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Towards a GPD fitting procedure

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- ***Generalized parton distributions***
- ***How to get a realistic GPD ansatz?***
- ***Ready for a fitting procedure?***
- ***Conclusions***

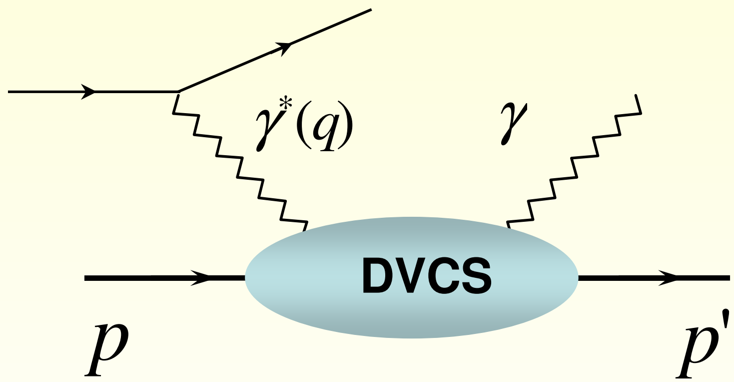
in collaboration with K. Kumerički and K. Passek-Kumerički (Zagreb)

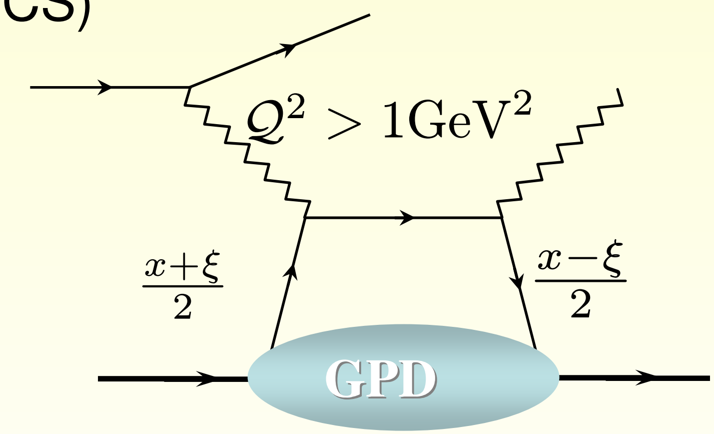
Generalized parton distributions

DM, Robaschik, Geyer,
Dittes, Hořejši (PhD 92,94)
A. Radyushkin (96)
X. Ji (96)

GPDs appear in various hard exclusive processes,

e.g., hard electroproduction of photons (DVCS)





$t = \Delta^2 \; - \; \text{fix}$

$$\mathcal{F}(\xi, \mathcal{Q}^2, \Delta^2) = \int_{-1}^{1} dx\, C(x, \xi, \alpha_s(\mu), \mathcal{Q}/\mu) F(x, \xi, \Delta^2, \mu)$$

Compton form factor

observable

hard scattering part

perturbation theory
(our conventions)

GPD

universal
(but conventional)

Definition of GPDs

Generically, GPDs are defined as matrix elements of light-ray operators

$$F(x,\eta,\Delta^2,\mu^2) = \int_{-\infty}^{\infty} d\kappa\, e^{i\kappa n\cdot P} \langle P_2|\phi(-\kappa n)\phi(\kappa n)_{(\mu^2)}|P_1\rangle\Big|_{\eta=\frac{n\cdot\Delta}{n\cdot P}}\,, \quad n^2 = 0$$

$$P = P_1 + P_2 \qquad \Delta = P_2 - P_1$$

For a nucleon (proton) target (mainly) four different twist-two GPDs appears:

$$\bar\psi_i\gamma_+\psi_i \quad \Rightarrow \quad {}^{i}q^{V} \;=\; \bar U(P_2,S_2)\gamma_+ U(P_1,S_1) H_i + \bar U(P_2,S_2)\frac{i\sigma_{+\nu}\Delta^\nu}{2M} U(P_1,S_1) E_i$$

$$\bar\psi_i\gamma_+\gamma_5\psi_i \quad \Rightarrow \quad {}^{i}q^{A} \;=\; \bar U(P_2,S_2)\gamma_+\gamma_5 U(P_1,S_1) \widetilde H_i + \bar U(P_2,S_2)\frac{\gamma_5}{2M} U(P_1,S_1) \widetilde E_i$$

shorthand for GPDs: $F = \{H, E, \widetilde H, \widetilde E\}$ & CFFs: $\mathcal{F} = \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}$

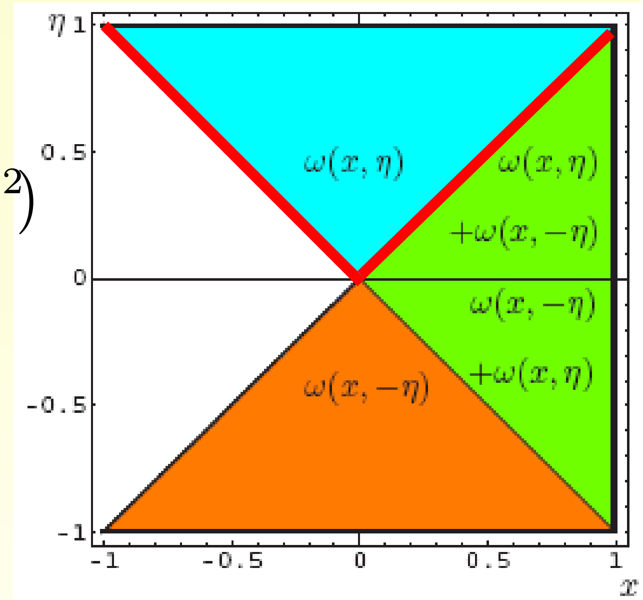
$$\Delta^2 \equiv t$$

Support of GPDs – a hint for duality

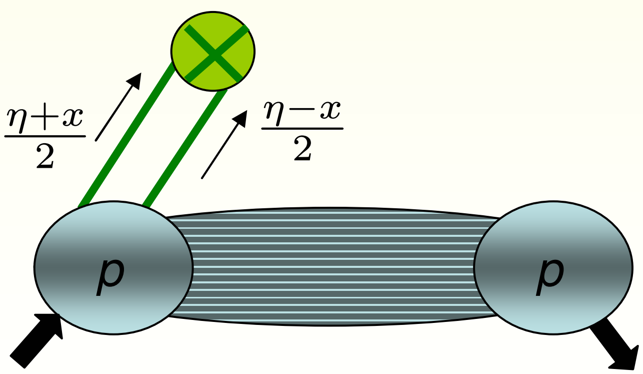
consider a quark GPD (anti-quark $x \to -x$)

$$F = \theta(-\eta \le x \le 1)\,\omega\big(x,\eta,\Delta^2\big) + \theta(\eta \le x \le 1)\,\omega\big(x,-\eta,\Delta^2\big)$$

$$\omega\left(x,\eta,\Delta^2\right) = \frac{1}{\eta}\int_0^{\frac{x+\eta}{1+\eta}} dy\,(1-x)^p f(y,(x-y)/\eta,\Delta^2)$$

