Lattice results on GPDs and TMDs

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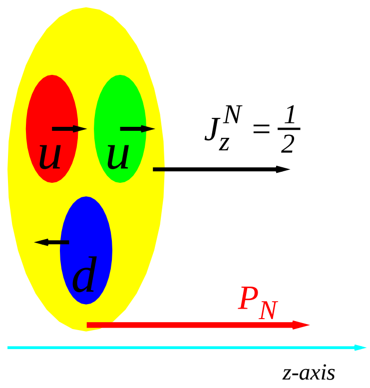
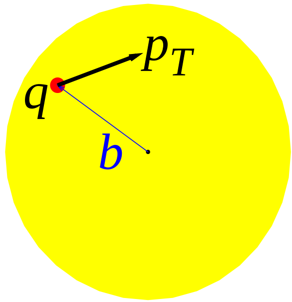
QCDSF Collaboration

GPDs (generalised parton distributions)
TMDs (transverse momentum dependent parton distribution functions)

describe (complementary aspects of) hadron structure





naive picture of a proton
with large momentum $P_N \to \infty$

proton moving towards us

pictures from H. Avakian et al., arXiv:1008.1921

low-energy (long-distance) quantities $\to$ not accessible in perturbation theory

calculation in models, lattice QCD , . . .

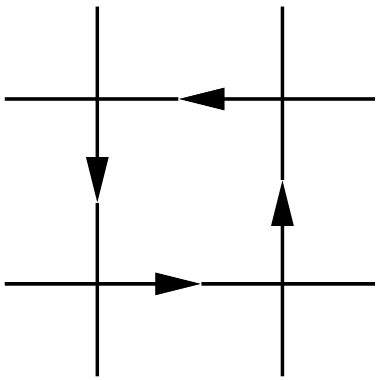
Plan of the talk

- Lattice QCD
- TMDs
- Lattice results for TMDs
- GPDs
- Lattice results for GPDs
  Distributions in impact parameter space, transverse spin structure, quark angular momentum in the nucleon

Lattice QCD

starting point: path-integral formulation of quantum field theory

Minkowski space-time $\rightarrow$ Euclidean space-time continuum $\rightarrow$ hypercubic lattice
with lattice constant a
(non-perturbative regularisation)
classical statistical mechanics
in 4 dimensions



gauge fields (gluons):
parallel transporters $U(x, \mu) \in$ SU(3) on links
fermion fields (quarks):
(Grassmann-valued) Dirac spinors ψ, $\bar{\psi}$ on lattice sites
integrated out analytically

$\rightarrow$ high-dimensional integral over gauge fields (link variables)
evaluated by Monte Carlo methods

expectation value of the observable A:

$$\langle A \rangle = Z^{-1} \int \mathrm{D}U\, \mathrm{D}\bar{\psi}\, \mathrm{D}\psi\, A(U, \bar{\psi}, \psi) \mathrm{e}^{-S_{\text{gauge}}[U] - \bar{\psi} M[U] \psi} \quad , \quad \mathrm{D}U = \prod_{x,\mu} \mathrm{d}U(x, \mu)$$

with the partition function $Z = \int \mathrm{D}U\, \mathrm{D}\bar{\psi}\, \mathrm{D}\psi\, \mathrm{e}^{-S_{\text{gauge}}[U] - \bar{\psi} M[U] \psi}$

$S_{\text{quarks}} = \bar{\psi} M[U] \psi$ with the (huge) fermion matrix M

(carrying position (x), spinor and colour indices)

integration over the (Grassmann-valued) fermion fields $\rightarrow \det M$, M^{-1}

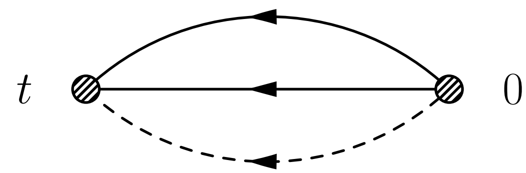
$$Z = \int \mathrm{D}U \underbrace{\det M[U]\, \mathrm{e}^{-S_{\text{gauge}}[U]}}_{\mathrm{e}^{-S_{\text{eff}}[U]}}$$

$$\langle A \rangle = Z^{-1} \int \mathrm{D}U\, (M[U]^{-1})_{ab} (M[U]^{-1})_{cd} \cdots \mathrm{e}^{-S_{\text{eff}}[U]}$$

↑ ↑ quark propagators

compute $\langle A \rangle$ by Monte Carlo methods, e.g., correlation functions of (composite) fields interpolating field for the proton: $$B_\alpha(t,\mathbf{p}) = \sum_{x,\, x_4=t} \mathrm{e}^{-\mathrm{i}\mathbf{p}\cdot\mathbf{x}} \epsilon_{ijk} u^i_\alpha(x) u^j_\beta(x) (C^{-1}\gamma_5)_{\beta\gamma} d^k_\gamma(x)$$

proton 2-point function $\langle B(t)\bar{B}(0)\rangle$ pictorially:



rewritten in the operator formalism for a lattice of time extent T:

$$\langle B(t)\bar{B}(0)\rangle = \frac{\mathrm{Tr}\left(\mathrm{e}^{-H(T-t)} B \mathrm{e}^{-Ht}\bar{B}\right)}{\mathrm{Tr}\,\mathrm{e}^{-HT}} \overset{T\to\infty}{=} \langle 0|B\mathrm{e}^{-Ht}\bar{B}|0\rangle \overset{t\to\infty}{=} \langle 0|B|N\rangle \mathrm{e}^{-E_N t}\langle N|\bar{B}|0\rangle + \cdots$$

(Hilbert space) traces Tr result from (anti)periodic boundary conditions in time for the fields

dependence of 2-point functions on (Euclidean) time $\rightarrow$ energies (masses)

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3-point function ($t > \tau > 0$)

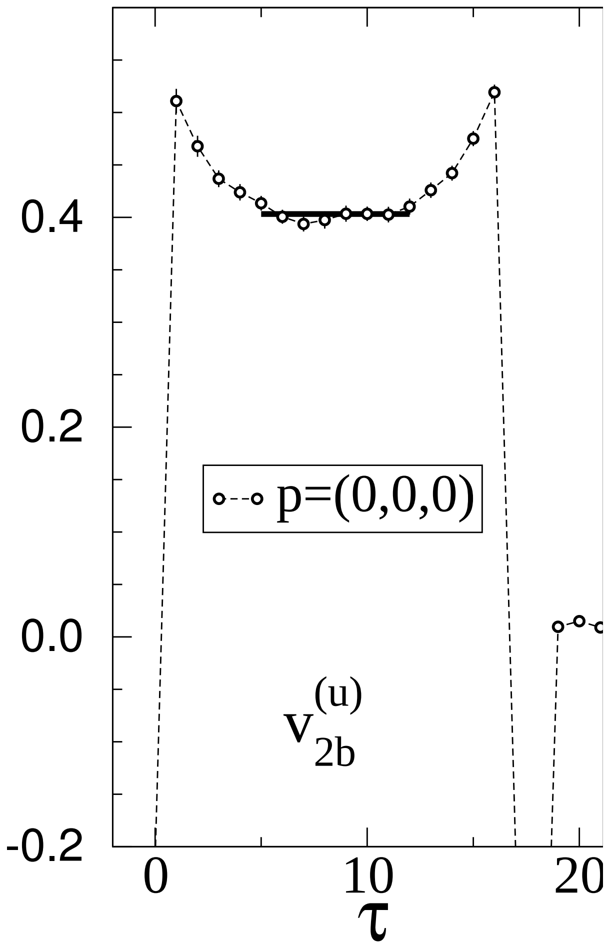
$$\langle B(t)\mathcal{O}(\tau)\bar{B}(0)\rangle \overset{T\to\infty}{=} \langle 0|B\mathrm{e}^{-H(t-\tau)}\mathcal{O}\mathrm{e}^{-H\tau}\bar{B}|0\rangle$$
$$= \langle 0|B|N\rangle \mathrm{e}^{-E_N(t-\tau)}\langle N|\mathcal{O}|N\rangle \mathrm{e}^{-E_N\tau}\langle N|\bar{B}|0\rangle + \cdots$$
$$= \langle 0|B|N\rangle \mathrm{e}^{-E_N t}\langle N|\bar{B}|0\rangle \langle N|\mathcal{O}|N\rangle + \cdots$$

$\rightarrow$ ratios 3-point-function/2-point function yield matrix elements

Some raw data

bare ratios $R = \frac{\langle B(t)\mathcal{O}(\tau)\bar{B}(0)\rangle}{\langle B(t)\bar{B}(0)\rangle} = \langle N|\mathcal{O}|N\rangle + \cdots$

for $\langle x\rangle^{(u)} = v_2^{(u)}$ (forward matrix element)



$\beta = 5.4$, $\kappa = 0.1356$, $t/a = 17$

horizontal line: fit to the data

τ in lattice units

τ, $t-\tau$ large enough?

