# Hard meson electroproduction.

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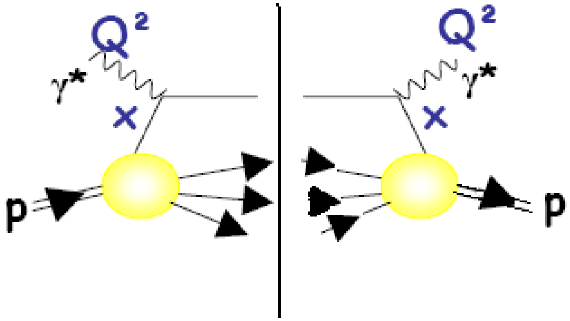
- Introduction : Generalized Parton Distributions (GPDs).
- GPDs & meson leptoproduction .
- Model for GPDs.
- Modified PA
- Cross section for vector meson production in a wide energy range.
- Results for $A_{UT}$ asymmetry at HERMES and COMPASS
- Results on $\pi$ mesons production.

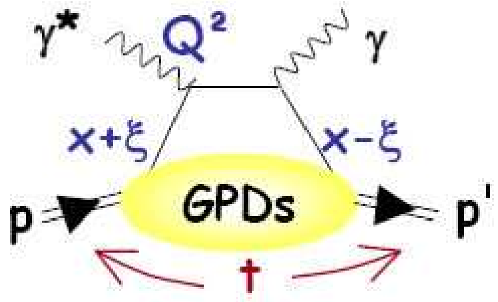
# DIS and DVCD

- Deep Inelastic scattering




Cross section -
expressed in terms of
ordinary parton
distributions $q(x)$

- Deeply Virtual Compton Scattering

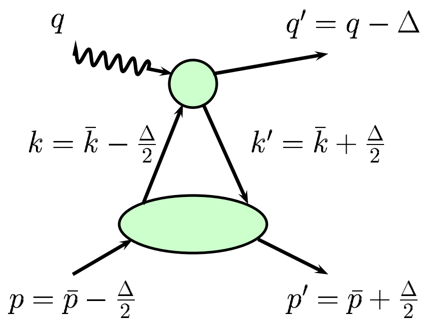



Amplitude - proportional to
Generalized Parton
Distributions
GPDs $H(x, \xi, t)$

# Handbag factorization of Mesons production amplitude

- Large $Q^2$- factorization into a hard meson photoproduction off partons, and GPDs. (LL )

Radyushkin, Collins,Frankfurt Strikman




$$k = ((\overline{x} + \xi)p_+, ....)$$

$$k' = ((\overline{x} - \xi)p_+, ....)$$

$$k \neq k'; \;\; \xi \sim \frac{x_B}{2 - x_B}$$

$L \to L$ transition - predominant. Other amplitudes are suppressed as powers $1/Q$

The process of VM production

- $\phi$ production (gluon&strange sea)
- $\rho$, $\omega$ production (gluon&sea&valence quarks)
- Charge mesons- transition GPDs

## GPDs – extensive information about hadron structure.

- Ordinary parton distribution connected with GPDs

$$H(x,0,0) = xg(x)$$

........................

- Hadron Form factors –are the GPDs moment

$$\int dx H(x,\xi,t) = F(t)$$

........................

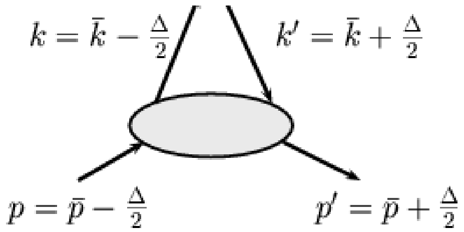
- Information on the parton angular momenta from Ji sum rules

$$\int x dx (H^q(x,\xi,0) + E^q(x,\xi,0)) = 2J^q$$

# Generalized Parton Distributions

D.Mueller, 1994; Ji, 1997; Radyushkin, 1997

$$\xi = \frac{(p-p')^+}{(p+p')^+} \sim \frac{x_b}{2}, \qquad \overline{x} = \bar{k}^+/\bar{p}^+\,, \qquad t$$




$$\langle p'\nu'| \sum_{a,a'} A^{a\rho}(0)\, A^{a'\rho'}(\bar{z})|p\nu\rangle \propto \int_0^1 \frac{d\overline{x}}{(\overline{x}+\xi-i\varepsilon)(\overline{x}-\xi+i\varepsilon)}\, \mathrm{e}^{-i(\overline{x}-\xi)p\cdot\bar{z}}$$

$$\times \left\{ \frac{\bar{u}(p'\nu')\, \not{n}\, u(p\nu)}{2\bar{p}\cdot n}\, H^g(\overline{x},\xi,t) + \frac{\bar{u}(p'\nu')\, i\,\sigma^{\alpha\beta}\, n_\alpha \Delta_\beta\, u(p\nu)}{4m\,\bar{p}\cdot n}\, E^g(\overline{x},\xi,t) \right.$$

$$+ \left. \frac{\bar{u}(p'\nu')\, \not{n}\gamma_5\, u(p\nu)}{2\bar{p}\cdot n}\, \widetilde{H}^g(\overline{x},\xi,t) + \frac{\bar{u}(p'\nu')\, n\cdot\Delta\, \gamma_5\, u(p\nu)}{4m\,\bar{p}\cdot n}\, \widetilde{E}^g(\overline{x},\xi,t) \right\}.$$

$$H^g(\overline{x},0,0) = \overline{x}\, g(\overline{x}); \qquad \widetilde{H}^g(\overline{x},0,0) = \overline{x}\, \Delta g(\overline{x}) \tag{1}$$

Distributions $E^g(\overline{x},\xi,t)$, $(\tilde{E})$ determine mainly proton spin-flip – Some contribution for unpolarized case at moderate $x$.

# Modelling the GPDs

The double distributions for GPDs Radyushkin '99 .
– simple for the double distributions. Regge form: $\alpha_i = \alpha_i(0) + \alpha' t$

$$f_i(\beta, \alpha, t) = h_i(\beta, t) \frac{\Gamma(2n_i + 2)}{2^{2n_i+1} \Gamma^2(n_i + 1)} \frac{[(1 - |\beta|)^2 - \alpha^2]^{n_i}}{(1 - |\beta|)^{2n_i+1}}, \tag{2}$$

$\star$ Gluon contribution (n=2).

$$h_g(\beta, 0) = |\beta| g(|\beta|) \tag{3}$$

$$\Downarrow$$

$$H_i(\overline{x}, \xi, t) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \, \delta(\beta + \xi \, \alpha - \bar{x}) \, f_i(\beta, \alpha, t) \tag{4}$$

Decomposition into valence and sea quarks (Diehl, '03)

$q_{val}(\beta, 0) = [q(\beta, 0) + q(-\beta, 0)]\Theta(\beta)$ $q_{sea}(\beta, 0) = [q(\beta, 0)\Theta(\beta) - q(-\beta, 0)\Theta(-\beta)]$

