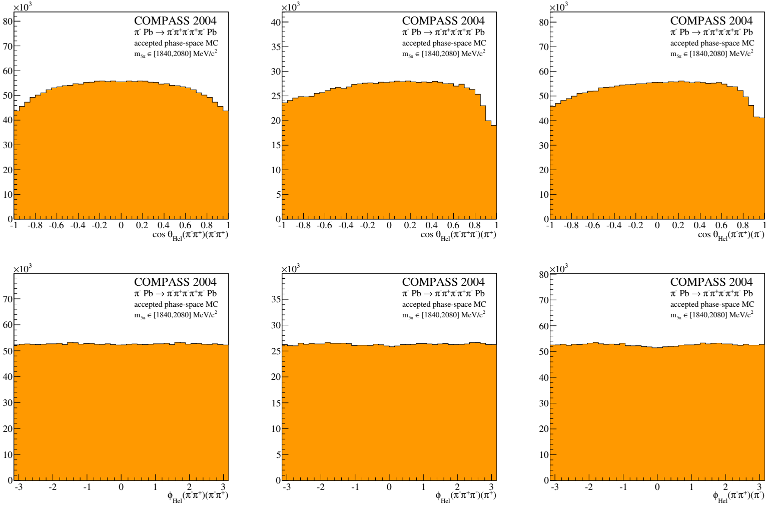

Acceptance Correction III

Accepted Phase-Space MC

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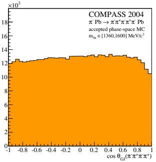
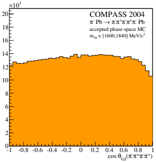
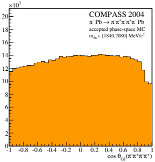


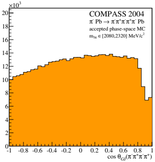
(a) $\cos\theta^{4\pi}_{\mathrm{GJ}}$



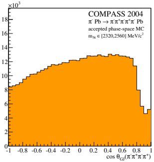
(b) $\cos\theta^{4\pi}_{\mathrm{GJ}}$



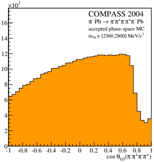
(c) $\cos\theta^{4\pi}_{\mathrm{GJ}}$



(d) $\cos\theta^{4\pi}_{\mathrm{GJ}}$



(e) $\cos\theta^{4\pi}_{\mathrm{GJ}}$

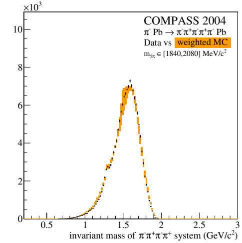


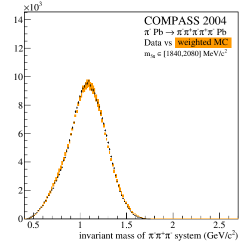
(f) $\cos\theta^{4\pi}_{\mathrm{GJ}}$

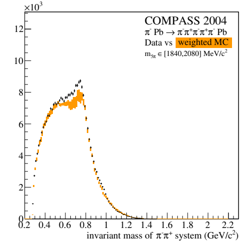
Figure:

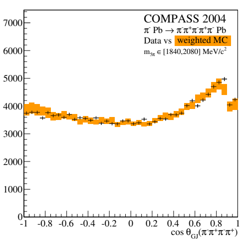
Kinematic Validation of Fit

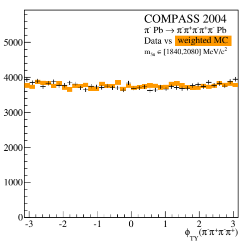
Data vs Weighted Monte Carlo $m_{5\pi} \in [1840, 2080]\,\mathrm{MeV}/c^2$

