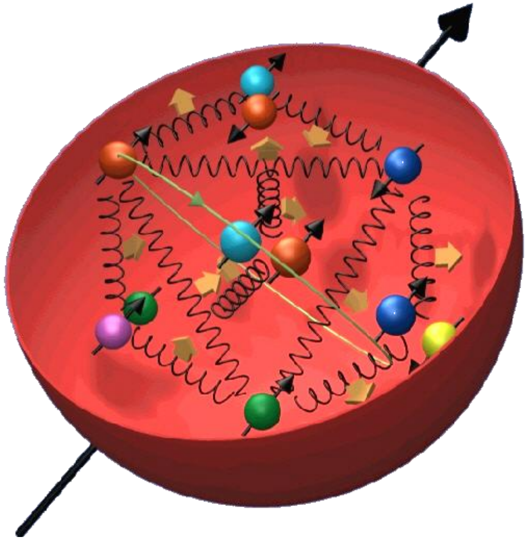
The Nucleon Spin Structure



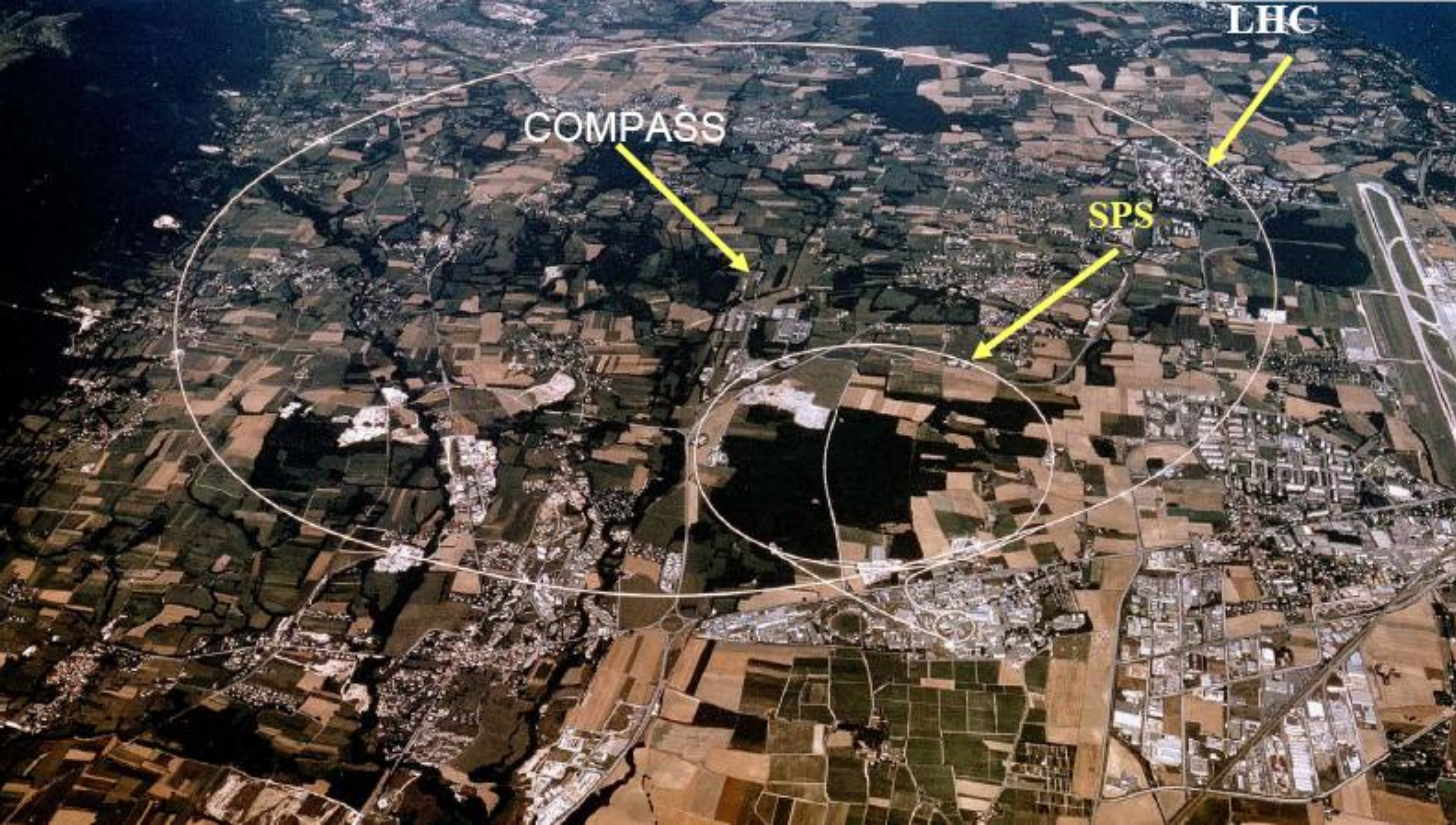
Gerhard Mallot

Lecture 2

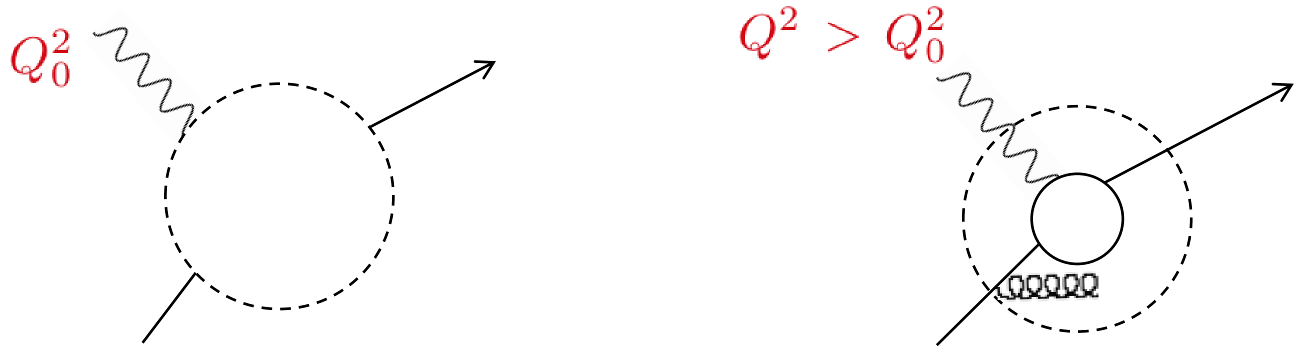
- Experimental status
  - Q^2 evolution, scaling violations, DGLAP
  - status of g_1 and QCD analyses
  - interplay: g_2
  - semi-inclusive data
  - ΔG from hadron pairs

- longitudinally polarised muon beam
- longitudinally or transversely polarised deuteron (^{6}LiD) target
- momentum and calorimetry measurement
- particle identification

<u>luminosity:</u> ~$5 \cdot 10^{32}$ cm^{-2} s^{-1}
<u>beam intensity:</u> $2 \cdot 10^8$ μ+/spill (4.8s/16.2s)
<u>beam momentum:</u> 160 GeV/*c*
<u>beam polarization</u>: ~76 %
<u>target polarization:</u> ~50 %

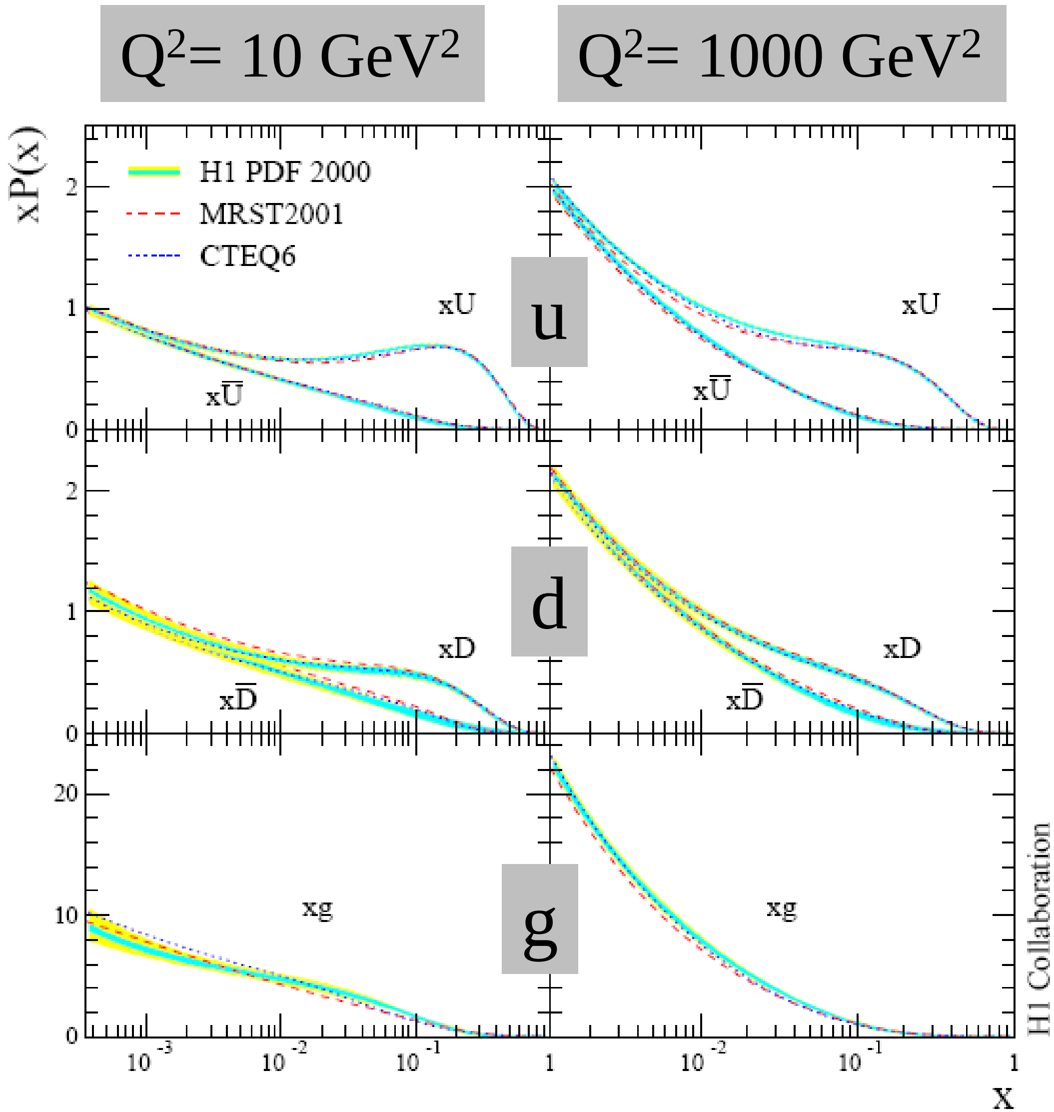


1. Q^2 evolution of structure functions



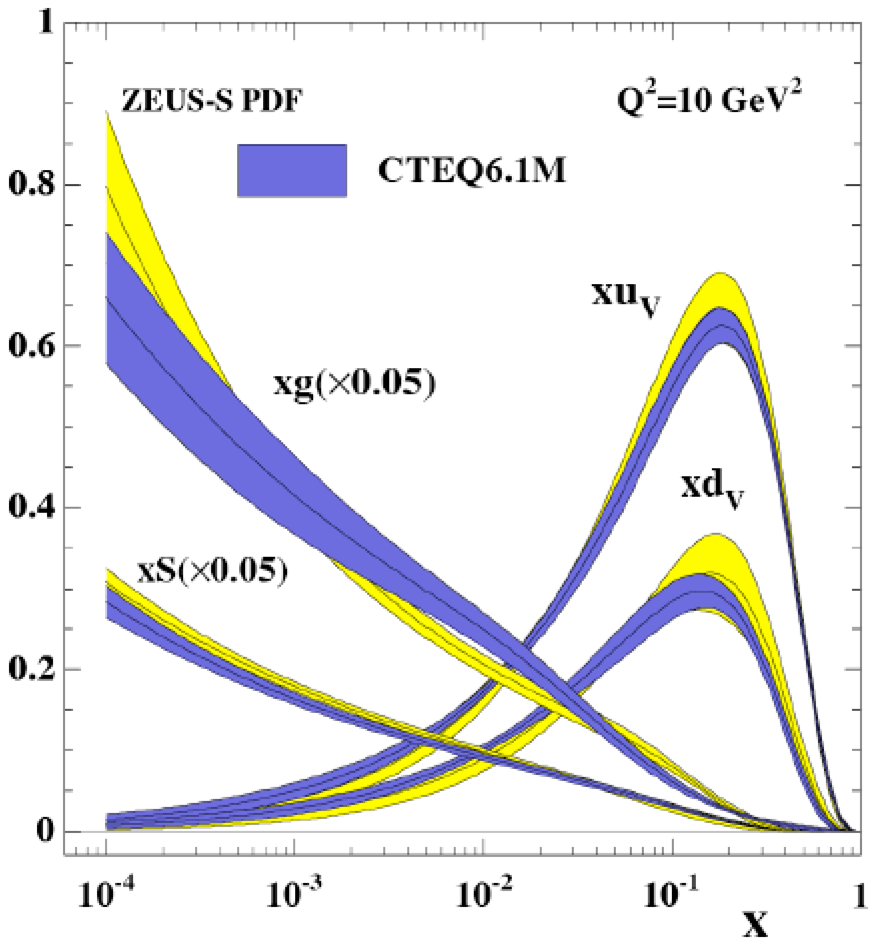
- with increasing Q^2 more details are resolved
- quarks/gluons split and produce more partons
- the 'new' partons have smaller x-Bjorken, since the new partons have to share the momentum
- scaling violations in PDFs and SFs: $P(x) \rightarrow P(x,Q^2)$
- the Q^2 evolution is calculable in perturbative QCD, if the PDFs $P(x,Q^2_0)$ are known at some Q^2_0
- x dependence is non-perturbative and not described in pQCD

Parton Distribution Functions



- unpolarised
- H1 analysis of HERA ep data
- strong increase at small *x* with Q^2
- the various fits agrees very well
- fully constrained by data

PDFs



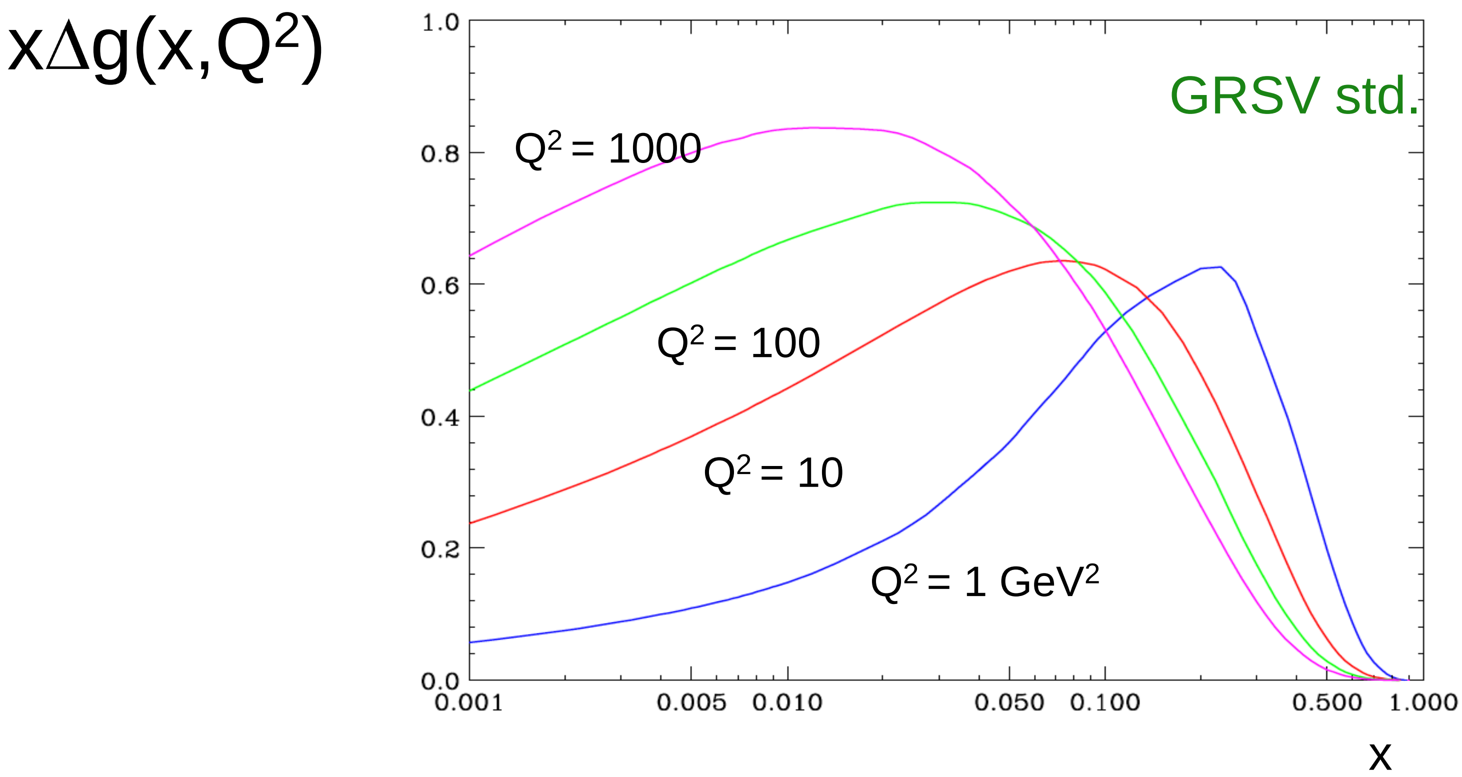
- Zeus analysis
- valence quark distr.

$$q_V = q - \bar{q}$$

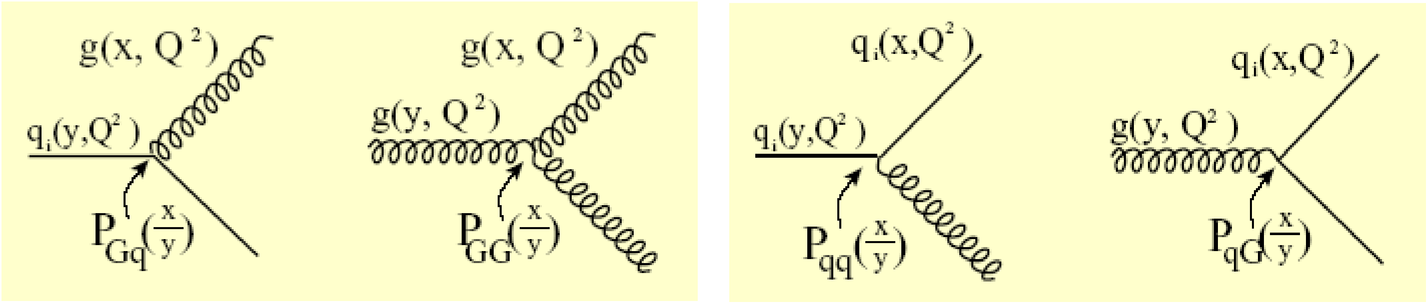
- sea quark distr.