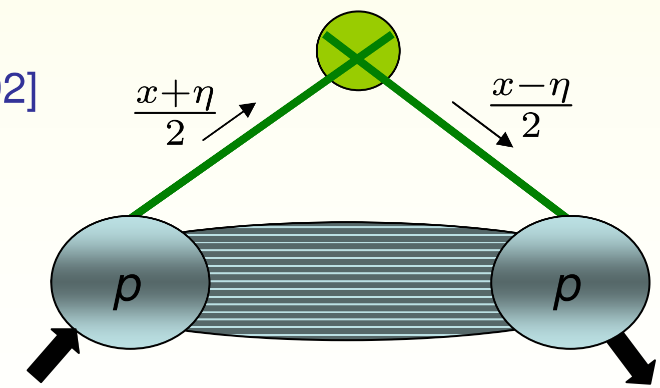


a naive ***dual*** interpretation on partonic level:



support extension is unique [DM et al. 88/92]

ambiguous (*D*-term)
[DM, A. Schäfer (05)]



central region $-\eta < x < \eta$

mesonic exchange in *t*-channel

outer region $\eta < x$

partonic exchange in *s*-channel

Overlap representation of GPDs

QCD bound state problem might be formulated in LC quantization:

Drell, Yan (69)
Drell, Brodsky

$$P^-|P,S\rangle = \frac{M^2}{P^+}|P,S\rangle\,, \quad \text{with} \quad P^- = P^0 - P^3\,,\ P^+ = P^0 + P^3\,,\ \mathbf{P}_\perp = 0$$

formally, solution is expanded with respect to *partonic degrees of freedom:*

$$|P, S = \{\uparrow,\downarrow\}\rangle = \sum_{n,\lambda_i}\int [dx\, d^2\mathbf{k}]_n\, \psi^{\uparrow,\downarrow}_{n,\lambda_i}(x_i, \mathbf{k}_\perp, \lambda_i)|n, x_i P^+, x_i \mathbf{P}_\perp + \mathbf{k}_{\perp i}, \lambda_i\rangle$$

GPDs defined as overlap of LC-wave functions (outer region):

Diehl, Feldmann, Jakob, Kroll (98)

$$F(x \geq \eta, \eta, \Delta^2) \propto \sum_{n,\lambda_i}\left(\frac{1-\eta}{1+\eta}\right)^{\frac{2-n}{2}}\int [dx\, d^2\mathbf{k}]_n\, \delta\!\left(\frac{x+\eta}{1+\eta} - x_1\right)\psi^{\uparrow *}_{n,\lambda_i}(x'_i, \mathbf{k}'_{\perp i})\psi^{\uparrow(\downarrow)}_{n,\lambda_i}(x_i, \mathbf{k}_{\perp i})$$

$$x'_1 = \frac{x-\eta}{1-\eta}\,, \quad \mathbf{k}'_{\perp,1} = \mathbf{k}_{\perp,1} - \frac{1-x}{1-\eta}\boldsymbol{\Delta}_\perp$$

Note: $x'_1 \to 0 \text{ for } x \to \eta$

positivity constraints* [Pobylitsa (02)] *are satisfied

if Lorentz symmetry is correctly implemented,
central region follows from duality

D.S. Hwang
D.M., to appear

Constraints on GPDs

! polynomiality conditions arise from *hidden* Lorentz covariance

$$\int_{-\eta}^{1} dx\, x^n F(x,\eta,t) = \text{polynom of order } n \text{ or } n+1 \text{ in } \eta$$

satisfied within spectral representation (*D*-term is misleading)

$$F(x,\eta,t) = (1-x)^p \int_0^1 dy \int_{-1+y}^{1-y} dz\, \delta(x-y-z\eta) f(y,z,t),\ p = \{0,1\}$$

! lowest moment reduction to partonic form factor – related to observables

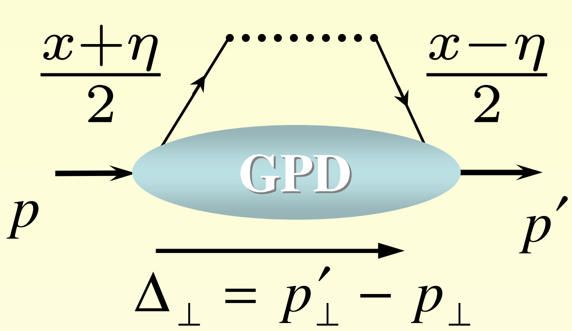
! first moment is given by the expectation value of the energy-momentum tensor

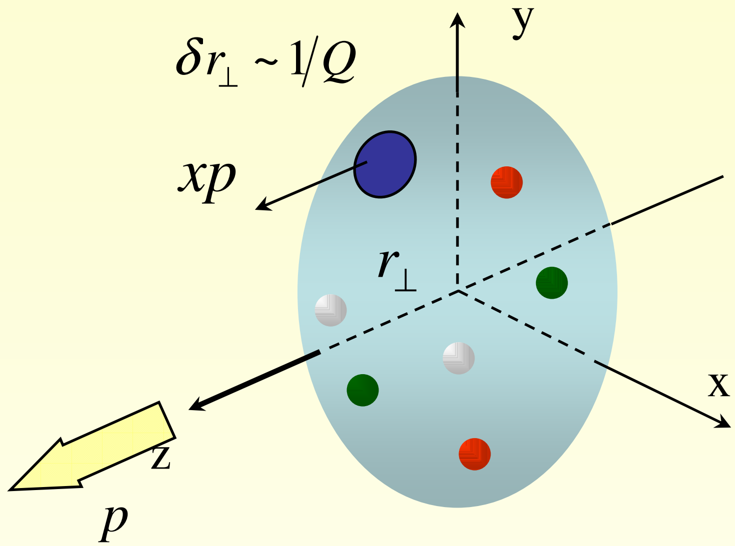
! reduction to parton densities (PDs)

$$q(x) = \lim_{\Delta\to 0} H(x,\eta,t), \quad \Delta q(x) = \lim_{\Delta\to 0} \widetilde{H}(x,\eta,t)$$