Systematic problems

bare lattice results (matrix elements) $\rightarrow \rightarrow \rightarrow$ value to be compared with experiment

- renormalisation (and mixing)
  perturbative $\leftrightarrow$ nonperturbative
- finite size effects
  volume large enough?
- chiral extrapolation (in m_π)
  quark masses in the simulations larger than in reality
- continuum extrapolation
  lattice spacing small enough?
- flavour singlet quantities difficult
  (quark-line) disconnected contributions (closed quark loops) hard to evaluate accurately

TMDs

based on work by B.U. Musch, Ph. Hägler, A. Schäfer, D.B. Renner, J.W. Negele
Europhys. Lett. 88 (2009) 61001; arXiv:0811.1536; arXiv:0907.2381

TMDs: transverse momentum dependent parton distribution functions
defined in terms of forward matrix elements between nucleon states $|P,S\rangle$

$$\tilde{\Phi}_\Gamma(z;P,S) = \langle P,S|\bar{q}(z)\Gamma\mathcal{U}_{\mathcal{C}(z,0)}q(0)|P,S\rangle$$

$\mathcal{U}_{\mathcal{C}(z,0)}$: Wilson line $\quad z \leftrightarrow l$

through
$$\Phi_\Gamma(x,\mathbf{k}_\perp;P,S) = \int \mathrm{d}(\bar{n}\cdot k)\int \frac{\mathrm{d}^4 z}{2(2\pi)^4}\mathrm{e}^{-\mathrm{i}k\cdot z}\langle P,S|\bar{q}(z)\Gamma\mathcal{U}_{\mathcal{C}(z,0)}q(0)|P,S\rangle$$

n, $\bar{n}$: light-cone vectors with $n\cdot\bar{n}=1$, $P = P^+\bar{n} + \frac{m_N^2}{2P^+}n$

parametrisation for $\Gamma_V^\mu = \gamma^\mu$, $\Gamma_A^\mu = \gamma^\mu\gamma_5$, $\Gamma_T^{\mu\nu} = \mathrm{i}\sigma^{\mu\nu}\gamma_5$ in terms of distributions $f(x,\mathbf{k}_\perp^2)$, ...

$$n_\mu\Phi_V^\mu = f_1 + \frac{\mathbf{S}_i\epsilon_{\perp ij}\mathbf{k}_j}{m_N}f_{1T}^\perp \quad , \quad n_\mu\Phi_A^\mu = \Lambda g_1 + \frac{\mathbf{k}_\perp\cdot\mathbf{S}_\perp}{m_N}g_{1T}$$

Λ: nucleon helicity

$$n_\mu\Phi_T^{\mu j} = -\mathbf{S}_j h_1 - \frac{\epsilon_{\perp ji}\mathbf{k}_i}{m_N}f_1^\perp - \frac{\Lambda\mathbf{k}_j}{m_N}h_{1L}^\perp - \frac{(2\mathbf{k}_j\mathbf{k}_i - \mathbf{k}_\perp^2\delta_{ji})\mathbf{S}_i}{2m_N^2}h_{1T}^\perp$$

relation between TMDs and ordinary parton distributions?

at least formally for the unpolarised (f_1), polarised (g_1) and transversity (h_1) distributions:

$$f_1(x) = \int \mathrm{d}^2 k_\perp \, f_1(x, \mathbf{k}_\perp^2) \qquad \text{etc.}$$

subtle issue: choice of the path $\mathcal{C}(z,0)$ in the Wilson line $\mathcal{U}_{\mathcal{C}(z,0)}$

here: straight path at equal times ($z^0 = 0$)

matrix elements $\tilde{\Phi}_\Gamma(z; P, S) = \langle P, S | \bar{q}(z) \Gamma \mathcal{U}_{\mathcal{C}(z,0)} q(0) | P, S \rangle$

$\rightarrow$ 8 independent amplitudes $\tilde{A}_i(z^2, z \cdot P)$

$$\begin{aligned}
\tilde{\Phi}_V^\mu &= 4P^\mu \tilde{A}_2 + 4\mathrm{i} m_N^2 z^\mu \tilde{A}_3 \\
\tilde{\Phi}_A^\mu &= -4m_N S^\mu \tilde{A}_6 - 4\mathrm{i} m_N P^\mu z \cdot S \tilde{A}_7 + 4m_N^3 z^\mu z \cdot S \tilde{A}_8 \\
\tilde{\Phi}_T^{\mu\nu} &= 4S^{[\mu} P^{\nu]} \tilde{A}_{9m} + 4\mathrm{i} m_N^2 S^{[\mu} z^{\nu]} \tilde{A}_{10} - 2m_N^2 \left[2z \cdot S z^{[\mu} P^{\nu]} - z^2 S^{[\mu} P^{\nu]} \right] \tilde{A}_{11}
\end{aligned}$$

TMDs f_1, g_1, ... $\leftrightarrow$ amplitudes $\tilde{A}_2$, $\tilde{A}_6$, ... via Fourier integrals

$z \cdot P$ conjugate to x, $\mathbf{z}_\perp$ conjugate to $\mathbf{k}_\perp$

Lattice results for TMDs

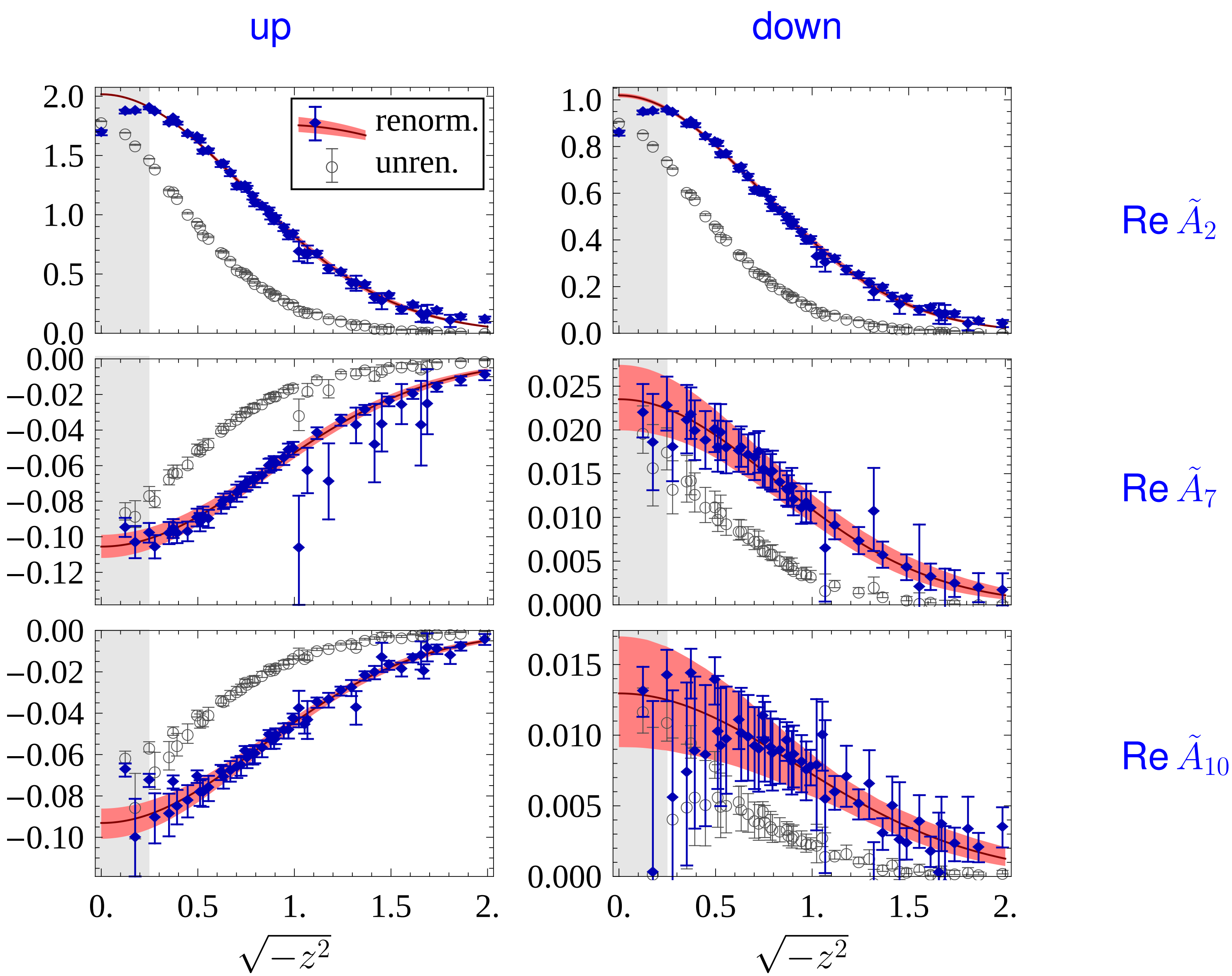
- Wilson line: product of connected link variables (parallel transporters)
  oblique angles: zig-zag path
- renormalisation nontrivial
- only quark-line connected diagrams considered (no problem for isovector quantities)
- MILC gauge configurations: $m_\pi \approx 500\,\mathrm{MeV}$, $m_N = 1.291(23)\,\mathrm{GeV}$, $a = 0.124\,\mathrm{fm}$

straight path at equal times
→ purely spatial quark separations $z^2 < 0$, $|z \cdot P| \leq \sqrt{-z^2}|\mathbf{P}|$
→ full dependence on x and $\mathbf{k}_\perp$ ($z \cdot P$ and z^2) cannot be studied

here: focus on the z^2 dependence
present results for the lowest x moments corresponding to $z \cdot P = 0$, e.g.