$$S = 2\bar{u} + 2\bar{d} + s + \bar{s}$$

Polarised gluon evolution $\Delta g(x,Q^2)$



Splitting functions ΔP



$$\Delta\mathcal{P}_{ij} = \frac{\alpha_s}{2\pi}\Delta\mathcal{P}^{(0)}_{ij}(z) + \frac{\alpha_s}{2\pi}^2\Delta\mathcal{P}^{(1)}_{ij}(z) + \ldots$$

LO NLO

define:

$$(a \otimes b)(x) := \int_x^1 \frac{\mathrm{d}y}{y} a\left(\frac{x}{y}\right) b(y)$$

$$\Delta q^{\mathrm{ns}}(x,Q^2) = \sum_{i=1}^{n}\left(\frac{e_i^2}{\langle e^2\rangle} - 1\right)\Delta q_i(x,Q^2)$$

$$\Delta q^{\mathrm{s}}(x,Q^2) = \sum_{i=1}^{n}\Delta q_i(x,Q^2)$$

Q^2 evolution & DGLAP equations

Dokshitzer '77; Gribov, Lipatov '75; Altarelli, Parisi '77

$$\frac{d}{d\ln Q^2}\Delta q^{ns} = \Delta\mathcal{P}^{ns}_{qq}\otimes\Delta q^{ns}$$

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\Delta q^s\\ \Delta g\end{pmatrix} = \begin{pmatrix}\Delta\mathcal{P}^s_{qq} & \Delta\mathcal{P}^s_{qg}\\ \Delta\mathcal{P}^s_{gq} & \Delta\mathcal{P}^s_{gg}\end{pmatrix}\otimes\begin{pmatrix}\Delta q^s\\ \Delta g\end{pmatrix}$$

$$g_1 = \frac{1}{2}\langle e^2\rangle\left\{C_{ns}\otimes\Delta q^{ns} + C_s\otimes\Delta q^s + C_g\otimes\Delta g\right\}.$$

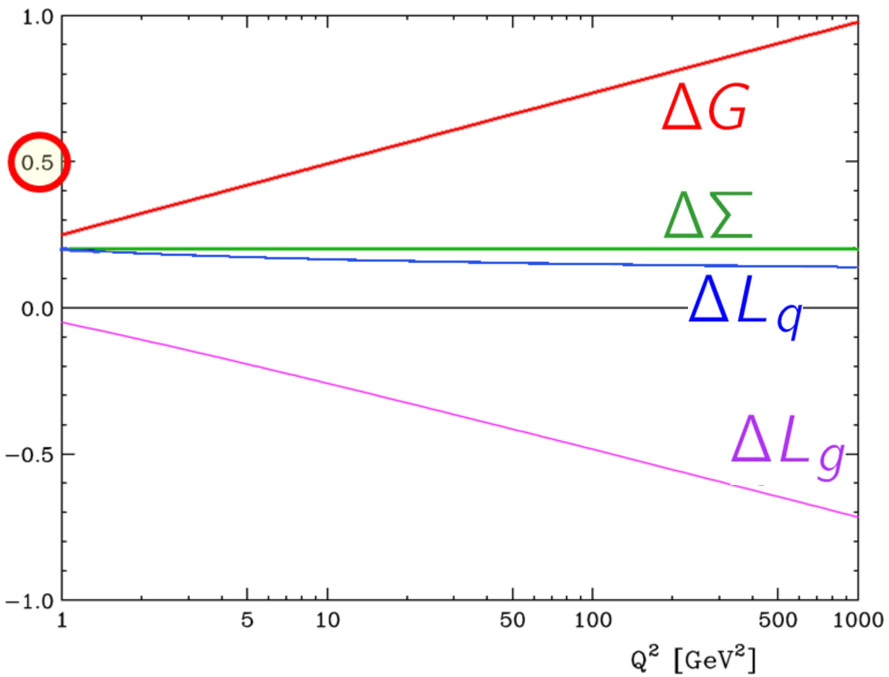
LO coefficient functions: $C_s^{(0)} = C_{ns}^{(0)} = \delta(1-x),\ C_g^{(0)} = 0$

back to PM expression:

$$g_1(x) = \frac{1}{2}\sum_i e_i^2\left\{q_i^+(x) - q_i^-(x)\right\}$$

Evolution of first moments (LO)

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix} = \frac{\alpha_s}{2\pi}\begin{pmatrix}0 & 0 \\ 2 & \frac{1}{2}\beta_0\end{pmatrix}\begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix}$$



$$J_q = \frac{1}{2}\Delta\Sigma + L_q \rightarrow \frac{3n_f}{2(16+3n_f)} = 0.43/2$$

$$J_g = \Delta G + L_g \rightarrow \frac{16}{2(16+3n_f)} = 0.57/2$$

for $Q^2 \rightarrow \infty \quad (n_f = 4)$

Ji

Ratcliffe; Ji, Tang, Hoodbhoy; Hägler, Schäfer

Q^2 evolution

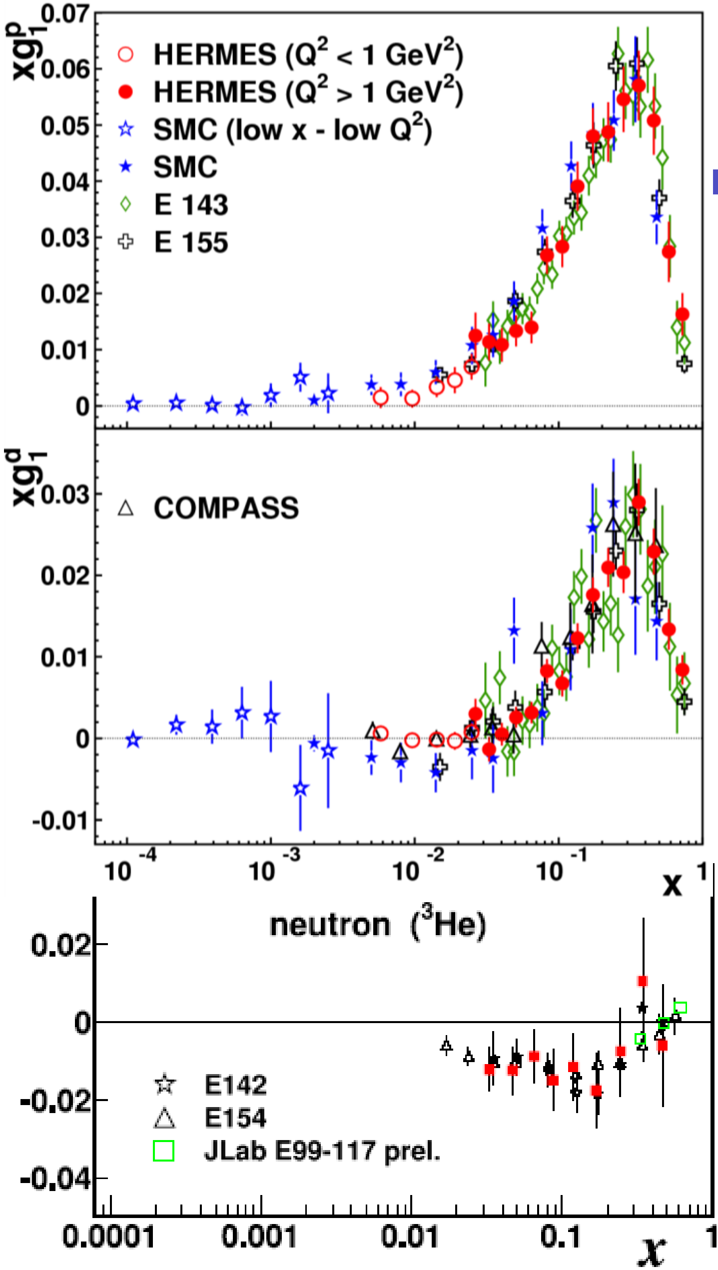
- non-singlet distributions decouple from gluon evolution, moments are Q^2-independent , e.g. $\Delta u - \Delta d = g_a$
- evolution of flavour singlet Δq^s and gluon are coupled
- $\alpha_s \Delta G$ constant in LO for $Q^2 \to \infty$, $\alpha_s \to 0$ therefore scheme ambiguity large ($\overline{\mathrm{MS}} \leftrightarrow$ AB)

$$\Delta\Sigma_{AB} = \Delta\Sigma_{\overline{MS}} + n_f \frac{\alpha_s}{2\pi}\Delta G$$

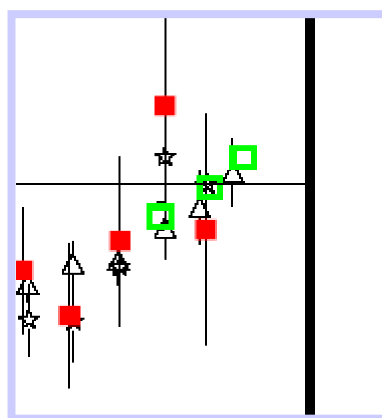
(axial anomaly)

- in principle Δg can be determined from the Q^2 evolution of $g_1(x,Q^2)$ like g in the unpolarised case (DGLAP fit)!
- need reasonable range in Q^2 at fixed x
- we lack a polarised ep collider

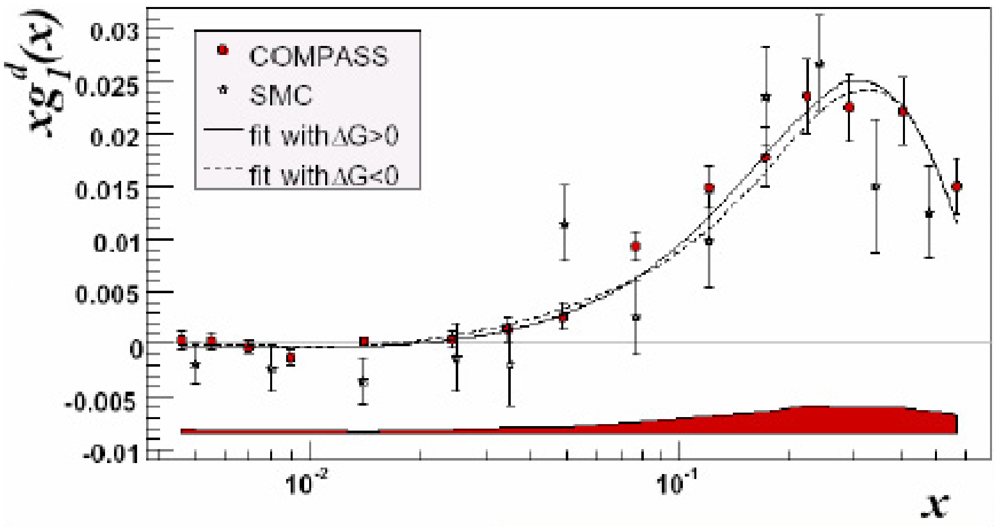
2. Status of g_1

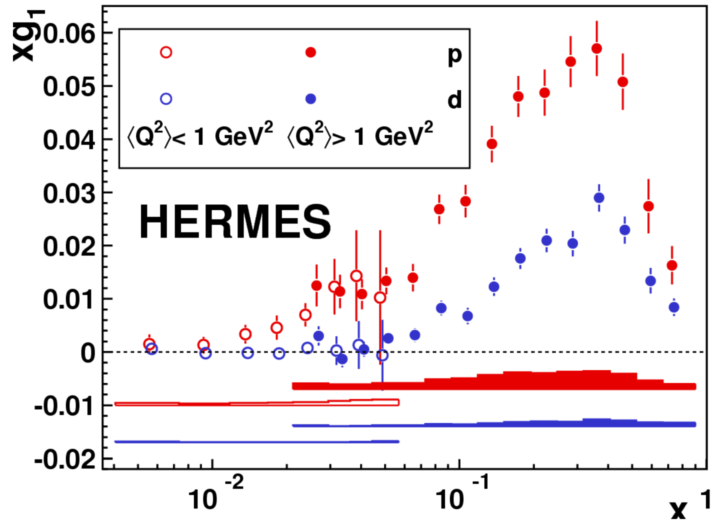


- Wealth of data g_1 data for p, n and d
- Data taken at different Q^2
- Only weak Q^2-dependence in overlap region
- large x neutron data from JLAB for the neutron (^{3}He): $g_1^n > 0$



New g_1 data

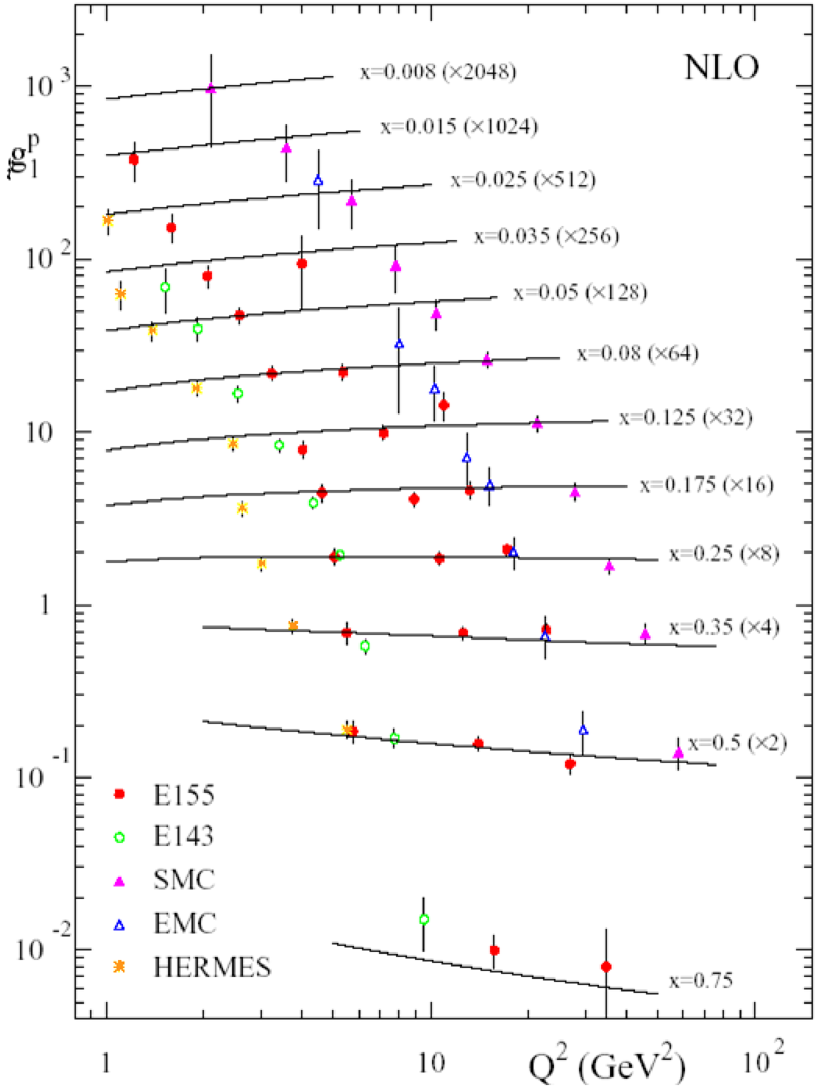




$$a_0 = 0.35 \pm 0.03 \text{ (stat.)} \pm 0.05 \text{ (syst.)}$$

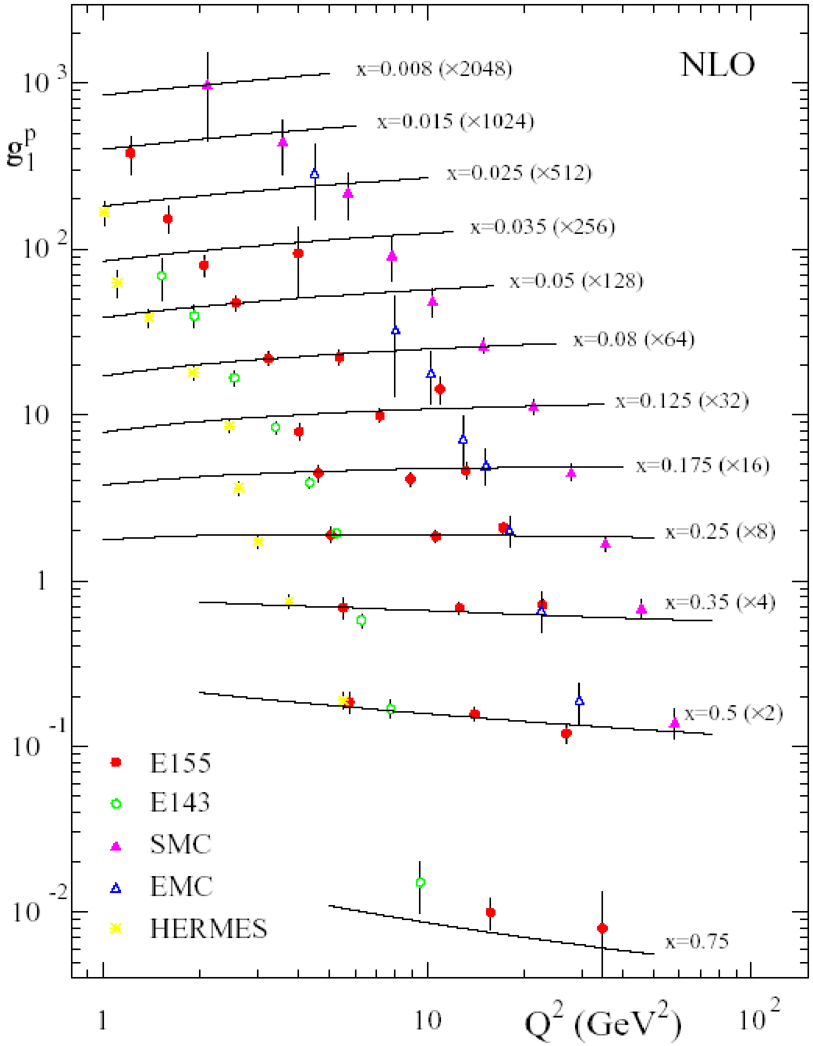
$$a_0 = 0.330 \pm 0.025 \text{ (stat.)} \pm 0.030 \text{ (syst.)}$$

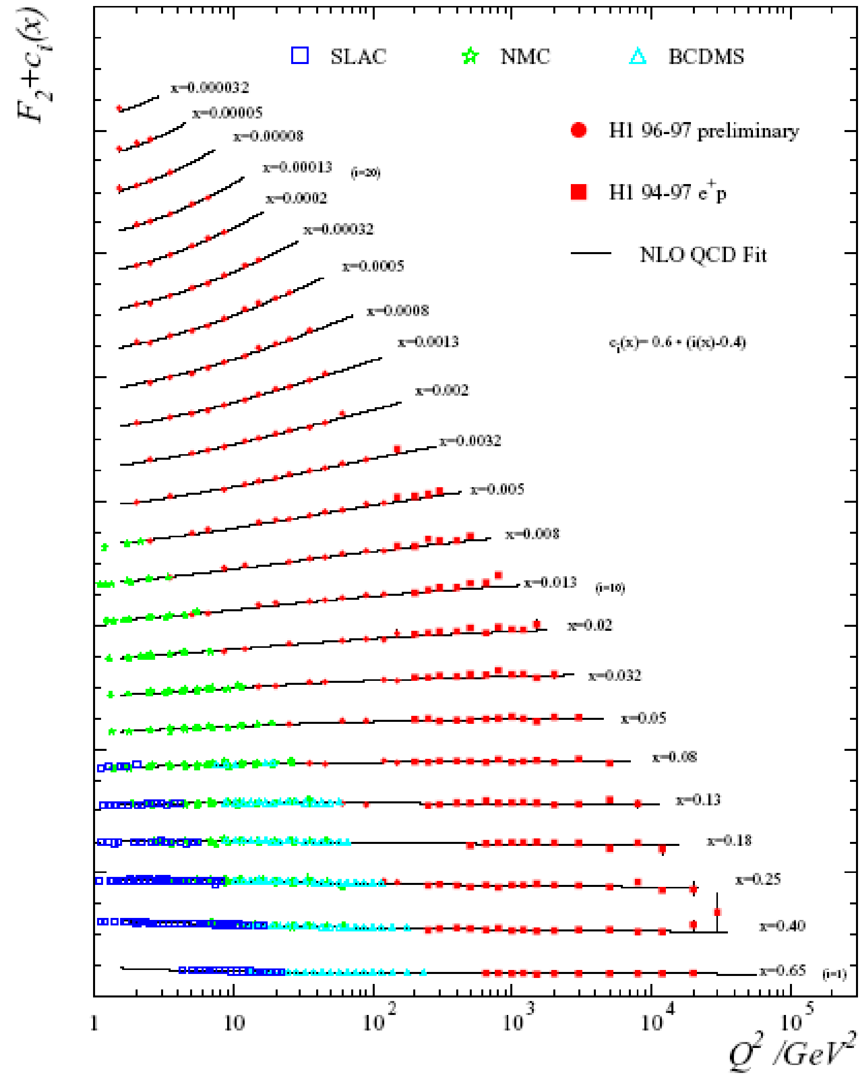
QCD fits to $g_1(x,Q^2)$



Looks quite nice, but...

