Beyond Collins and Sivers: further measurements of the target transverse spin-dependent azimuthal asymmetries in semi-inclusive DIS from COMPASS

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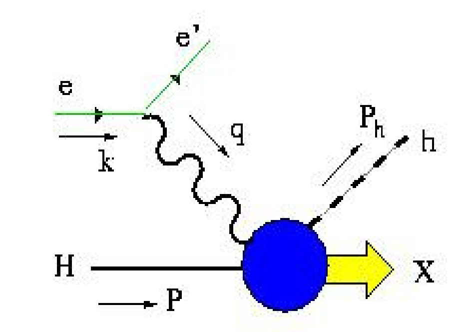
on behalf of the COMPASS Collaboration

- General expression for polarized SIDIS cross-section
- Target transverse spin dependent azimuthal asymmetries
- COMPASS results
- Parton model for SIDIS in CFR
- Conclusions

General expression of polarized SIDIS cross-section



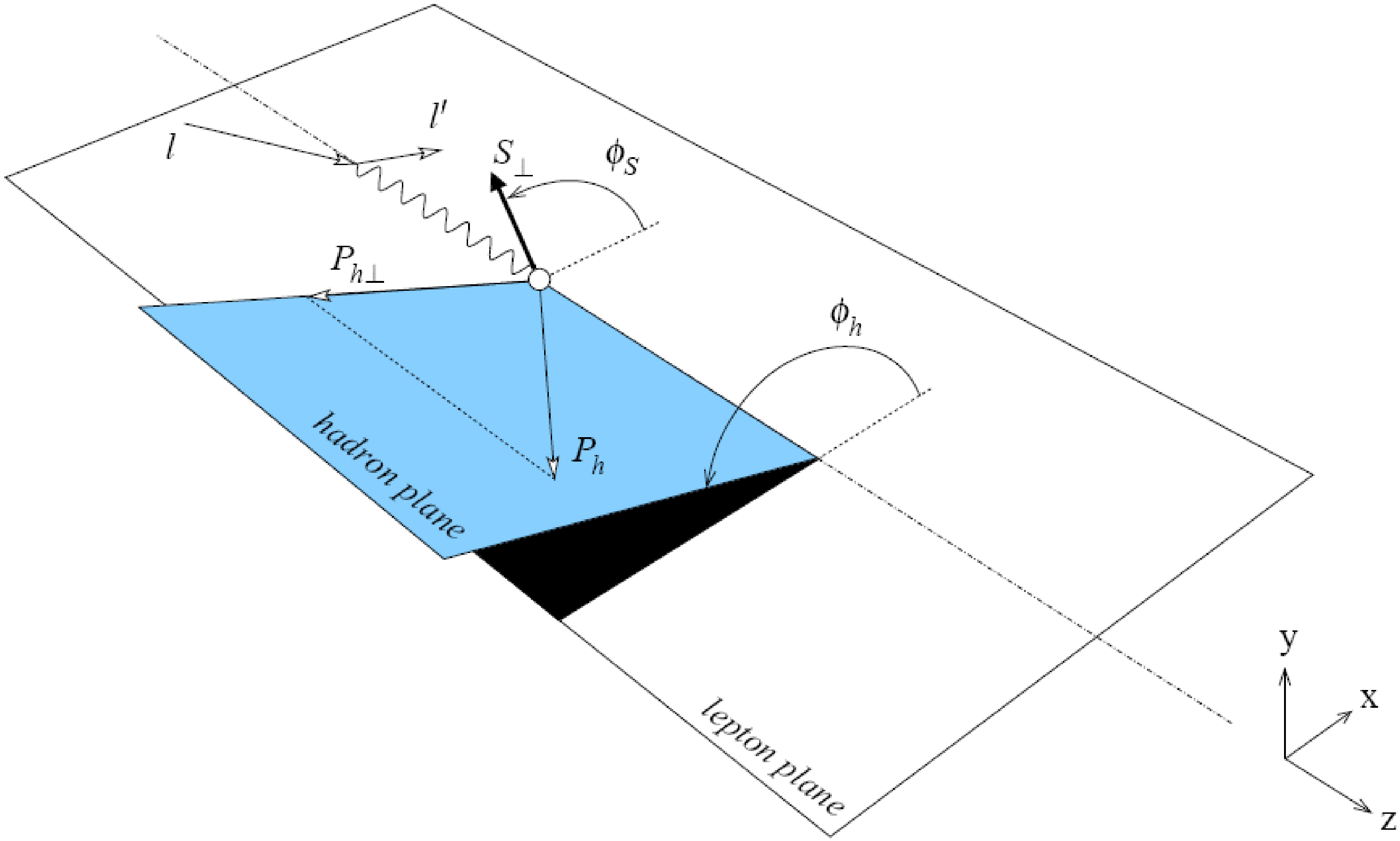
X can be one-particle state

One photon exchange approximation:

$$\mathcal{M} \propto J_{\mu}^{lept} \frac{1}{q^2} J_{hadr}^{\mu}$$

- 1) New quark distributions and semi-inclusive electroproduction on polarized nucleons. A.K. *NP B441 (1995) 234*
  - General expression is derived
  - Parton model: 6 twist-two TMD DFs & FFs + ~k_T/Q kinematical (Cahn) corrections
  - All possible (except Sivers and Boer-Mulders) azimuthal asymmetries appear in this approximation
- 2) Semi-inclusive deep inelastic scattering at small transverse momentum. Bacchetta, Diehl, Goeke, Metz, Mulders and Schlegel JHEP 0702:093,2007
  - General expression: new notations
  - Parton model: all twist-two and twist-three tree level contributions are considered

General expression of polarized SIDIS cross-section (2)



Using current conservation + parity conservation + hermiticity one can show that

18 independent Structure Functions describe one particle SIDIS.

Moreover, the dependences on azimuthal angle of produced hadron and of the target nucleon polarization were calculated explicitly and factorized

General expression of polarized SIDIS cross-section (3)

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =$$

$$\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h F_{UU}^{\cos\phi_h}$$

$$+\varepsilon\cos(2\phi_h)F_{UU}^{\cos 2\phi_h}+P_{beam}\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h F_{LU}^{\sin\phi_h}$$

$$+P_L\left[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)F_{UL}^{\sin 2\phi_h}\right]$$

$$+P_LP_{beam}\left[\sqrt{1-\varepsilon^2}F_{LL}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h F_{LL}^{\cos\phi_h}\right]$$

$$+|\boldsymbol{P}_T|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)$$

$$+\varepsilon\sin(\phi_h+\phi_S)F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\sin(3\phi_h-\phi_S)F_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S F_{UT}^{\sin\phi_S}+\sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]$$

$$+|\boldsymbol{P}_T|P_{beam}\Big[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S F_{LT}^{\cos\phi_S}$$

$$+\sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\},$$

$$\varepsilon=\frac{1-y-\frac{1}{4}y^2\gamma^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}y^2\gamma^2}$$

$$\gamma=2x_BM_p/Q$$

This is a general expression which is also valid for exclusive reactions and for entire phase space of SIDIS (TFR, CFR)

Azimuthal modulations:

2 polarization independent

1 single beam polarization dependent

2 single target longitudinal polarization dependent

1 double beam + target longitudinal polarization dependent

5 single target transverse polarization dependent

3 double beam + target transverse polarization dependent

Target transverse spin dependent azimuthal asymmetries

$$d\sigma(\phi_h, \phi_s, ...) \propto (1 + |\boldsymbol{S}_T| \sum_{i=1}^{5} D^{w_i(\phi_h,\phi_s)} A_{UT}^{w_i(\phi_h,\phi_s)} w_i(\phi_h, \phi_s)$$
$$+ P_{beam} |\boldsymbol{S}_T| \sum_{i=6}^{8} D^{w_i(\phi_h,\phi_s)} A_{LT}^{w_i(\phi_h,\phi_s)} w_i(\phi_h, \phi_s) + ...)$$

$$A_{BT}^{w_i(\phi_h,\phi_s)} \equiv \frac{F_{BT}^{w_i(\phi_h,\phi_s)}}{F_{UU,T}}$$

Collins

Sivers

$$w_1(\phi_h, \phi_s) = \sin(\phi_h - \phi_s),$$
$$w_2(\phi_h, \phi_s) = \sin(\phi_h + \phi_s),$$
$$w_3(\phi_h, \phi_s) = \sin(3\phi_h - \phi_s),$$
$$w_4(\phi_h, \phi_s) = \sin(\phi_s),$$
$$w_5(\phi_h, \phi_s) = \sin(2\phi_h - \phi_s),$$
$$w_6(\phi_h, \phi_s) = \cos(\phi_h - \phi_s),$$
$$w_7(\phi_h, \phi_s) = \cos(\phi_s),$$
$$w_8(\phi_h, \phi_s) = \cos(2\phi_h - \phi_s)$$

$$D^{\sin(\phi_h-\phi_s)}(y) = 1,$$
$$D^{\sin(\phi_h+\phi_s)}(y) = D^{\sin(3\phi_h+\phi_s)}(y) = D_{NN}(y) = \frac{2(1-y)}{1+(1-y)^2},$$
$$D^{\sin(2\phi_h-\phi_s)}(y) = D^{\sin(\phi_s)}(y) = \frac{2(2-y)\sqrt{1-y}}{1+(1-y)^2},$$
$$D^{\cos(\phi_h-\phi_s)}(y) = D(y) = \frac{y(2-y)}{1+(1-y)^2},$$
$$D^{\cos(2\phi_h-\phi_s)}(y) = D^{\cos(\phi_s)}(y) = \frac{2y\sqrt{1-y}}{1+(1-y)^2}.$$