$$f_1^{(0)}(\mathbf{k}_\perp^2) = \int_{-1}^{1} \mathrm{d}x\, f_1(x, \mathbf{k}_\perp^2) = \int \frac{\mathrm{d}^2 z_\perp}{(2\pi)^2}\, \mathrm{e}^{\mathrm{i}\mathbf{z}_\perp \cdot \mathbf{k}_\perp}\, 2\tilde{A}_2(-\mathbf{z}_\perp^2, 0)$$

representative results for Re $\tilde{A}_{2,7,10}(-\mathbf{z}_\perp^2, 0)$ with Gaussian fits $\tilde{A}(-\mathbf{z}_\perp^2, 0) = c e^{-\mathbf{z}_\perp^2/\sigma^2}$



define $\mathbf{k}_\perp^2$ moments: $$f^{(0,n)} = \int \mathrm{d}^2 k_\perp \left(\frac{\mathbf{k}_\perp^2}{2m_N^2}\right)^n f^{(0)}(\mathbf{k}_\perp^2)$$ etc.

	c	$2/\sigma$ (GeV)	$\sigma/2$(fm)
$\tilde{A}_2^u$	$2.0159(86) = f_{1,u}^{(0,0)}$	$0.3741(72)$	0.527
$\tilde{A}_2^d$	$1.0192(90) = f_{1,d}^{(0,0)}$	$0.3839(78)$	0.514
$\tilde{A}_6^u$	$-0.920(35) = -g_{1,u}^{(0,0)}$	$0.311(11)$	0.634
$\tilde{A}_6^d$	$0.291(19) = -g_{1,d}^{(0,0)}$	$0.363(18)$	0.544
$\tilde{A}_{9m}^u$	$0.931(29) = h_{1,u}^{(0,0)}$	$0.3184(90)$	0.620
$\tilde{A}_{9m}^d$	$-0.254(16) = h_{1,d}^{(0,0)}$	$0.327(15)$	0.603
$\tilde{A}_7^u$	$-0.1055(66) = -g_{1T,u}^{(0,1)}$	$0.328(14)$	0.602
$\tilde{A}_7^d$	$0.0235(38) = -g_{1T,d}^{(0,1)}$	$0.346(36)$	0.570
$\tilde{A}_{10}^u$	$-0.0931(73) = h_{1L,u}^{\perp(0,1)}$	$0.340(14)$	0.580
$\tilde{A}_{10}^d$	$0.0130(40) = h_{1L,d}^{\perp(0,1)}$	$0.301(48)$	0.656

- $g_{1,u-d}^{(0,0)} = 1.209(36)$ close to the physical $g_A = 1.2695(29)$
  in agreement (within errors) with a direct calculation
- $g_{1T}^{(0,1)} \sim -h_{1L}^{\perp(0,1)}$ as in (some) quark models

$\mathbf{k}_\perp$ densities of longitudinally (L) and transversely (T) polarised quarks in form of a multipole expansion

$$\rho_L = \frac{1}{2}\left(f_1 + \lambda\Lambda g_1 + \frac{\mathbf{S}_j \epsilon_{ji} \mathbf{k}_i}{m_N} f_{1T}^\perp + \lambda \frac{\mathbf{k}_\perp \cdot \mathbf{S}_\perp}{m_N} g_{1T}\right)$$

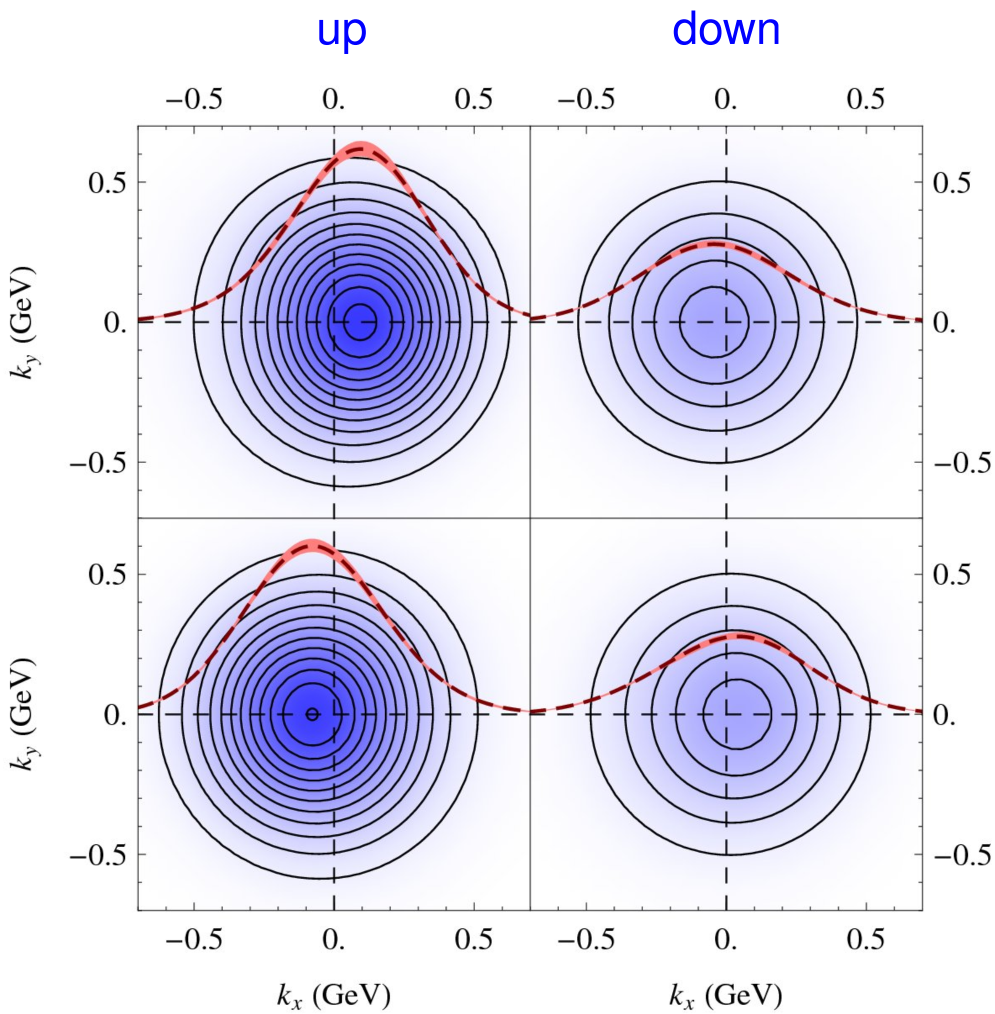
$$\rho_T = \frac{1}{2}\left(f_1 + \mathbf{s}_\perp \cdot \mathbf{S}_\perp h_1 + \frac{\mathbf{s}_j \epsilon_{ji} \mathbf{k}_i}{m_N} h_1^\perp + \Lambda \frac{\mathbf{k}_\perp \cdot \mathbf{s}_\perp}{m_N} h_{1L}^\perp + \frac{\mathbf{s}_j (2\mathbf{k}_j \mathbf{k}_i - \mathbf{k}_\perp^2 \delta_{ji}) \mathbf{S}_i}{2m_N^2} h_{1T}^\perp\right)$$

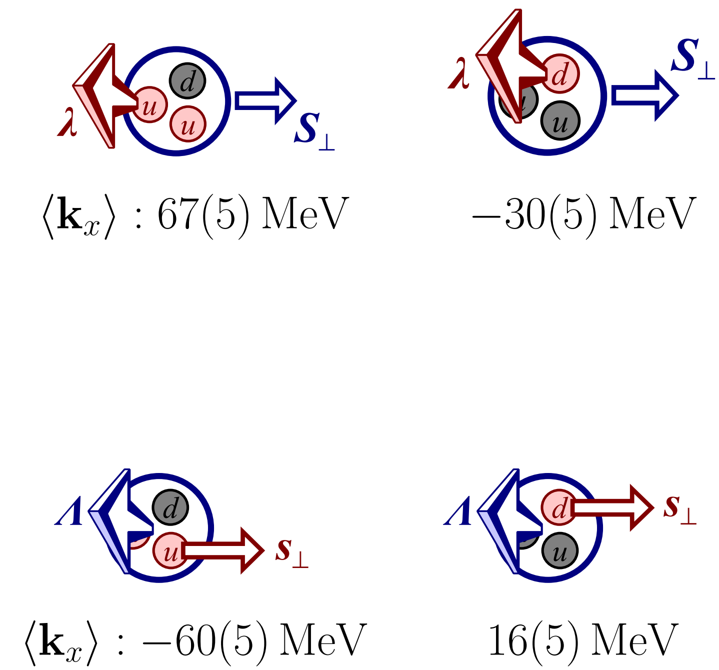
λ: quark helicity, Λ: nucleon helicity

- monopole terms $\propto f_1, g_1, h_1$
- dipole terms $\propto f_{1T}, g_{1T}, h_1^\perp, h_{1L}^\perp$
- quadrupole term $\propto h_{1T}^\perp$

terms $\propto f_{1T}^\perp$ and $\propto h_1^\perp$ (proportional to the T-odd Sivers and Boer-Mulders functions) absent for straight Wilson lines

