Proposal for a measurement of Semi-inclusive deep inelastic scattering off transversely polarised deuterons

The COMPASS Collaboration

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1 Introduction

In collinear QCD, when the transverse momentum of the partons is neglected, three parton distribution functions (PDFs) fully describe the nucleon at the twist-two level: the momentum distributions $f_1^q(x)$, the helicity distributions $g_1^q(x)$ and the transversity distributions $h_1^q(x)$ [1], where x is the Bjorken variable. On the other hand, a sizable transverse momentum of quarks was derived from the measured azimuthal asymmetries of hadrons produced in unpolarised semi-inclusive deep inelastic scattering (SIDIS) and of the lepton pairs produced in Drell-Yan (DY) processes. Taking into account a finite intrinsic transverse momentum k_T , in total eight transverse momentum dependent (TMD) distribution functions are required to fully describe the nucleon at leading twist [2]. Presently, PDFs that describe non-perturbative properties of hadrons are not yet calculable in QCD from first principles, but they can already be computed in lattice QCD. In the SIDIS cross section they appear convoluted with fragmentation functions (FFs) [3, 4], so that they can be extracted from the data.

Since transverse spin couples naturally to intrinsic transverse momentum, the resulting correlations are encoded in various TMD PDFs and fragmentation functions (FFs). Particularly interesting is therefore the measurement of the SIDIS cross section when the target nucleon is transversely polarized. In this case 8 (5 in case of unpolarised lepton beam) different spindependent azimuthal modulations are expected, from which invaluable information on the TMD PDFs can be extracted ¹. In this domain the HERMES and the COMPASS collaborations have performed pioneering measurements and shown beyond any possible doubt the correctness of three most interesting recent conjectures:

- The Sivers function: in a nucleon that is polarized transversely to its momentum the quark distribution is not left-right symmetric with respect to the plane defined by the directions of the nucleon spin and momentum. This asymmetry of the distribution function is called the Sivers effect, and the asymmetric distribution is known as the Sivers distribution function [6].
- The Transversity function: the quarks in a transversely polarized nucleon are transversely polarized. Their polarization is described by the h_1 PDFs which a priori are different and have different properties than the helicity PDFs.
- The Collins function: the hadronization of a transversely polarized quark is not left-right symmetric with respect to the plane defined by the quark momentum and the quark

¹For a review of the notation we refer to the Appendix A of the memo CERN-SPSC-2009-025 SPSC-M-769, SPSLC-P-297 Add.2 [5], which for completeness is also added to this document as section 6.

spin [7]. This fact has been exploited to measure the quark transverse polarization in a transversely polarized nucleon, namely the quark transversity PDF.

HERMES [8] and COMPASS [9, 10, 11] are up to now the only SIDIS experiments that have shown that the Sivers function, the Transversity function and the Collins function are different from zero. Independent evidence that the Collins effect is non zero has been provided at the e^+e^- colliders, by looking at the azimuthal correlations of hadrons produced in opposite jets [12, 13], so that global analyses using the SIDIS and the e^+e^- data could result in the extraction of the quark transversity PDF [14, 15].

The non zero results for the Collins and the Sivers asymmetries were obtained on proton targets. COMPASS has also measured transverse spin asymmetries using a deuteron target [16]. The accuracy of the data is definitely inferior to that of the proton data, and all the results were compatible with zero, hinting at a possible cancellation between u and d quarks. More recently data have been collected at much lower energy at JLab on a ³He target, essentially a transversely polarized neutron target: the measured asymmetries [17, 18] are also compatible with zero, but the error bars are fairly large. The COMPASS data are still today the only SIDIS data ever taken on a transversely polarised deuteron target, they are necessary to flavor separate the PDFs, and provide constraints on the d-quark contribution.

We propose to perform a one-year measurement scattering the M2 muon beam at 160 GeV/c momentum on a transversely polarized deuteron target, so that, combining the new deuteron data with the good precision proton data collected in the year 2010, the u- and d-distribution functions can be extracted from the SIDIS asymmetry data with comparable accuracies. Due to the late delivery of the COMPASS polarized target magnet, this precise measurement could not be carried through in the early years of data taking when the low statistics sample was collected. Also, the knowledge gained in the last few years (thanks also to the COMPASS results) has by now made the physics case very clear and strong, and we regard this measurement necessary to complete the exploratory COMPASS program on transverse spin.

