RECENT CALCULATIONS IN CHIRAL PERTURBATION THEORY

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Abstract
There are several processes that can be measured at COMPASS and that are of relevance for chiral perturbation theory. I discuss, in particular, pion polarizabilities and photon–meson transition amplitudes. In addition, I point out that more precise experimental information on the $I = 2$ S-wave phase shift in elastic $\pi\pi$ scattering would be very welcome.

1. INTRODUCTION

At low energies, it is useful to replace QCD by an effective quantum field theory that has the same physical content as QCD but is formulated in terms of asymptotically observable fields. This method to calculate physical observables in QCD is called chiral perturbation theory (ChPT) [1, 2, 3].

At COMPASS, one can test several ChPT predictions. This concerns in particular the electric and magnetic polarizabilities of the charged pions and kaons, and photon–meson transition amplitudes like γ + meson $\to$ meson + meson. The relation of the latter process to the chiral anomaly makes it particularly attractive. Furthermore, in case that COMPASS can provide precise data on low-energy S-wave phase shifts in elastic $\pi\pi$ scattering, or information on $\pi K \to \pi K$, one should not miss the opportunity to measure these processes.

In the following, I concentrate on

$$
\begin{aligned}
\pi X &\to \pi\gamma X' \\
K X &\to K\gamma X' \\
\pi X &\to \pi\pi X', K\bar{K}X' \\
K X &\to K\pi X' \\
K X &\to K X \,.
\end{aligned}
$$

These processes are generated in the COMPASS experiment through the Primakoff reaction or through one-pion exchange. I shall not discuss

$$K_{l3}, K_{l4}, K \to 2\pi, 3\pi, \dots \,,$$

because these are (will be) investigated, for example, at NA48.

2. CHIRAL PERTURBATION THEORY

Chiral perturbation theory [1, 2, 3] has been developed over the last two decades into a method that allows one to calculate several hadronic quantities with high precision. The method can be summarized as follows. One replaces the original Lagrangian of QCD by a Lagrangian that contains the asymptotically observable fields like pions, kaons, etas, $\dots$,

$$
\begin{array}{ccc}
\mathcal{L}_{QCD} & \to & \mathcal{L}_{eff} \\
\text{quarks, gluons} & & \pi, K, \eta, p, n, \dots
\end{array}
\tag{1}
$$

$$\mathcal{L}_{eff} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \cdots \,. \tag{2}$$

Here, $\mathcal{L}_n$ generates terms of order (external momentum)^n in the low-energy expansion of S-matrix elements.

Comments on this construction:

- chiral symmetry determines the structure of $\mathcal{L}_n$
- the low-energy coupling constants (LECs) in $\mathcal{L}_n$ must be determined from experiment or lattice calculations, they are in general not determined by chiral symmetry alone
- for appropriately chosen LECs, the effective theory reproduces the S-matrix elements of QCD at low energies [4]

I illustrate the method with the $I = 0$ S-wave scattering length of elastic $\pi\pi$ scattering. The result of the χ calculation is

$$a_0^0 = \frac{7x}{32\pi}\left[1 + c_1 x + c_2 x^2 + O(x^3)\right] \tag{3}$$

where

$$x = M_\pi^2/F_\pi^2 \; ; \; c_{1,2} = \text{numbers + }\chi\text{logs + LECs}\,,$$

and where $F_\pi = 92.4$ MeV is the pion decay constant. The symbol χlogs stands for non-analytic terms in the chiral expansion, of the form log M_π^2, and powers thereof. A similar expression holds for the $I = 2$ S-wave scattering length a_0^2. The leading terms in the scattering length expansion were determined by Weinberg [5] in 1966, whereas the coefficients c_1 and c_2 and the analogous ones for a_0^2 have been worked out algebraically at a later stage [6, 7]. Recently, the relevant LECs have been determined numerically, such that [8]

$$a_0^0 - a_0^2 = 0.265 \pm 0.004\,. \tag{4}$$

Note that the uncertainty is less than two per cent in this case. Once $a_0^0 - a_0^2$ is known, one can predict the lifetime of the ground state of the $\pi^+\pi^-$ atom [9],

$$\tau = (2.9 \pm 0.1) \times 10^{-15}\text{s}\,. \tag{5}$$

Needless to say, we anxiously await the result of the lifetime measurement at DIRAC [10].

This is obviously not the place to review ChPT. I refer the interested reader to one of the many reviews available, for example, one may search at *arXiv:hep-ph* with *find title chiral perturbation theory*.