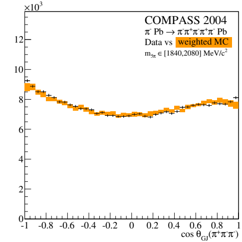












Kinematic Validation of Fit

Data vs Weighted Monte Carlo $m_{5\pi} \in [1840, 2080]\,\mathrm{MeV}/c^2$

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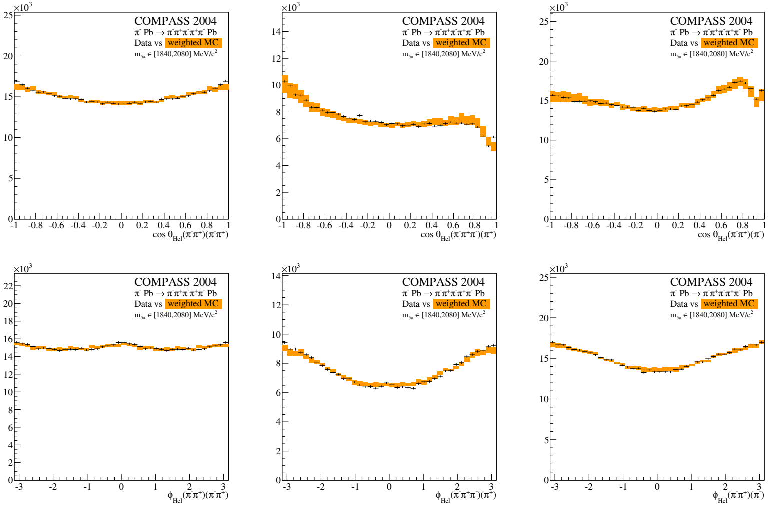
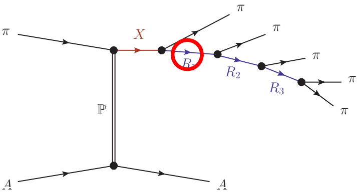


4π Isospin Symmetrization

Isospin Clebsch Gordan Coefficients



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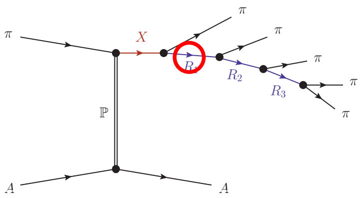

2π decay amplitude is isospin-symmetric, independent of $I_{(\pi\pi)}$

$$\langle 1^{\pm}_{1}; 1^{\mp}_{1} \mid \mathcal{D} \mid I^{0}_{2} \rangle = \frac{2}{\sqrt{c}} \langle 1^{+}_{1}; 1^{-}_{1} \mid \mathcal{D} \mid I^{0}_{2} \rangle .$$

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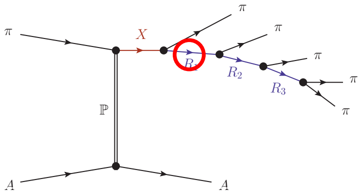

2π decay amplitude is isospin-symmetric, independent of $I_{(\pi\pi)}$

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For three and four pion systems this is not true:

$$\langle \pi^{\pm}\sigma \mid \mathcal{D} \mid 1_3^{\pm} \rangle \quad \text{symmetric}$$

$$\langle \pi^{\pm}\rho^{0} \mid \mathcal{D} \mid 1_3^{\pm} \rangle \quad \text{antisymmetric}$$

4π Isospin Symmetrization

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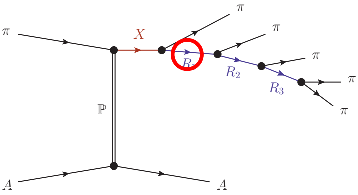

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$$\langle 4\pi \mid \mathcal{D} \mid 1_4^0 \rangle = \left(\frac{1}{\sqrt{2}}\right) \langle 1_1^{-}; 1_3^{+} \mid \mathcal{D} \mid 1_4^0 \rangle \cdot \left(\frac{1}{\sqrt{2}}\right) \langle 1_1^{+}; 1_2^{0} \mid \mathcal{D} \mid 1_3^{+} \rangle \cdot \frac{2}{\sqrt{2}} \langle 1_1^{+}; 1_1^{-} \mid \mathcal{D} \mid 1_2^0 \rangle$$
$$+ \left(\frac{-1}{\sqrt{2}}\right) \langle 1_1^{+}; 1_3^{-} \mid \mathcal{D} \mid 1_4^0 \rangle \cdot \left(\frac{-1}{\sqrt{2}}\right) \langle 1_1^{-}; 1_2^{0} \mid \mathcal{D} \mid 1_3^{-} \rangle \cdot \frac{2}{\sqrt{2}} \langle 1_1^{+}; 1_1^{-} \mid \mathcal{D} \mid 1_2^0 \rangle$$

