Study of the exclusive ρ^0 with polarized deuteron target

$$\mu + N^{\uparrow} \rightarrow \mu' + N' + \rho^0$$

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On behalf of the COMPASS collaboration

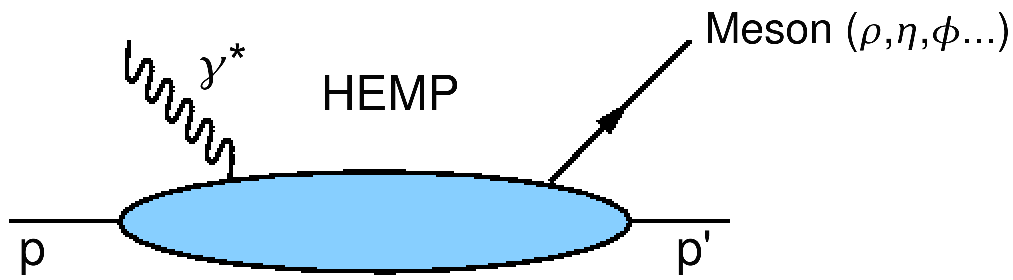
GPD08, Trento

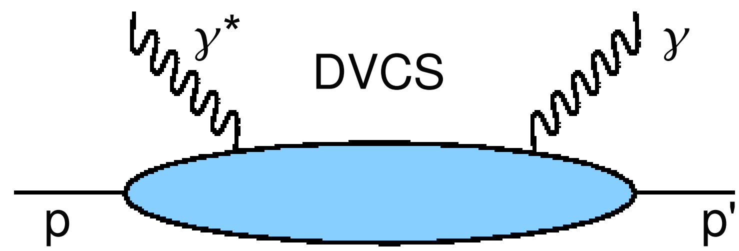
Outline :

- Motivations : Generalized parton distributions (GPD)
- COMPASS
- Exclusive ρ^0 production
- Transverse Target Spin Asymmetries

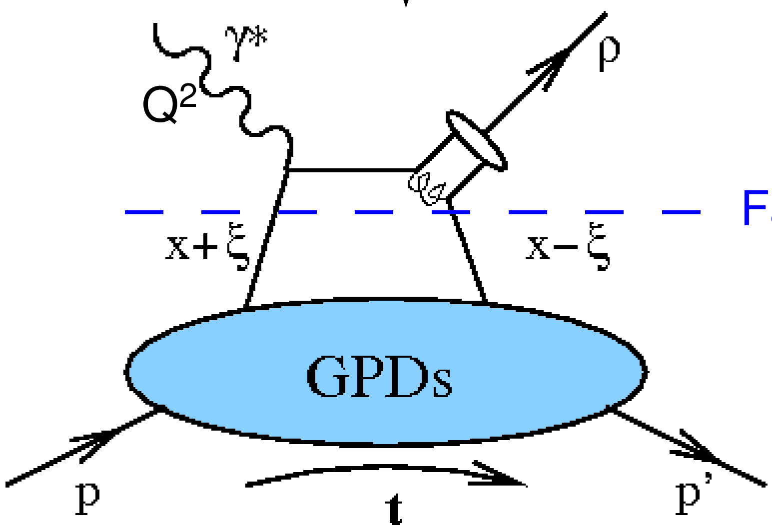
Generalized parton distributions

How to obtain information about GPDs : Exclusive interactions with the proton





At twist 2 : handbag diagram



Factorization for
- High Q^2 (>1GeV^2)
- Low t ($t/Q^2 << 1$)
- Longitudinal γ^*

For vector meson production :
sensitivity to the GPDs H and E for quarks and gluons

How to extract constrains on GPDs from exclusive ρ production

$$\sigma \sim \sigma(x_B, Q^2, t)(W_{UU} + P_l W_{LU} + S_L W_{UL} + P_l S_L W_{LL} + S_T W_{UT} + P_l S_T W_{LT})$$

M.Diehl; hep-ph/0704.1565

σ : $\sigma(\mu p \rightarrow \mu' p' \rho)$

P_l : beam longitudinal polarization

S_L : target longitudinal polarization

S_T : target transverse polarization

W_{XY} : Angular dependence for a beam polarization X and a target polarization Y

X,Y = U : Unpolarized
= L : Longitudinal
= T : Transverse

How to extract constrains on GPDs

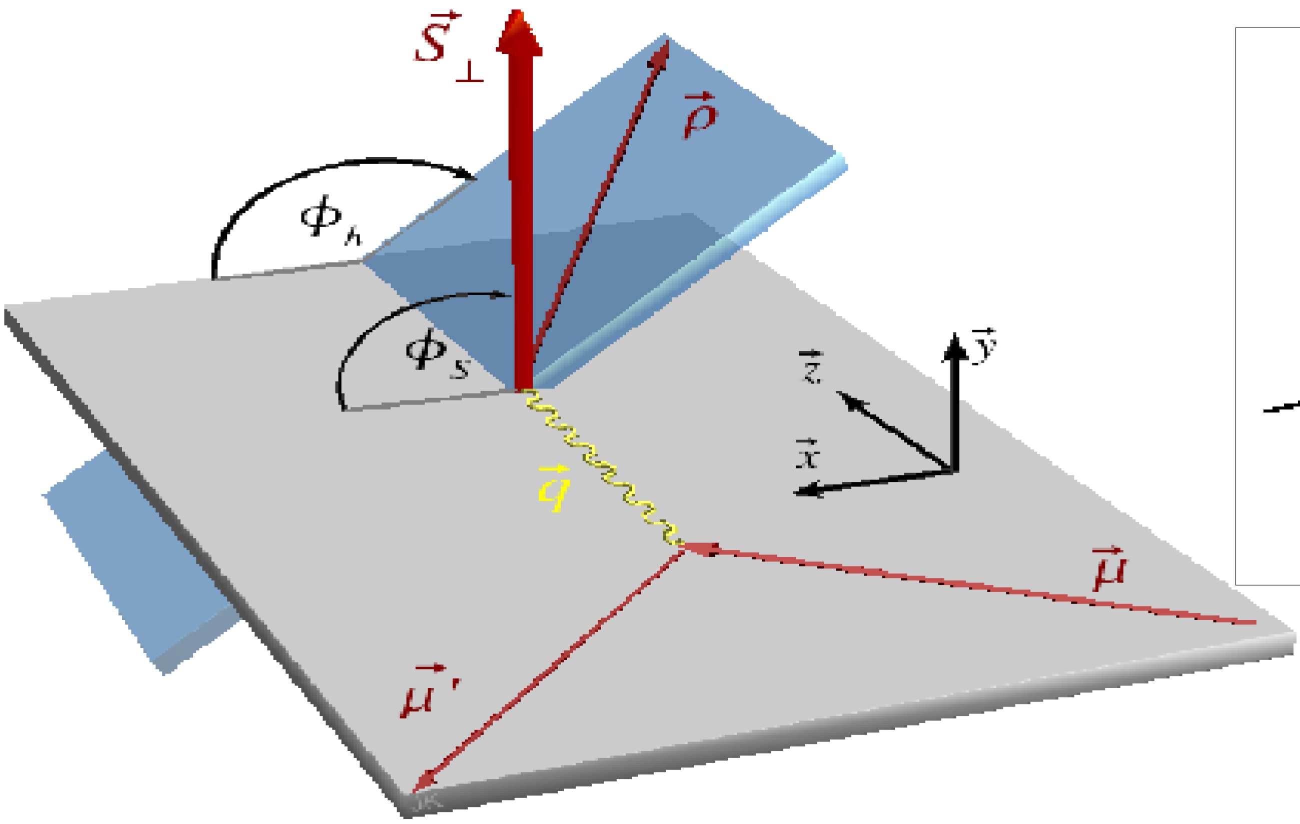
$$\sigma \sim \sigma(x_B, Q^2, t)(W_{UU} + P_l W_{LU} + S_L W_{UL} + P_l S_L W_{LL} + S_T W_{UT} + P_l S_T W_{LT})$$

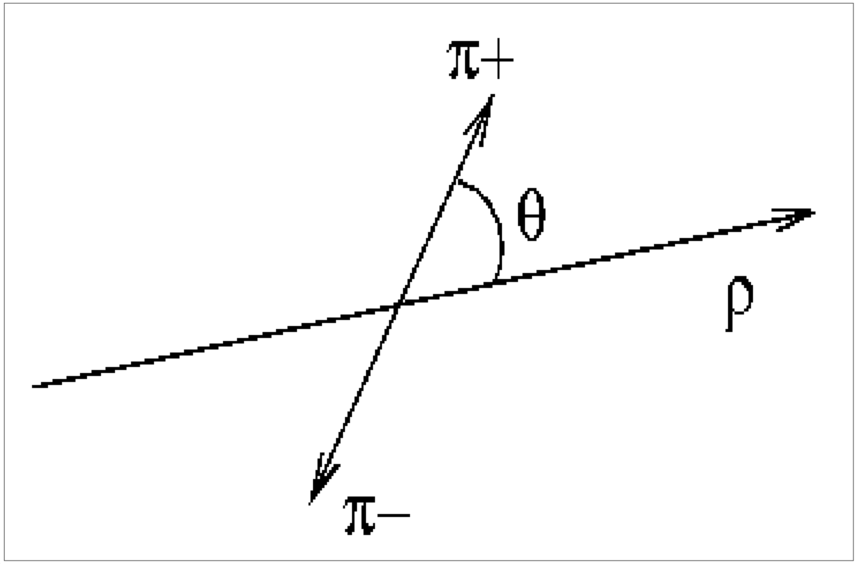
Unpolarized angular dependence of the cross section

$$W_{UU}(\theta) \simeq u_L + u_T$$

Transverse target angular dependence of the cross section

$$W_{UT}(\phi - \phi_S, \theta) \simeq \sin(\phi - \phi_S)\, Im(n_L + n_T)$$





How to extract constrains on GPDs

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Unpolarized angular dependence

Transverse target angular dependence

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$$W_{UT}(\phi - \phi_S, \theta) \simeq \sin(\phi - \phi_S)\, Im(n_L + n_T)$$

$$\frac{Im\, n_L}{u_L} = \frac{\sqrt{t_0 - t}}{M_N} \frac{\sqrt{1-\xi^2}\, Im(\mathcal{E}^*\mathcal{H})}{(1-\xi^2)|\mathcal{H}|^2 - (\xi^2 + t/(4M_N^2))|\mathcal{E}|^2 - 2\xi^2\, Re(\mathcal{E}^*\mathcal{H})}$$

$\mathcal{E}, \mathcal{H}$: Integrals of GPDs E(x, ξ ,t) and H(x, ξ ,t)

How to extract constrains on GPDs

$$\sigma \sim \sigma(x_B, Q^2, t)\left(W_{UU} + P_l W_{LU} + S_L W_{UL} + P_l S_L W_{LL} + S_T W_{UT} + P_l S_T W_{LT}\right)$$

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$\mathcal{E}, \mathcal{H}$: Integrals of GPDs E(x, ξ ,t) and H(x, ξ ,t)

Extraction of the γ^*_L component in the $\sin(\phi - \phi_S)$ modulation in W_{UT}

→ Constrains on the GPD E

Directive line

Data selection : Exclusive ρ production

↓

Observable extraction : Transverse target spin asymmetries

↓

Results : Constraint GPDs, an experimental value for $\frac{Im(\mathcal{E}^*\mathcal{H})}{|\mathcal{H}|^2}$

