

# The deuteron spin-dependent structure function $g_1^d$



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on behalf of the COMPASS Collaboration

## DIS 2007

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XV International Workshop on Deep-Inelastic Scattering and Related Subjects

# COCommon Muon Proton Apparatus for Structure and Spectroscopy

## COMPASS

Bielefeld, Bochum, Bonn, Burdwan/Calcutta, CERN, Dubna, Erlangen,  
Freiburg, Lisbon, Mainz, Moscow, Munich, Nagoya, Prague, Protvino,  
Saclay, Tel Aviv, Turin, Trieste, Warsaw, ~240 physicists

### ■ Muon beam program:

- gluon polarisation,
- spin-dependent structure function
- polarised quark distributions,
- transversity,
- Lambda polarisation,
- vector meson production,
- GPD (future)

- longitudinally polarised muon beam
- longitudinally or transversely polarised deuteron ( ${}^6\text{LiD}$ ) target
- momentum and calorimetry measurement
- particle identification

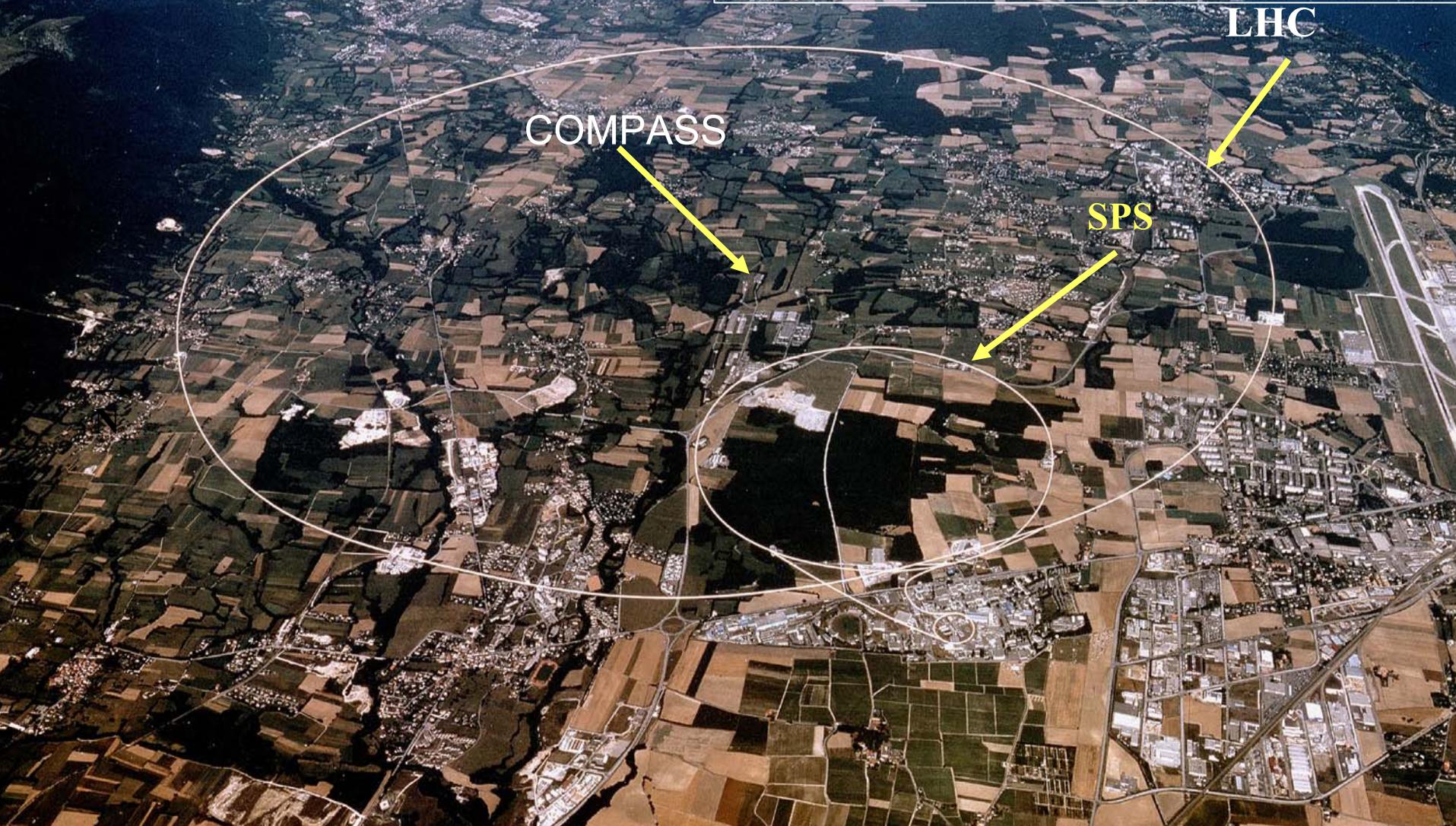
**luminosity:**  $\sim 5 \cdot 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$