2 The case for muon scattering on transversely polarized deuterons

High energy muon scattering on transversely polarized deuterons will provide in a standard 150 days run a wealth of data and complement the data sample collected in 2010 on transversely polarized protons. In the previous section the case for the transversity and the Sivers PDFs was singled out since these effects represent novel and unexpected features, and because there is still hope that these two phenomena could explain the very large transverse spin asymmetries observed since more than 40 years in hadron-hadron scattering. From the present data several extractions of the transversity and of the Sivers PDFs have been performed. As an example Fig. 1 shows the results of the point-by-point extractions of the transversity and the Sivers PDFs using all the existing COMPASS p and d data [19, 20] compared to the extractions done using also the HERMES data [21, 22]. It is immediately apparent that the accuracy of the d-quark PDFs is considerably inferior to that of the u-quark and this is the straightforward motivation for this proposal.

The case for the Collins asymmetry will be detailed in the next section. Here we will summarize the other measurements which will be performed in parallel using the new deuteron data.

In the SIDIS regime the data will allow the extraction of

- the Sivers function. As underlined in Ref. [20], and clear from fig. 1, the d_v Sivers function is poorly determined from the present data, and even its sign is not unambiguously fixed by the measurements. The new data will allow to improve on its accuracy by the same



Figure 1: The transversity and the Sivers PDFs extracted point-by-point using the existing COMPASS p and d data from Ref. [19, 20]. The curves are the results of fits to the COMPASS and HERMES data. Note that the uncertainty band for the d-quark transversity would be larger if the Soffer bound was not imposed.

factor as for the transversity PDF, namely by a factor 2 to 4 as explained in the next sections.

On the theoretical side, the interest on the Sivers function is steadily growing, and particularly so the interest on the gluon Sivers distribution. An analysis of all the COMPASS data has provided some indication that the gluon Sivers function might be different from zero [23], but the accuracy of the existing deuteron data is worse by a factor of about 2 than that of the proton data, and also for this specific case new data would be helpful.

In a related analysis the Sivers asymmetry of the J/Ψ has also been determined, since in some models it is related to the gluon Sivers asymmetry [24]. That analysis can also be repeated with the new deuteron data.

- two hadron asymmetries. The high energy of the beam and the large acceptance of the COMPASS spectrometer have allowed us to collect in 2010 a considerably large sample of (oppositely charged) hadron pairs and to perform a unique and original comparison [11, 25] between the single-hadron Collins asymmetry and the di-hadron asymmetry. The conclusion of this investigation was that both the single hadron and the di-hadron transverse-spin dependent fragmentation functions are driven by the same elementary mechanism, which is very well described in the 3P0 recursive string fragmentation model [26, 27].

A corresponding analysis with the deuteron data was not possible because of the small statistics of the two hadron data sample due to the use of the SMC small acceptance PT magnet in the first three years of COMPASS running with the deuteron target.

- the g_2 structure function. In the naive parton model g_2 is expected to be zero, thus its measurement provides information on the quark-gluon interaction. In COMPASS we have started an analysis to extract g_2 from the 2010 proton data, which will be repeated with the new deuteron data.

Moreover, COMPASS has performed the first ever multidimensional extraction of the whole set of target transverse spin dependent azimuthal asymmetries using the proton data collected in 2010 [28]. Various multi-differential configurations have been tested exploring the $x - Q^2 - z - p_T$ phase-space. Very interesting correlations have been noticed in particular for the Sivers function, This analysis was not possible with the existing deuteron data and will be done with the new data.

Recently COMPASS has extracted p_T weighted Sivers asymmetries from the 2010 proton data. Also in this case only new accurate data will allow the same analysis to be performed the deuteron.

Finally, precise results on deuteron will be produced for all the other 6 SIDIS TSAs.

