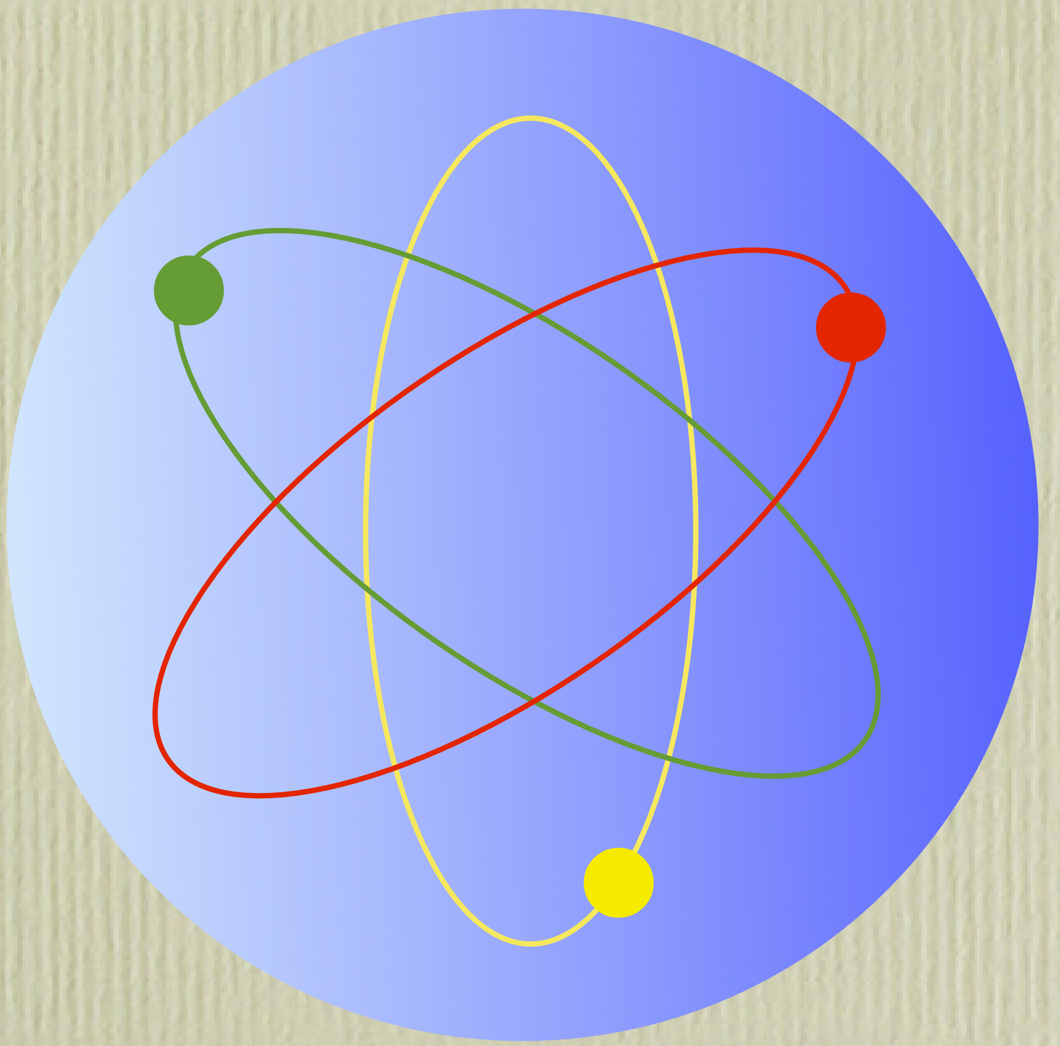
The Sivers and Collins functions



Mauro Anselmino
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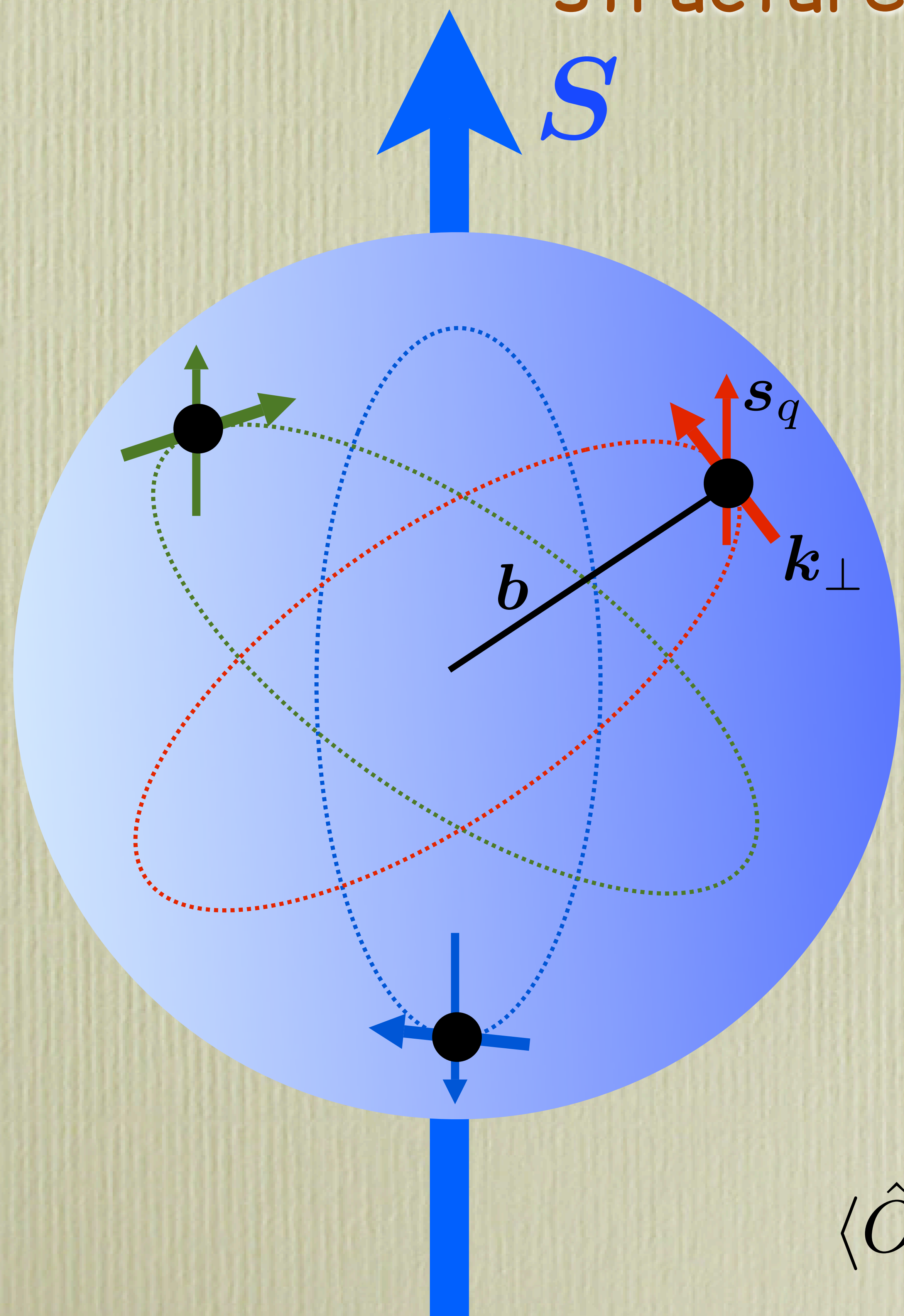
in collaboration with M. Boglione, U. D'Alesio, A. Kotzinian, S. Melis, F. Murgia, A. Prokudin

Hard Meson and Photon Production

ECT*, 11-15 October, 2010

exploring the 3-dimensional phase-space structure of the nucleon



intrinsic motion
spin-$k_\perp$ correlations?
orbiting quarks?