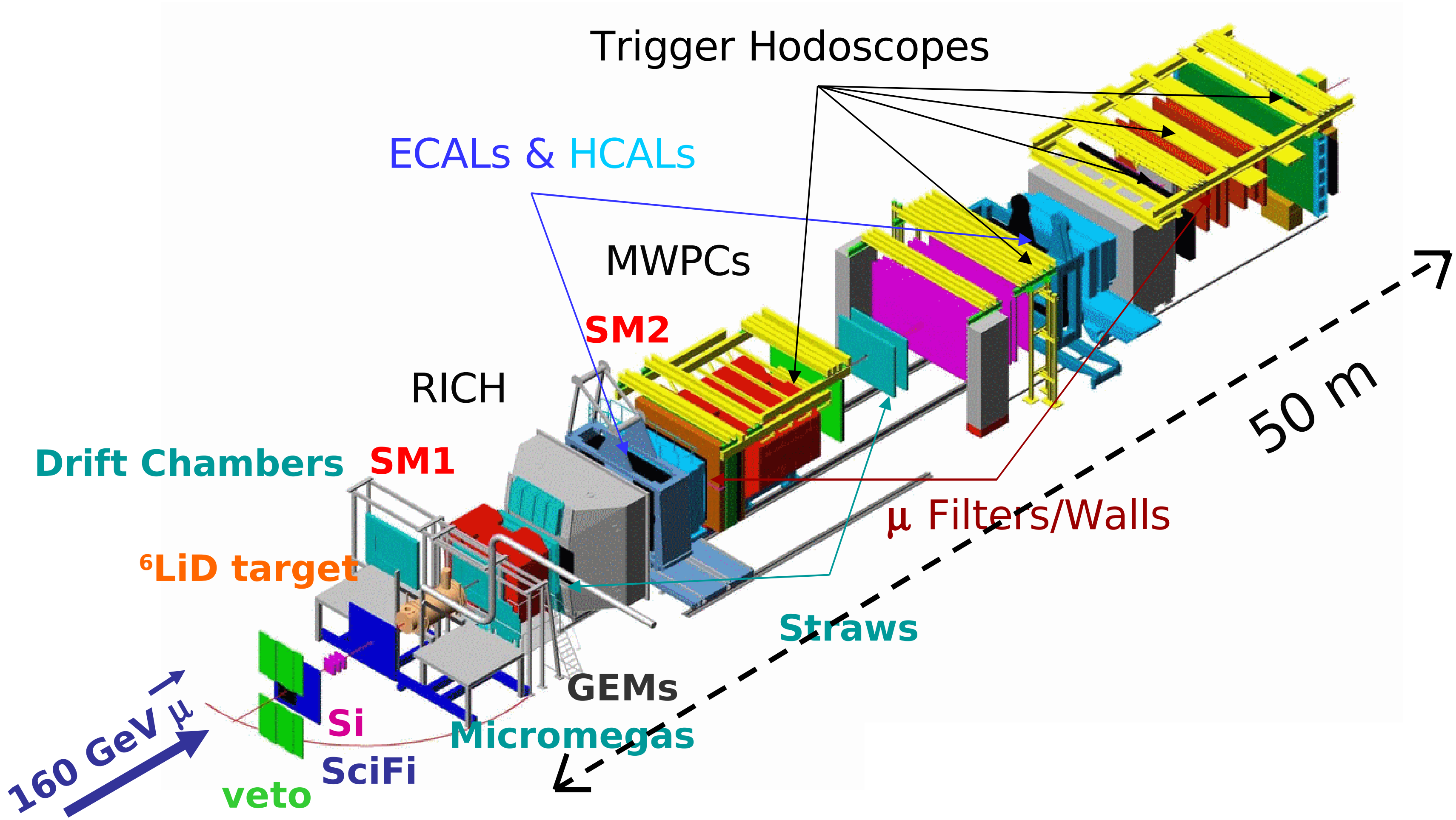
↓

Use : Check of models

The experiment : COMPASS

Longitudinally polarized muons : 160 GeV/c
2.10^8 μ/spill (4.8s / 16.8s)
P_{Beam}= -80%

Luminosity: ~ 5. 10^{32} $cm^{-2}s^{-1}$

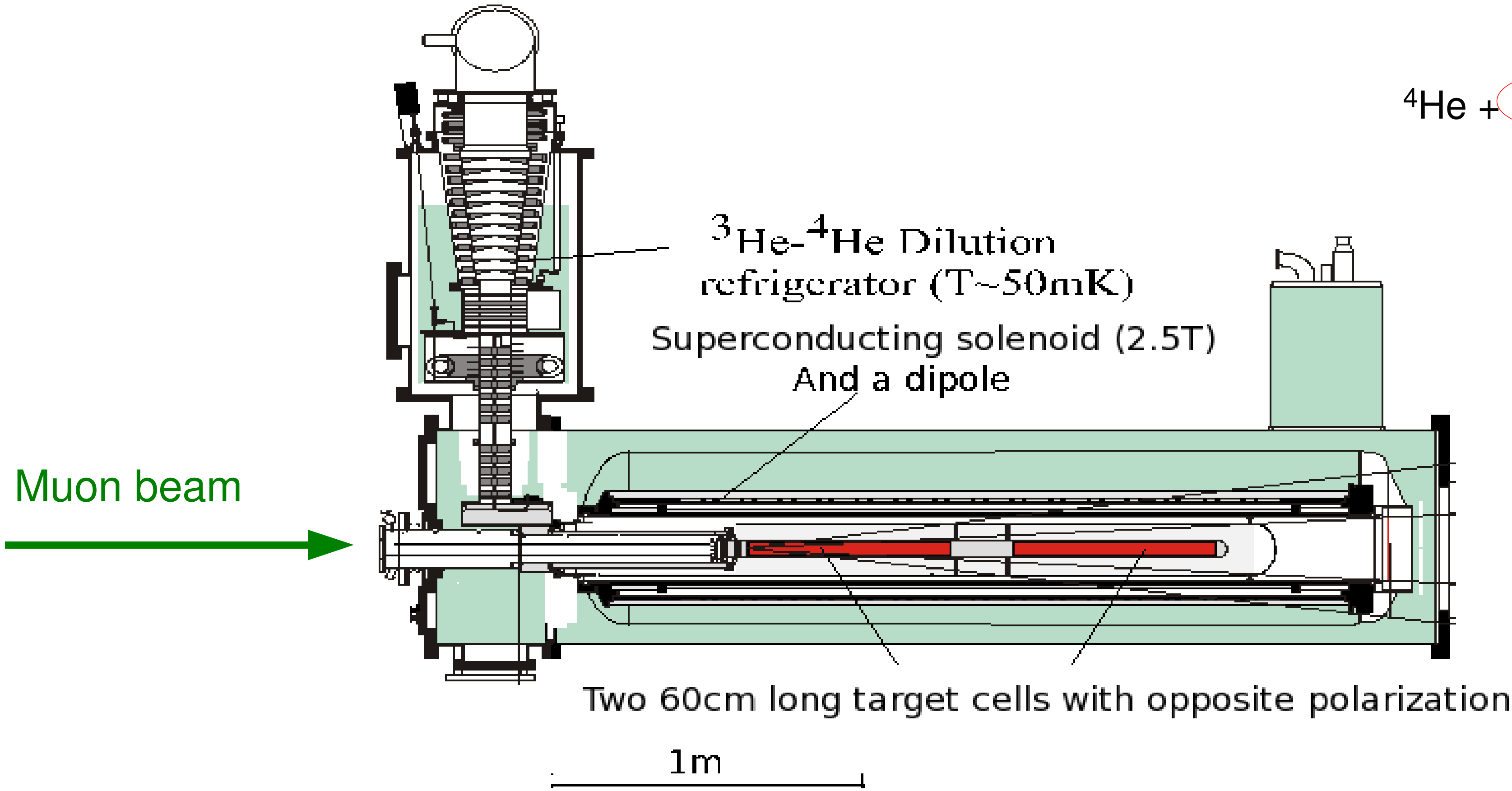


The COMPASS polarized ^{6}LiD target (2002-3-4)

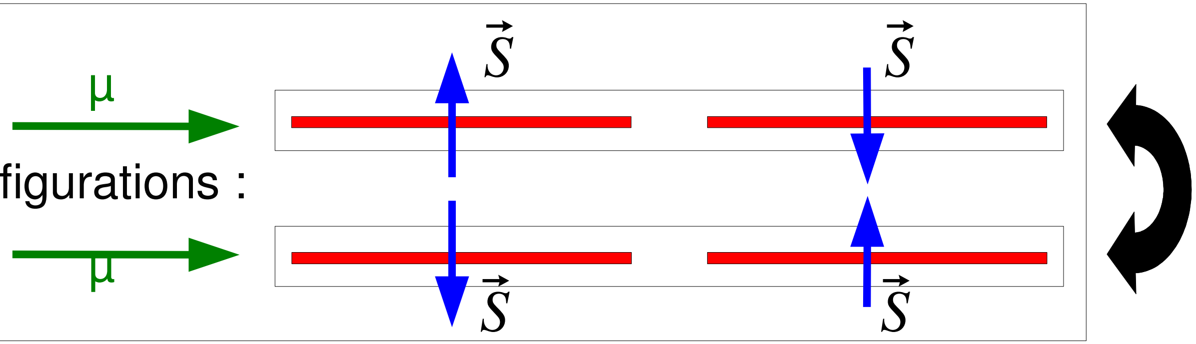
Transversally (or longitudinally) polarized deuteron target : ^{6}LiD $P_T \sim 50\%$

$^4He + D + D$

Polarizable

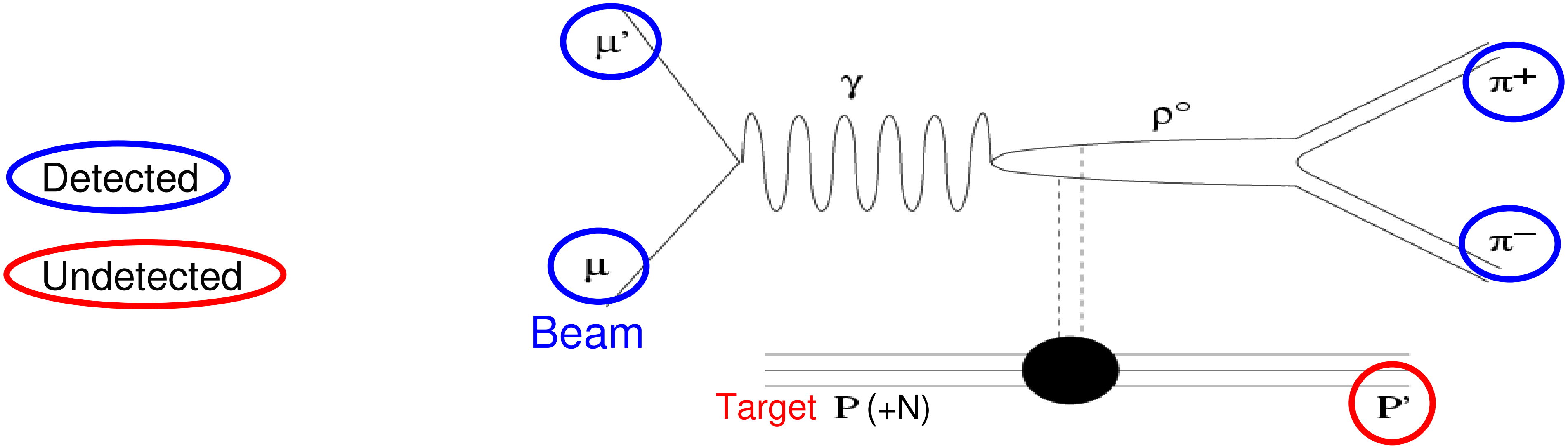


2 polarization configurations :



By changing MW frequencies (Every week)

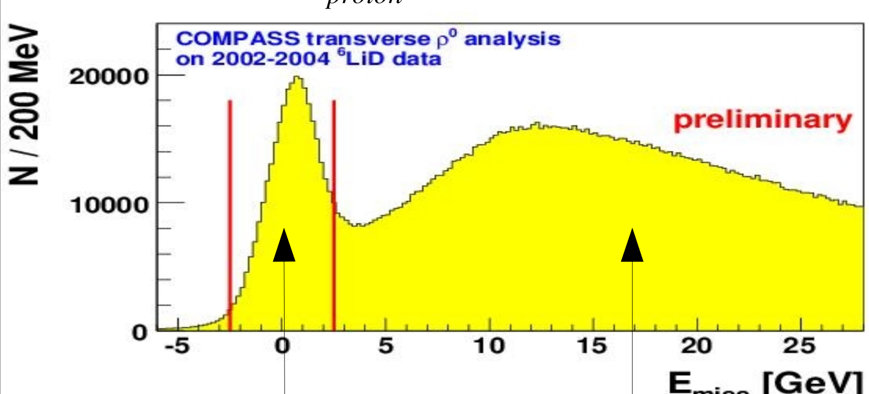
Exclusive ρ^0 production : Event selection



Exclusive ρ_0 Production

Recoil proton (p') is not detected,
Check if the proton is intact :