In exclusive vector meson production COMPASS has produced several interesting results. In a first paper [29] we published the transverse target spin azimuthal asymmetry $A_{UT}^{\sin(\phi-\phi_S)}$ in hard exclusive production of ρ^0 mesons which we measured both on transversely polarized protons and deuterons. The measured asymmetry is sensitive to the nucleon helicity-flip generalized parton distributions E_q , which are related to the orbital angular momentum of quarks in the nucleon. A second publication [30] used the high statistics proton data collected in 2010, and presented results for all 8 possible transverse target spin asymmetries. In particular a specific combination of two of these asymmetries indicates a signal from the so called "transversity GPD" (i.e. GPD with the helicity flip of exchanged quark). Concerning deuterons, only the results on the $A_{UT}^{\sin(\phi-\phi_S)}$ asymmetry are published [29], due to the poor statistics of the existing deuteron COMPASS data. Thus measuring exclusive production of ρ^0 mesons on transversely polarized deuteron target in 2021 will allow us to get a more precise and complete results (8 asymmetries) for deuteron as well.

3 The case for transversity

In this section the case for the Collins asymmetry and the extraction of transversity for the u and d quarks will be detailed. The measurement of the quark transversity distributions, which are defined in terms of the nucleon matrix element of the quark tensor current, is particularly important because it provides access to the quark tensor charges δq , which are given by the integral

$$\delta q(Q^2) = \int_0^1 dx [h_1^q(x, Q^2) - h_1^{\bar{q}}(x, Q^2)] \tag{1}$$

In a non-relativistic quark model, h_1^q is equal to g_1^q , and δq is equal to the valence quark contribution to the nucleon spin. The difference between h_1^q and g_1^q provides important constraints to any model of the nucleon. Knowing the quark tensor charges one can construct the isovector nucleon tensor charge $g_T = \delta u - \delta d$, a fundamental property of the nucleon which, together with the vector and axial charge, characterizes the nucleon as a whole. Since many years the tensor charge is being calculated with steadily increasing accuracy by lattice QCD [31]. More recently, its connection with possible novel tensor interactions at the TeV scale in neutron and nuclear β -decays and its possible contribution to the neutron EDM have also been investigated [32], and possible constraints on new physics beyond the standard mode have also been derived [33].

The present knowledge on g_T is well summarized in Fig. 2, from Ref. [32]. The huge difference between the accuracy of the extractions from the existing data and from the QCD lattice simulations is striking and more experimental data are needed. In the near future the only planned and approved experiments will run at JLab12 [34, 35], with very good statistics but x > 0.05 and relatively small Q^2 . In the longer term the planned Electron Ion Collider (EIC) has the potential to carry on a very good program scattering at high \sqrt{s} electrons on transversely polarized protons, but it is not yet on the real axis, and storing polarized deuterons is not in the core program.

We propose to measure during one full year, as soon as the LS2 will be over, SIDIS on a transversely polarized deuteron target in the M2 muon beam line. The polarized target will be



Figure 2: A summary plot showing the current estimates of g_T^{u-d} from Ref. [32].

reassembled at the end of the DVCS/SIDIS run, this fall, to be used for the Drell-Yan run of 2018, and will be left on the floor in Hall 888 for this new measurement. The main objective of the measurement is to improve (considerably) the accuracy of the extraction of h_1^d , but the precision of h_1^u will also improve, by a factor 1.5-2 in the valence region (x > 0.1), as will be shown in the next section.

The COMPASS data will provide large Q^2 data in the x-range covered by JLab, which is very important to evaluate the size of the Q^2 evolution, and will provide lower-x data (down to x = 0.003) which are essential both to perform the integrals necessary to evaluate the tensor charges and to estimate the transversity of the sea quarks. The phase space covered by the different experiments is shown in Fig. 3. Clearly the experiment we propose is unique and complementary to the JLab12 experiments.

3.1 Present COMPASS data and extrapolated errors

The transversity PDF is chiral-odd and thus not directly observable in inclusive deep inelastic lepton-nucleon scattering. In 1993 Collins suggested [7] that it could be measured in SIDIS processes, where it appears coupled with another chiral-odd function, which by now is known as "Collins fragmentation function" $H_{1q}^{\perp h}$. It is the chiral-odd transverse-spin dependent FF that describes the correlation of quark (q) transverse polarisation and hadron (h) transverse momentum. This mechanism leads to a left-right asymmetry in the distribution of hadrons produced in the fragmentation of transversely polarized quarks, which in SIDIS shows up as an azimuthal transverse spin asymmetry A_{Coll} (the "Collins asymmetry") in the distribution of



Figure 3: $x - Q^2$ scatter plot for SIDIS experiments from ref. [36].