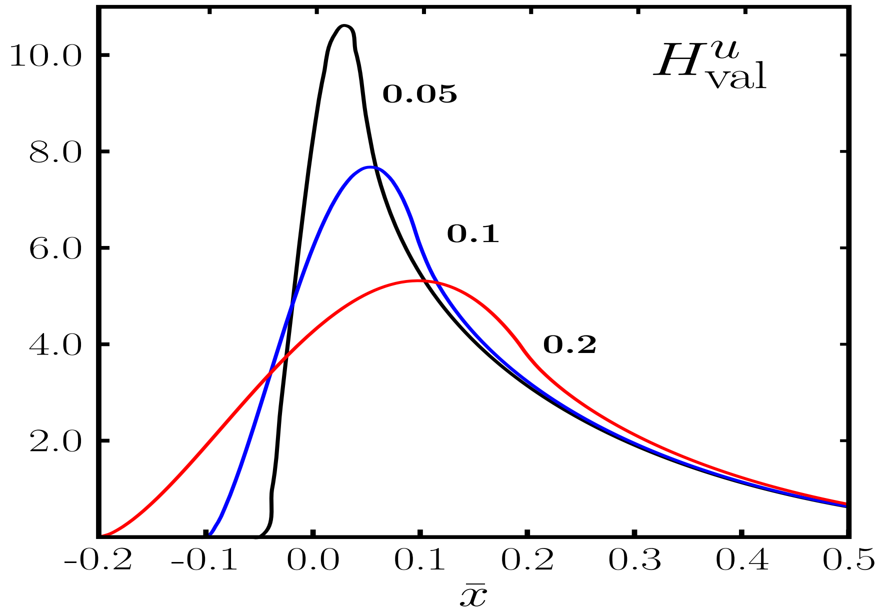
$$\Downarrow$$

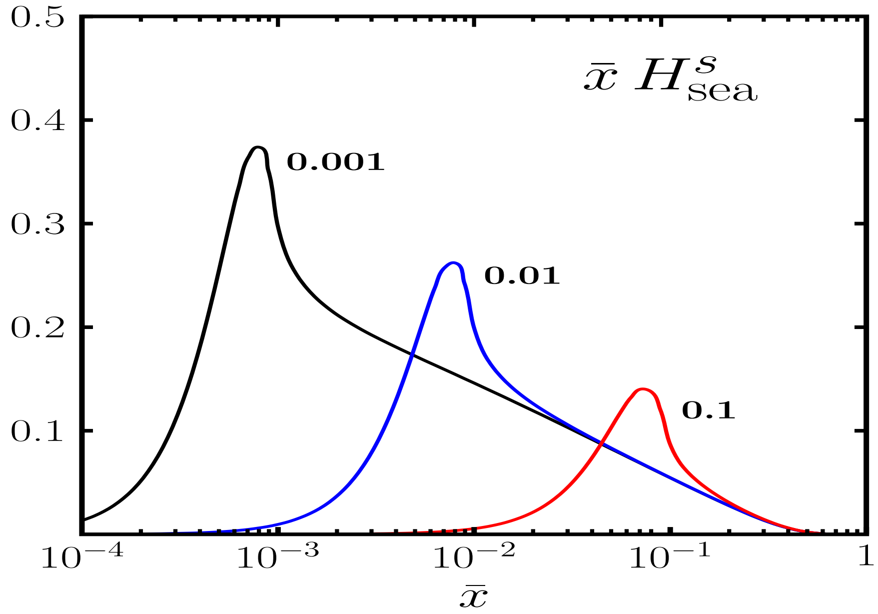
$\star$ $h^q_{sea}(\beta, 0) = q_{sea}(|\beta|) \, \mathrm{sign}(\beta)$ - sea quark contribution (n=2).

$\star$ $h^q_{val}(\beta, 0) = q_{val}(|\beta|) \, \Theta(\beta)$ –valence contribution (n=1).

## ⋆ Results for valence and sea quarks GPDs

-CTEQ6 parameterization of PDFs




$H^u_{val}$ GPD at $t = 0$and $Q^2 = 4\mathrm{GeV}^2$. Lines- for $\xi = 0.05, 0.1, 0.2$.

$H^s_{sea}$ GPD at $t = 0$and $Q^2 = 4\mathrm{GeV}^2$.

$$H^u_{sea} = H^d_{sea} = \kappa_s\, H^s_{sea}$$

The flavor symmetry breaking factor is from the fit if CTEQ6M

$$\kappa_s = 1 + 0.68/(1 + 0.52\ln(Q^2/Q_0^2))$$

# Combinations of quark contribution to different reactions

Uncharged VM production - Standard $p \to p$ GPDs for $\rho, \omega$: Transition $p \to \Sigma^+$ GPDs for $K^{*0}$:

$$\begin{aligned}
\rho: &\quad \sim\ e^u H^u - e^d H^d = \frac{2}{3} H^u + \frac{1}{3} H^d \\
\omega: &\quad \sim\ \frac{2}{3} H^u - \frac{1}{3} H^d \\
K^{*0}: &\quad \sim\ H^d - H^s
\end{aligned}$$

Charged VM production - Transition $p \to n$ GPDs :

$$\rho^+: \quad \sim H^{(3)} \ = \ H^u - H^d;$$

Pseudoscalar mesons production contribution of polarized distributions:

$$\begin{aligned}
\pi^+: &\quad \sim \tilde{H}^{(3)} \ = \ \tilde{H}^u - \tilde{H}^d; \\
\pi^0: &\quad \sim \quad \frac{2}{3} \tilde{H}^u + \frac{1}{3} \tilde{H}^d
\end{aligned}$$

These combinations can be tested in mentioned reactions. GPDs in amplitudes are in integrated form. Direct GPDs extraction is impossible .

Our way: parameterization of GPDs and comparison with experiment.

# Modified PA for vector meson production

We calculate the $L \to L$ and $T \to T$ amplitudes – important in analyses of spin observales.

⋆ **Wave function**

The $k$- dependent wave function is used

$$\hat{\Psi}_V = \hat{\Psi}_V^0 + \hat{\Psi}_V^1.$$

$$\hat{\Psi}_V^0 = (\not{V} + m_V)\not{\epsilon}_V \phi_V(k,\tau).$$

$$\hat{\Psi}_V^1 = [\frac{2}{M_V}\not{V}\not{\epsilon}_V\not{K} - \frac{2}{M_V}(\not{V} - m_V)(\epsilon_V \cdot K)]\phi_V'(k,\tau).$$

J. Bolz, J. Körner and P. Kroll, 1994

- $\hat{\Psi}_V^0$ - leading twist wave function -L polarized meson
- $\hat{\Psi}_V^1$ - higher twist wave function -T polarized meson
- $V$ is a vector meson momentum and $m_V$ is its mass
- $\epsilon_V$ is a meson polarization vector and $K$ is a quark transverse momentum
- $M_V$ is a scale in the $\hat{\Psi}_V^1$. We use $M_V = m_V/2$

## $\star$ Structure of the amplitudes of meson production

$\gamma^*_\mu \to V'_\mu$ quark & gluons contributions

$$
\begin{aligned}
M_{\mu'+,\mu+} \;\sim\; & \sum_a e_a\, B_a^V \\
& \times \Big\{ C_F \int_{-1}^{1} d\overline{x}\; \hat{H}^a(\overline{x},\xi,t)\, F^a_{\mu',\mu}(\overline{x},\xi) \left[\frac{1}{\overline{x}+\xi-i\hat{\varepsilon}} + \frac{1}{\overline{x}-\xi+i\hat{\varepsilon}}\right] \\
& + 2\,(1+\xi) \int_0^1 d\overline{x}\, \frac{\hat{H}^g(\overline{x},\xi,t)\, F^g_{\mu',\mu}(\overline{x},\xi\ )}{(\overline{x}+\xi)(\overline{x}-\xi+i\hat{\varepsilon})} \Big\}\,.
\end{aligned} \tag{5}
$$

$\hat{H}^i = [H^i - \frac{\xi^2}{1-\xi^2}E^i] + [\tilde{H}^i \ \ (\tilde{E}^i)$ contributions$]$. $\quad M_{\mu'-,\mu+} \sim \frac{\sqrt{-t}}{2m}[< E > + ...\xi < \tilde{E} >]$

# Modified PA amplitudes

The hard scattering amplitudes-transverse quark motion

$$F^{a(g)}_{\mu',\mu}(\overline{x},\xi) = \frac{8\pi\alpha_s(\mu_R)}{\sqrt{2N_c}} \int_0^1 d\tau \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\phi_{V\mu'}(\tau,k_\perp^2)\; f^{a(g)}_{\mu',\mu}(\mathbf{k}_\perp,\overline{x},\xi,\tau)\hat{D}(\tau,Q,\mathbf{k}_\perp)\,. \tag{6}$$

$$\phi_V(\mathbf{k}_\perp,\tau) = 8\pi^2\sqrt{2N_c}\,f_V a_V^2\,\exp\left[-a_V^2\,\frac{\mathbf{k}_\perp^2}{\tau\bar{\tau}}\right]. \tag{7}$$