**beam intensity:**  $2 \cdot 10^8 \mu^+/\text{spill}$  (4.8s/16.2s)

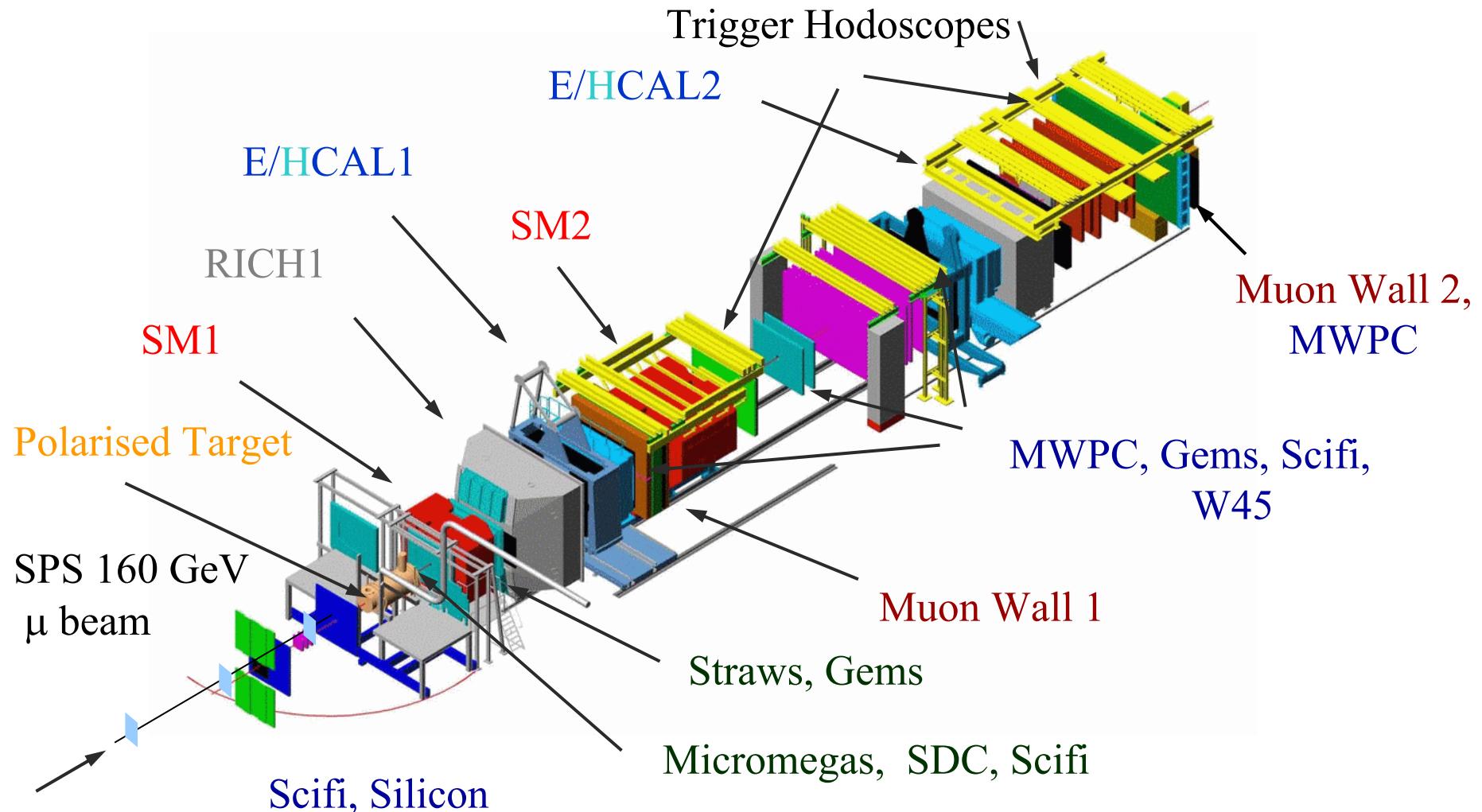
**beam momentum:** 160 GeV/c

**beam polarization:** ~76 %

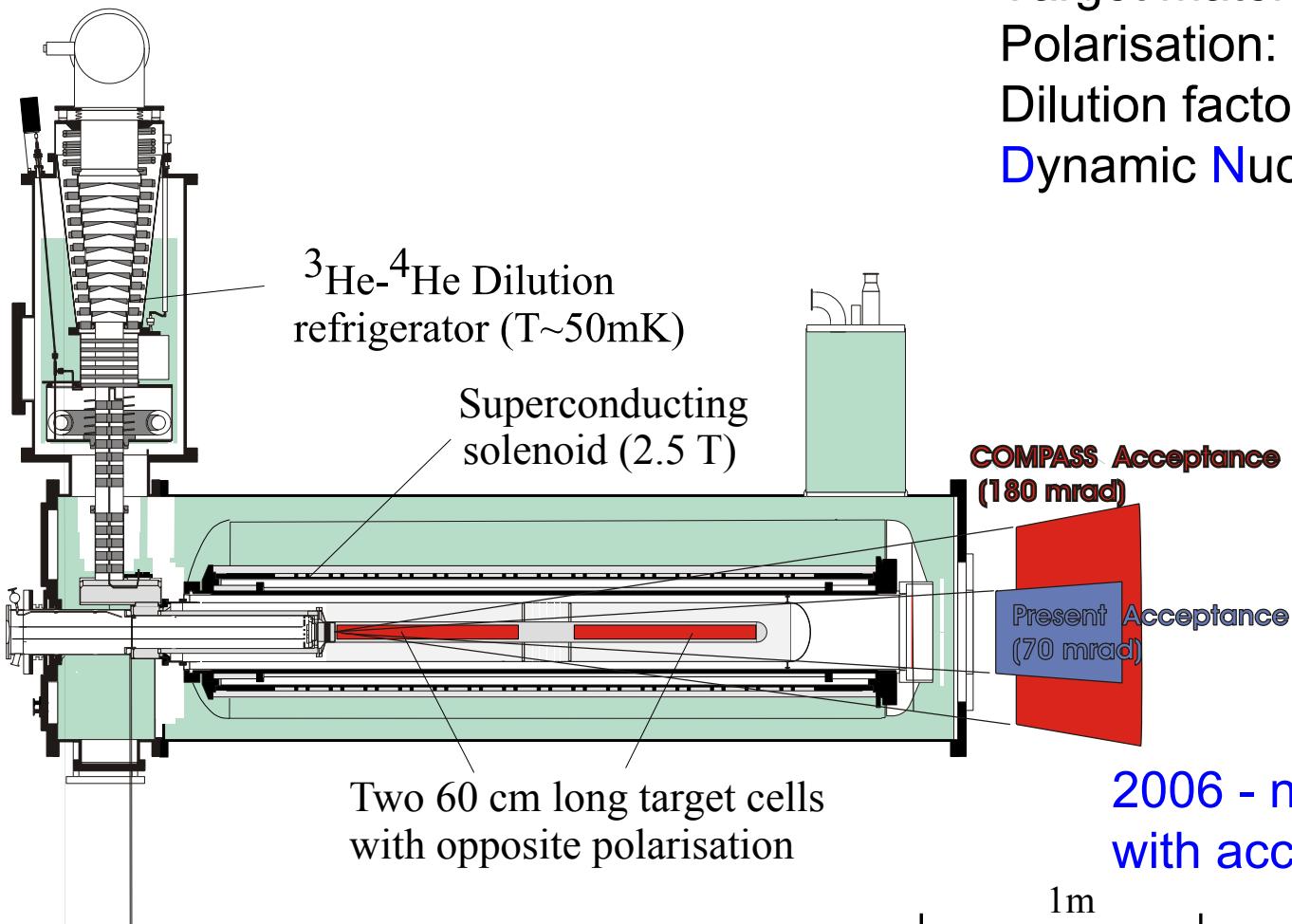
**target polarization:** ~50 %



# The COMPASS Spectrometer



# The COMPASS polarised target



Target material:  ${}^6\text{LiD}$   
Polarisation: >50%  
Dilution factor: ~0.4  
Dynamic Nuclear Polarisation

2006 - new solenoid with acceptance 180 mrad

# Content

-  **Definitions.**
-  Inclusive asymmetry  $A_1^d$  and structure function  $g_1^d$  for  $Q^2 < 1 \text{ (GeV/c)}^2$  (quasi-real photon).
-  Inclusive asymmetry  $A_1^d$ , structure function  $g_1^d$  and QCD analysis for  $Q^2 > 1 \text{ (GeV/c)}^2$  (fits).
-  First moment of  $g_1^d$ .

## Conclusions.



# Definitions

# $A_1^d$ and structure function $g_1^d$

$$A^{\mu d} = A_{||} = \frac{\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow}} = D (A_1^d + \eta A_2^d) \implies A_1^d \approx \frac{A_{||}}{D}$$

$$|\eta A_2^{d,p,n}| \ll |A_1^{d,p,n}|, \quad \eta = \frac{2(1-y)}{y(2-y)} \sqrt{Q^2}/E,$$

$$A_1^{p,n} = A^{\gamma N} = \frac{\sigma^{1/2} - \sigma^{3/2}}{\sigma^{1/2} + \sigma^{3/2}} \quad \text{for nucleon}$$

$$A_1^d = A^{\gamma d} = \frac{\sigma^0 - \sigma^2}{\sigma^0 + \sigma^2} \quad \text{for deuteron}$$

$$A_{\text{meas}} \sim A^{\mu d} \sim A_1^d;$$

Measurement of  $A_1$  gives access to  $g_1$  structure function

$$g_1^d = \frac{1}{2} (g_1^p + g_1^n) (1 - \frac{3}{2} \omega_d) \simeq A_1^d F_1^d = A_1^d \frac{F_2^d}{2x(1+R)}$$

# $A_1^d$ and structure function $g_1^d$

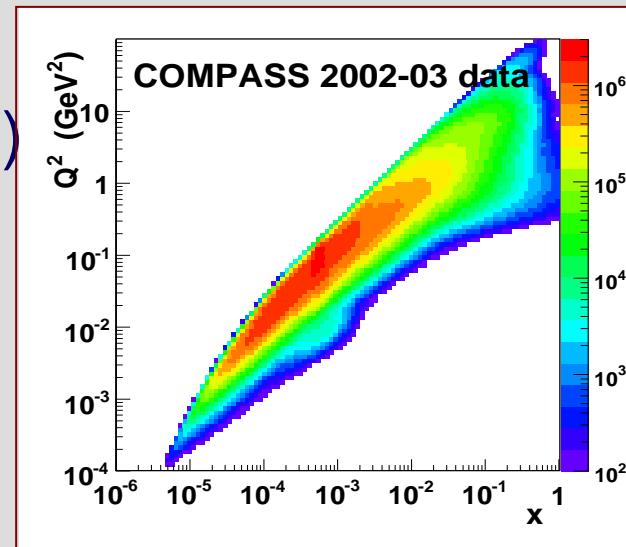
- $A_1^d$  and  $g_1^d$  for small  $Q^2$  ( $Q^2 < 1 \text{ (GeV/c)}^2$ ):  
physics at small  $x$ , parton saturation,  
non-perturbative models (Regge,VDM)  
poorly known (only SMC data)
- $A_1^d$  and  $g_1^d$  for high  $Q^2$  ( $Q^2 > 1 \text{ (GeV/c)}^2$ ):  
QCD analysis possible: $\Delta G$  estimation

# $A_1^d$ and structure function $g_1^d$

■  $A_1^d$  and  $g_1^d$  for small  $Q^2$  :  
physics at small  $x$ , parton saturation,  
non-perturbative models (Regge,VDM)  
poorly known (only SMC data)

■  $A_1^d$  and  $g_1^d$  for high  $Q^2$  :  
QCD analysis possible:  $\Delta G$  estimation

$Q^2$  and  $x$  are strongly correlated for  
small  $Q^2$  in COMPASS (fixed target)