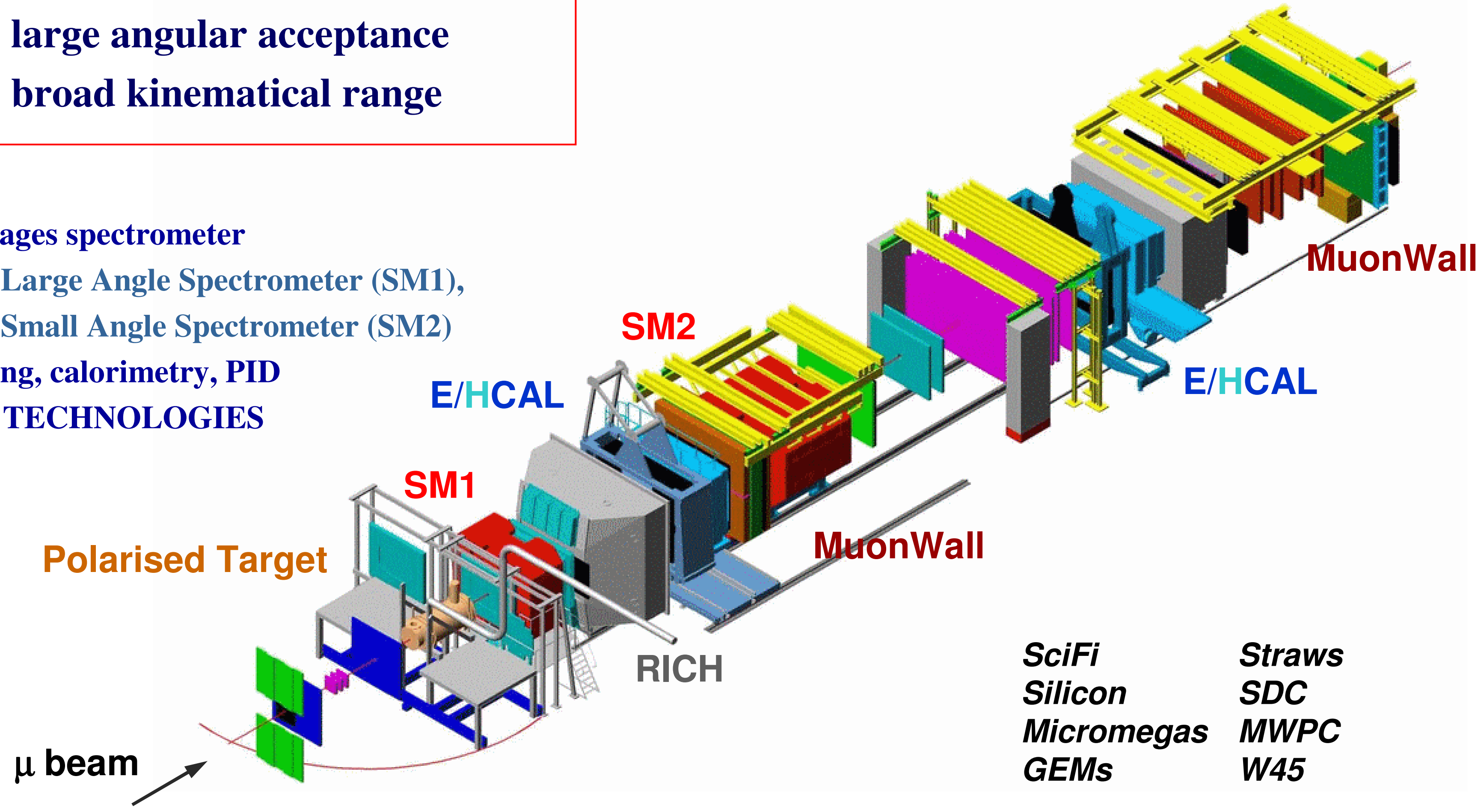
The COMPASS experiment @ CERN

hep-ex/0703049

beam: 160 GeV/c
longitudinal polarisation -76%
intensity $2{\cdot}10^{8}$ μ^{+}/spill (4.8s/16.2s)

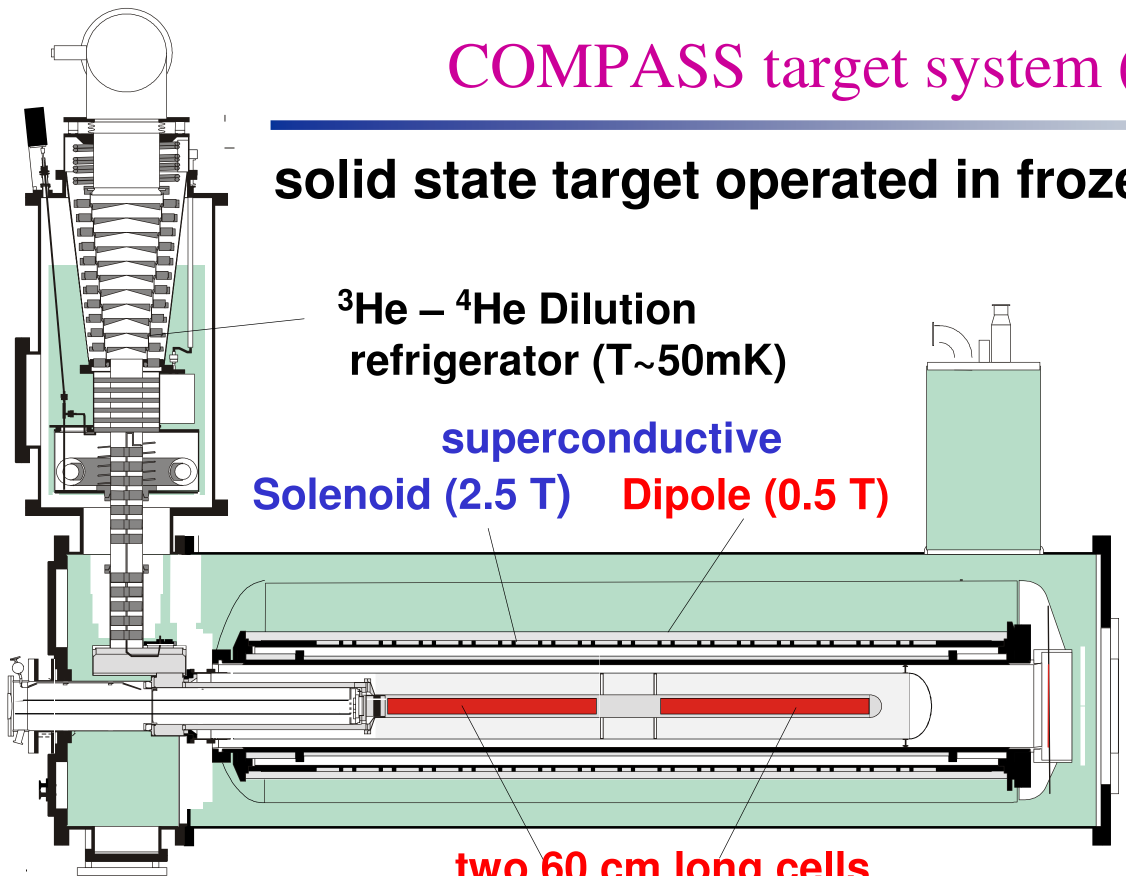
- high energy beam
- large angular acceptance
- broad kinematical range

two stages spectrometer
Large Angle Spectrometer (SM1),
Small Angle Spectrometer (SM2)
tracking, calorimetry, PID
NEW TECHNOLOGIES



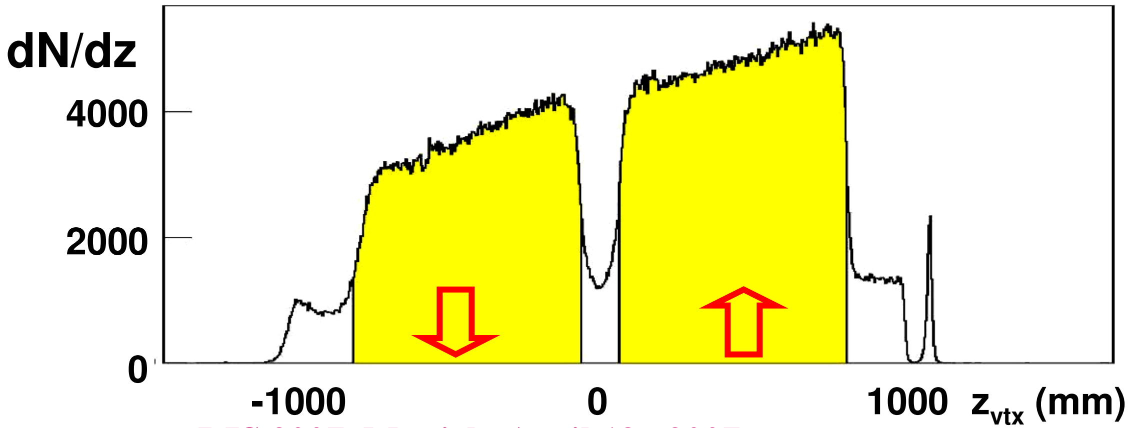
COMPASS target system (2002-2004)

solid state target operated in frozen spin mode



two 60 cm long cells
with *opposite polarisation* (systematics)