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Isospin Clebsch Gordan Coefficients

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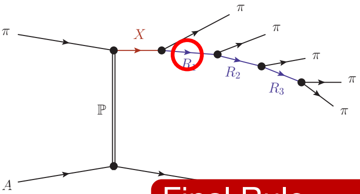

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For three and four pion systems this is not true:

$\langle 4\pi \mid \mathcal{D} \mid$ … etric … $\mathcal{D} \mid 1^{0}_{2} \rangle$ … $\mathcal{D} \mid 1^{0}_{2} \rangle$

Final Rule

$I(4\pi)$	$I(2\pi) = 1$	$I(2\pi) = 0$
0	$\eta = -1$	$\eta = +1$
1	$\eta = +1$	$\eta = -1$

Exotic 4π System ... or excited ρ?

Correct isospin assignment essential!

$$G = (-1)^I \cdot C$$

For the 4π system $G = +$.

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Consider a $J^P = 1^-$ state

$$\boxed{\begin{array}{lll} I = 0 & \Rightarrow & J^{PC} = 1^{-+} \\ I = 1 & \Rightarrow & J^{PC} = 1^{--} \end{array}} \quad (5)$$

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$$4\pi \to \pi^\pm a_1^\mp \to \pi^\pm(\pi^\mp \rho^0)$$

$$\begin{aligned} \langle 4\pi \mid \mathcal{D} \mid I_4^0 \rangle = & \left(\frac{1}{\sqrt{2}}\right) \langle 1_1^-; 1_3^+ \mid \mathcal{D} \mid 1_4^0 \rangle \cdot \left(\frac{1}{\sqrt{2}}\right) \langle 1_1^+; 1_2^0 \mid \mathcal{D} \mid 1_3^+ \rangle \cdot \frac{2}{\sqrt{2}} \langle 1_1^+; 1_1^- \mid \mathcal{D} \mid 1_2^0 \rangle \\ & \pm \left(\frac{-1}{\sqrt{2}}\right) \langle 1_1^+; 1_3^- \mid \mathcal{D} \mid 1_4^0 \rangle \cdot \left(\frac{1}{\sqrt{2}}\right) \langle 1_1^-; 1_2^0 \mid \mathcal{D} \mid 1_3^- \rangle \cdot \frac{2}{\sqrt{2}} \langle 1_1^+; 1_1^- \mid \mathcal{D} \mid 1_2^0 \rangle \end{aligned}$$

Analysis of the 4π Subsystem

Analysis of the 4π Subsystem

Taking out Model Dependence

Problems:

- More than one resonance in an isobar-channel (Unitarity!)
- Rescattering

Idea: (c.f. E791 $D^+ \to K^-\pi^+\pi^+$)

- Do NOT put any model
- Replace 4-body amplitude $\to$ with piecewise constant amplitude
- Free fit of amplitude in isobar channel

Caveat:

- Need another isobar to act as interferometer
- Needs huge statistics (many fit-parameters)

