The Goal

Structure and dynamics of hadrons
⇒ non-perturbative regime of QCD

Theory:
• Models: QM, bag, flux tube, ...
• Effective theories: \( \chi \)PT, ...
• Lattice-QCD

Q²

Hard processes:
⇒ Nucleon structure
  • Helicity
  • Transversity
  • GPDs

Spectroscopy:
⇒ Hadron mass spectrum
  ⇒ Gluonic excitations
⇒ Multi-quark systems

Scattering at very low Q²:
⇒ Polarizabilities of \( \pi \), K
⇒ Chiral anomaly: \( F_{3\pi} \)
COMPASS at CERN

COmmon Muon and Proton Apparatus for Structure and Spectroscopy

- p up to 400 GeV
- secondary hadrons ($\pi$, K, ...): $2 \times 10^7$/s
- tertiary $\mu$ (polarized): $4 \times 10^7$/s
The COMPASS Experiment

Two-stage spectrometer
- large angular acceptance
- broad kinematical range
- ~250000 channels
- up to 580 TB/year

Dipole magnets
Tracking detectors
RICH
El.-mag. calorimeter
Hadronic calorimeter
Muon identification

[hep-ex/0703049, NIM A 577, 455 (2007)]
The COMPASS Experiment

Two-stage spectrometer
• large angular acceptance
• broad kinematical range
• ~250000 channels
• up to 580 TB/year

Data taking periods:
• 2002-2004: 160 GeV/c $\mu^+$
• 2004: 2 weeks 190 GeV/c $\pi^-$
• 2006-2007: 160 GeV/c $\mu^+$

[hep-ex/0703049, NIM A 577, 455 (2007)]
Two-stage spectrometer
- large angular acceptance
- broad kinematical range
- ~250000 channels
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Data taking periods:
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- 2004: 2 weeks 190 GeV/c $\pi^-$
- 2006-2007: 160 GeV/c $\mu^+$
The COMPASS Experiment

Beam: 190 GeV/c $\pi^-$, $10^6$/s
Targets: Pb, Cu, C
Describe response to external e.m. fields ⇒ stiffness of system

- electric polarizability \[ d = \alpha \vec{E} \]
- magnetic polarizability \[ \mu = \beta \vec{H} \]

\[ \Delta V \sim 10 \text{ MV} \]

<table>
<thead>
<tr>
<th>( \chi^\text{PT} )</th>
<th>( \alpha_\pi (10^{-4} \text{ fm}^3) )</th>
<th>( \alpha_\pi + \beta_\pi (10^{-4} \text{ fm}^3) )</th>
<th>( \alpha_\pi - \beta_\pi (10^{-4} \text{ fm}^3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buergi, Gasser</td>
<td>0.2±0.1</td>
<td>5.7±1.0</td>
<td></td>
</tr>
<tr>
<td>( \gamma p \rightarrow \gamma \pi^+ n )</td>
<td>20±12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lebedev (86)</td>
<td></td>
<td>11.6±1.5±3.0±0.5</td>
<td></td>
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<tr>
<td>MAMI A2 (05)</td>
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<tr>
<td>( \gamma \gamma \rightarrow \pi^+ \pi^- )</td>
<td>19.1±4.8±5.7</td>
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<tr>
<td>PLUTO (84)</td>
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<tr>
<td>DM1 (86)</td>
<td>17.2±4.6</td>
<td></td>
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<tr>
<td>DM2 (86)</td>
<td>26.3±7.4</td>
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</tr>
<tr>
<td>Mark II (92)</td>
<td>2.2±1.6</td>
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<td></td>
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<tr>
<td>( \pi Z \rightarrow \gamma \pi Z )</td>
<td>6.8±1.4±1.2</td>
<td>1.4±3.1±2.8</td>
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</tr>
<tr>
<td>Serpukhov (85)</td>
<td></td>
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</tbody>
</table>
Measurement of Polarizabilities

- Measurable in **Compton** scattering
- **Primakoff** reaction: inverse kinematics
- Assume \( \overline{\alpha}_\pi + \overline{\beta}_\pi = 0 \)
- Cross section

\[
\frac{d\sigma_{\gamma\pi}^{\text{Prim}}}{d\omega} = \frac{d\sigma_{\gamma\pi}^{\text{Thomson}}}{d\omega} + 4Z^2 \alpha^2 m_\pi \frac{\omega}{E_{\text{beam}}} \overline{\beta}_\pi \left( \ln \frac{Q_{\text{max}}^2}{Q_{\text{min}}^2} - 3 + 4 \sqrt{\frac{Q_{\text{min}}^2}{Q_{\text{max}}^2}} \right)
\]