! positivity constraints (requirement on GPDs and scheme) **[Pobylitsa (02)]**
are automatically satisfied in the overlap representation

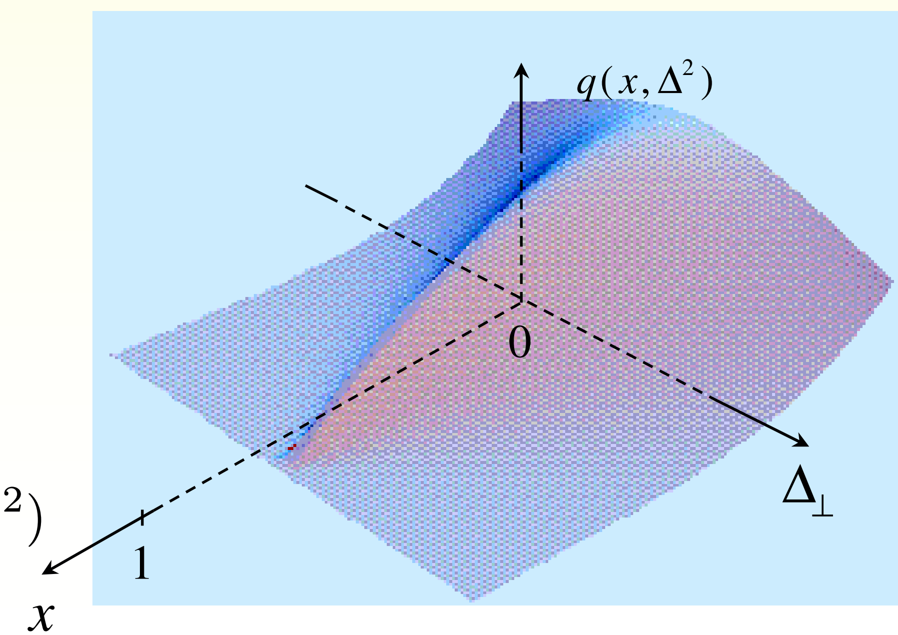
Partonic interpretation of GPDs





- GPDs simultaneously carry information on both *longitudinal* and *transverse* distribution of partons in a proton

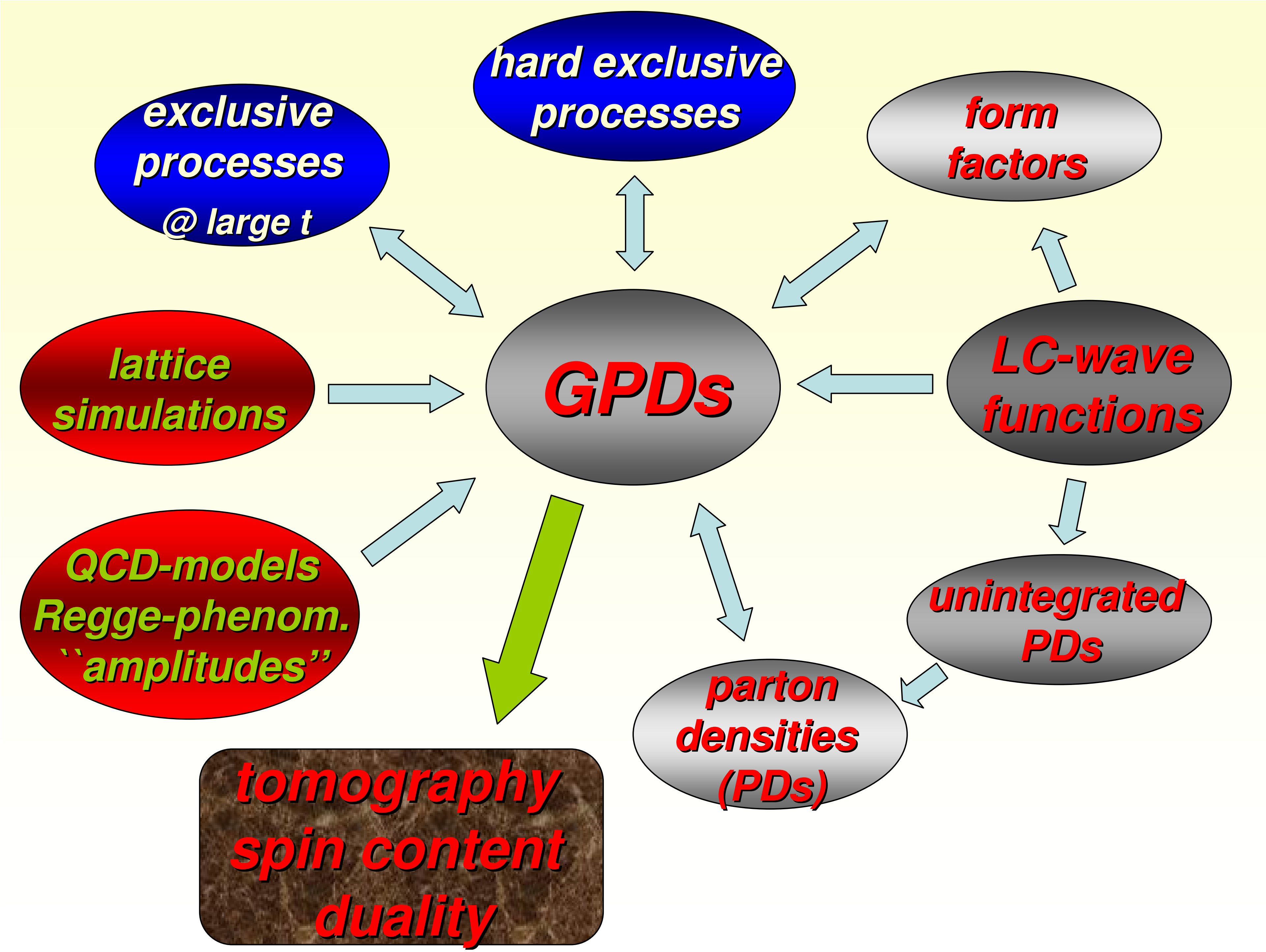
  for *η=0* they have a probabilistic interpretation (infinite momentum frame) **[Burkhardt (00)]**



- GPDs contain also information on partonic angular momentum **[X. Ji (96)]**

$$\frac{1}{2} = \sum_{a=q,G} J_a^z$$

$$J_a^z = \lim_{\Delta \to 0} \frac{1}{2} \int_{-1}^{1} dx\, x\, (H_a + E_a)(x, \eta, \Delta^2)$$