Ideally: obtain a quantum phase-space distribution (like the Wigner function)

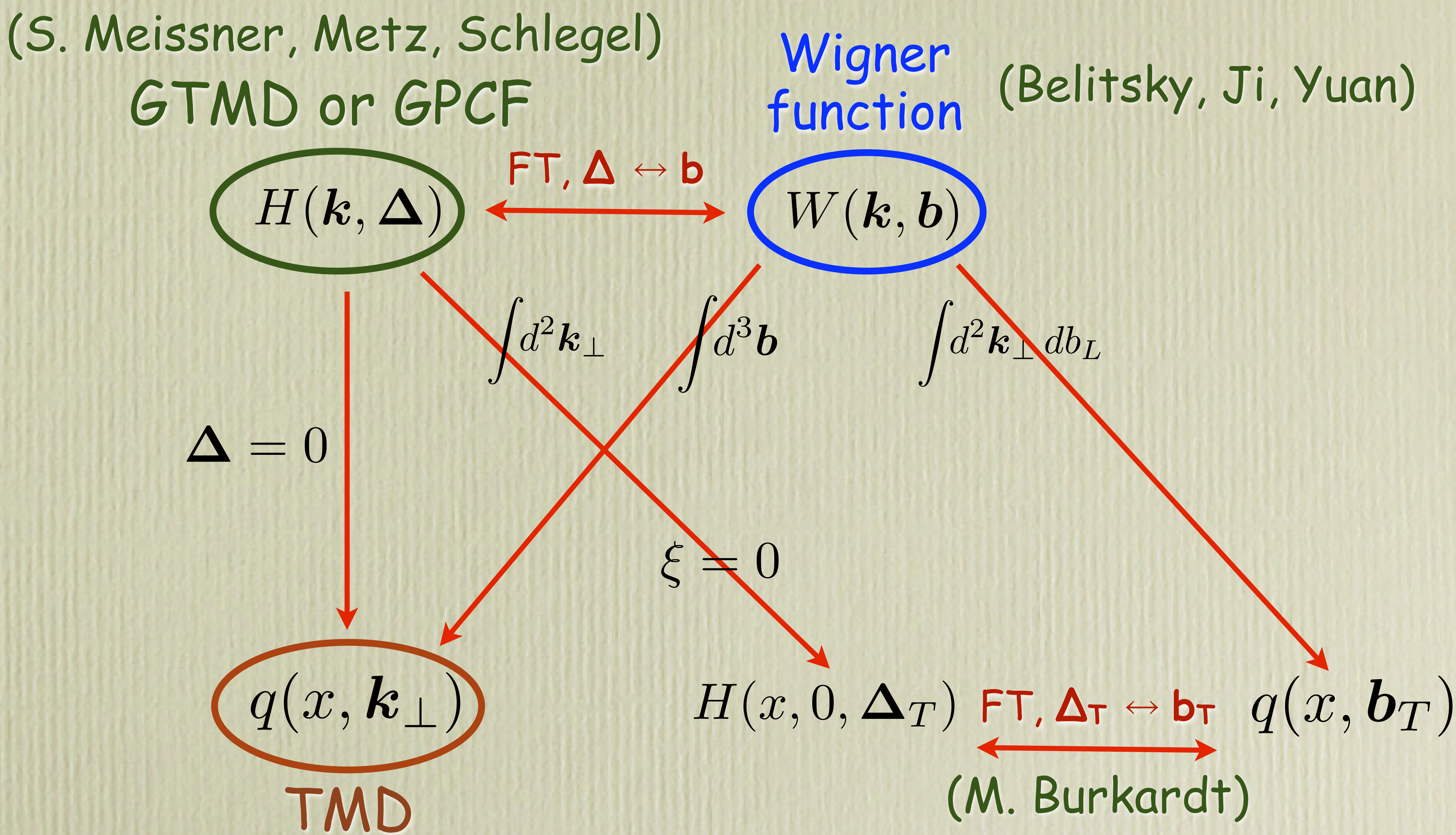
in 1-dimensional QM:

$$\int dp\, W(x,p) = |\psi(x)|^2$$

$$\int dx\, W(x,p) = |\phi(p)|^2$$

$$\langle \hat{O}(x,p) \rangle = \int dx\, dp\, W(x,p)\, O(x,p)$$

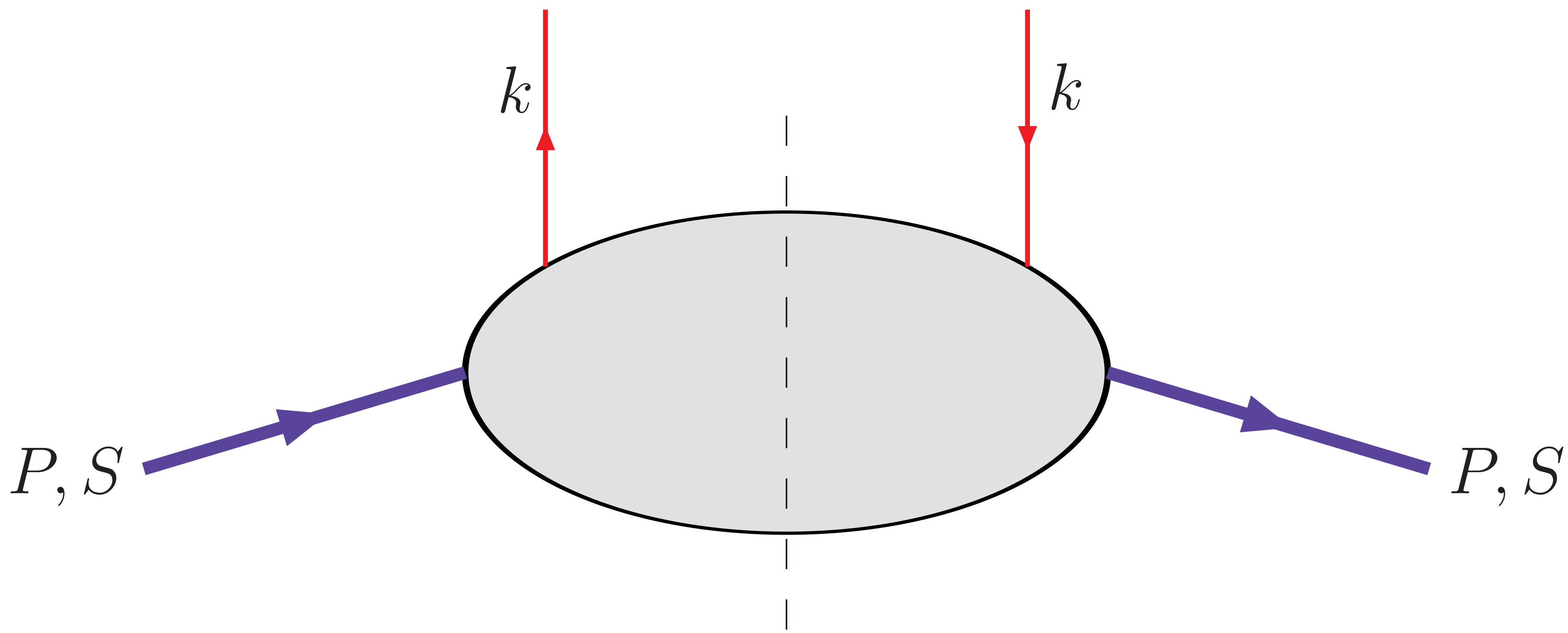
phase-space parton distribution, $W(\boldsymbol{k}, \boldsymbol{b})$



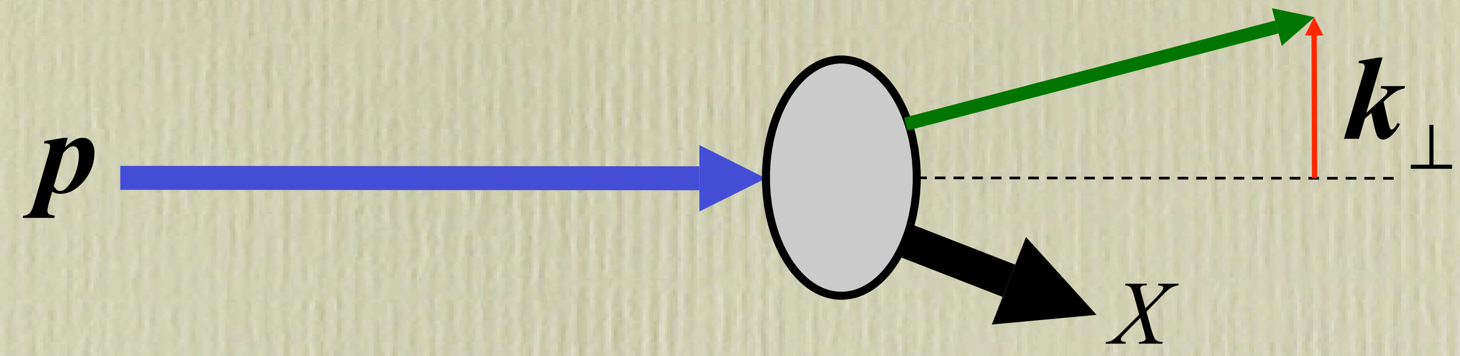
$$\int d^2\boldsymbol{k}_\perp\, H(\boldsymbol{k}, \boldsymbol{\Delta}) = H(x, \xi, \boldsymbol{\Delta}_T)$$

the leading-twist correlator, with intrinsic $k_\perp$, contains eight independent functions

$$
\begin{aligned}
\Phi(x, \boldsymbol{k}_\perp) \quad = \quad & \frac{1}{2}\Big[f_1 \not{n}_+ + f_{1T}^\perp \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu k_\perp^\rho S_T^\sigma}{M} + \left(S_L g_{1L} + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} g_{1T}^\perp \right) \gamma^5 \not{n}_+ \\
+ \quad & h_{1T} i\sigma_{\mu\nu}\gamma^5 n_+^\mu S_T^\nu + \left(S_L h_{1L}^\perp + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_T}{M} h_{1T}^\perp \right) \frac{i\sigma_{\mu\nu}\gamma^5 n_+^\mu k_\perp^\nu}{M} \\
+ \quad & h_1^\perp \frac{\sigma_{\mu\nu} k_\perp^\mu n_+^\nu}{M} \Big]
\end{aligned}
$$