2002-2004: ^{6}LiD
- **dilution factor f = 0.38**
- **polarization P_T = 50%**
- **~20% of the time transversely polarised**



during data taking with transverse polarization
polarization reversal in the 2 cells after ~ 5 days

EVENT SELECTION

DIS cuts

***Q*² > 1 GeV²**

***W*² > 25 GeV²**

0.1 < *y* < 0.9

Hadron cuts

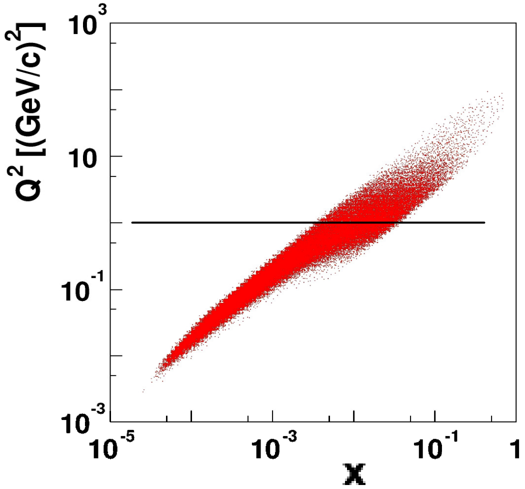
P_T^h > 0.1 GeV/c

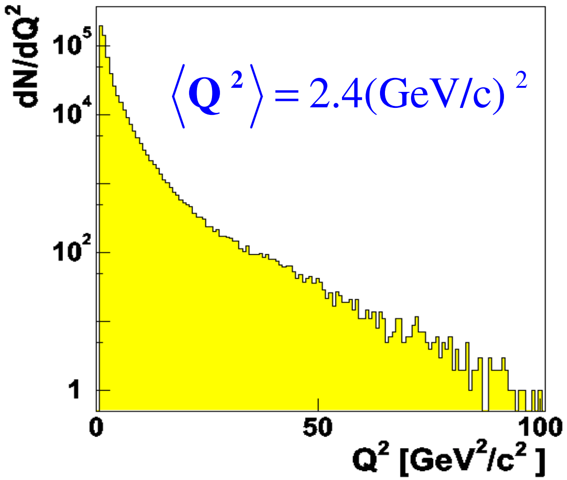
***z* > 0.2 (CFR)**

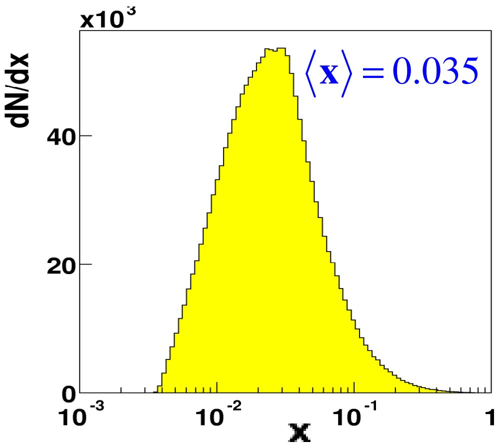
Final statistics for 2002 – 2004

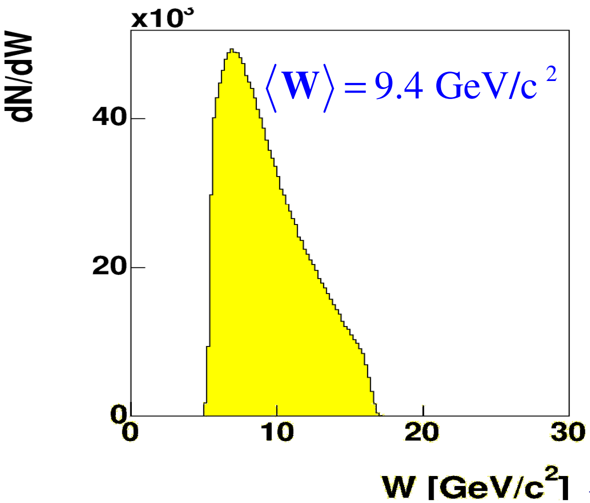
positive hadrons: 8.4·10⁶

negative hadrons: 7.0·10⁶









More details in

A. Bressan's talk

and

NP B765 (2007) 31

2002 data

Asymmetry extraction

The number-of-events asymmetries

$$A_{UT,\,raw}^{w_i(\phi_h,\phi_s)} = D^{w_i(\phi_h,\phi_s)}(y) f |S_T| A_{UT}^{w(\phi_h,\phi_s)}, \quad (i = 1, 5),$$
$$A_{LT,\,raw}^{w(\phi_h,\phi_s)} = D^{w(\phi_h,\phi_s)}(y) f P_{beam} |S_T| A_{LT}^{w(\phi_h,\phi_s)}, \quad (i = 6, 8)$$

Independent angles

$$\Phi_1 = \phi_h - \phi_s$$
$$\Phi_2 = \phi_h + \phi_s$$
$$\Phi_3 = 3\phi_h - \phi_s$$
$$\Phi_4 = \phi_s$$
$$\Phi_5 = 2\phi_h - \phi_s$$

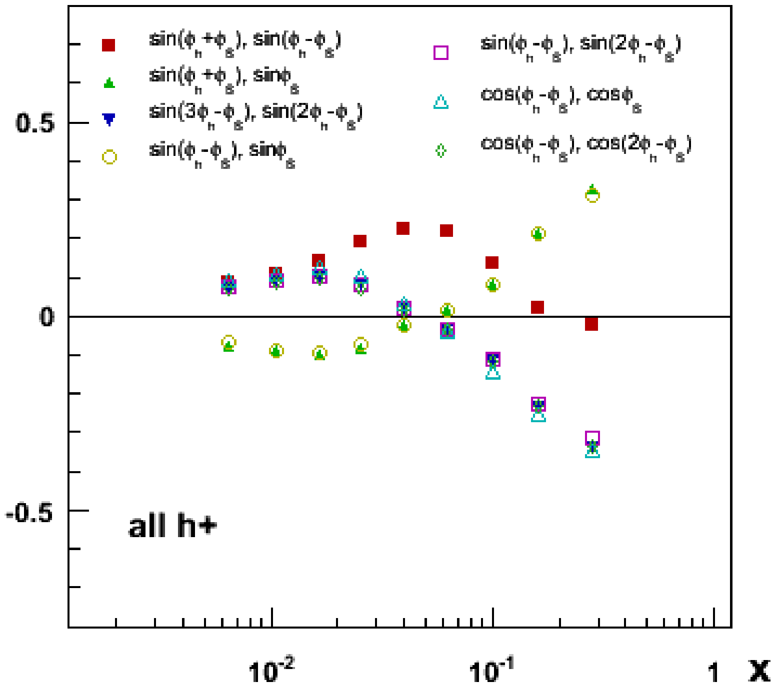
Azimuthal modulations

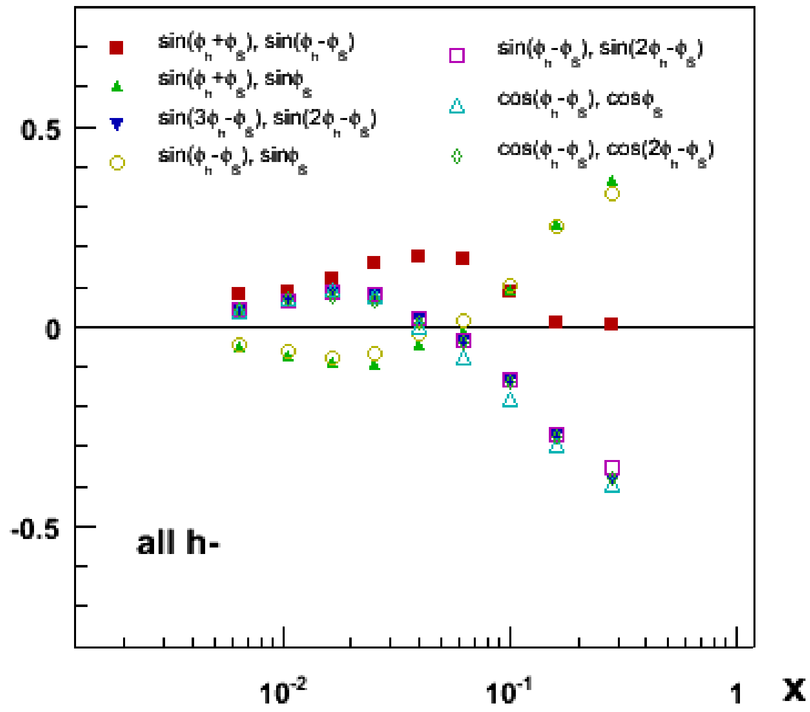
$$W_1(\Phi_1) = A_{raw}^{w_1(\phi_h,\phi_s)} \sin(\Phi_1) + A_{raw}^{w_6(\phi_h,\phi_s)} \cos(\Phi_1)$$
$$W_2(\Phi_2) = A_{raw}^{w_2(\phi_h,\phi_s)} \sin(\Phi_2)$$
$$W_3(\Phi_3) = A_{raw}^{w_3(\phi_h,\phi_s)} \sin(\Phi_3)$$
$$W_4(\Phi_4) = A_{raw}^{w_4(\phi_h,\phi_s)} \sin(\Phi_4) + A_{raw}^{w_7(\phi_h,\phi_s)} \cos(\Phi_4)$$
$$W_5(\Phi_5) = A_{raw}^{w_5(\phi_h,\phi_s)} \sin(\Phi_5) + A_{raw}^{w_8(\phi_h,\phi_s)} \cos(\Phi_5)$$