$g_1(x,Q^2)$ $F_2(x,Q^2)$


NLO
g_1^p
x=0.008 (×2048)
x=0.015 (×1024)
x=0.025 (×512)
x=0.035 (×256)
x=0.05 (×128)
x=0.08 (×64)
x=0.125 (×32)
x=0.175 (×16)
x=0.25 (×8)
x=0.35 (×4)
x=0.5 (×2)
x=0.75
E155
E143
SMC
EMC
HERMES
Q^2 (GeV^2)


$F_2+c_i(x)$
SLAC
NMC
BCDMS
H1 96-97 preliminary
H1 94-97 e$^+$p
NLO QCD Fit
Q^2 /GeV^2

NLO QCD Fits

- choose scheme, usually $\overline{\mathrm{MS}}$
- choose start value for evolution, $Q^2=Q^2_0$
- choose parametrisations for

$$\Delta q^s, \Delta q^{ns}, \Delta g\ (x,Q^2_0)$$

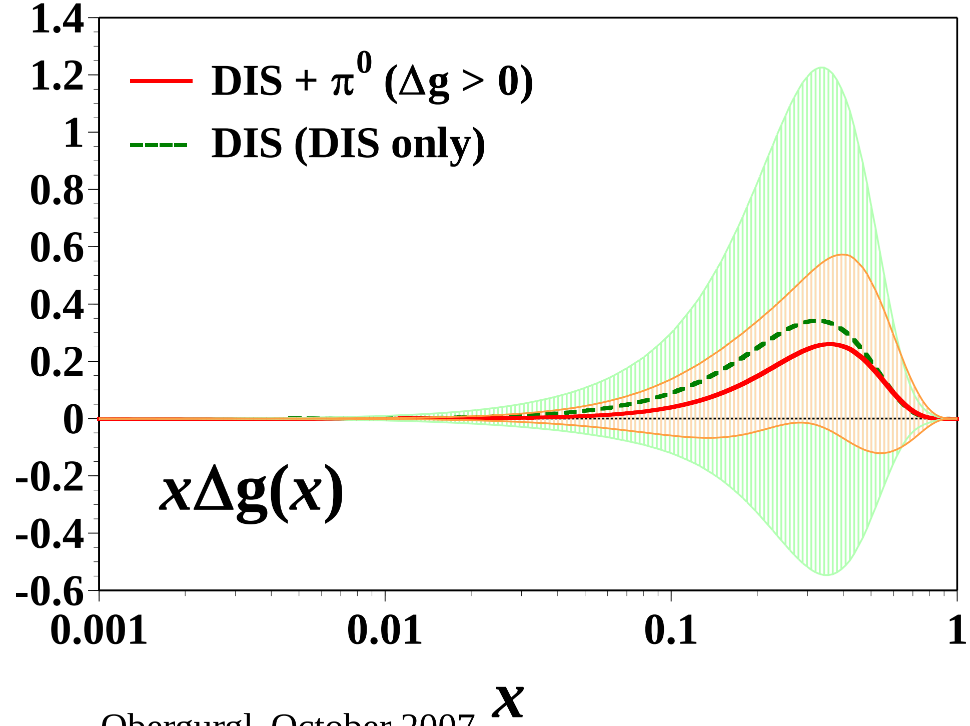
- fit parameters of these parametrisations using the DLGAP equations (NLO)
- extra problems in polarised case
  - no positivity condition
  - no momentum sum rule

NLO QCD fits

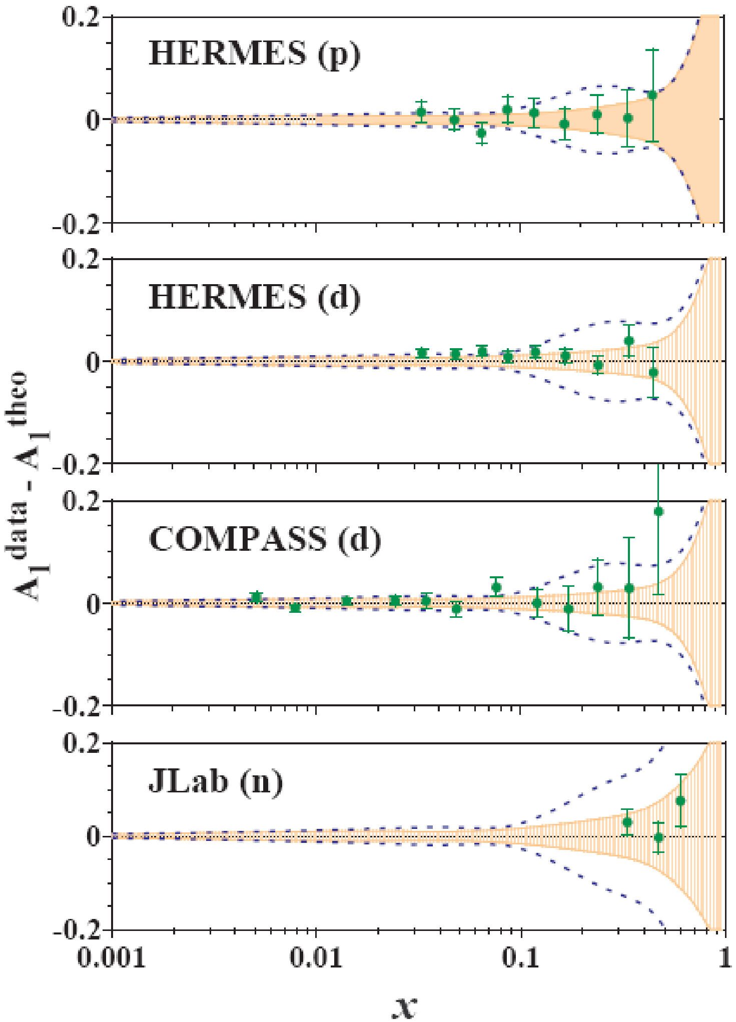
- Many groups, example AAC 2006

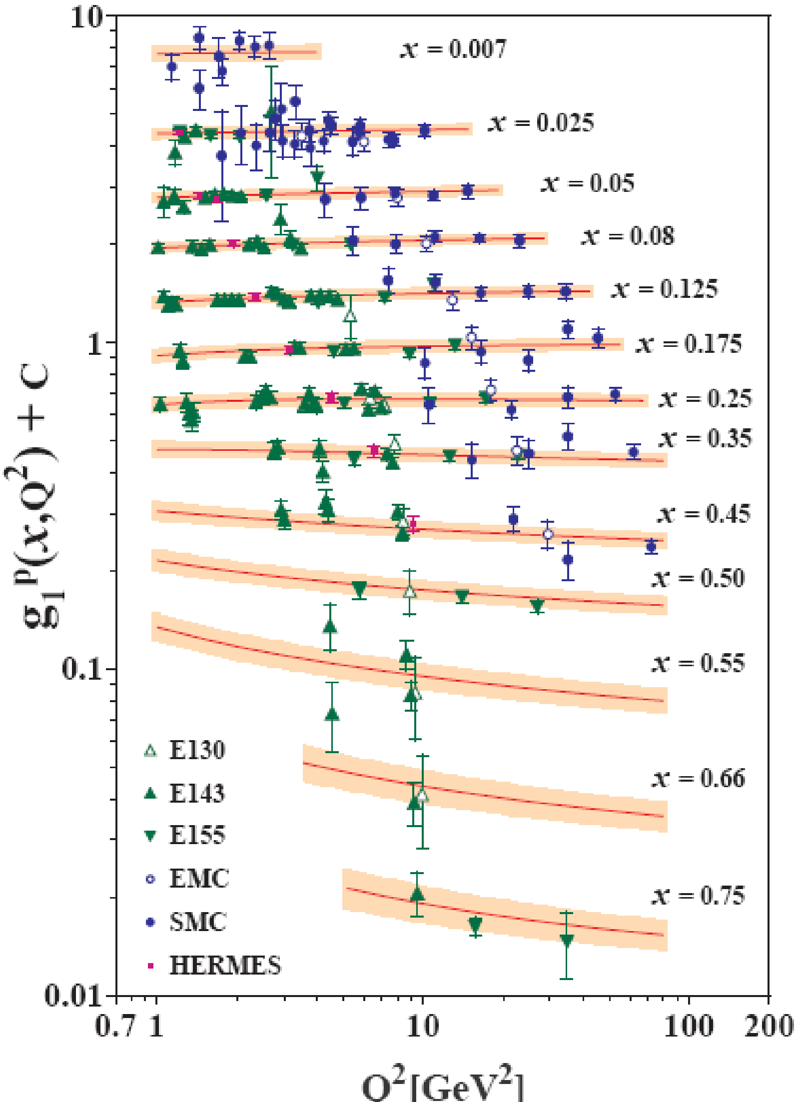
  AAC: Asymmetry analysis collaboration, Japan
  GRSV: Glück, Reya, Stratmann, Vogelsang
  BB: Blümlein, Böttcher
  LSS: Leader, Sidorov, Stamenov
  …

- Still large uncertainty in Δg, even sign not determined

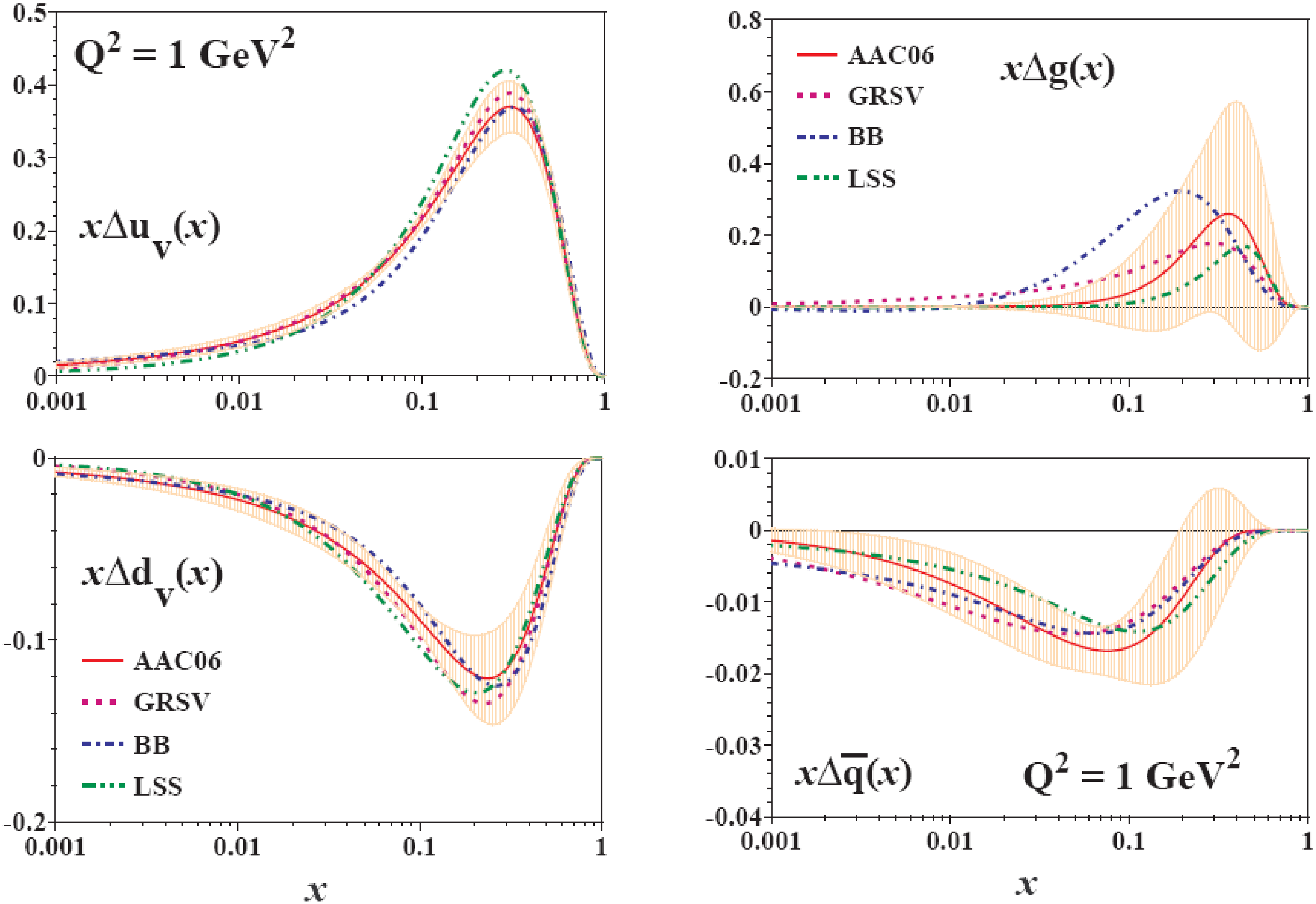


AAC06 analysis





AAC 2006



$Q^2 = 1\ GeV^2$
$x\Delta u_v(x)$
$x\Delta g(x)$
AAC06
GRSV
BB
LSS
$x\Delta d_v(x)$
AAC06
GRSV
BB
LSS
$x\Delta\bar{q}(x)$
$Q^2 = 1\ GeV^2$
0.5
0.4
0.3
0.2
0.1
0
0.8
0.6
0.4
0.2
0
-0.2
-0.1
-0.2
0.01
0
-0.01
-0.02
-0.03
-0.04
0.001
0.01
0.1
1
x
x

First moments

$Q^2 = 1\ \mathrm{GeV}^2$

	ΔG	$\Delta\Sigma$
AAC06	0.47 ± 1.08	0.25 ± 0.10
GRSV01	0.420	0.204
LSS	0.680	0.210
BB	1.026	0.138

only DIS

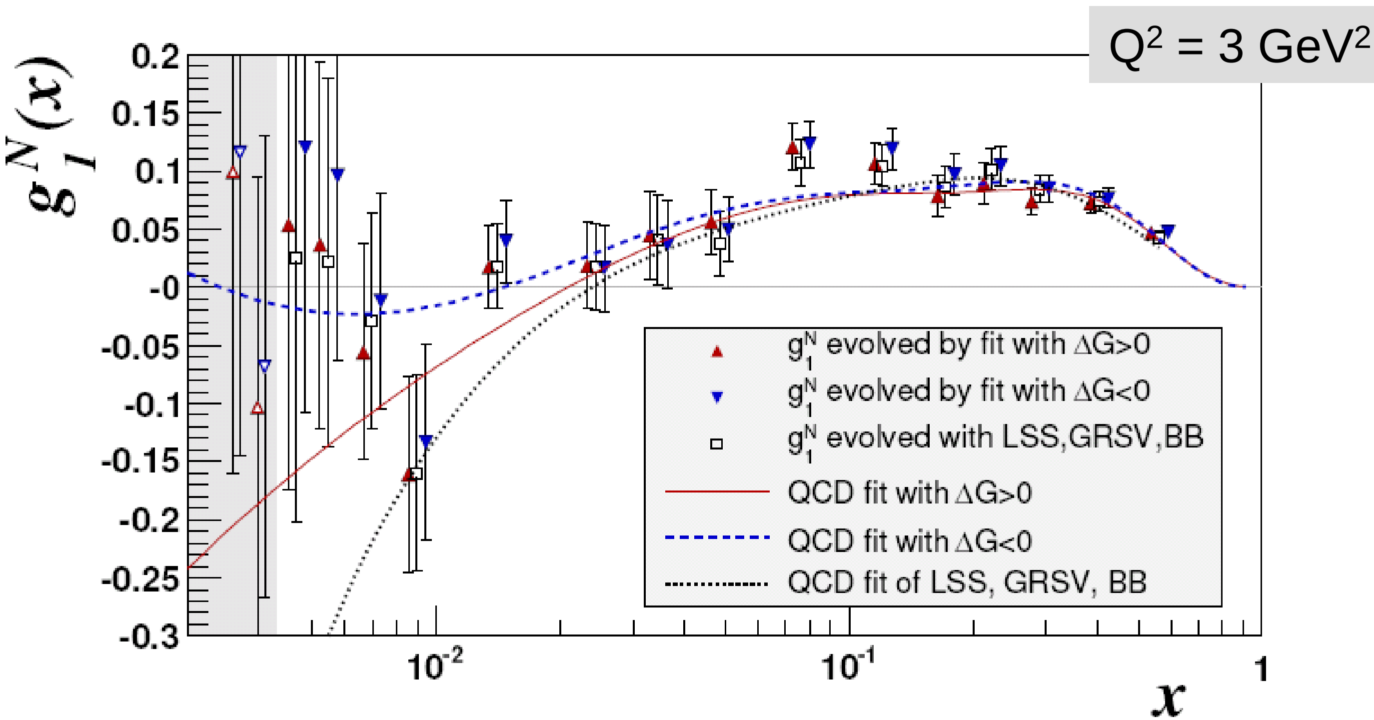
– AAC06 [Phys. Rev. D74 (2006) 014015]
– GRSV01 [Phys. Rev. D63 (2001) 094005]
– LSS01 [Eur.Phys.J. C23 (2002) 479]
– BB02 [Nucl. Phys. B636 (2002) 225]