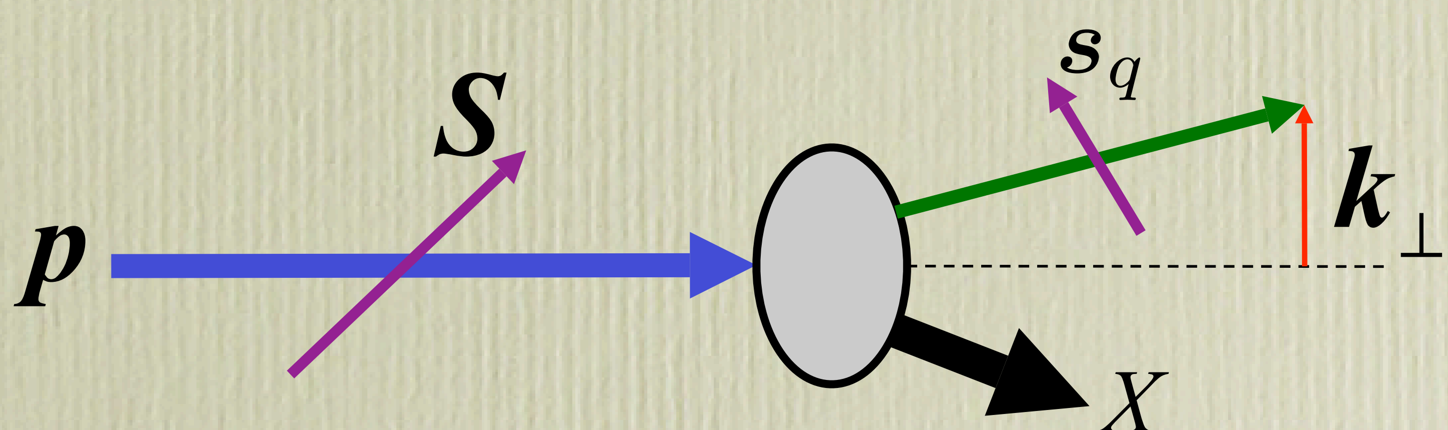


with partonic interpretation



$$f_1^q(x, k_\perp^2) \qquad\qquad q(x) = f_1^q(x) = \int \mathrm{d}^2\boldsymbol{k}_\perp \, f_1^q(x, k_\perp^2)$$

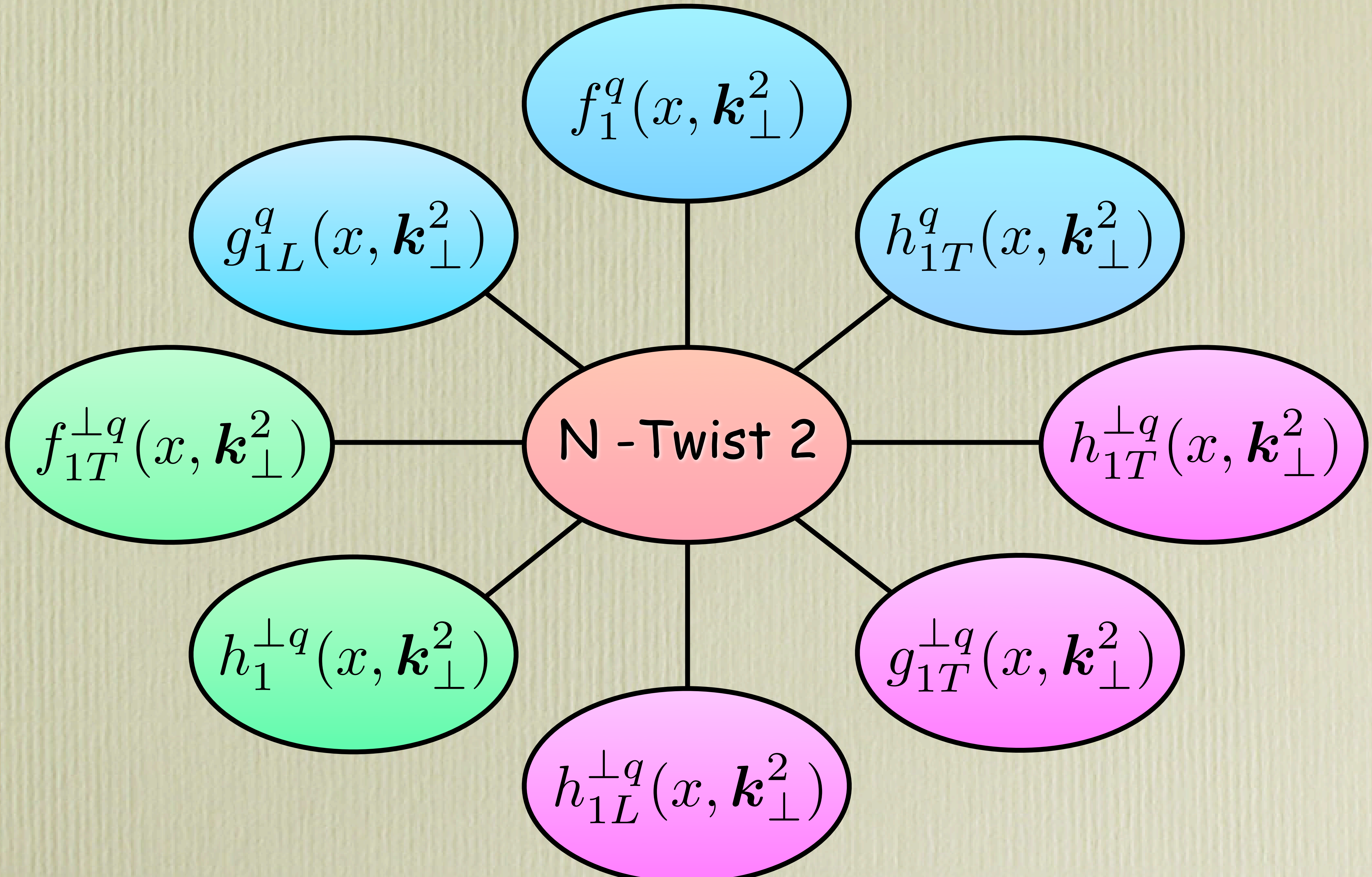
several spin-$\mathbf{k}_\perp$ correlations in TMDs



$$\boldsymbol{S}\cdot(\boldsymbol{p}\times\boldsymbol{k}_\perp) \qquad \boldsymbol{s}_q\cdot(\boldsymbol{p}\times\boldsymbol{k}_\perp) \qquad (\boldsymbol{p}\cdot\boldsymbol{S})(\boldsymbol{p}\cdot\boldsymbol{s}_q) \qquad \cdots$$

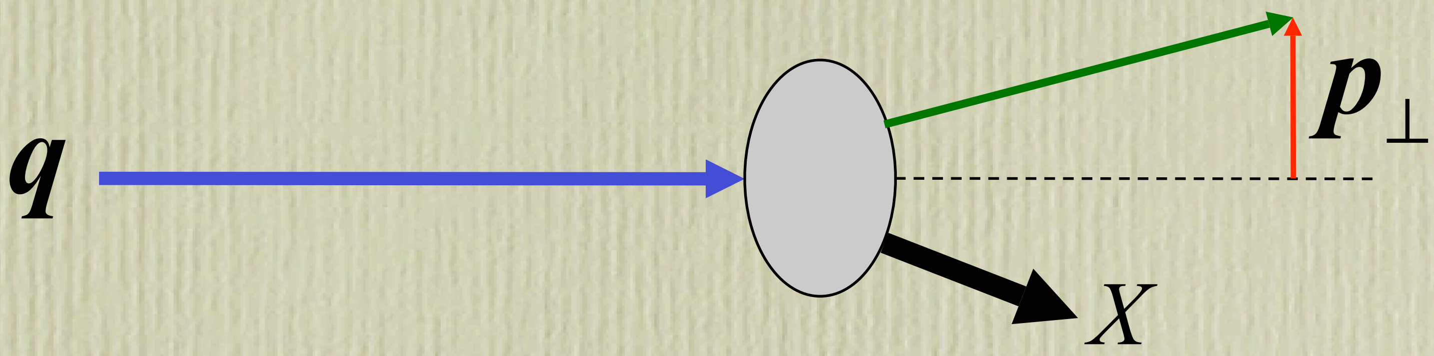
"Sivers effect" "Boer-Mulders effect"