We have used the double ratio method (as for Sivers and Collins asymmetries extraction)

Correlations

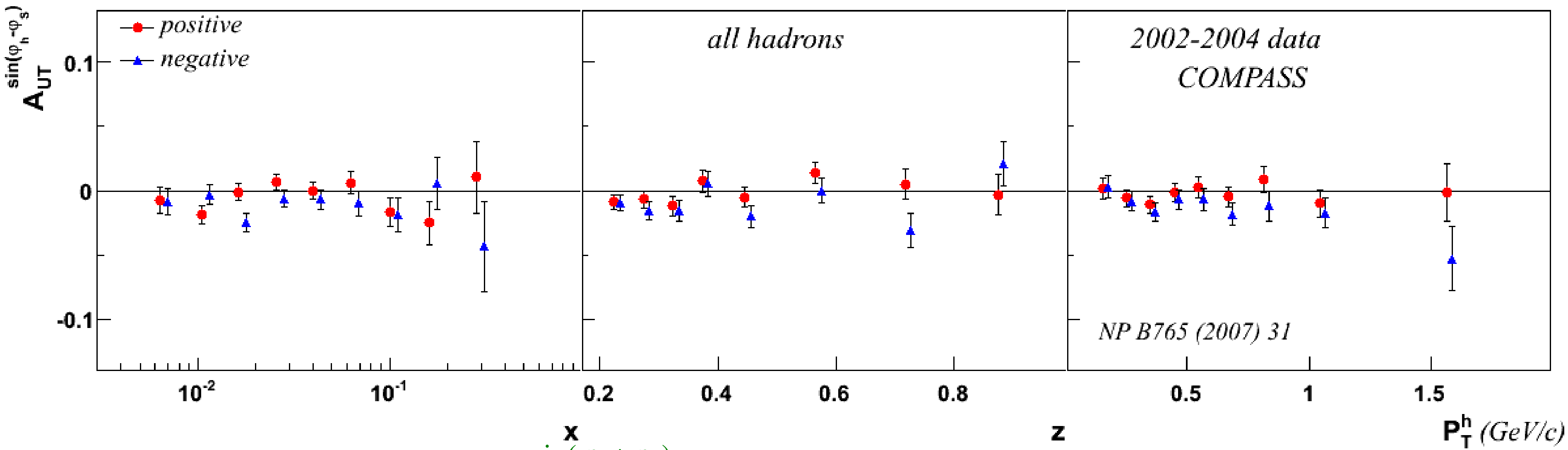
2D fit: correlations between different modulations are small



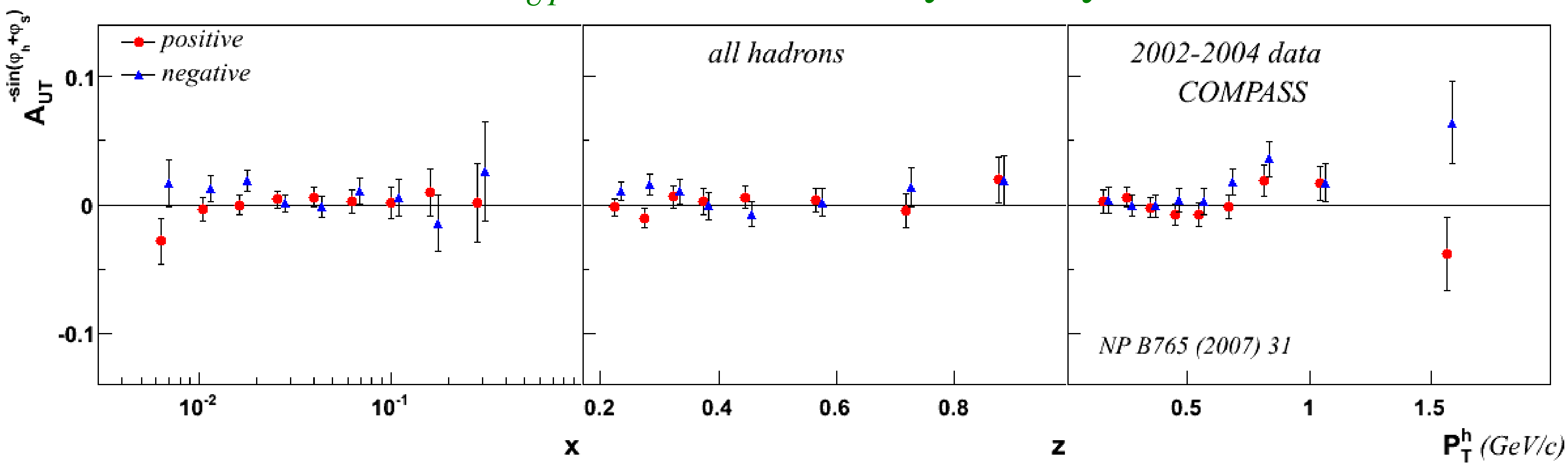


Results (talks by Bressan & D'Alesio)