M. Diehl, Ph. Hägler, Eur. Phys. J. C44 (2005) 87





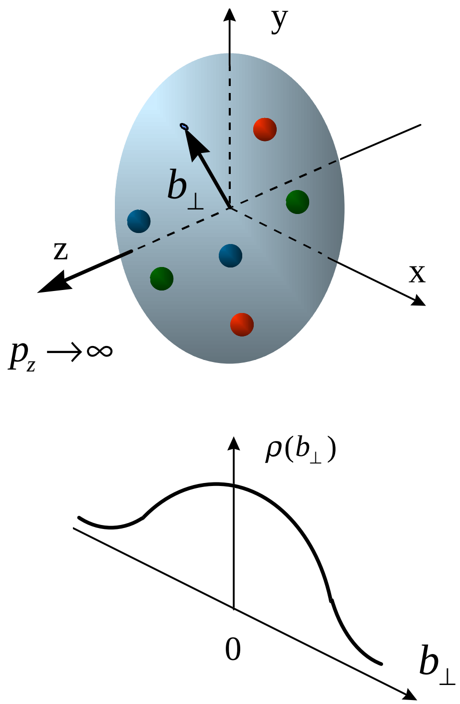
lowest x moment of ρ_L for $\lambda = 1$, $\mathbf{S}_\perp = (1, 0)$
lowest x moment of ρ_T for $\Lambda = 1$, $\mathbf{s}_\perp = (1, 0)$

density profile at $\mathbf{k}_y = 0$ as a function of $\mathbf{k}_x$

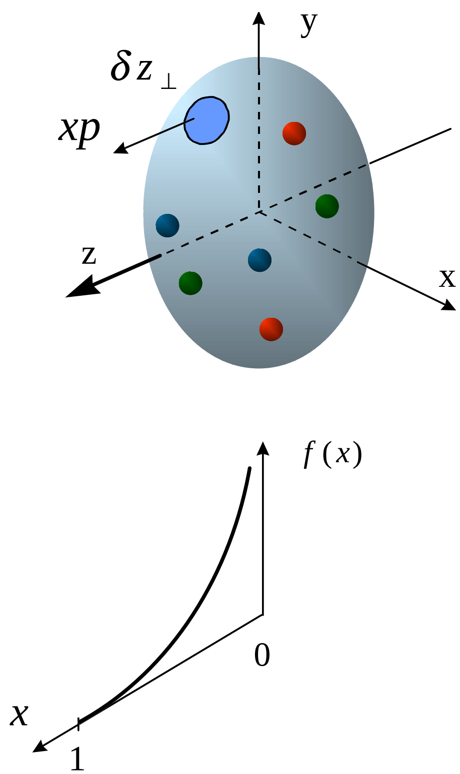
$\mathbf{k}_\perp$ shifts orthogonal to the dipole deformations of densities in impact parameter space

Generalised parton distributions (GPDs)

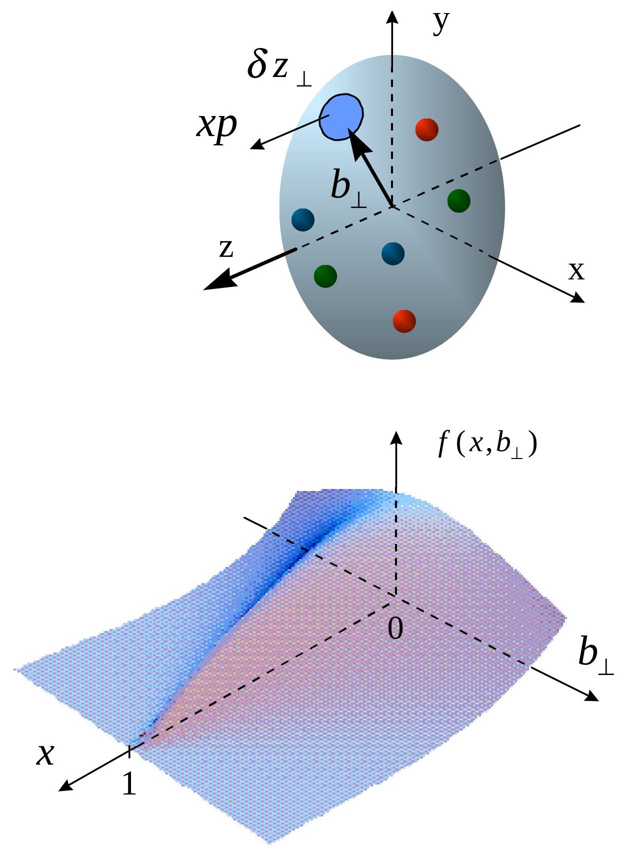
form factor



PDF



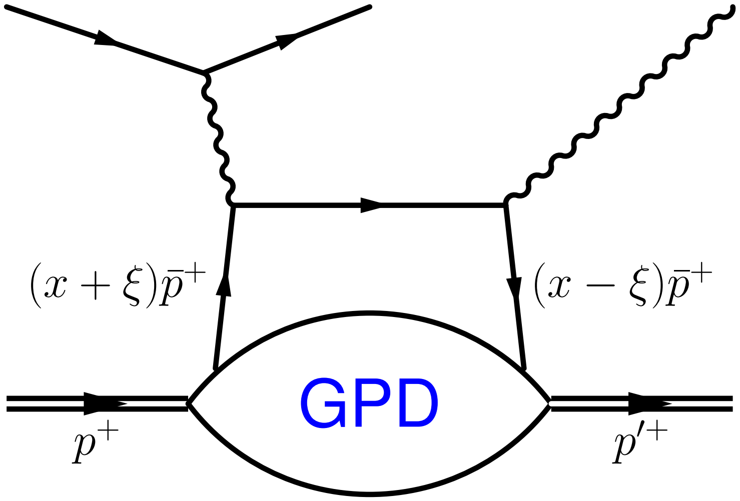
GPD at $\xi = 0$



pictures by Dieter Müller

for $\xi = 0$: probabilistic interpretation in impact parameter space (M. Burkardt)

Formal definition of GPDs



Wilson line ↓

$$\int \frac{\mathrm{d}\lambda}{2\pi} \mathrm{e}^{\mathrm{i}\lambda x} \langle p' | \bar{q}(-\tfrac{1}{2}\lambda n)\, \not{n}\, \mathcal{U} q(\tfrac{1}{2}\lambda n) | p \rangle$$
$$= H_q(x,\xi,t)\, \bar{u}(p')\, \not{n} u(p)$$
$$+ E_q(x,\xi,t)\, \bar{u}(p') \frac{\mathrm{i}\sigma^{\mu\nu} n_\mu \Delta_\nu}{2m_N} u(p)$$

$\bar{p} = \frac{1}{2}(p'+p)$, $\Delta = p' - p$, n: light-like vector with $\bar{p}\cdot n = 1$, $\xi = -n\cdot\Delta/2$, $t = \Delta^2$

(dependence on renormalisation scale suppressed)

ordinary parton distributions
electromagnetic form factors } special cases

for example: $H_q(x,0,0) = \begin{cases} q(x) & \text{for } x > 0 \\ -\bar{q}(-x) & \text{for } x < 0 \end{cases}$

moments w.r.t. x in terms of generalised form factors (GFFs) A, B, C:

$$\int_{-1}^{1} \mathrm{d}x\, x^{n-1} H_q(x,\xi,t) = \sum_{i=0}^{\left[\frac{n-1}{2}\right]} A^q_{n,2i}(t)(-2\xi)^{2i} + \mathrm{Mod}(n+1,2) C^q_n(t)(-2\xi)^n$$

$$\int_{-1}^{1} \mathrm{d}x\, x^{n-1} E_q(x,\xi,t) = \sum_{i=0}^{\left[\frac{n-1}{2}\right]} B^q_{n,2i}(t)(-2\xi)^{2i} - \mathrm{Mod}(n+1,2) C^q_n(t)(-2\xi)^n$$