The nucleon at twist-2

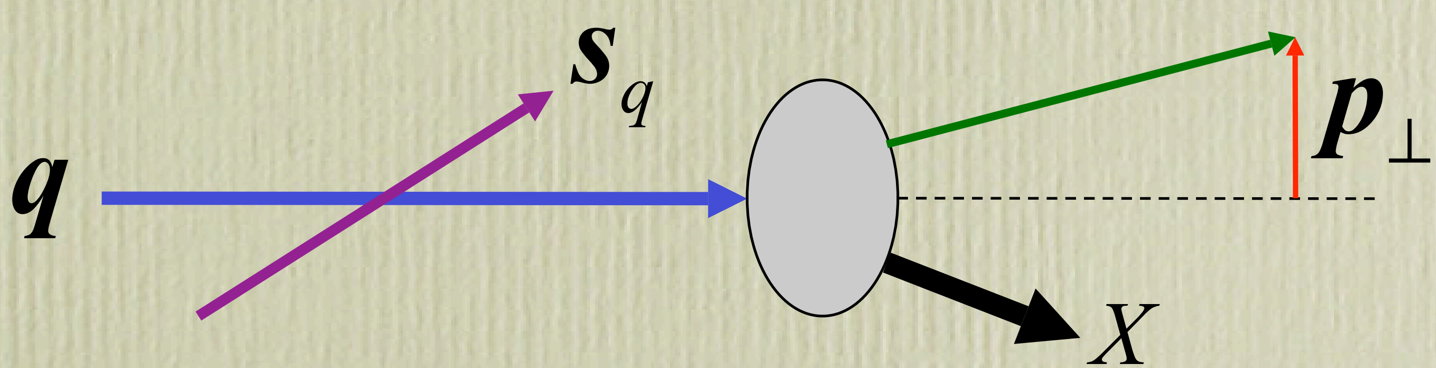


talks by Chen, Schnell and Schlegel

similar spin-$\mathbf{p}_\perp$ correlations in fragmentation process (case of final spinless hadron)



$$D_1^q(x, \boldsymbol{p}_\perp^2)$$

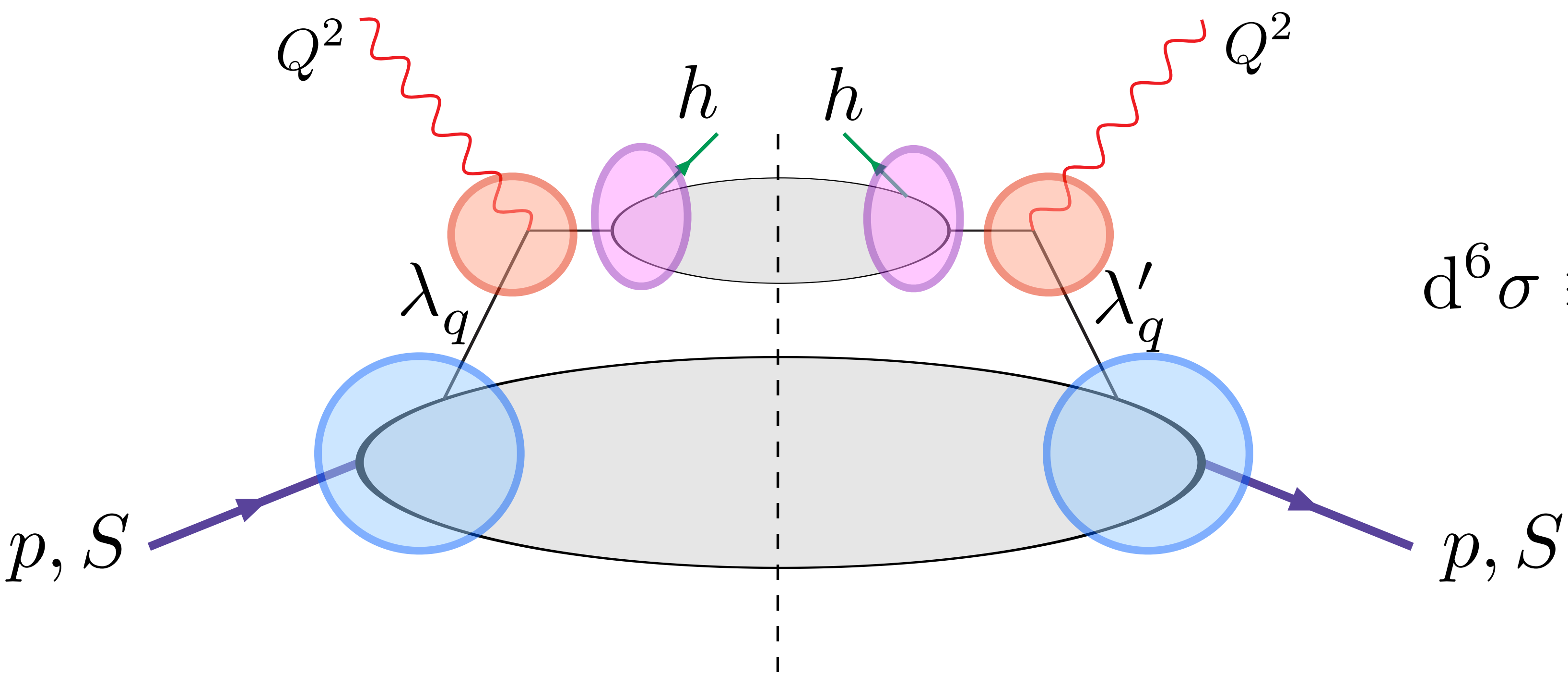


$$H_1^{\perp q}(x, \boldsymbol{p}_\perp^2)$$

$$\boldsymbol{s}_q \cdot (\boldsymbol{p}_q \times \boldsymbol{p}_\perp)$$

"Collins effect"

TMDs in SIDIS



$$d^6\sigma \equiv \frac{d^6\sigma^{\ell p^\uparrow \to \ell h X}}{dx_B\, dQ^2\, dz_h\, d^2\boldsymbol{P}_T\, d\phi_S}$$

factorization holds at large Q^2, and $P_T \approx k_\perp \approx \Lambda_{\rm QCD}$

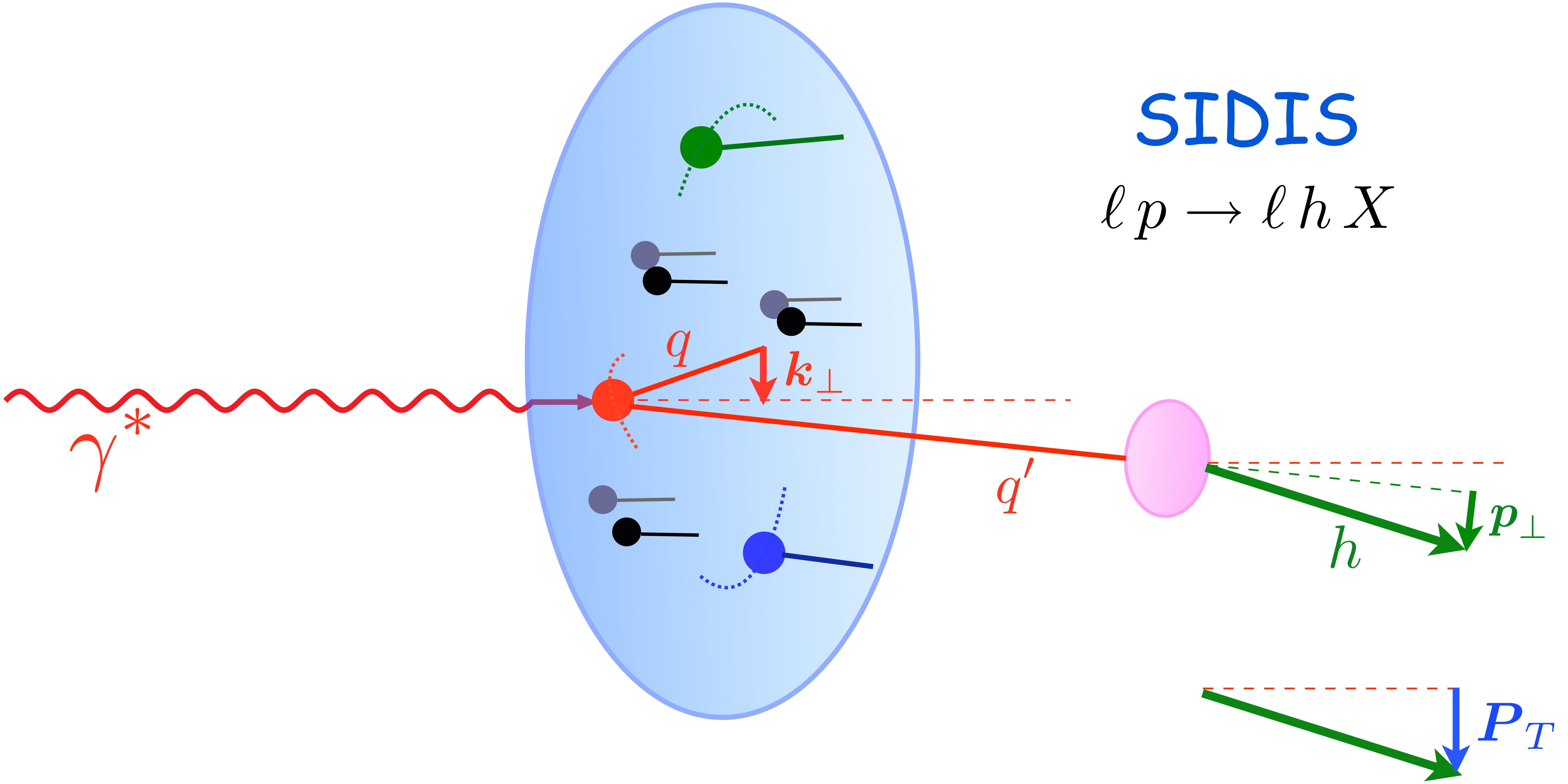
Two scales: $P_T \ll Q^2$

$$d\sigma^{\ell p \to \ell h X} = \sum_q f_q(x, \boldsymbol{k}_\perp; Q^2) \otimes d\hat{\sigma}^{\ell q \to \ell q}(y, \boldsymbol{k}_\perp; Q^2) \otimes D_q^h(z, \boldsymbol{p}_\perp; Q^2)$$