$A_{UT}^{\sin(\varphi_h-\varphi_s)}$ - Sivers asymmetry

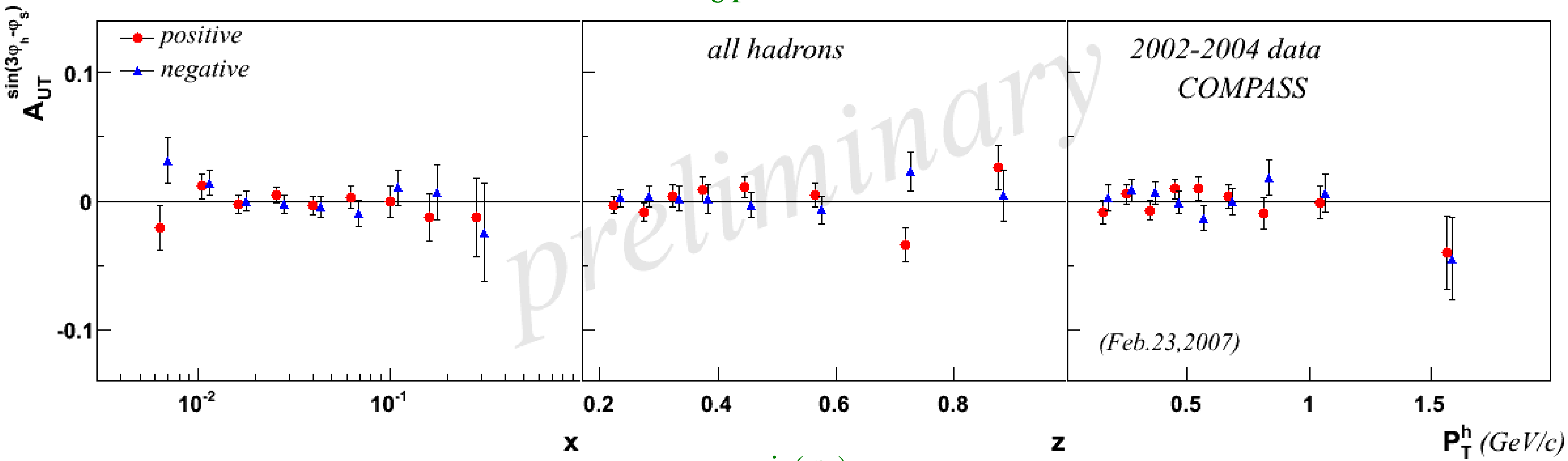


$-A_{UT}^{\sin(\varphi_h+\varphi_s)}$ - Collins asymmetry

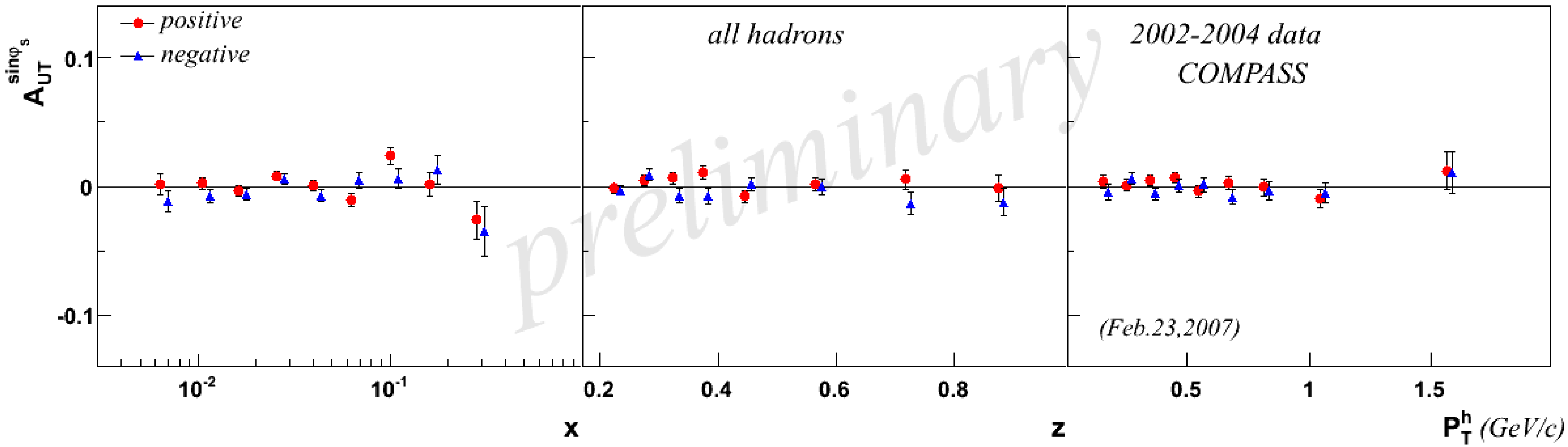


Results

$A_{UT}^{\sin(3\varphi_h-\varphi_s)}$

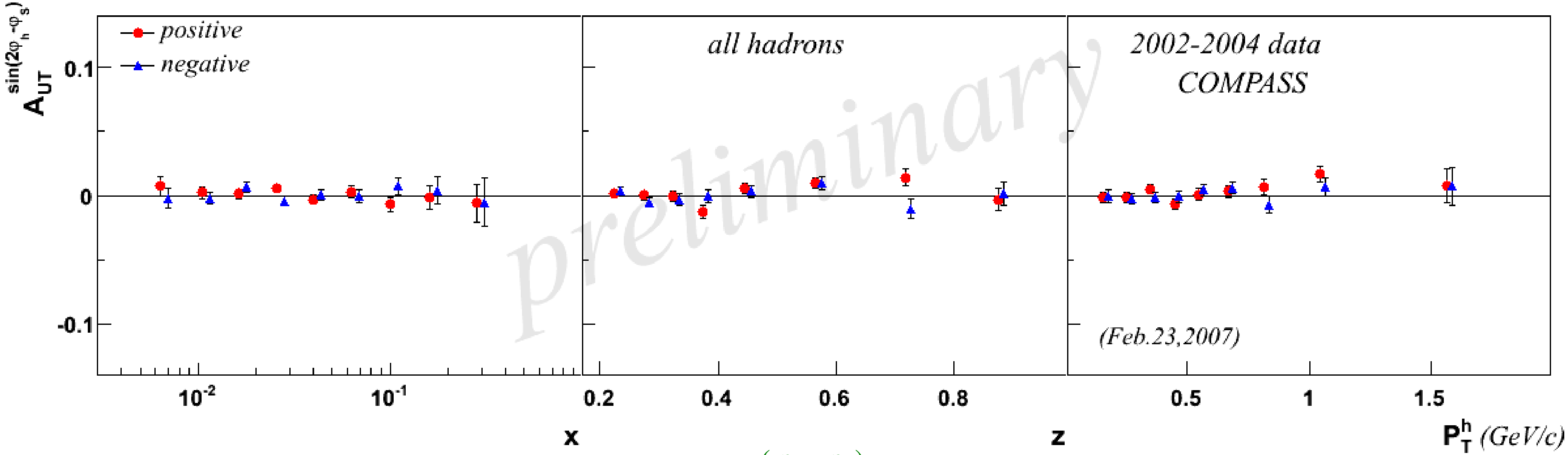


$A_{UT}^{\sin(\varphi_s)}$

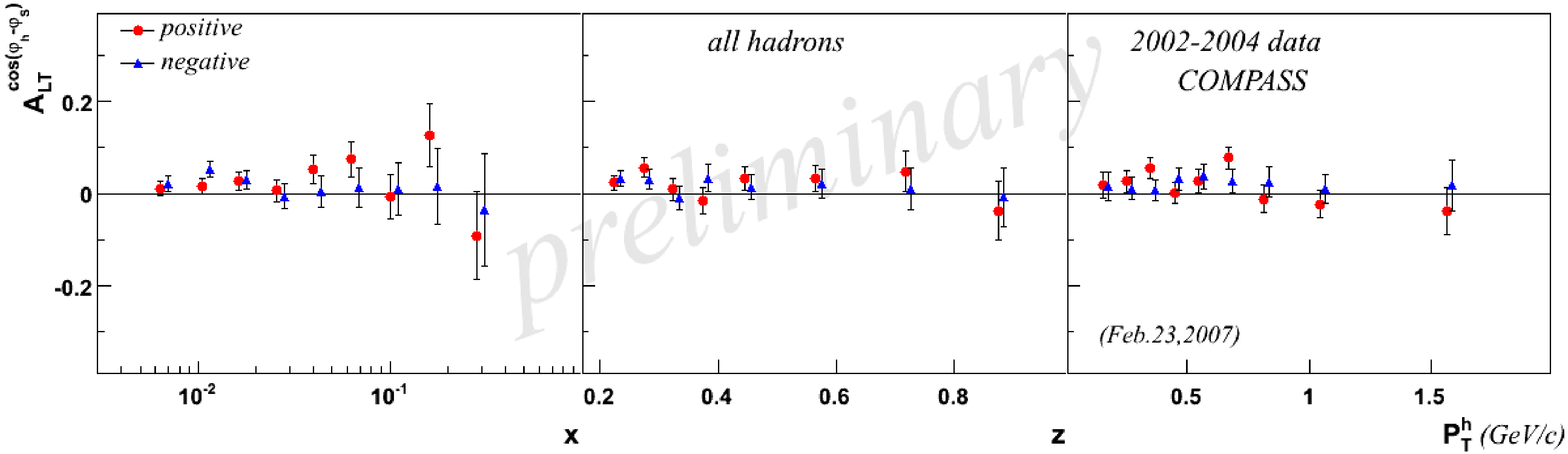


Results

$A_{UT}^{\sin(2\varphi_h-\varphi_s)}$

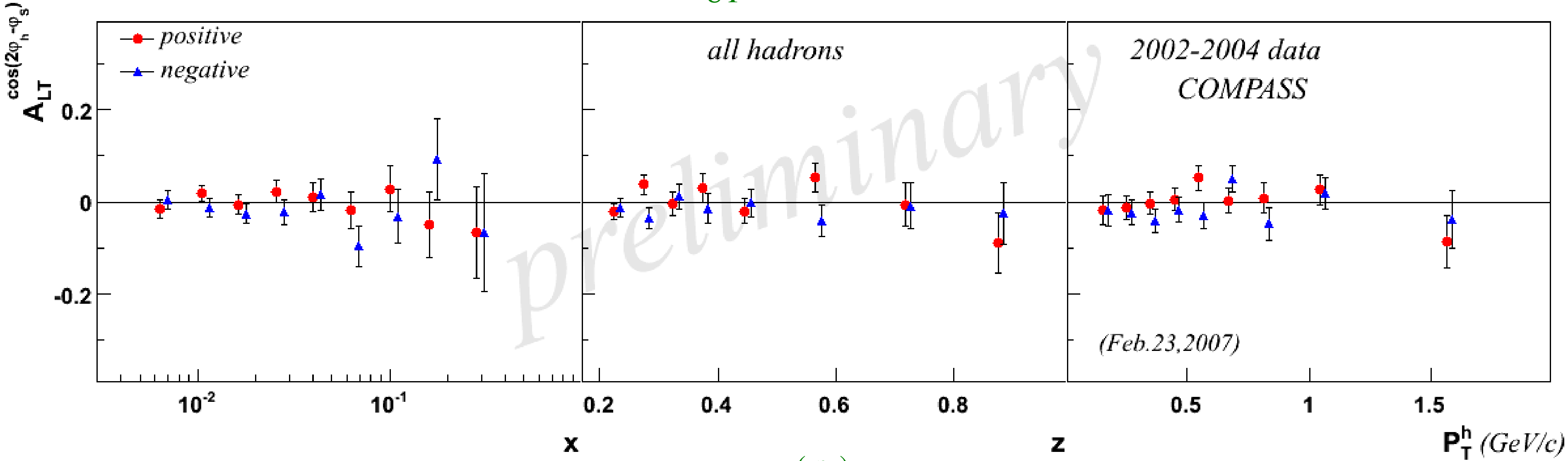


$A_{LT}^{\cos(\varphi_h-\varphi_s)}$

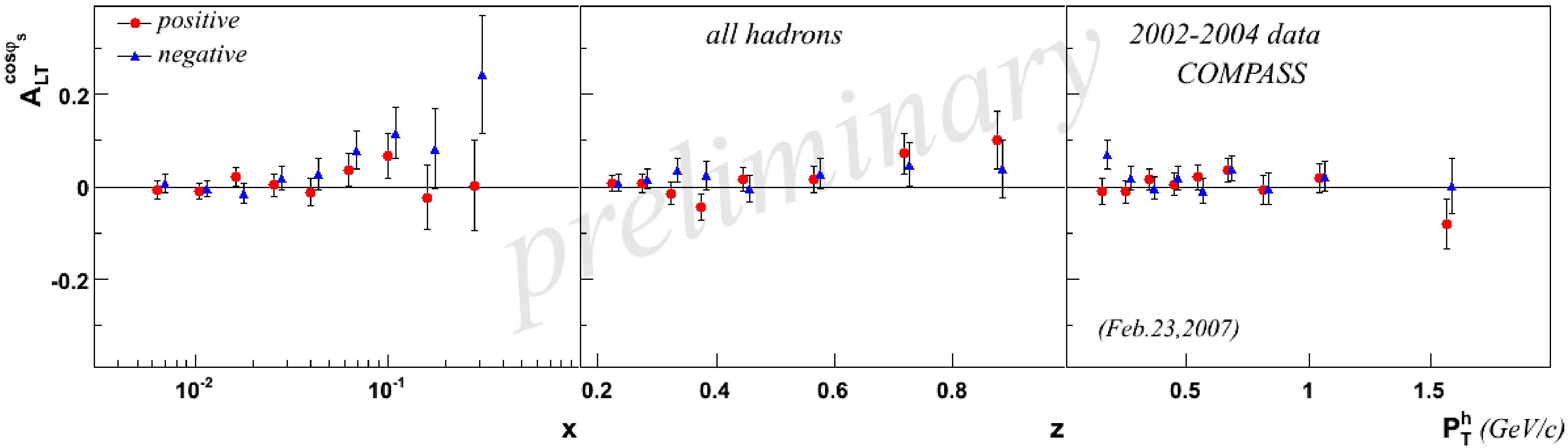


Results

$A_{UT}^{\sin(2\varphi_h - \varphi_s)}$



$A_{UT}^{\cos(\varphi_s)}$



Parton model for SIDIS in CFR

$$d\sigma^{l+N\to l'+h+X} \propto DF \otimes d\sigma^{l+q\to l'+q'} \otimes FF$$

Factorization theorem for TMD SIDIS is proven only at twist-two

At twist-two

Sivers

$$\mathcal{P}_N^q(x,\mathbf{k}_T) = f_1^q(x,k_T^2) + f_{1T}^{\perp q}(x,k_T^2)\frac{[\mathbf{k}_T \times \hat{\mathbf{P}}_N]\cdot S_T^N}{M},$$

$$f_1^q(x,k_T^2)s_L^q(x,\mathbf{k}_T) = g_{1L}^q(x,k_T^2)\lambda_N + g_{1T}^{\perp q}(x,k_T^2)\frac{\mathbf{k}_T\cdot\mathbf{S}_T^N}{M},$$

$$f_1^q(x,k_T^2)\mathbf{s}_T^q(x,\mathbf{k}_T) = h_{1T}^q(x,k_T^2)\mathbf{S}_T^N + [h_{1L}^{\perp q}(x,k_T^2)\lambda_N + h_{1T}^{\perp q}(x,k_T^2)\frac{\mathbf{k}_T\cdot\mathbf{S}_T^N}{M}]\frac{\mathbf{k}_T}{M} + h_1^{\perp q}(x,k_T^2)\frac{[\mathbf{k}_T\times\hat{\mathbf{P}}_N]}{M}$$

Boer-Mulders

Often used:

$$h_1^q(x,k_T^2) = h_{1T}^q(x,k_T^2) + \frac{k_T^2}{2M^2}h_{1T}^{\perp q}(x,k_T^2)$$

Collins

$$\mathcal{P}_{q\uparrow}^h(z,\mathbf{P}_{Tq}^h) = D_q^h(z,P_{Tq}^h) + H_{1q}^{\perp h}(z,P_{Tq}^h)\frac{[\mathbf{P}_{Tq}^h\times\hat{\mathbf{k}}']\cdot\mathbf{s}_T'}{M} = D_q^h(z,P_{Tq}^h) + s_T'\frac{P_{Tq}^h}{M}H_{1q}^{\perp h}(z,P_{Tq}^h)\sin(\phi_{Collins})$$

Twist-two contributions

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \\
&\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
&\quad + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} + P_{beam}\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
