

# $\pi^-\gamma \rightarrow \pi^-\pi^-\pi^+$ at Low Masses compared to ChPT Prediction at COMPASS

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XIV International Conference on Hadron Spectroscopy  
June 17<sup>th</sup> 2011

supported by: Maier-Leibnitz-Labor der TU und LMU München,  
Excellenzcluster: Origin and Structure of the Universe, BMBF



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## Meson Spectroscopy at Low Momentum Transfer

## Diagrams from ChPT

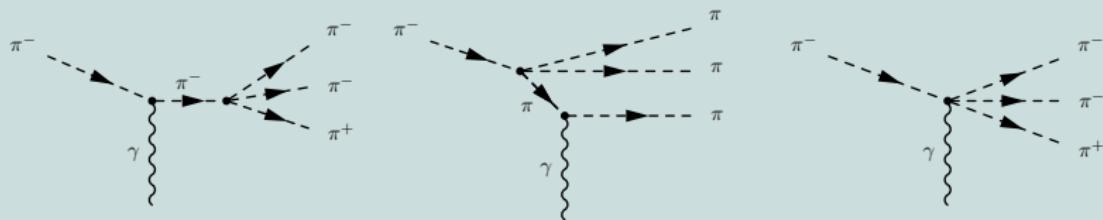
## Absolute Cross-Section and ChPT Prediction

## Motivation for Analysis

Meson spectrum at lowest momentum transfer:

Contribution of **photon-exchange**

- At low masses: "pions only" → predictions by **ChPT**



tree diagrams of  $3\pi$  production in  $\pi\gamma$  scattering

- At higher masses: **resonances**:

- radiative coupling of  $a_2(1320)$  / heavier mesons
- interference between diffractive and Coulomb production



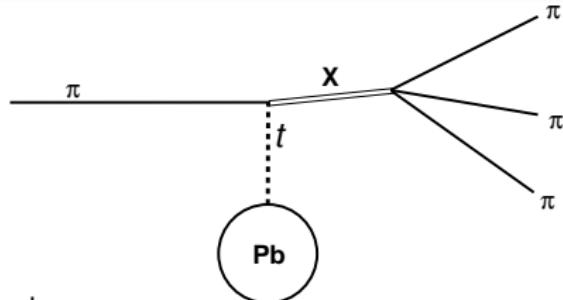
## Interaction mechanisms

- Diffraction:

- target particle remains intact
- $I = 0$  **Reggeon t-channel exchange**  
exchange particle: (mostly) Pomeron
- Assumptions:** Factorization of meson and Pb vertex, no final state interaction

- Primakoff: **photon exchange ( $J^{PC} = 1^{--}$ )**

dominant contribution at smallest momentum transfer  
quasi-real  $\gamma \rightarrow M = \pm 1$

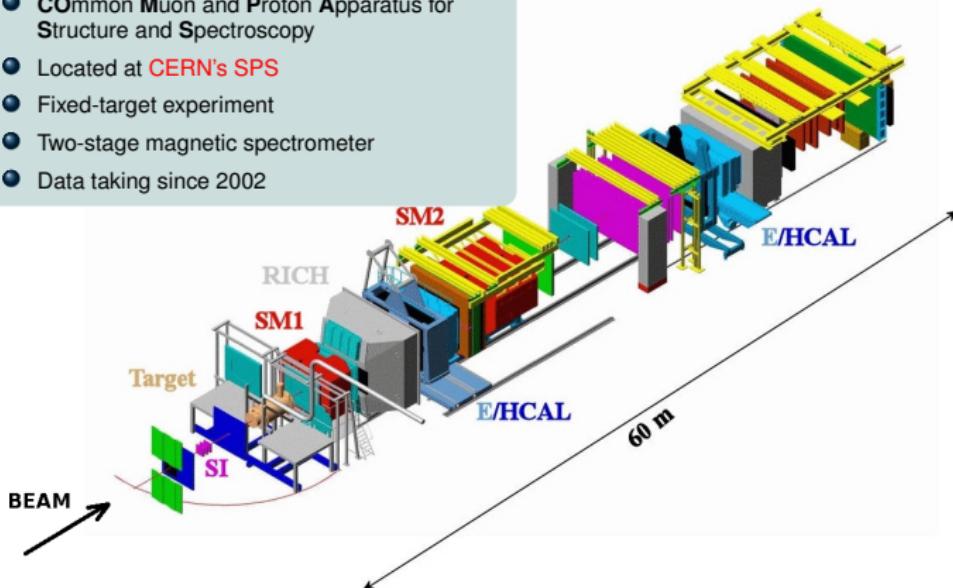


# COMPASS 2004 Pilot Hadron Run

## Experimental Setup

### Overview

- COmmon Muon and Proton Apparatus for Structure and Spectroscopy
- Located at CERN's SPS
- Fixed-target experiment
- Two-stage magnetic spectrometer
- Data taking since 2002



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### Pilot Hadron Run 2004

- 190 GeV  $\pi^-$  beam
- Nuclear targets (Pb)
- Tracking: Silicons for vertexing
- Trigger: Multiplicity trigger, ECAL trigger

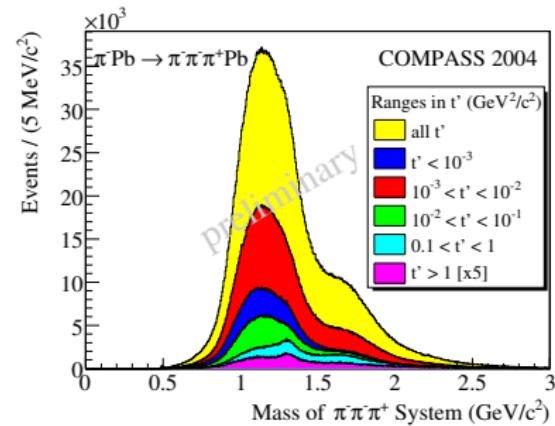
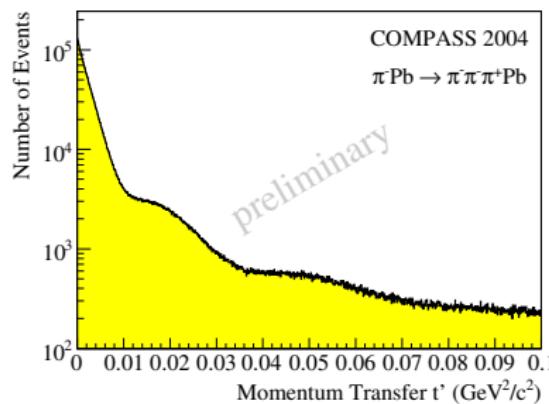
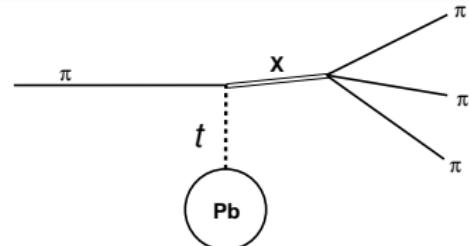