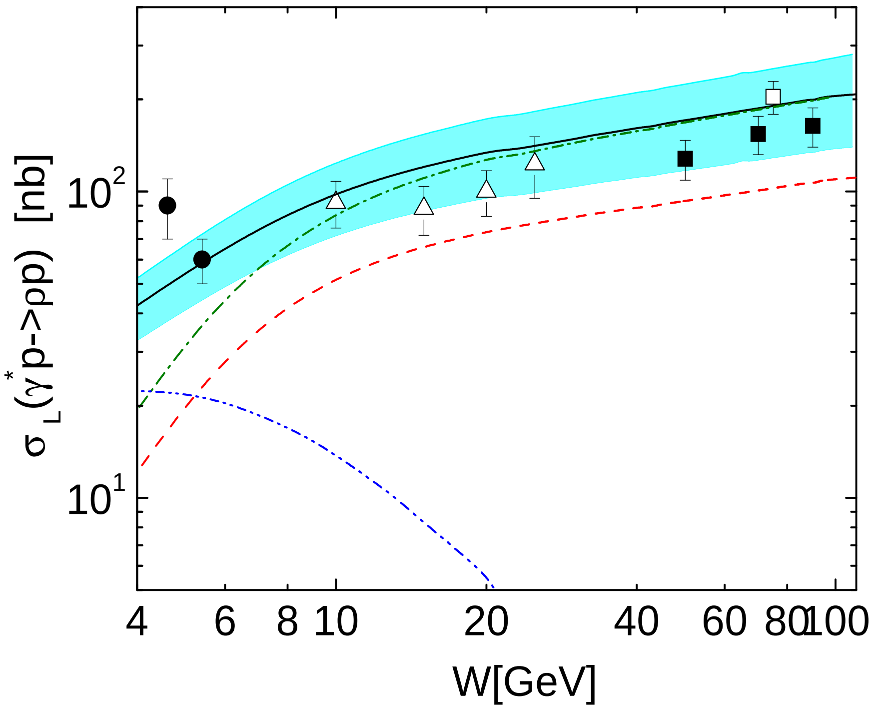
We consider Sudakov suppression of large quark-antiquark separations. These effects suppress contributions from the end-point regions , where factorization breaks down . Since the Sudakov factor is exponentiate in the impact parameter space- we have to work in this space.

The helicity amplitudes for vector-meson electroproduction read

$$\mathcal{M}^H_{\mu'+,\mu+} = \frac{e}{\sqrt{2N_c}}\mathcal{C}_V \int d\overline{x}d\tau\; f_{\mu'\mu}\,H(\overline{x},\xi,t)\times\int d^2\mathbf{b}\,\phi_V(\tau,b^2)\,\hat{D}(\tau,Q,b)\,\alpha_s(\mu_R)\,\exp\left[-S(\tau,b,Q)\right]. \tag{8}$$

The renormalization scale $\mu_R$ is taken to be the largest mass scale appearing in the hard scattering amplitude, i.e. $\mu_R = \max\,(\tau Q,\bar{\tau}Q,1/b)$.

# Analyses of different contributions to the $\rho$ production.




Full line- cross section.

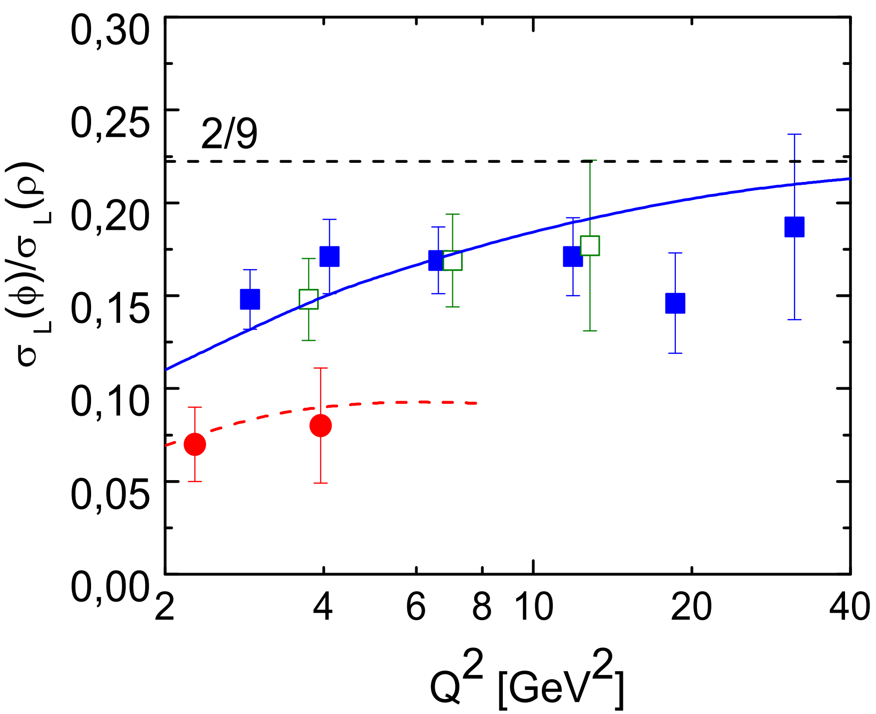
Red-gluon contribution

Green- gluon+ sea contribution.

Blue -Valence- gluon+sea interference

At HERMES energies valence quarks contribution is essential. At energies higher 5GeV$^2$ it decreases rapidly with energy growing . As a result, COMPASS (unpolarized cross sectoin) is closed to HERA physics- gluon and sea is essential .

# Ratio of cross sections of $\phi/\rho$ production.



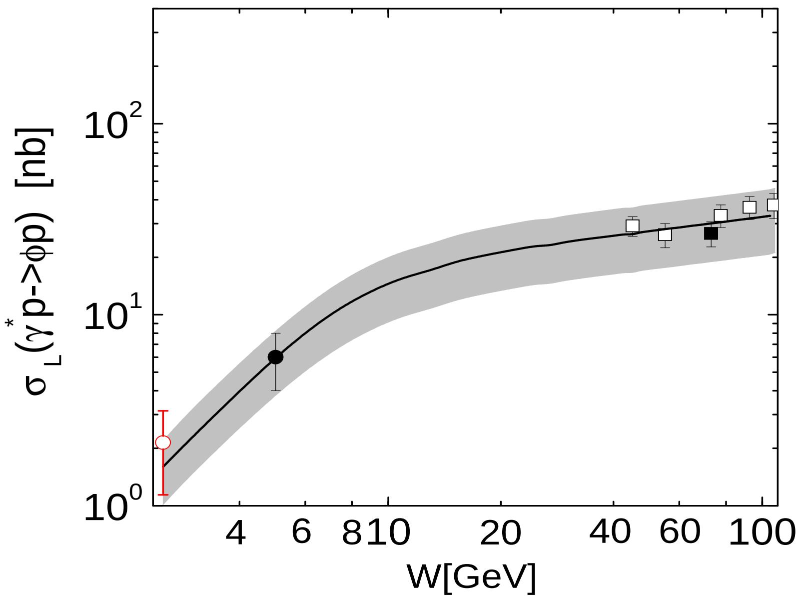

Show strong violation of $\sigma_\phi/\sigma_\rho = 2/9$ at HERA energies and low $Q^2$ is caused by the flavor symmetry breaking between $\bar{u}$ and $\bar{s}$

$$\bar{u}(x) = \bar{d}(x) = \kappa_s s(x)$$

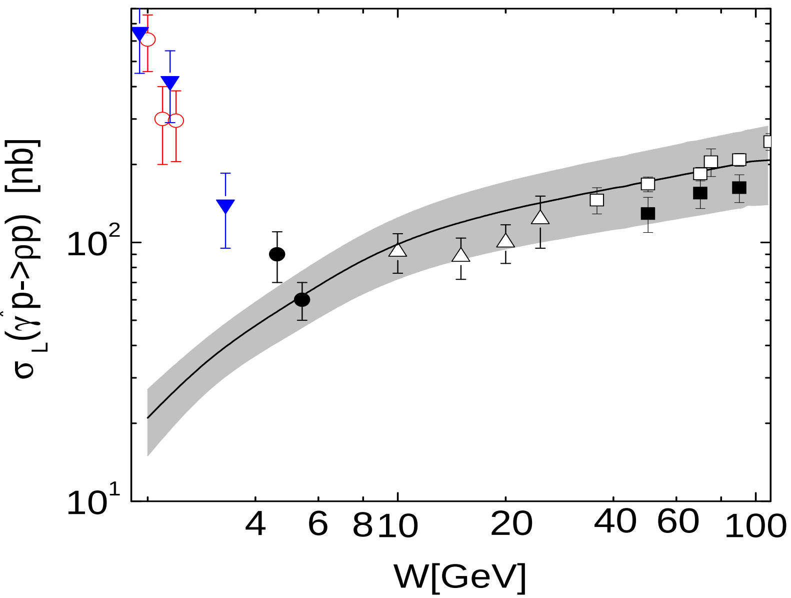
The flavor symmetry breaking factor is from the fit of CTEQ6M PDFs

$Q^2$ dependence of $\sigma_\phi/\sigma_\rho$ at HERA is determined by $\kappa_s$ factor completely.
At HERMES energies we have valence quarks contribution which gives additional suppression of $\sigma_\phi/\sigma_\rho$ ratio.