\( \omega = \frac{E_\gamma}{E_{\text{beam}}}, \)

\[
Q_{\text{min}} = \frac{m_\pi^2}{2E_{\text{beam}}} \frac{\omega}{1 - \omega}
\]

minimum Q to produce real photon

\[
Q_{\text{max}}^2 = 0.0075 \text{ (GeV/c)}^2
\]

defined by analysis cut

Hadron Physics at COMPASS
B. Ketzer
Cross Section Ratios

\[ R(\omega) = \frac{N_{\text{exp}}(\omega)}{N_{\text{MC}}(\omega)} = \frac{d\sigma_{\gamma \pi}^{\text{Prim}}}{d\sigma_{\gamma \pi}^{\text{Thomson}}} \approx 1 + \frac{3}{2} \frac{m_{\pi}^3}{\alpha} \frac{\omega^2}{1 - \omega} \beta_{\pi} \]

Status of analysis:

- Improve calibration of detectors (Si, ECAL)
- Improve MC description of setup
- Radiative corrections
  - $\pi^-$ Compton
  - Pb nuclear form factor
- Suppress $\pi^- \pi^0$ background

waiting for final result...
Meson Spectroscopy

**Quark model:** bound state of $q\bar{q}$

Quantum numbers: $I^G (J^{PC})$

$$P=(-1)^{l+1}, \quad C=(-1)^{l+s}, \quad G=(-1)^{I+l+s}$$

- $(q\bar{q})_0$
- $(q\bar{q})(q\bar{q})$
- $(q\bar{q})_8 g$
- $gg$

**QCD:** other color-neutral configurations

with same quantum numbers $\Rightarrow$ mixing

- Decoupling only possible for
  - narrow states
  - vanishing leading $q\bar{q}$ term

$\Rightarrow$ exotic $J^{PC}: 0^{--}, 0^{+-}, 1^{+-}, 2^{++}, ...$

Hadron Physics at COMPASS

B. Ketzer
Glueballs

Quenched L-QCD prediction

Lightest glueballs:
- $M \sim 1.7 \text{ GeV}/c^2$ ($J^{PC} = 0^{++}$)
- $M \sim 2.4 \text{ GeV}/c^2$ ($J^{PC} = 2^{++}$)

Experimental candidates:
- $f_0(1500)$ (Crystal Barrel, WA102)
  - $J^{PC}=0^{++}$ \Rightarrow mixing with isoscalar mesons!
  - \Rightarrow COMPASS

Higher masses:
- exotic: $J^{PC} = 2^{-+}, 0^{+-}$
- $M \sim 4.3 \text{ GeV}/c^2$
  - \Rightarrow PANDA
  - (talks by S. Lange et al.)
Hybrids

Light meson sector exotics $J^{PC}=1^{-+}$:

- $\pi_1(1400)$ (VES, E852, Crystal Barrel)
- $\pi_1(1600)$ (E852, VES)

still controversial...

⇒ COMPASS
Hybrids

**Light meson sector** (< 2.2 GeV/c²):
- \( \pi_1(1400) \) (VES, E852, Crystal Barrel)
- \( \pi_1(1600) \) (E852, VES)

still controversial...

\( \Rightarrow \) COMPASS

**Charmonium region**: less populated
- \((\bar{c}c)g\) ground state

predicted at 4.3 – 4.4 GeV/c²

exotic: \( J^{PC}=1^{-+} \)

narrow: 5 – 50 MeV/c²?

\( \Rightarrow \) PANDA
**π_1(1600) – Positive Results in 3π**

**BNL E852:** \( \pi^- + p \rightarrow \pi^+ \pi^- \pi^- + p \)
- \( p_\pi = 18 \text{ GeV/c} \)
- limited statistics: 250k ev.
- rank 2
- mass dependent fit

\[
\begin{align*}
\text{VES: } & \quad \pi^- + A \rightarrow \pi^+ \pi^- \pi^- + A \\
& \quad p_\pi = 37 \text{ GeV/c} \\
& \quad \text{full coherence}
\end{align*}
\]