COMPASS QCD fit

- parton distributions

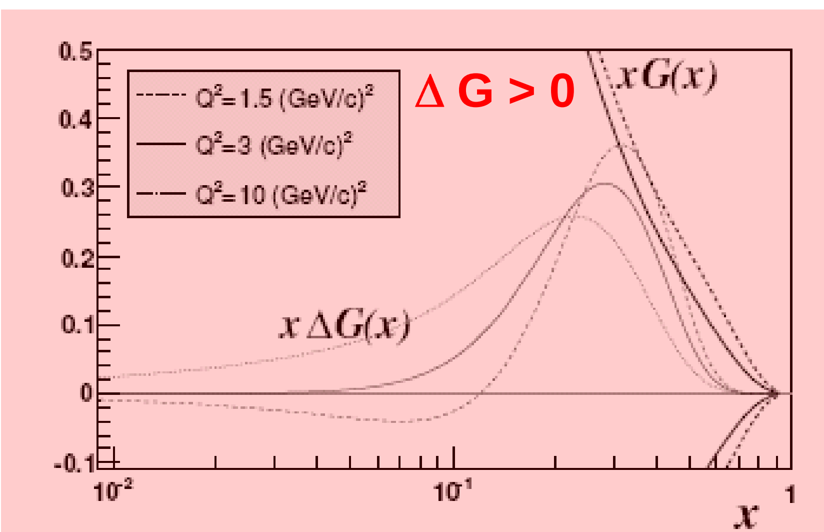
$$\Delta f = \eta x^{\alpha}(1-x)^{\beta}(1+\gamma x), \quad Q_0^2 = 3\ \mathrm{GeV}^2$$

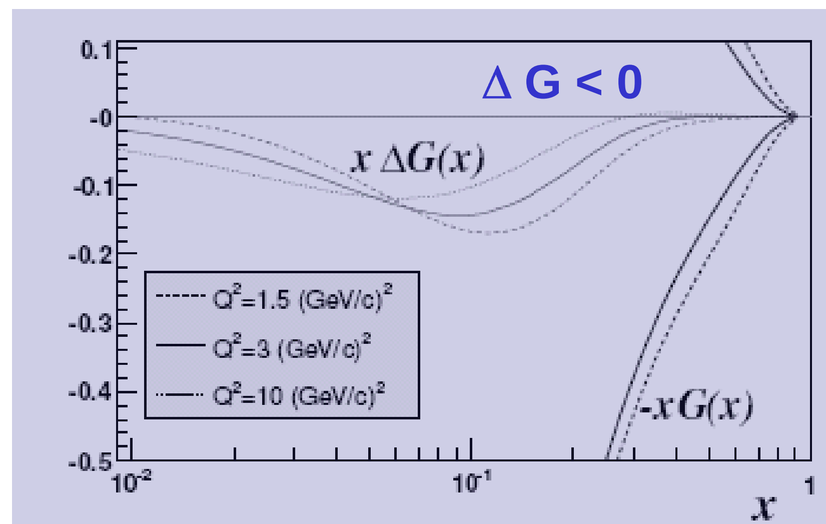
- New g_1^d data + world data
- $\Delta G > 0$ and $\Delta G < 0$



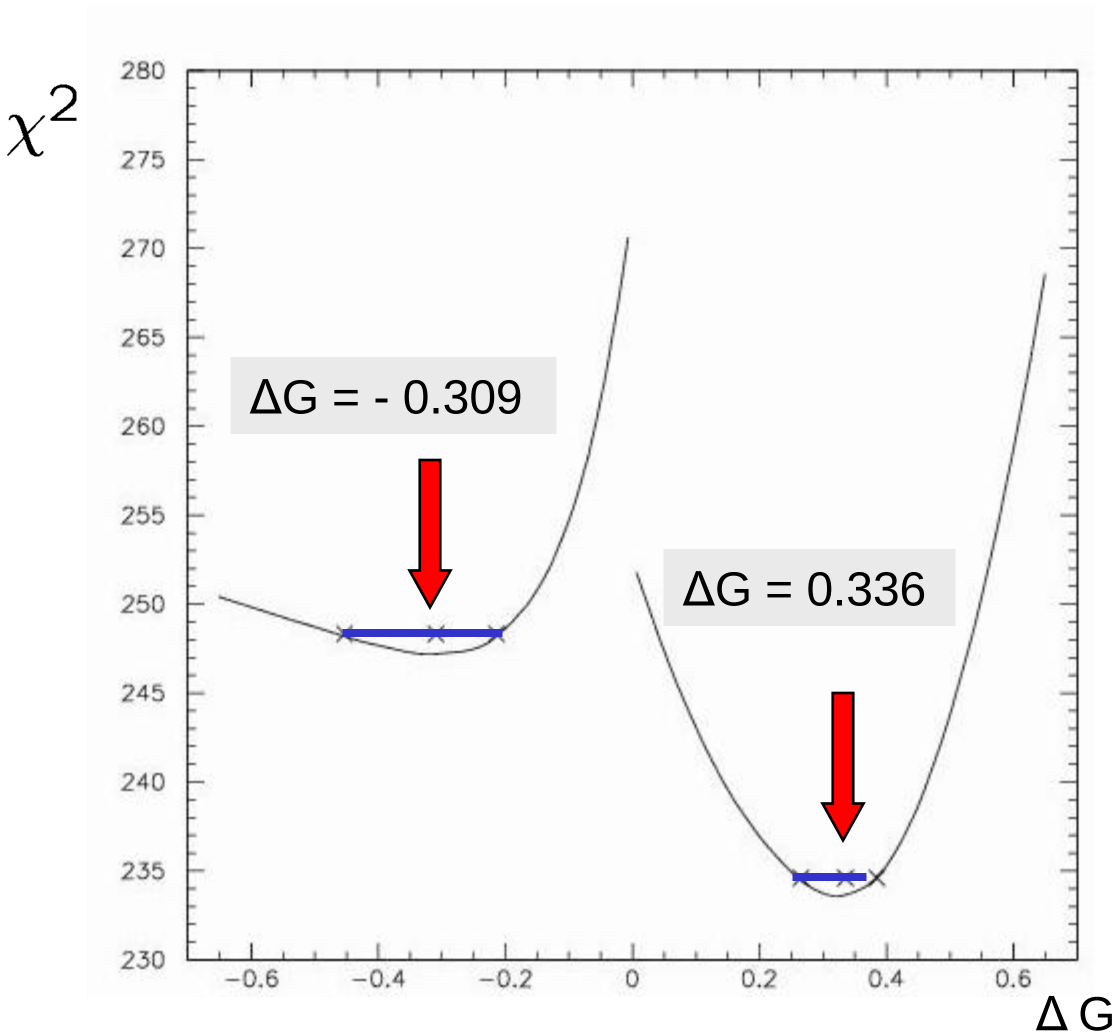
COMPASS QCD fit

- two solutions: $\Delta G > 0$ and $\Delta G < 0$
- $|\Delta G| \simeq 0.2–0.3$





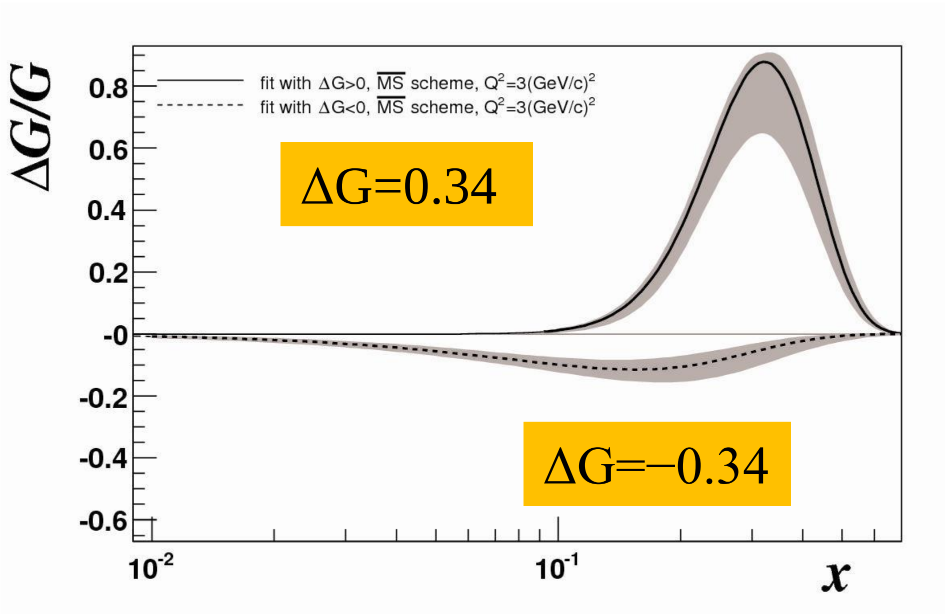
χ^2 as Function of ΔG



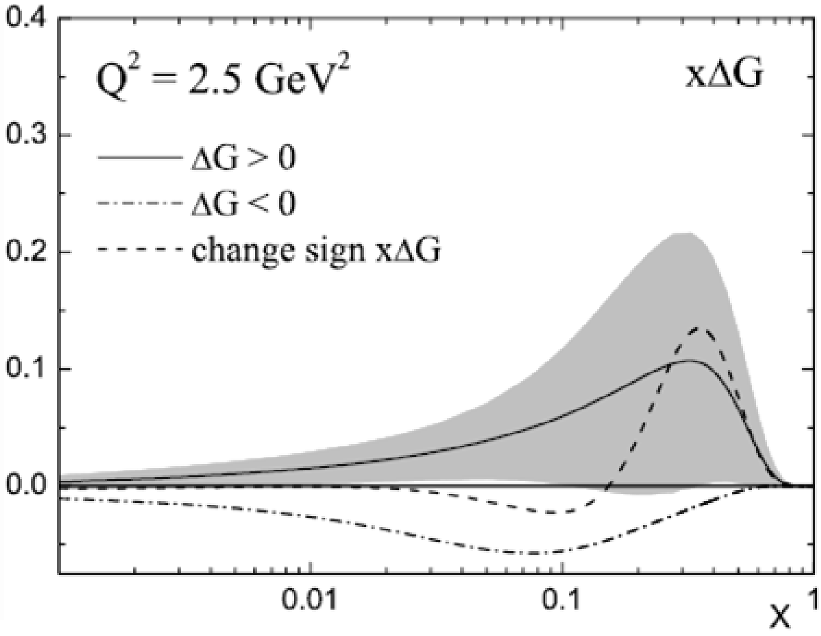
- Two distinct solutions
- $\Delta G > 0$ preferred
- $\Delta G < 0$ preferred by small-x points

COMPASS QCD fit

COMPASS

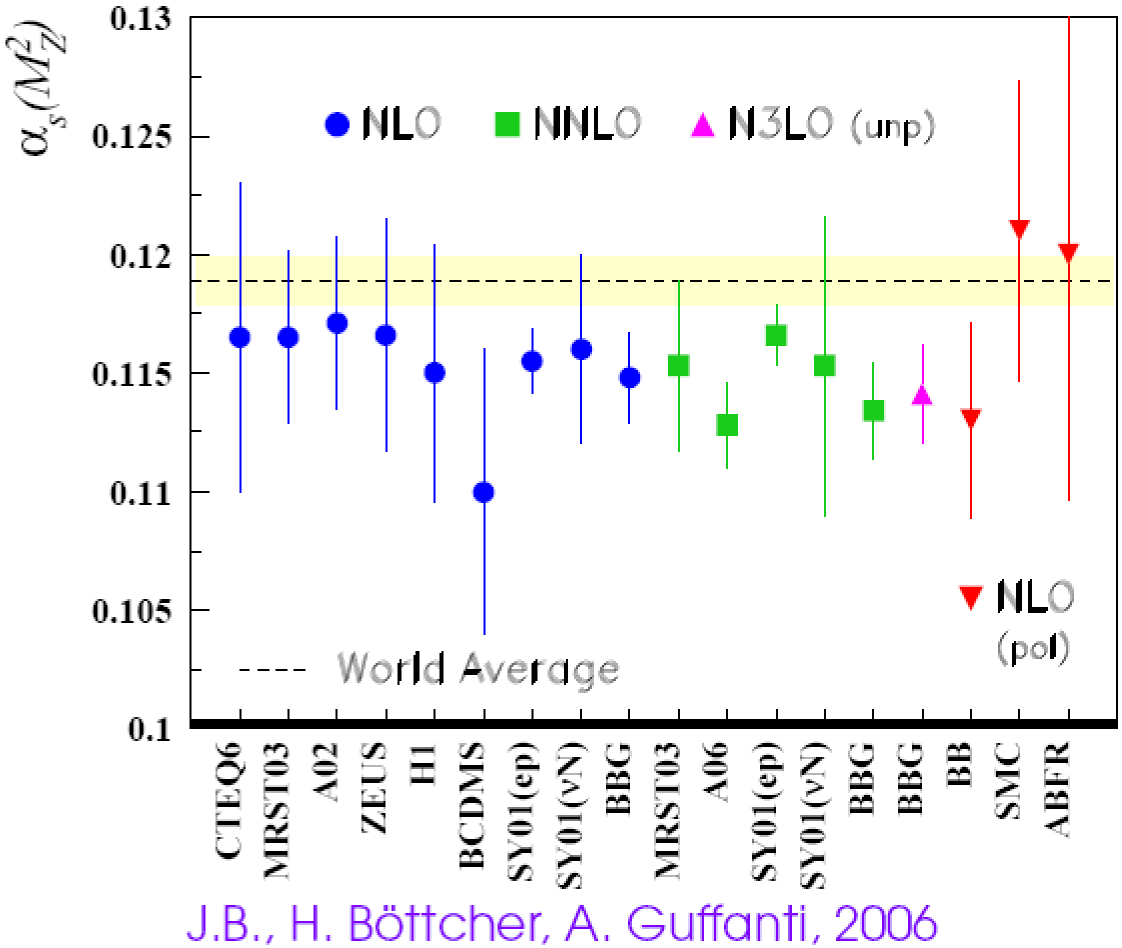


LSS 2006



- Not yet included in fits: final Hermes g_1^d
- Uncertainty due to parametrisation not included

α_s from pol. DIS



3. Interplay: g_2

unpolarised:

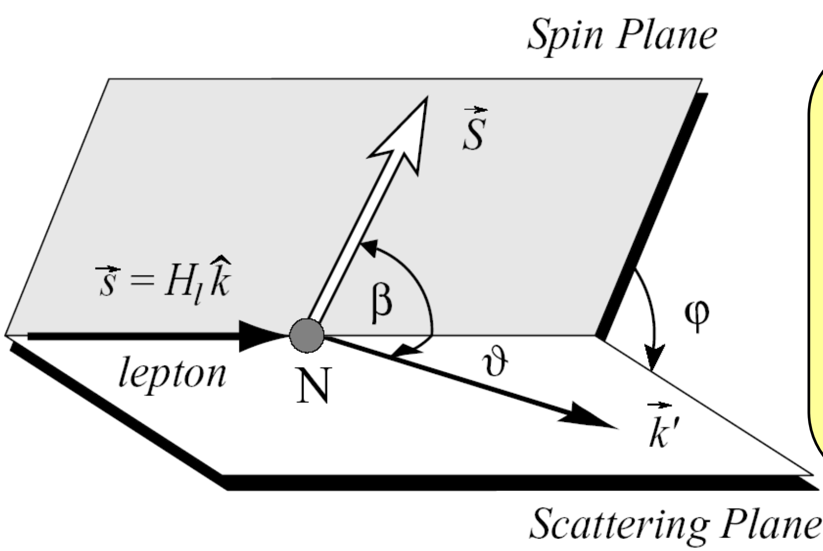
$$\frac{d^3\bar{\sigma}}{dx\,dy\,d\varphi} = \frac{4\alpha^2}{Q^2}\left\{\frac{y}{2}F_1 + \frac{1}{2xy}\left(1-y-\frac{y^2\gamma^2}{4}\right)F_2\right\}$$

longitudinally polarised nucleon: β=0,π

$$\frac{d^3\Delta_{\parallel}\sigma}{dx\,dy\,d\varphi} = \frac{4\alpha^2}{Q^2}\left\{\left(1-\frac{y}{2}-\frac{y^2\gamma^2}{4}\right)g_1 - \frac{y}{2}\gamma^2 g_2\right\}$$

transversely polarised nucleon: β= π/2

$$\frac{d^3\Delta_{\perp}\sigma}{dx\,dy\,d\varphi} = \frac{4\alpha^2}{Q^2}\left\{\gamma\sqrt{1-y-\frac{y^2\gamma^2}{4}}\left(\frac{y}{2}g_1 + g_2\right)\right\}$$



Measure asymmetries:

$$A_{\parallel}(x,Q^2;E) = \frac{\Delta_{\parallel}\sigma}{\bar{\sigma}} = \frac{\sigma^{\rightleftarrows} - \sigma^{\rightrightarrows}}{\sigma^{\rightleftarrows} + \sigma^{\rightrightarrows}},$$

$$A_{\perp}(x,Q^2;E) = \frac{\Delta_{\perp}\sigma}{\bar{\sigma}} = \frac{\mathcal{H}_\ell}{\cos\varphi}\cdot\frac{\sigma(\varphi)-\sigma(\pi\pm\varphi)}{\sigma(\varphi)+\sigma(\pi\pm\varphi)}$$

Wandzura-Wilczek

Twist 3 g2 (quark-gluon corr.)