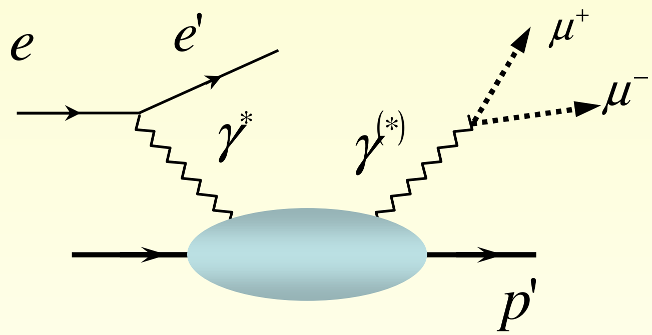
hard exclusive processes
exclusive processes @ large t
form factors
lattice simulations
GPDs
LC-wave functions
QCD-models Regge-phenom. ``amplitudes''
unintegrated PDs
parton densities (PDs)
tomography spin content duality

GPD related hard exclusive processes

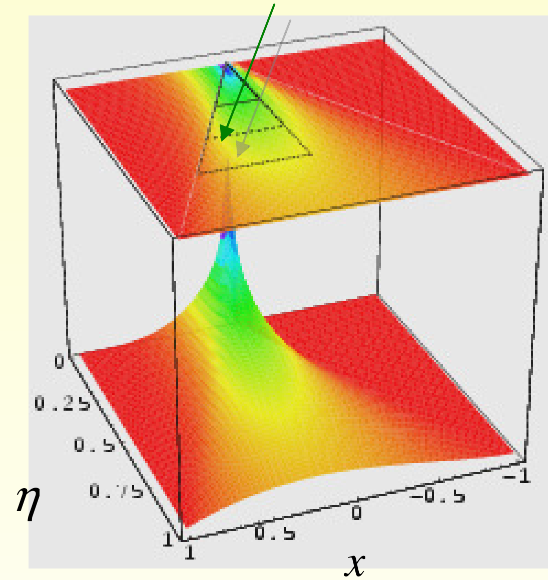
- Deeply virtual Compton scattering

  (clean probe)

$$ep \to e'p'\gamma$$
$$ep \to e'p'\mu^+\mu^-$$
$$\gamma p \to p'e^+e^-$$



scanned area of the surface as a functions of lepton energy

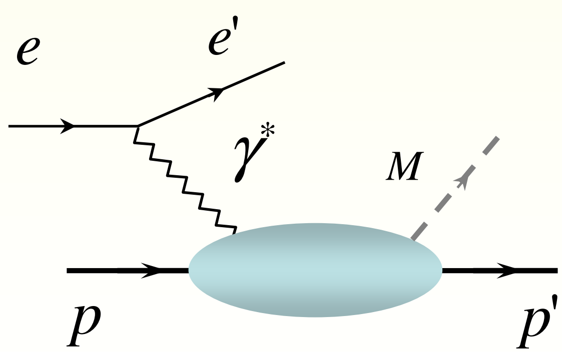


$$ep \to e'\,p'\,\mu^+\mu^-$$

- Hard exclusive meson production

  (flavor filter)

$$ep \to e'p'\pi$$
$$ep \to e'p'\rho$$
$$ep \to e'n\pi^+$$
$$ep \to e'n\rho^+$$



<u>twist-two observables:</u>

cross sections

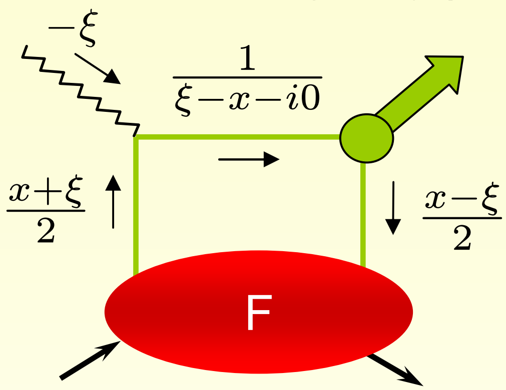
transverse target spin asymmetries

- etc.

measured from H1, ZEUS, HERMES; Hall A & B (CLAS) @ JLAB

Which partonic information can be accessed?

Real and imaginary part of CFFs have to LO the following partonic interpretation:



$$\Im\mathrm{m}\mathcal{F}(\xi, \mathcal{Q}^2, \Delta^2) = \pi F(x=\xi, \xi, \Delta^2, \mathcal{Q}^2)$$

$$\Re\mathrm{e}\mathcal{F}(\xi, \Delta^2, \mathcal{Q}^2) = \mathrm{PV}\int_{-1}^{1} dx \frac{1}{\xi - x} F(x, \xi, \Delta^2, \mathcal{Q}^2)$$

Real part is given by a dispersion relation:

$$\Re\mathrm{e}\mathcal{F}(\xi, \Delta^2, \mathcal{Q}^2) = \mathrm{PV}\int_{-1}^{1} d\xi' \frac{1}{\xi - \xi'} F(x=\xi', \xi', \Delta^2, \mathcal{Q}^2) + \mathcal{C}(\Delta^2, Q^2)$$

CFFs to LO are entirely determined:

$$\mathcal{F}(\xi, \mathcal{Q}^2, \Delta^2) \Leftrightarrow F(x=\xi, \eta=\xi, \Delta^2, \mathcal{Q}^2),\ \theta(|x| \le \xi) D(x/\xi, \Delta^2, \mathcal{Q}^2)$$

How strong is the skewness dependence?

in DIS: $q(x, \mathcal{Q}^2) = F(x, \eta=0, \Delta^2=0, \mathcal{Q}^2)$

How to get a realistic GPD model?

❖ lattice simulations of GPD moments (first few, heavy pion world) [QCDSF,LHPC,…]

❖ bag model [Ji et al.], quark soliton model [Göke et al,…], BS-equation [Miller,…], ….