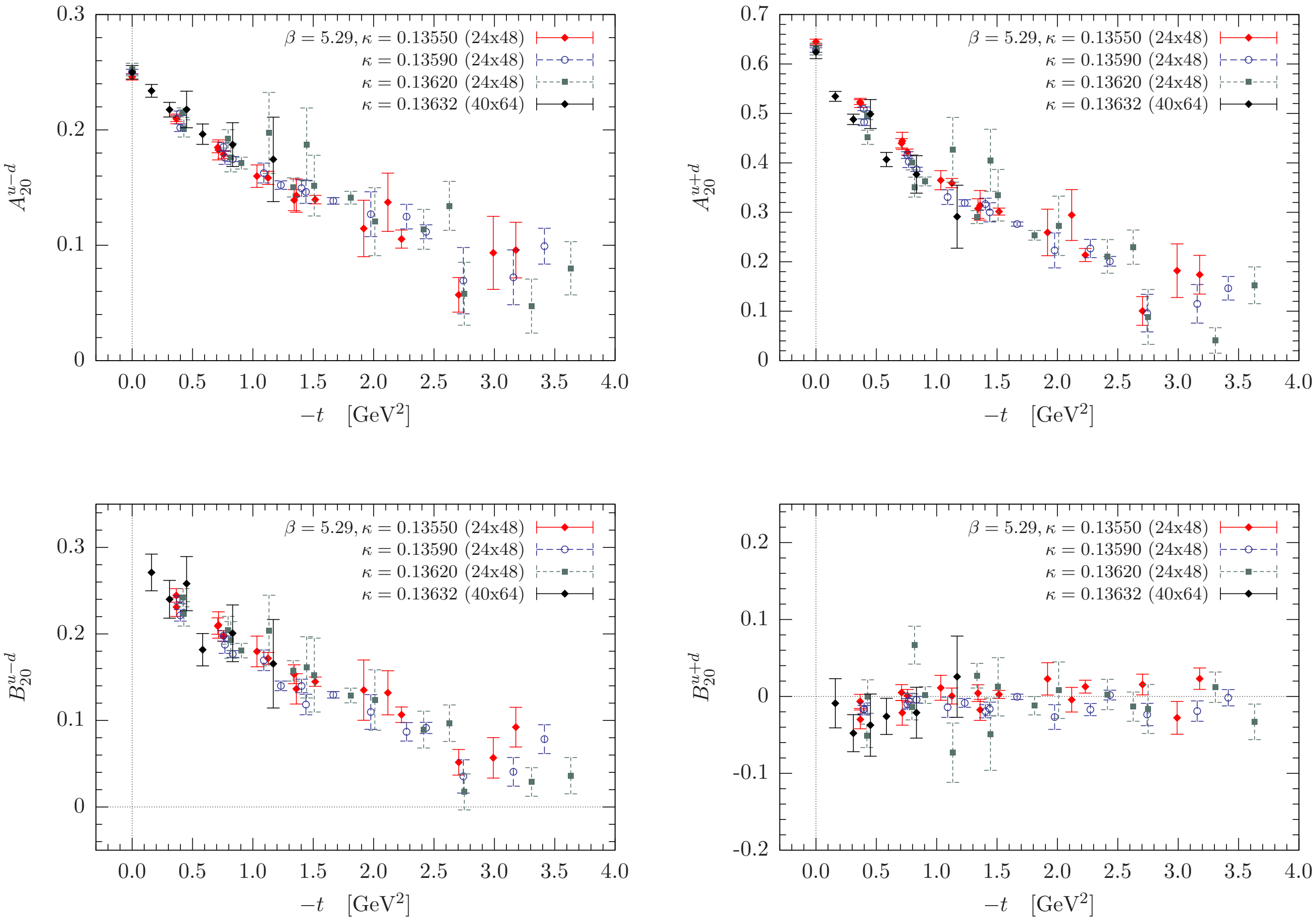
GFFs from matrix elements of local (twist 2) operators (momentum transfer $\Delta = p' - p \neq 0$)

$$\begin{aligned} \langle p'|\mathcal{O}^q_{(\mu_1\cdots\mu_n)}|p\rangle = {} & \bar{u}(p')\gamma_{(\mu_1} u(p) \sum_{i=0}^{\left[\frac{n-1}{2}\right]} A^q_{n,2i}(t) \Delta_{\mu_2}\cdots\Delta_{\mu_{2i+1}} \overline{p}_{\mu_{2i+2}}\cdots\overline{p}_{\mu_n)} \\ & - \frac{\bar{u}(p')\mathrm{i}\Delta^\alpha \sigma_{\alpha(\mu_1} u(p)}{2m_N} \sum_{i=0}^{\left[\frac{n-1}{2}\right]} B^q_{n,2i}(t) \Delta_{\mu_2}\cdots\Delta_{\mu_{2i+1}} \overline{p}_{\mu_{2i+2}}\cdots\overline{p}_{\mu_n)} \\ & + C^q_n(t)\mathrm{Mod}(n+1,2)\frac{1}{m_N}\bar{u}(p')u(p)\Delta_{(\mu_1}\cdots\Delta_{\mu_n)} \end{aligned}$$

with $\mathcal{O}^q_{\mu_1\cdots\mu_n} = (\mathrm{i}/2)^{n-1}\ \bar{q}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n} q$

analogous: $\mathcal{O}^{q,5}_{\mu_1\cdots\mu_n} = (\mathrm{i}/2)^{n-1}\ \bar{q}\gamma_{\mu_1}\gamma_5\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n} q$ and $(\mathrm{i}/2)^{n-1}\ \bar{q}\mathrm{i}\sigma_{\lambda\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n} q$

Lattice results for GPDs: distributions in impact parameter space



QCDSF results

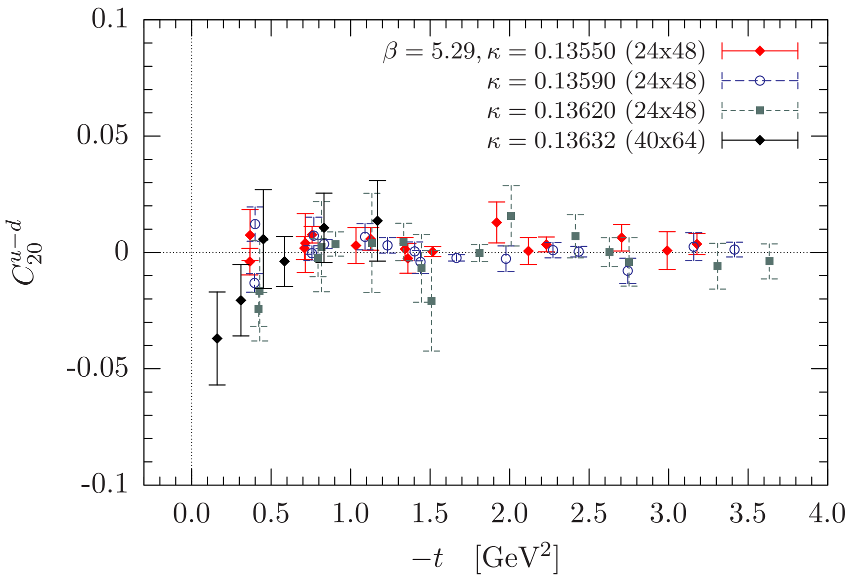
$\overline{\text{MS}}$ scheme
at $\mu = 2\,\text{GeV}$

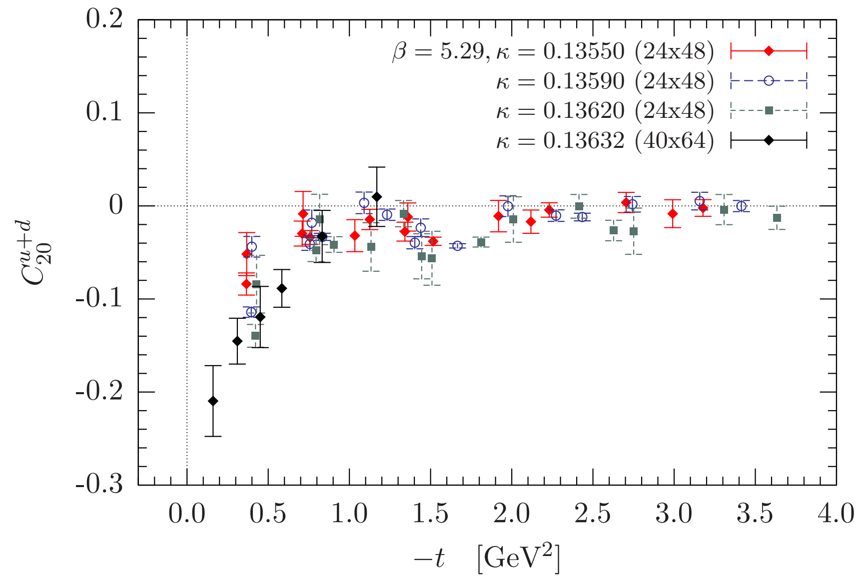
$m_\pi \approx 860\,\text{MeV}$
$m_\pi \approx 630\,\text{MeV}$
$m_\pi \approx 410\,\text{MeV}$
$m_\pi \approx 270\,\text{MeV}$