Wandzura-Wilczek :

$$g_2(x,Q^2) = g_2^{\mathrm{WW}} + \bar{g}_2(x,Q^2)$$

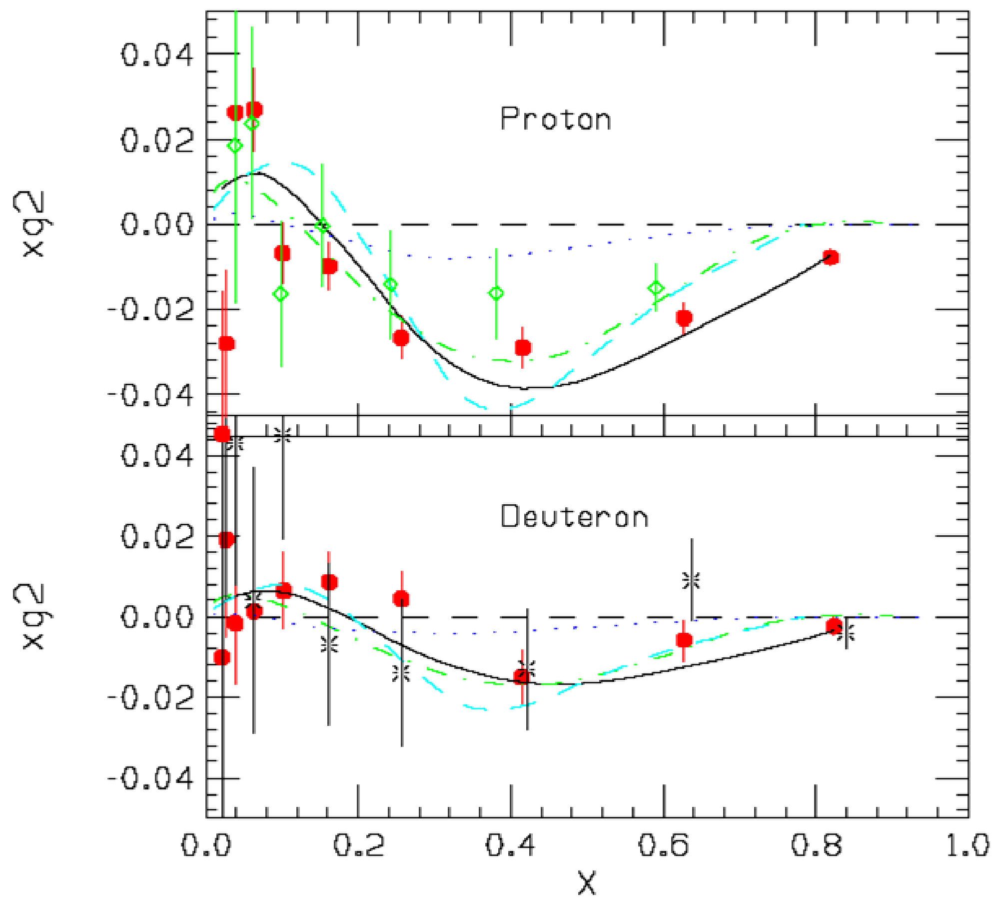
$$g_2^{\mathrm{WW}}(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \frac{g_1(y,Q^2)}{y}\,\mathrm{d}y$$

- twist-3 term $\bar{g}_2$, matrix element d_2

$$d_2 = 3\int_0^1 x^2\,\bar{g}_2(x,Q^2)\,\mathrm{d}x$$

g_2 from SLAC

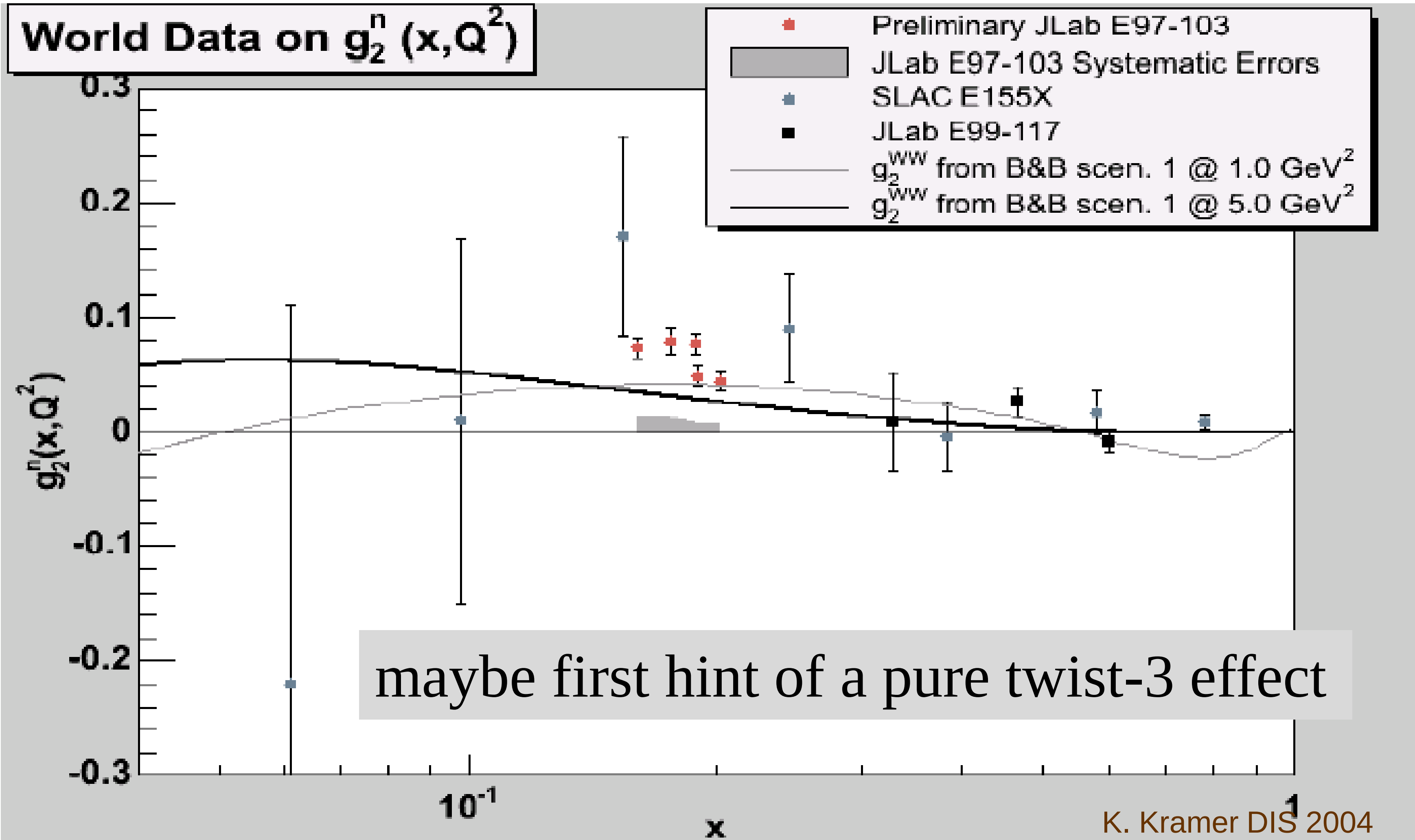
- —— twist-2 term: g_2^{WW}
- -- -- bag model calculations



E155X PRELIMINARY
E143 AVERAGE 29 GeV
E155 AVERAGE 38 GeV

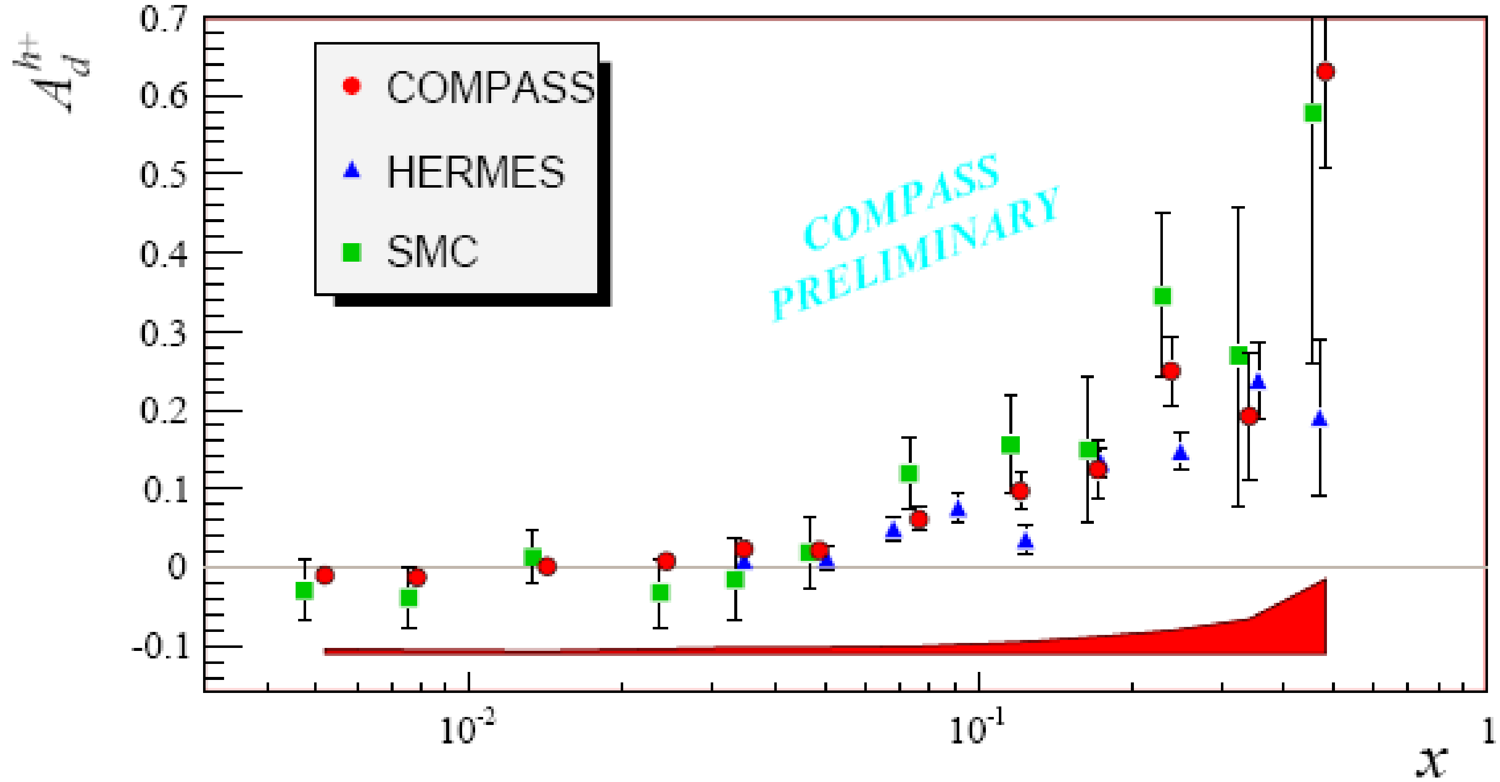
xg_2^{WW} solid
Stratmann: dot-
Song: dot
Weigel: DASH

Neutron g_2 from JLAB

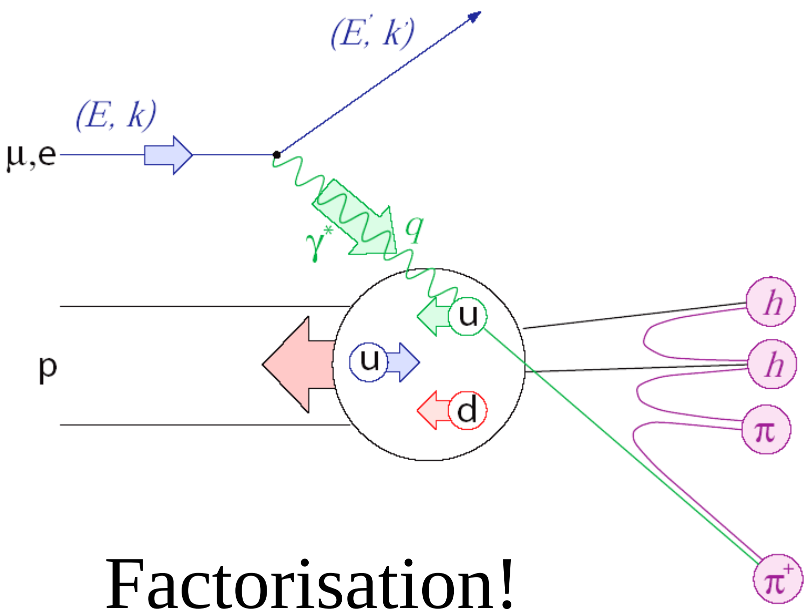


4. Semi-inclusive DIS

- additional hadron observed in final state



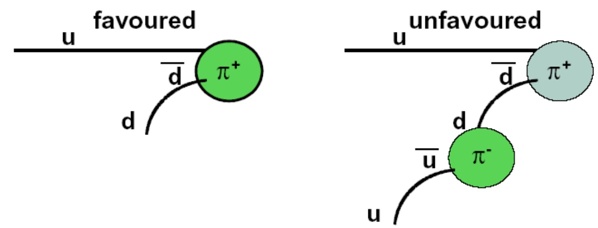
Fragmentation Function $D_f^h(z, Q^2)$



Factorisation!

D_q^h from quark q into hadron h

$z = \frac{E_h}{\nu}$ energy fraction carried by h

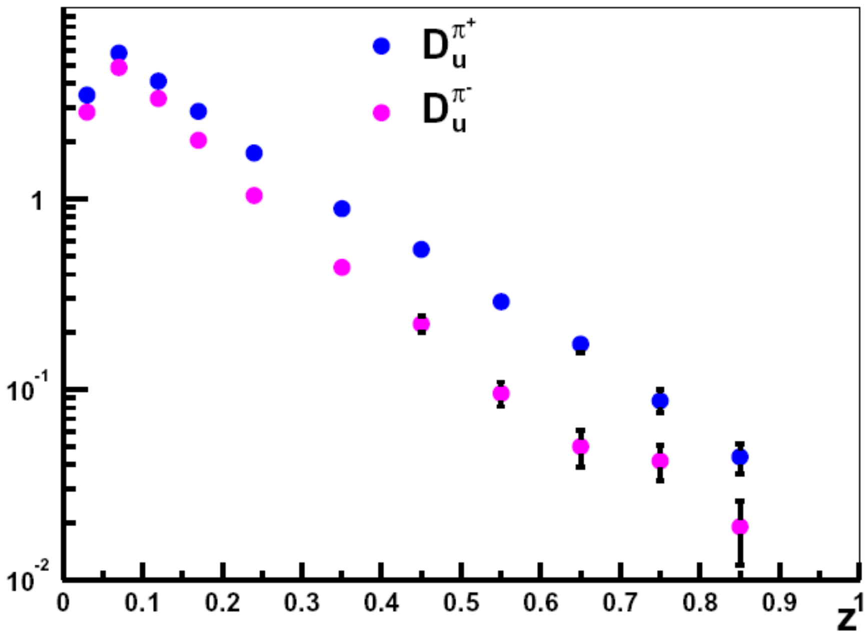


$$D_u^{\pi^+} \overset{\text{CC}}{=} D_{\bar{u}}^{\pi^-} \overset{\text{IS}}{=} D_{\bar{d}}^{\pi^+} \overset{\text{CC}}{=} D_d^{\pi^-}$$

$$D_d^{\pi^+} \overset{\text{CC}}{=} D_{\bar{d}}^{\pi^-} \overset{\text{IS}}{=} D_{\bar{u}}^{\pi^+} \overset{\text{CC}}{=} D_u^{\pi^-}$$

$$\frac{1}{\sigma_0} \frac{d\sigma^h}{d\,z} = \frac{\sum_f e_f^2 \; q_f(x, Q^2) \cdot D_f^h(z, Q^2)}{\sum_f e_f^2 \; q_f(x, Q^2)}$$

Semi-inclusive DIS

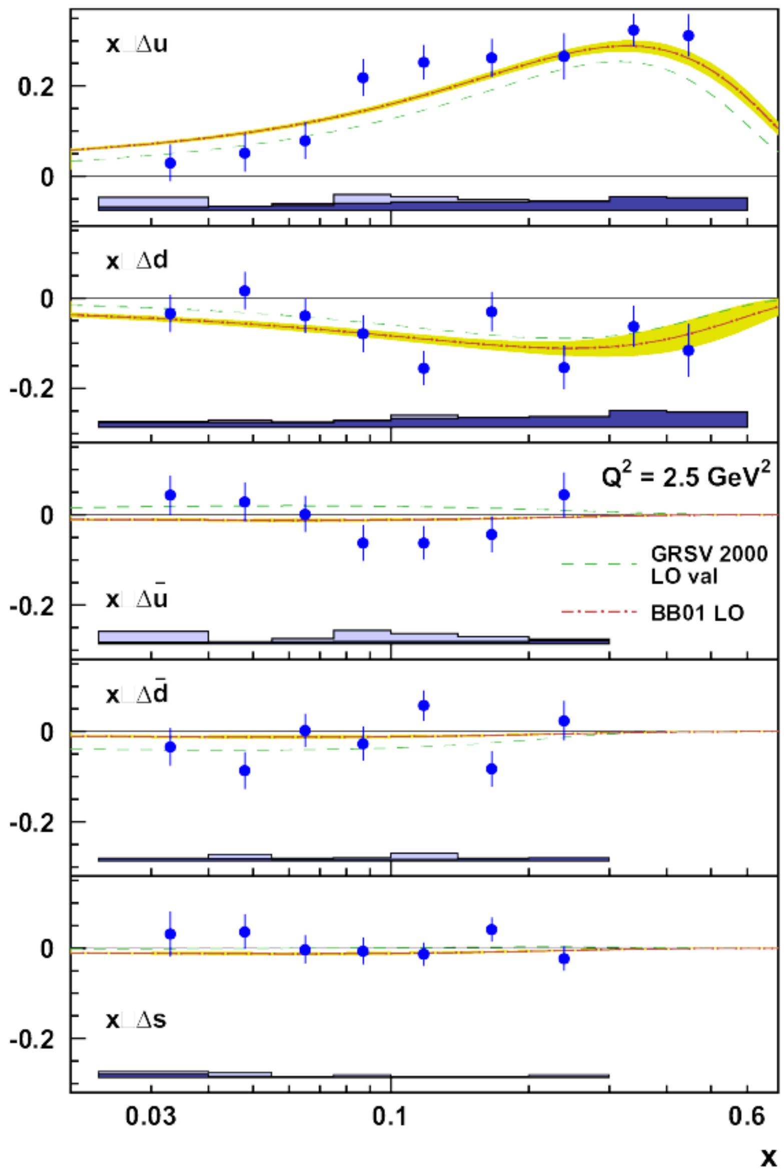


final hadron "remembers" flavour of initially struck!

poorly known

$$A_1^h(x, Q^2) = \frac{\int \mathrm{d}z \sum_f e_f^2 \Delta q_f(x, Q^2) \cdot D_f^h(z, Q^2)}{\int \mathrm{d}z \sum_f e_f^2 \; q_f(x, Q^2) \cdot D_f^h(z, Q^2)}$$

Flavour separated polarisation



Asymmetries can in LO be related to Δq by

$$\vec{\mathcal{A}} = \mathcal{P}\vec{\mathcal{Q}}$$

where
$$\vec{\mathcal{A}} = (A^h_{1,t}, \ldots)$$
$$\vec{\mathcal{Q}} = (\Delta q_f, \ldots)$$

$$\mathcal{P}^h_f = \frac{e_f^2 q_f(x) \int dz D^h_f}{\sum_i e_i^2 q_i(x) \int dz D^h_i(z)}$$

Alternative: difference asymmetries

Semi-inclusive asymmetries

$$A^+ = \frac{\sigma^{h+}_{\uparrow\downarrow} - \sigma^{h+}_{\uparrow\uparrow}}{\sigma^{h+}_{\uparrow\downarrow} + \sigma^{h+}_{\uparrow\uparrow}} \quad A^- = \frac{\sigma^{h-}_{\uparrow\downarrow} - \sigma^{h-}_{\uparrow\uparrow}}{\sigma^{h-}_{\uparrow\downarrow} + \sigma^{h-}_{\uparrow\uparrow}}$$