❖ overlap of LC wave functions [Brodsky, Feldmann, Diehl, Hwang, Jakob, Kroll]

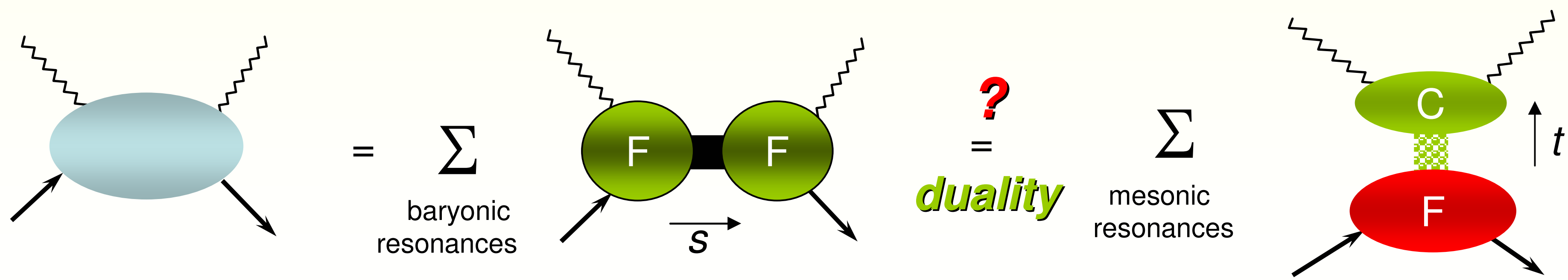
❖ models for amplitudes (perhaps better understanding as for GPDs)

- resumming *s*-channel resonances [Close, Zhao]

- vector dominance & Regge inspired description [Guidal et al., M. Capua et al., …]

s-channel contributions (resonance region, large *x*)

t-channel contributions (Regge phenomenology, small *x*)

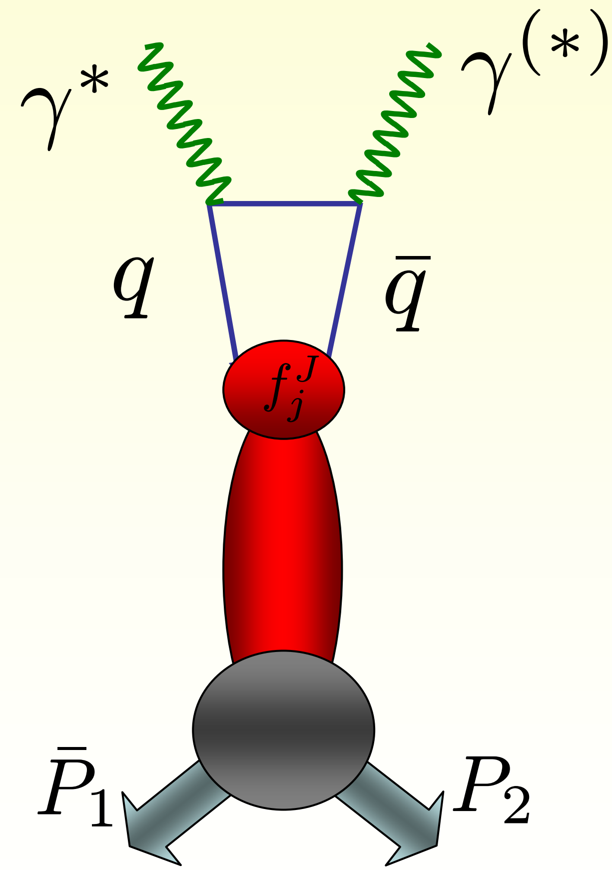


➡ take models (`knowledge') for the amplitude and extract GPDs

Ansatz for partonic partial wave amplitudes

We work in **conformal Mellin-space** and use **SO(3) *t*-channel partial waves**

$$F_j(\eta,\Delta^2,\mu^2) = \int_{-1}^{1} dx\, \eta^j C_j^{3/2}(x/\eta) F(x,\eta,\Delta^2,\mu^2) = \sum_J F_j^J(\Delta^2,\mu^2)\hat{d}_J(\eta)$$



- at short distance a quark/anti-quark state is produced, labeled by ***conformal spin*** *j+2*
- they form an intermediate mesonic state with total angular momentum ***J***
  strength of ***coupling*** is $f_j^J,\ J \leq j+1$
- mesons propagate with $\frac{1}{m^2(J)-t} \propto \frac{1}{J-\alpha(t)}$
- decaying into a nucleon anti-nucleon pair with given spin *S* and angular momentum *L*, described by an ***impact form factor***

$$F_j^J(\Delta^2) = \frac{f_j^J}{J-\alpha(\Delta^2)} \frac{1}{(1-\frac{\Delta^2}{M^2(J)})^p}$$

! *D*-term arises from the SO(3) partial wave *J=j+1* *(j → -1)*

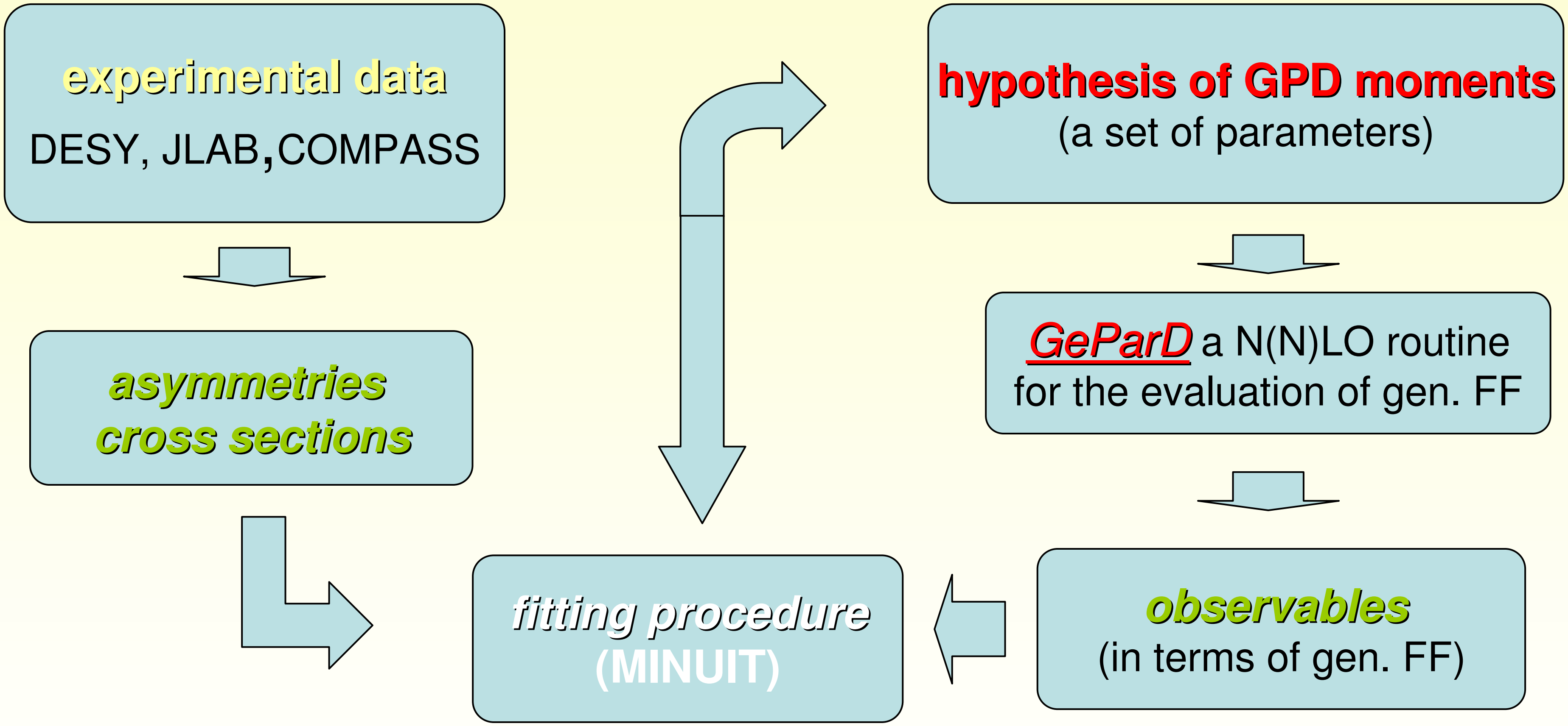
Perturbative and higher twist corrections