[Y. Khokhlov, Nucl. Phys. A 663, 596c (2000)]
π₁(1600) – Negative Results in 3π

**BNL E852:** π⁻ + p → π⁺π⁻π⁻ + p

- pπ = 18 GeV/c
- full statistics: 2.6M ev.
- rank 1
- extended wave set (2+ waves)
- no mass dependent fit

**VES:** π⁻ + A → π⁺π⁻π⁻ + A

- pπ = 37 GeV/c
- unlimited rank


Meson Spectroscopy at COMPASS

Two production mechanisms studied in parallel using proton, pion and kaon projectiles

Central production

- Large rapidity gap between $p_{\text{slow}}$, $h_{\text{fast}}$, $X$
- Beam particle loses $\sim$10% of its energy
- Particles at large angles from $X$ decays
- Possible source of glueballs

Diffractive scattering

- Forward kinematics
- Large cross section ($\sim$mb)
- Need to separate particles at very small angles
- Study of $J^{PC}$-exotic mesons
Diffractive Reactions in COMPASS

Example: $\pi^- + \text{Pb} \rightarrow \pi^- \pi^- \pi^+ + \text{Pb}$
- $4\pi$ vertex in Pb target
- Exclusivity $\Rightarrow$ target stays intact
- Momentum transfer

$$-t \equiv Q^2 = -(p_a - p_c)^2$$

Diffraction on Pb nuclei

Diffraction on nucleons

COMPASS 2004
$\pi\text{Pb} \rightarrow \pi\pi\pi^+\text{Pb}$
Invariant Mass of $3\pi$ System

**COMPASS:** $p_\pi = 190$ GeV/c
- No acceptance correction
- 4M events in 3 days (full $t$ range)
- 450k events in $0.1 < t' < 1.0$ GeV/c$^2$

**BNL852:** $p_\pi = 18$ GeV/c
- PRD 65, 072001 (2002):
  - 250k events $\Rightarrow \pi_1(1600)$
PWA Technique

• t-channel Reggeon exchange
• Reflectivity basis in G-J frame
• At high $s$: $\varepsilon = \eta$ of Regge trajectory
• Isobar model

1. Mass-independent PWA of angular distributions in 40 MeV mass bins

$$\sigma_{\text{indep}}(\tau) = \frac{1}{N_r} \sum_{\varepsilon=-1}^{N_r} \sum_{r=1}^{N_r} \left| \sum_i T_{ir}^{\varepsilon} \psi_i^{\varepsilon}(\tau) \right|^2 \sqrt{\int \psi_i^{\varepsilon}(\tau')^2 d\tau'}^2$$

• 42 partial waves $i=J^{PC}M^\varepsilon[...L$

$[...]=$isobar $(\pi\pi)_S$, $f_0(980)$, $\rho(770)$, $f_2(1270)$, $\rho_3(1690)$
PWA Technique

- t-channel Reggeon exchange
- Reflectivity basis in G-J frame
- At high $s$: $\varepsilon = \eta$ of Regge trajectory
- Isobar model

1. **Mass-independent PWA** of angular distributions in 40 MeV mass bins

2. **Mass-dependent $\chi^2$ fit** to results of step 1
   - 7 waves
   - Parameterized by BW
   - Coherent background for some waves
\( \pi_1(1600) \) in COMPASS

- BW parameters for \( \pi_1(1600) \)

\[
M = \left( 1.660 \pm 0.010 ^{+0.000} \_{-0.064} \right) \text{GeV/c}^2
\]

\[
\Gamma = \left( 0.269 \pm 0.021 ^{+0.042} \_{-0.064} \right) \text{GeV/c}^2
\]

- Leakage negligible
B. Ketzer

**π₁(1600) in COMPASS**

- BW parameters for π₁(1600)

\[
M = \left(1.660 \pm 0.010^{+0.000}_{-0.064}\right) \text{GeV}/c^2
\]

\[
\Gamma = \left(0.269 \pm 0.021^{+0.042}_{-0.064}\right) \text{GeV}/c^2
\]