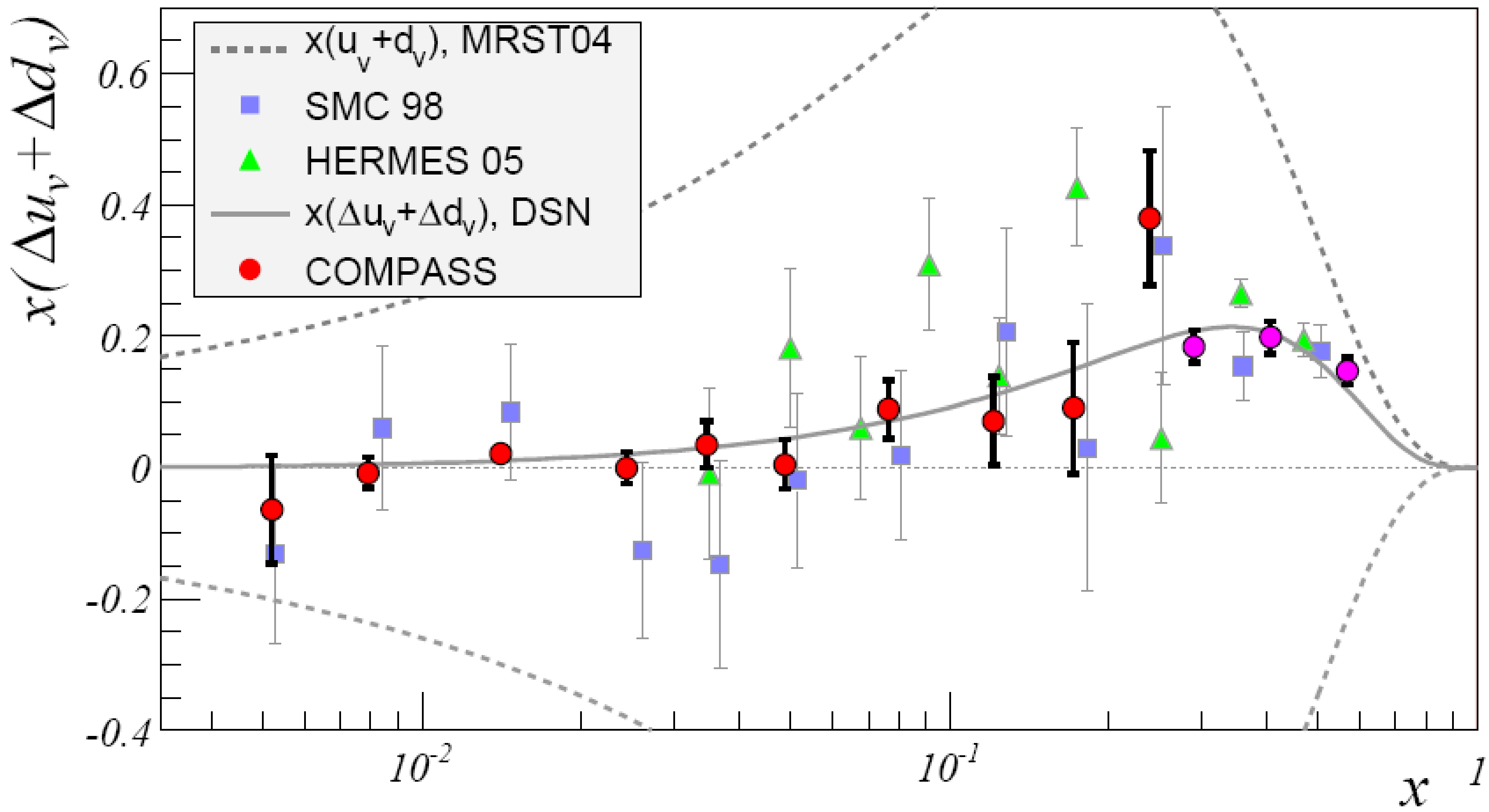
$$A_1^h(x) = \frac{\sum_q e_q^2(\Delta q(x) D_q^h + \Delta\bar{q}(x) D_{\bar{q}}^h)}{\sum_q e_q^2(q(x) D_q^h + \bar{q}(x) D_{\bar{q}}^h)}$$

Difference asymmetry

$$A^{+-} = \frac{(\sigma^{h+}_{\uparrow\downarrow} - \sigma^{h-}_{\uparrow\downarrow}) - (\sigma^{h+}_{\uparrow\uparrow} - \sigma^{h-}_{\uparrow\uparrow})}{(\sigma^{h+}_{\uparrow\downarrow} - \sigma^{h-}_{\uparrow\downarrow}) + (\sigma^{h+}_{\uparrow\uparrow} - \sigma^{h-}_{\uparrow\uparrow})}$$

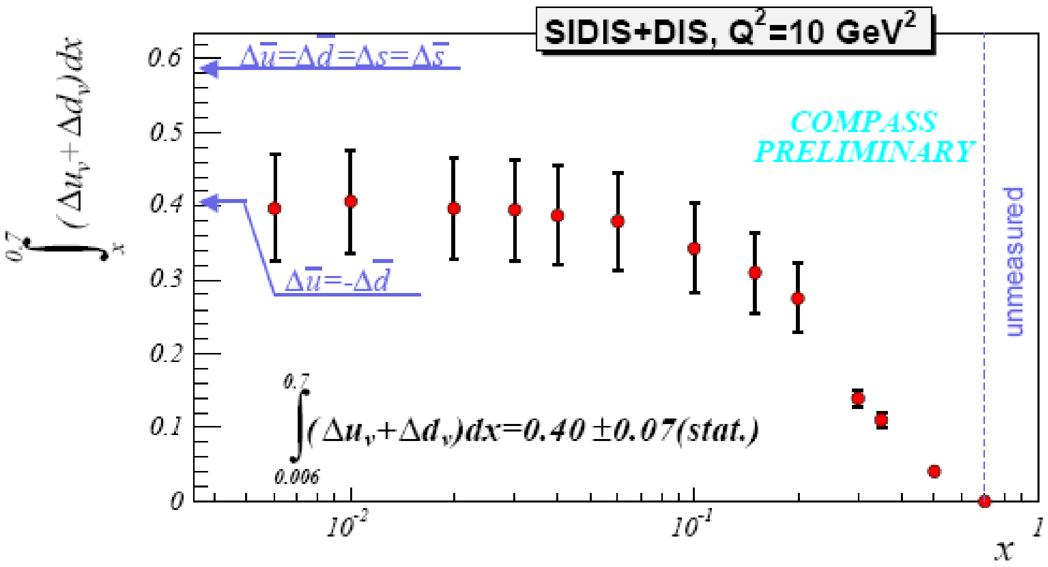
$$A_d^{\pi^+-\pi^-}(x) = A_d^{K^+-K^-}(x) = \frac{\Delta u_v(x) + \Delta d_v(x)}{u_v(x) + d_v(x)}$$

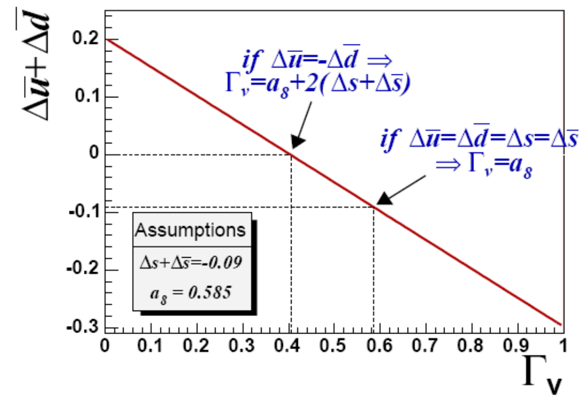
Valence quark polarisation without fragmentations function



$x(\Delta u_v+\Delta d_v)$
0.6
0.4
0.2
0
-0.2
-0.4
x(u_v+d_v), MRST04
SMC 98
HERMES 05
x(Δu_v+Δd_v), DSN
COMPASS
10^{-2}
10^{-1}
1
x

Valence quark polarisation

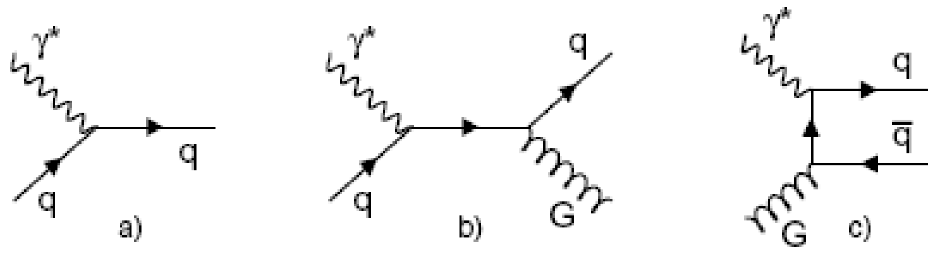




	x-range	Q^2 GeV^2	$\Delta u_v + \Delta d_v$ measurem.	$\Delta u_v + \Delta d_v$ DNS	$\Delta\bar{u} + \Delta\bar{d}$ measurem.	$\Delta\bar{u} + \Delta\bar{d}$ DNS
SMC 98	0.003–0.7	10	$0.26 \pm 0.21 \pm 0.11$	0.386	$0.02 \pm 0.08 \pm 0.06$	-0.009
HERMES 05	0.023–0.6	2.5	$0.43 \pm 0.07 \pm 0.06$	0.363	$-0.06 \pm 0.04 \pm 0.03$	-0.005
COMPASS	0.006–0.7	10	$0.40 \pm 0.07 \pm 0.05$	0.385	$0.0 \pm 0.04 \pm 0.03$	-0.007

5. Δ G from high-p_T hadron pairs

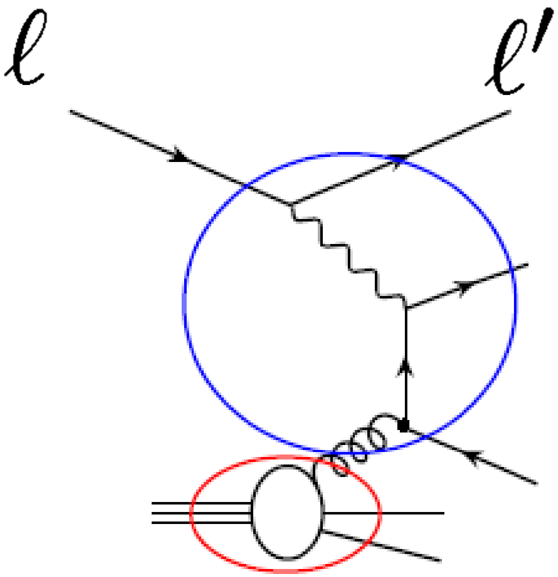
- Contributions: a) LO b) QCD Compton c) PGF



$$A_{\mathrm{LL}}^{\ell N} \simeq \langle \hat{a}_{\mathrm{LL}}^{\gamma g \to qg} \rangle \frac{\Delta q}{q} + \langle \hat{a}_{\mathrm{LL}}^{\gamma g \to q\bar{q}} \rangle \frac{\Delta g}{g}$$

Photon-gluon fusion (PGF)

- Gluon polarisation is measurable in PGF



$$A_{\parallel} = R_{pgf} \langle \hat{a}_{pgf} \rangle \frac{\Delta G}{G}$$

- measure $A_{\parallel}$
- calculate R_{pgf} and $\langle \hat{a}_{pgf} \rangle$

  using Monte Carlo

Hadron production

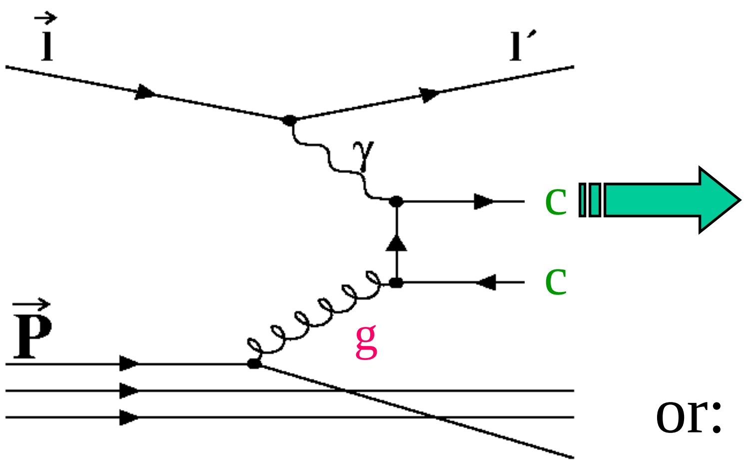
- LO analysis of hadron-pair asymmetries:
  - open charm: single D meson — **AROMA, RAPGAP**
    cleanest process wrt physics background
  - high-p_T hadron pairs with $Q^2 > 1$ GeV^2 — **LEPTO**
  - high-p_T hadron pairs with $Q^2 < 1$ GeV^2 — **PYTHIA**

- NLO (photo production)
  - open charm — Bojak, Stratmann
  - single incl. high-p_T hadron — Jaeger. Stratmann, Vogelsang
  - hadron pairs: LO done, — Hendlmeier, Stratmann, Schäfer
  - NLO underway

- All analyses up to now in LO (plus parton showers)

Open charm at COMPASS

- Photon-gluon fusion: 1.2 D^0 per PGF $c\bar{c}$ event



$$D^0 \Rightarrow K^- \pi^+$$ BR 4 %

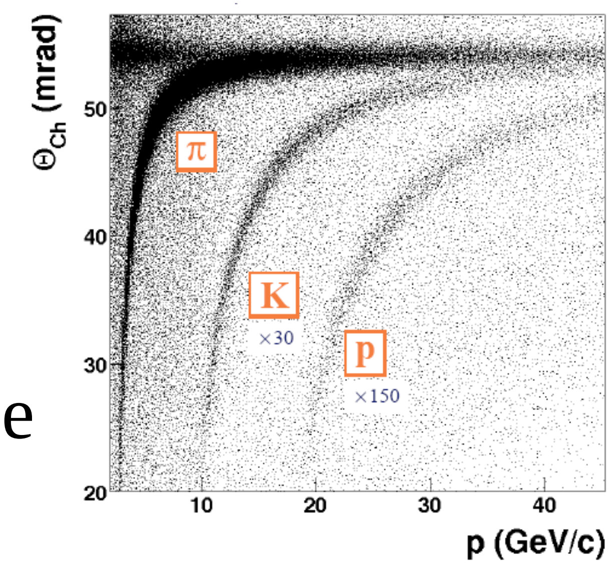
or: $$D^{*+} \Rightarrow D^0 \pi_s^+$$

$$\Rightarrow K^- \pi^+ \pi_s$$

D^0 from D^* about 20 %

Kπ separation

- kaon identification by RICH
- cleaner $D^* \rightarrow D\,\pi_s \rightarrow K\,\pi\,\pi_s$
  additional slow pion π_s
- no D decay vertex due to multiple scattering in solid target
- define



$$\Delta M_{K\pi\pi} = M_{K\pi\pi_s} - (M_{K\pi} + M_{\pi_s})$$

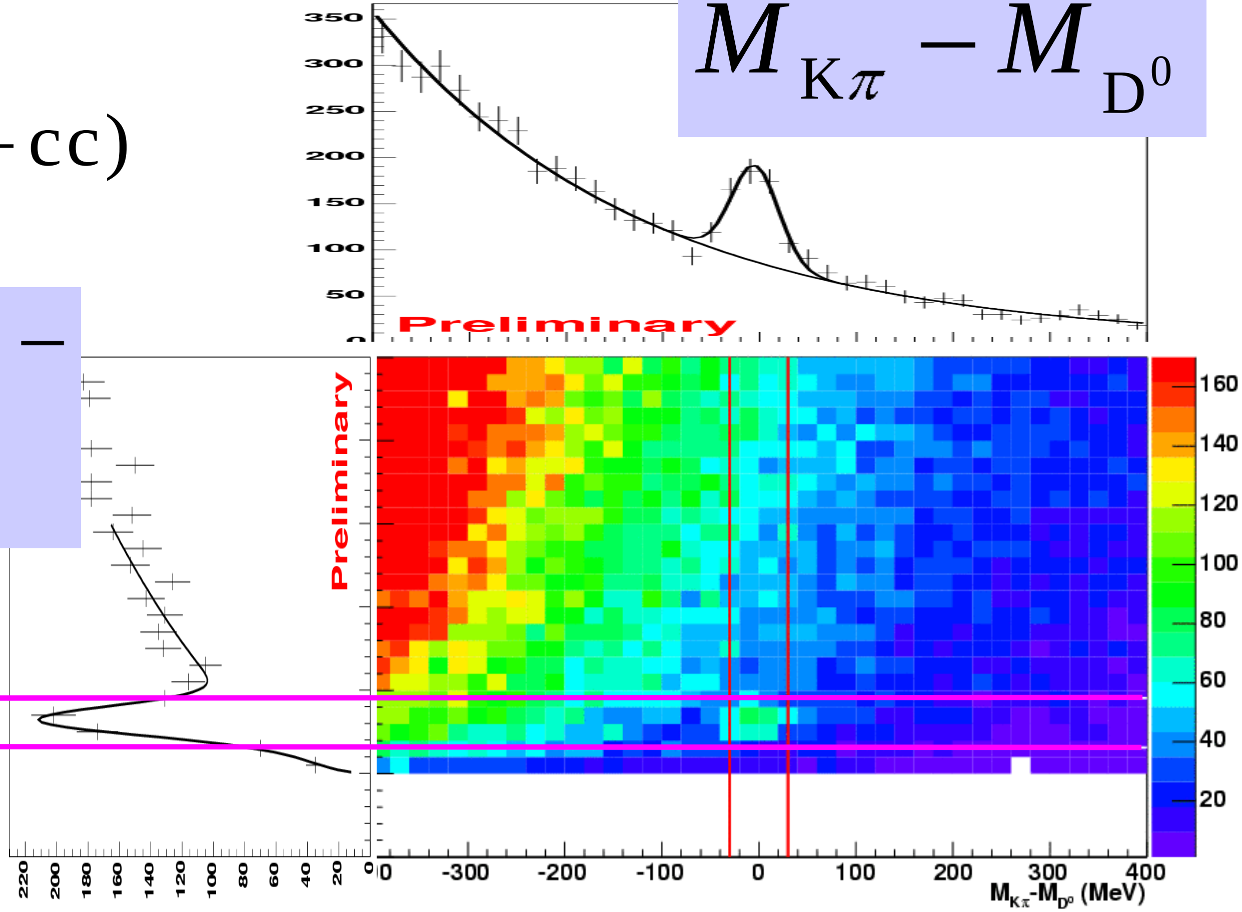
- sharp peak for D^* in $\Delta M_{K\pi\pi s}$