 $\pi^- \text{Pb} \rightarrow \pi^- \pi^- \pi^+ \text{Pb}$  Data

## Pilot Hadron Run 2004

$\approx 4\,000\,000$  exclusive  $3\pi$  events

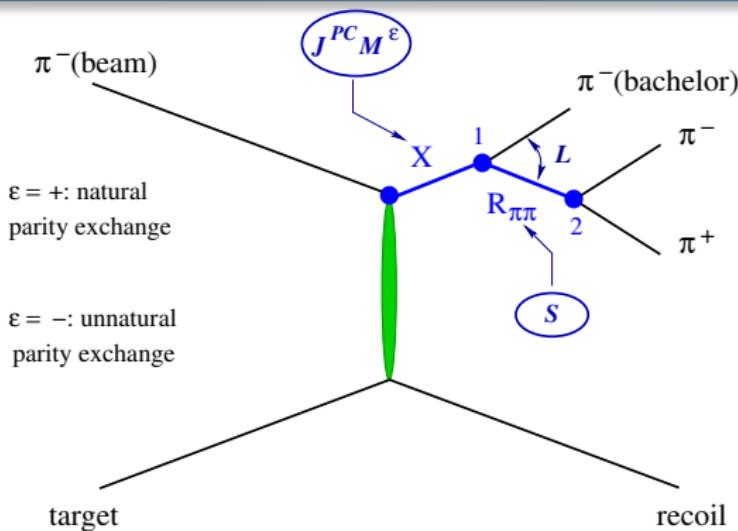
$\approx 1\,000\,000$  with  $t' < 10^{-3} (\text{GeV}/c)^2$

$\approx 2\,000\,000$  with  $10^{-3} < t' < 10^{-2} (\text{GeV}/c)^2$   
 with  $t' = |t| - |t|_{\min} (m_{3\pi})$



# Partial Wave Analysis Formalism

## Isobar Model



- Isobar model: Intermediate 2-particle decays
- Partial wave in reflectivity basis:  $J^{PC} M^\epsilon [isobar] L$

- Mass-independent PWA ( $40 \text{ MeV}/c^2$  mass bins): 38 waves  
Fit of angular dependence of partial waves, interferences
- Acceptance corrections from MC
- (Mass-dependent  $\chi^2$ -fit)



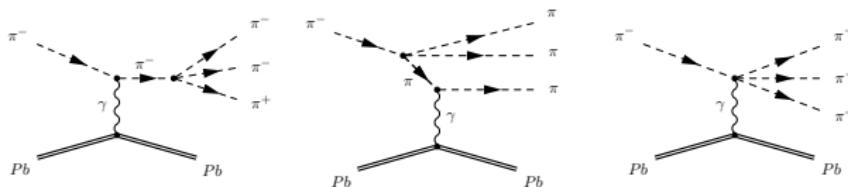
Meson Spectroscopy at Low Momentum Transfer

Diagrams from ChPT

Absolute Cross-Section and ChPT Prediction

# ChPT Amplitude (Leading Order)

- $t' < 10^{-3} (\text{GeV}/c)^2$ : Coulomb production in  $M = 1$  waves ( $\pi\gamma \rightarrow 3\pi$ )
- Low masses: no resonances, governed by pion-scattering  
→ **tree diagrams** from ChPT prediction<sup>1</sup>

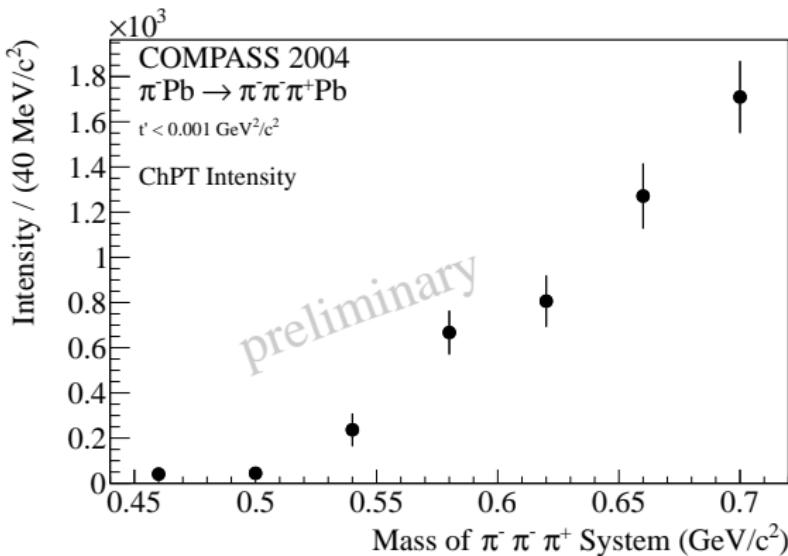


- ChPT amplitude introduced as single partial wave
  - not orthogonal to  $M = 1$  waves in isobar model
  - replaces (up to 6)  $(\pi\pi)_S\pi$  and  $\rho\pi$  waves by a **single amplitude**
  - Fit quality in low mass region compatible
- ChPT valid for  $3m_\pi \approx 0.42 \text{ GeV}/c^2 < m_{3\pi} < 0.7 \text{ GeV}/c^2$
- Higher masses: Isobaric decays