- Leakage negligible
### Summary of Waves

<table>
<thead>
<tr>
<th>State</th>
<th>(GeV)</th>
<th>COMPASS $\pm$ stat $\pm$ syst</th>
<th>PDG</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1(1260)$</td>
<td>$M$</td>
<td>$1.256 \pm 0.006 + 0.007 - 0.017$</td>
<td>$1.230 \pm 0.040$</td>
</tr>
<tr>
<td></td>
<td>$\Gamma$</td>
<td>$0.366 \pm 0.009 + 0.028 - 0.025$</td>
<td>$0.250$ to $0.600$</td>
</tr>
<tr>
<td>$a_2(1320)$</td>
<td>$M$</td>
<td>$1.321 \pm 0.001 + 0.000 - 0.007$</td>
<td>$1.3183 \pm 0.0006$</td>
</tr>
<tr>
<td></td>
<td>$\Gamma$</td>
<td>$0.110 \pm 0.002 + 0.002 - 0.015$</td>
<td>$0.107 \pm 0.005$</td>
</tr>
<tr>
<td>$\pi_1(1600)$</td>
<td>$M$</td>
<td>$1.660 \pm 0.010 + 0.000 - 0.064$</td>
<td>$1.653^{+0.018}_{-0.015}$</td>
</tr>
<tr>
<td></td>
<td>$\Gamma$</td>
<td>$0.269 \pm 0.021 + 0.042 - 0.064$</td>
<td>$0.225^{+0.045}_{-0.028}$</td>
</tr>
<tr>
<td>$\pi_2(1670)$</td>
<td>$M$</td>
<td>$1.659 \pm 0.003 + 0.024 - 0.008$</td>
<td>$1.6724 \pm 0.0032$</td>
</tr>
<tr>
<td></td>
<td>$\Gamma$</td>
<td>$0.271 \pm 0.009 + 0.022 - 0.024$</td>
<td>$0.259 \pm 0.009$</td>
</tr>
<tr>
<td>$\pi(1800)$</td>
<td>$M$</td>
<td>$1.785 \pm 0.009 + 0.012 - 0.006$</td>
<td>$1.812 \pm 0.014$</td>
</tr>
<tr>
<td></td>
<td>$\Gamma$</td>
<td>$0.208 \pm 0.022 + 0.021 - 0.037$</td>
<td>$0.207 \pm 0.013$</td>
</tr>
<tr>
<td>$a_4(2040)$</td>
<td>$M$</td>
<td>$1.884 \pm 0.013 + 0.050 - 0.002$</td>
<td>$2.001 \pm 0.010$</td>
</tr>
<tr>
<td></td>
<td>$\Gamma$</td>
<td>$0.295 \pm 0.024 + 0.046 - 0.019$</td>
<td>$0.313 \pm 0.031$</td>
</tr>
</tbody>
</table>

**Details: talk of Q. Weitzel, Sat 1C**
COMPASS in 2008

Hadron Physics at COMPASS

B. Ketzer
COMPASS in 2008

Hadron Physics at COMPASS

B. Ketzer
• 190 GeV/c hadron beam
  • positive: 75% p, 25% π+
  • negative: 96% π, 3.5% K-, 0.5% ⌹
• Beam intensity $5 \cdot 10^6$/s, $5 \cdot 10^7$/spill (10s)
• Target: 40cm liquid hydrogen
• $\mathcal{L}=0.15$ pb$^{-1}$/day
• Expected event rates:
  • $\pi^- p \to \pi_1(1600) p, \pi_1(1600) \to \pi^+ \pi^- \pi^-$
  • $\sigma(\pi_1) \times \text{BR}(\pi_1 \to \pi^+ \pi^- \pi^-)=0.6 \mu$b
  • $\varepsilon(\text{DAQ}) \times \varepsilon(\text{reco})=5\%$
  • $\mathcal{E}(\text{DAQ}) \times \mathcal{E}(\text{reco})=2\%$
  • $\pi^- p \to \pi^- f_0(1500) p, f_0(1500) \to \eta \eta \to 4\gamma$
  • $\sigma(f_0) \times \text{BR}(f_0 \to \eta \eta \to 4\gamma)=1.5 \mu b \cdot 5\% \cdot 15\%=11\text{nb}$
  • $\mathcal{E}(\text{DAQ}) \times \mathcal{E}(\text{reco})=2\%$

$\Rightarrow 4.5 \cdot 10^3 \text{ ev./day}$

$\Rightarrow 30 \text{ ev./day}$
Conclusions

- **COMPASS**: data taking with $\mu$ beam: 2002-2004, 2006-2007
- Pilot run with $\pi$ beam: 2004 *(3 days)*
  - **Coulomb interactions** being studied
    - Pion polarizability
    - $F_{3\pi}$ being studied