(Collins, Soper, Ji, J.P. Ma, Yuan, Qiu, Vogelsang, Collins, Metz)

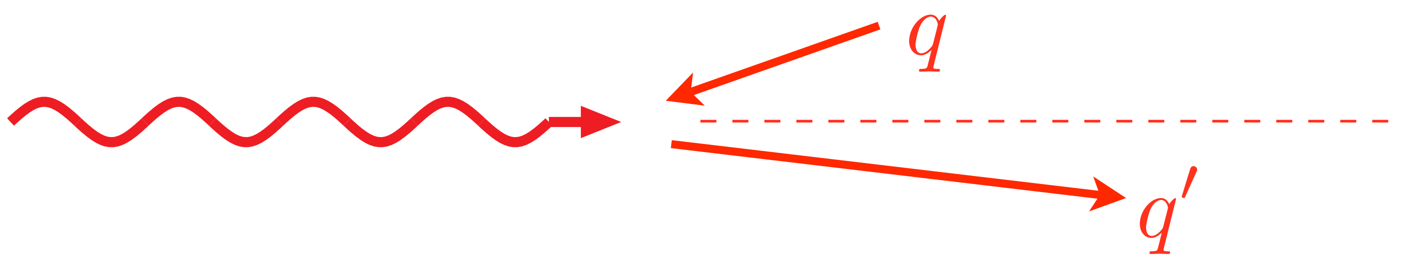
SIDIS

$$\ell\, p \to \ell\, h\, X$$



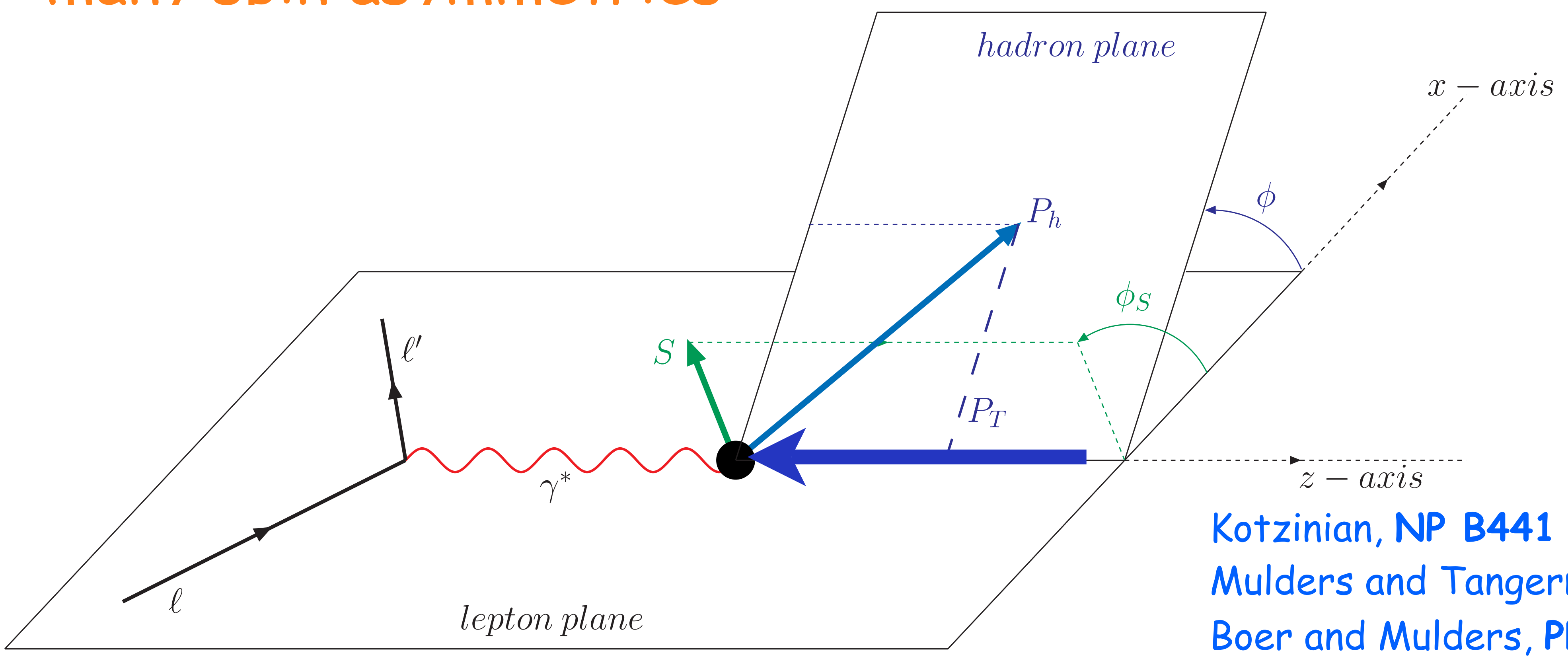
$$\Lambda_{\rm QCD} \simeq k_\perp \simeq P_T \ll Q \qquad \boldsymbol{P}_T \simeq \boldsymbol{p}_\perp + z_h\, \boldsymbol{k}_\perp$$

elementary interaction: $\gamma^*\, q \to q'$

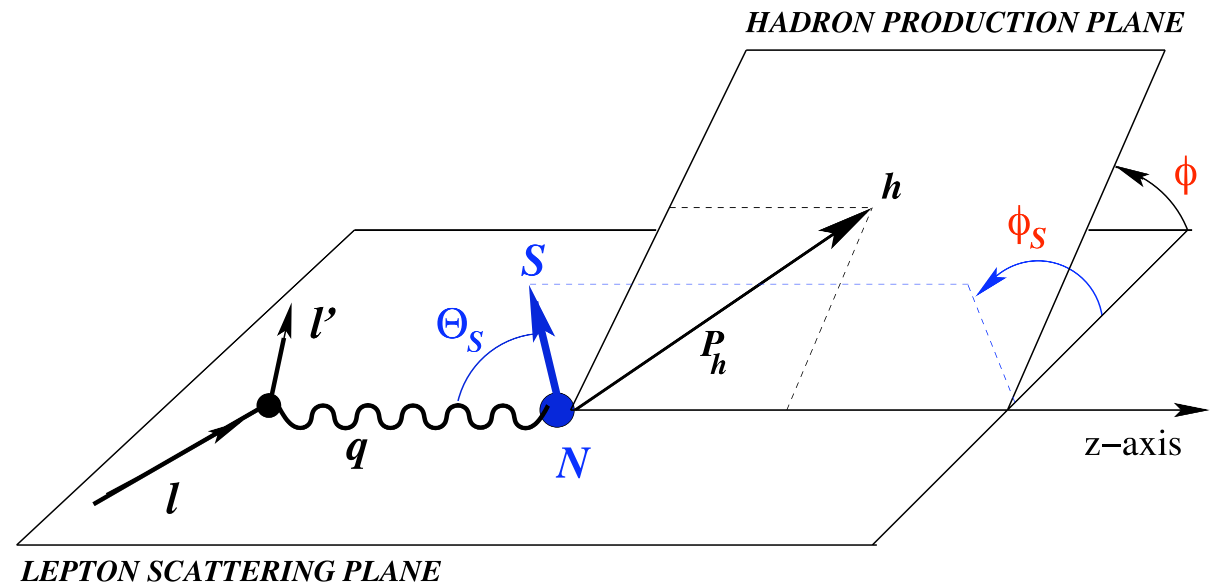


$$
\begin{aligned}
\frac{d\sigma}{d\phi} = & \; F_{UU} + \cos(2\phi)\, F_{UU}^{\cos(2\phi)} + \frac{1}{Q}\, \cos\phi\, F_{UU}^{\cos\phi} + \lambda\, \frac{1}{Q}\, \sin\phi\, F_{LU}^{\sin\phi} \\
& + S_L \left\{ \sin(2\phi)\, F_{UL}^{\sin(2\phi)} + \frac{1}{Q}\, \sin\phi\, F_{UL}^{\sin\phi} + \lambda \left[F_{LL} + \frac{1}{Q}\, \cos\phi\, F_{LL}^{\cos\phi} \right] \right\} \\
& + S_T \Bigg\{ \sin(\phi-\phi_S)\, F_{UT}^{\sin(\phi-\phi_S)} + \sin(\phi+\phi_S)\, F_{UT}^{\sin(\phi+\phi_S)} + \sin(3\phi-\phi_S)\, F_{UT}^{\sin(3\phi-\phi_S)} \\
& + \frac{1}{Q} \left[\sin(2\phi-\phi_S)\, F_{UT}^{\sin(2\phi-\phi_S)} + \sin\phi_S\, F_{UT}^{\sin\phi_S} \right] \\
& + \lambda \left[\cos(\phi-\phi_S)\, F_{LT}^{\cos(\phi-\phi_S)} + \frac{1}{Q} \left(\cos\phi_S\, F_{LT}^{\cos\phi_S} + \cos(2\phi-\phi_S)\, F_{LT}^{\cos(2\phi-\phi_S)} \right) \right] \Bigg\}
\end{aligned}
$$