Analysis of the 4π Subsystem

Taking out Model Dependence

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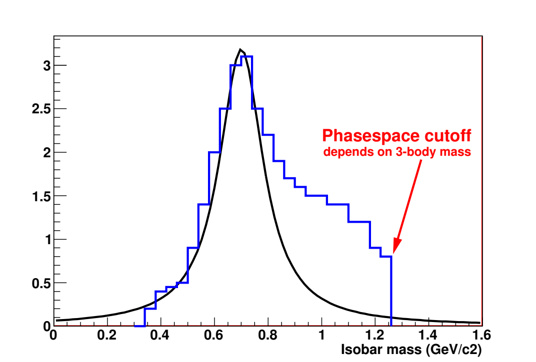
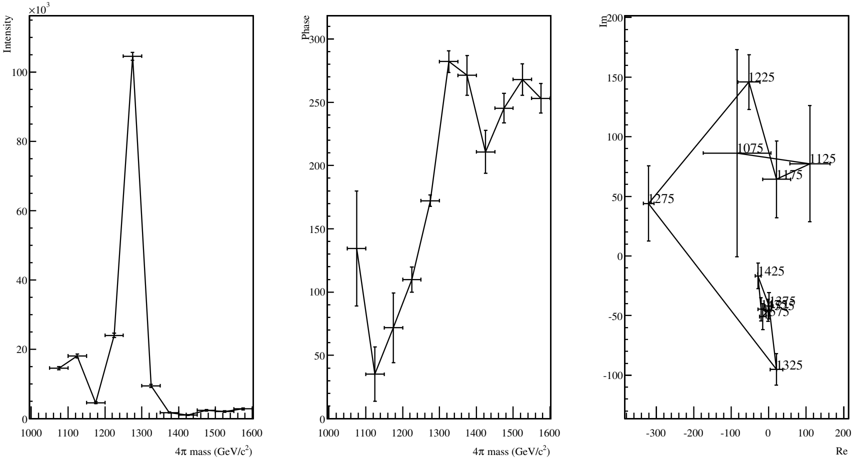
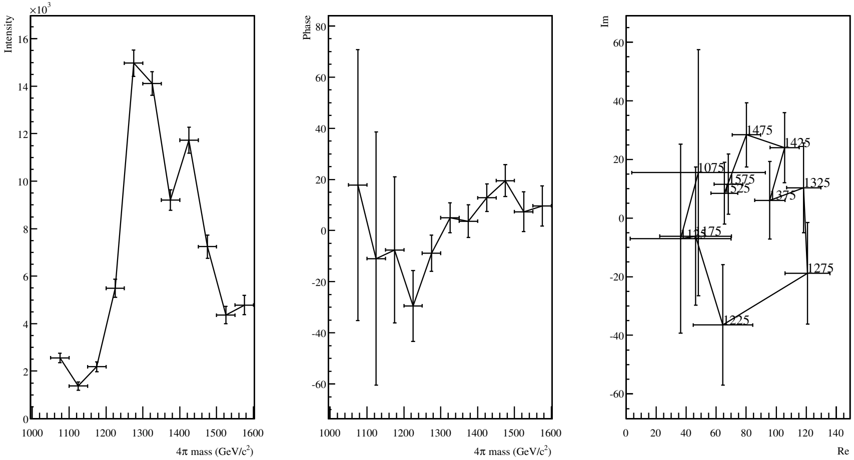
Piecewise amplitude definition



4π decay of $I^G(J^{PC}) = 0^+(1^{++})f_1$



4π decay of $I^G(J^{PC}) = 0^+(2^{++}) f_2$



Previous Search for $0^+(1^{-+})$ in $\bar{p}n \to 5\pi$

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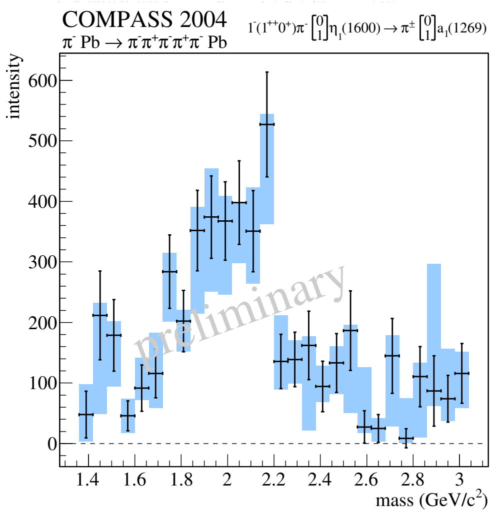
Abele et Al. Eur. Phys. J. C 21 (2001) 261

- Initial state (at rest) dominated by $I^G = 1^-$ $J^{PC} = 0^{-+}$ ($\bar{p}n$ s-wave)
- 4π subsystem dominated by 0^+0^{++}
- $\rho(1450)$ and $\rho(1700)$ found with PDG values
- Search for $\eta_1(1400)$ as Partner to $\pi_1(1400)$
  - Cannot be established (although slight increase in loglikelihood)
  - But: $0^{-+} \to \pi\eta_1$ requries P-Wave!
  - and: η_1 might be heavier while PhaseSpace is limited in $\bar{p}n$