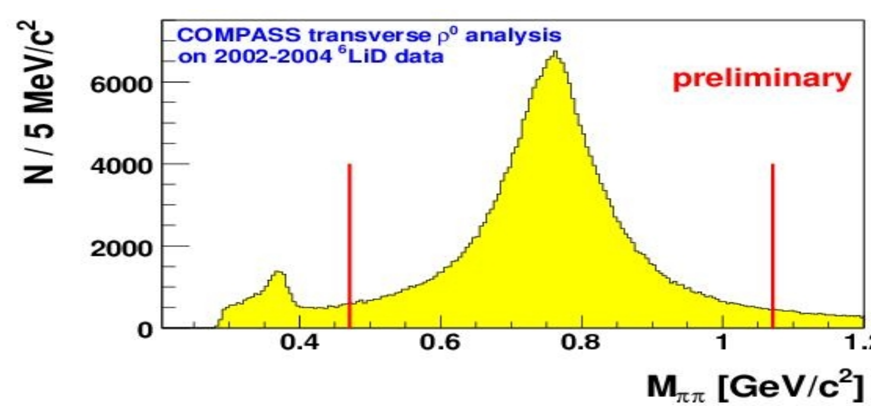
$$E_{miss} = \frac{M_X^2 - M_{proton}^2}{2\,M_{proton}} \in [-2.5, 2.5]\,GeV$$



Exclusive peak   Non exclusive background

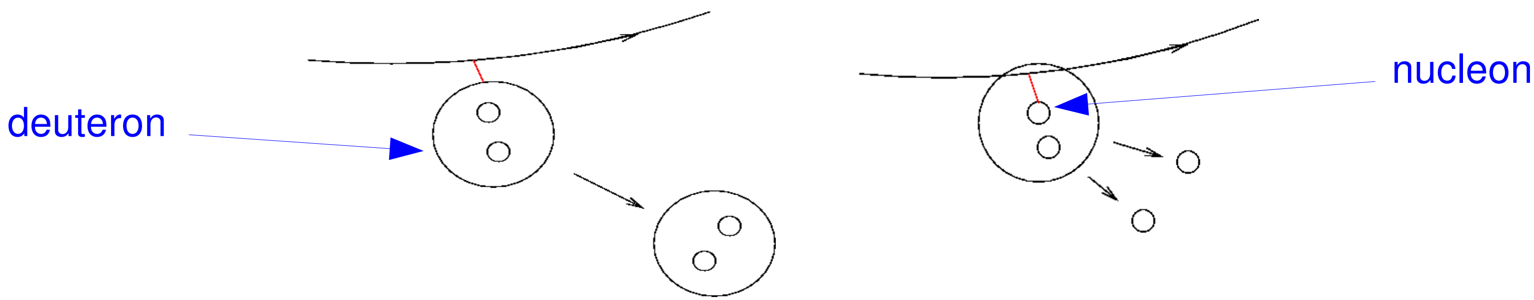
Invariant mass selection

$$0.3 < M_{\pi\pi} - M_{\rho} < 0.3\,GeV$$



Exclusive ρ^0 production : What remains in the sample

What we have : ◆ **Coherent** + **incoherent** scattering



- **Transverse** ($J_z=\pm1$) + **longitudinal** ($J_z=0$) polarization of γ^*
- Scattering off **protons** and **neutrons**
- **Non-exclusive** background

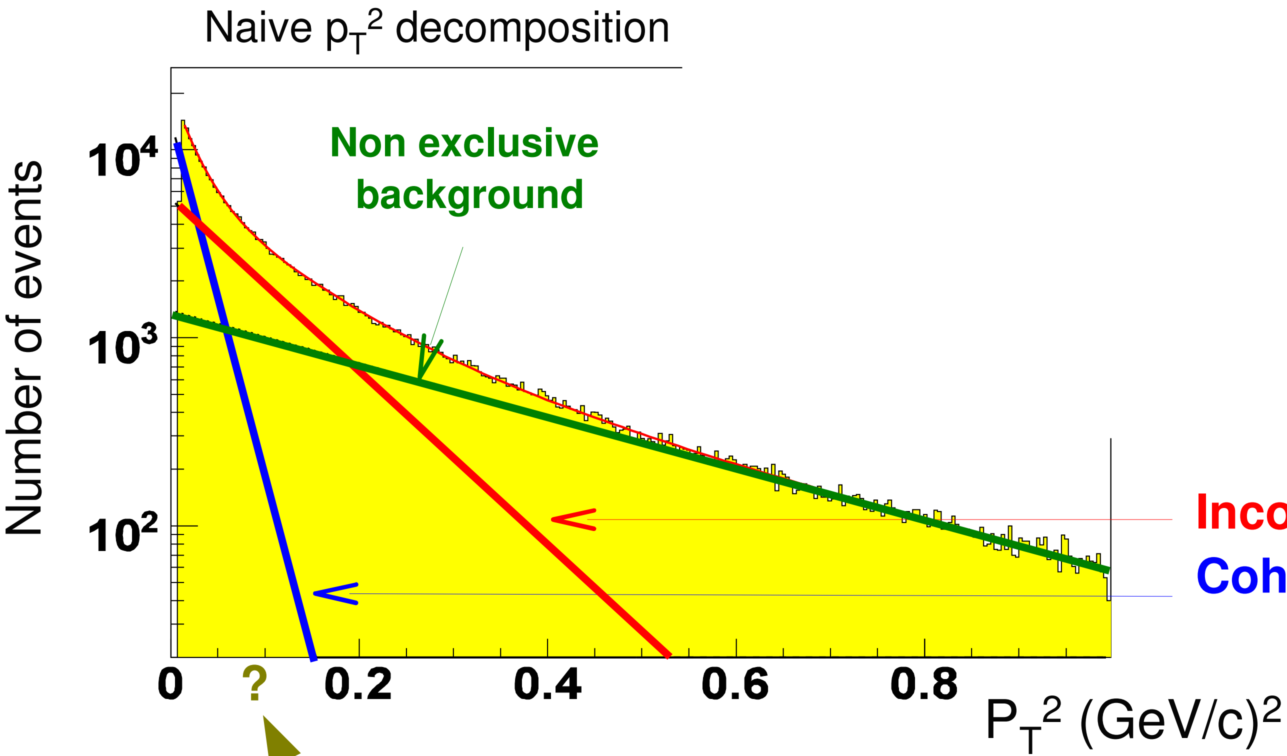
What we want :

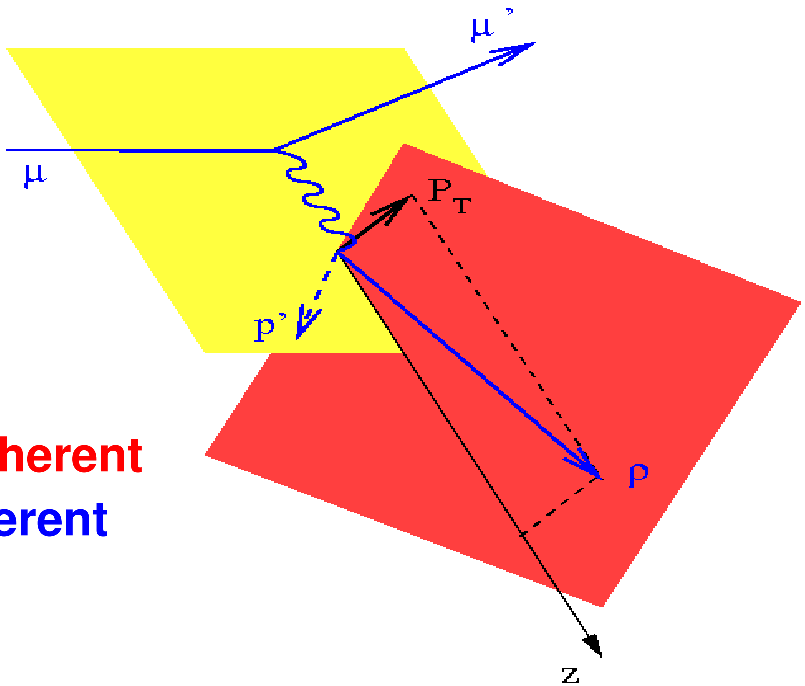
- **Incoherent** scattering for study of nucleon GPD
- **longitudinal** γ^* for factorization

What we have to do :

- **Coherent** / **incoherent** and **Transverse** / **longitudinal** separation
- Background estimation

Exclusive ρ^0 production : Coherent / incoherent separation





Find the p_T^2 selection to : Reject coherent sample
Keep incoherent sample
Reject Non exclusive background

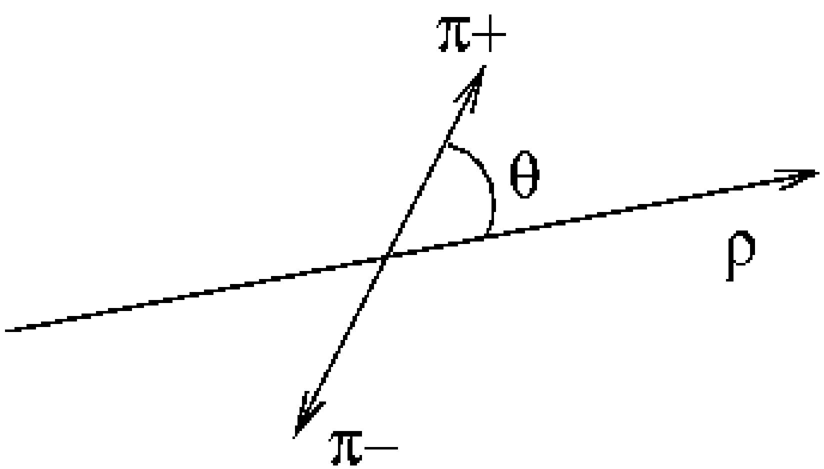
p_T^2>0.1(GeV/c)^2 -> we keep 70% of incoherent production and less than 10% of coherent
p_T^2<0.5 (GeV/c)^2 To reduce non exclusive background

Exclusive ρ^0 production : Longitudinal / Transverse separation

M.Diehl, S.Sapeta; EPJC41 (2005)

$$W_{UT}(\ldots,\theta,\ldots)=\frac{3}{4\pi}\left(\cos^2\theta\, W_{UT}^{LL}+\sin^2\theta\, W_{UT}^{TT}+\sqrt{2}\cos\theta\sin\theta\, W_{UT}^{LT}\right)$$

longitudinal / transverse separation of ρ polarization



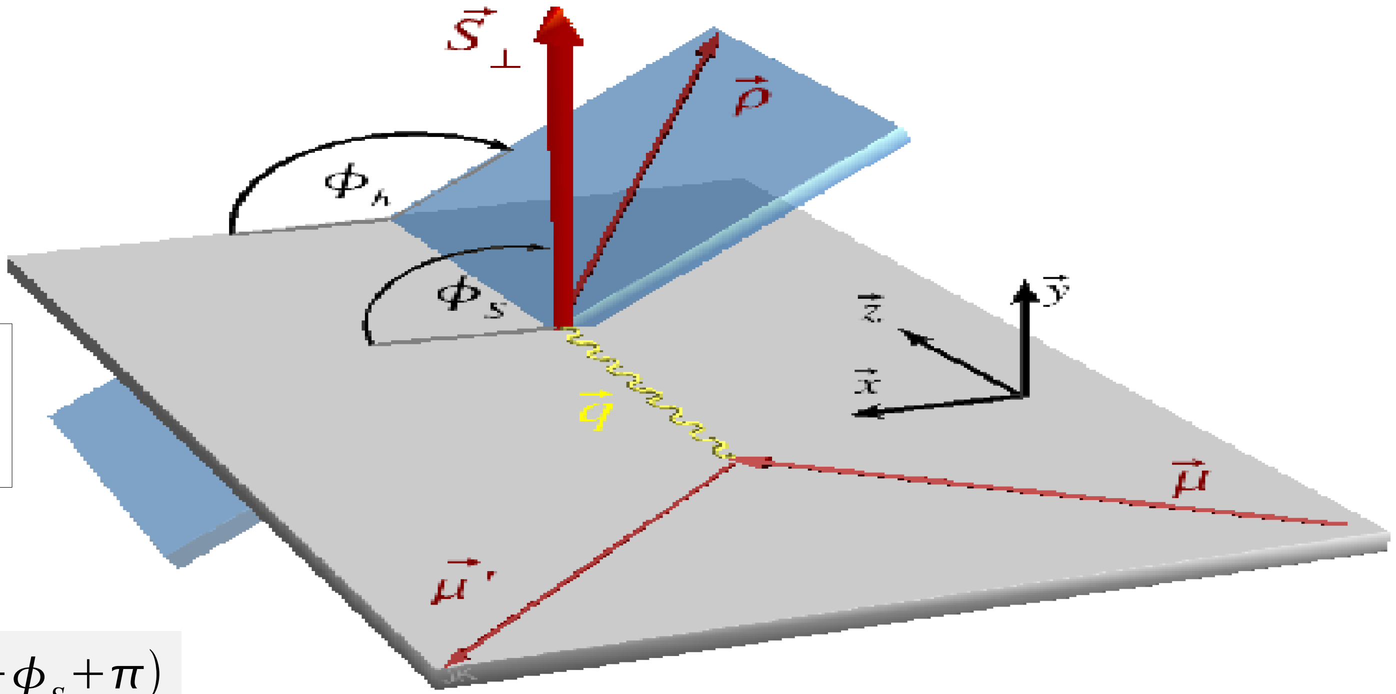
LL : ρ longitudinal
TT : ρ transverse
LT : interference longitudinal/transverse