**Stay tuned for new results from COMPASS...**

- PWA for $\pi^+\pi^-\pi^-$ at high $t$ completed:
  - Statistics 2×higher than BNL E852 in 1 year
  - Strong signal in *exotic* $1^{-+}$ wave at 1.6 GeV/c$^2$
- PWA for $\pi^+\pi^+\pi^-\pi^-\pi^-$ started

- **Hadron beam** running from 2008 on with IH2 target
  - Central production: 10× WA102 statistics in 60 days
  - Diffractive reactions: 10× BNL E852 statistics in 10 days
Describe response to external e.m. fields ⇒ stiffness of system

- electric polarizability \( \vec{d} = \alpha \vec{E} \)
- magnetic polarizability \( \vec{\mu} = \beta \vec{H} \)

\[ \chi_{\text{PT (2-loop):}} \]
\[ \bar{\alpha}_\pi = (2.9 \pm 0.5) \cdot 10^{-4} \text{ fm}^3 \]
\[ \bar{\beta}_\pi = (-2.8 \pm 0.5) \cdot 10^{-4} \text{ fm}^3 \]

\[ \Delta V \sim 10 \text{ MV} \]

Experiments:
- PLUTO
- DM1
- DM2
- Mark II
- Lebedev
- Mami A2
- Serpukhov
- COMPASS
$Q^2$ Distributions

$$\frac{d^3 \sigma}{d Q^2 \, d \omega \, d \cos \theta} = \frac{\alpha Z^2}{\pi \omega} \cdot \frac{Q^2 - Q_{\text{min}}^2}{Q^4} \cdot \left| F_Z(Q^2) \right|^2 \cdot \frac{d \sigma_{\gamma \pi}(\omega, \theta)}{d \cos \theta}$$
Analysis Procedure

Selection of Primakoff events:
• One primary vertex in target
• One outgoing track with neg. charge and p<170 GeV/c
• One photon cluster in ECAL2 with $E_\gamma>7$ GeV
• Exclusivity: $|E_\pi+E_\gamma-E_{\text{beam}}|<25$ GeV
• $Q^2<0.0075$ GeV$^2$/c$^2$

Background suppression:
• $\rho$ decay: $M_{\pi\gamma}<3.75$ m$_\pi$
• Beam K decay: empty target subtraction
• Diffractive channels: different $Q^2$ dependence

Extraction of polarizability:
• Assume $\alpha_\pi+\beta_\pi=0$
• Fit ratio $R(\omega)=1+d\sigma(\beta_\pi)/d\sigma_{\text{pointlike}}$
• $d\sigma_{\text{pointlike}}$ from Monte-Carlo, cross-checked with $\mu$ beam
Radiative Corrections

\[ \frac{\sigma_{\text{Born}} + \sigma_{\text{corr}}}{\sigma_{\text{Born}}} \]

- Vac. pol.
- Compton
- Mult. exch.
- Screening
- SUM

\[ \Pi \]

Preliminary

\[ 0.9 \quad 0.92 \quad 0.94 \quad 0.96 \quad 0.98 \quad 1 \quad 1.02 \quad 1.04 \]

\[ 0.5 \quad 0.55 \quad 0.6 \quad 0.65 \quad 0.7 \quad 0.75 \quad 0.8 \quad 0.85 \quad 0.9 \]

Hadron Physics at COMPASS
## Systematic Error Estimate

<table>
<thead>
<tr>
<th>Description</th>
<th>Error, $10^{-4}$fm$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Setup description in MC ($\mu$ data)</td>
<td>±0.5</td>
</tr>
<tr>
<td>Diffractive and empty target background subtraction</td>
<td>±0.3</td>
</tr>
<tr>
<td>Muons background</td>
<td>+0.2</td>
</tr>
<tr>
<td>Electrons background</td>
<td>&lt;+0.1</td>
</tr>
<tr>
<td>SYSTEMATIC TOTAL</td>
<td>±0.6</td>
</tr>
</tbody>
</table>
**F^3π Coupling Constant**

**Primakoff π⁰ production**

- **Prediction:**
  \[ F^{3\pi}(0) = 9.7 \pm 0.1 \text{ GeV}^{-3} \]

- **Experiment (Serpukhov):**
  \[ F^{3\pi}(0) = 12.9 \pm 0.9 \pm 0.5 \text{ GeV}^{-3} \]