- ***perturbative next--to--leading order corrections*** [conformal approach D.M. (94)]
  - ✓ hard scattering part for photon/meson electroproduction [A. Belitsky, D.M. (00,01)
  - ✓ flavor singlet part for meson electroproduction [D. Ivanov, L. Szymanowski (04)]
  - ✓ for all then flavor singlet twist--two anomalous dimensions [A. Belitsky, D.M. (98)]
  - ✓ and flavor singlet twist--two evolution kernels [A. Belitsky, D.M., A. Freund (99,00)]
- ***evaluation of higher twist contributions***
  - ✓ completing the twist-three sector [A. Belitsky, D.M. (00)]
  - ❖ target mass corrections (twist-4) to photon electroproduction [A.Belitsky,D.M.(01)
  - ❖ WW-approximation to helicity flip DVCS contribution [N. Kivel, L. Mankiewicz (01)]

  o power suppressed corrections are not well understood
- ***perturbative next--to--next--to--leading order corrections to DVCS***
  [D.M. (05); K.Kumerićki, K.Passek-Kumerićki, D.M., A. Schäfer (06/07)]

Ready for a GPD fitting procedure?

[K. Kumerički, D.M., K. Passek-Kumerički, hep-ph/0703179]



partially *YES* but it is ***NOT*** completed yet:

- reasonable well motivated hypotheses of GPD moments must be implemented
- some technical, however, straightforward work is left
  (like a reevaluation of observables)

Lessons from DVCS fits for H1 and ZEUS data

DVCS cross section has been measured in the small $\xi = \mathcal{Q}^2/(2W^2 + \mathcal{Q}^2)$ region

$$40\mathrm{GeV} \lesssim W \lesssim 150\mathrm{GeV}, \quad 2\mathrm{GeV}^2 \lesssim \mathcal{Q}^2 \lesssim 80\mathrm{GeV}^2, \quad |t| \lesssim 0.8\mathrm{GeV}^2$$

and it is predicted by

$$\frac{d\sigma}{d\Delta^2}(W, \Delta^2, \mathcal{Q}^2) \approx \frac{4\pi\alpha^2}{\mathcal{Q}^4} \frac{W^2\xi^2}{W^2 + \mathcal{Q}^2} \left[|\mathcal{H}|^2 - \frac{\Delta^2}{4M_\mathrm{p}^2} |\mathcal{E}|^2 + \left|\widetilde{\mathcal{H}}\right|^2 \right] (\xi, \Delta^2, \mathcal{Q}^2) \Big|_{\xi = \frac{\mathcal{Q}^2}{2W^2 + \mathcal{Q}^2}}$$

suppressed contributions *<<0.05>>* relative *O(ξ)*

LO [Belitsky, DM, Kirchner (01), Guzey, Teckentrup (06)]

data are described within ***questionable*** *t*-slope parameters

NLO [Freund, M. McDermott (02)]

results strongly depend on used parton density parameterization

do a simultaneous fit to DIS and DVCS

Ansatz for conformal GPD moments

$$H_j^{\Sigma}(\eta, \Delta^2, \mu_0^2) = N_{\Sigma} \frac{B(1-\alpha_{\Sigma}(0)+j, 8)}{B(2-\alpha_{\Sigma}(0), 8)} \frac{1}{1-\frac{\Delta^2}{(m_j^{\Sigma})^2}} \frac{1}{\left(1-\frac{\Delta^2}{(M_j^{\Sigma})^2}\right)^3} + \mathcal{O}(\eta^2)$$

PD momentum fraction

PD Mellin moments

Regge inspired
t-dependence

impact form factor
(counting rules, lattice)

some simplifications in the ansatz:

- neglecting η dependence
- only designed for small *x* (no momentum sum rule, N_{Σ}, N_G free parameters)
- flavor non-singlet contribution is neglected (< 5% effect)
- fixed numbers of quarks $(n_f=4)$

parameters @ fixed input scale $Q^2 = 4\ GeV^2$

- 2x normalization *N*, 2x intercept α, 2x cut-off mass M_0
- little sensitivity of slope α' (=0.15/GeV^2)
- little sensitivity on *j*-dependence in M_j

order (scheme)	$\alpha_s(M_Z)$	N_Σ	$\alpha_\Sigma(0)$	M^2_Σ	N_G	$\alpha_G(0)$	M^2_G	χ^2	χ^2/d.o.f.	$\chi^2_{\Delta^2}$
LO	0.130	0.157	1.17	0.228	0.527	1.25	0.263	100	0.85	38.5
NLO ($\overline{\text{MS}}$)	0.116	0.172	1.14	1.93	0.472	1.08	4.45	109	0.92	4.2
NLO ($\overline{\text{CS}}$)	0.116	0.167	1.14	1.34	0.535	1.09	1.59	95	0.80	2.2
NNLO ($\overline{\text{CS}}$)	0.114	0.167	1.14	1.17	0.571	1.07	1.39	91	0.77	2.2