produced hadrons. At leading order this asymmetry can be written as

$$A_{Coll} = \frac{\sum_{q,\bar{q}} e_q^2 x h_1^q \otimes H_{1q}^{\perp}}{\sum_{q,\bar{q}} e_q^2 x f_1^q \otimes D_{1q}}$$
(2)

where the sum is over all (anti)quark flavours, D_q^h is the usual FF and \otimes indicates the (different for numerator and denominator) convolutions over the intrinsic transverse momenta. The Collins effect shows up as a modulation $[1 + a_C \sin(\phi_h + \phi_S - \pi)]$ in the hadron azimuthal distribution. Here $\Phi_C = \phi_h + \phi_S - \pi$ is the Collins angle, and ϕ_h and ϕ_S are the azimuthal angles of the hadron transverse momentum \vec{p}_{hT} and of the spin direction of the target nucleon with respect to the lepton scattering plane, in a reference system in which the z axis is the virtual-photon direction. The amplitude of the modulation is $a_C = D_{NN} f P A_{Coll}$, where D_{NN} is the transverse spin transfer coefficient from target quark to struck quark, f the dilution factor of the target material, and P is the proton (or deuteron) polarization. In Fig. 4 the results [10] for A_{Coll} we have obtained from the 2010 data collected using as target NH₃, a polarized proton target, are shown as a function of x and compared to the results we obtained [16] from the deuteron runs of 2002, 2003, and 2004, when as target we used ⁶LiD.

It is clear that the accuracy of the data is considerably better for the proton run, in particular at large x, where the Collins asymmetry is large. In order to quantify this fact, it is instructive to plot the ratio of the errors. In Fig. 5 this ratio is shown as a function of x. In order to understand this plot, one has to remind that, for small asymmetries, the statistical error is given by

$$\sigma_A \simeq \frac{1}{fP} \frac{1}{\sqrt{N}} = \frac{1}{FOM} \frac{1}{\sqrt{N}} \tag{3}$$

where N is the total number of hadrons and FOM is the figure of merit of the polarised target. Using $N_{d,h} = 15.5 \cdot 10^6$ and $N_{p,h} = 80 \cdot 10^6$ as the figures for the number of hadrons collected on p and d, and the known FOM values for the two targets, one gets

$$\frac{\sigma_{A_d}}{\sigma_{A_p}} = \frac{0.155 \cdot 0.80}{0.40 \cdot 0.50} \frac{\sqrt{80}}{\sqrt{15.5}} = 0.62 \cdot 2.3 = 1.4 \tag{4}$$



Figure 4: A_{Coll} obtained from the 2010 data with the polarized proton NH₃ target as a function of x (left plot) compared to the results we obtained [16] from the runs of 2002, 2003 and 2004 with polarised deuteron ⁶LiD target (right plots). The red (black) points refer to positive (negative) hadrons. The full points at -0.06 in the left plot show the extrapolated statistical error from the proposed deuteron run (see text).



Figure 5: Ratio of the A_{Coll} statistical uncertainties on deuteron and proton.



Figure 6: Values of u_v -quark (left) and d_v -quark (right) transversity extracted from the existing p and d data (open points), and the corresponding error bars estimated using the existing p data and the new d data (closed points).

At small x, where most of the events cluster, the ratio between the deuteron and the proton asymmetries is indeed constant, since the spectrometer acceptance was essentially the same in the two data taking, and its value is close to 1.4. Here the better FOM of the deuteron target largely compensates the factor of 5 in statistics in favor of the proton target. The remaining 10% difference is due to the fact that the PT cells diameter in the deuteron runs was 3 cm while for the proton runs it was 4 cm, which resulted in a 20% larger beam acceptance in the proton runs. Our plan is to run in 2021 with 4 cm target cells diameter as long as enough ⁶LiD material will be available. At large x on the contrary the ratio increases dramatically, reflecting the difference in acceptance of the COMPASS PT magnet, which has a polar angle acceptance of 200 mrad as seen from the upstream end of the target, while in the earlier measurements with the ⁶LiD target we had utilized the SMC magnet, which has a corresponding polar angle acceptance of 70 mrad.