An Interesting Amplitude

With an exotic 4π isobar

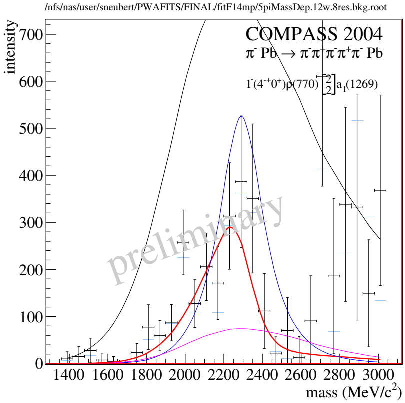


$\pi_2(1880)$ Mass Measurements

Mass (MeV/c^2)	Experiment	Reaction
$1929 \pm 24 \pm 18$	E852	$\pi^- p \to \eta\eta\pi^- p$
$1876 \pm 11 \pm 67$	E852	$\pi^- p \to \omega\pi^-\pi^0 p$
$2003 \pm 88 \pm 148$	E852	$\pi^- p \to \eta\pi^-\pi^+\pi^- p$
$1880 \pm 20 \pm 148$	CB	$\bar{p}p \to \eta\eta\pi^0\pi^0$
$1836 \pm 13 + 0 - 44$	COMPASS	$\pi^- Pb \to \pi^-\pi^+\pi^- Pb$
1876 ± 13	COMPASS	$\pi^- Pb \to \pi^-\pi^+\pi^-\pi^+\pi^- Pb$

Table: Measured values for the mass of the $\pi_2(1880)$ resonances. As reported in [**?**] and compared to the COMPASS results. It is interesting that for both the 3π [**?**] and the new 5π result agree very well.

4^{-+}


/nfs/nas/user/sneubert/PWAFITS/FINAL/fitF14mp/5piMassDep.12w.8res.bkg.root
COMPASS 2004
π⁻ Pb → π⁻π⁺π⁻π⁺π⁻ Pb
1⁻(4⁻⁺0⁺)ρ(770) [2 2] a₁(1269)
preliminary
intensity
mass (MeV/c²)
0
100
200
300
400
500
600
700
1400
1600
1800
2000
2200
2400
2600
2800
3000

5π Phase Space Parameterization

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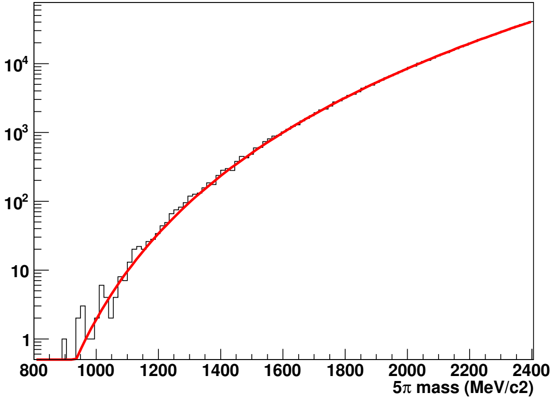
$$\rho_{5\pi} = a(m - m_{thresh})^5 \cdot [1 + b(m - m_{thresh})] \quad (6)$$



PWA Formalism Redux

2Stage Isobar-Model Fit

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Mass-Independent PWA

- Fit angular distributions + isobar systems in independent mass bins

$$\sigma(\tau, m) = \sum_{\epsilon=\pm1} \sum_{r=1}^{N_r} \left| \sum_i T_{ir}^{\epsilon}(m)\, f_i^{\epsilon}(t')\, \psi_i^{\epsilon}(\tau, m) \right|^2$$

- Production amplitude
- t'-dependence (helicity “flip”)
- Decay amplitude (Helicity formalism, reflectivity basis)

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Mass-Dependent χ^2 fit → Extract Resonance Parameters

- Parameterization of spin-density matrix elements $\sum_r T^{\epsilon}_{ir} T^{\epsilon *}_{jr}(m_x)$
- Takes into account interference terms
- Coherent background for some waves