3. PHOTON-INDUCED REACTIONS

3.1 Pion polarizabilities

We consider Compton scattering

$$\gamma\pi^\pm \to \gamma\pi^\pm$$

and expand the amplitude in powers of photon momenta. The quadratic terms contain the polarizabilities $\bar{\alpha}_\pi, \bar{\beta}_\pi$. I refer the reader to Bürgi's article [11] for further notation. Here, I note that

a) The Compton amplitude and hence the polarizabilities are known to one [12] and two loops [11] in the chiral expansion.

b) Once the Compton amplitude is known, the polarizabilities may be expanded in powers of the pion mass – the structure of this expansion is very similar to the expansion (3) for the scattering length,

$$\bar{\alpha}_\pi \pm \bar{\beta}_\pi = \frac{\alpha}{16\pi^2 M_\pi F_\pi^2}\left\{a_\pm + b_\pm x + O(x^2)\right\} , \tag{6}$$

where $\alpha = 1/137.036$ is the fine structure constant of QED. The quantities $a_\pm$ and $b_\pm$ denote the one- and two-loop contributions, respectively [the tree contributions vanish]. The coefficients $a_\pm, b_\pm$ again contain pure numbers, χlogs and LECs. At one loop, the sum of the polarizabilities vanishes, whereas the difference is given by a particular combination $\bar{l}_6 - \bar{l}_5$ of two LECs from the effective Lagrangian $\mathcal{L}_4$ [11, 13],

$$a_+ = 0\,,\; a_- = \frac{2}{3}(\bar{l}_6 - \bar{l}_5)\,. \tag{7}$$

At two-loop order, both the sum and the difference receive a non-vanishing contribution $b_\pm \neq 0$. *Chiral symmetry does not, therefore, predict that $\bar{\alpha}_\pi + \bar{\beta}_\pi = 0$.* For an explicit expression of $b_\pm$ in terms of χlogs and LECs, I refer the reader to Ref. [11].

c) The two-loop coefficients $b_\pm$ contain LECs from $\mathcal{L}_6$. They had been estimated in Ref. [11] by use of resonance saturation. Bürgi's final result is

$$\begin{aligned} \bar{\alpha}_\pi + \bar{\beta}_\pi &= (0.3 \pm 0.1) \times 10^{-4}\,\mathrm{fm}^3\,, \\ \bar{\alpha}_\pi - \bar{\beta}_\pi &= (4.4 \pm 1.0) \times 10^{-4}\,\mathrm{fm}^3\,. \end{aligned} \tag{8}$$

The uncertainties stem from the uncertainties in the low-energy constants and do not contain estimates of the higher order terms (three loops and beyond).

d) The knowledge of LECs has improved over the last years, e.g., the structure of $\mathcal{L}_6$ is now known [14], and some of the low-energy constants at order p^4 have been reevaluated [15, 16, 17]. To illustrate the effect on (8), I consider the one-loop contribution (7). The combination $\bar{l}_6 - \bar{l}_5$ can be determined from the decay $\pi \to e\nu\gamma$. Its value depends on the data, and on the accuracy to which the chiral expansion is carried out. Here I compare the values used in Ref. [11] with the more recent determination by Bijnens and Talavera [15],

$$\bar{l}_6 - \bar{l}_5 = \begin{cases} 2.7 \pm 0.4 & [11] \\ 3.0 \pm 0.3 & [15]\;. \end{cases} \tag{9}$$

Evaluating $\bar{\alpha}_\pi - \bar{\beta}_\pi$ at one-loop order with the two central values displayed in Eq. (9) generates a difference of 0.6×10^{-4} fm^3. Furthermore, the constants $\bar{l}_{5,6}$ also enter the coefficients $b_\pm$ at two-loop order, which in addition contain $\bar{l}_{1,2,3,4}$ from $\mathcal{L}_4$. These observations make it evident that an update of (8) is needed before firm conclusions from a comparison with new data from the COMPASS experiment can be drawn [18].

3.2 Kaon polarizabilities

Kaon polarizabilities have been worked out to one loop in Ref. [19]. I am not aware of a calculation at two-loop order. Since the chiral expansion now also contains an expansion in powers of the strange quark mass, the predictions will be less precise. However, it is clear that one should perform such an analysis in view of the fact that the kaon polarizabilities will be measured at COMPASS [20].

The motivations for measuring the meson polarizabilities are for example

i) Polarizabilities start being nonzero at one-loop order–one explores chiral loop effects directly.

ii) In general, a comparison of the data with the chiral expansion is of similar interest as the $\pi\pi$ scattering lengths.