3.2 Projected errors after 1 year of deuteron run

Since target density and packing factors are essentially identical for ⁶LiD and NH₃, it can be safely assumed that in one year of deuteron run in the conditions of the 2010 proton run $80 \cdot 10^6$ "good" events will be collected, so that the errors on the new deuteron asymmetries will be equal to the present errors for the 2010 proton asymmetries scaled by the ratio of the FOM, namely they will be smaller by a factor of 0.62. The projected errors for the deuteron asymmetries are also plotted in Fig. 4, together with the existing deuteron and proton asymmetries. We neglect the systematic errors which were estimated to be at most 0.5 times the statistical errors in the 2010 data. Using the 2010 proton data and the projections of Fig. 4 for the new deuteron data it is possible to extract the u- and d-quark transversity, and quantify the gain in statistical error in these fundamental PDFs. To carry through this evaluation we have followed the procedure of Ref. [19], which allows a point by point extraction of transversity directly from the measured SIDIS and $e^+e^- \rightarrow hadrons$ asymmetries. The results of such extractions are given in Fig. 6, which shows both the values of transversity (open points) extracted from the existing p and d data, and the corresponding error bars (closed points) estimated using the existing p data and the new d data. Also, Fig. 7 gives the ratio, at each x value, of the existing errors on the extracted transversities and the projected errors, taking the existing proton asymmetries from 2010 and the projected errors for the deuteron asymmetries obtained after 1 year of running.



Figure 7: Ratio of the existing errors on the extracted transversities and the projected errors for u_v -quark (left) and d_v -quark (right).

One can see that the gain in precision is quite good for the d-quark and important also for the u-quark.

3.3 Projections for the tensor charge

In order to evaluate the tensor charge it is convenient to introduce a functional dependence for h_1^q , to be fixed by fitting the extracted values of $xh_1^{u_v}$ and $xh_1^{d_v}$. We neglect the Q^2 dependence of h_1 and take

$$xh_1^q(x) = a_q x^{b_q} (1-x)^{c_q}.$$
(5)

Unfortunately, the present statistical accuracy on xh_1^q with $q = u_v, d_v$ does not allow to safely determine all the parameters a_q , b_q and c_q and in particular their covariance matrix, needed for this exercise. We thus assumed $c_q = 4$, as suggested by the central values given by the fit. For the remaining two free parameters we get

$$a_{u_v} = 3.5 \pm 1.6, \ b_{u_v} = 1.3 \pm 0.2, \ a_{d_v} = -5.2 \pm 5.3, \ b_{d_v} = 1.5 \pm 0.5.$$
 (6)

The comparisons between the fitted $xh_1^{u_v}$ and $xh_1^{d_v}$ and the extracted transversity values are shown in Fig. 8, together with the 68% uncertainty bands.

To estimate the impact of our measurements on the extraction of the tensor charge, the curves have been numerically integrated in the two ranges 0 < x < 1 and 0.003 < x < 0.21. The reduced range of integration, which excludes our last measured x bin, is meant to not overlap with the precise data from JLab12 which should come in the future. The results are given in Tab. 1 together with the corresponding value of $g_T = \int dx h_1^{u_v}(x) - \int dx h_1^{d_v}(x)$. While the evaluation of the *d*-quark tensor charge presently has no statistical significance, the new measurement should provide more than a 4σ effect with respect to the presently estimated value, and the extraction of g_T in the *x*-interval in which COMPASS can measure is more than respectable. For completeness we have integrated the fitted functions also in the entire domain 0 < x < 1, to give an idea of the contribution that COMPASS can give to the determination of the tensor charge. Needless to say, this estimates are meant only to propagate the statistical uncertainties from the measured PDF to the integrated tensor charge in order to evaluate the impact of the new data, and not to give a value for the tensor charge itself.



Figure 8: Extracted values of the valence quark transversity distributions $xh_1^{u_v}$ and $xh_1^{d_v}$ with the curves from the fits.

	0.003 < x < 0.21		
errors	$\int dx h_1^{u_v}(x)$	$\int dx h_1^{d_v}(x)$	g_T
old	0.255 ± 0.043	-0.202 ± 0.112	0.45 ± 0.12
proj	0.211 ± 0.027	-0.212 ± 0.042	0.423 ± 0.050
	0 < x < 1		
old	0.59 ± 0.13	-0.61 ± 0.35	1.20 ± 0.37
proj	0.587 ± 0.077	-0.585 ± 0.119	1.172 ± 0.142

Table 1: Integrated values of h_1 and result for g_T from the fits with the present and the projected uncertainties.