If S-Channel Helicity Conservation :
ρ polarization → γ^* polarization

longitudinal / transverse separation of γ^* polarization

(SCHC holds for $\geq$ 90% of exclusive ρ production)

Transverse Target Spin Asymmetry : The extraction



$(\phi\text{-}\phi_S)$: Angle between target spin and hadronic plane

$$A_{UT}(\phi-\phi_S) \sim \frac{\sigma(\phi-\phi_S)-\sigma(\phi-\phi_S+\pi)}{\sigma(\phi-\phi_S)+\sigma(\phi-\phi_S+\pi)}$$

Flux — Acceptance — Dilution factor — Mean target polarization

$$N(\phi-\phi_S) = F\, a(\phi-\phi_S)\,\sigma_0 \left(1 \pm f \langle P_T \rangle A_{UT}^{exp} \sin(\phi-\phi_S)\right)$$

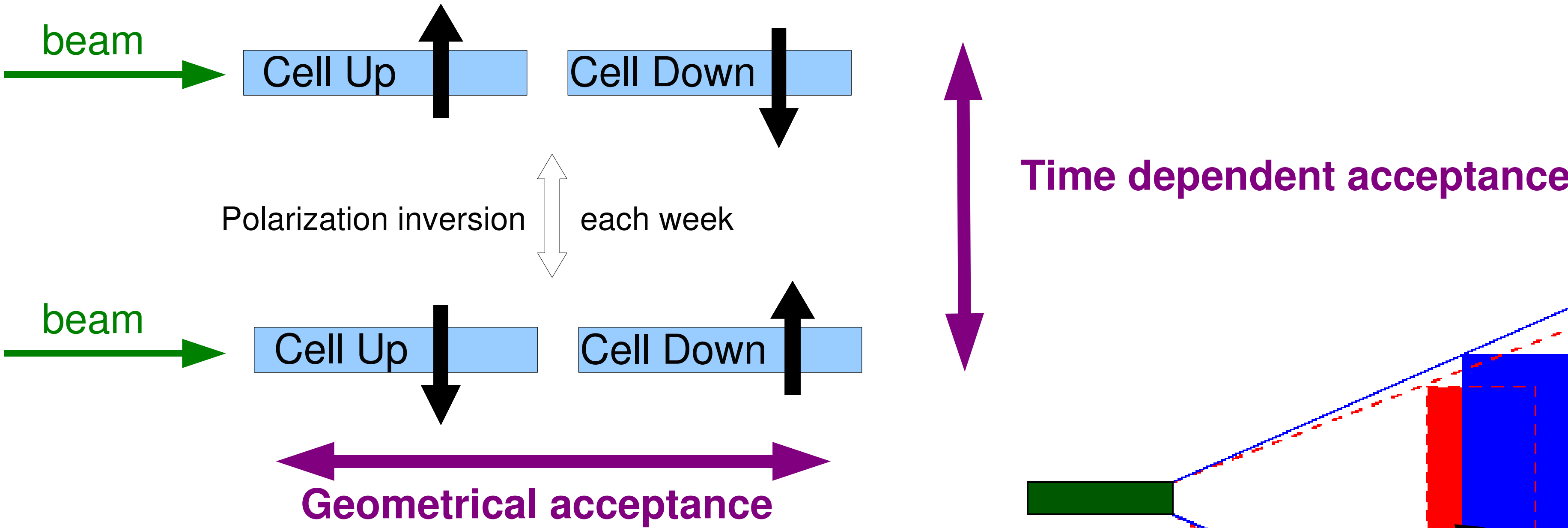
A_{UT} measurable by one target and one polarization

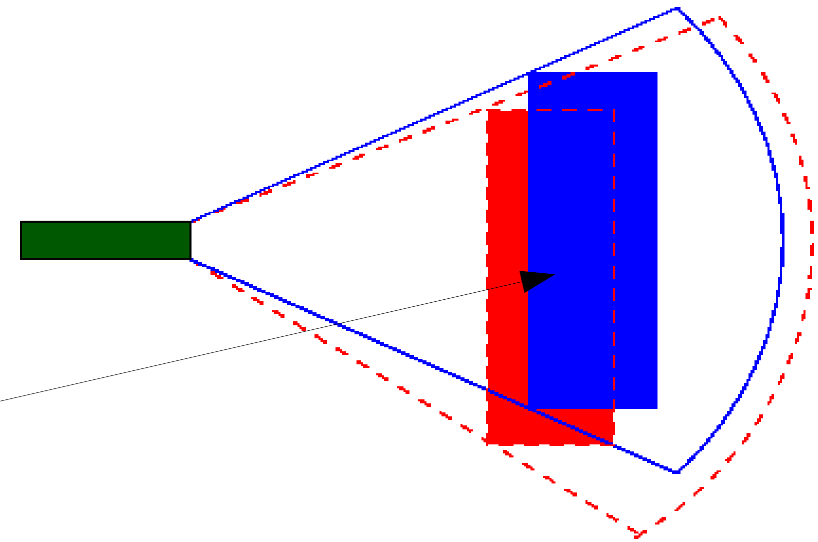
With only one target with one polarization, $A_{UT}^{(exp)}$ is sensitive acceptance effects

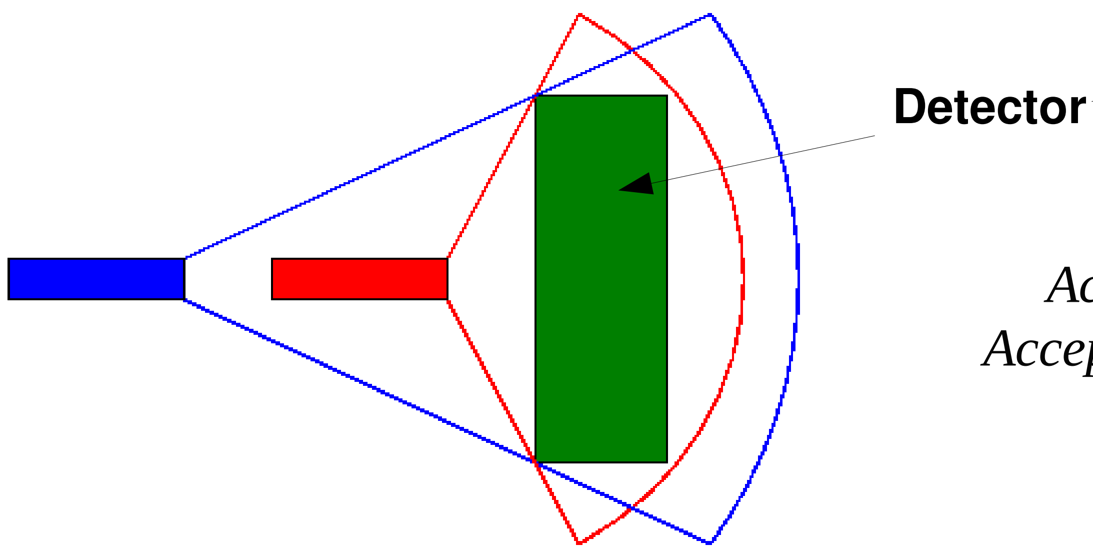
➔ **Asymmetry extraction from double ratio method with 2 targets with 2 polarizations**

$$\frac{N^{\uparrow}_{Up}(\phi-\phi_S)\,N^{\uparrow}_{Down}(\phi-\phi_S)}{N^{\downarrow}_{Down}(\phi-\phi_S+\pi)\,N^{\downarrow}_{Up}(\phi-\phi_S+\pi)} = \frac{F^{\uparrow}_{Up}\,F^{\uparrow}_{Down}}{F^{\downarrow}_{Down}\,F^{\downarrow}_{Up}}\;\frac{a^{\uparrow}_{Up}(\phi-\phi_S)\,a^{\uparrow}_{down}(\phi-\phi_S)}{a^{\downarrow}_{Down}(\phi-\phi_S+\pi)\,a^{\downarrow}_{Up}(\phi-\phi_S+\pi)}\;\frac{\left(1+f\langle P_T\rangle A_{UT}^{exp}\sin(\phi-\phi_S)\right)^2}{\left(1-f\langle P_T\rangle A_{UT}^{exp}\sin(\phi-\phi_S)\right)^2}$$

Transverse Target Spin Asymmetry : Acceptance effects





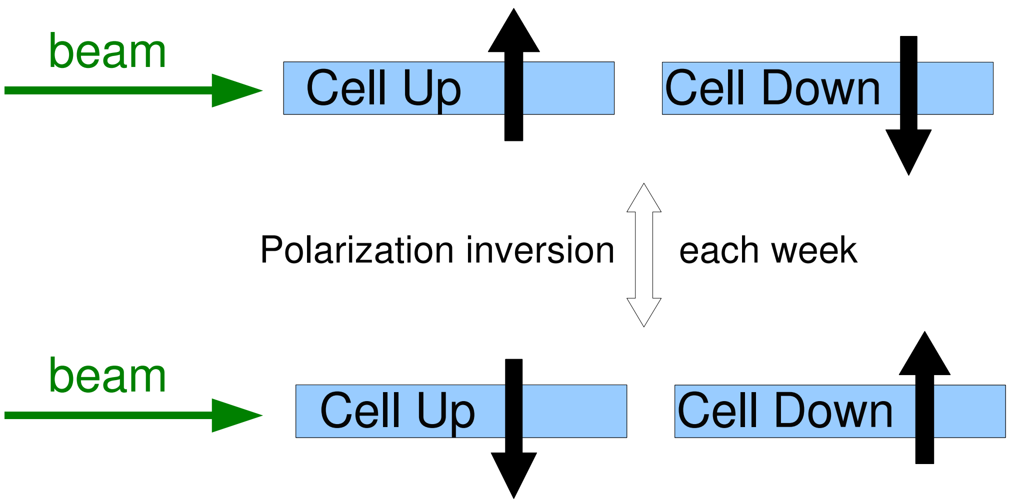


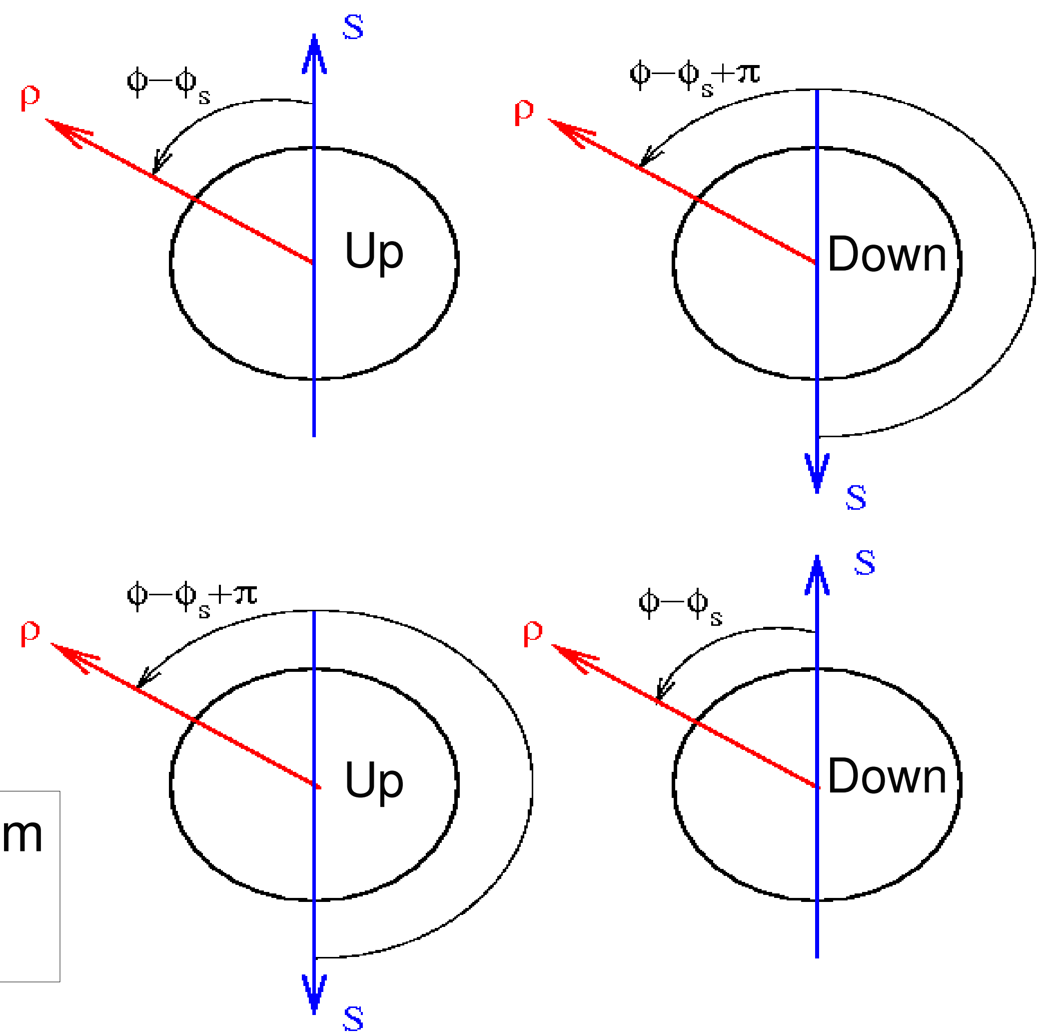
$$Acceptance(Up,\uparrow) \neq Acceptance(Up,\downarrow)$$
$$Acceptance(Down,\uparrow) \neq Acceptance(Down,\downarrow)$$

$$Acceptance(Up,\uparrow) \neq Acceptance(Down,\downarrow)$$
$$Acceptance(Up,\downarrow) \neq Acceptance(Down,\uparrow)$$