3.3 Kaon electromagnetic form factor

If COMPASS can measure kaon electromagnetic form factors [21] to larger values of $|q^2|$ than exist at present and more precisely, this will probably be useful also in constraining the theory for V_{us} via K_{l3} decays [22].

4. THE ANOMALY

The word *anomaly* refers to anomalous Ward identities for the Green functions of quark currents.

4.1 $\pi^0 \to \gamma\gamma$

The amplitude for $\pi \to \gamma\gamma$ has been evaluated [23] in the framework of $U(3) \times U(3)$ at order e^2, including one chiral loop. Recently, higher order terms in isospin breaking effects have been taken into account, both, in $U(2) \times U(2)$ [24], and in $U(3) \times U(3)$ [25]. The predictions are

$$\Gamma_{\pi^0\to\gamma\gamma} = \left\{ \begin{array}{l} 8.06 \pm 0.02 \pm 0.06 \;\text{ eV } \;[24] \\ 8.10 \pm 0.08 \;\text{ eV } \;[25] \;, \end{array} \right. \tag{10}$$

whereas the PDG [26] quotes $\Gamma_{\pi^0\to\gamma\gamma}$ = 7.74 ± 0.55 eV. The measurement of this decay is in progress at JLAB (PRIMEX) [27].

4.2 $\gamma\pi^\pm \to \pi^\pm\pi^0$

We write the matrix element for the process $\gamma\pi \to \pi\pi$ in the form [28]

$$F^{3\pi}(s,\cos\theta) = F_0^{3\pi}\left\{f^{(0)}(s,\cos\theta) + f^{(1)}(s,\cos\theta) + f^{(2)}(s,\cos\theta) + \cdots\right\} \;,$$

where $f^{(0)}, f^{(1)}$ and $f^{(2)}$ denote the tree, one-loop [29] and two-loop [30] contribution in the chiral expansion, and where $s, \cos\theta$ are the standard kinematic variables. The constant $F_0^{3\pi}$ is fixed by the chiral anomaly. Recently, electromagnetic corrections have been calculated to $f^{(0)}$ and $f^{(1)}$ [28, 24]. Interestingly, the correction to $f^{(0)}$ in the $\gamma\pi^\pm \to \pi^\pm\pi^0$ channel is sizeable, for the following reason. This term is generated by the two graphs a) and b) in Fig. 1, and is given [28] by $f^{(0)} = 1 - 2e^2F_\pi^2/t$, where t denotes the (momentum transfer)^2 of the virtual photon in graph b). As t may become small, this contribution can be sizeable. In addition, it is rather sensitive to the scattering angle, see Fig. 2 in Ref. [28].

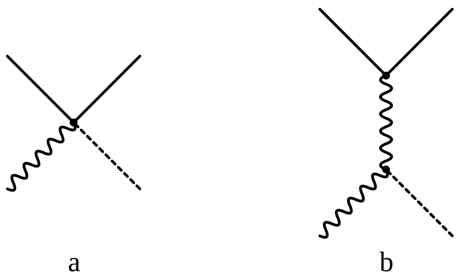

Fig. 1: Leading order contributions to the process $\gamma\pi^\pm \to \pi^\pm\pi^0$. The solid (dashed) lines denote charged (neutral) pions, a wiggly line the photon. Graph a) displays the standard anomalous vertex, and b) contains both, an ordinary vertex $\gamma\pi^\pm\pi^\pm$ and an anomalous one $\gamma\gamma\pi^0$. The two vertexes in b) are connected by the exchange of a virtual photon in the t-channel. Both graphs occur at order p^4 in the chiral expansion.

Remarks:

i) Including electromagnetic interactions in the chiral Lagrangian requires the charge to be counted as a quantity of order p for consistency. In this case, the two graphs a) and b) are algebraically of the same order in the low-energy expansion, because the propagator in b) contributes at order p^{-2}.

ii) In graph b), an ordinary vertex ($\pi^{\pm}\pi^{\pm}\gamma$) as well as an anomalous one ($\pi^0\gamma\gamma$) are present.

iii) Both graphs change sign under $\pi^+ \leftrightarrow \pi^-$.

At one-loop order, infrared divergences occur – these are cancelled in the standard manner by including [28] also the process

$$\gamma\pi \to \pi\pi\gamma \, .$$

The authors of Ref. [28] find that the result is rather insensitive to the photon energy of the outgoing photon. For explicit formulas for the cross sections, I refer the reader to this reference. All in all, after including electromagnetic interactions in the manner just described, theory agrees with the available experimental information according to Ref. [28].