⇒ COMPASS

**Chiral Perturbation Theory**

\[ F^{3\pi}(0) = \frac{F^\pi(0)}{\sqrt{\pi \alpha f_{\pi^\pm}^2}}, \quad f_{\pi^\pm} = (130.7 \pm 0.4) \text{ MeV} \]
### Light Exotics

<table>
<thead>
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<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>E852</td>
<td>$\pi_1(1400)$</td>
<td>1$^{++}$</td>
<td>1359 ±16 +10 $-14$</td>
<td>314 $+31$ $-29$</td>
<td>$\eta\pi$</td>
</tr>
<tr>
<td>Crystal Barrel</td>
<td>$\pi_1(1400)$</td>
<td>1$^{-+}$</td>
<td>1400 ±20±20</td>
<td>310 ±50 $+50$ $-30$</td>
<td>$\eta\pi$</td>
</tr>
<tr>
<td>Crystal Barrel</td>
<td>$\pi_1(1400)$</td>
<td>1$^{+-}$</td>
<td>1360 ±25</td>
<td>220 ±90</td>
<td>$\eta\pi$</td>
</tr>
<tr>
<td>Obelix</td>
<td>$\pi_1(1400)$</td>
<td>1$^{--}$</td>
<td>1384 ±28</td>
<td>378 ±58</td>
<td>$\rho\pi$</td>
</tr>
<tr>
<td>E852</td>
<td>$\pi_1(1600)$</td>
<td>1$^{--}$</td>
<td>1593 ±8 $+20$ $-47$</td>
<td>168 ±20 ±150 $-12$</td>
<td>$\rho\pi$</td>
</tr>
<tr>
<td>E852</td>
<td>$\pi_1(1600)$</td>
<td>1$^{-+}$</td>
<td>1597 ±10 $+45$ $-10$</td>
<td>340 ±40±50</td>
<td>$\eta'\pi$</td>
</tr>
<tr>
<td>Crystal Barrel</td>
<td>$\pi_1(1600)$</td>
<td>1$^{+-}$</td>
<td>1590 ±50</td>
<td>280 ±75</td>
<td>$b_1\pi$</td>
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<tr>
<td>Crystal Barrel</td>
<td>$\pi_1(1600)$</td>
<td>1$^{--}$</td>
<td>1555 ±50</td>
<td>468 ±80</td>
<td>$\eta'\pi$</td>
</tr>
<tr>
<td>E852</td>
<td>$\pi_1(1600)$</td>
<td>1$^{-+}$</td>
<td>1709 ±24±41</td>
<td>403 ±80±115</td>
<td>$f_1\pi$</td>
</tr>
<tr>
<td>E852</td>
<td>$\pi_1(1600)$</td>
<td>1$^{--}$</td>
<td>1664 ±8±10</td>
<td>185 ±25±28</td>
<td>$\omega\pi\pi$</td>
</tr>
<tr>
<td>E852</td>
<td>$\pi_1(2000)$</td>
<td>1$^{--}$</td>
<td>2001 ±30±92</td>
<td>333 ±52±49</td>
<td>$f_1\pi$</td>
</tr>
<tr>
<td>E852</td>
<td>$\pi_1(2000)$</td>
<td>1$^{-+}$</td>
<td>2014 ±20±16</td>
<td>230 ±32±73</td>
<td>$\omega\pi\pi$</td>
</tr>
<tr>
<td>E852</td>
<td>$h_2(1950)$</td>
<td>2$^{+-}$</td>
<td>1954 ±8</td>
<td>138 ±3</td>
<td>$\omega\pi\pi$</td>
</tr>
</tbody>
</table>
# Waves used in PWA

<table>
<thead>
<tr>
<th>$J^{PC} M^\pi$</th>
<th>$L$</th>
<th>Isobar $\pi$</th>
<th>Cut [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^{-+}0^+$</td>
<td>$S$</td>
<td>$f_0 \pi$</td>
<td>1.40</td>
</tr>
<tr>
<td>$0^{-+}0^+$</td>
<td>$S$</td>
<td>$(\pi \pi)_s \pi$</td>
<td>-</td>
</tr>
<tr>
<td>$0^{-+}0^+$</td>
<td>$P$</td>
<td>$\rho \pi$</td>
<td>-</td>
</tr>
<tr>
<td>$1^{-+}1^+$</td>
<td>$P$</td>
<td>$\rho \pi$</td>
<td>-</td>
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