4 samples of data → 4 different acceptances

Transverse Target Spin Asymmetry : Acceptance effects





If the direction of $\gamma^* \simeq$ direction of the beam

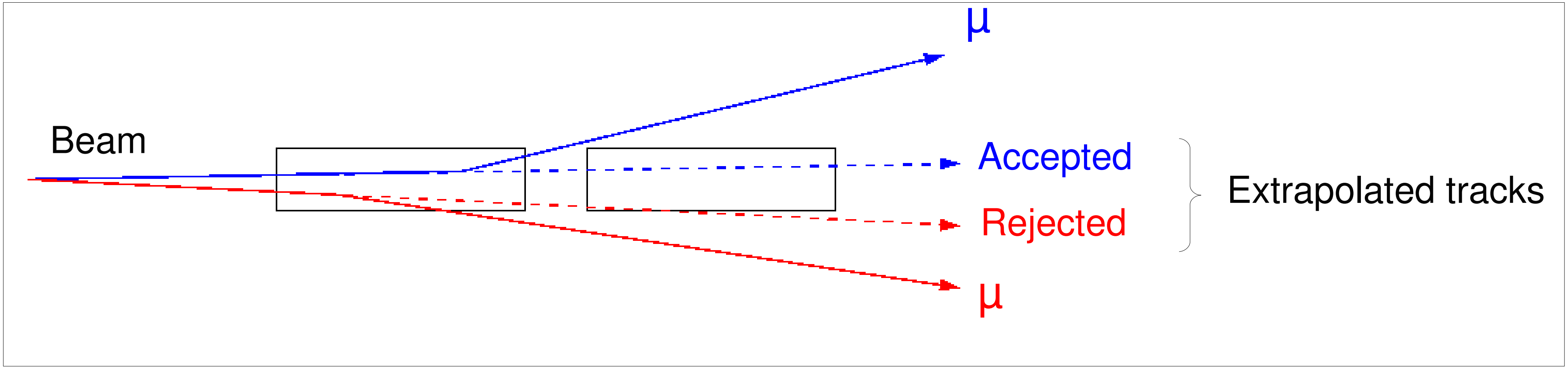
$$(\theta_{Beam\,\gamma^*} = 0.033\,rad\,;\,RMS = 0.021)$$

$$\frac{a^{\uparrow}_{Up}(\phi-\phi_S)}{a^{\downarrow}_{Down}(\phi-\phi_S+\pi)} \simeq \frac{a^{\downarrow}_{Up}(\phi-\phi_S+\pi)}{a^{\uparrow}_{Down}(\phi-\phi_S)} \;\longrightarrow\; \frac{a^{\uparrow}_{Up}(\phi-\phi_S)}{a^{\downarrow}_{Down}(\phi-\phi_S+\pi)}\,\frac{a^{\uparrow}_{Down}(\phi-\phi_S)}{a^{\downarrow}_{Up}(\phi-\phi_S+\pi)} \simeq 1$$

Acceptance cancellation

Transverse Target Spin Asymmetry : Flux term

In the event selection : extrapolated incoming track of the beam pass through both cells



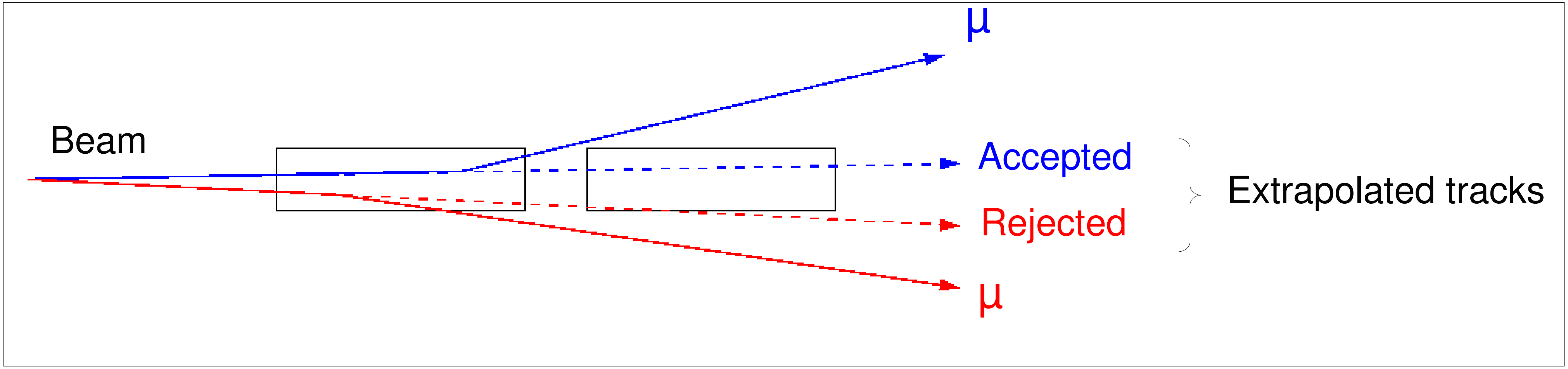
$$F^{\downarrow}_{Down} = F^{\uparrow}_{Up}\, e^{-\alpha L}$$
$$F^{\uparrow}_{Down} = F^{\downarrow}_{Up}\, e^{-\alpha L}$$

$$\Rightarrow \quad \frac{F^{\uparrow}_{Up}}{F^{\downarrow}_{Down}} \frac{F^{\uparrow}_{Down}}{F^{\downarrow}_{Up}} \simeq 1$$

Flux Cancellation

Transverse Target Spin Asymmetry : Flux term

In the event selection : extrapolated incoming track of the beam pass through both cells



$$F^{\downarrow}_{Down} = F^{\uparrow}_{Up} e^{-\alpha L}$$
$$F^{\uparrow}_{Down} = F^{\downarrow}_{Up} e^{-\alpha L}$$

$$\frac{F^{\uparrow}_{Up}}{F^{\downarrow}_{Down}} \frac{F^{\uparrow}_{Down}}{F^{\downarrow}_{Up}} \simeq 1$$ Flux Cancellation

$$\frac{N^{\uparrow}_{Up}(\phi-\phi_S) N^{\uparrow}_{Down}(\phi-\phi_S)}{N^{\downarrow}_{Down}(\phi-\phi_S+\pi) N^{\downarrow}_{Up}(\phi-\phi_S+\pi)} = \frac{F^{\uparrow}_{Up} F^{\uparrow}_{Down}}{F^{\downarrow}_{Down} F^{\downarrow}_{Up}} \frac{a^{\uparrow}_{Up}(\phi-\phi_S) a^{\uparrow}_{down}(\phi-\phi_S)}{a^{\downarrow}_{Down}(\phi-\phi_S+\pi) a^{\downarrow}_{Up}(\phi-\phi_S+\pi)} \frac{(1+f\langle P_T\rangle A^{exp}_{UT}\sin(\phi-\phi_S))^2}{(1-f\langle P_T\rangle A^{exp}_{UT}\sin(\phi-\phi_S))^2}$$

Flux Cancellation   Acceptance cancellation

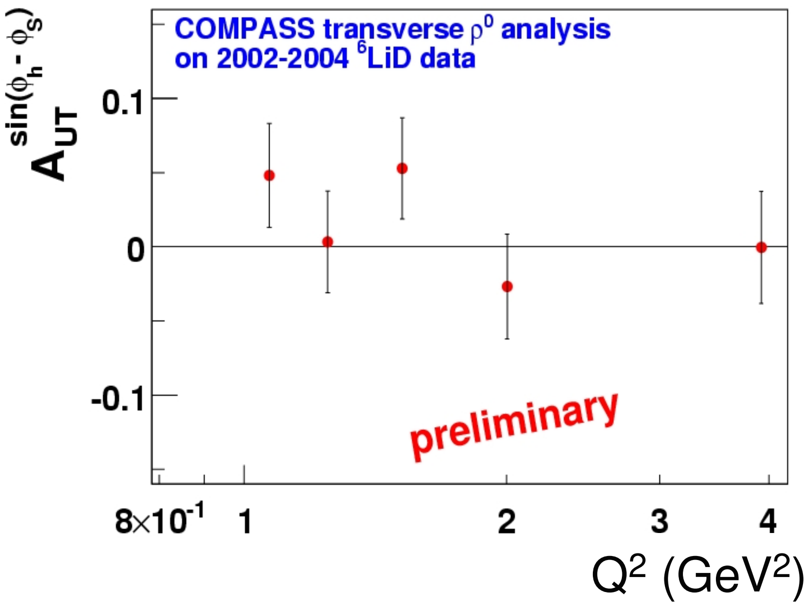
$f = 0.36$
P_T = target polarization

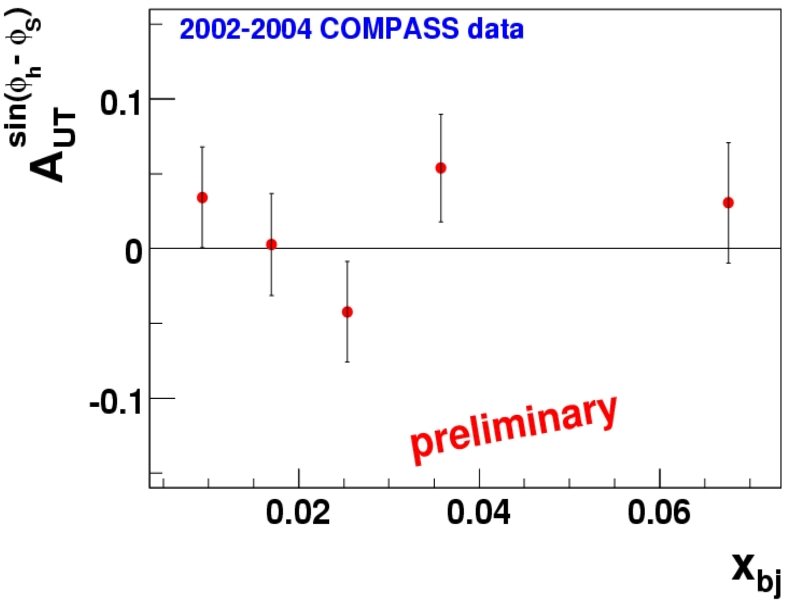
$$\frac{N^{\uparrow}_{Up}(\phi-\phi_S) N^{\uparrow}_{Down}(\phi-\phi_S)}{N^{\downarrow}_{Down}(\phi-\phi_S+\pi) N^{\downarrow}_{Up}(\phi-\phi_S+\pi)} = \frac{(1+f\langle P_T\rangle A_{UT}\sin(\phi-\phi_S))^2}{(1-f\langle P_T\rangle A_{UT}\sin(\phi-\phi_S))^2}$$