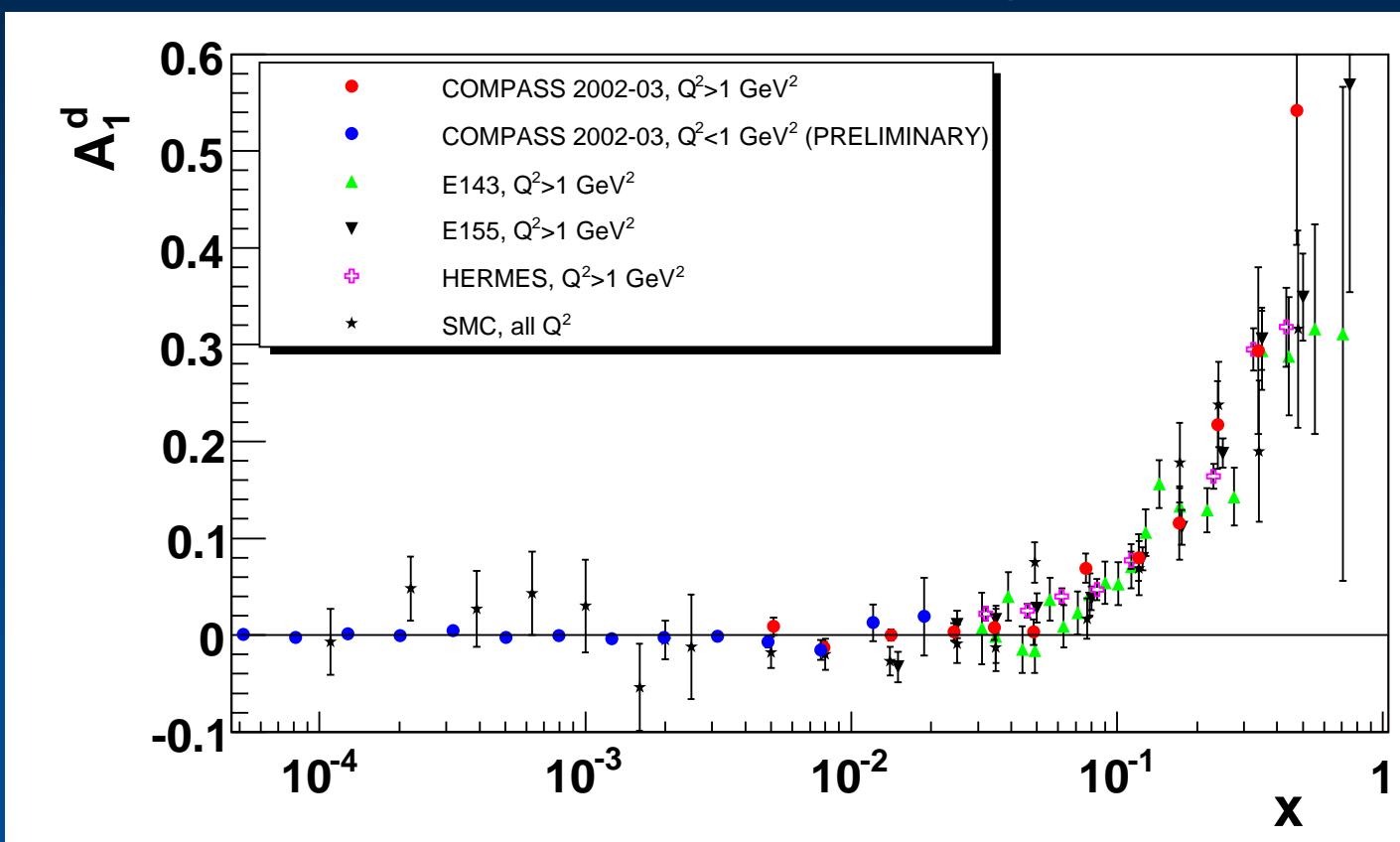




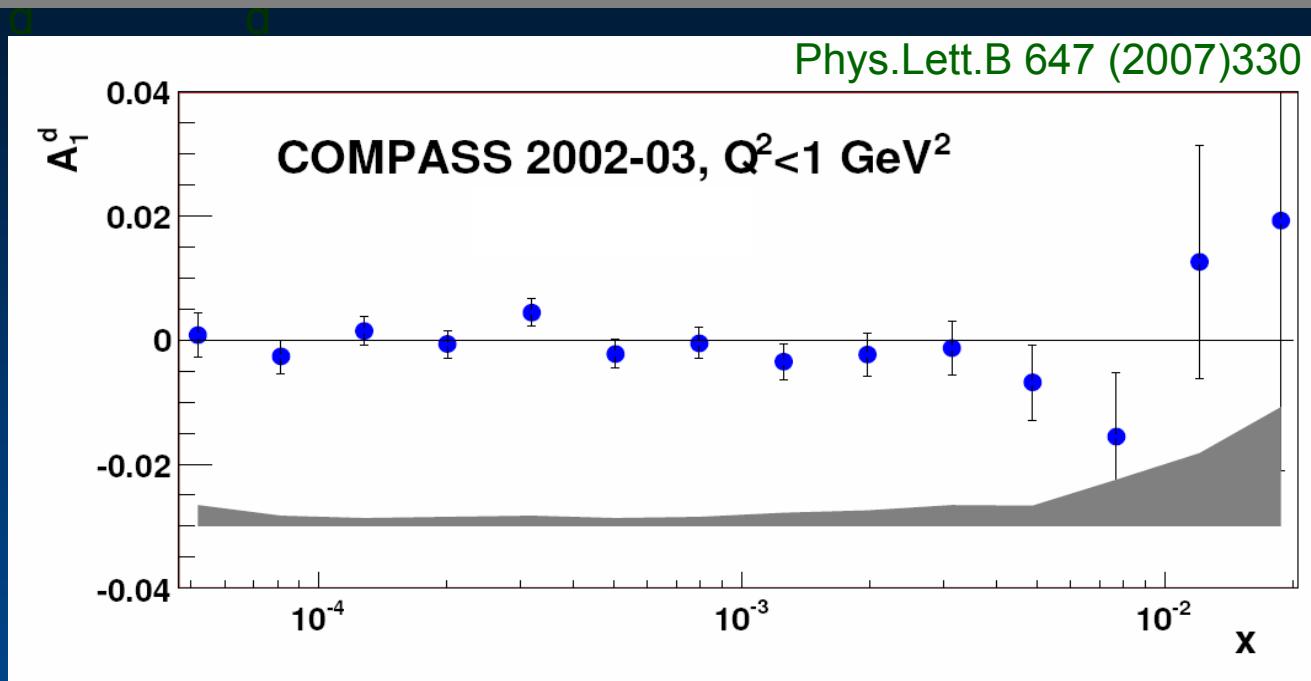
Inclusive asymmetry  $A_1^d$   
and structure function  $g_1^d$   
for  $Q^2 < 1 \text{ (GeV/c)}^2$

# Inclusive asymmetry $A_1^d$ and structure function $g_1^d$ for $Q^2 < 1$ (GeV/c) $^2$

blue points – Compass 2002-2003 data ( $Q^2 < 1$  (GeV/c) $^2$ )  
10-20 times lower statistical errors compared to SMC



# Inclusive asymmetry $A_1^d$ and structure function $g_1^d$ for $Q^2 < 1$ (GeV/c) $^2$



$A_1^d$  asymmetry compatible with 0 at low  $x$  range ( $0.0005 < x < 0.02$ )

At low  $x$   $A_1^d$  has been measured only by COMPASS and SMC

Systematic errors are mainly due to false asymmetries

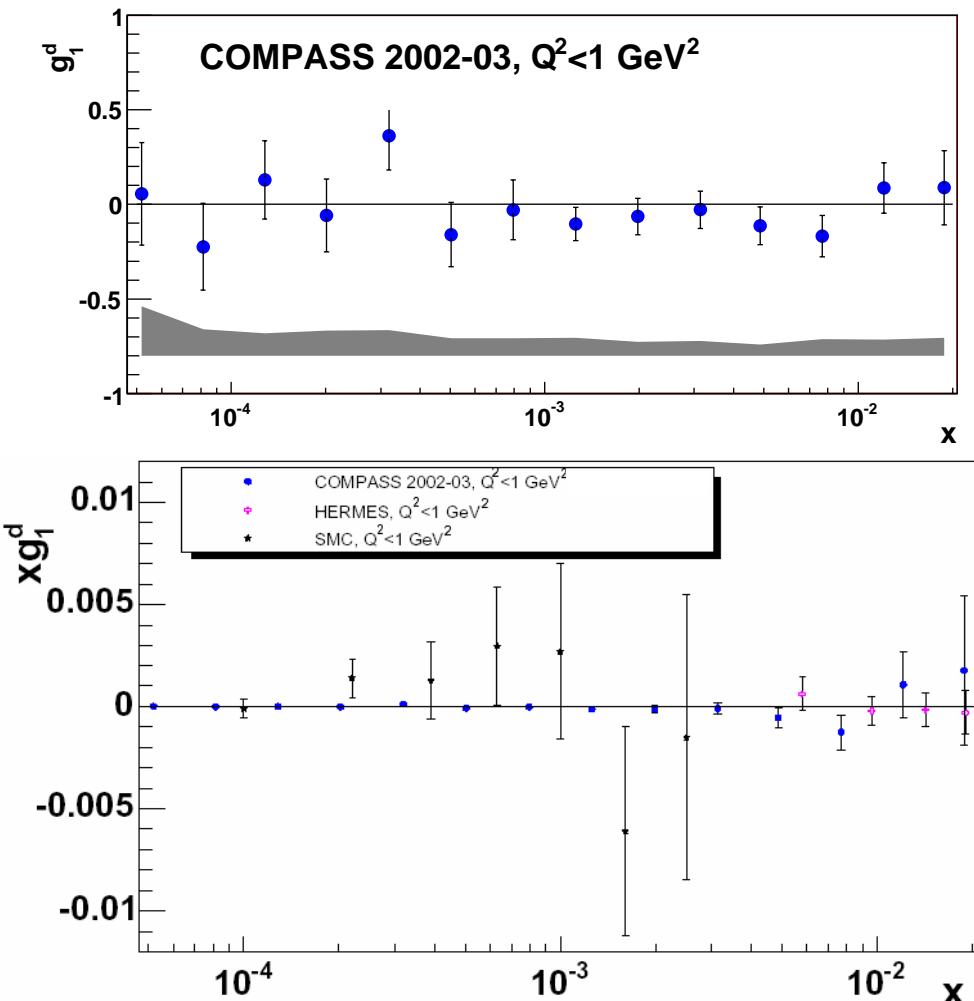
# Inclusive asymmetry $A_1^d$ and structure function $g_1^d$ for $Q^2 < 1$ (GeV/c) $^2$

d

d

$$g_1(x) = A_1(x) \frac{F_2(x)}{2x(1+R)}$$

$F_2$  taken from SMC param.,  
 R depends on x:  $x > 0.12$   
 SLAC (Phys.Lett.B250(1990)193,  
 B52(1999)194)  
 $0.003 < x < 0.12$  NMC  
 (R param. unpublished)  
 $x < 0.003$  ZEUS  
 (Eur.Phys.JC7(1999)609,  $\sigma_L$ ,  $\sigma_T$   
 cross sections param.)

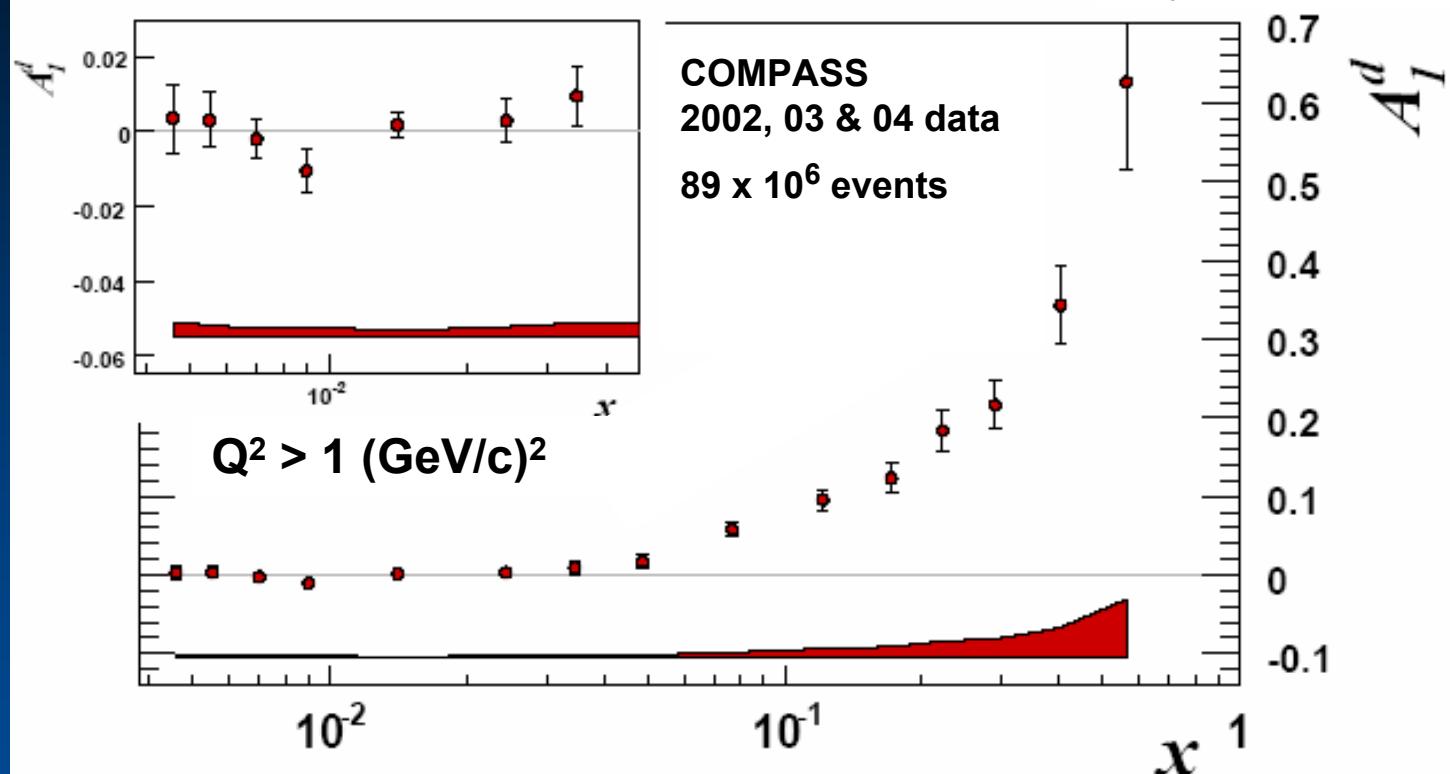




Inclusive asymmetry  $A_1^d$   
and structure function  $g_1^d$   
for  $Q^2 > 1$   $(\text{GeV}/c)^2$