&\quad + P_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&\quad + P_L P_{beam}\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&\quad + |P_T|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&\quad\quad + \varepsilon\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\quad\quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
&\quad + |P_T|P_{beam}\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
&\quad\quad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\},
\end{aligned}
$$

$F_{UU,T}$ → $f_1^q \otimes D_q^h$

$\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$ → $h_1^{\perp q} \otimes H_{1q}^{\perp h}$

$\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}$ → $h_{1L}^{\perp q} \otimes H_{1q}^{\perp h}$

$\sqrt{1-\varepsilon^2}\,F_{LL}$ → $g_{1L}^q \otimes D_q^h$

$\sin(\phi_h-\phi_S)\,F_{UT,T}^{\sin(\phi_h-\phi_S)}$ → $f_1^{\perp q} \otimes D_q^h$ — Sivers

$\varepsilon\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}$ → $h_1^q \otimes H_{1q}^{\perp h}$ — Collins

$\varepsilon\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}$ → $h_{1T}^{\perp q} \otimes H_{1q}^{\perp h}$ — New (no name)

$\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}$ → $g_{1T}^{\perp q} \otimes D_q^h$ — New (no name)

Interpretation of target transverse spin asymmetries

Twist-2:

$$A_{UT}^{\sin(\varphi_h-\varphi_s)} \propto f_{1T}^{\perp q} \otimes D_{1q}^{h}$$

$$A_{UT}^{\sin(\varphi_h+\varphi_s)} \propto h_1^{q} \otimes H_{1q}^{\perp h}$$

$$A_{LT}^{\cos(\varphi_h-\varphi_s)} \propto g_{1T}^{q} \otimes D_{1q}^{h}$$

$$A_{UT}^{\sin(3\varphi_h-\varphi_s)} \propto h_{1T}^{\perp q} \otimes H_{1q}^{\perp h}$$

BDGMMSch: Whether and how the tree-level factorization used in the present paper extends to subleading level in $1/Q$ is presently not known.

Twist-2 + k_T/Q kinematical corrections:

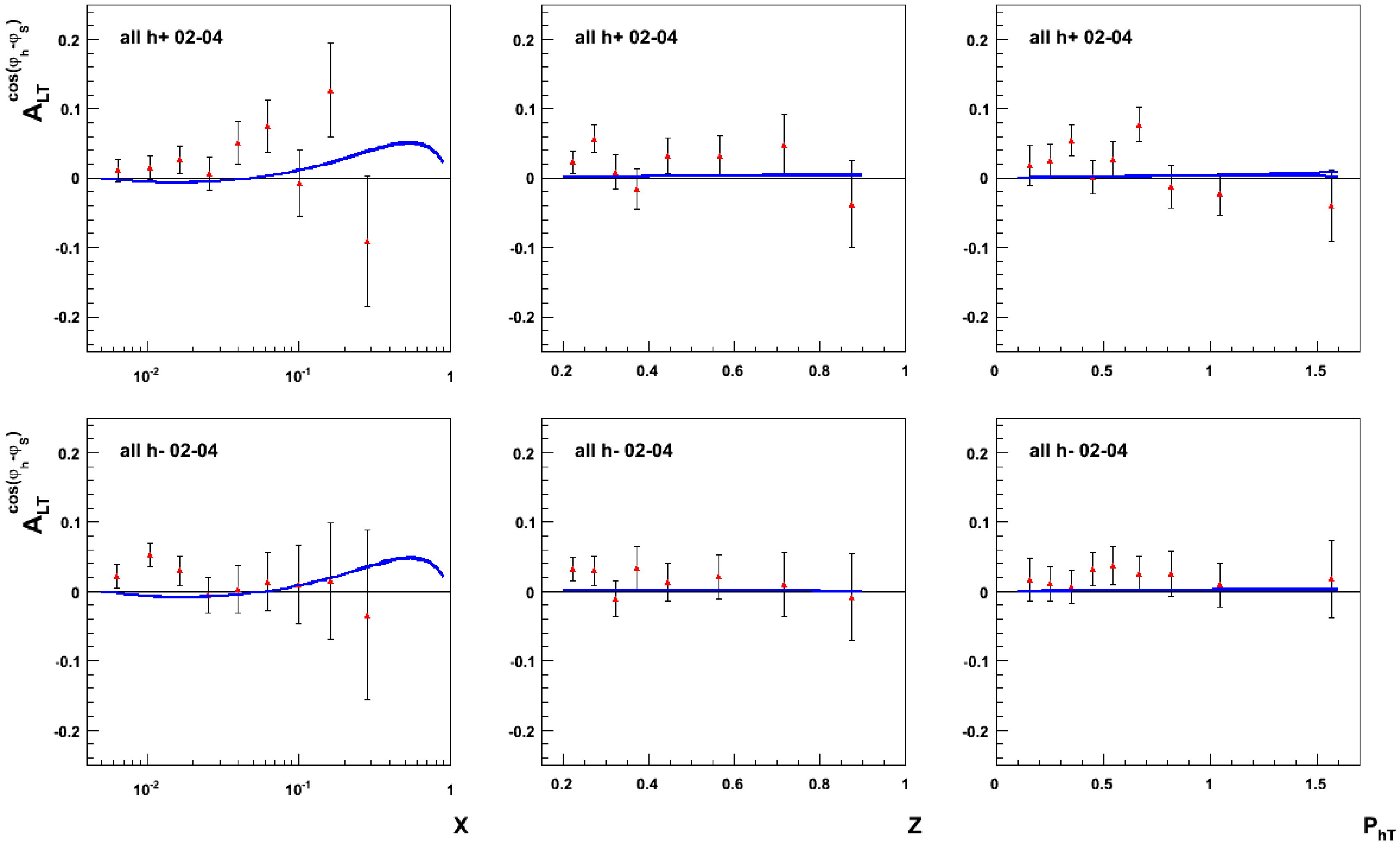
$$A_{LT}^{\cos(\varphi_s)} \propto \frac{M}{Q} g_{1T}^{q} \otimes D_{1q}^{h}$$