<sup>1</sup>EPJ A36 (2008) 181

# Total Intensity of Chiral Amplitude

Chiral amplitude at low masses equivalent to fit with 6 waves  
**but** much less parameters (i.e. basically intensity)

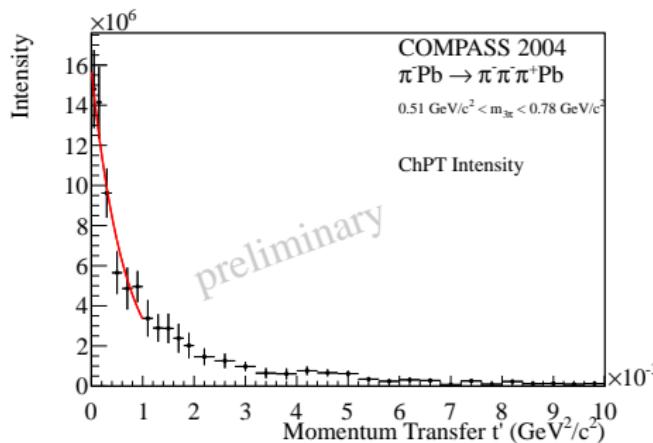


...is this really photon exchange?



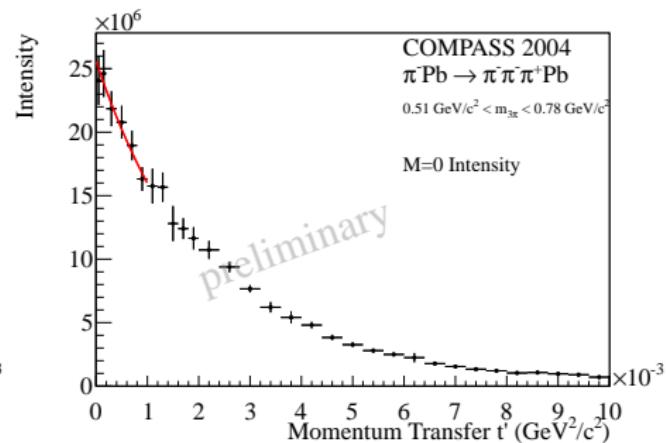
# $t'$ Dependence of Chiral Amplitude

- Fits in small bins of  $t'$ , but only 1 mass bin ( $0.51 < m_{3\pi} < 0.78$  GeV/c $^2$ )
- $t'$  dependence of intensity  $\sigma \propto e^{-bt'}$



$$b_{\text{chiral}} \approx 1560 \text{ (GeV/c)}^{-2}$$

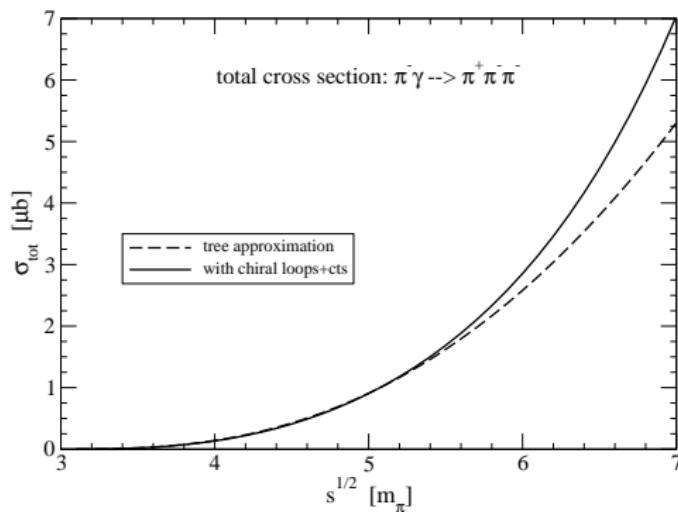
→ Primakoff (Photon)



$$b_{M=0} \approx 400 \text{ (GeV/c)}^{-2}$$

→ diffractive (Pomeron)

- **Goal:** Compare to



[Nucl. Phys. A 848 (2010) 198]

- Experimental determination of absolute cross-section  $\pi\gamma \rightarrow 3\pi$
- **Normalization** via  $K^- \rightarrow \pi^+ \pi^- \pi^-$  decays in target region



Meson Spectroscopy at Low Momentum Transfer

Diagrams from ChPT

Absolute Cross-Section and ChPT Prediction

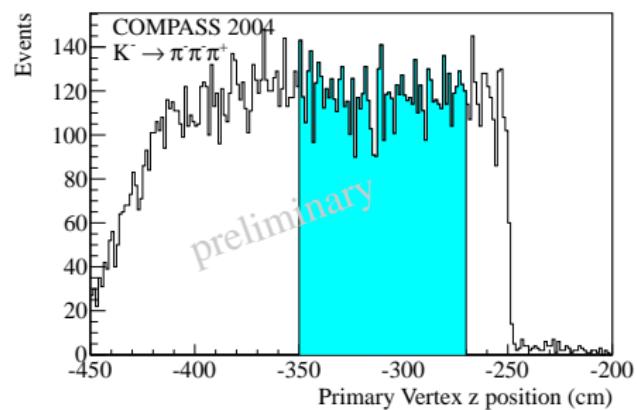
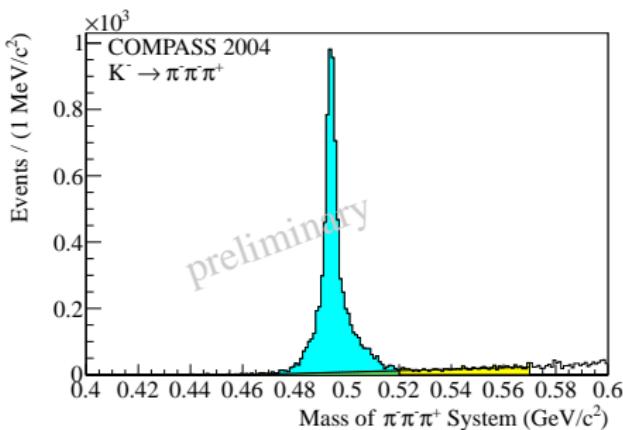
# From Intensity to Cross-Section