# Cross section of $\rho$ and $\phi$ production cross



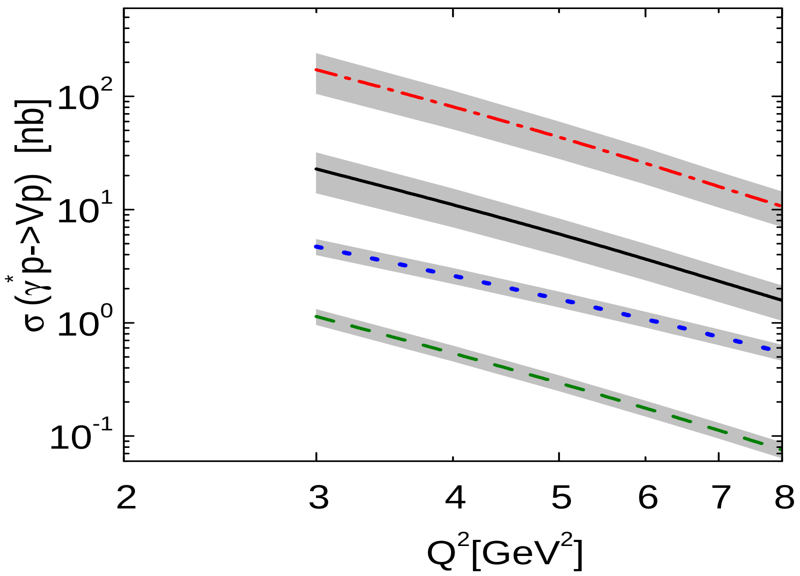

The longitudinal cross section for $\phi$ at $Q^2 = 3.8\,\mathrm{GeV}^2$. Data: HERMES (solid circle), ZEUS (open square), H1 (solid square), open circle- CLAS data point



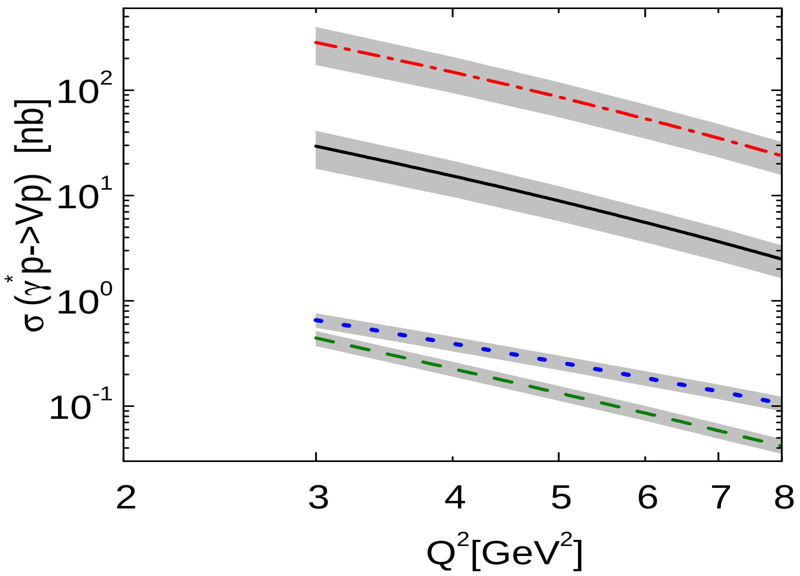

The longitudinal cross section for $\rho$ at $Q^2 = 4.0\,\mathrm{GeV}^2$. Data: HERMES (solid circle), ZEUS (open square), H1 (solid square), E665 (open triangle), open circles- CLAS, CORNEL -solid triangle

Such problem appears in all the cases when valence quark distributions are essential at low $W$: $\rho^0$, $\rho^+$, $\omega$ production.- Break in DD, handbag, other effects???

# Hierarchy of cross sections for various meson production.




Our predictions for HERMES energy $W = 5$GeV.




Our predictions for COMPASS energy $W = 10$GeV

- Red- dot-dashed line $\rho^0$ ; Black-full line $\omega$. –gluon contribution here - growing with energy .
- Blue- dotted line $\rho^+$ ; Green-dashed line $K^{*0}$. $K^{*0}$ - sea contribution .

## Effects of $\tilde{H}$ in $\sigma_U$ & $A_{LL}$ asymmetry.

$\tilde{H}$ determines amplitudes with unnatural parity and cross section

$$\sigma_U \propto |\mathcal{M}^{\widetilde{H}}|^2$$

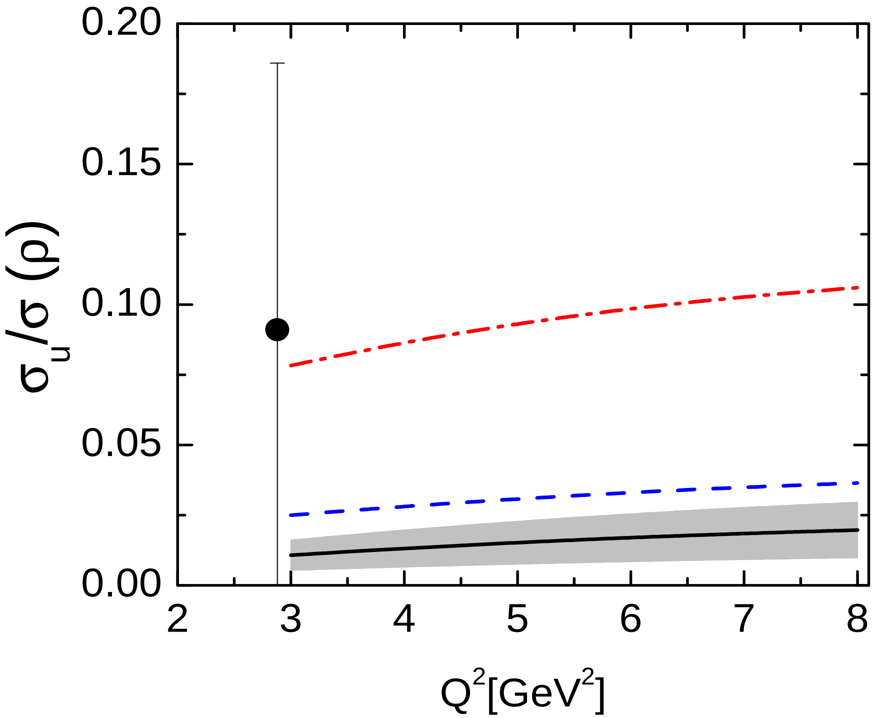
The leading term in $A_{LL}$ is an interference between the $H$ and the $\widetilde{H}$ terms.