# Inclusive asymmetry $A_1^d$ and structure function $g_1^d$ for $Q^2 > 1$ (GeV/c) $^2$

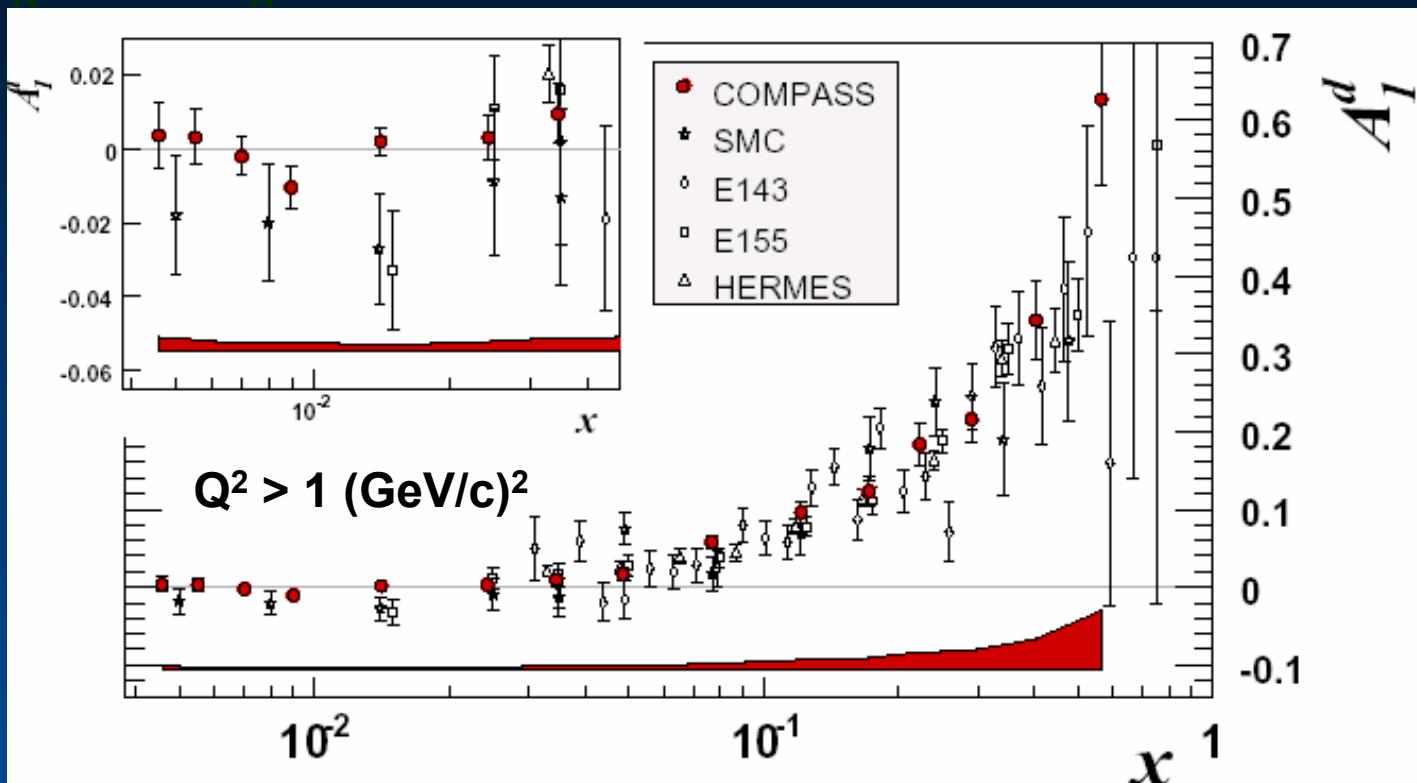
Phys.Lett.B 647(2007)8



$A_1$  compatible with 0 for  $x < 0.05$

Large asymmetry at large  $x$

# Inclusive asymmetry $A_1^d$ and structure function $g_1^d$ for $Q^2 > 1$ (GeV/c) $^2$



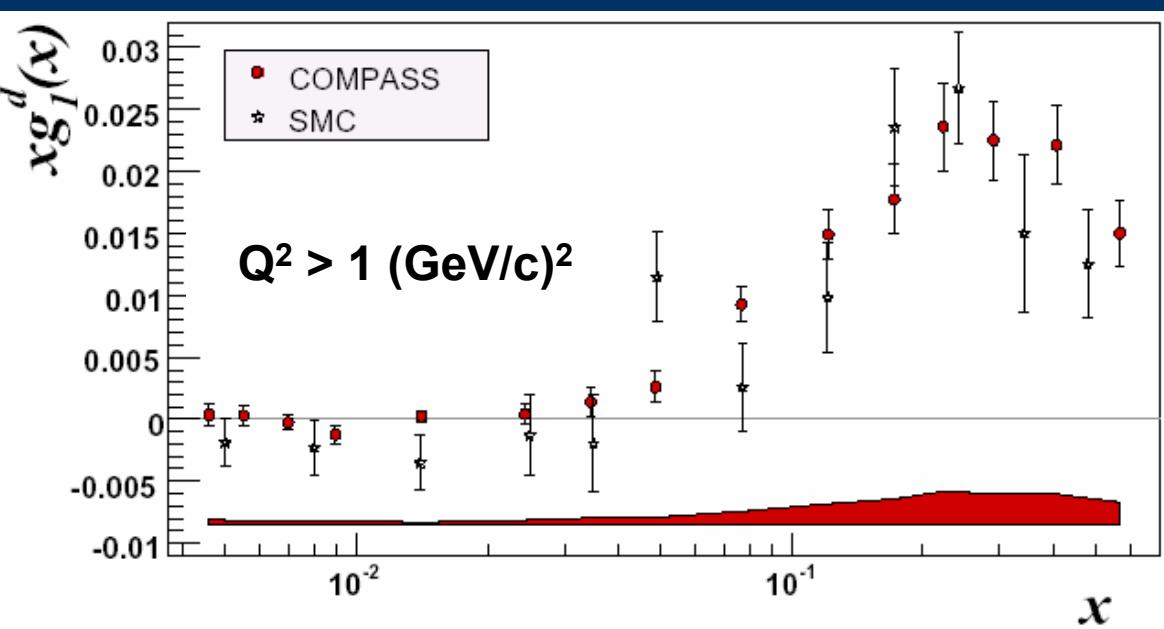
Good agreement with previous experiments

Improved significantly statistics at low  $x$

No tendency towards negative values at  $x < 0.03$

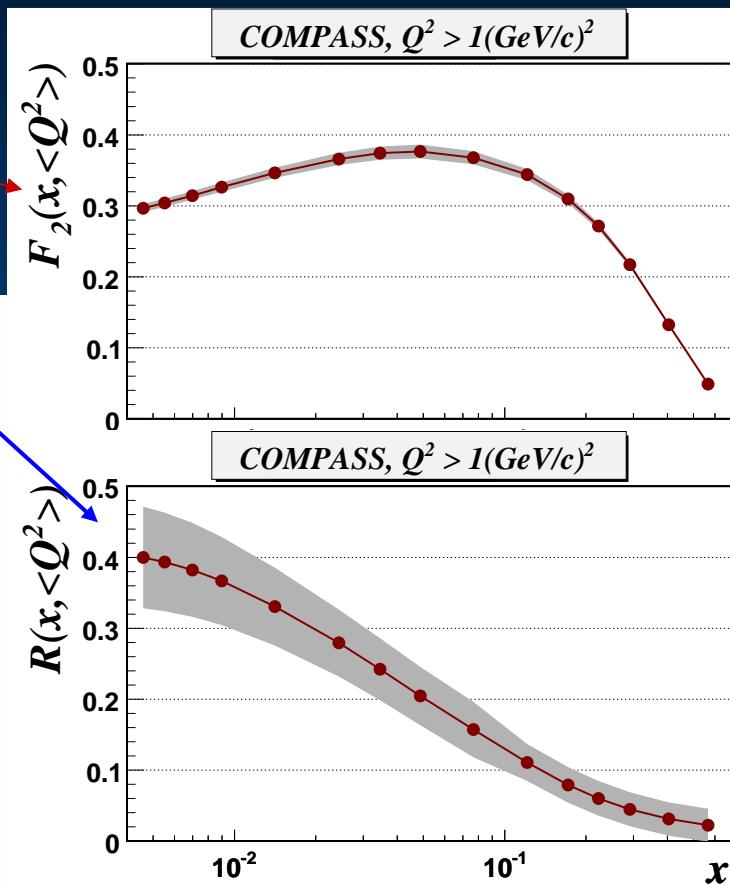
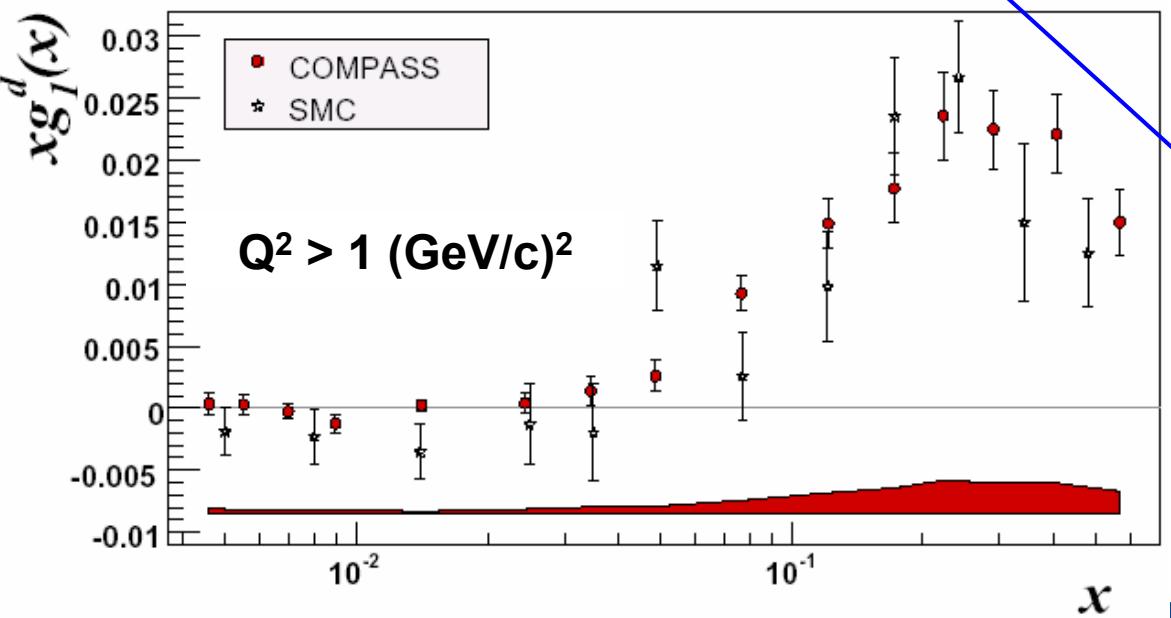
# Inclusive asymmetry $A_1$ and structure function $g_1$ for $Q^2 > 1$ (GeV/c) $^2$

$$g_1^d = g_1^N \left(1 - \frac{3}{2} \omega_d\right) = \frac{F_2^d}{2x(1+R)} A_1^d$$



# Inclusive asymmetry $A_1$ and structure function $g_1$ for $Q^2 > 1 \text{ (GeV/c)}^2$

$$g_1^d = g_1^N \left(1 - \frac{3}{2} \omega_d\right) = \frac{F_2^d}{2x(1+R)} A_1^d$$



R(1998)



# QCD analysis of the world data on structure function $g_1$

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$$g_1(x, Q^2) = \frac{1}{2} \langle e^2 \rangle \left[ C_q^s \otimes \Delta\Sigma + C_q^{NS} \otimes \Delta q^{NS} + 2n_f C_G \otimes \Delta G \right]$$