Beam  $K^-$  Decays from Data

In each bin of  $m_{3\pi}$ :

$$\text{Number of reactions (events)} = \epsilon_{\text{bin}} \cdot \int \mathcal{L} dt \cdot \sigma_{\text{abs}}$$

COMPASS  $\pi^-$  beam features  $K^-$  component  
 $\rightarrow$  beam flux from  $K^- \rightarrow \pi^+ \pi^- \pi^-$  decays

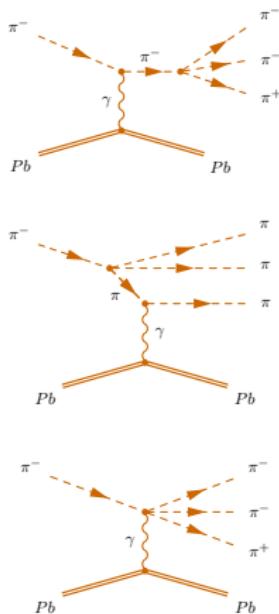


Cross-section of  $\pi^-\gamma \rightarrow \pi^-(q_1)\pi^-(q_2)\pi^+(q_3)$  from ChPT

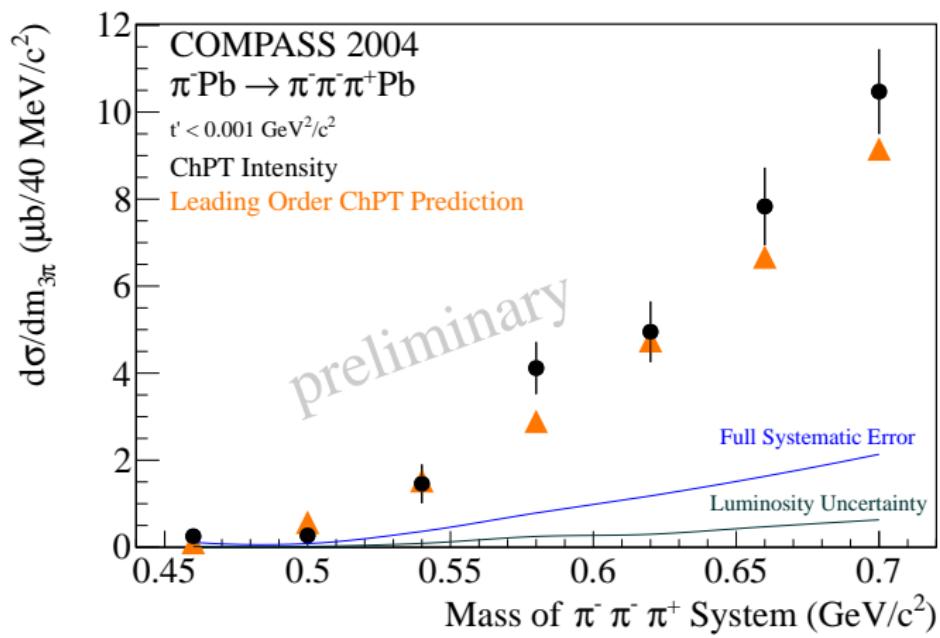
$$\begin{aligned}
 d\sigma_r(s) = & \frac{8\pi\alpha \cdot s}{(s - m_\pi^2)^3 f_\pi^4} \cdot d\Phi_3(p + k, q_1, q_2, q_3) \cdot \\
 & \left[ \frac{\vec{q}_1 \times \hat{k}}{\omega_1 - \vec{q}_1 \cdot \hat{k}} \left( p_0(\sqrt{s} - \omega_2) - \sqrt{s}\omega_1 - \vec{k} \cdot \vec{q}_2 \right) + \right. \\
 & \frac{\vec{q}_2 \times \hat{k}}{\omega_2 - \vec{q}_2 \cdot \hat{k}} \left( p_0(\sqrt{s} - \omega_1) - \sqrt{s}\omega_2 - \vec{k} \cdot \vec{q}_1 \right) + \\
 & \left. \frac{\vec{q}_3 \times \hat{k}}{\omega_3 - \vec{q}_3 \cdot \hat{k}} \left( \sqrt{s}k_0 - p_0\omega_3 - \vec{k} \cdot \vec{q}_3 \right) \right]^2
 \end{aligned}$$

Cross-section of  $\pi^- \text{Pb} \rightarrow \pi^-\pi^-\pi^+ \text{Pb}$

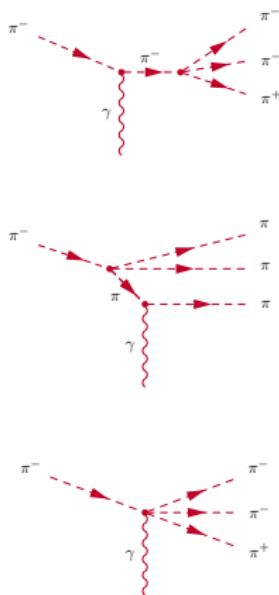
$$\frac{d\sigma}{ds dt'} = \underbrace{\frac{\alpha \cdot Z^2}{\pi(s - m_\pi^2)} \cdot F_{\text{eff}}^2(t') \cdot \frac{t'}{(t' + t_{\min})^2}}_{\text{Weizsäcker-Williams photon density}} \cdot \sigma_r(s)$$