$a \approx 0.075\,\text{fm}$

$$\int_{-1}^{1} \mathrm{d}x\, x H_q(x,\xi,t) = A_{20}^q(t) + 4\xi^2 C_2^q(t)$$

$$\int_{-1}^{1} \mathrm{d}x\, x E_q(x,\xi,t) = B_{20}^q(t) - 4\xi^2 C_2^q(t)$$

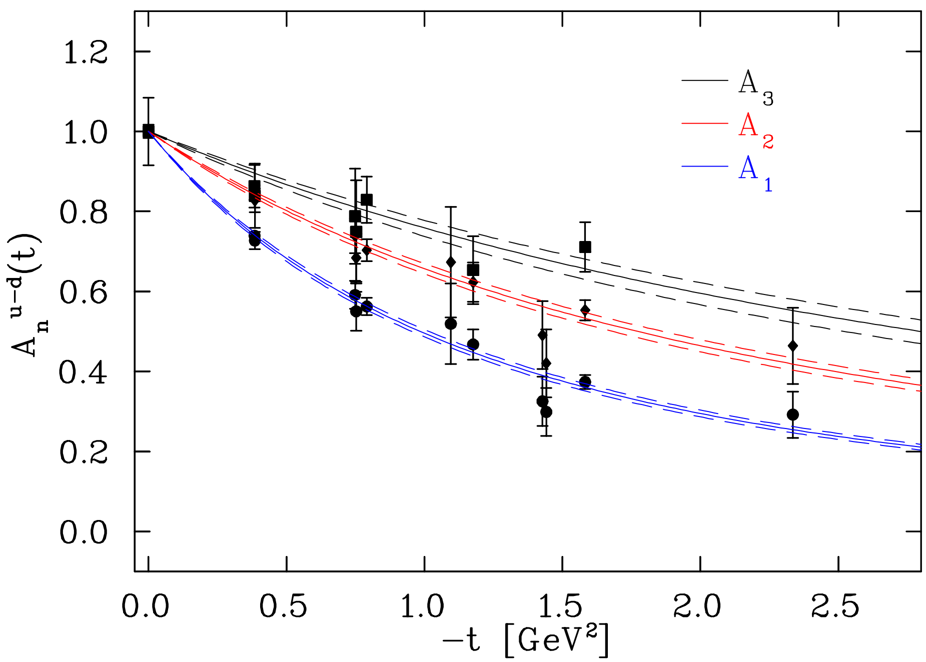




$$\int_{-1}^{1} \mathrm{d}x\, x H_q(x,\xi,t) = A_{20}^q(t) + 4\xi^2 C_2^q(t)$$

$$\int_{-1}^{1} \mathrm{d}x\, x E_q(x,\xi,t) = B_{20}^q(t) - 4\xi^2 C_2^q(t)$$

GFFs $A_{10}^{u-d} = F_1^{u-d}(t)$, A_{20}^{u-d}, A_{30}^{u-d} (non-singlet), normalised to unity at $t = 0$



$\beta = 5.4$, $\kappa = 0.1350$
$24^3 \times 48$ lattice

dipole fit:

$$A_{n0}(t) = \frac{A_{n0}(0)}{(1 - t/M_n^2)^2} = \frac{\langle x^{n-1} \rangle}{(1 - t/M_n^2)^2}$$

form factor $A_{n0}(t)$ flattens as n grows
$\leftrightarrow$ dipole mass M_n grows with n

$$\int_{-1}^{1} \mathrm{d}x\, x^{n-1} H_q(x, \xi = 0, t) = A_{n0}^q(t)$$

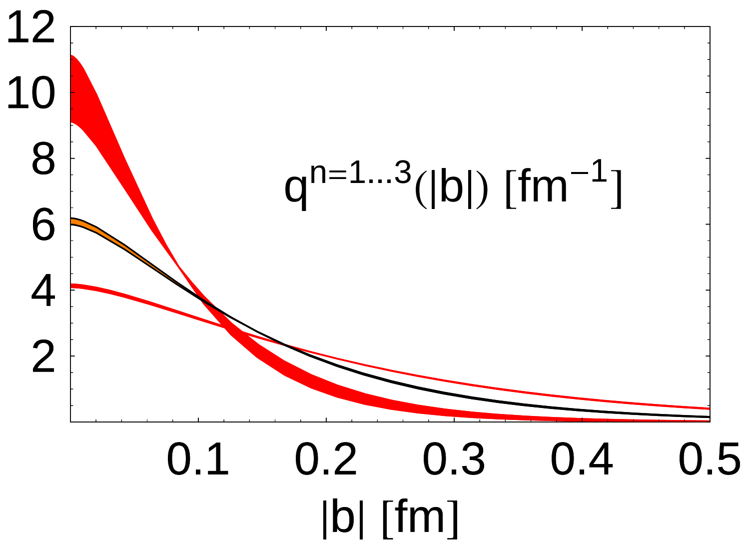
H_q (as a function of t) becomes wider as x grows

$$q(x, \mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\, \mathrm{e}^{\mathrm{i}\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp} H_q(x, 0, -\mathbf{\Delta}_\perp^2)$$

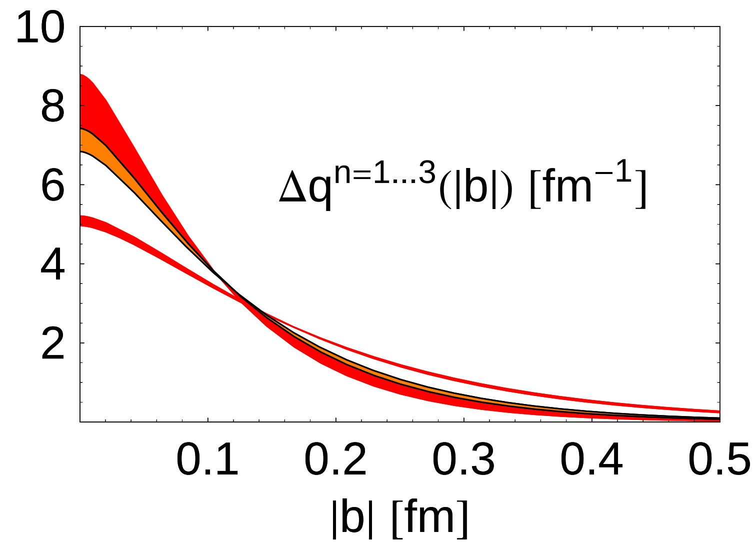
q (as a function of $\mathbf{b}_\perp$) becomes narrower as x grows (as expected)

lowest three moments of $H(x, \xi = 0, t)$ and $\tilde{H}(x, \xi = 0, t)$
Fourier transform to impact parameter space
with the help of the dipole ansatz extrapolated linearly to the chiral limit:

$$\int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp} \int_{-1}^{1} dx\, x^{n-1} H_q(x, 0, -\mathbf{\Delta}_\perp^2)$$
$$= \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp} \frac{A_{n0}^q(0)}{(1 + \mathbf{\Delta}_\perp^2 / M_n^2)^2} = \int_{-1}^{1} dx\, x^{n-1} q(x, \mathbf{b}_\perp)$$



H



$\tilde{H}$

larger n corresponds to a narrower distribution

flavour $u - d$

M. G. et al., Eur. Phys. J. A32 (2007) 445 [hep-lat/0609001]

Lattice results for GPDs: transverse spin structure

what about the GPDs (GFFs) connected with the tensor operators $(\mathrm{i}/2)^{n-1}\ \bar{q}\mathrm{i}\sigma_{\lambda\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n}q$?

together with the vector operators $(\mathrm{i}/2)^{n-1}\ \bar{q}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n}q$