many spin asymmetries



Kotzinian, **NP B441** (1995) 234
Mulders and Tangermann, **NP B461** (1996) 197
Boer and Mulders, **PR D57** (1998) 5780
Bacchetta et al., **PL B595** (2004) 309
Bacchetta et al., **JHEP 0702** (2007) 093
Anselmino et al., in preparation



$$F_{UU} \sim \sum_a e_a^2 f_1^a \otimes D_1^a \qquad F_{LT}^{\cos(\phi-\phi_S)} \sim \sum_a e_a^2 g_{1T}^{\perp a} \otimes D_1^a$$

$$F_{LL} \sim \sum_a e_a^2 g_{1L}^a \otimes D_1^a \qquad F_{UT}^{\sin(\phi-\phi_S)} \sim \sum_a e_a^2 f_{1T}^{\perp a} \otimes D_1^a$$

chiral-even TMDs

$$F_{UU}^{\cos(2\phi)} \sim \sum_a e_a^2 h_1^{\perp a} \otimes H_1^{\perp a} \qquad F_{UT}^{\sin(\phi+\phi_S)} \sim \sum_a e_a^2 h_{1T}^a \otimes H_1^{\perp a}$$

$$F_{UL}^{\sin(2\phi)} \sim \sum_a e_a^2 h_{1L}^{\perp a} \otimes H_1^{\perp a} \qquad F_{UT}^{\sin(3\phi-\phi_S)} \sim \sum_a e_a^2 h_{1T}^{\perp a} \otimes H_1^{\perp a}$$

chiral-odd TMDs

$$\frac{1}{Q} \cos\phi \, F_{UU}^{\cos\phi} \sim f_1^q \otimes D_1^q \otimes \mathrm{d}\hat{\sigma} + \left(h_1^{q\perp} \otimes H_1^{q\perp} \otimes \mathrm{d}\Delta\hat{\sigma} \right)$$

Cahn kinematical effects

integrated $f_1^q(x)$ and $g_{1L}^q(x)$ can be measured in usual DIS

Spin dependent TMDs

Sivers function

in momentum space

$$\begin{aligned} f_{q/p,\boldsymbol{S}}(x,\boldsymbol{k}_\perp) &= f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp) \\ &= f_{q/p}(x,k_\perp) - \frac{k_\perp}{M} f_{1T}^{\perp q}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp) \end{aligned}$$

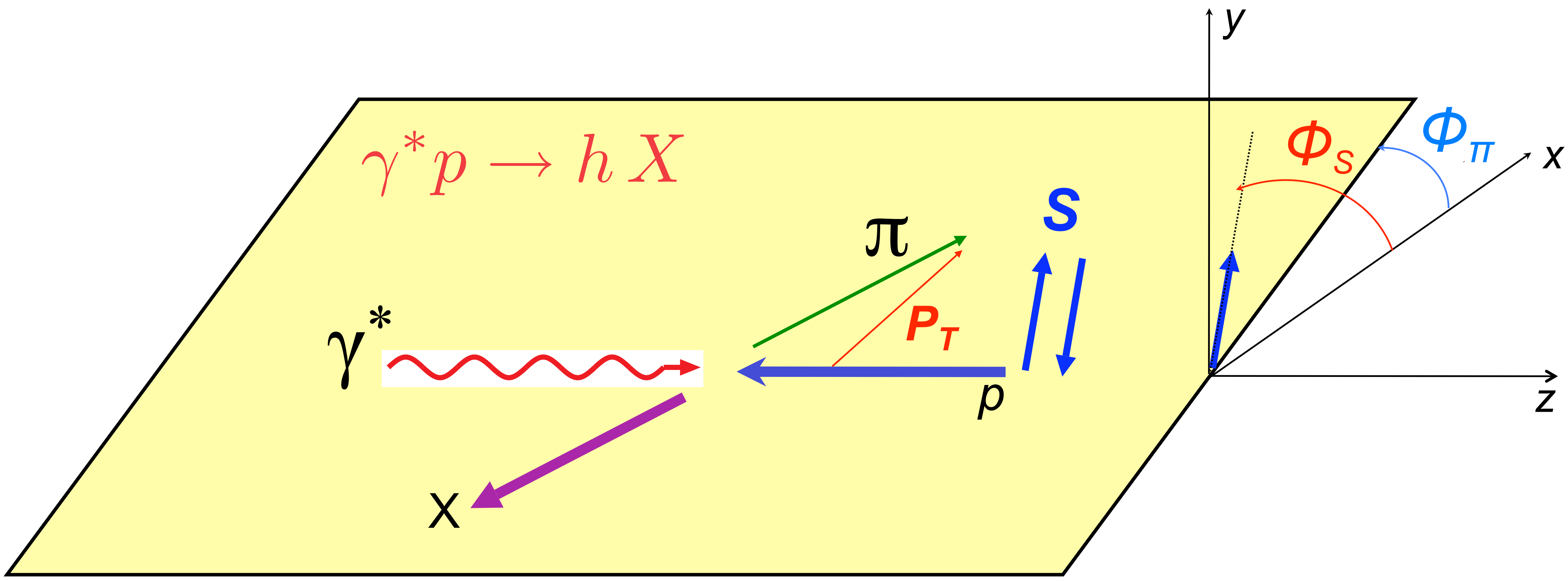
in configuration space

$$\begin{aligned} f_{q/p,\boldsymbol{S}}(x,\boldsymbol{b}_T) &= \int \frac{d^2\boldsymbol{\Delta}_T}{(2\pi)^2}\, e^{-i\,\boldsymbol{b}_T\cdot\boldsymbol{\Delta}_T} \\ &\times \left[H_q(x,0,-\boldsymbol{\Delta}_T^2) + i\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{\Delta}}_T)\,E_q(x,0,-\boldsymbol{\Delta}_T^2)\right] \end{aligned}$$

Sivers SSA in SIDIS

probing polarized nucleons: transverse single spin asymmetries in SIDIS

$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$



$$A_N \propto \boldsymbol{S} \cdot (\boldsymbol{p} \times \boldsymbol{P}_T) \propto P_T \sin(\phi_\pi - \phi_S)$$

Large Q^2: the virtual photon explores the nucleon structure.

In collinear configurations there cannot be (at LO) any P_T

Sivers effect in SIDIS - $F_{UT}^{\sin(\phi-\phi_S)}$

$f_{1T}^{\perp q}(x, \boldsymbol{k}_\perp^2)$

$$d\sigma^{\uparrow,\downarrow} = \sum_q f_{q/p^{\uparrow,\downarrow}}(x, \boldsymbol{k}_\perp; Q^2) \otimes d\hat{\sigma}(y, \boldsymbol{k}_\perp; Q^2) \otimes D_{h/q}(z, \boldsymbol{p}_\perp; Q^2)$$