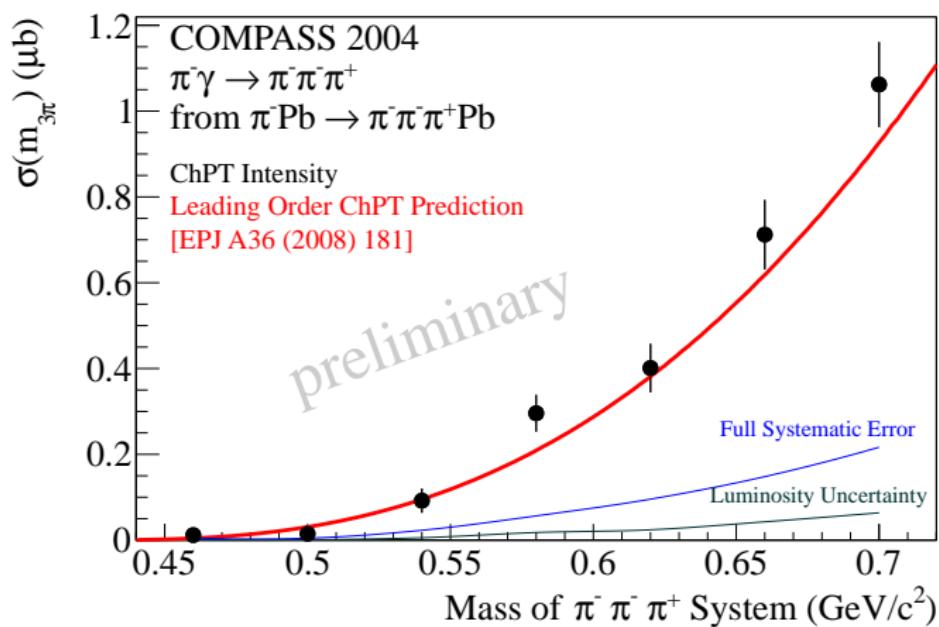
Absolute cross-section of  $\pi^- \text{Pb} \rightarrow \pi^- \pi^- \pi^+ \text{Pb}$   
 induced by  $\gamma$  exchange



# First Measurement of $\pi\gamma \rightarrow 3\pi$ Absolute Cross-Section



Measured absolute cross-section of  $\pi^-\gamma \rightarrow \pi^-\pi^-\pi^+$



- Test of ChPT calculations in low mass region ( $m_{3\pi} < 0.72 \text{ GeV}/c^2$ )
  - Intensity shows good agreement with predicted cross-section (tree diagrams)
  - First Measurement of  $\pi\gamma \rightarrow 3\pi$  absolute cross section in low mass region
- 
- Next-order (chiral loop) contribution
  - Extension to adjacent mass region, including (chiral)  $\rho$  contributions
  - Radiative coupling of resonances, e.g.  $a_2$  (see next talk)
  - COMPASS 2009 data



## BACKUP SLIDES

# BACKUP: Partial Wave Analysis Formalism

- Mass-independent PWA (narrow mass bins):

$$\sigma_{\text{indep}}(\tau, m, t') = \sum_{\epsilon=\pm 1} \sum_{r=1}^{N_r} \left| \sum_i T_{ir}^{\epsilon} f_i^{\epsilon}(t') \psi_i^{\epsilon}(\tau, m) \right|^2 / \sqrt{\int |f_i^{\epsilon}(t')|^2 dt'} \sqrt{\int |\psi_i^{\epsilon}(\tau', m)|^2 d\tau'}$$

- Production strength assumed constant in single bins
  - Decay amplitudes  $\psi_i^{\epsilon}(\tau, m)$ , with  $t'$  dependence  $f_i^{\epsilon}(t')$
  - Production amplitudes  $T_{ir}^{\epsilon}$  → Extended log-likelihood fit
  - Acceptance corrections included
- Spin-density matrix:  $\rho_{ij}^{\epsilon} = \sum_r T_{ir}^{\epsilon} T_{jr}^{\epsilon*}$

→ Physical parameters:

$$\text{Intens}_i^{\epsilon} = \rho_{ii}^{\epsilon}, \\ \text{relative phase } \Phi_{ij}^{\epsilon}$$

$$\text{Coh}_{i,j}^{\epsilon} = \sqrt{(\text{Re } \rho_{ij}^{\epsilon})^2 + (\text{Im } \rho_{ij}^{\epsilon})^2} / \sqrt{\rho_{ii}^{\epsilon} \rho_{jj}^{\epsilon}}$$

- Mass-dependent  $\chi^2$ -fit (not presented here):

- $X$  parameterized by Breit-Wigner (BW) functions
- Background can be added

BACKUP: Chiral amplitude replacing other waves ( $M=1$ )

Waveset in low mass region

Comparison of two **fitting models** (for  $m_{3\pi} < 0.7 \text{ GeV}/c^2$ ):

- Several  $M = 0$  waves
- 

- $1^{++} 1^\pm \rho \pi S$
- $1^{++} 1^\pm (\pi\pi)_S \pi P$
- $1^{-+} 1^\pm \rho \pi P$
- $2^{++} 1^\pm \rho \pi D$
- $2^{-+} 1^\pm \rho \pi P$
- $2^{-+} 1^\pm (\pi\pi)_S \pi D$

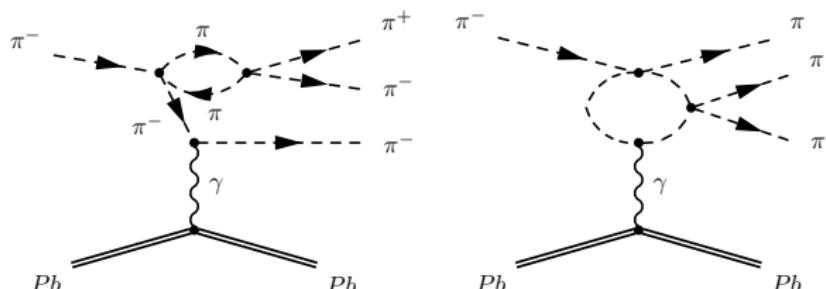
- Several  $M = 0$  waves
- 

- **ChPT amplitude**

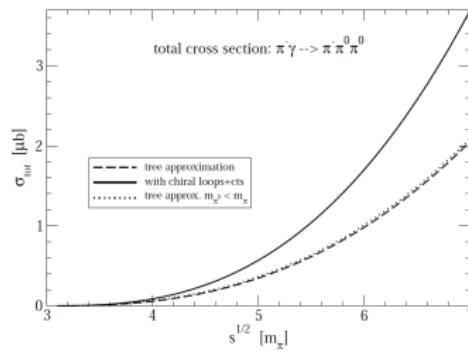
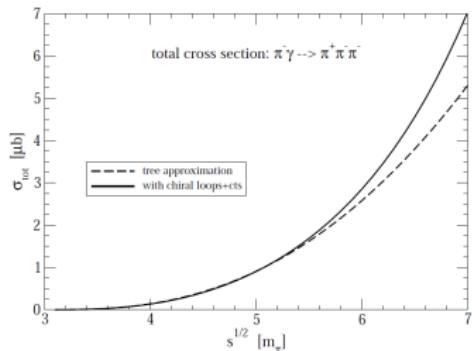
Both reflectivities are taken into account

- Fit without chiral amplitude, but “other” waves starting at  $0.5 \text{ GeV}/c^2$
- Chiral amplitude starting at threshold (i.e.  $0.5 \text{ GeV}/c^2$ )  
 $M=1 (\pi\pi)_S \pi$  and  $\rho \pi$  waves  
thresholded as indicated

- Loop diagrams<sup>2</sup>, e.g.