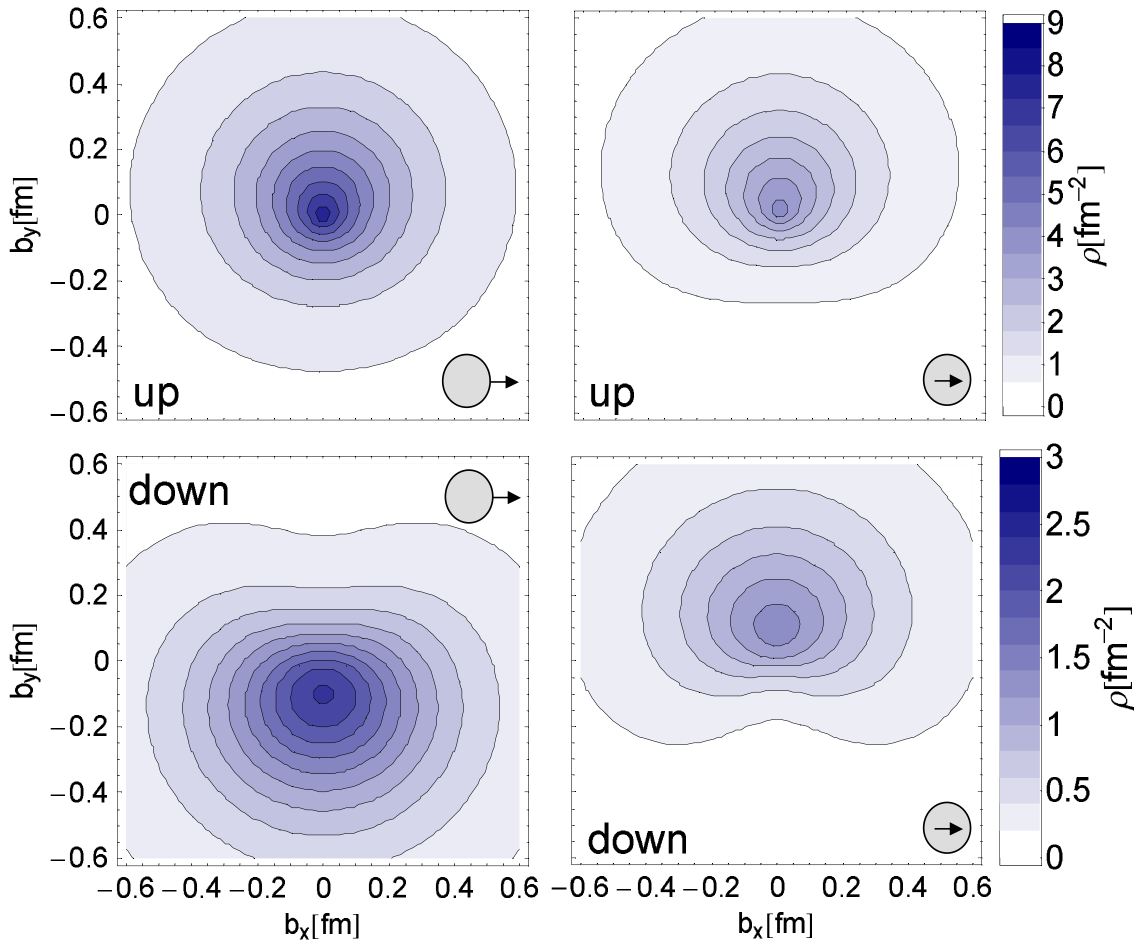
$\rightarrow$ (moments of) the density of transversely polarised quarks in a transversely polarised nucleon in impact parameter space

M. Diehl, Ph. Hägler, Eur. Phys. J. C44 (2005) 87

$$
\begin{aligned}
&\int_{-1}^{1}\mathrm{d}x\, x^{n-1}\rho(x,\mathbf{b}_\perp,\mathbf{s}_\perp,\mathbf{S}_\perp) \\
&= \frac{1}{2}\left\{A_{n0}(b_\perp^2) + s_\perp^i S_\perp^i\left(A_{Tn0}(b_\perp^2) - \frac{1}{4m_N^2}\Delta_{b_\perp}\widetilde{A}_{Tn0}(b_\perp^2)\right)\right. \\
&\left.+\frac{b_\perp^j\epsilon^{ji}}{m_N}\left(S_\perp^i B_{n0}'(b_\perp^2) + s_\perp^i\overline{B}_{Tn0}'(b_\perp^2)\right) + s_\perp^i(2b_\perp^i b_\perp^j - b_\perp^2\delta^{ij})S_\perp^j\frac{1}{m_N^2}\widetilde{A}_{Tn0}''(b_\perp^2)\right\}
\end{aligned}
$$

$\mathbf{s}_\perp$: transverse spin of the quark $\quad$ $\mathbf{S}_\perp$: transverse spin of the nucleon

unpolarised quark in a $\perp$ polarised nucleon: only contributions from vector operators
$\perp$ polarised quark in an unpolarised nucleon: also contributions from tensor operators



QCDSF/UKQCD,
PRL 98 (2007) 222001

(gen.) dipole parametrisation
+ linear chiral extrapolation

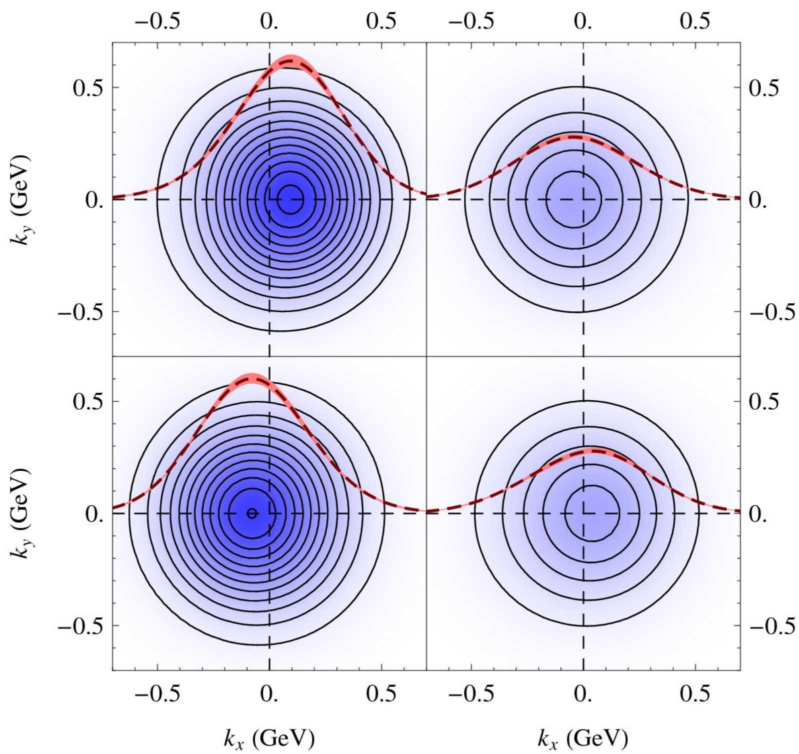
x^0 moment ($q - \bar{q}$)
quark spins $\leftrightarrow$ inner arrows
nucleon spins $\leftrightarrow$ outer arrows

transversely polarised quarks
in an unpolarised nucleon:
distortion in positive y-direction
for u and d quarks

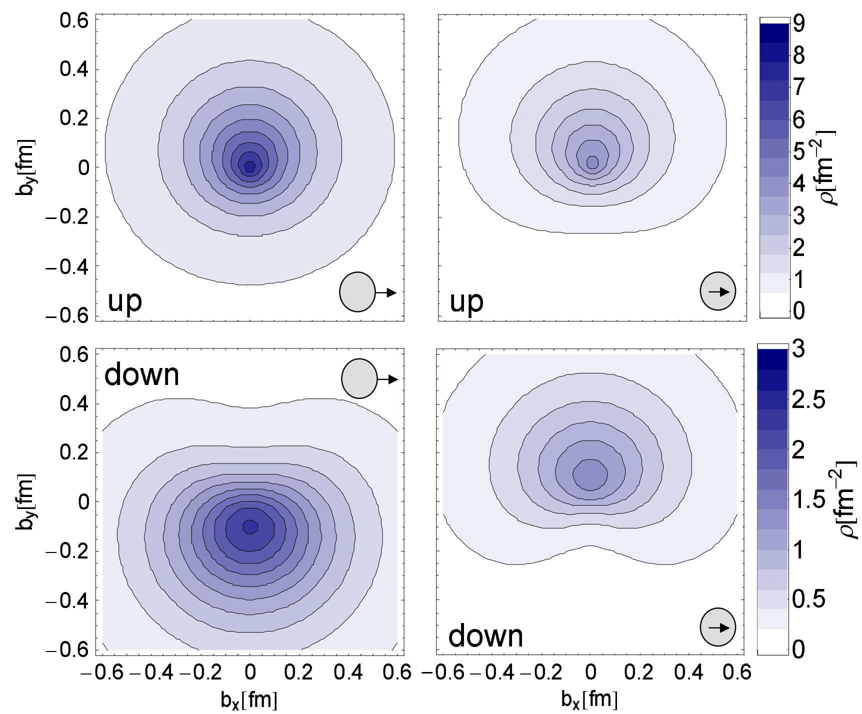
$\downarrow ?$

unpolarised quark
in a polarised nucleon:
distortion $\stackrel{?}{\longrightarrow}$ Sivers effect

sizable negative Boer-Mulders function for u and d quarks
(correlation of quark $\perp$ momentum and the $\perp$ quark spin)
M. Burkardt, Phys. Rev. D72 (2005) 094020



$\mathbf{k}_\perp$ shifts



orthogonal to the
dipole deformations of densities
in impact parameter space

Lattice results for GPDs: quark angular momentum in the nucleon

Ji's sum rule for the total angular momentum of quarks of flavour q in the nucleon:

$$J_q = \frac{1}{2}\int_{-1}^{1} \mathrm{d}x\, x\, (H_q(x,\xi,0) + E_q(x,\xi,0)) = \frac{1}{2}\left(A_{20}^q(t=0) + B_{20}^q(t=0)\right)$$

quark spin contribution to the nucleon spin:

$$S_q = \frac{1}{2}\int_{-1}^{1} \mathrm{d}x\, \tilde{H}_q(x,\xi,0) = \frac{1}{2}\tilde{A}_{10}^q(t=0) = \frac{1}{2}\Delta q$$