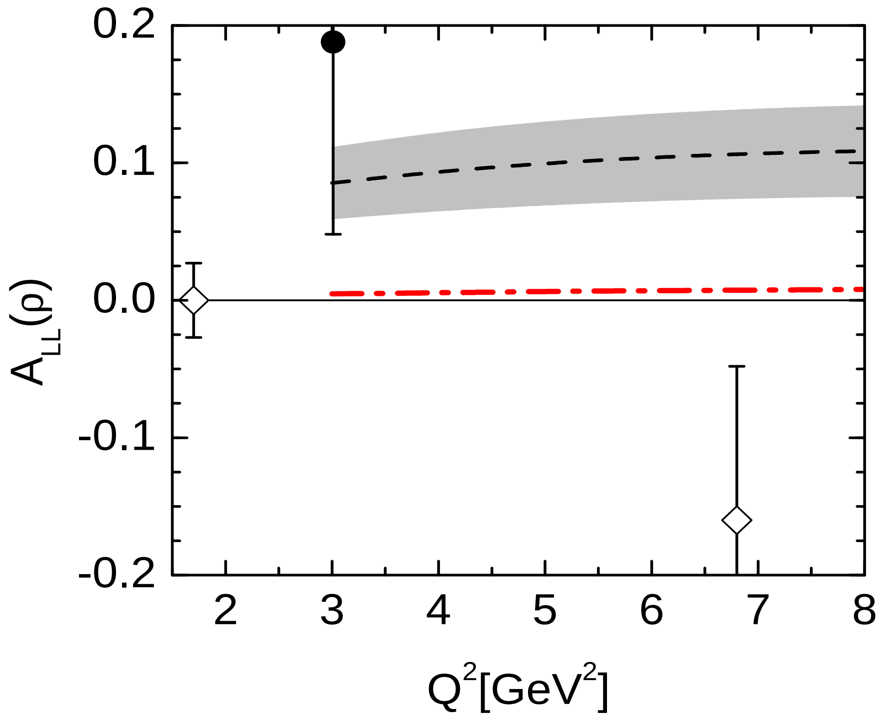
$$A_{LL}[ep \to epV] = 2\sqrt{1-\varepsilon^2}\,\frac{\mathrm{Re}\left[\mathcal{M}^{H}_{++,++}\,\mathcal{M}^{\widetilde{H}*}_{++,++}\right]}{\varepsilon|\mathcal{M}^{H}_{0+,0+}|^2 + |\mathcal{M}^{H}_{++,++}|^2}\,. \tag{9}$$

This ratio is of order $\langle\widetilde{H}\rangle/\langle H\rangle$ .
$A_{LL}$ asymmetry can be measured with longitudinally polarized beam and target.

Gluon and sea polarized distributions compensate each other essentially.

The dominant contribution comes from quark polarized distributions We construct $\Delta u$ and $\Delta d$ distributions. The standard values $\delta_{val} = 0.48; \alpha'_{val} = 0.9\mathrm{GeV}^{-2}$ are used. GPDs $\widetilde{H}$ is estimated using double distribution.






The ratio of $\sigma_U$ and $\sigma$ for $\rho$ production versus $Q^2$ at $W = 5$ GeV. Data taken from HERMES . The solid (dashed, dash-dotted) line represents our estimate (obtained with $e_u\tilde{H}^u_{\rm val} - e_d\tilde{H}^d_{\rm val}$, with $e_u\tilde{H}^u_{\rm val} + e_d\tilde{H}^d_{\rm val}$). .

Larger result is expected for this ratio for $\omega$ cross section ratio.

The $A_{LL}$ asymmetry for $\rho$ production at $W = 5\,(10)$ GeV dashed (dash-dotted) line.
Data taken from COMPASS and HERMES

## $\star$ Spin-flip contribution. Effects of $E$ GPDs.

The proton spin-flip amplitude is associated with $E$ GPDs.

$$\mathcal{M}_{\mu'-,\mu+} \propto \frac{\sqrt{-t}}{2m} \int_{-1}^{1} d\overline{x}\, E^a(\overline{x},\xi,t)\, F^a_{\mu',\mu}(\overline{x},\xi)$$

Double distribution model is used to construct $E$ GPDs. $E$ parameters- from Pauli form factor.

M. Diehl, ...,P.Kroll '04

Standard connection with ordinary distribution :

$$E^a(x,0,0) = e^a(x)$$

And

$$\int_0^1 dx e^a_{val}(x) = \kappa^a, \quad \kappa^u \sim 1.67, \quad \kappa^d \sim -2.03$$

$\kappa^a$ -anomalous magnetic moments of valence quarks.

This means that $E^a \propto \kappa^a$ and $E^u$ and $E^d$ have different signs.

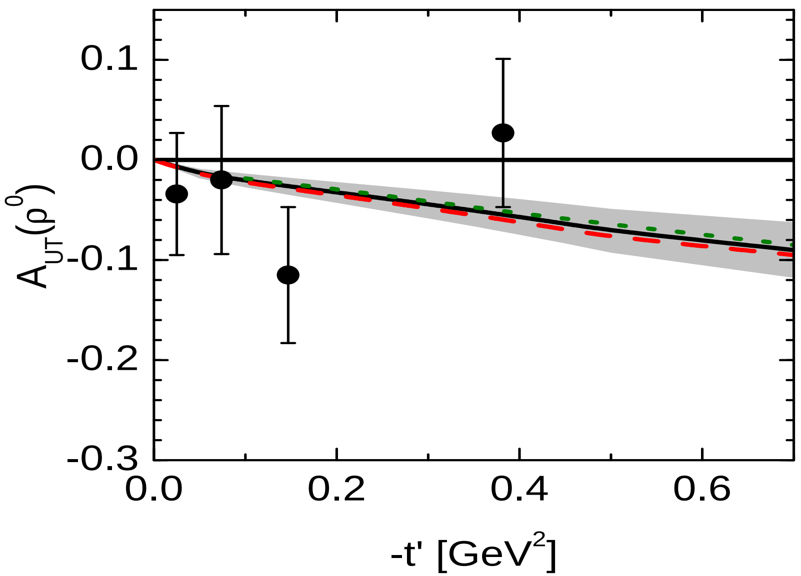
$\star$ $\rho$ production- $M^\rho \propto e_u E^u_{val} - e_d E^d_{val} = \frac{2}{3} E^u_{val} + \frac{1}{3} E^d_{val}$ different signs- compensation .

$\star$ $\omega$ production- $M^\omega \propto e_u E^u_{val} + e_d E^d_{val} = \frac{2}{3} E^u_{val} - \frac{1}{3} E^d_{val}$ same signs- enhancement.

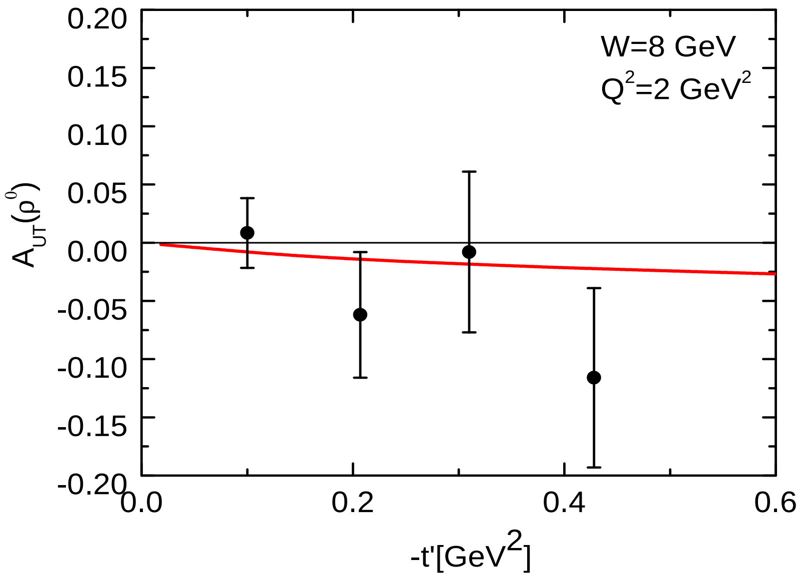
## $A_{UT}$ asymmetry for $\rho$ production.

$$A_{UT} = -2\frac{\mathrm{Im}[M^*_{+-,++}\, M_{++,++} + \varepsilon M^*_{0-,0+} M_{0+,0+}]}{\sum'_\nu [|M_{+\nu',++}|^2 + \varepsilon |M_{0\nu',0+}|^2]} \propto \frac{\mathrm{Im} < E >^* < H >}{| < H > |^2}$$

$E$ distributions needed to calculate $M_{+-,++}$ – from Pauli FF of nucleon. +DD form for GPD.