Transverse Target Spin Asymmetry : Results on deuteron

COMPASS results on a DEUTERON target
without coherent/incoherent and transverse/longitudinal separation

$$\langle Q^2 \rangle \simeq 2.0\, GeV^2 \qquad \langle x_{bj} \rangle \simeq 0.03$$





A_{UT} results on deuteron target, compatible with 0
both protons and neutrons contribute and might cancel asymmetry

Summary :

COMPASS (2002-3-4): A_{UT} compatible to zero with a deuteron target

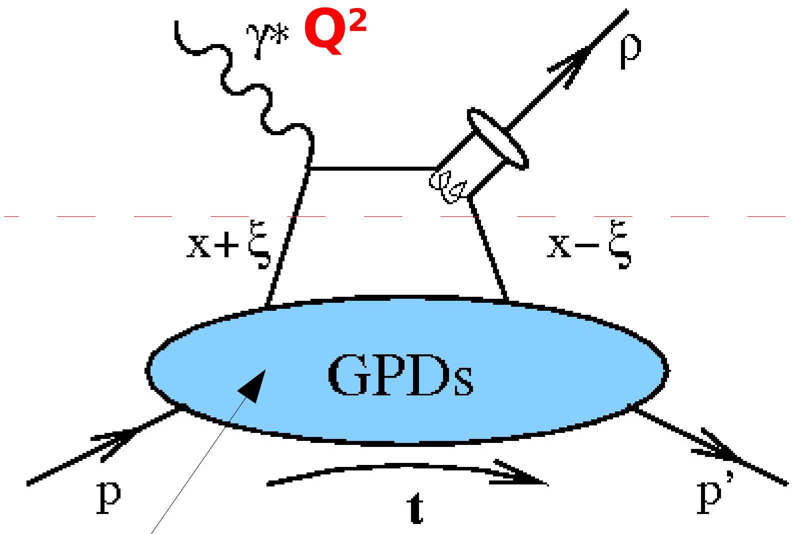
In progress :
- MC to measure acceptance (based on DIPSI generator)
- MC to understand background
- Coherent / incoherent separation (by p_t^2 selection)
- Transverse / longitudinal γ^* separation (by angular distribution)

Future results :
- A_{UT} extraction for a proton target (2007 data)
- Determination of SDME and control the SCHC
- Same work for exclusive ω and ϕ production

Backup

1/ Generalized partons distributions (GPDs)

At leading order : ρ production dominated by handbag diagram



Factorisation: Q^2 large, -t small
And γ^* longitudinal

$$4\,GPDs: H(x,\xi,t), E(x,\xi,t), \tilde{H}(x,\xi,t), \tilde{E}(x,\xi,t)$$

ρ production is only sensitive to H and E

$$\int_{-1}^{1} dx\, H(x,\xi,t) = F_1(t)$$
$$\int_{-1}^{1} dx\, E(x,\xi,t) = F_2(t)$$

GPDs contains : Form factors
Partons distributions
Angular momentum of quarks (Ji sum rule)

$$H(x,\xi=0,t=0) = q(x)$$

No equivalent continuity condition for E(x,0,0)

1/ Generalized partons distributions (GPDs)

How partons contribute to the proton spin : $\frac{1}{2} = J_q + J_g = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$

quarks: $\frac{1}{2}\Delta\Sigma + L_q$ — $\frac{1}{2}\Delta\Sigma$ ≈30%, L_q unknown

gluons: $\Delta G + L_g$ — ΔG: Large values excluded (COMPASS, RHIC, HERMES), L_g unknown

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L_q is measurable from GPDs via the Ji sum rule

$$J_q = \frac{1}{2}\Delta\Sigma + L_q = \frac{1}{2}\int dx\, x\,\left(H_q(x,\xi,0) + E_q(x,\xi,0) \right)$$

How to extract constrains on GPDs

$$\sigma \sim \sigma_{unpolarized}\left(W_{UU} + P_l W_{LU} + S_L W_{UL} + P_l S_L W_{LL} + S_T W_{UT} + P_l S_T W_{LT}\right)$$

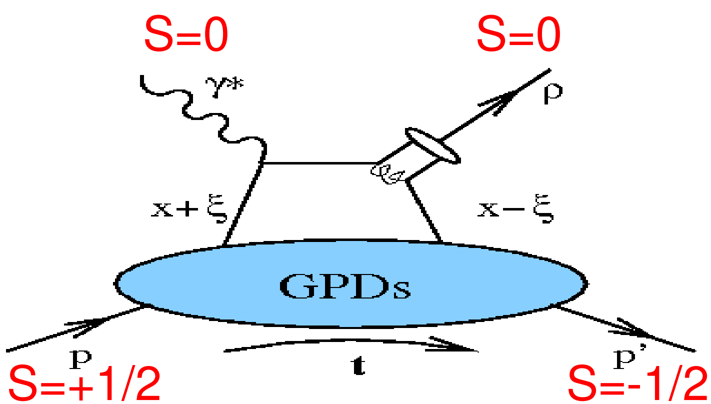
Unpolarized angular dependence of the cross section

Transverse target angular dependence of the cross section

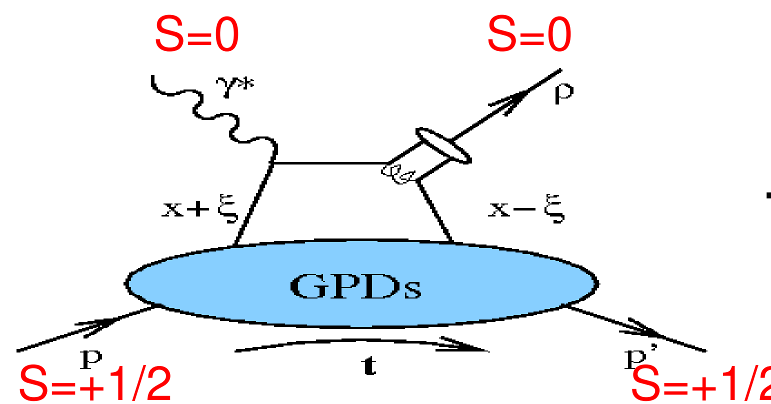
$$W_{UU} = \ldots + \left(u_{++}^{00} + \epsilon\, u_{00}^{00}\right) + \ldots$$

$$W_{UT} = \ldots + \sin(\phi - \phi_S)\, Im\left(n_{++}^{00} + \epsilon\, n_{00}^{00}\right) + \ldots$$

$n_{00}^{00} \sim$

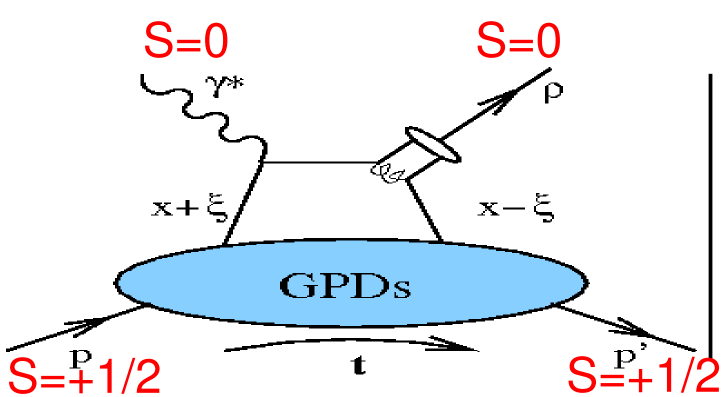


$\times$

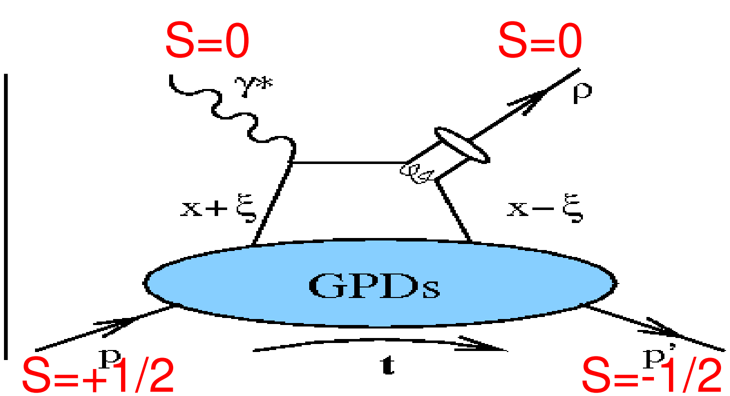


+...

$u_{00}^{00} \sim \Big|$



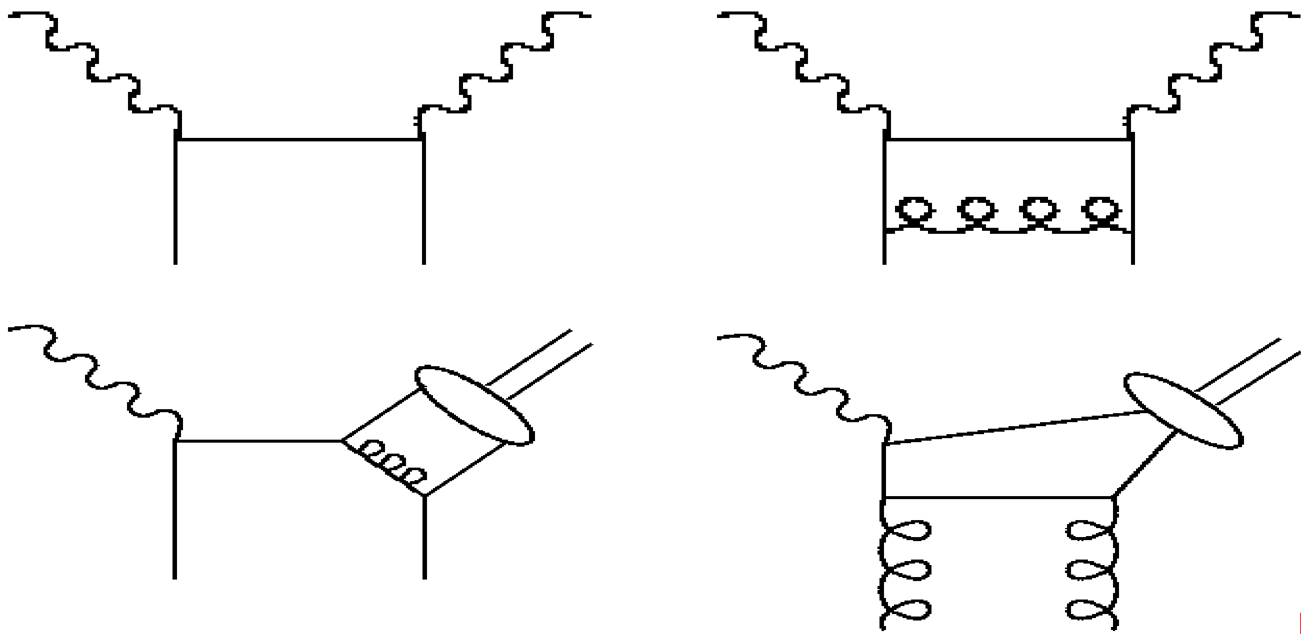
$\Big|^2 + \Big|$



$\Big|^2$ +...