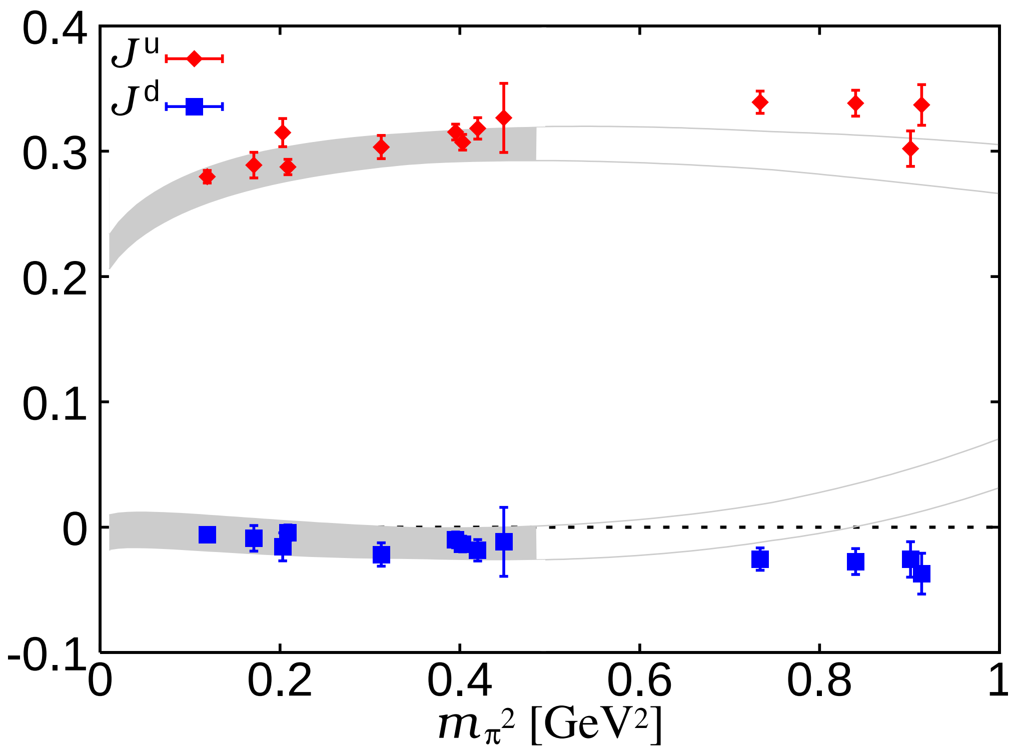
quark orbital angular momentum: $L_q = J_q - S_q$

difficult problem:

- disconnected contributions (not yet included)
- $B_{20}^q(t=0)$ requires an extrapolation from $t \neq 0$ to the forward limit
- chiral extrapolation and finite size corrections

for GFFs at vanishing momentum transfer t

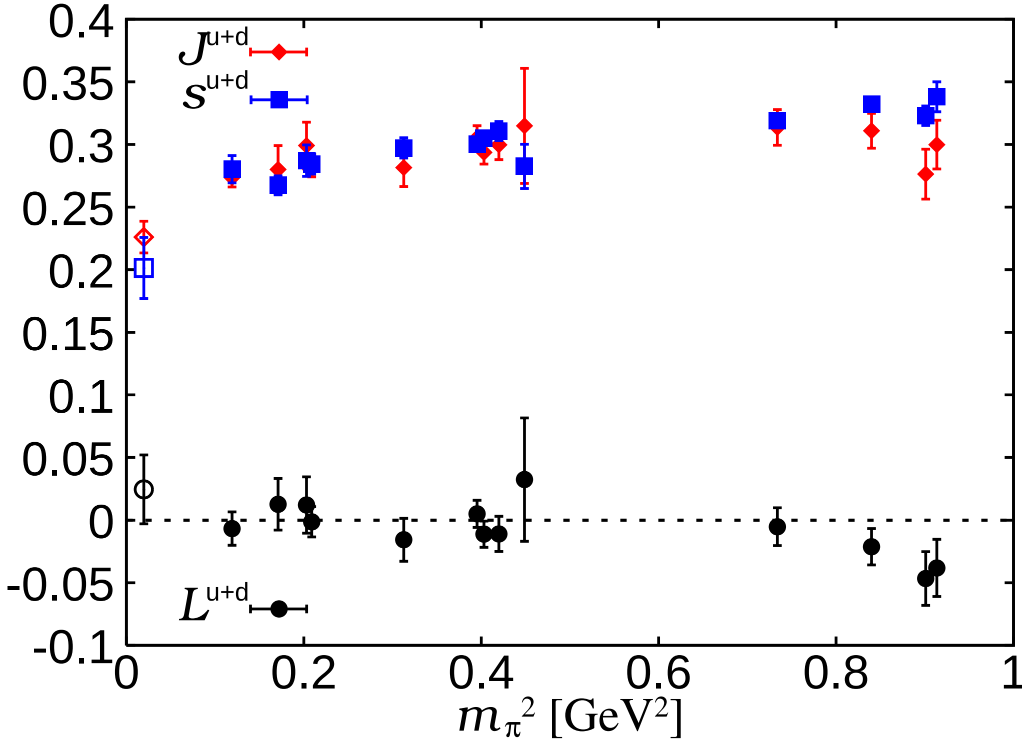
- heavy-baryon chiral perturbation theory
  e.g., M. Diehl, A. Manashov, A. Schäfer, Eur. Phys. J. A31 (2007) 335
- covariant chiral perturbation theory in the baryon sector
  e.g., M. Dorati, T.A. Gail, T.R. Hemmert, Nucl. Phys. A798 (2008) 96



total angular momentum of quarks in the nucleon with χPT fit

QCDSF-UKQCD, arXiv:0710.1534

note: $J_d \approx 0$



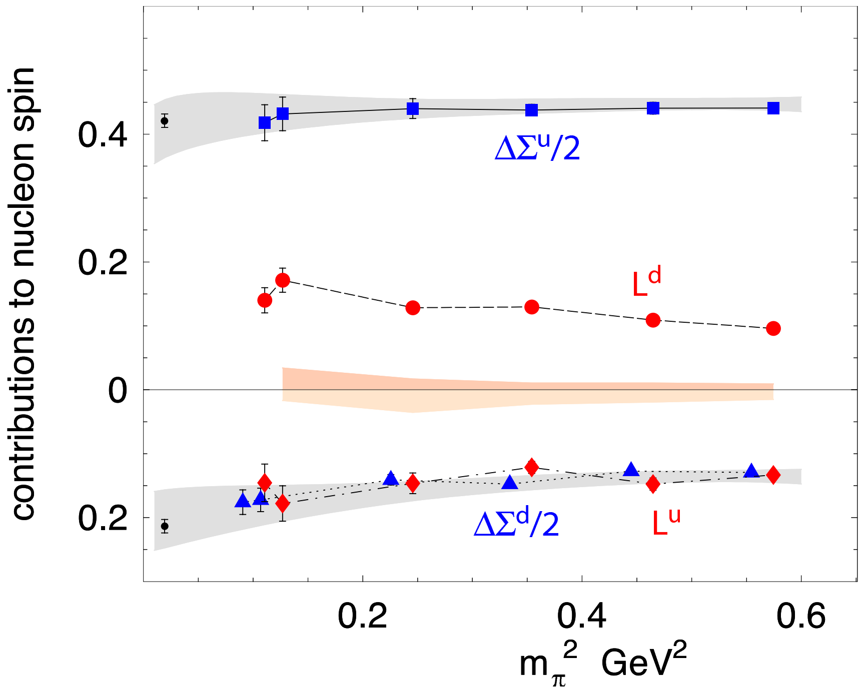
spin and orbital angular momentum of quarks in the nucleon

QCDSF-UKQCD, arXiv:0710.1534

note: $L_u + L_d \approx 0$

open symbols: extrapolated values at the physical pion mass

LHPC: Ph. Hägler et al., Phys. Rev. D77 (2008) 094502



grey bands:

(preliminary) chiral extrapolations

brown bands:

errors for L_q from the extrapolation in t

stars:

experimental results from HERMES 2007

similar findings as QCDSF: signs of $\frac{1}{2}\Delta\Sigma^q = S_q$ and L_q opposite

$$J_d = L_d + S_d \approx 0$$

$L_u + L_d \approx 0$ in strong disagreement with relativistic quark models

strong scale dependence? lattice data at a scale of $4\,\mathrm{GeV}^2$!

Summary and outlook

- first steps towards a three-dimensional picture of the nucleon!
- lattice simulations $\rightarrow$ quantities that are hard to obtain otherwise
- input from other fields important, e.g., chiral perturbation theory

ongoing developments:

- smaller (through twisted boundary conditions) and larger (through a variational analysis) momenta in (generalised) form factors
- disconnected contributions from all-to-all propagators
- larger lattices to cope with finite size effects
- smaller quark masses to make the chiral extrapolation more reliable
- continuum extrapolation remains difficult
- simulations with $n_f = 2 + 1(+1)$ dynamical quarks

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