$D^{*+} \to D^0 \pi_s^{\ +}$ tagging

$$D^* \to D^0 \pi_s$$
$$\to K\pi\pi_s \quad (+cc)$$

$$M_{K\pi} - M_{D^0}$$

$$\Delta M_{K\pi\pi} = M_{K\pi\pi_s} - (M_{K\pi} + M_{\pi_s})$$



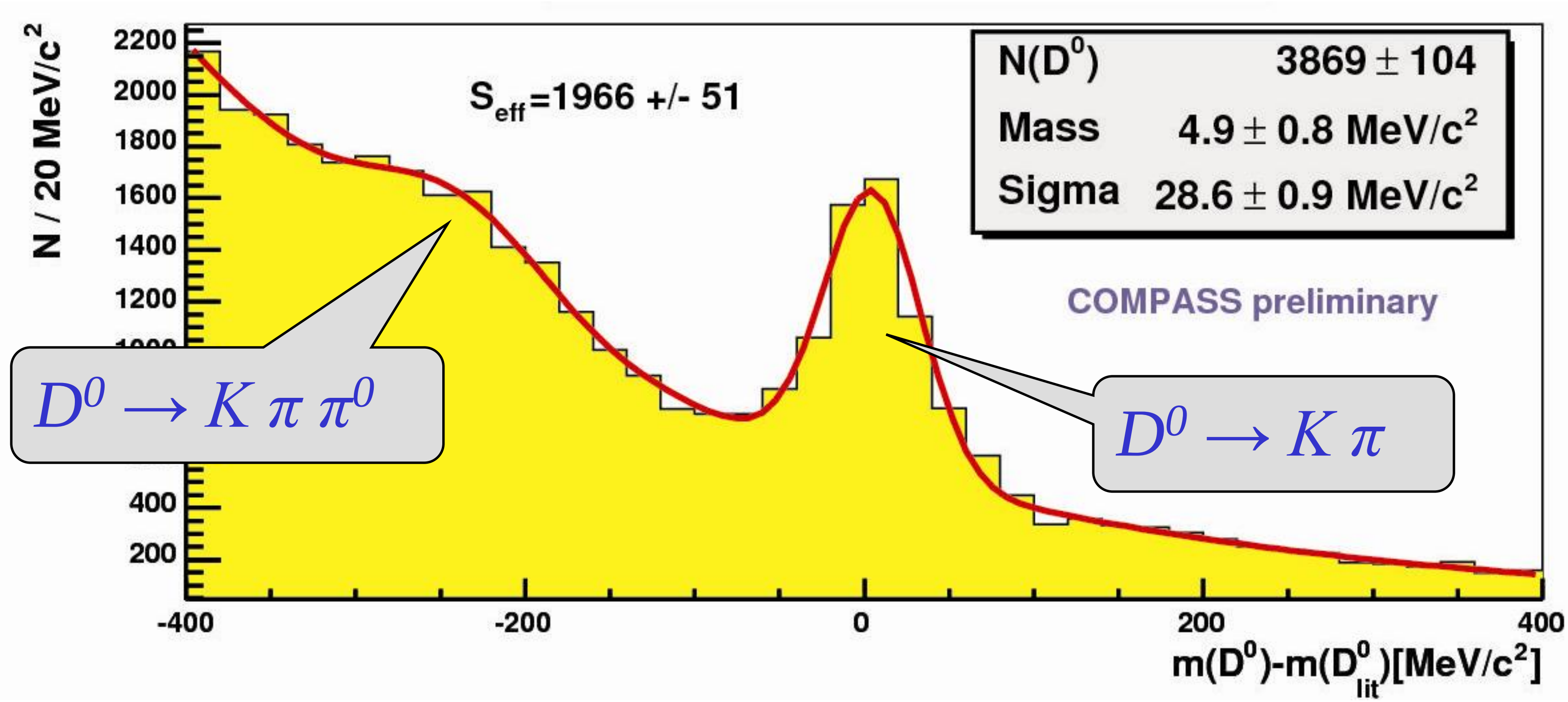
Choose:

$3.1 < \Delta M_{K\pi\pi} < 9.1$ MeV

Open Charm: D's from D*s

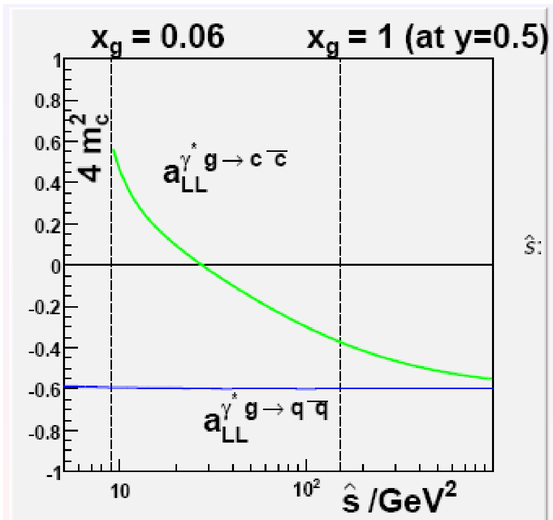
$D^* \rightarrow D\,\pi_s \rightarrow K\,\pi\,\pi_s$ slow pion required

2002–2004

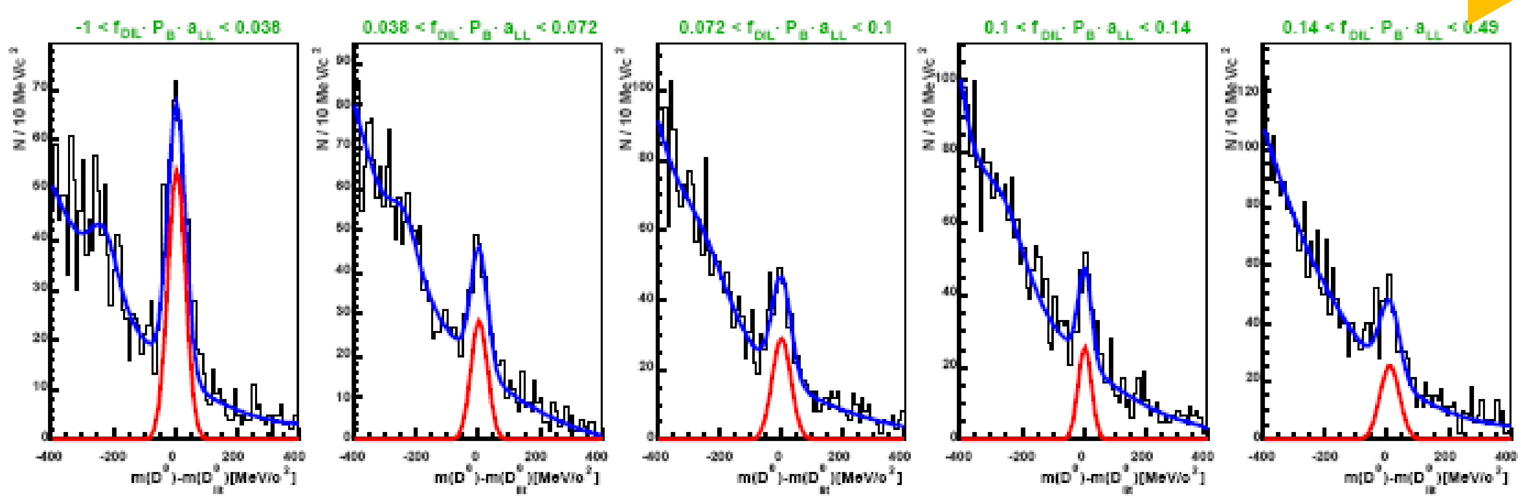


Analysing power A_{LL}

a_{LL}

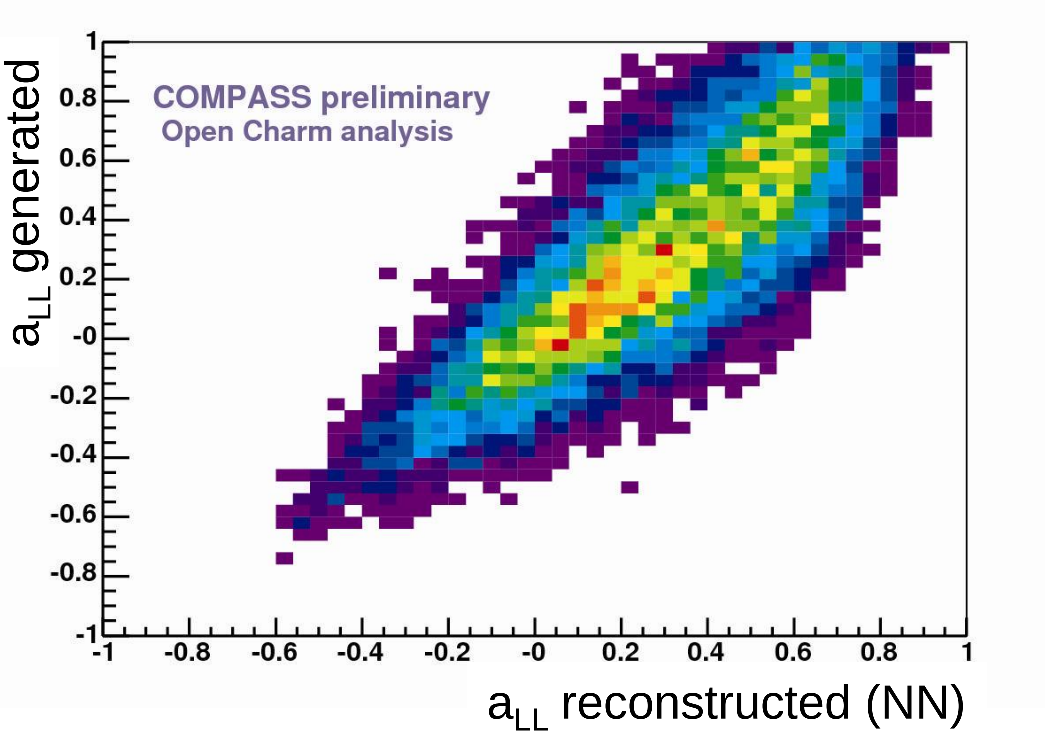


a_{LL}



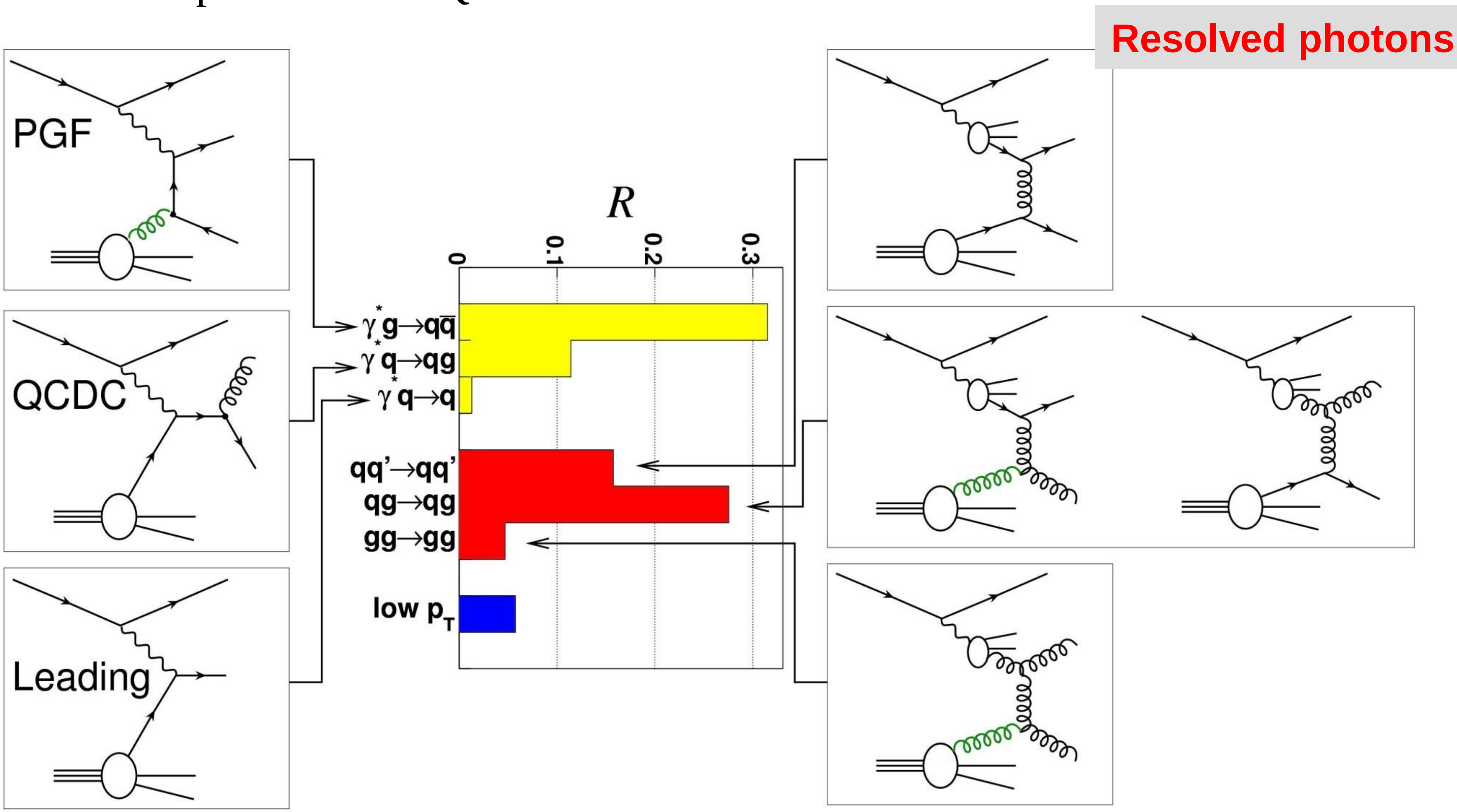
Open charm: MC

- analysis uses event a_{LL} weighting for statistical precision
- a_{LL} estimated with NN from event kinematics
- indispensable due to large variation of a_{LL}
- good correlation of 0.82 between generated and reconstructed a_{LL}



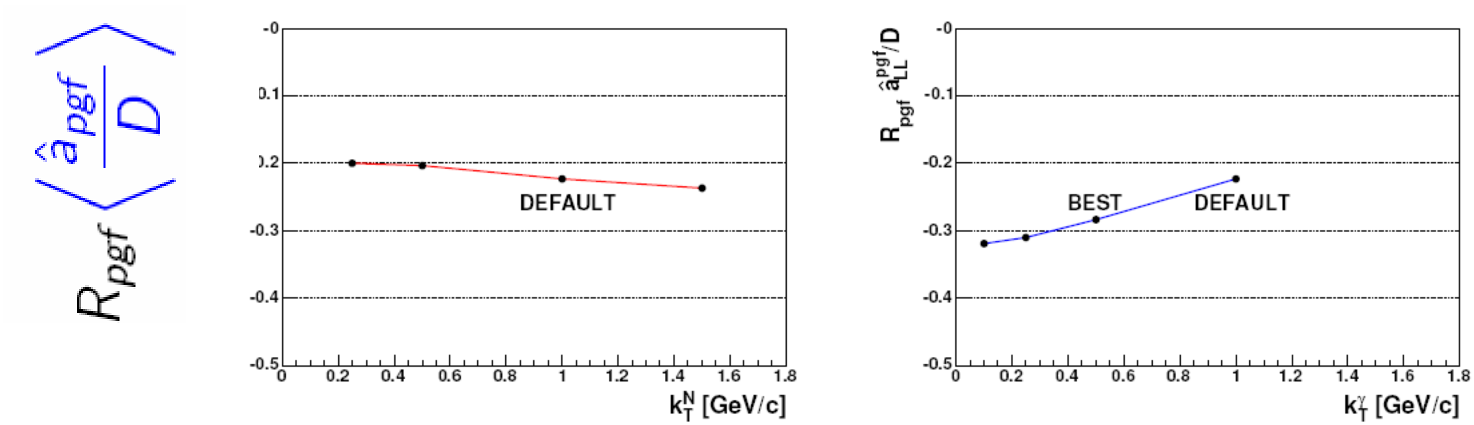
Light hadron production

Ratios for processes for $Q^2 < 1$



Example: k_T tuning

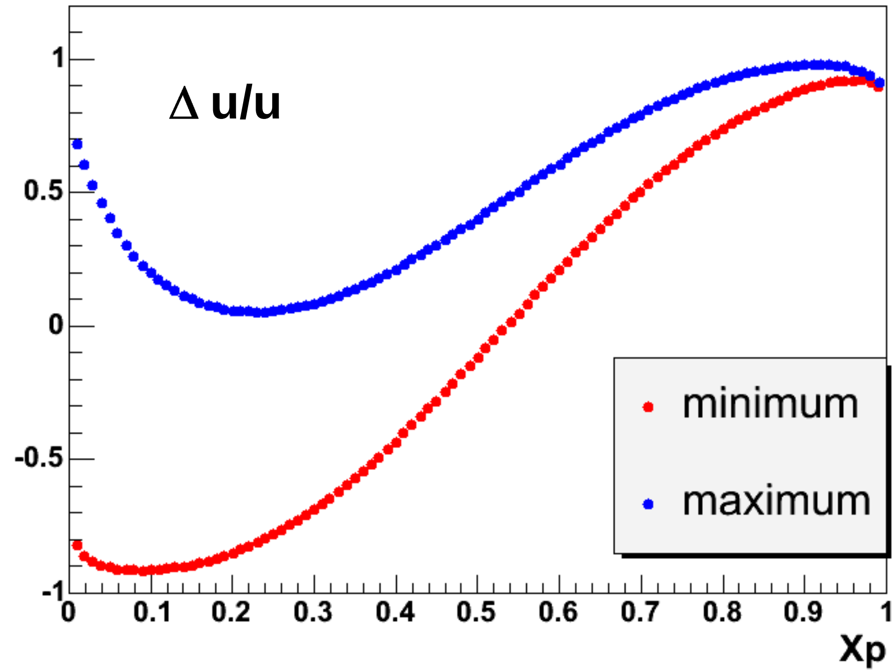
nucleon photon



- systematic error:
  - determined using 15 independent MC simulations
  - exploring the parameter space
    - in k_T of nucleon and photon
    - fragmentation functions
    - parton shower on/off,
    - renormalisation scale

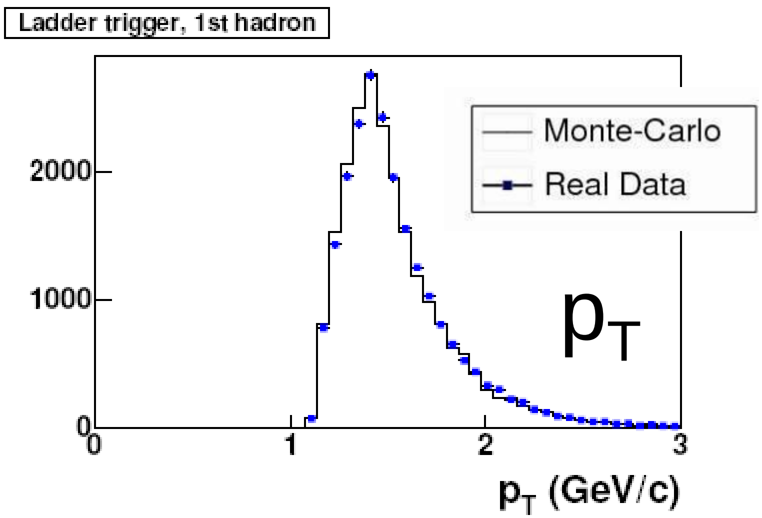
Resolved photons

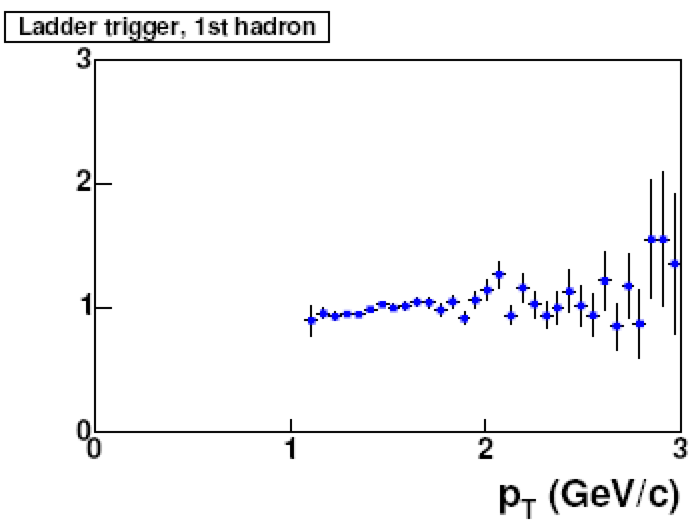
- More than 50%, however assuming a min and max scenario, shows little difference.
- Probing photon at large *x*, where photon PDF rather well determined