█ DGLAP equations:  $t = \log\left(\frac{Q^2}{\Lambda^2}\right)$ 

$$\begin{cases} \frac{d}{dt} \Delta q^{NS} = \frac{\alpha_s(t)}{2\pi} P_{qq}^{NS} \otimes \Delta q^{NS} \\ \frac{d}{dt} \begin{pmatrix} \Delta\Sigma \\ \Delta G \end{pmatrix} = \frac{\alpha_s(t)}{2\pi} \begin{pmatrix} P_{qq}^S & 2n_f P_{qG}^S \\ P_{Gq}^S & P_{GG}^S \end{pmatrix} \otimes \begin{pmatrix} \Delta\Sigma \\ \Delta G \end{pmatrix} \end{cases}$$

█ Initial parametrization:  
 x dependence at **fixed  $Q^2$**   
 $(\gamma \neq 0$  for singlet only for  $\Delta G > 0)$

█ Minimization routine

$$(\Delta\Sigma, \Delta q_s, \Delta q_8, \Delta G) = \eta \frac{x^\alpha (1-x)^\beta (1+\gamma x)}{\int_0^1 x^\alpha (1-x)^\beta (1+\gamma x) dx}$$

$$\chi^2 = \sum_{i=1}^N \frac{[g_1^{calc}(x, Q^2) - g_1^{\exp}(x, Q^2)]^2}{[\sigma_{stat}^{\exp}(x, Q^2)]^2}$$

# QCD analysis of the world data on structure function $g_1$

Two different approaches in NLO  $\overline{\text{MS}}$  scheme have been used:

- grid in  $(Q^2, x)$  space (Phys.Rev.D58(1998)112002)
- Mellin transform + moments space (Phys.Rev.D70(2004)074032)

World data fit: 9 experiments, total 230 points, 43 from COMPASS

Experiment	Target nucleon	Nb of points	Reference
EMC	p	10	Nucl. Phys. B 328 (1989) 1
SMC	p	12	Phys.Rev. D 58 (1998) 112001
SMC	d	12	Phys.Rev. D 58 (1998) 112001
COMPASS	d	43	Phys.Lett. B 647 (2007)8
E143	p	28	Phys.Rev. D 58 (1998) 112003
E143	d	28	Phys.Rev. D 58 (1998) 112003
E155	d	24	Phys. Lett. D 463 (1999) 339
E155	p	24	Phys.Lett. B 493 (2000) 19
JLAB	n	3	Phys. Rev. Lett. 92 (2004) 012004
E142	n	8	Phys.Rev. D 54 (1996) 6620
E154	n	11	Phys.Rev. Lett. 79 (1997) 26
HERMES	n	9	Phys.Lett. B 404 (1997) 383
HERMES	p	9	Phys.Rev. D75 (2005) 012003
HERMES	d	9	Phys.Rev. D75 (2005) 012003

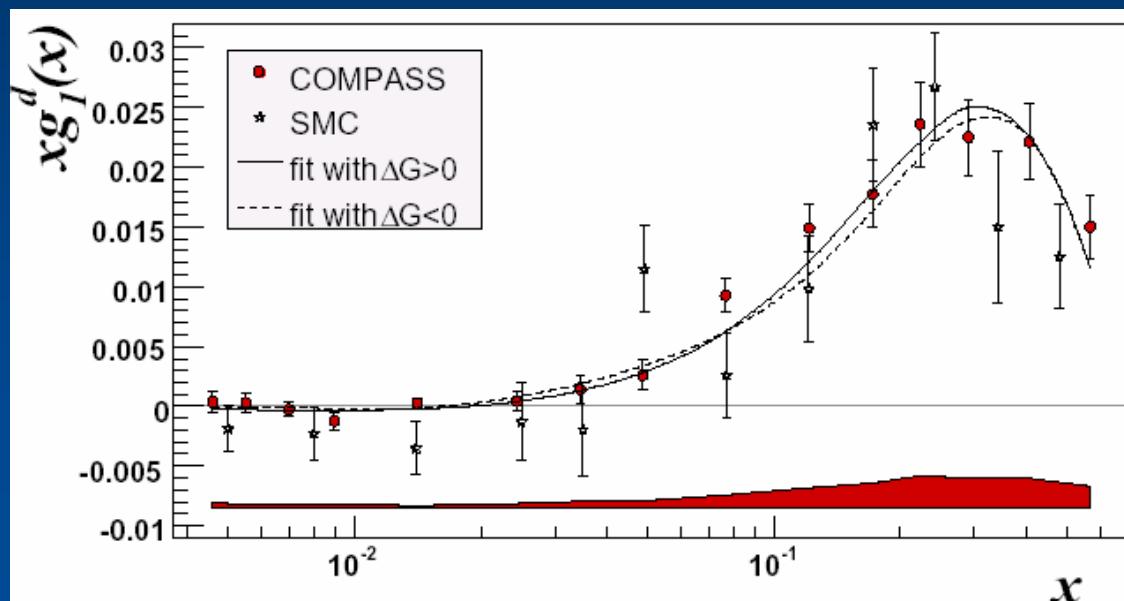
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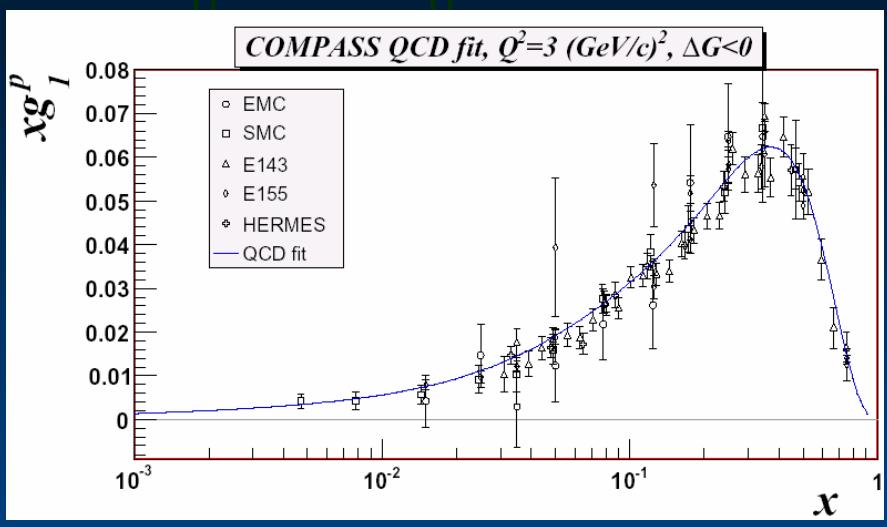
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World data fit: 9 experiments, 230 points, 43 from COMPASS

Two solutions describe data equally well:  $\Delta G > 0$  and  $\Delta G < 0$ .

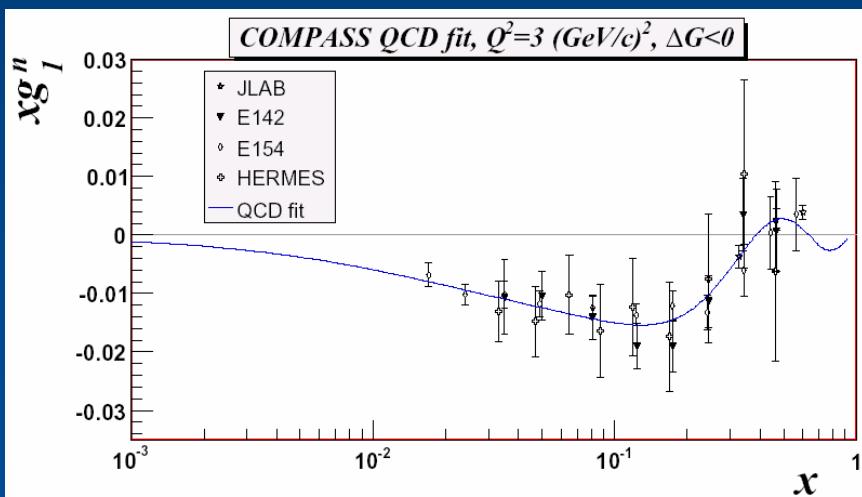
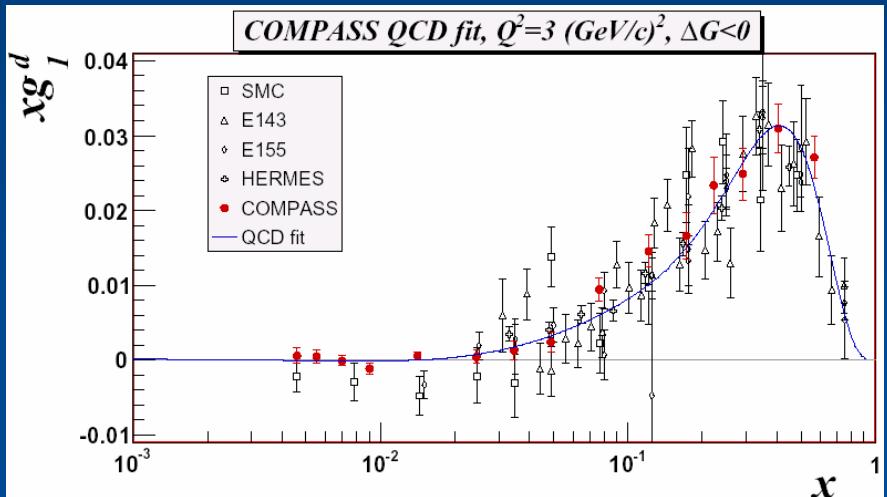