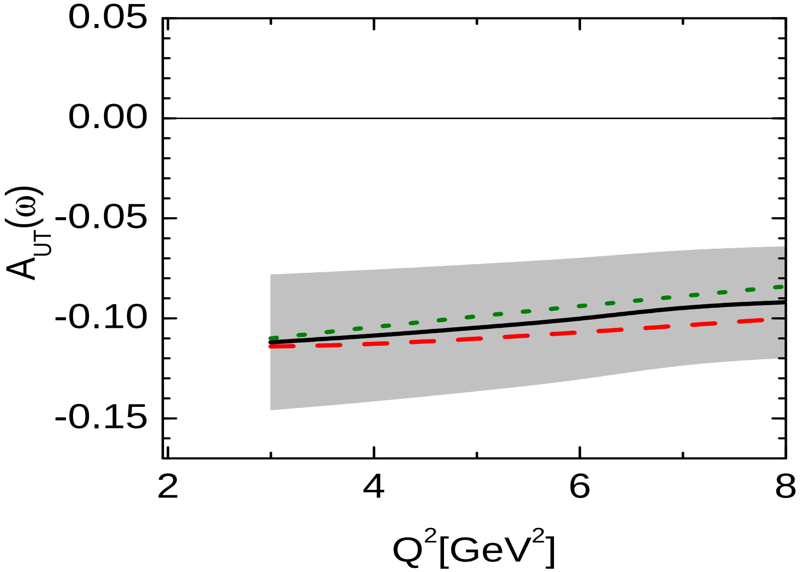


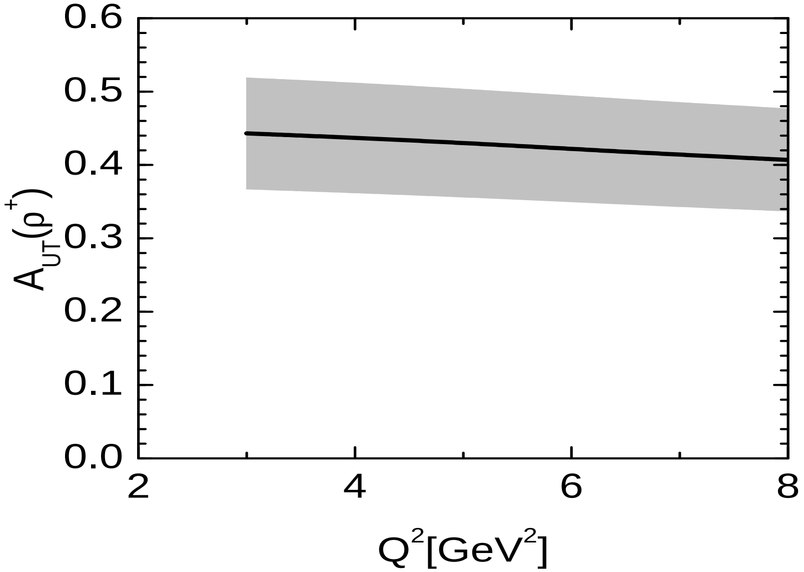

Predictions for HERMES energy $W = 5\mathrm{GeV}$, $Q^2 = 3\mathrm{GeV}^2$. Preliminary HERMES data are shown.




Predictions for COMPASS energy $W = 8\mathrm{GeV}$. Preliminary COMPASS data at $W = 8\mathrm{GeV}$ Sandacz, Photon 09, Bradamante, TPSH 09

# $A_{UT}$ asymmetry for $\omega$ and $\rho^+$ production.

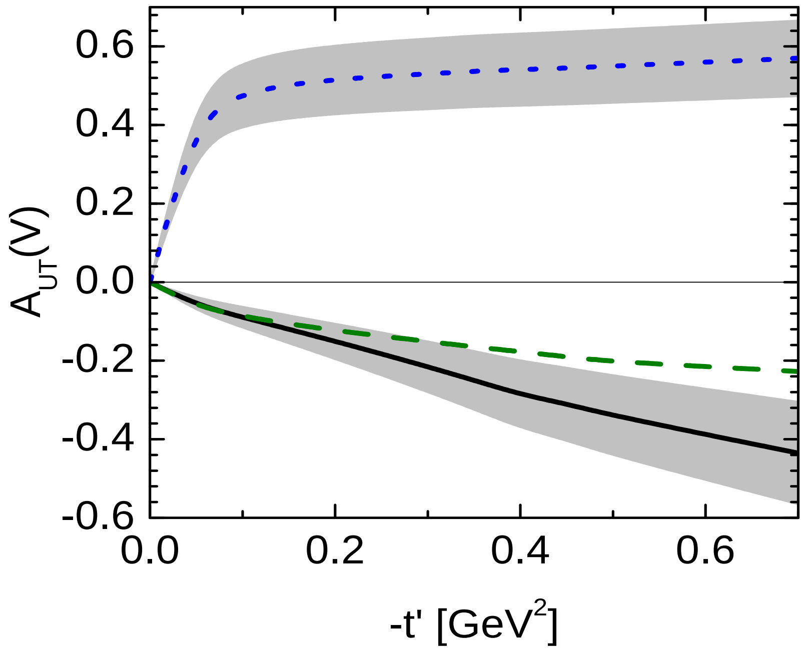


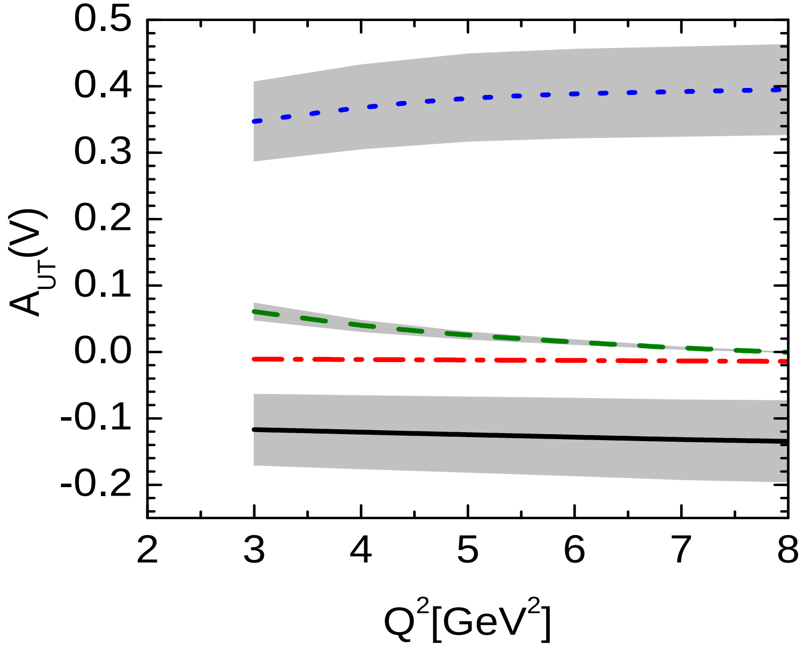

Our predictions for $\omega$ at HERMES energy $W = 5$GeV.
Not small negative asymmetry: no compensation in $e_uE^u + e_dE^d$.

Our predictions for $\rho^+$ at HERMES energy $W = 5$GeV .
Large positive asymmetry- no compensation in $E^u - E^d$ .

# Hierarchy of $A_{UT}$ asymmetry for various meson production.




Predictions for HERMES energy $W = 5\text{GeV}$, $Q^2 = 3\text{GeV}^3$. $\rho^0$ , $\omega$. $\rho^+$ , $K^{*0}$




Predictions for COMPASS energy $W = 10\text{GeV}$.

At small $-t$ $A_{UT} \propto \sqrt{-t}$. For $\rho^+$ flatness caused by large spin-flip contribution E to cross section $\propto t$. (No compensation between $u$ and $d$ quarks here) .