4.3 $\gamma K \to \pi K$

This process is again dominated by anomalous terms in the effective Lagrangian (SU(3) version of anomaly). The calculations are yet to be performed. Since the chiral expansion also involves expansions in m_s, the prediction will be less precise than in the pion case discussed above. Note that for the process $\gamma K^0 \to K^0\pi^0$, the t-channel singularity is absent. Therefore, at leading order, this reaction gives direct access to the chiral anomaly.

4.4 N_c-dependence

The N_c dependence of anomalous processes has been investigated in the framework of the standard model in [31, 32, 33]. These authors find the following N_c dependence for the amplitudes:

$\pi^0 \to \gamma\gamma, \gamma\pi \to \pi\pi$	independent of N_c	[31, 32, 33]
$\gamma K^{\pm} \to \pi^0 K^{\pm}$	$N_c + 1$	[33]
$\gamma K^0 \to \pi^0 K^0$	$N_c - 1$	[33] .

5. STRONG INTERACTIONS

5.1 $\pi K \to \pi K$

Recent calculations that involve the reaction $\pi K \to \pi K$ include

i) a dispersive analysis of available data to determine LECs in $\mathcal{L}_4$ [34]

ii) a relation between the spectrum of $\pi^- K^+$ atoms and the elastic πK scattering amplitude [35].

What is the motivation for having more precise data on this reaction?

a) to pin down LECs

b) to get information on the character of chiral symmetry breakdown: is the vacuum affected by heavy $\bar{s}s$ pairs [36] ?

It would be useful to have data that allow one to separate isospin 1/2 and 3/2 amplitude. Furthermore, information on $\pi\pi \to K\bar{K}$ in the region of $K\bar{K}$ threshold and beyond would be very welcome [37].

5.2 $\pi\pi \to \pi\pi$

Elastic $\pi\pi$ scattering is an ideal observable to access the chiral structure of QCD vacuum for the following reason. The quark mass expansion of the pion mass reads

$$M_\pi^2 = M^2 - M^4\bar{l}_3/(32\pi^2 F_\pi^2) + O(M^6) \tag{11}$$

where M^2 is proportional to the quark condensate and to the quark mass, and $\bar{l}_3$ is a low-energy constant from $\mathcal{L}_4$. Standard chiral perturbation theory assumes that the first term in this expansion is dominant, and that further terms induce small corrections. This assumption has been put in question by Stern and collaborators [36] who pointed out that there is no experimental evidence for the quark condensate to be different from zero. The LEC $\bar{l}_3$ also occurs in the elastic $\pi\pi$ scattering amplitude and has recently been determined from high precision data on K_{e4} decays [38, 39]. It has turned out that indeed the first term in Eq. (11) is dominant – a rearrangement of the chiral expansion as proposed in [36] is not needed (see also Ref. [40]).

The point I wish to make concerning the COMPASS experiment and $\pi\pi$ scattering is the following. The Roy-equation analysis of $\pi\pi$ scattering [41], together with chiral symmetry requirements, predicts the $I = 2$ S-wave phase shift below 800 MeV, to within rather small uncertainties, see Fig. 9 in Ref. [17]. *It would therefore be very instructive to have more precise experimental information on this phase shift.*

6. SUMMARY

The following table reflects my personal view of the theoretical and experimental status of some of the processes described above.

	Ref. theory	Status of theory	Status of experiment
photons:			
$\gamma\pi \to \gamma\pi$	[11, 13]	**(*)	*
$\gamma K \to \gamma K$	[19]	*	
anomaly:			
$\pi^0 \to \gamma\gamma$	[23, 24, 25]	***	**
$\gamma\pi \to \pi\pi$	[28, 24]	***	*
$\gamma K \to K\pi$			
strong interactions:			
$\pi K \to \pi K$	[34]	**	*
$\pi\pi \to \pi\pi$	[17, 38, 39, 40, 41]	****	***

Acknowledgments

I thank the organizers of the COMPASS workshop for the invitation to give this talk and for giving me the opportunity to express the interest of the chiral community in low-energy experiments carried out at COMPASS. In addition, I thank A. Ananthanarayan, J. Bijnens, P. Büttiker, M. Knecht, B. Moussallam and M.E. Sainio for correspondence, J. Bijnens and M. Knecht for useful comments concerning the manuscript, and G. Colangelo and H. Leutwyler for informative discussions on the $I = 2$ S-wave phase shift. This work was supported in part by the Swiss National Science Foundation, and by RTN, BBW-Contract No. 01.0357 and EC-Contract No. HPRN-CT-2002-00311 (EURIDICE).

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