$$A_{LT}^{\cos(2\varphi_h-\varphi_s)} \propto \frac{M}{Q} g_{1T}^{q} \otimes D_{1q}^{h}$$

$$A_{UT}^{\sin(\varphi_s)} \propto \frac{M}{Q}\left(h_1^{q} \otimes H_{1q}^{\perp h} + f_{1T}^{\perp q} \otimes D_{1q}^{h}\right)$$

$$A_{UT}^{\sin(2\varphi_h-\varphi_s)} \propto \frac{M}{Q}\left(h_{1T}^{\perp q} \otimes H_{1q}^{\perp h} + f_{1T}^{\perp q} \otimes D_{1q}^{h}\right)$$

Double spin $\cos(\varphi_h-\varphi_s)$ asymmetry



Predictions by

A.K., Parsamyan & Prokudin,

Phys.Rev.D73:114017,2006

$$g_{1T}^{q(1)}(x) \approx x \int_x^1 dy \frac{g_1^q(y)}{y}$$

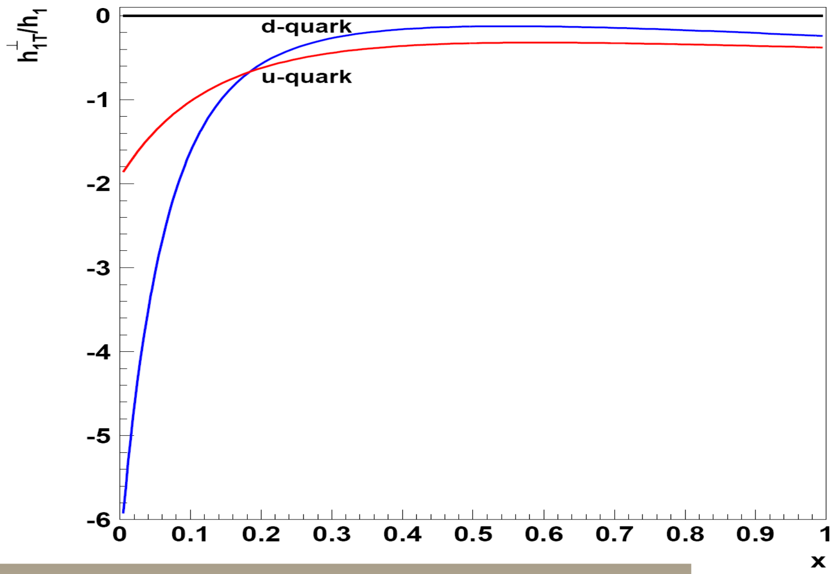
Estimation of $\sin(3\varphi_h - \varphi_s)$ asymmetry

Quark-Diquark model

R. Jakob, P. Mulders & Rodrigues

NP A626, 937 (1997)

$$R = \frac{A_{UT}^{\sin(3\varphi_h - \varphi_s)}}{A_{UT}^{\sin(\varphi_h + \varphi_s)}}$$



$$\mathrm{R}(x,z,P_T^h) \approx \frac{\langle k_T^2 \rangle^2 z^2}{(\langle p_T^2 \rangle + z^2 \langle k_T^2 \rangle)^2} \frac{(P_T^h)^2}{2M^2} \frac{\sum_q h_{1T}^{\perp q}(x) H_q^h(z)}{\sum_q h_1^q(x) H_q^h(z)}$$

At COMPASS

$\langle z \rangle \approx 0.4, \quad \langle P_T^h \rangle \approx 0.5$ GeV/c

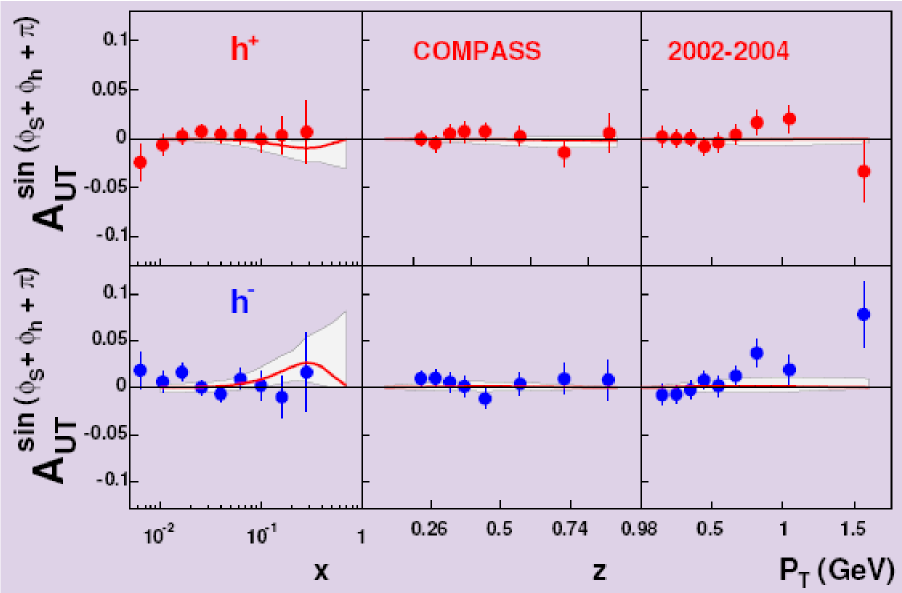
$$\langle \mathrm{R}(x) \rangle \approx 0.02 \frac{\sum_q h_{1T}^{\perp q}(x) H_q^h(0.4)}{\sum_q h_1^q(x) H_q^h(0.4)} \ll 1$$

Conclusions

- There are 8 target transverse spin dependent azimuthal modulations in one particle SIDIS
  - HERMES & COMPASS have already published the results on Collins and Sivers asymmetries
  - Here we presented the remaining 6 physical asymmetries extracted from COMPASS data with transversely polarized deuteron target
    - Two twist-2 asymmetries can be interpreted in QCD parton model and will allow to extract unexplored DFs g_{1T}^{q} and $h_{1T}^{\perp q}$
    - Remaining four can be interpreted as twist-3 contributions
- There are more data to come soon from COMPASS and other experiments that will yield further understanding of TMD formulation of SIDIS and other processes in QCD

Additional slides

Collins



Analysis by

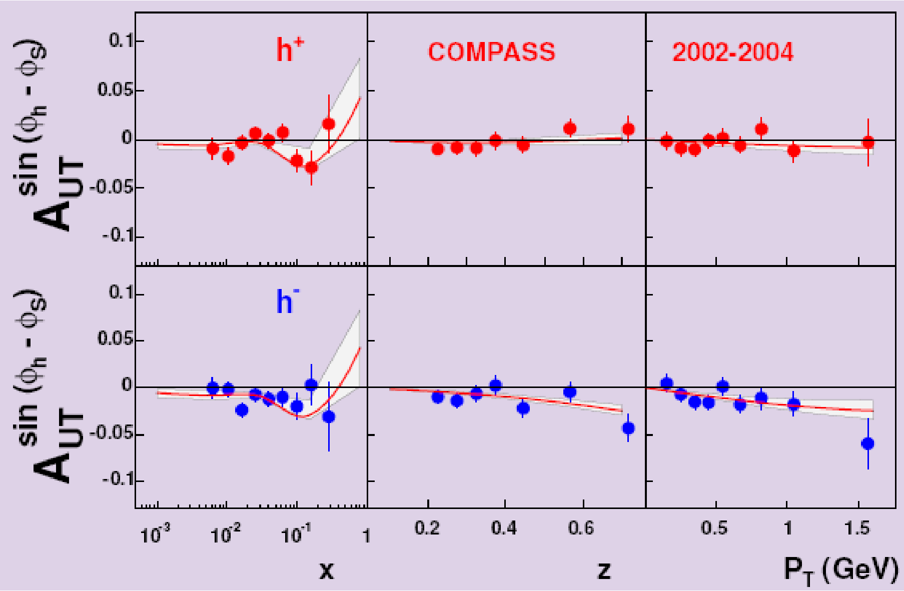
M. Anselmino, M. Boglione,

U. D'Alesio, F. Murgia, A.K.,

A. Prokudin, C. Türk

Phys.Rev.D75:054032,2007.