simultaneous NNLO fit to DVCS and DIS

LO & MS NLO fits are not optimal

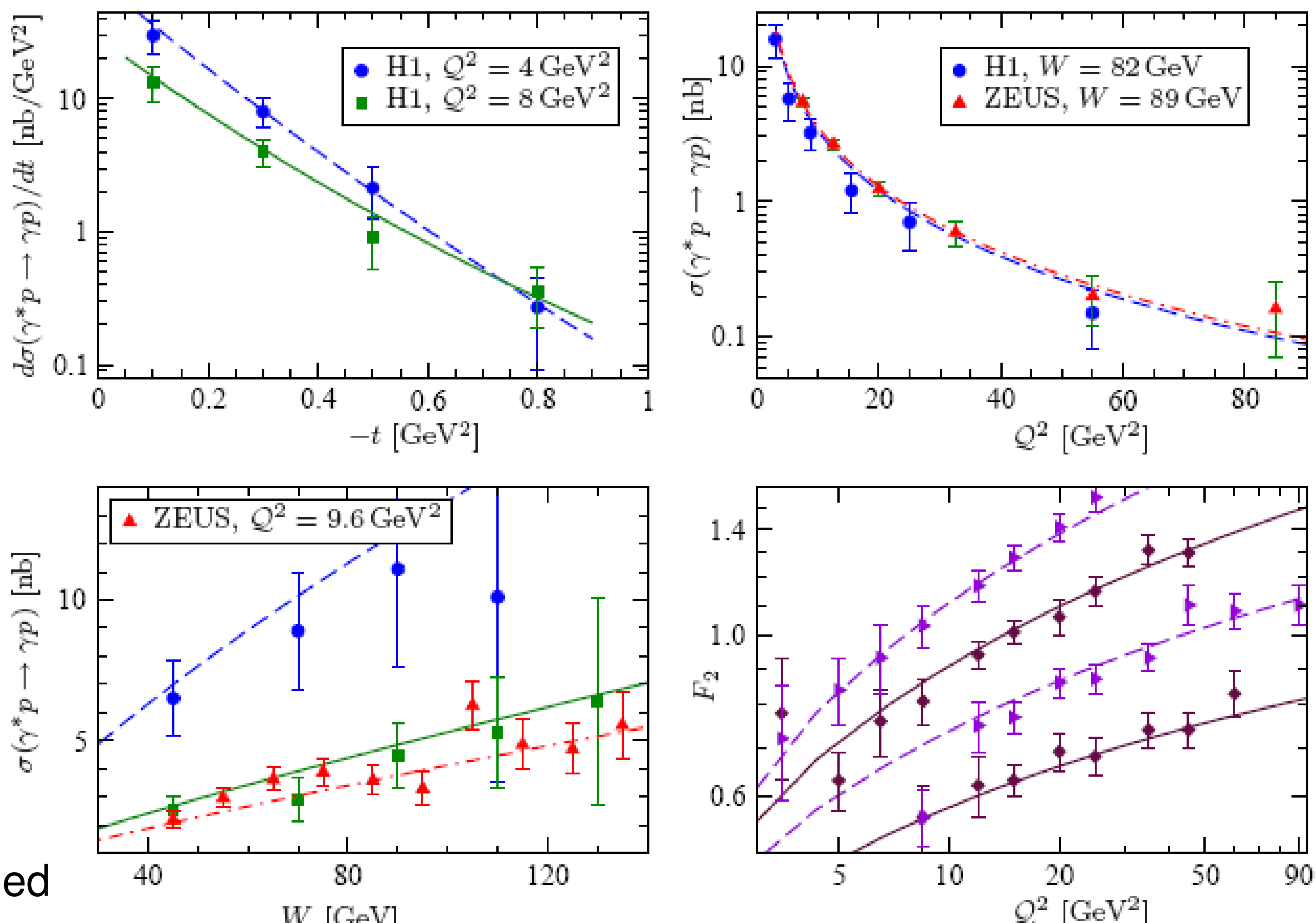
⇩

missing parameter

CS beyond LO yields good fits

⇩

neglecting η is justified ? just luck



Can one do better?

Yes, introduce a distribution of SO(3) partial waves in conformal GPD moments

toy example: take two partial waves

η dependence can be safely neglected

$$F_j(\eta,\Delta^2) = \frac{f_j^{j+1}}{(1-\frac{\Delta^2}{M^2(j+1)})^p}\left(\frac{1}{j+1-\alpha(\Delta^2)}\hat{d}_{j+1}(\eta) + \frac{s\,\eta^2}{j-1-\alpha(\Delta^2)}\hat{d}_{j-1}(\eta)\right)$$

effective relative strength of remaining partial waves

now we get a very good LO fit:

- fixed $s_G = 0$, $M = M_G = M_\Sigma$
- X^2/d.o.f. = 0.52, $s_\Sigma = -0.75$,
- *other parameters are consistent with previous fits*

  $N_\Sigma = 0.14$, $\alpha_\Sigma = 1.20$, $N_G = 0.8$, $\alpha_G = 1.16$
- $X^2_t = 2.61$, $M^2 = 0.86$

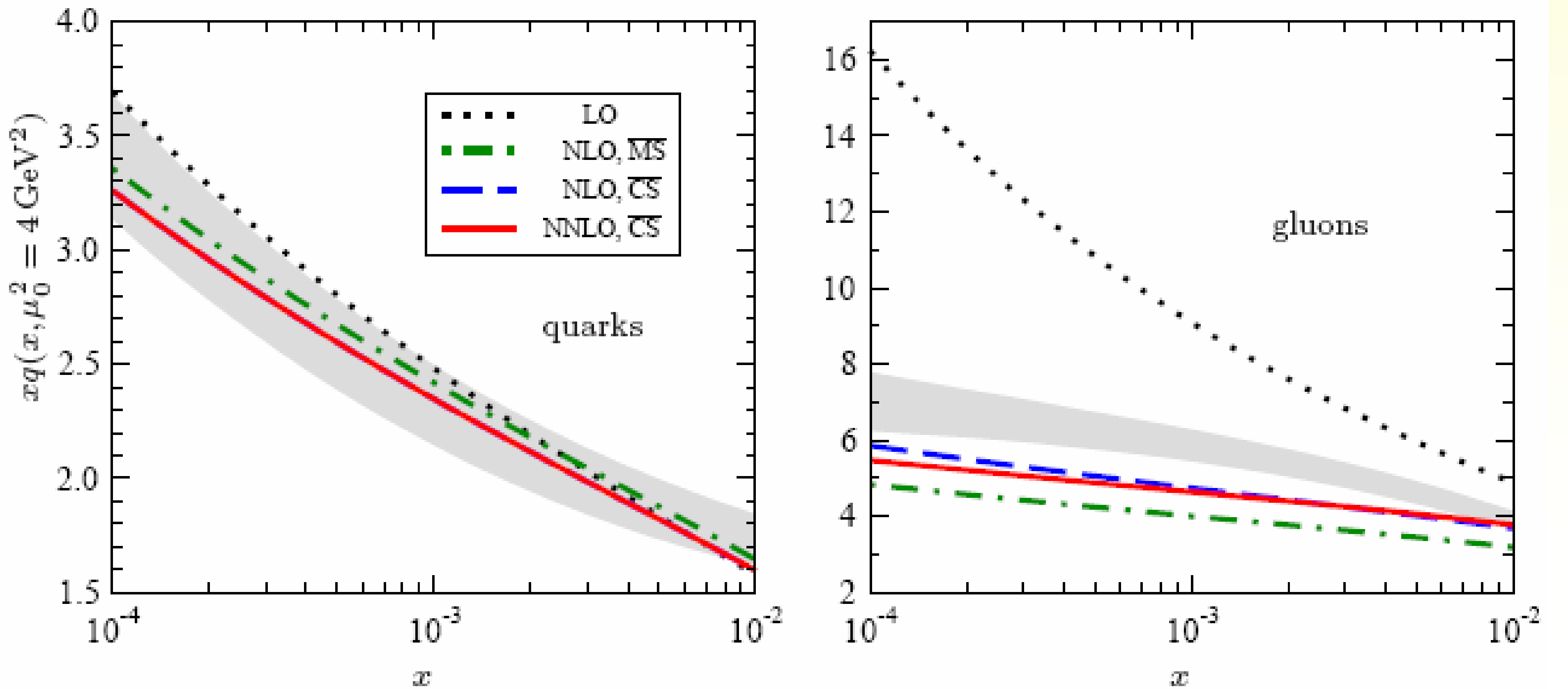
`negative' skewness dependence is ***required*** at LO