$$d\sigma^{\uparrow} - d\sigma^{\downarrow} =$$

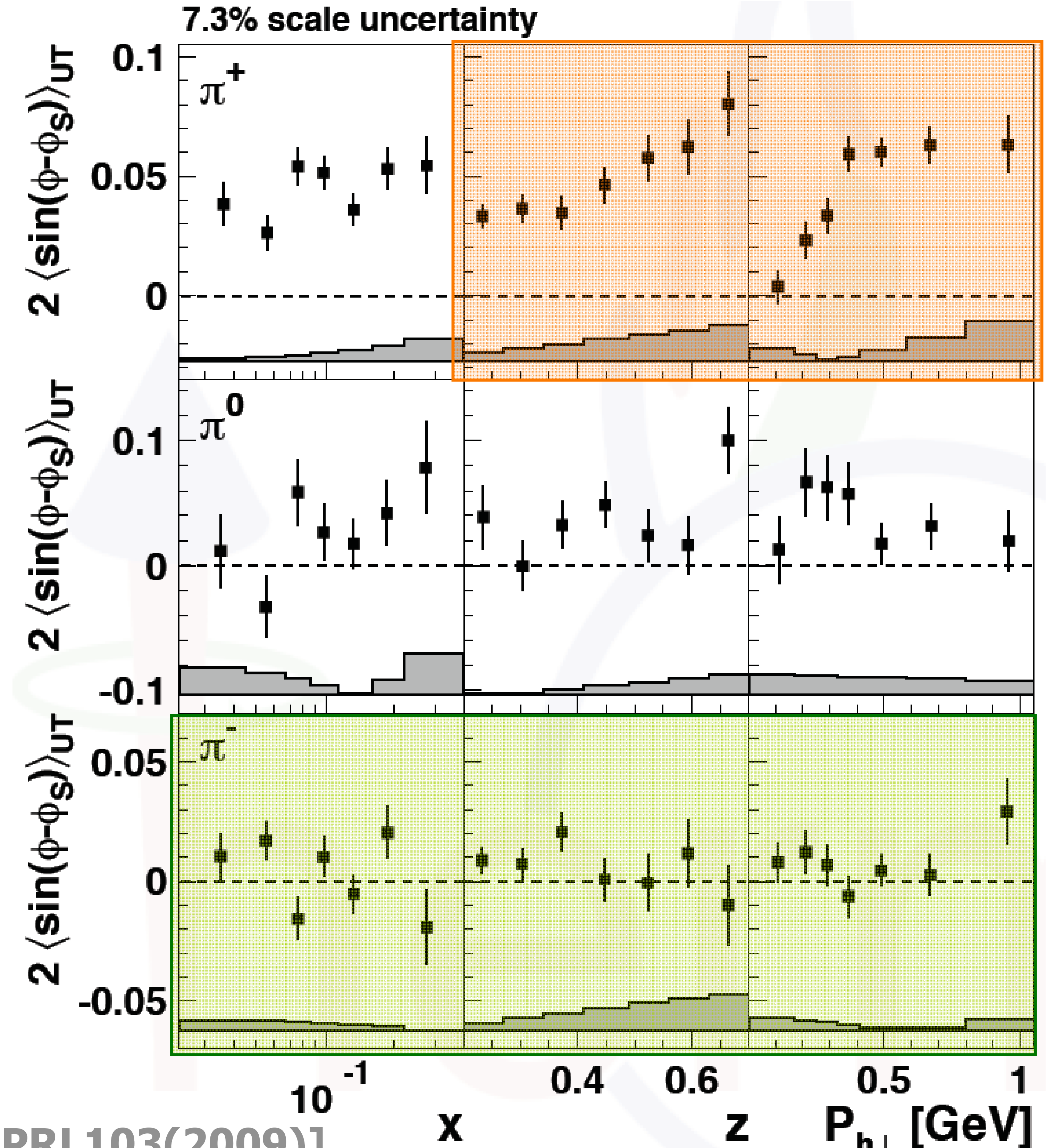
$$\sum_q \Delta^N f_{q/p^\uparrow}(x, k_\perp) \underbrace{\boldsymbol{S} \cdot (\hat{\boldsymbol{p}} \times \hat{\boldsymbol{k}}_\perp)}_{\sin(\varphi - \phi_S)} \otimes d\hat{\sigma}(y, \boldsymbol{k}_\perp) \otimes D_{h/q}(z, \boldsymbol{p}_\perp)$$

$$\sim F_{UT}^{\sin(\phi-\phi_S)} \sin(\phi - \phi_S)$$

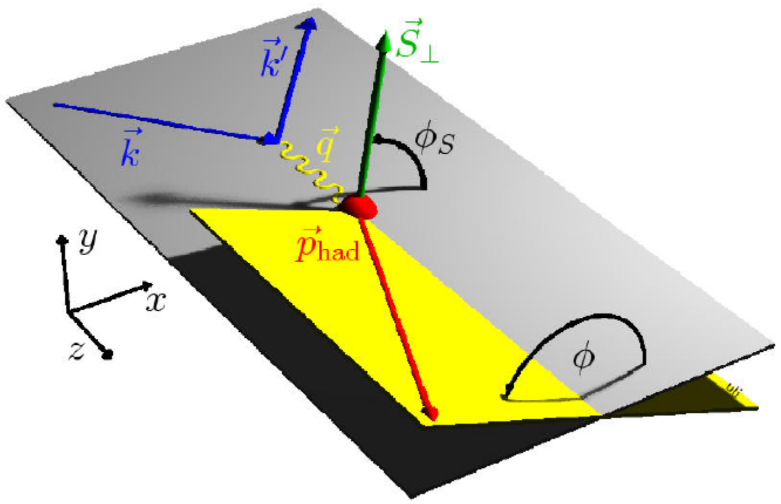
measured quantity

$$\left\{ \begin{array}{l} 2\langle \sin(\phi - \phi_S) \rangle = A_{UT}^{\sin(\phi-\phi_S)} \\ \equiv 2 \dfrac{\int d\phi \, d\phi_S \, [d\sigma^\uparrow - d\sigma^\downarrow] \sin(\phi - \phi_S)}{\int d\phi \, d\phi_S \, [d\sigma^\uparrow + d\sigma^\downarrow]} \end{array} \right.$$

HERMES new data on pion Sivers asymmetry



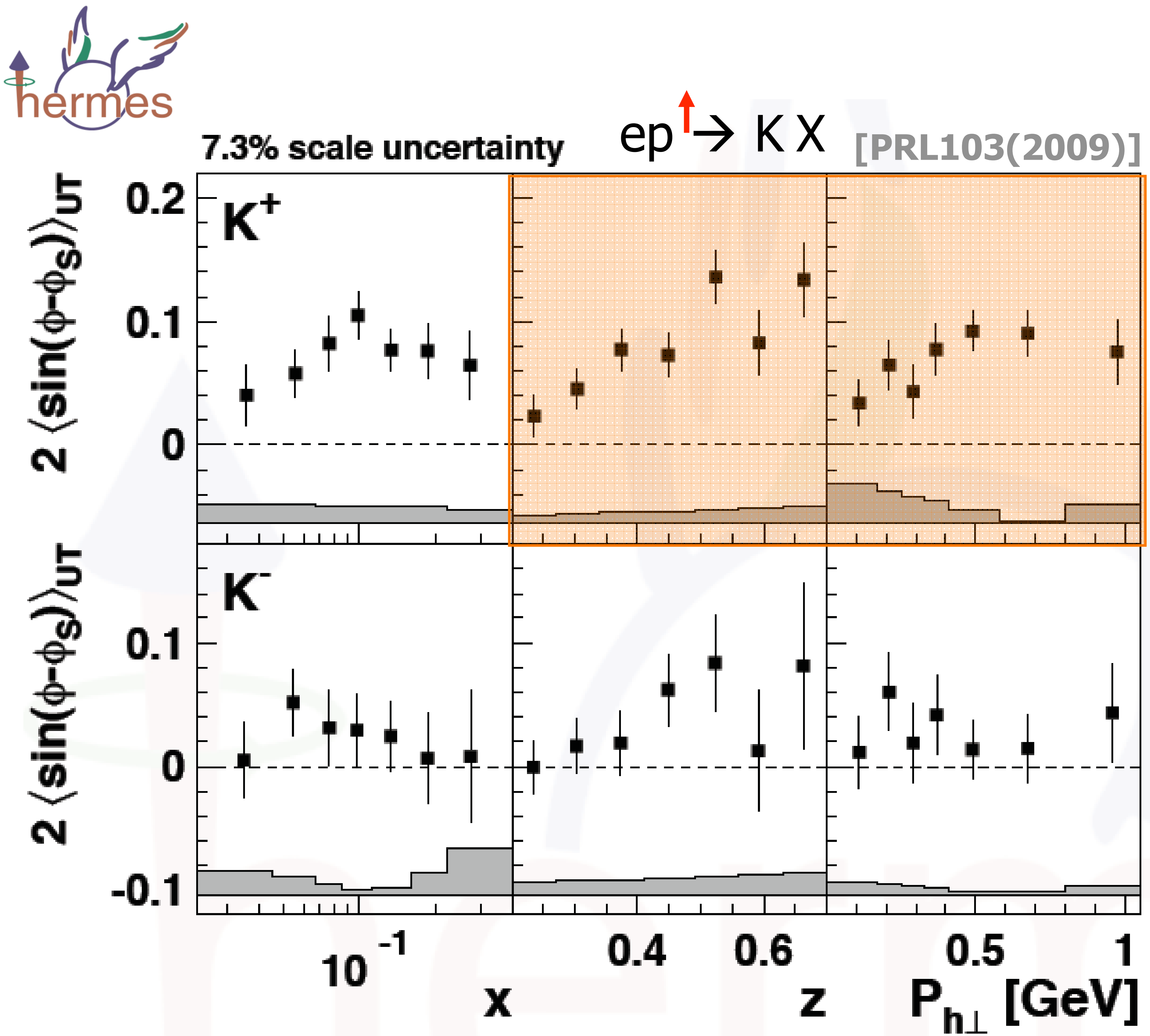
[PRL103(2009)]



$$2\left\langle \sin(\phi-\phi_S)\right\rangle = A_{UT}^{\sin(\phi-\phi_S)}$$

$$\equiv 2\,\frac{\int d\phi\, d\phi_S\, (d\sigma^{\uparrow}-d\sigma^{\downarrow})\,\sin(\phi-\phi_S)}{\int d\phi\, d\phi_S\, (d\sigma^{\uparrow}+d\sigma^{\downarrow})}$$