1/ Generalized partons distributions (GPDs)

How to obtain information about GPDs : exclusive interactions with the proton

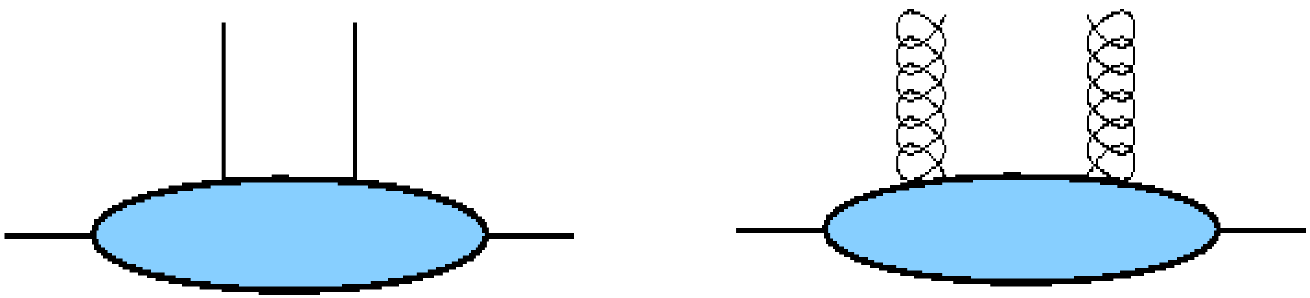


← DVCS

Electromagnetic probe
Calculable perturbatively

← HEMP

Factorization for : - Hight Q2
- Low t
- Longitudinal γ^*



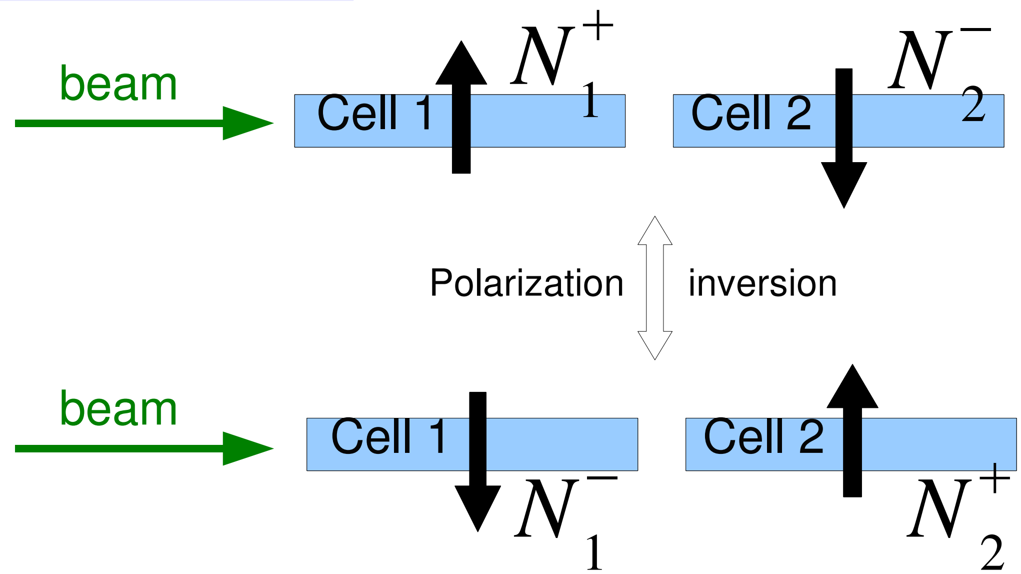
Quark Gpd
(twist 2)

Gluon Gpd
(twist 2)

Nucleon structure
Non perturbative

4/ Transverse Target Spin Asymmetries (A_{UT})

Double ratio method



$$N^{\pm}_{1,2}(\phi-\phi_S)=F^{\pm}_{1,2}\,a^{\pm}_{1,2}(\phi-\phi_S)\,\sigma_0\left(1\pm f\langle P_T\rangle A_{UT}\sin(\phi-\phi_S)\right)$$

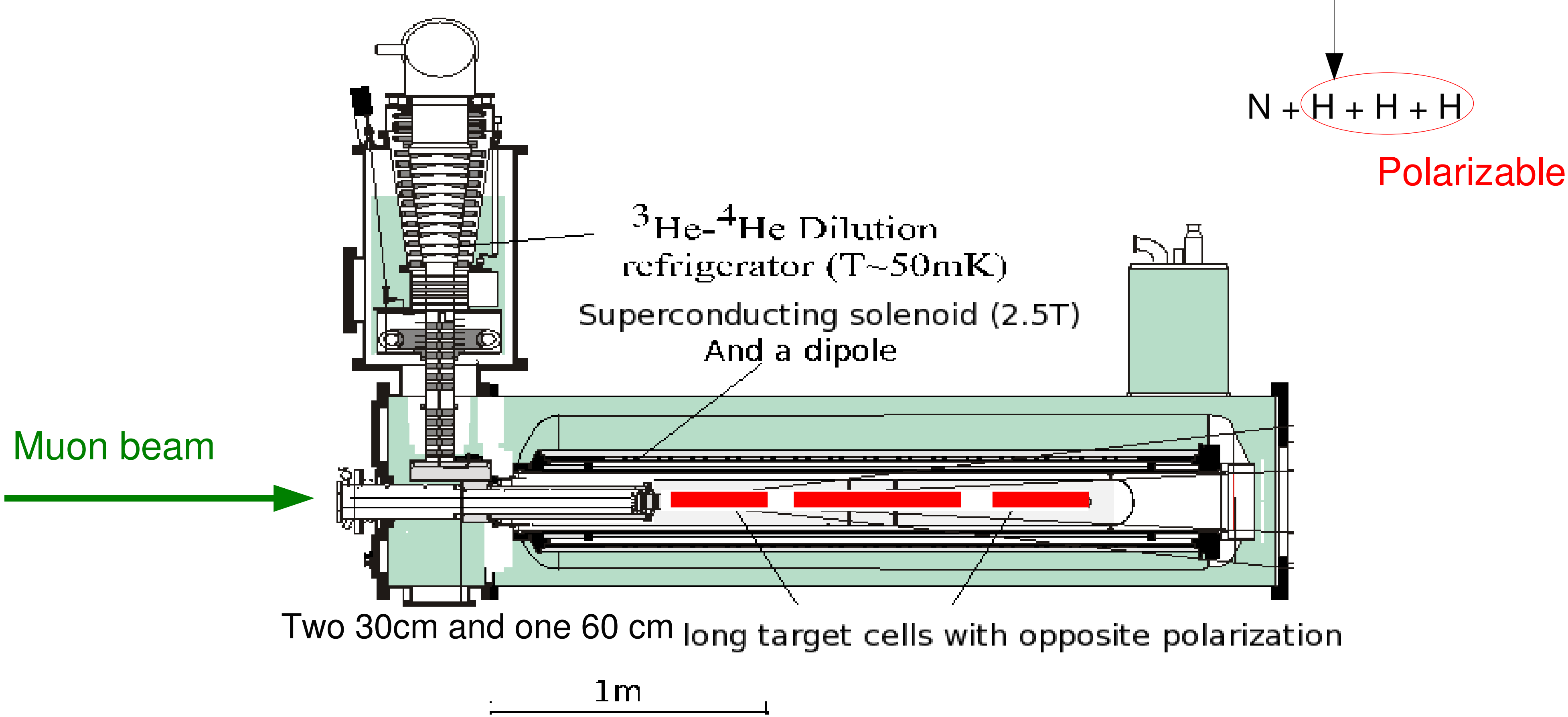
Flux — Acceptance — Dilution factor — Mean target polarization

$$\frac{N_1^+(\phi-\phi_S)N_2^+(\phi-\phi_S)}{N_1^-(\phi-\phi_S+\pi)N_2^-(\phi-\phi_S+\pi)}=\frac{F_1^+\,F_2^+}{F_2^-\,F_1^-}\,\frac{a_1^+(\phi-\phi_S)a_2^+(\phi-\phi_S)}{a_2^-(\phi-\phi_S+\pi)a_1^-(\phi-\phi_S+\pi)}\,\frac{(1+f\langle P_T\rangle A_{UT}\sin(\phi-\phi_S))^2}{(1-f\langle P_T\rangle A_{UT}\sin(\phi-\phi_S))^2}$$

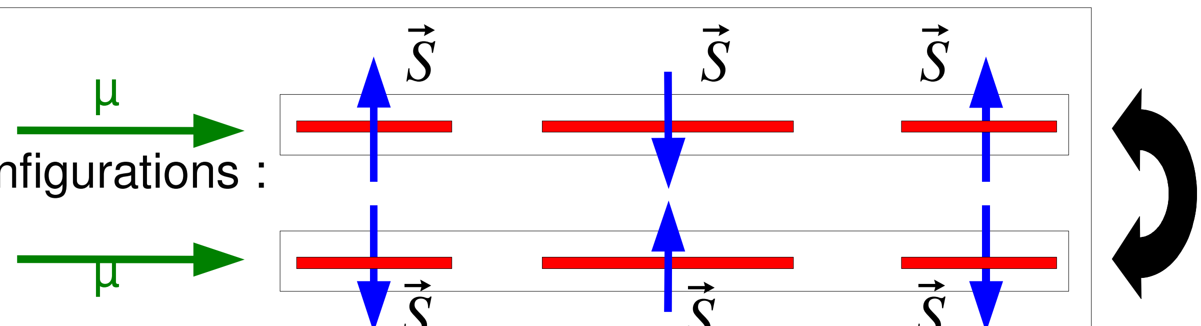
$$\frac{N_1^+(\phi-\phi_S)N_2^+(\phi-\phi_S)}{N_1^-(\phi-\phi_S+\pi)N_2^-(\phi-\phi_S+\pi)}\approx C\left[1+4f\langle|P_T|\rangle A_{UT}\sin(\phi-\phi_S)\right]$$

The COMPASS polarized ammonia target (2007)

Transversally (or longitudinally) polarized proton target : NH_3 $P_T \sim 90\%$



2 polarization configurations :



From M.Dielh : Vector meson production from a polarized nucleon, 0704.1565 [hep-ph]

$$\frac{d\sigma}{d\psi\, d\phi\, d\varphi\, d(\cos\vartheta)\, dx_B\, dQ^2\, dt} = \frac{1}{(2\pi)^2}\, \frac{d\sigma}{dx_B\, dQ^2\, dt}$$

$$\times \Big(W_{UU} + P_\ell W_{LU} + S_L W_{UL} + P_\ell S_L W_{LL} + S_T W_{UT} + P_\ell S_T W_{LT} \Big)$$

$$W_{XY}(\phi, \varphi, \vartheta)$$

$$= \frac{3}{4\pi} \Big[\cos^2\vartheta\; W_{XY}^{LL}(\phi) + \sqrt{2}\cos\vartheta\; \sin\vartheta\; W_{XY}^{LT}(\phi, \varphi) + \sin^2\vartheta\; W_{XY}^{TT}(\phi, \varphi) \Big]$$

$$
\begin{aligned}
W_{UU}^{LL}(\phi) &= \big(u_{++}^{0\,0} + \epsilon u_{0\,0}^{0\,0}\big) - 2\cos\phi\,\sqrt{\epsilon(1+\epsilon)}\,\mathrm{Re}\,u_{0+}^{0\,0} - \cos(2\phi)\,\epsilon u_{-+}^{0\,0}\,, \\
W_{UU}^{LT}(\phi,\varphi) &= \cos(\phi+\varphi)\,\sqrt{\epsilon(1+\epsilon)}\,\mathrm{Re}\big(u_{0+}^{0+} - u_{0+}^{-0}\big) \\
&\quad - \cos\varphi\;\mathrm{Re}\big(u_{++}^{0+} - u_{++}^{-0} + 2\epsilon u_{0\,0}^{0+}\big) + \cos(2\phi+\varphi)\,\epsilon\,\mathrm{Re}\,u_{-+}^{0+} \\
&\quad - \cos(\phi-\varphi)\,\sqrt{\epsilon(1+\epsilon)}\,\mathrm{Re}\big(u_{0+}^{0-} - u_{0+}^{+0}\big) + \cos(2\phi-\varphi)\,\epsilon\,\mathrm{Re}\,u_{-+}^{+0}\,, \\
W_{UU}^{TT}(\phi,\varphi) &= \tfrac{1}{2}\big(u_{++}^{++} + u_{++}^{--} + 2\epsilon u_{0\,0}^{++}\big) + \tfrac{1}{2}\cos(2\phi+2\varphi)\,\epsilon u_{-+}^{-+} \\
&\quad - \cos\phi\,\sqrt{\epsilon(1+\epsilon)}\,\mathrm{Re}\big(u_{0+}^{++} + u_{0+}^{--}\big) + \cos(\phi+2\varphi)\,\sqrt{\epsilon(1+\epsilon)}\,\mathrm{Re}\,u_{0+}^{-+} \\
&\quad - \cos(2\varphi)\;\mathrm{Re}\big(u_{++}^{-+} + \epsilon u_{0\,0}^{-+}\big) - \cos(2\phi)\,\epsilon\,\mathrm{Re}\,u_{-+}^{++} \\
&\quad + \cos(\phi-2\varphi)\,\sqrt{\epsilon(1+\epsilon)}\,\mathrm{Re}\,u_{0+}^{+-} + \tfrac{1}{2}\cos(2\phi-2\varphi)\,\epsilon u_{-+}^{+-}\,. \qquad (4.10)
\end{aligned}
$$