Glück, Reya, Sieg

Data versus MC





- excellent to good agreement for all kinematics variables

Gluon polarisation

high-pT pairs; $Q^2 > 1\text{GeV}^2$:

2002–2003

$$\frac{\Delta G}{G} = 0.06 \pm 0.31(\text{stat.}) \pm 0.06(\text{syst.}) \qquad \langle x_g \rangle = 0.13$$

high-pT pairs; $Q^2 < 1\text{GeV}^2$:

2002–2004

$$\frac{\Delta G}{G} = 0.016 \pm 0.058(\text{stat.}) \pm 0.055(\text{syst.})$$

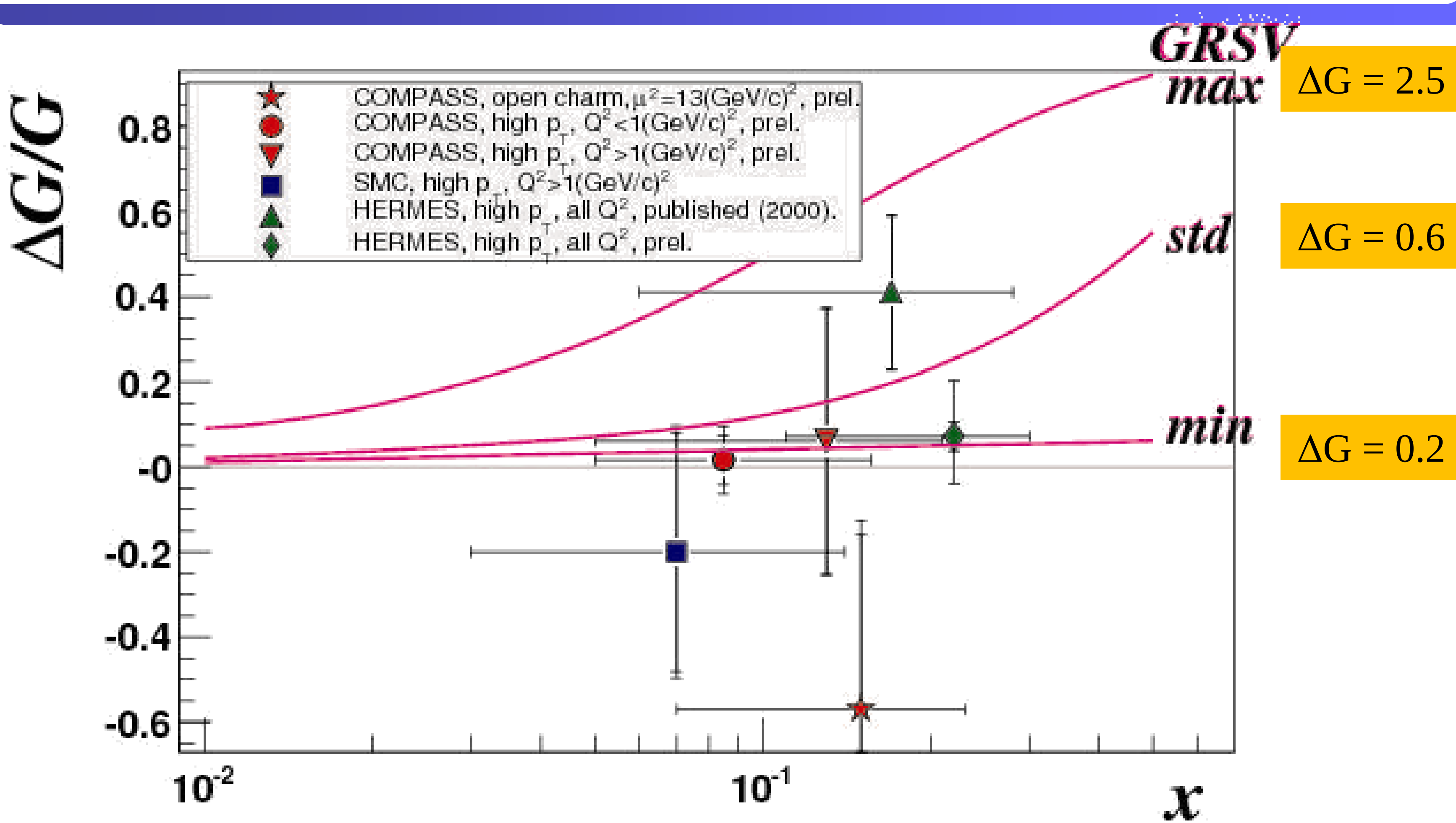
$$\langle x_g \rangle = 0.085 \qquad \langle \mu^2 \rangle = 3\ \text{GeV}^2$$

Open charm:

$$\frac{\Delta G}{G} = -0.57 \pm 0.41(\text{stat.}) \pm 0.17(\text{syst.})$$

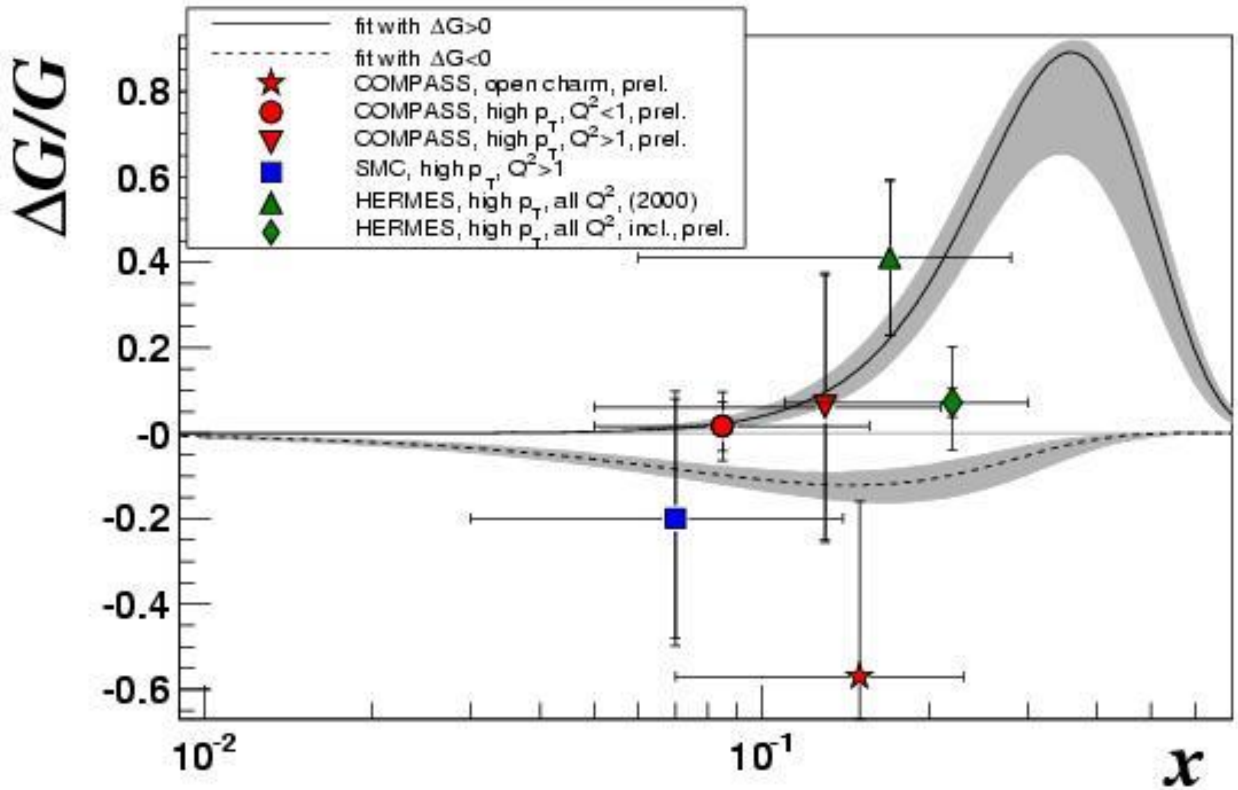
$$\langle x_g \rangle = 0.15 \quad \langle \mu^2 \rangle = 13\ \text{GeV}^2$$

$\Delta G/G$ from high-p_T pairs



- GRSV-max strongy disfavoured

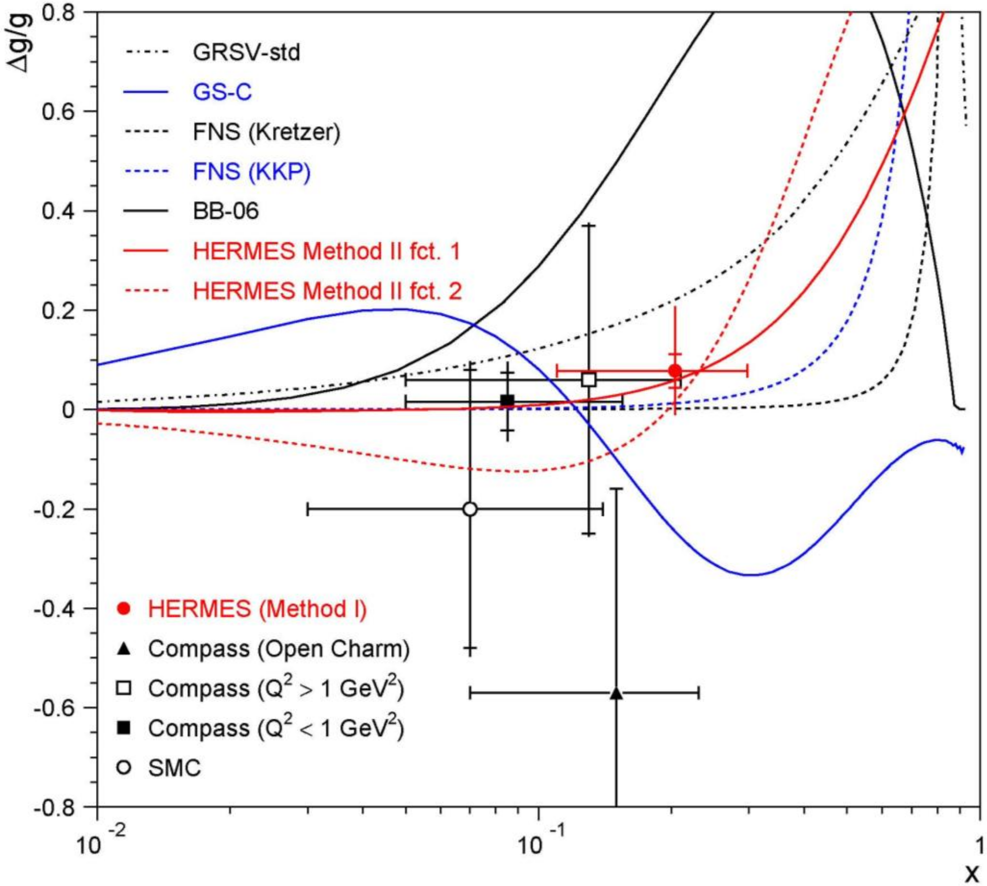
COMPASS QCD fit



- Note NLO fits, LO data

New Hermes analysis

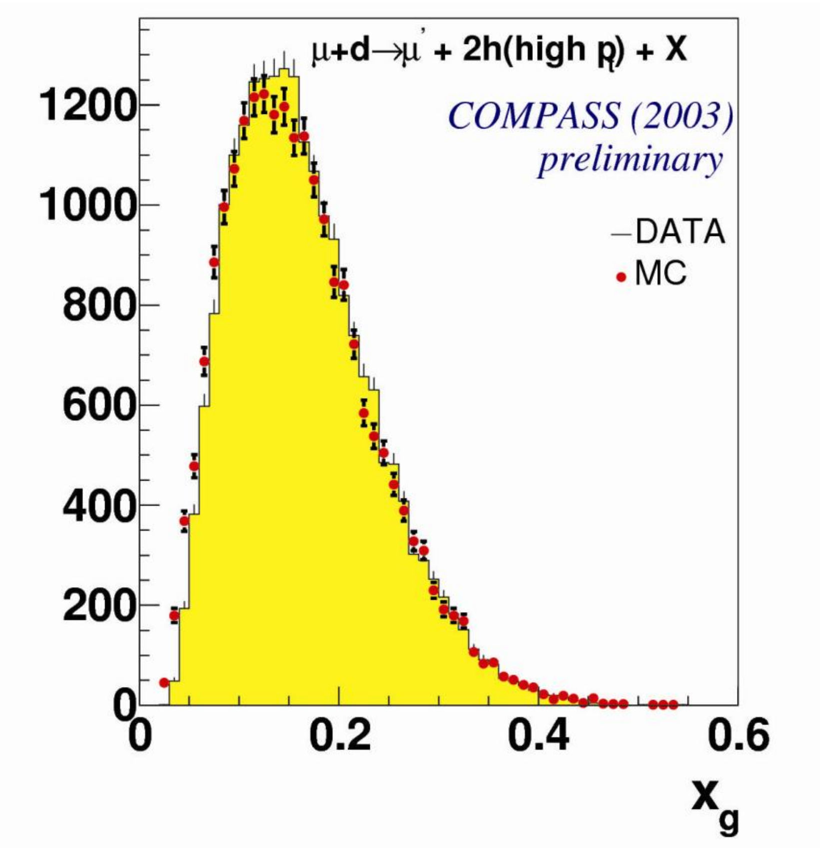
Single inclusive hadrons



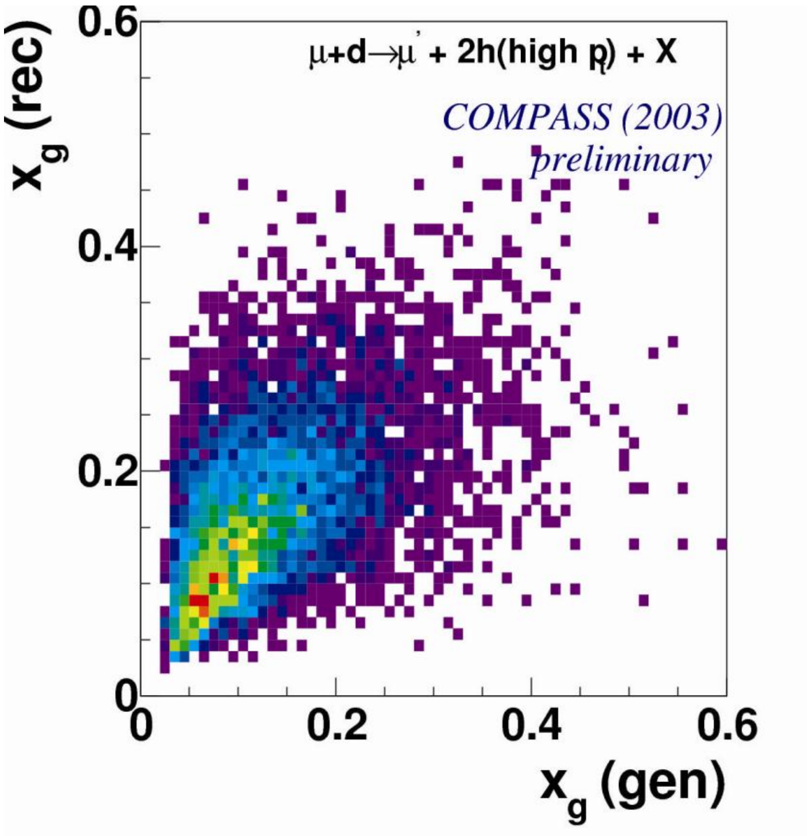
P. Liebing at spin 2006

Can we learn more about x?

here Lepto and Q2>1

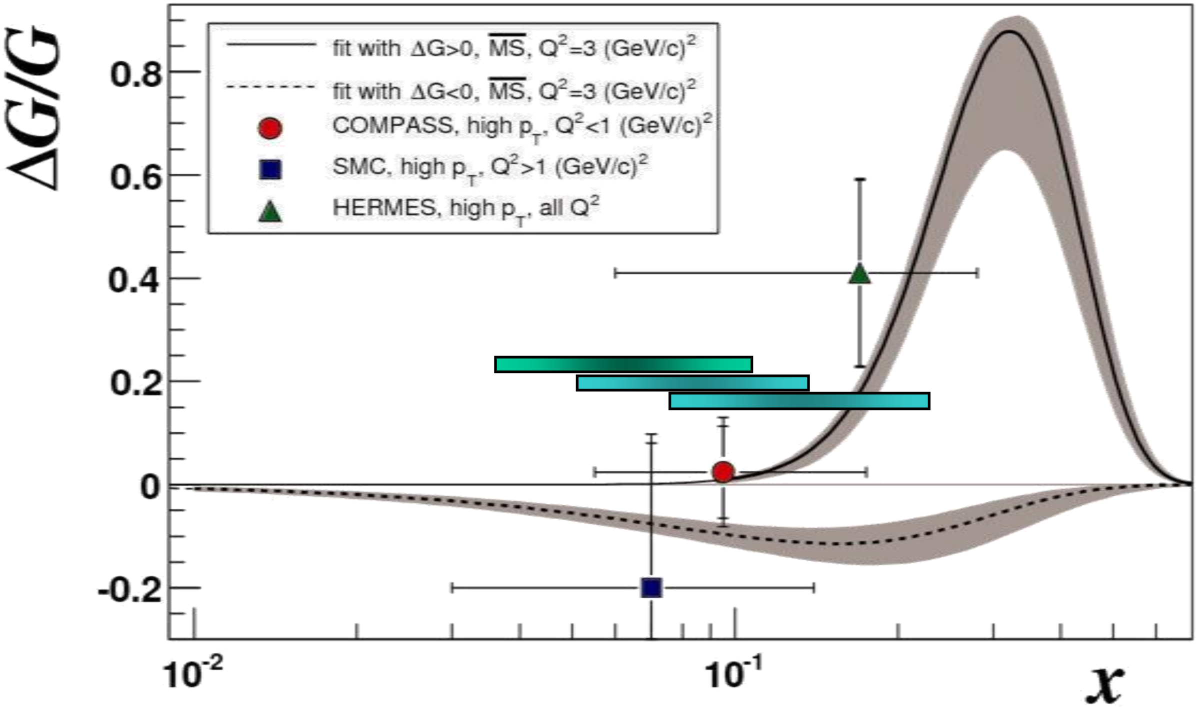


All processes



PGF events (MC)

Splitting in x_g bins?



Compass

- Splitting of high-p_T, $Q^2<1$ data in 3 x_g bins under study
- Optimizing correlation of rec. and `true' x_g (NN)
- More significant with 2006 data

Lecture 3

- Experimental status
  - RHIC pp data
  - transverse asymmetries
- Excursion: pion polarisability