Mass Independent Amplitude Fit

Intensity distribution parameterization

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Intensity distribution $\mathcal{I}$ as a function of decay-kinematic variables τ:

$$\mathcal{I}(\tau) = \sum_{\epsilon=\pm1} \sum_{r} \left| \sum_{\alpha \in M} T^{\epsilon}_{\alpha r} \bar{\psi}^{\epsilon}_{\alpha}(\tau) \right|^2$$

- Finite *waveset M*
- Production amplitude
- Decay amplitude

Mass Independent Amplitude Fit

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- Finite *waveset M*
- Production amplitude
- Decay amplitude

The likelihood $\mathcal{L}$ to observe (a specific set of) N events in a bin with finite acceptance $\eta(\tau)$ (assuming a model M, parameters T^{ϵ}_{ir}) is:

$$P(\mathrm{Data}|T_{ir}, M) = \mathcal{L} = \left[\frac{\bar{N}^N}{N!} e^{-\bar{N}}\right] \prod_{i}^{N} \frac{\mathcal{I}(\tau_i)\eta(\tau_i)f(\tau_i)}{\underbrace{\int \mathcal{I}(\tau)\eta(\tau)d\rho(\tau)}_{=\bar{N}}} \quad \text{with} \quad d\rho(\tau) = f(\tau)d\tau$$

Mass Independent Amplitude Fit

Definition of LogLikelihood Function

$$\mathcal{L} = \left[\frac{\bar{N}^N}{N!} e^{-\bar{N}}\right] \prod_i^N \frac{\mathcal{I}(\tau_i)}{\bar{N}} \eta(\tau_i) f(\tau_i) = \frac{1}{N!} \prod_i^N \mathcal{I}(\tau_i) \cdot \prod_i^N \eta(\tau_i) f(\tau_i) \cdot e^{-\bar{N}}$$

Mass Independent Amplitude Fit

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Taking the logarithm and inserting for $\bar{N}$:

$$\ln\mathcal{L} = -N\ln N + \sum_i^N \eta(\tau_i)f(\tau_i) + \sum_i^N \ln\mathcal{I}(\tau_i) - \int \mathcal{I}(\tau)\eta(\tau)d\rho(\tau)$$

Mass Independent Amplitude Fit

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drop $(-N \ln N + \sum_i^N \eta(\tau_i) f(\tau_i))$ and insert intensity parameterization

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Mass Independent Amplitude Fit

Definition of LogLikelihood Function

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$$IA^{\epsilon}_{\alpha\beta} = \int \bar{\psi}^{\epsilon}_{\alpha}(\tau_n)\bar{\psi}^{\epsilon}_{\beta}(\tau_n)^*\eta(\tau)\mathrm{d}\tau$$

Which waves to include into the waveset?

Which waves to include into the waveset?

Avoid overfitting

Which waves to include into the waveset?

Avoid overfitting

$\rightarrow$ Data driven method

How to Measure the Goodness of a Model

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Marginal Likelihood Definition

Bayes' Theorem (for the Model Probability after Observation)

$$P(M_k|\mathrm{Data}) = \frac{P(\mathrm{Data}|M_k)P(M_k)}{\sum_{k'} P(\mathrm{Data}|M_{k'})P(M_{k'})}$$

with model-priors $P(M_k)$ $\sum_{k'} P(M_{k'}) = 1$

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Marginal Likelihood or Evidence

$$P(\mathrm{Data}|M_k) = \int \underbrace{P(\mathrm{Data}|T^k, M_k)}_{\mathcal{L}} \underbrace{P(T^k|M_k)}_{\mathrm{Prior}} dT^k$$

$P(T^k|M_k)$ contains any pre-knowledge on the model-parameters T

- Marginalization ($= \int dT$) is not trivial in high-dimensional spaces
- Numerically stable is only the LogLikelihood

The Occam Factor Approximation

David J. C. MacKay, 2003 “Information Theory, Inference and Learning Algorithms”

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$$P(\mathrm{Data}|M_k) = \int \underbrace{P(\mathrm{Data}|T^k, M_k)}_{\mathcal{L}} \underbrace{P(T^k|M_k)}_{\mathrm{Prior}} dT^k$$

Approximate with Laplace’s method:

$$P(\mathrm{Data}|M_k) \approx P(\mathrm{Data}|T^k_{\mathrm{ML}}, M_k) \cdot \underbrace{P(T^k_{\mathrm{ML}}|M_k) \cdot \sqrt{(2\pi)^d |\mathbf{C}_{T|\mathrm{Data}}|}}_{\textit{Occam factor}}$$