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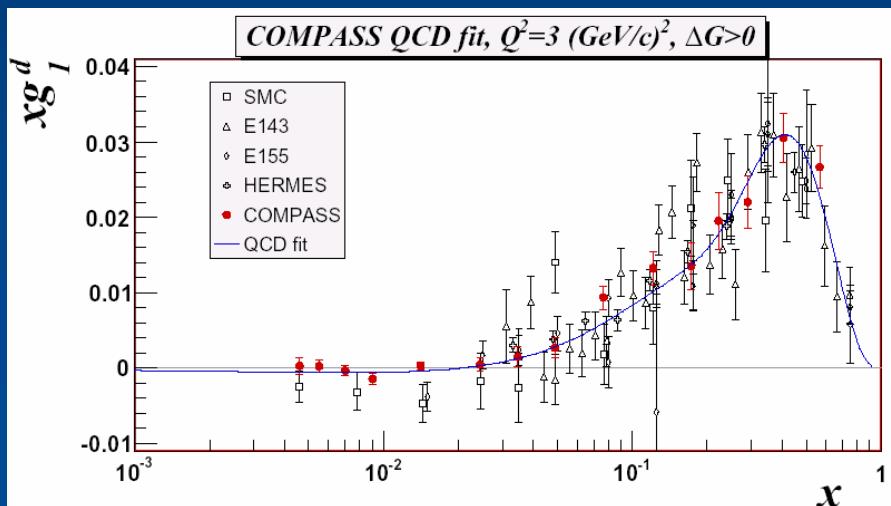
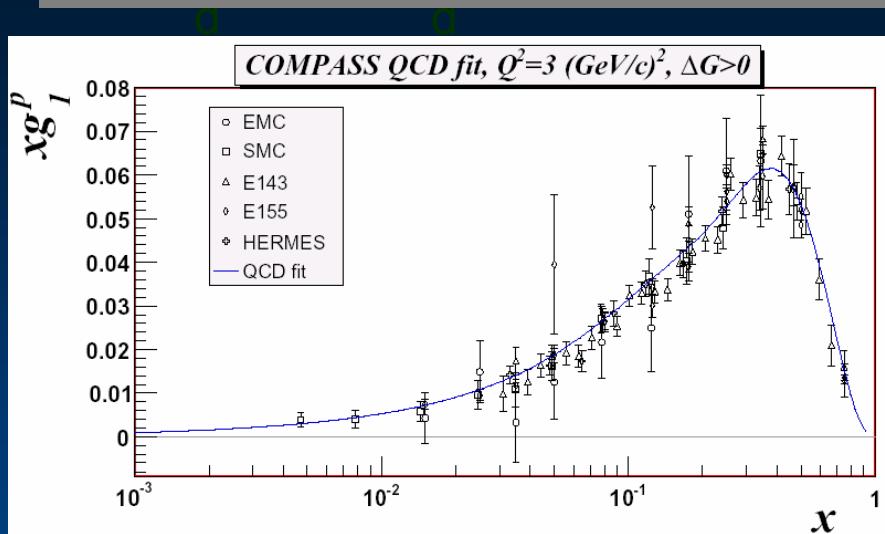


World data and QCD fits at  
 $Q_0^2 = 3 \text{ (GeV/c)}^2$

Solutions with  $\Delta G < 0$

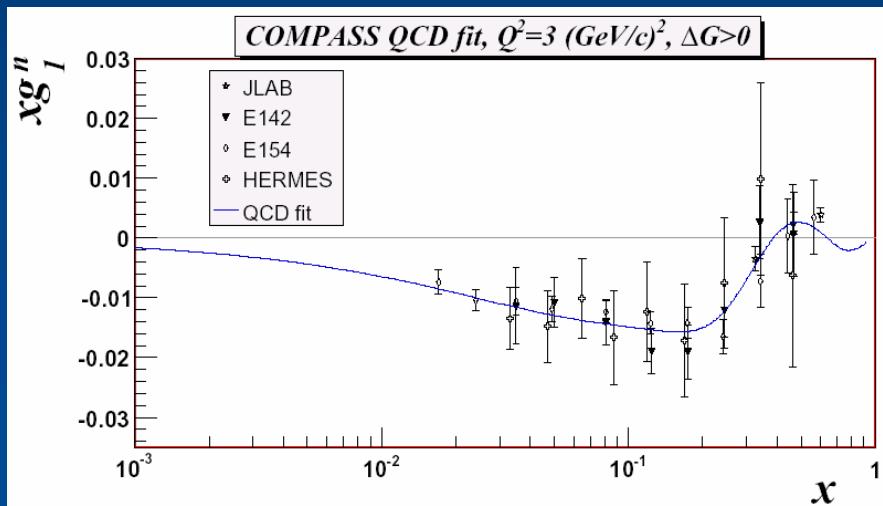


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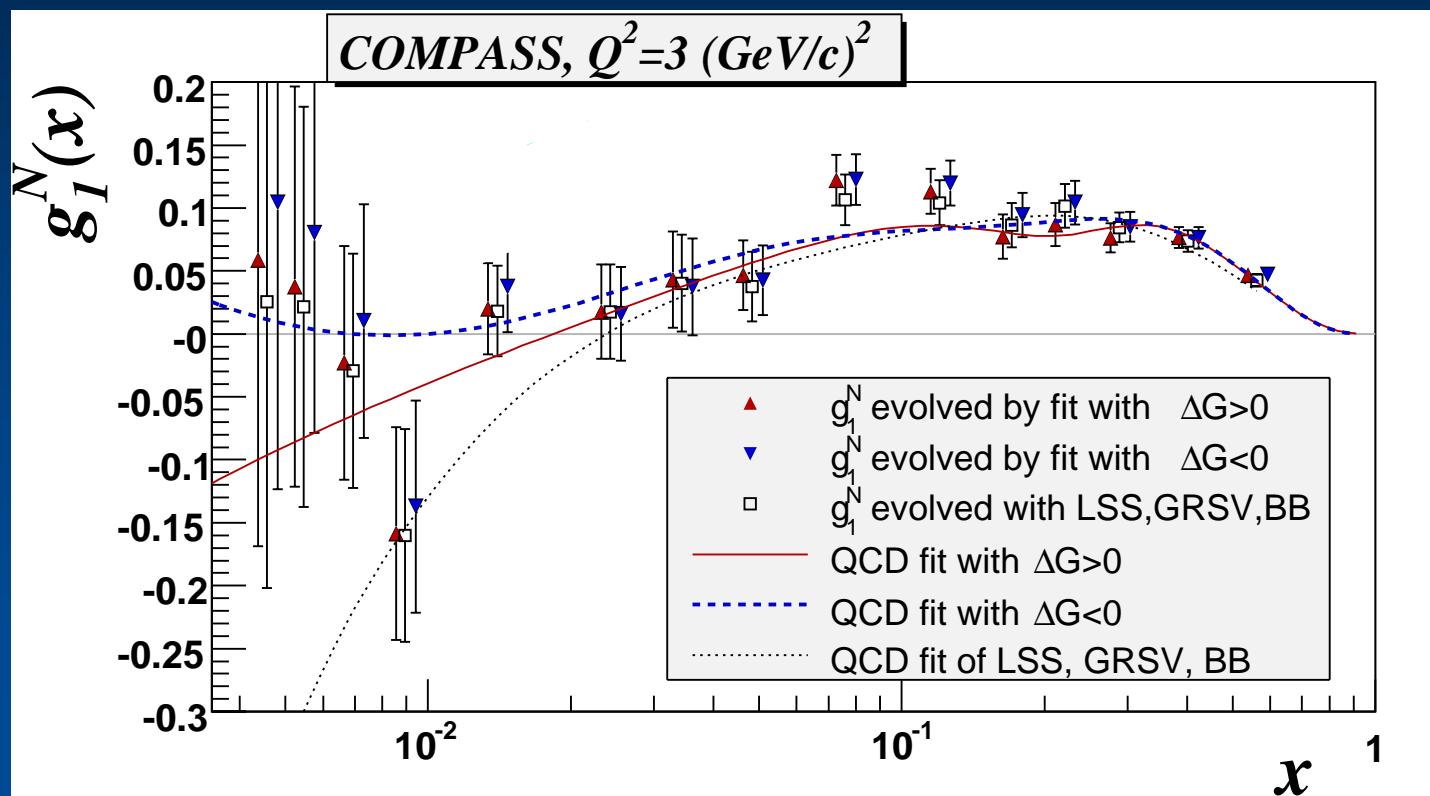
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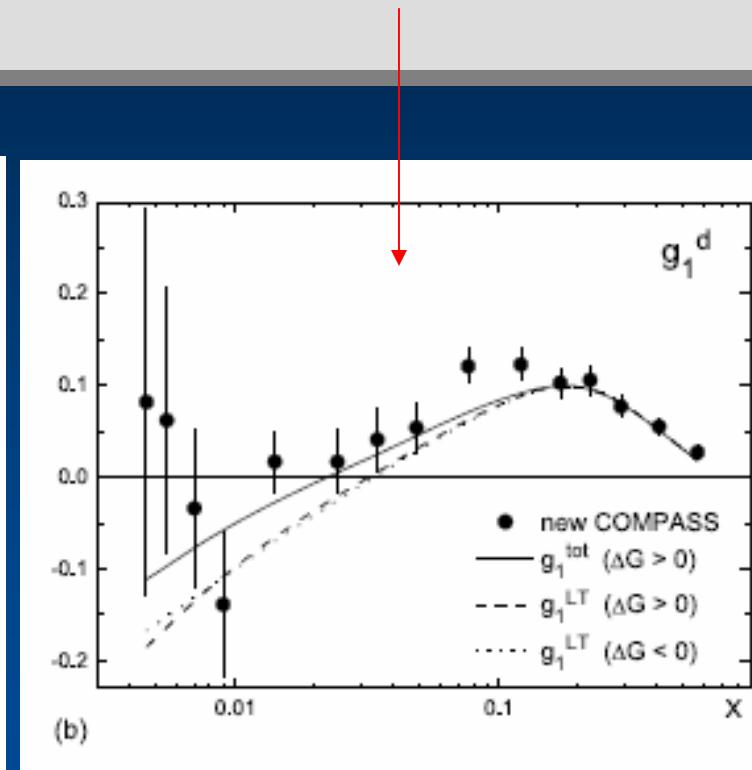
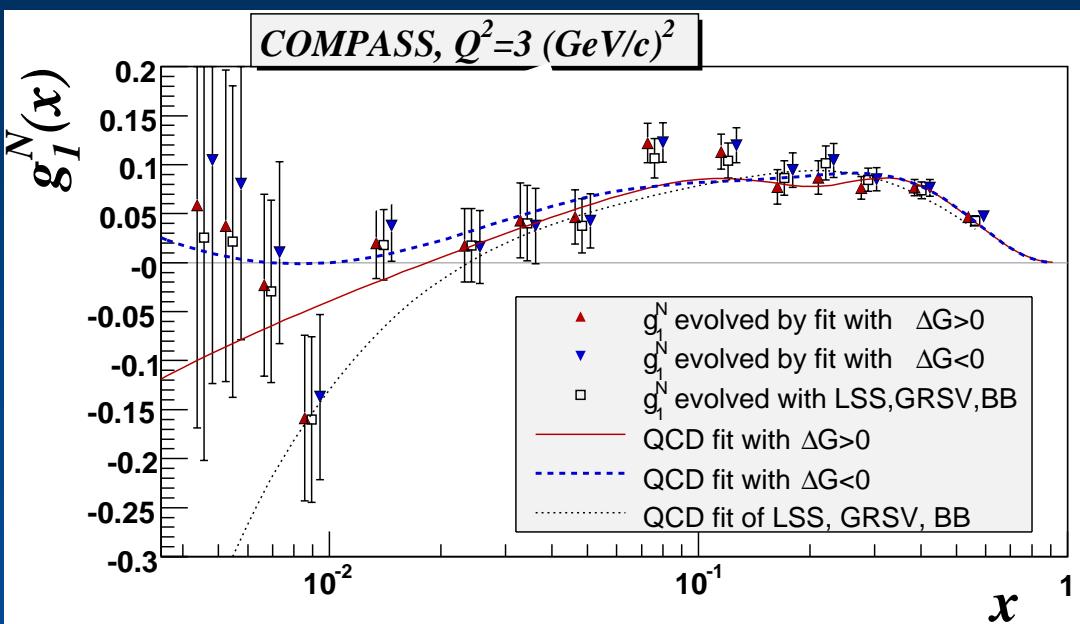
# QCD analysis of the world data on structure function $g_1$