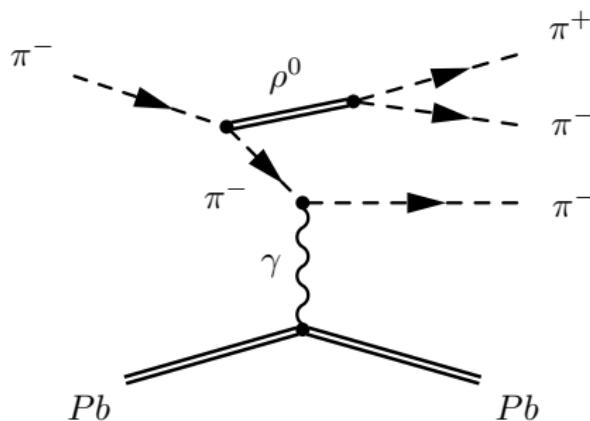
- Prediction of total cross-section ( $\pi^- \pi^- \pi^+$  vs.  $\pi^- \pi^0 \pi^0$ ):



<sup>2</sup>N. Kaiser, Nucl. Phys. A 848 (2010) 198

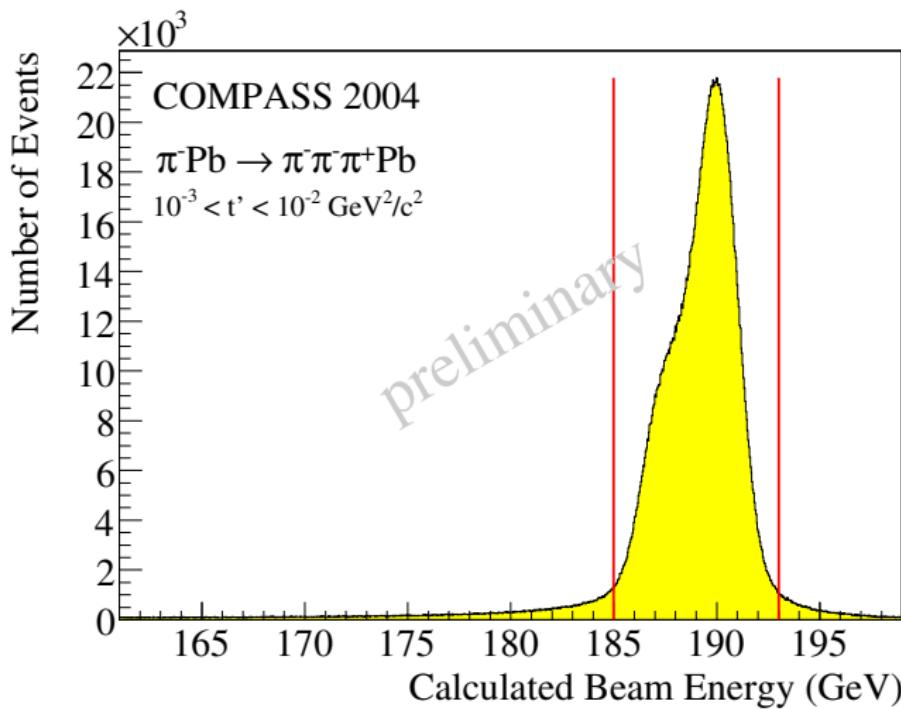
Contributions from  $\rho$  exchange

- explicit contribution from the  $\rho$  ( $m_{3\pi} > 0.7 \text{ GeV}/c^2$ ), e.g.



BACKUP:  $3\pi$  Data Sample (2004)

Reconstructed beam energy distribution

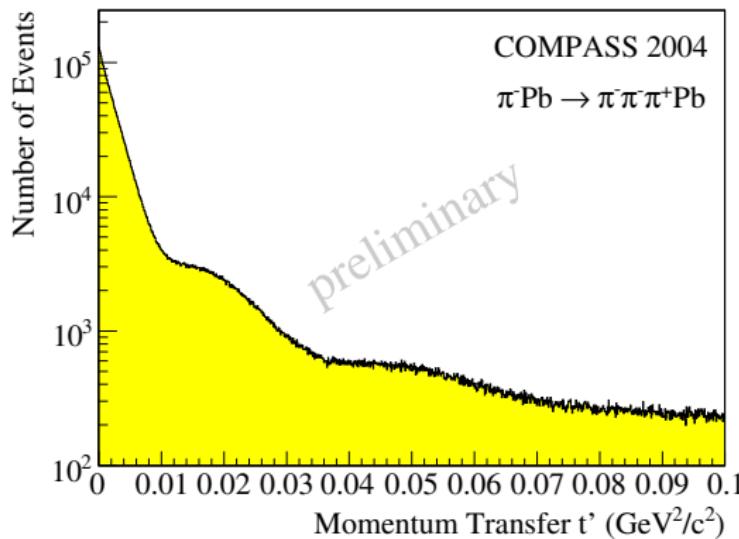


# BACKUP: $3\pi$ Data Sample (2004)

## Momentum Transfer

Momentum transfer to target:

$$-t = -(p_{\text{beam}} - p_{(\pi^-\pi^-\pi^+)})^2 \Rightarrow t' = |t| - |t|_{\min}$$