# Parton angular momenta.

Evaluation from Ji's sum rule

$$
\begin{aligned}
< J^a > &= \frac{1}{2}\left[e_{20}^{a_v} + h_{20}^{a_v}\right] + e_{20}^{\bar{a}} + h_{20}^{\bar{a}} \\
< J^g > &= \frac{1}{2}\left[e_{20}^{g} + h_{20}^{g}\right]
\end{aligned}
\tag{10}
$$

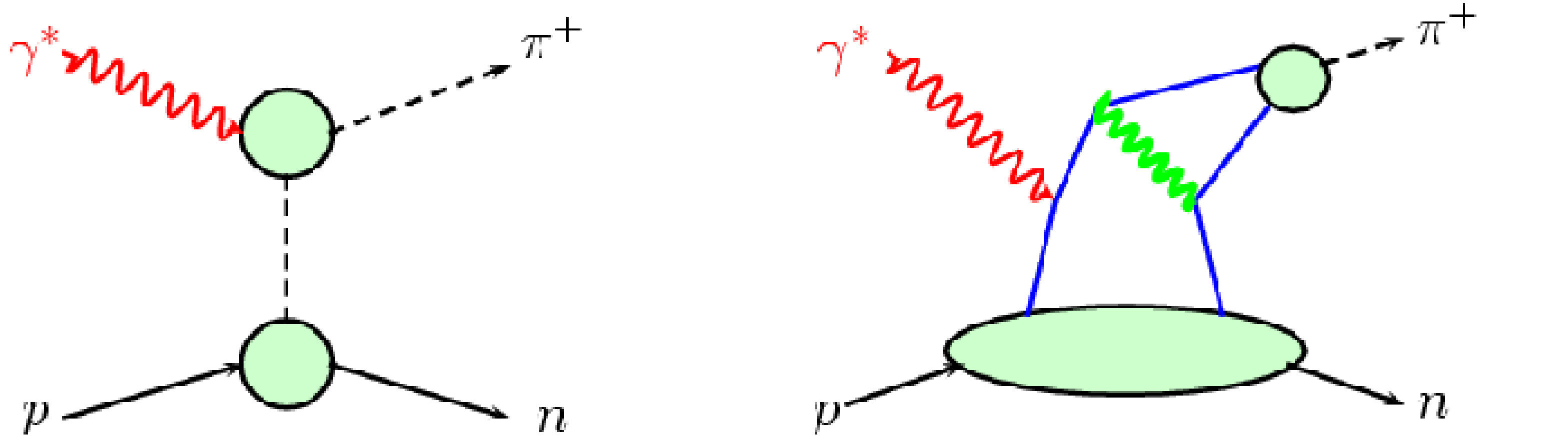
$e(h)_{20}^{a}$ - second moments of corresponding parton distributions.

In our model we find the angular momenta for valence quarks:

$$
< J_v^u >= 0.222, \quad < J_v^d >= -0.015, \quad < J^g >= 0.214. \tag{11}
$$

These results are closed to other estimations (Lattice) ...

# $\pi^+$ production at HERMES.




Pion pole and handbag

contributions to $\pi^+$ production.

$$\mathcal{M}^{\pi^+}_{0+,0+} \propto \sqrt{1-\xi^2}\,[\langle \tilde{H}^{(3)}\rangle - \frac{2\xi m Q^2}{1-\xi^2}\frac{\rho_\pi}{t-m_\pi^2}];\; \mathcal{M}^{\pi^+}_{0-,0+} \propto \frac{\sqrt{-t'}}{2m}\left[\xi\langle \widetilde{E}^{(3)}\rangle + 2mQ^2\frac{\rho_\pi}{t-m_\pi^2}\right].$$

$$<\tilde{F}>=\sum_\lambda \int_{-1}^{1} d\overline{x}\mathcal{H}_{0\lambda,0\lambda}(\overline{x},...)\tilde{F}(\overline{x},\xi,t),\;\; F^{(3)} = F^u - F^d.$$

## Calculation of $M_{0-,++}$ – special case.

$M_{\mu'\nu',\mu\nu} \propto \sqrt{-t'}^{|\mu-\nu-\mu'+\nu'|}$ from angular momentum conservation.

$M_{0-,++} \propto \sqrt{-t'}^{\,0} \propto const$ but handbag amplitude $\propto t'$

$M_{0-,++}$ -is determined by twist 3 contribution $\to const$ .

Transversity GPDS $(H_T, E_T, ...)$ contributes

$$\mathcal{M}^{twist-3}_{0-,\mu+} \propto \sqrt{1-\xi^2}\int_{-1}^{1} d\overline{x}\mathcal{H}_{0-,\mu+}(\overline{x},...)[H^3_T + ...O(\xi^2\, E^3_T)].$$

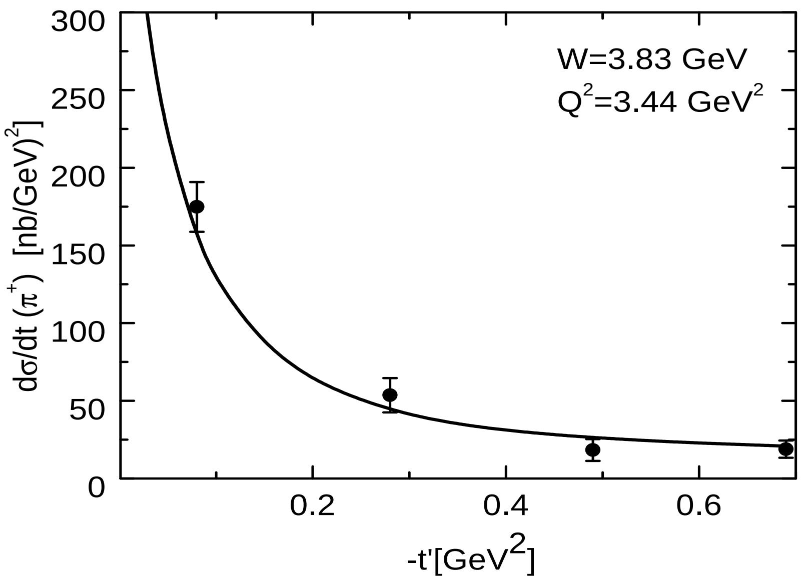
Double distribution model

$$H^a_T(x,0,0) = \delta^a(x)$$

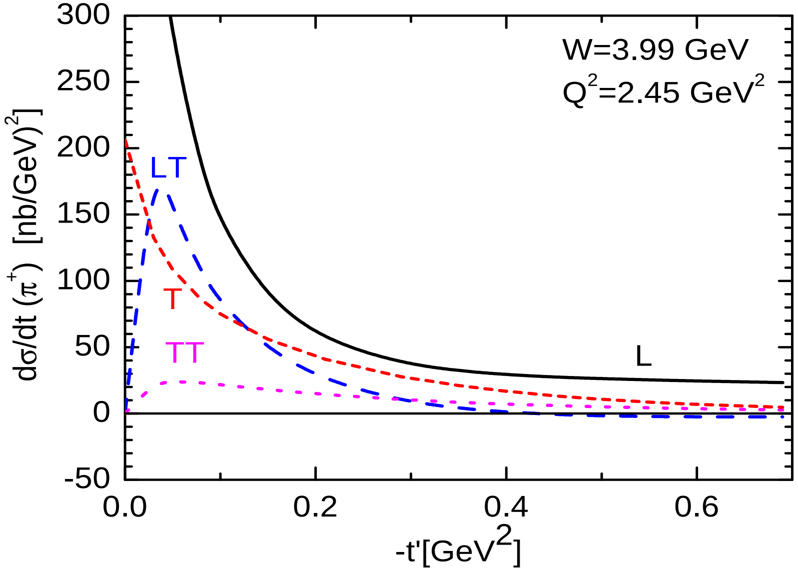
transversity PDFs   Anselmino model

$$\delta^a(x) = 7.46\, N^a_T\, x\,(1-x)^5\,[q_a(x)+\Delta q_a(x)], \quad N^u_T = 0.5, \; N^d_T = -0.6$$