Sivers



Analysis by

M. Anselmino, M. Boglione,

U. D'Alesio, F. Murgia, A.K.,

A. Prokudin

Phys.Rev.D72:094007,2005.

Asymmetry extraction (2)

Counting rates:

$$N^{\pm}_{u/d}(\Phi_j) = F^{\pm}_{u/d} n^{\pm}_{u/d} a^{\pm}_{u/d}(\Phi_j)\sigma(1 \pm W_j(\Phi_j))$$

Double ratio method:

$$F(\Phi_j) = \frac{N_u^+(\Phi_j)N_d^+(\Phi_j)}{N_u^-(\Phi_j)N_d^-(\Phi_j)},$$

with

$$\sigma_F(\Phi_j) = \sqrt{\frac{1}{N_u^+(\Phi_j)} + \frac{1}{N_d^+(\Phi_j)} + \frac{1}{N_u^-(\Phi_j)} + \frac{1}{N_d^-(\Phi_j)}}$$

Assuming for acceptance: $\dfrac{a_u^+(\Phi_j)}{a_d^-(\Phi_j)} = \dfrac{a_u^-(\Phi_j)}{a_d^+(\Phi_j)}$

$$F(\Phi_j) = const\frac{(1+W_j(\Phi_j))(1+W_j(\Phi_j))}{(1-W_j(\Phi_j))(1-W_j(\Phi_j))}$$

$$F(\Phi_j) = par(0)(1 + 4par(1)\sin\Phi_j + 4par(2)\cos\Phi_j)$$

Subleading twist (from paper (2))

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right],$$

$$F_{UT,L}^{\sin(\phi_h-\phi_S)} = 0,$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h} h_1 H_1^{\perp}\right],$$

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{C}\left[\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)\,(\boldsymbol{p}_T\cdot\boldsymbol{k}_T) + \boldsymbol{p}_T^2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T) - 4\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)}{2M^2 M_h} h_{1T}^{\perp} H_1^{\perp}\right],$$

$$F_{UT}^{\sin\phi_S} = \frac{2M}{Q}\,\mathcal{C}\left\{\left(x f_T D_1 - \frac{M_h}{M} h_1 \frac{\tilde{H}}{z}\right) - \frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\left[\left(x h_T H_1^{\perp} + \frac{M_h}{M} g_{1T} \frac{\tilde{G}^{\perp}}{z}\right) - \left(x h_T^{\perp} H_1^{\perp} - \frac{M_h}{M} f_{1T}^{\perp} \frac{\tilde{D}^{\perp}}{z}\right)\right]\right\},$$

$$F_{UT}^{\sin(2\phi_h-\phi_S)} = \frac{2M}{Q}\,\mathcal{C}\left\{\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2 - \boldsymbol{p}_T^2}{2M^2}\left(x f_T^{\perp} D_1 - \frac{M_h}{M} h_{1T}^{\perp} \frac{\tilde{H}}{z}\right) - \frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\left[\left(x h_T H_1^{\perp} + \frac{M_h}{M} g_{1T} \frac{\tilde{G}^{\perp}}{z}\right) + \left(x h_T^{\perp} H_1^{\perp} - \frac{M_h}{M} f_{1T}^{\perp} \frac{\tilde{D}^{\perp}}{z}\right)\right]\right\},$$

$$F_{LT}^{\cos(\phi_h-\phi_S)} = \mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} g_{1T} D_1\right],$$

$$F_{LT}^{\cos\phi_S} = \frac{2M}{Q}\,\mathcal{C}\left\{-\left(x g_T D_1 + \frac{M_h}{M} h_1 \frac{\tilde{E}}{z}\right) + \frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\left[\left(x e_T H_1^{\perp} - \frac{M_h}{M} g_{1T} \frac{\tilde{D}^{\perp}}{z}\right) + \left(x e_T^{\perp} H_1^{\perp} + \frac{M_h}{M} f_{1T}^{\perp} \frac{\tilde{G}^{\perp}}{z}\right)\right]\right\},$$

$$F_{LT}^{\cos(2\phi_h-\phi_S)} = \frac{2M}{Q}\,\mathcal{C}\left\{-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2 - \boldsymbol{p}_T^2}{2M^2}\left(x g_T^{\perp} D_1 + \frac{M_h}{M} h_{1T}^{\perp} \frac{\tilde{E}}{z}\right) + \frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\left[\left(x e_T H_1^{\perp} - \frac{M_h}{M} g_{1T} \frac{\tilde{D}^{\perp}}{z}\right) - \left(x e_T^{\perp} H_1^{\perp} + \frac{M_h}{M} f_{1T}^{\perp} \frac{\tilde{G}^{\perp}}{z}\right)\right]\right\}.$$

$$x e = x\tilde{e} + \frac{m}{M} f_1,$$

$$x f^{\perp} = x\tilde{f}^{\perp} + f_1,$$

$$x g_T' = x\tilde{g}_T' + \frac{m}{M} h_{1T},$$

$$x g_T^{\perp} = x\tilde{g}_T^{\perp} + g_{1T} + \frac{m}{M} h_{1T}^{\perp},$$

$$x g_T = x\tilde{g}_T - \frac{p_T^2}{2M^2} g_{1T} + \frac{m}{M} h_1,$$

$$x g_L^{\perp} = x\tilde{g}_L^{\perp} + g_{1L} + \frac{m}{M} h_{1L}^{\perp},$$

$$x h_L = x\tilde{h}_L + \frac{p_T^2}{M^2} h_{1L}^{\perp} + \frac{m}{M} g_{1L},$$

$$x h_T = x\tilde{h}_T - h_1 + \frac{p_T^2}{2M^2} h_{1T}^{\perp} + \frac{m}{M} g_{1T},$$

$$x h_T^{\perp} = x\tilde{h}_T^{\perp} + h_1 + \frac{p_T^2}{2M^2} h_{1T}^{\perp}.$$

$$x e_L = x\tilde{e}_L,$$

$$x e_T = x\tilde{e}_T,$$

$$x e_T^{\perp} = x\tilde{e}_T^{\perp} + \frac{m}{M} f_{1T}^{\perp},$$

$$x f_T' = x\tilde{f}_T' + \frac{p_T^2}{M^2} f_{1T}^{\perp},$$

$$x f_T^{\perp} = x\tilde{f}_T^{\perp} + f_{1T}^{\perp},$$

$$x f_T = x\tilde{f}_T + \frac{p_T^2}{2M^2} f_{1T}^{\perp},$$

$$x f_L^{\perp} = x\tilde{f}_L^{\perp},$$

$$x g^{\perp} = x\tilde{g}^{\perp} + \frac{m}{M} h_1^{\perp},$$

$$x h = x\tilde{h} + \frac{p_T^2}{M^2} h_1^{\perp}.$$

Physics with heavy notations

Asymmetry extraction (3)

Two dimensional fit

$$N^{\pm}_{u/d}(\phi_h,\phi_s) = F^{\pm}_{u/d} n^{\pm}_{u/d} a^{\pm}_{u/d}(\phi_h,\phi_s)\sigma\{1 \pm \sum_{i=1}^{8} A_{raw}^{w_i(\phi_h,\phi_s)} w_i(\phi_h,\phi_s)\}$$

$$F(\phi_h,\phi_s) = \frac{N^{\uparrow}_{up}(\phi_h,\phi_s) N^{\uparrow}_{down}(\phi_h,\phi_s)}{N^{\downarrow}_{up}(\phi_h,\phi_s) N^{\downarrow}_{down}(\phi_h,\phi_s)},$$

$$\sigma_F(\phi_h,\phi_s) = \sqrt{\frac{1}{N^{\uparrow}_{up}(\phi_h,\phi_s)} + \frac{1}{N^{\uparrow}_{down}(\phi_h,\phi_s)} + \frac{1}{N^{\downarrow}_{up}(\phi_h,\phi_s)} + \frac{1}{N^{\downarrow}_{down}(\phi_h,\phi_s)}}$$

Fitting function:

$$\begin{aligned} F(\phi_h,\phi_s) = par(0)[1 + 4[&par(1)sin(\phi_h+\phi_s-\pi) + par(2)sin(3\phi_h-\phi_s) + \\ &par(3)sin(\phi_h-\phi_s) + par(4)cos(\phi_h-\phi_s) + par(5)sin(\phi_s) + \\ &par(6)sin(2\phi_h-\phi_s) + par(7)\cos(\phi_s) + par(8)cos(2\phi_h-\phi_s)]] \end{aligned}$$

For each of 9 x-bins, 8 z-bins and 9 P_{hT}-bins -- 64 (φ_h, φ_S)-bin