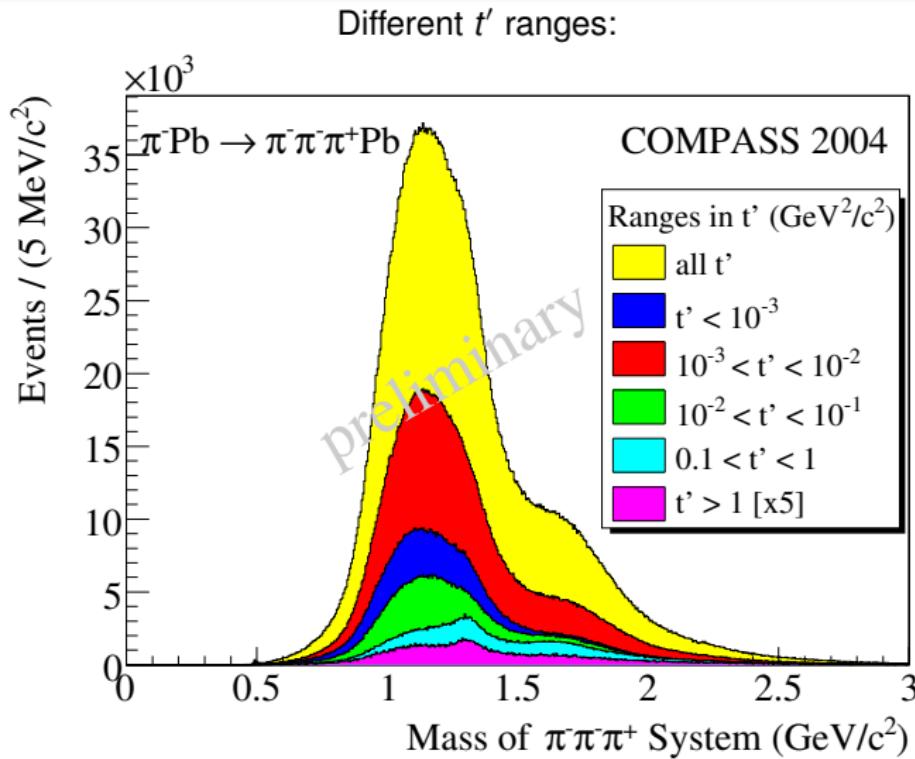


with

$$|t|_{\min} = \frac{(m_{3\pi}^2 - m_\pi^2)^2}{4|\vec{p}_{\text{beam}}|_{\text{lab}}^2}$$

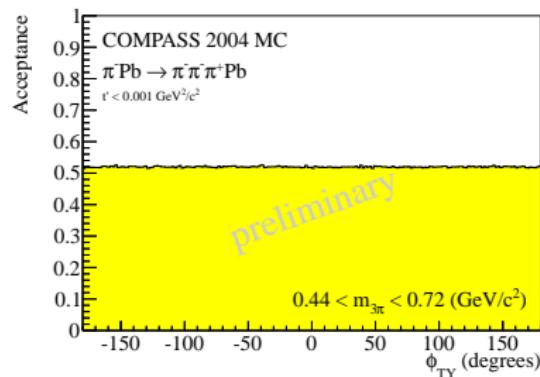
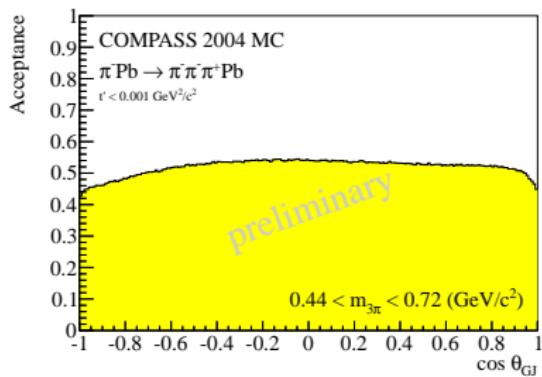
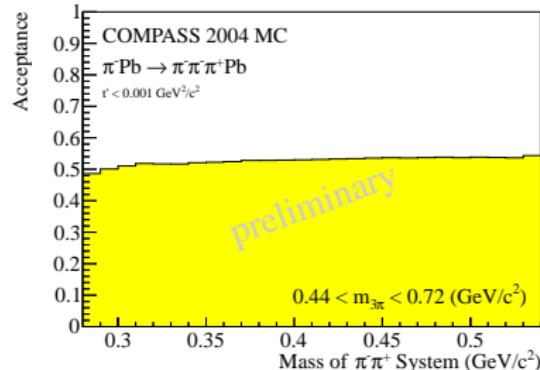
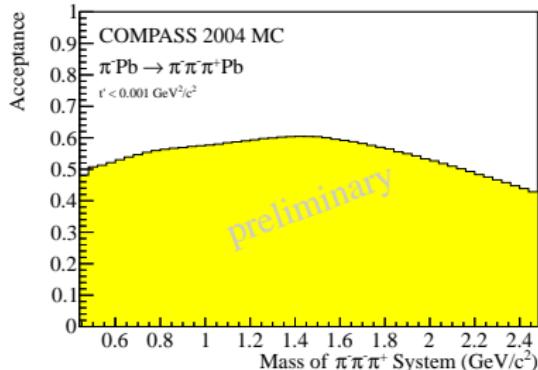
**Diffraction pattern:**  
 Pb nucleus acts like  
**"black disc"** in optics

- "Low  $t'$ ":  $10^{-3} (\text{GeV}/c)^2 < t' < 10^{-2} (\text{GeV}/c)^2$   $\sim 2\,000\,000$  events
- "Primakoff region":  $t' < 10^{-3} (\text{GeV}/c)^2$   $\sim 1\,000\,000$  events