$$
\begin{aligned}
W_{LU}^{LL}(\phi) &= -2\sin\phi\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Im}\,u_{0+}^{0\,0}\,, \\
W_{LU}^{LT}(\phi,\varphi) &= \sin(\phi+\varphi)\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Im}\big(u_{0+}^{0+} - u_{0+}^{-0}\big) \\
&\quad - \sin\varphi\,\sqrt{1-\epsilon^2}\,\mathrm{Im}\big(u_{++}^{0+} - u_{++}^{-0}\big) \\
&\quad - \sin(\phi-\varphi)\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Im}\big(u_{0+}^{0-} - u_{0+}^{+0}\big)\,, \\
W_{LU}^{TT}(\phi,\varphi) &= -\sin\phi\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Im}\big(u_{0+}^{++} + u_{0+}^{--}\big) + \sin(\phi+2\varphi)\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Im}\,u_{0+}^{-+} \\
&\quad - \sin(2\varphi)\,\sqrt{1-\epsilon^2}\,\mathrm{Im}\,u_{++}^{-+} \\
&\quad + \sin(\phi-2\varphi)\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Im}\,u_{0+}^{+-}\,.
\end{aligned}
\tag{4.12}
$$

$$
\begin{aligned}
W_{UL}^{LL}(\phi) &= -2\sin\phi\,\sqrt{\epsilon(1+\epsilon)}\,\operatorname{Im} l_{0+}^{0\,0} - \sin(2\phi)\,\epsilon\,\operatorname{Im} l_{-+}^{0\,0}\,, \\
W_{UL}^{LT}(\phi,\varphi) &= \sin(\phi+\varphi)\,\sqrt{\epsilon(1+\epsilon)}\,\operatorname{Im}\big(l_{0+}^{0+} - l_{0+}^{-0}\big) \\
&\quad - \sin\varphi\;\operatorname{Im}\big(l_{++}^{0+} - l_{++}^{-0} + 2\epsilon\, l_{0\,0}^{0+}\big) + \sin(2\phi+\varphi)\,\epsilon\,\operatorname{Im} l_{-+}^{0+} \\
&\quad - \sin(\phi-\varphi)\,\sqrt{\epsilon(1+\epsilon)}\,\operatorname{Im}\big(l_{0+}^{0-} - l_{0+}^{+0}\big) + \sin(2\phi-\varphi)\,\epsilon\,\operatorname{Im} l_{-+}^{+0}\,, \\
W_{UL}^{TT}(\phi,\varphi) &= \tfrac{1}{2}\sin(2\phi+2\varphi)\,\epsilon\,\operatorname{Im} l_{-+}^{-+} \\
&\quad - \sin\phi\,\sqrt{\epsilon(1+\epsilon)}\,\operatorname{Im}\big(l_{0+}^{++} + l_{0+}^{--}\big) + \sin(\phi+2\varphi)\,\sqrt{\epsilon(1+\epsilon)}\,\operatorname{Im} l_{0+}^{-+} \\
&\quad - \sin(2\varphi)\;\operatorname{Im}\big(l_{++}^{-+} + \epsilon\, l_{0\,0}^{-+}\big) - \sin(2\phi)\,\epsilon\,\operatorname{Im} l_{-+}^{++} \\
&\quad + \sin(\phi-2\varphi)\,\sqrt{\epsilon(1+\epsilon)}\,\operatorname{Im} l_{0+}^{+-} + \tfrac{1}{2}\sin(2\phi-2\varphi)\,\epsilon\,\operatorname{Im} l_{-+}^{+-}
\end{aligned}
\tag{4.13}
$$

$$
\begin{aligned}
W_{LL}^{LL}(\phi) &= -2\cos\phi\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Re}\,l_{0+}^{0\,0} + \sqrt{1-\epsilon^2}\;l_{++}^{0\,0}\,, \\
W_{LL}^{LT}(\phi,\varphi) &= \cos(\phi+\varphi)\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Re}\big(l_{0+}^{0+} - l_{0+}^{-0}\big) \\
&\quad - \cos\varphi\,\sqrt{1-\epsilon^2}\;\mathrm{Re}\big(l_{++}^{0+} - l_{++}^{-0}\big) \\
&\quad - \cos(\phi-\varphi)\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Re}\big(l_{0+}^{0-} - l_{0+}^{+0}\big)\,, \\
W_{LL}^{TT}(\phi,\varphi) &= \sqrt{1-\epsilon^2}\;\tfrac{1}{2}\big(l_{++}^{++} + l_{++}^{--}\big) \\
&\quad - \cos\phi\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Re}\big(l_{0+}^{++} + l_{0+}^{--}\big) + \cos(\phi+2\varphi)\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Re}\,l_{0+}^{-+} \\
&\quad - \cos(2\varphi)\,\sqrt{1-\epsilon^2}\;\mathrm{Re}\,l_{++}^{-+} \\
&\quad + \cos(\phi-2\varphi)\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Re}\,l_{0+}^{+-}
\end{aligned}
\qquad (4.14)
$$

$$
\begin{aligned}
W_{UT}^{LL}(\phi_S,\phi) &= \sin(\phi-\phi_S)\Big[\,\mathrm{Im}\big(n_{++}^{0\,0}+\epsilon n_{0\,0}^{0\,0}\big) \\
&\qquad -2\cos\phi\,\sqrt{\epsilon(1+\epsilon)}\,\mathrm{Im}\,n_{0+}^{0\,0}-\cos(2\phi)\,\epsilon\,\mathrm{Im}\,n_{-+}^{0\,0}\Big] \\
&\quad +\cos(\phi-\phi_S)\Big[-2\sin\phi\,\sqrt{\epsilon(1+\epsilon)}\,\mathrm{Im}\,s_{0+}^{0\,0}-\sin(2\phi)\,\epsilon\,\mathrm{Im}\,s_{-+}^{0\,0}\Big], \\
W_{UT}^{LT}(\phi_S,\phi,\varphi) &= \sin(\phi-\phi_S)\Big[\cos(\phi+\varphi)\,\sqrt{\epsilon(1+\epsilon)}\,\mathrm{Im}\big(n_{0+}^{0+}-n_{0+}^{-0}\big) \\
&\qquad -\cos\varphi\;\mathrm{Im}\big(n_{++}^{0+}-n_{++}^{-0}+2\epsilon n_{0\,0}^{0+}\big)+\cos(2\phi+\varphi)\,\epsilon\,\mathrm{Im}\,n_{-+}^{0+} \\
&\qquad -\cos(\phi-\varphi)\,\sqrt{\epsilon(1+\epsilon)}\,\mathrm{Im}\big(n_{0+}^{0-}-n_{0+}^{+0}\big)+\cos(2\phi-\varphi)\,\epsilon\,\mathrm{Im}\,n_{-+}^{+0}\Big] \\
&\quad +\cos(\phi-\phi_S)\Big[\sin(\phi+\varphi)\,\sqrt{\epsilon(1+\epsilon)}\,\mathrm{Im}\big(s_{0+}^{0+}-s_{0+}^{-0}\big) \\
&\qquad -\sin\varphi\;\mathrm{Im}\big(s_{++}^{0+}-s_{++}^{-0}+2\epsilon s_{0\,0}^{0+}\big)+\sin(2\phi+\varphi)\,\epsilon\,\mathrm{Im}\,s_{-+}^{0+} \\
&\qquad -\sin(\phi-\varphi)\,\sqrt{\epsilon(1+\epsilon)}\,\mathrm{Im}\big(s_{0+}^{0-}-s_{0+}^{+0}\big)+\sin(2\phi-\varphi)\,\epsilon\,\mathrm{Im}\,s_{-+}^{+0}\Big],
\end{aligned}
$$

$$
\begin{aligned}
W_{UT}^{TT}(\phi_S,\phi,\varphi) = \sin(\phi-\phi_S)\Big[& \tfrac{1}{2}\,\mathrm{Im}\big(n_{++}^{++}+n_{++}^{--}+2\epsilon n_{0\,0}^{++}\big)+\tfrac{1}{2}\cos(2\phi+2\varphi)\,\epsilon\,\mathrm{Im}\,n_{-+}^{-+} \\
& -\cos\phi\,\sqrt{\epsilon(1+\epsilon)}\,\mathrm{Im}\big(n_{0+}^{++}+n_{0+}^{--}\big)+\cos(\phi+2\varphi)\,\sqrt{\epsilon(1+\epsilon)}\,\mathrm{Im}\,n_{0+}^{-+} \\
& -\cos(2\varphi)\;\mathrm{Im}\big(n_{++}^{-+}+\epsilon n_{0\,0}^{-+}\big)-\cos(2\phi)\,\epsilon\,\mathrm{Im}\,n_{-+}^{++} \\
& +\cos(\phi-2\varphi)\,\sqrt{\epsilon(1+\epsilon)}\,\mathrm{Im}\,n_{0+}^{+-}+\tfrac{1}{2}\cos(2\phi-2\varphi)\,\epsilon\,\mathrm{Im}\,n_{-+}^{+-}\Big] \\
+\cos(\phi-\phi_S)\Big[& \tfrac{1}{2}\sin(2\phi+2\varphi)\,\epsilon\,\mathrm{Im}\,s_{-+}^{-+} \\
& -\sin\phi\,\sqrt{\epsilon(1+\epsilon)}\,\mathrm{Im}\big(s_{0+}^{++}+s_{0+}^{--}\big)+\sin(\phi+2\varphi)\,\sqrt{\epsilon(1+\epsilon)}\,\mathrm{Im}\,s_{0+}^{-+} \\
& -\sin(2\varphi)\;\mathrm{Im}\big(s_{++}^{-+}+\epsilon s_{0\,0}^{-+}\big)-\sin(2\phi)\,\epsilon\,\mathrm{Im}\,s_{-+}^{++} \\
& +\sin(\phi-2\varphi)\,\sqrt{\epsilon(1+\epsilon)}\,\mathrm{Im}\,s_{0+}^{+-}+\tfrac{1}{2}\sin(2\phi-2\varphi)\,\epsilon\,\mathrm{Im}\,s_{-+}^{+-}\Big]
\end{aligned}
\qquad (4.17)
$$

$$
\begin{aligned}
W_{LT}^{LL}(\phi_S,\phi) &= \sin(\phi-\phi_S)\Big[\ 2\sin\phi\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Re}\,n_{0+}^{0\,0}\ \Big] \\
&\quad + \cos(\phi-\phi_S)\Big[-2\cos\phi\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Re}\,s_{0+}^{0\,0} + \sqrt{1-\epsilon^2}\,s_{++}^{0\,0}\Big]\,, \\
W_{LT}^{LT}(\phi_S,\phi,\varphi) &= \sin(\phi-\phi_S)\Big[-\sin(\phi+\varphi)\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Re}\big(n_{0+}^{0+} - n_{0+}^{-0}\big) \\
&\qquad + \sin\varphi\,\sqrt{1-\epsilon^2}\,\mathrm{Re}\big(n_{++}^{0+} - n_{++}^{-0}\big) \\
&\qquad + \sin(\phi-\varphi)\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Re}\big(n_{0+}^{0-} - n_{0+}^{+0}\big)\Big] \\
&\quad + \cos(\phi-\phi_S)\Big[\cos(\phi+\varphi)\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Re}\big(s_{0+}^{0+} - s_{0+}^{-0}\big) \\
&\qquad - \cos\varphi\,\sqrt{1-\epsilon^2}\,\mathrm{Re}\big(s_{++}^{0+} - s_{++}^{-0}\big) \\
&\qquad - \cos(\phi-\varphi)\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Re}\big(s_{0+}^{0-} - s_{0+}^{+0}\big)\Big]\,,
\end{aligned}
$$

$$
\begin{aligned}
W_{LT}^{TT}(\phi_S,\phi,\varphi) = \sin(\phi-\phi_S) \\
&\times \Big[\sin\phi\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Re}\big(n_{0+}^{++}+n_{0+}^{--}\big) - \sin(\phi+2\varphi)\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Re}\,n_{0+}^{-+} \\
&\quad + \sin(2\varphi)\,\sqrt{1-\epsilon^2}\,\mathrm{Re}\,n_{++}^{-+} \\
&\quad - \sin(\phi-2\varphi)\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Re}\,n_{0+}^{+-} \Big] \\
&+ \cos(\phi-\phi_S)\Big[\sqrt{1-\epsilon^2}\,\tfrac{1}{2}\big(s_{++}^{++}+s_{++}^{--}\big) \\
&\quad - \cos\phi\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Re}\big(s_{0+}^{++}+s_{0+}^{--}\big) + \cos(\phi+2\varphi)\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Re}\,s_{0+}^{-+} \\
&\quad - \cos(2\varphi)\,\sqrt{1-\epsilon^2}\,\mathrm{Re}\,s_{++}^{-+} \\
&\quad + \cos(\phi-2\varphi)\,\sqrt{\epsilon(1-\epsilon)}\,\mathrm{Re}\,s_{0+}^{+-} \Big] \qquad (4.18)
\end{aligned}
$$