Comparison of fits - disagreement of data with previous QCD fits (LSS05, BB, GRSV)



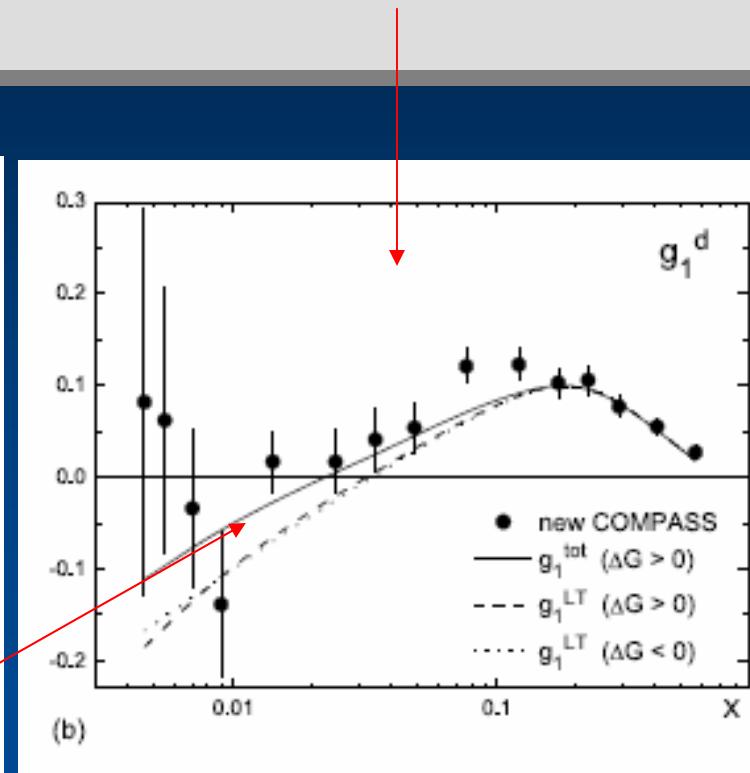
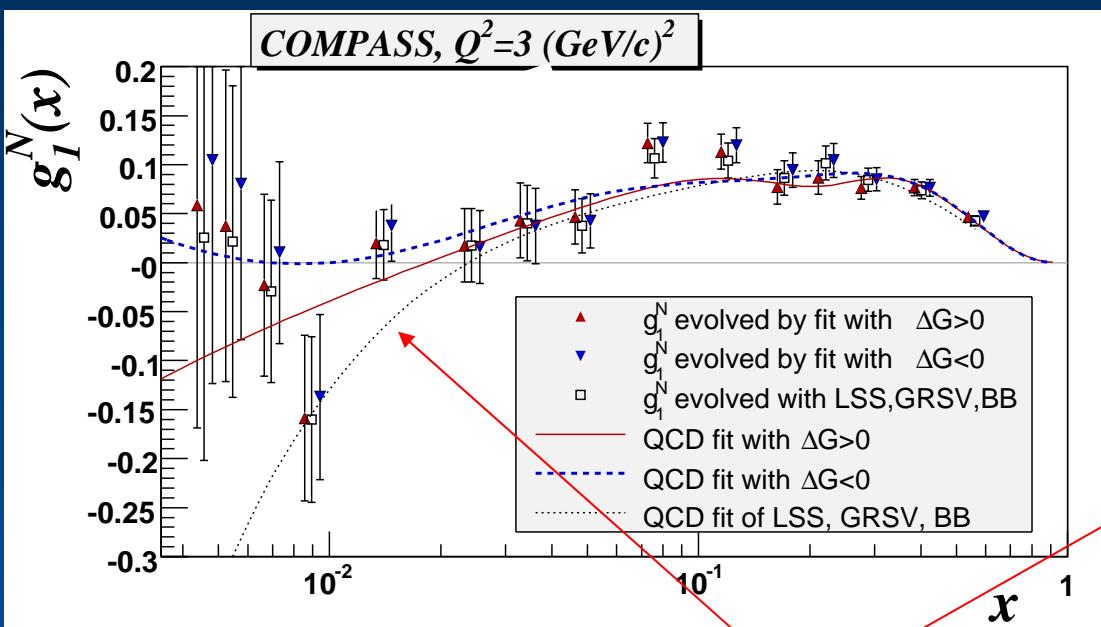
# QCD analysis of the world data on structure function $g_1$

Comparison of data and fits - LSS06 (hep-ph/0612360)



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Comparison of data and fits - LSS06 (hep-ph/0612360)

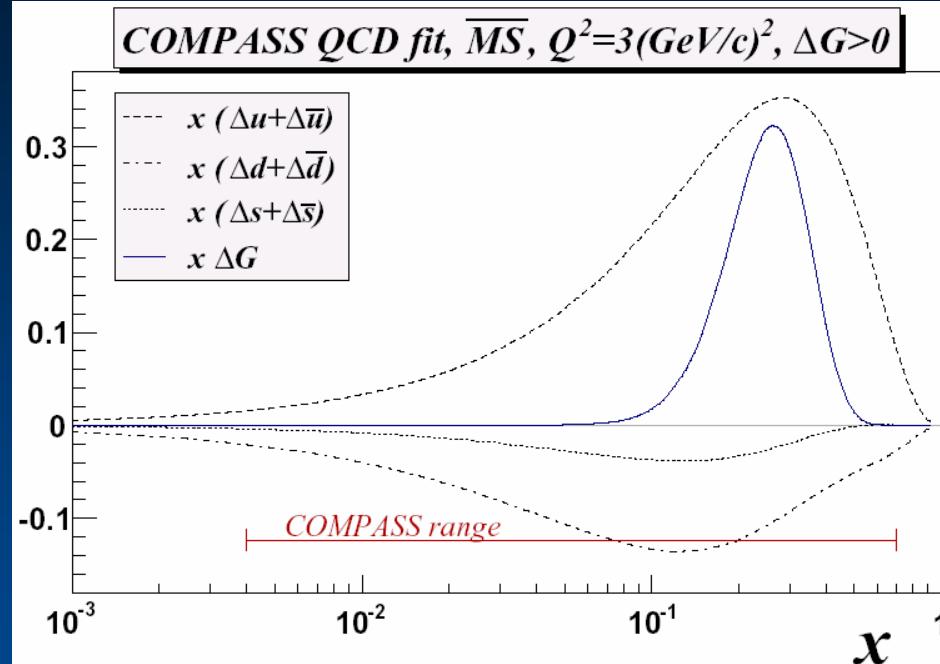
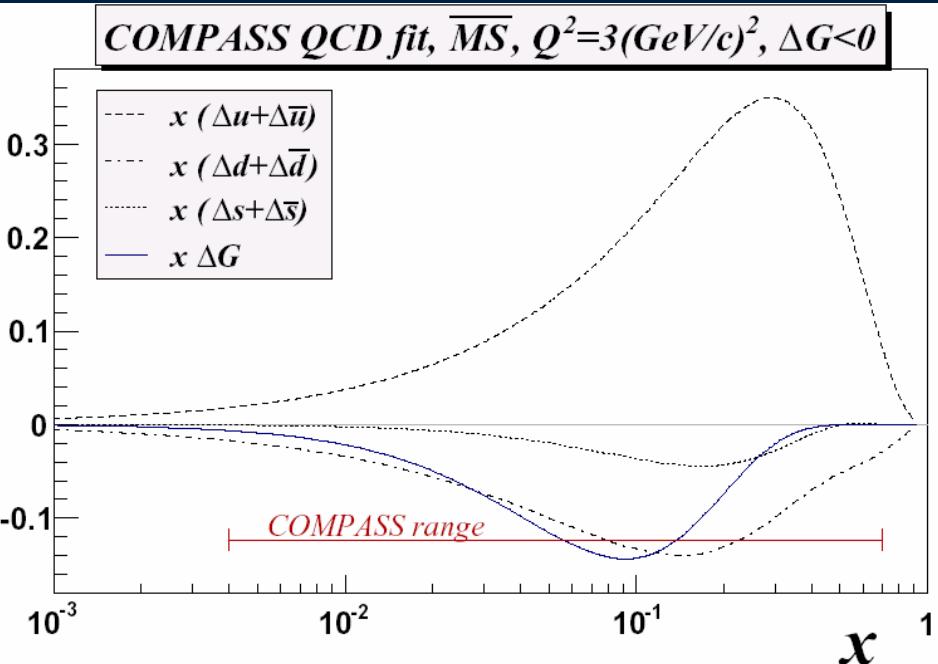


LSS05 vs LSS06

# QCD analysis Polarised parton distributions

d

d



Very small sensitivity of  $x(\Delta q + \Delta \bar{q})$  to  $x \Delta G$

# QCD fits results

(world data)

Quark polarisation:

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	$\eta_G > 0$	$\eta_G < 0$
$\eta_\Sigma$	$0.27 \pm 0.01$	$0.32 \pm 0.01$

$$\longrightarrow \eta_\Sigma = 0.30 \pm 0.01(\text{stat}) \pm 0.02(\text{evol})$$

$$\left( \eta_K = \int_0^1 \Delta k \, dx \right) \quad (\text{error} \approx \text{factor 2 larger without COMPASS data})$$

Gluon polarisation (indirect determination via DGLAP):

- Solutions with  $\eta_G > 0$ :