BACKUP:  $3\pi$  Data Sample (2004) $\pi^- \pi^- \pi^+$  mass distribution



# Acceptance of $\pi^- \pi^- \pi^+$ events ( $t' < 0.001 \text{ GeV}/c^2$ )



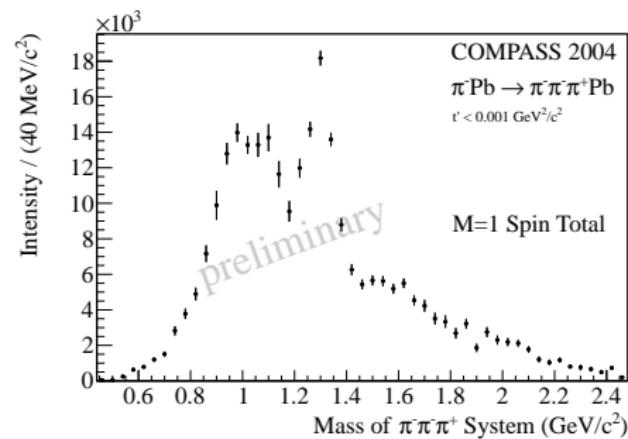
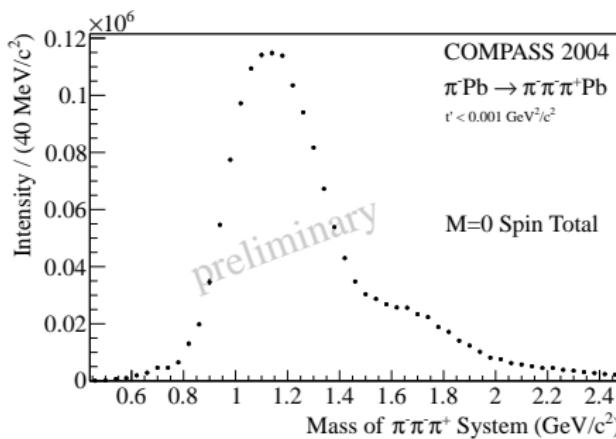
# BACKUP: Spin Totals for $t' < 10^{-3} (\text{GeV}/c)^2$

$t'$  dependent amplitudes:

Primakoff production: **M=1**:  $e^{-b_{\text{prim}}t'} \rightarrow$  arises for  $t' \approx 0$

Diffractive production: **M=0**:  $e^{-b(m)t'}$

**M=1**:  $t' e^{-b(m)t'} \rightarrow$  vanishes for  $t' \approx 0$



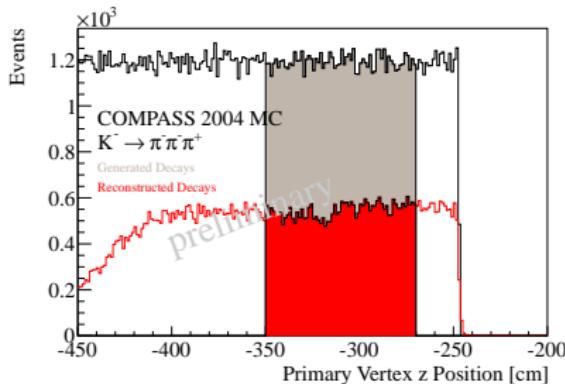
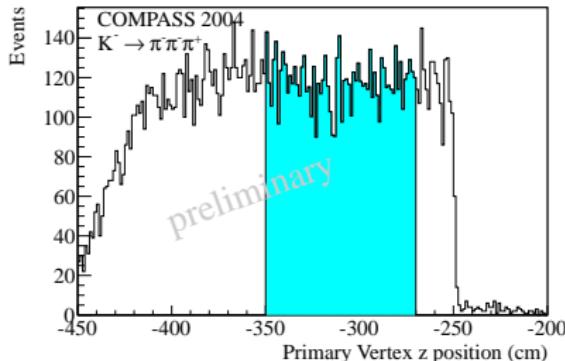
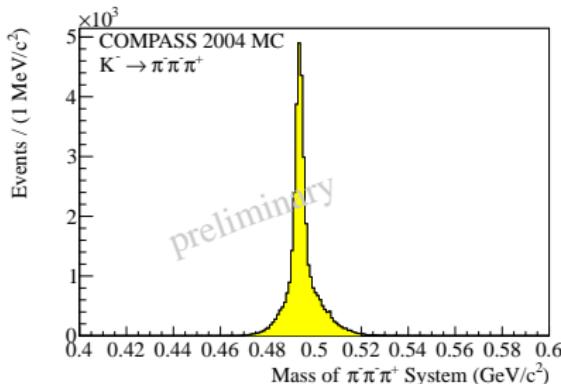
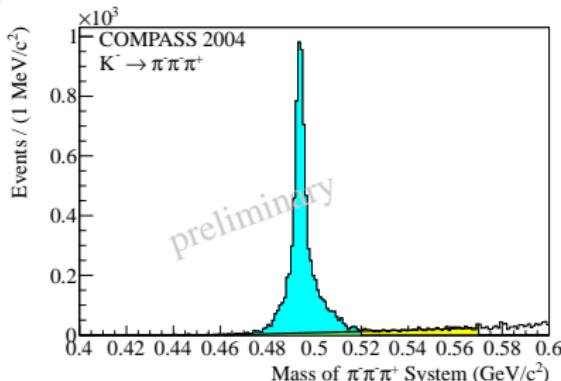


## BACKUP: Kaon mass resolution and vertex distribution



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Data vs. MC



# BACKUP: From intensity to cross-section

In each bin of  $m_{3\pi}$ :

$$\text{Number of reactions (events)} = \epsilon_{bin(3\pi)} \cdot \int \mathcal{L} dt \cdot \sigma_{abs}$$

COMPASS  $\pi^-$  beam is contaminated with  $K^-$

→ beam flux from  $K^-$  decays!

## Calculation of Integrated Flux

$$\epsilon \cdot \int \Phi dt = \frac{N(K^- \text{ decays})}{(1 - e^{-s/\beta\gamma\tau}) \cdot BR \cdot \left(\frac{K^-}{\pi^- K^- \bar{p}}\right)}$$

$\epsilon$  acceptance (from MC of kaon decay)

$\int \Phi dt$  integrated beam flux

$s$  length of decay volume

$\beta\gamma\tau$ :  $K^-$  decay length (1429 m at 190 GeV)

$BR$ :  $K^- \rightarrow \pi^+ \pi^- \pi^-$  branching ratio (5.59%)

$\frac{K^-}{\pi^- K^- \bar{p}}$ :  $K^-$  fraction at COMPASS target (2.43%, estimated relative uncertainty 5%)