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- $P(\mathrm{Data}|T^k_{\mathrm{ML}}, M_k)$ LogLikelihood at maximum likelihood solution T_{ML}
- $|\mathbf{C}_{T|\mathrm{Data}}|$ determinant of covariance matrix
- Dimension of parameter space: d

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- $P(\mathrm{Data}|T^k_{\mathrm{ML}}, M_k)$ LogLikelihood at maximum likelihood solution T_{ML}
- $|\mathbf{C}_{T|\mathrm{Data}}|$ determinant of covariance matrix
- Dimension of parameter space: d

Logarithmic evidence:

$$\ln P(\mathrm{Data}|M_k) \approx \ln P(\mathrm{Data}|T^k_{\mathrm{ML}}, M_k) + \ln P(T^k|M_k) + \ln \sqrt{(2\pi)^d |\mathbf{C}_{T|D}|}$$

Final Definition

Log-Evidence

$$\ln P(Data|M_k) \approx \ln \mathcal{L}_{ML} + \ln \sqrt{(2\pi)^d |\mathbf{C}_{T|\mathrm{Data}}|} - \ln V_T^k + \sum_{i \in M} \ln S_i$$

where V_T^k is the (prior) volume of parameter space

- Models (=wavesets) compared through the Bayes-Factor

$$B_{12} = \frac{P(Data|M_1)}{P(Data|M_2)}$$

- Interpretation according to Kass&Raftery:

$2\ln B_{12}$	B_{12}	Evidence
0 to 2	1 to 3	Not worth mentioning
2 to 6	3 to 20	Positive
6 to 10	20 to 150	Stong
> 10	> 150	Very strong

Kass, Raftery, *Bayes Factors*, J. Am. Stat. Assoc. 90 (1995) 773

Automatic Waveset Exploration

Genetic Algorithm

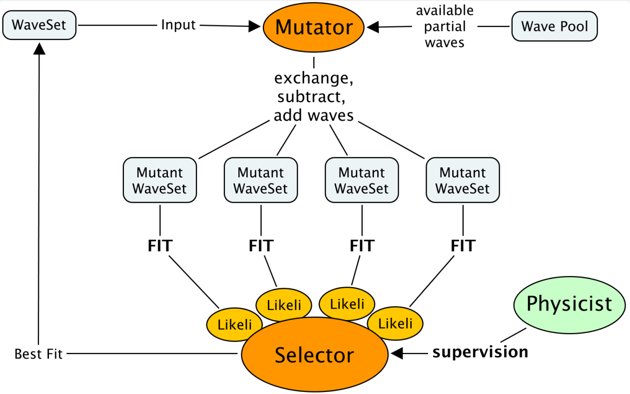
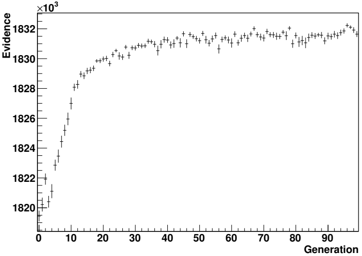


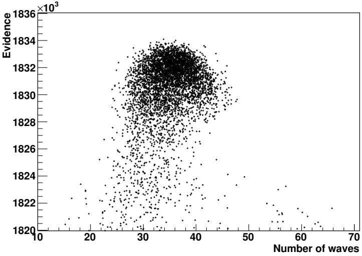
Figure of Merrit

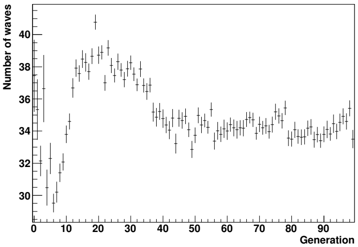
- Bayesian Statistics $\rightarrow$ regularized Log-Likelihood
- Takes into account model complexity

Automatic Waveset Exploration

Genetic Algorithm – 100 generations, population size 50







- Pool: $\sim$ 300 waves
- Small wave suppression 5σ
- Waveset size optimizes around 34 waves