Partonic picture: longitudinal degrees

our fits are ***compatible*** with Alekhin's NLO PDF parameterization:

✓ central value of our quark densities lies in Alekhin's error band

✓ gluons are less constrained by DIS fit
(error bands would overlap)

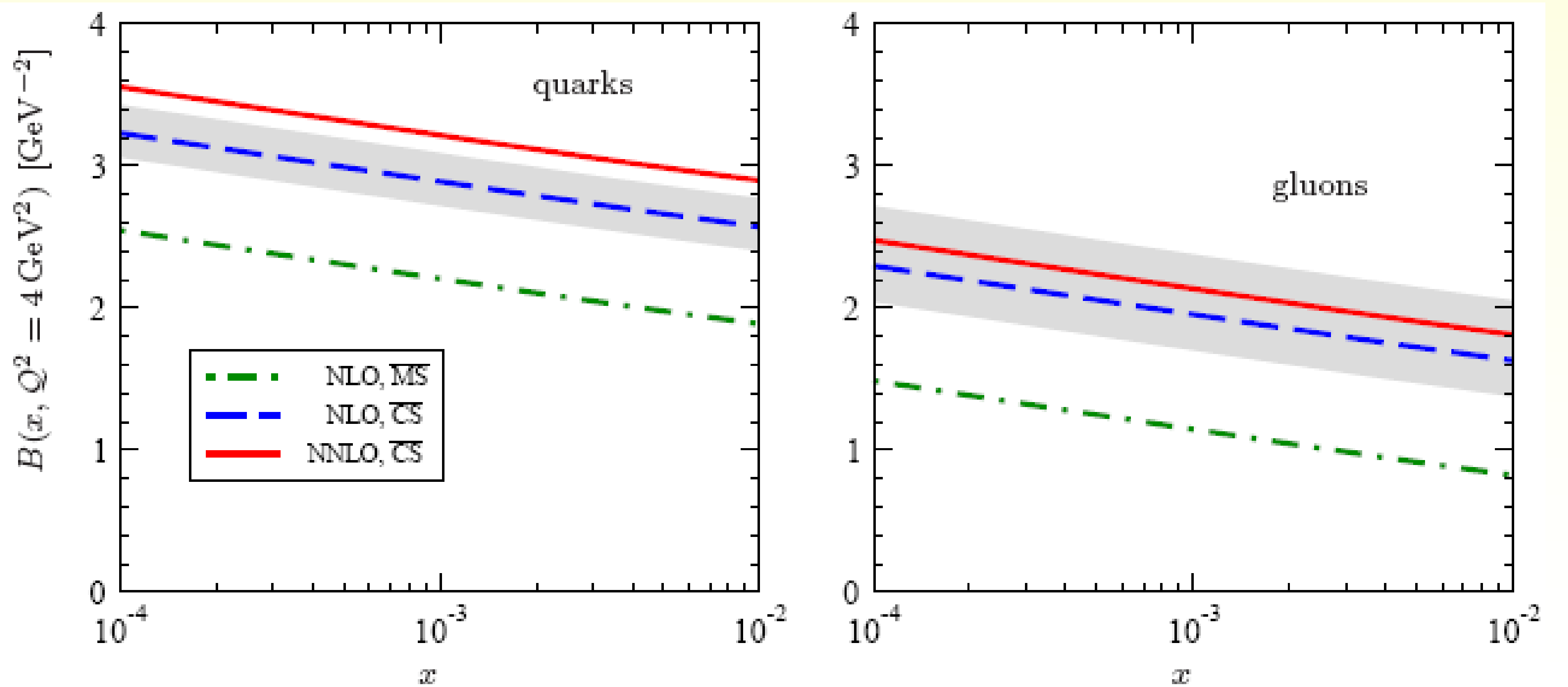


Partonic picture: transversal degrees

transversal distribution of partons in the infinite momentum frame:

$$H(x,\vec{b}) = \int \frac{d^2\vec{\Delta}}{(2\pi)^2}\, e^{-i\vec{b}\cdot\vec{\Delta}} H(x,\eta=0,\Delta^2=-\vec{\Delta}^2)$$

the average distance of partons is: $\langle \vec{b}^2\rangle(x,\mathcal{Q}^2) = \dfrac{\int d\vec{b}\,\vec{b}^2 H(x,\vec{b},\mathcal{Q}^2)}{\int d\vec{b}\, H(x,\vec{b},\mathcal{Q}^2)} = 4B(x,\mathcal{Q}^2)$



Conclusions

- useful to consider GPDs as overlap of wave functions

→ a tool to probe the wave functions of nucleon, hadrons, and nuclei

→ this point of view allows

  i. to connect uninegrated parton densities and GPDs
  ii. yields the question for the appropriate parameterization of wave functions

- only "realistic" GPD parameterizations provide insight into the proton

  - tomography -- 3D picture (realistic to do at present/future)
  - angular momentum of partons (a very long way)

- dual parameterization of GPDs based on *t*-channel exchanges formulated in conformal Mellin space

✓ parameterization of all degrees of freedoms of GPDs

✓ numeric is ***fast*** and ***reliable*** [even at NLO for MS scheme]

✓ perturbative expansion in DVCS ***works*** – except for ***evolution*** at ***small x***

✓ fitting procedure (better than comparing model A, B, ..., with data) ***can be set up***

✓ a `global' analysis of GPD related data ***requires NLO***