4 Experimental Apparatus and Beam request

The apparatus to be used for the deuteron run is basically the COMPASS Spectrometer as it was used in the 2010 muon run. The polarized target will be housed in the large acceptance COMPASS PT magnet, and the target material will be the same which was used in the years 2002, 2003, 2004 and 2006, namely ⁶LiD. For a better usage of the muon beam, the target cells diameter will be increased from 3 to 4 cm. The average polarization of the target is expected to be the same as in the past deuteron runs ($\leq 50\%$).

The beam request is the same as for the 2010 proton run, namely 2.5×10^{13} protons delivered to the T6 target of the M2 beam line every 40.8 s. With an accelerator chain efficiency of 90% and a running time of 150 days a total of 6.1×10^{18} protons at T6 is expected. This number of protons is the basis of all the projections presented in this document, which are obtained from the number of reconstructed hadrons in the 2010 run.

The COMPASS Spectrometer availability and efficiency are expected to be the same as in the 2010 run. Tracking will profit of the addition of several trackers over the past ten years, in particular the new large area DC5, the pixelized GEMs and Micromegas and several scintillating fiber hodoscopes. At variance with the past deuteron runs, electromagnetic calorimetry will also be available (ECAL1 and ECAL2). In addition some increase in the collected data is expected from hardware upgrades of the last years, in particular concerning the DAQ and trigger. Since no upgrades of the present spectrometer are necessary for this measurement, it can start soon and tale place in 2021.

5 Summary

We propose to improve our knowledge of the transverse spin structure of the nucleon by measuring 160 GeV muon semi inclusive DIS on a transversely polarized deuteron target. The proposed measurements will have a profound impact on the field, and their combination with the already taken proton data will allow to further clarify the properties of the up, down and sea quarks in the nucleon.

Quoting from our last proposal for a polarized SIDIS measurement [5], "the high intensity and polarization of the muon beam together with the COMPASS polarized target and spectrometer make CERN a unique place to perform such measurement. This will not change until the construction of a high energy and luminosity polarized electron-ion collider in the longer term future".

6 Appendix A from Ref. [5]

TMD PDFs and SIDIS scattering

The recent theoretical work on the nucleon structure points out the relevance of its transverse structure. A good knowledge of the transverse intrinsic momentum \mathbf{k}_T carried by the partons and of its connection with the spin is needed to understand the parton orbital motion and to progress towards a more structured picture, beyond the collinear partonic representation.

In the QCD parton model, at leading twist, the nucleon structure is described by eight TMD PDFs: $f_1(x, \mathbf{k}_T^2)$, $g_{1L}(x, \mathbf{k}_T^2)$, $h_1(x, \mathbf{k}_T^2)$, $g_{1T}(x, \mathbf{k}_T^2)$, $h_{1T}^{\perp}(x, \mathbf{k}_T^2)$, $h_{1L}^{\perp}(x, \mathbf{k}_T^2)$, $h_1^{\perp}(x, \mathbf{k}_T^2)$ and $f_{1T}^{\perp}(x, \mathbf{k}_T^2)$, using the so-called Amsterdam notation. After integrating over \mathbf{k}_T only the first three PDFs survive, yielding the number distribution $f_1(x)$ (or q(x)), the helicity distribution $g_1(x)$ (or $\Delta q(x)$), and the transversity distribution $h_1(x)$ (or $\Delta_T q(x)$ in the usual COMPASS notation). These three functions fully specify the quark structure of the nucleon at the twisttwo level. Today, a lot of attention is put in particular on the TMD functions f_{1T}^{\perp} , the Sivers function which gives the correlation between the nucleon transverse spin and the quark intrinsic transverse momentum, h_1^{\perp} , the Boer–Mulders function which gives the correlation between the transverse spin and the intrinsic transverse momentum of a quark inside an unpolarised nucleon, and g_{1T} , which is the only chiral-even and T-even leading twist function in addition to f_1 and g_1 .