# Cross section of $\pi^+$ production at HERMES.

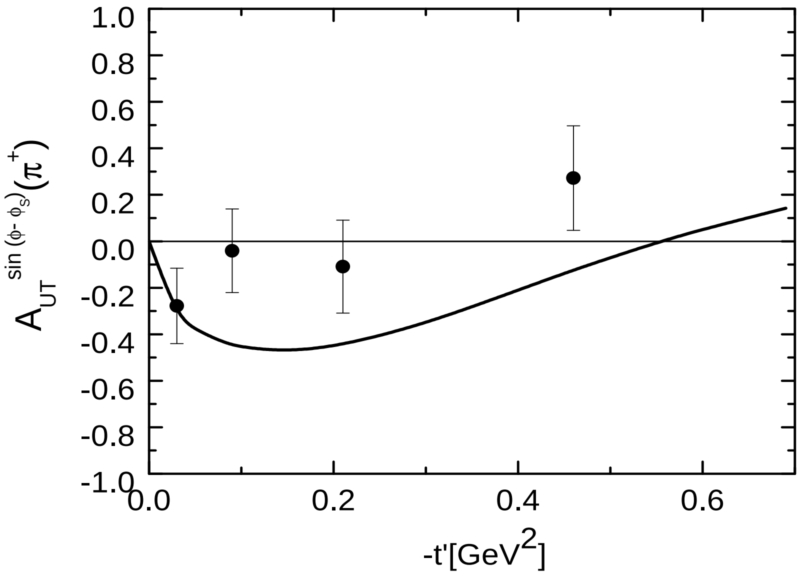


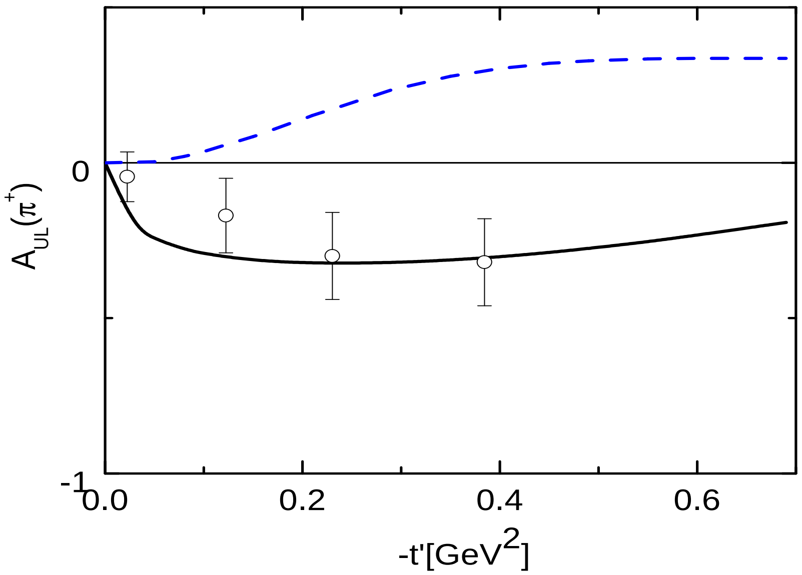

Cross section of $\pi^+$ production at HERMES with HERMES data.




The partial cross section $d\sigma_L/dt, d\sigma_T/dt, d\sigma_{LT}/dt, d\sigma_{TT}/dt$. Information about different amplitudes contribution.

# Asymmetry in $\pi^+$ production at HERMES.

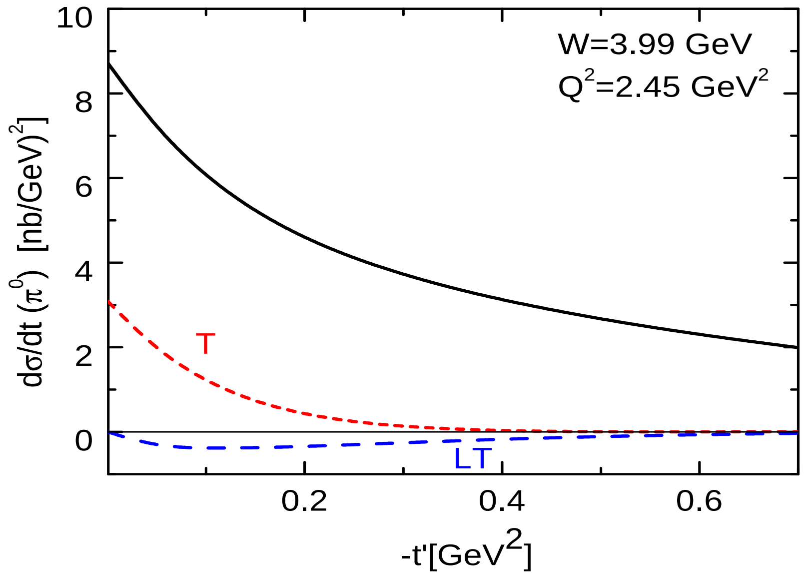


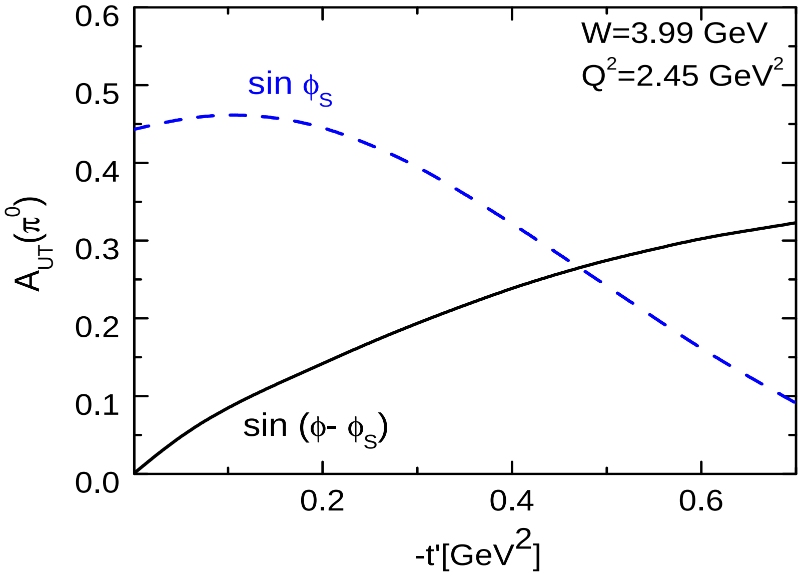

$A_{UT}$ asymmetry of $\pi^+$ production at HERMES with preliminary HERMES data.

$A_{UL}$ asymmetry of $\pi^+$ production at HERMES with preliminary HERMES data. Dashed line- twist 3 is omitted.

# $\pi^0$ production at HERMES.




Cross section of $\pi^0$ production. Unseparated cross section- black, red-transverse, blue-longitudinal-transverse interference cross section.




$\sin\phi_S$ and $\sin\phi - \phi_S$ moments of $A_{UT}$ asymmetry for $\pi^0$ production at HERMES

No pion pole contribution here $\to$ small cross section . At COMPASS $\pi^0$ cross section smaller then $1nb/GeV^2$

# Conclusion

- We analyse meson electroproduction within handbag approach.
- Modified PA is used to calculate hard subprocess amplitude.
- PDF:
  –gluon, sea, valence PDFs -from CTEQ6 parameterization.
  –polarized $\tilde{H}$ from BB parameterization.
  – $E$ distributions -from Pauli nucleon FF analyses.
- GPDs are calculated using these PDF on the bases of DD representation.
- Direct GPDs extraction is impossible .
  Our way: parameterization of GPDs , amplitudes calculation in handbag approach ; comparison with experiment.
- We describe fine :
  –cross section of light meson production: corrections $\sim k_\perp^2/Q^2$ in propagators are important.
  –spin observables: $R$ -ratio , SDME and different asymmetries for various meson production.
- Light meson electroproduction-can be an excellent object to study GPDs.
  In different energy ranges, information about quark and gluon GPDs can be extracted from the cross section and spin observables of the vector meson electroproduction.