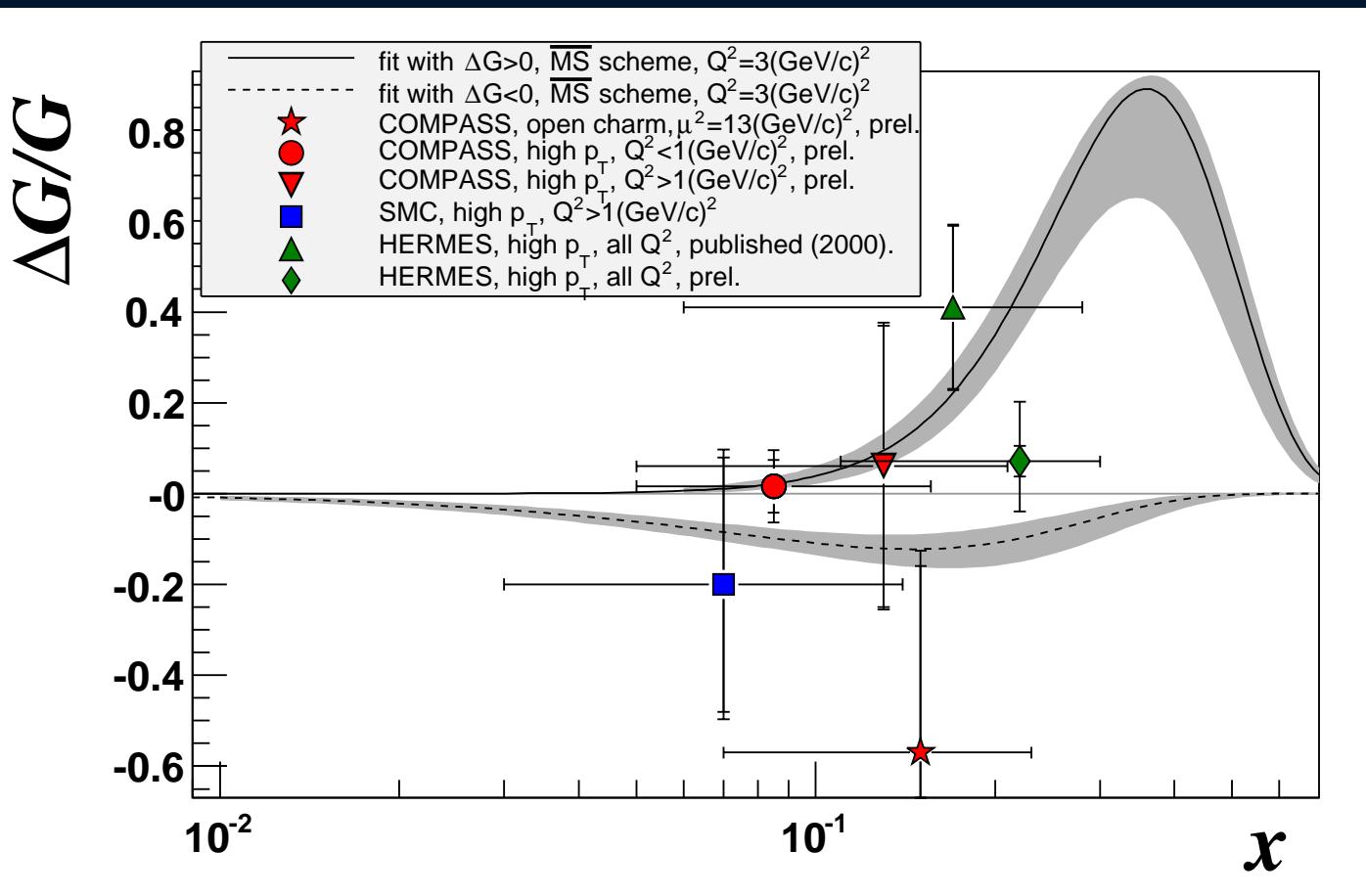
$$\eta_G^{\text{prog1}} = 0.34^{+0.05}_{-0.07}, \quad \eta_G^{\text{prog2}} = 0.23^{+0.04}_{-0.05}$$

- Solutions with  $\eta_G < 0$ :

$$\eta_G^{\text{prog1}} = -0.31^{+0.10}_{-0.14}, \quad \eta_G^{\text{prog2}} = -0.19^{+0.06}_{-0.11}$$

$$|\eta_G| \approx 0.2 - 0.3$$

# QCD fits results: Gluon polarisation



Unpolarised  $G(x)$  from MRST (NLO 2004)  
Bands correspond to errors from fit of  $\Delta G(x)$



# First moment of $g_1^d$

# First moment of $g_1^d$

(Compass data only)

Phys.Lett.B 647(2007)8

$$\Gamma_1^N(Q_0^2 = 3 \text{GeV}^2) = \int_0^1 g_1^N(x) dx = 0.050 \pm 0.003(\text{stat}) \pm 0.003(\text{evol}) \pm 0.005(\text{syst})$$

$$\Gamma_1^N(Q^2) = \frac{1}{9} \left( 1 - \frac{\alpha_s(Q^2)}{\pi} + O(\alpha_s^2) \right) \left( a_0(Q^2) + \frac{1}{4} a_8 \right) \quad (\text{NLO QCD})$$

from Y. Goto *et al.*, PRD62 (2000) 034017: ( $SU(3)_f$  assumed for weak decays)

$$a_8 = 0.585 \pm 0.025$$

$$a_{0|Q_0^2=3(\text{GeV}/c)^2} = 0.35 \pm 0.03(\text{stat}) \pm 0.05(\text{syst})$$

Contribution from unmeasured x range  $\approx 4\%$

# First moment of $g_1^d$ (COMPASS data only)

Another notation: in the limit  $Q^2 \rightarrow \infty$  (beyond NLO)

$$\hat{a}_0 \equiv a_{0|Q^2 \rightarrow \infty}$$

$$\Gamma_1^N(Q^2) = \frac{1}{9} C_1^S(Q^2) \hat{a}_0 + \frac{1}{36} C_1^{NS}(Q^2) a_8 \quad C_1 \text{ calculated behind 3 loops app.}$$

S.A.Larin *et al.*, Phys.Lett.B404(1997)153

$$a_{0|Q^2 \rightarrow \infty} = 0.33 \pm 0.03(stat) \pm 0.05(syst)$$

$$(\Delta s + \Delta \bar{s}) = \frac{1}{3} (\hat{a}_0 - a_8) = -0.08 \pm 0.01(stat) \pm 0.02(syst)$$



# Summary

# Summary

- New measurements of  $A_1^d$ ,  $g_1^d$  have been presented.
- Good agreement with results from previous experiments in the region of middle and high  $x$ .
- Improvement in statistical precision factor 4 for  $x < 0.03$ .
- No tendency toward negative values at  $x < 0.03$ .

- New QCD fits have been performed.
- Fits have produced consistent results and yield two solutions for PDF with  $\Delta G(x) > 0$  and  $\Delta G(x) < 0$  which equally well describe the present  $g_1$  data. The shapes of  $\Delta G(x)$  are very different in two cases.
- The first moment of the polarised gluon distribution has been estimated from the QCD fits.
- Polarised strange quark distribution has been found.

# Spares

# DIS 2007

April 16-20, 2007, Munich, Germany

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# Spare 0 - Nucleon spin decomposition

$$\Gamma_1 = \int g_1(x) dx$$

$$\Gamma_1^p - \Gamma_1^n = \frac{g_A}{6g_V} C_1^{NS} \quad (\text{Bjorken sum rule})$$

$$\Gamma_1^{p,n} = \left( \pm a_3 + \frac{a_8}{\sqrt{3}} \right) \frac{C_1^{NS}}{12} + a_0 \frac{C_1^S}{9} \quad (\text{Ellis-Jaffe sum rule})$$

$a_3, a_8, g_{A,V}$  - hyperon  $\beta$  decay + SU<sub>f</sub>(3);

$C_1^{S,NS}$  - calculable in QCD

But - due to  anomaly -  $a_0 = \Delta\Sigma - (3\alpha_S/2\pi) \Delta G$  and  
If  $\Delta G \approx 2.5$   $\rightarrow \Delta\Sigma \approx 0.6$   $\rightarrow$  can “solve the spin crisis”



Need direct measurement of  $\Delta G$

# Spare 1 – QCD fits

1.  $\gamma \neq 0$  only for singlet in the case of  $\Delta G > 0$  (and  $\beta_G$  fixed to 10);  
for keeping same number of parameters as for  $\Delta G < 0$ .  
From the fit for  $\Delta G < 0$   $\beta_G$  is 13 with large error  
so fixed number 10 is reasonable.
2.  $\chi^2$  – statistical errors because systematical is highly correlated.
3. p24: curve is made from points taken from fits for  $Q^2$  corresponds  
to  $Q^2$  of the experimental point and interpolated between by spline
4. p31: for  $Q^2 = 11$  ( $\text{GeV}/c^2$ ) (close to charm scale)  
 $\Delta G/G(x=0.13) = -0.072$  instead of 0.082 for  $Q^2 = 3$  ( $\text{GeV}/c^2$ )  
 $x\Delta G (x=0.13) = -0.077$  instead of -0.127  
 $xG(x=0.13) = 1.0678$  instead of 1.1551

# Spare 2 – QCD fits

$\alpha_S(M_Z)$	0.1137 (lower)	0.1187(standard)	0.1237(high)	$\alpha_S$ fitted
$\eta_G < 0$	-0.342±0.117	-0.329±0.107	-0.326±0.1104	$\alpha_S = 0.1276 + 0.0015 - 0.0017$ $\eta_G = -0.34 \pm 0.1$
$\eta_G > 0$	0.253±0.08	0.231±0.049	0.216±0.054	$\alpha_S = 0.1269 + 0.0016 - 0.0018$ $\eta_G = 0.20 \pm 0.05$

Effect of  $a_8$  on evaluation of  $a_0$ :

$a_8 \pm 20\% \rightarrow \pm 0.11 \rightarrow$  changes  $a_0$  by  $\pm 0.03$

$$a_0 = 9 \Gamma_1^N - \frac{1}{4} a_8$$

$$(\Delta s + \Delta \bar{s}) = 1/3 (a_0 - a_8) = 3 \Gamma_1^N - 1/12 a_8$$

$$a_8 \pm 0.11 \rightarrow \text{changes } (\Delta s + \Delta \bar{s}) \text{ by } \pm 0.048$$

# Spare 3 – R for $Q^2 < 1$ GeV $^2$

## 2.1 The $R$ function

The  $R$  function which was previously used by the SMC, and it is commonly used by COMPASS [2] is composed of three different parameterizations in different regions of  $x$  (see [4] for references and explanations):

- SLAC,  $x > 0.12$ ,
- NMC,  $0.003 < x < 0.12$ ,
- ZEUS,  $x < 0.003$ .

Values of  $R$  have large discontinuities close to the validity limits of the parametrizations, Fig.4. To partially overcome the problem, a new SLAC parametrization was used for  $Q^2 > 0.5$  GeV $^2$ , [5]. Below the  $Q^2 = 0.5$  GeV $^2$  the following formula was employed:

$$R(Q^2 < 0.5, x) = R_{SLAC}(0.5, x) \times \beta(1 - \exp(-Q^2/\alpha)) \quad (1)$$

where  $\alpha = 0.2712$ ,  $\beta = 1/(1 - \exp(-0.5/\alpha)) = 1.1880$ . At  $Q^2 = 0.5$  GeV $^2$  the function and its first derivative are continuous. In the  $Q^2=0$  limit:  $R \sim Q^2$ , which is expected from the current conservation. The new  $R$  parametrization is shown in the right plot of Fig.4. The error on  $R$ ,  $\delta R$ , above  $Q^2 = 0.5$  GeV $^2$  was taken from [5] and below  $Q^2 = 0.5$  GeV $^2$  was set to  $\delta R = 0.2$ . For that value and for the simplest assumption about  $R$  for  $Q^2 < 0.5$  GeV $^2$  and any  $x$ , e.g.  $R = 0.2$ , there is an approximate agreement (within  $1\sigma$ ) with the value at the photo-production limit where  $R=0$  and with measurements at higher  $Q^2$  from HERA, where  $R \approx 0.4$ .