A powerful method to access the poorly known TMD PDF is SIDIS on transversely polarised targets. In fact, on the basis of general principles of quantum field theory in the one photon exchange approximation, the SIDIS cross-section in the COMPASS kinematical range can be written in a model independent way as:

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dp_T^h} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \Big\{ F_{UU} + \\
+ \sqrt{2\epsilon(1+\epsilon)}\cos\phi_h F_{UU}^{\cos\phi_h} + \epsilon\cos 2\phi_h F_{UU}^{\cos 2\phi_h} + \\
+ \lambda\sqrt{2\epsilon(1-\epsilon)}\sin\phi_h F_{LU}^{\sin\phi_h} + \\
+ S_L \Big[\sqrt{2\epsilon(1+\epsilon)}\sin\phi_h F_{UL}^{\sin\phi_h} + \epsilon\sin 2\phi_h F_{UL}^{\sin 2\phi_h} + \\
+ \lambda \left(\sqrt{1-\epsilon^2}F_{LL} + \sqrt{2\epsilon(1-\epsilon)}\cos\phi_h F_{LL}^{\cos\phi_h} \right) \Big] + \\
+ S_T \Big[\sin(\phi_h - \phi_S)F_{UT}^{\sin(\phi_h - \phi_S)} + \epsilon\sin(\phi_h + \phi_S)F_{UT}^{\sin(\phi_h + \phi_S)} + \\
+ \epsilon\sin(3\phi_h - \phi_S)F_{UT}^{\sin(3\phi_h - \phi_S)} + \\
+ \sqrt{2\epsilon(1+\epsilon)}\sin\phi_S F_{UT}^{\sin\phi_S} + \\
+ \sqrt{2\epsilon(1+\epsilon)}\sin(2\phi_h - \phi_S)F_{UT}^{\cos(\phi_h - \phi_S)} \\
+ \lambda \Big(\sqrt{1-\epsilon^2}\cos(\phi_h - \phi_S)F_{LT}^{\cos(\phi_h - \phi_S)} + \\
+ \sqrt{2\epsilon(1-\epsilon)}\cos\phi_S F_{LT}^{\cos\phi_S} + \\
+ \sqrt{2\epsilon(1-\epsilon)}\cos(2\phi_h - \phi_S)F_{LT}^{\cos(2\phi_h - \phi_S)} \Big) \Big] \Big\}. (1)$$

Here ϕ_S and ϕ_h are the azimuthal angles of the nucleon transverse spin and of the hadron transverse momentum $\vec{p}_T^{\,h}$ in the Gamma–Nucleon System, α is the fine structure constant, λ is the lepton helicity, S_T and S_L are the nucleon transverse and longitudinal polarisation. Neglecting the terms in $\gamma^2 = (2Mx/Q)^2$, the quantity ϵ is given by $\epsilon = (1-y)/(1-y+y^2/2)$.

The r.h.s. structure functions F's in general depend on Q^2 , x, z and p_T^h . Their superscripts

refer to the corresponding azimuthal asymmetries. The subscripts refer to the beam and to the target polarisation (U means unpolarised, L longitudinally polarised, and T transversely polarised). Since the modulations which appear in the cross-section for unpolarised, longitudinally polarised and transversely polarised nucleons are independent combinations of the azimuthal angles, all of them can be measured using data taken with unpolarised, longitudinally polarised and transversely polarised targets.

In the S_T -dependent part of the cross-section, only four of the eight structure functions are of leading order. They are:

- $F_{UT}^{\sin(\phi_h + \phi_S)} \propto h_1 \otimes H_1^{\perp}$, where h_1 is the transversity distribution, H_1^{\perp} is the Collins fragmentation function and \otimes indicates the convolution over the quark intrinsic transverse momentum summed over the quark flavors. When divided by F_{UU} it is the Collins asymmetry measured by COMPASS and HERMES;
- $F_{UT}^{\sin(\phi_h \phi_S)} \propto f_{1T}^{\perp} \otimes D$, where f_{1T}^{\perp} is the Sivers function and D is the unpolarised fragmentation function. When divided by F_{UU} it is the Sivers asymmetry measured by COMPASS and HERMES;
- $F_{UT}^{\sin(3\phi_h \phi_S)} \propto h_{1T}^{\perp} \otimes H_1^{\perp}$, and
- $F_{LT}^{\cos(\phi_h \phi_S)} \propto g_{1T} \otimes D.$

A complete list of the TMD PDFs which appear in all the structure functions can be found e.g. in Ref. [4]

References

- We are following the "Amsterdam notation", see P. J. Mulders and R. D. Tangerman, Nucl. Phys. B 461 (1996) 197 Erratum: [Nucl. Phys. B 484 (1997) 538] doi:10.1016/S0550-3213(96)00648-7, 10.1016/0550-3213(95)00632-X [hep-ph/9510301].
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