HERMES kaon Sivers asymmetry



Sivers parameterization

$$\Delta^N f_{q/p^\uparrow}(x,k_\perp) = 2\mathcal{N}_q(x)\, h(k_\perp)\, f_{q/p}(x,k_\perp)$$

$$\mathcal{N}_q(x) = N_q\, x^{\alpha_q}(1-x)^{\beta_q}\, \frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

$$h(k_\perp) = \sqrt{2e}\,\frac{k_\perp}{M_1}\, e^{-k_\perp^2/M_1^2} \qquad |N_q| \le 1$$

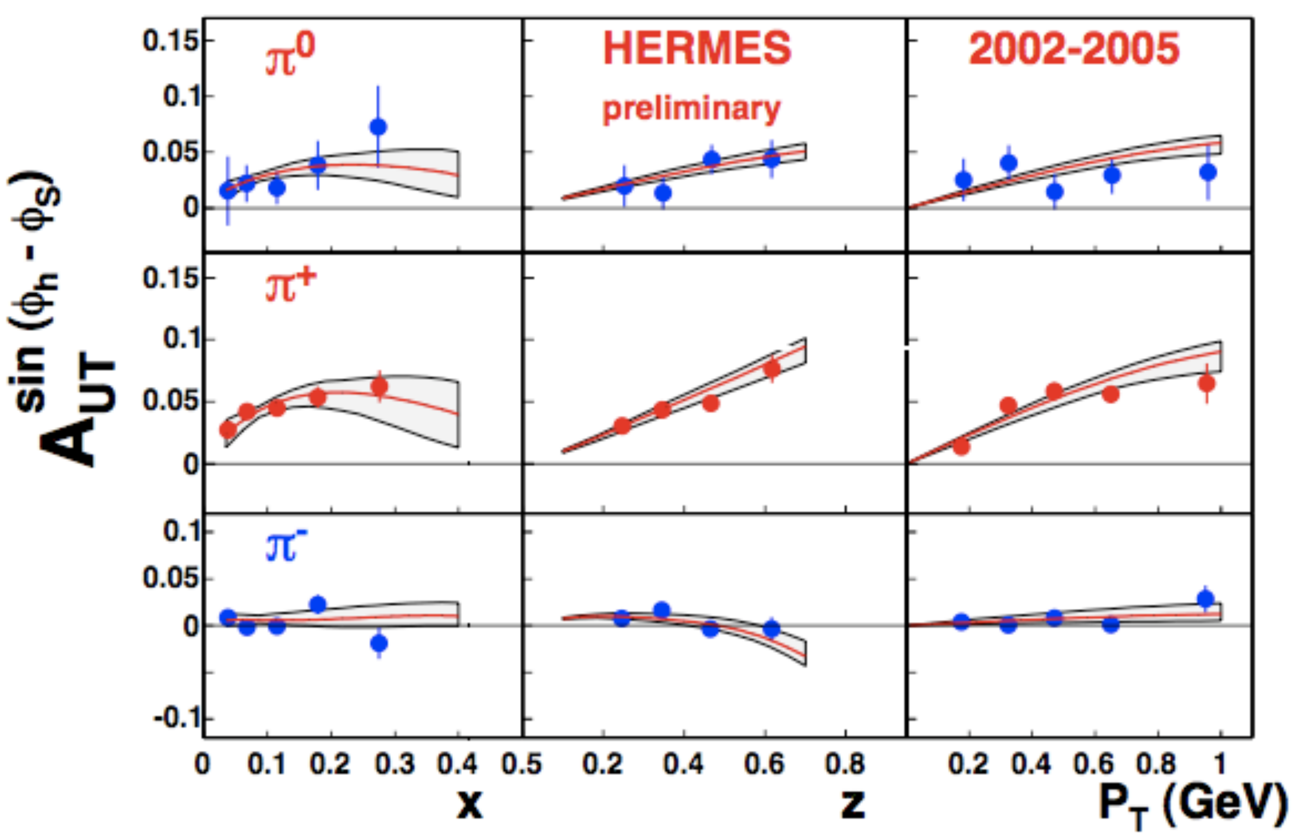
$$f_{q/p}(x,k_\perp) = f_q(x)\,\frac{1}{\pi\langle k_\perp^2\rangle}\, e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

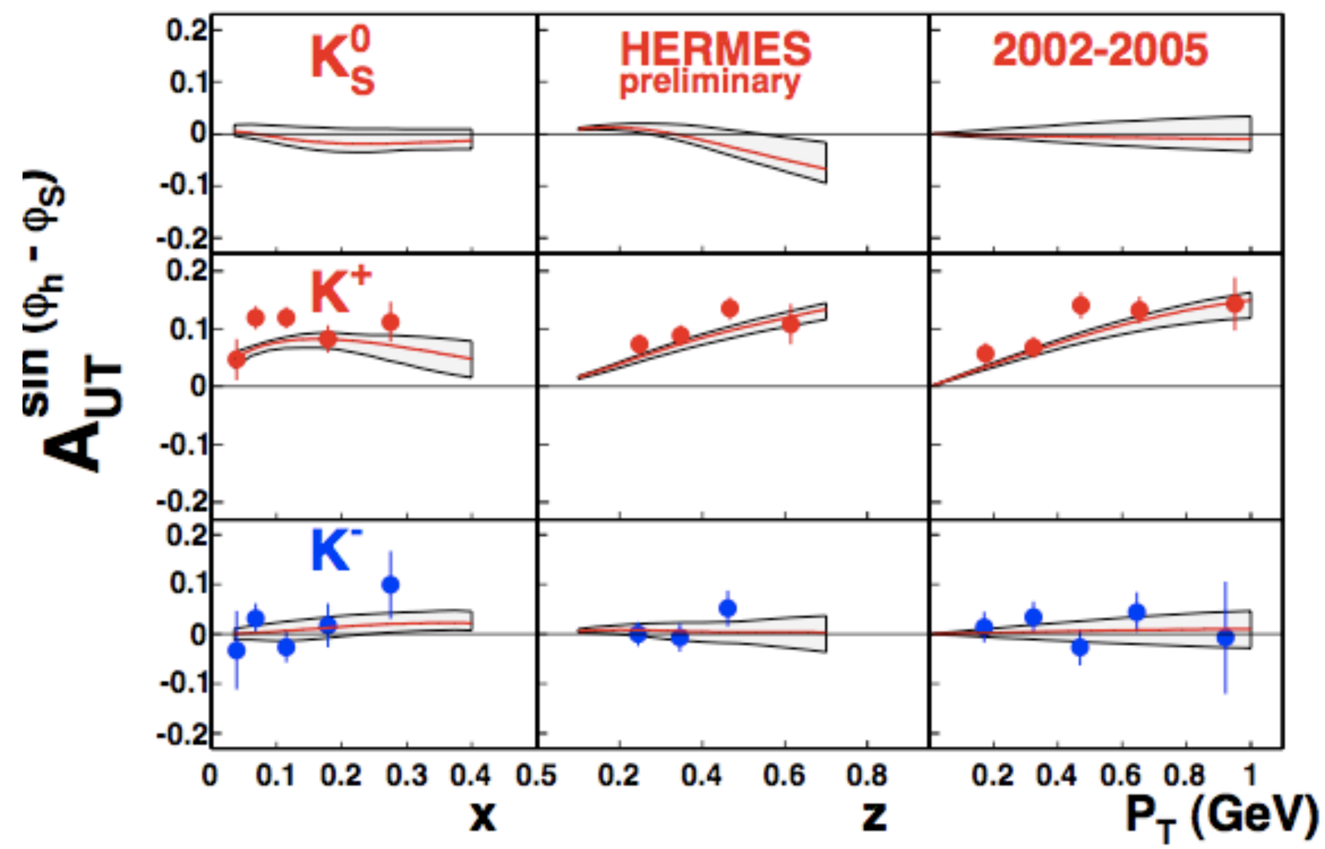
$$D_q^h(z,p_\perp) = D_q^h(z)\,\frac{1}{\pi\langle p_\perp^2\rangle}\, e^{-p_\perp^2/\langle p_\perp^2\rangle}$$

$$\langle k_\perp^2\rangle = 0.25\ (\mathrm{GeV}/c)^2 \qquad \langle k_\perp^2\rangle = 0.20\ (\mathrm{GeV}/c)^2$$

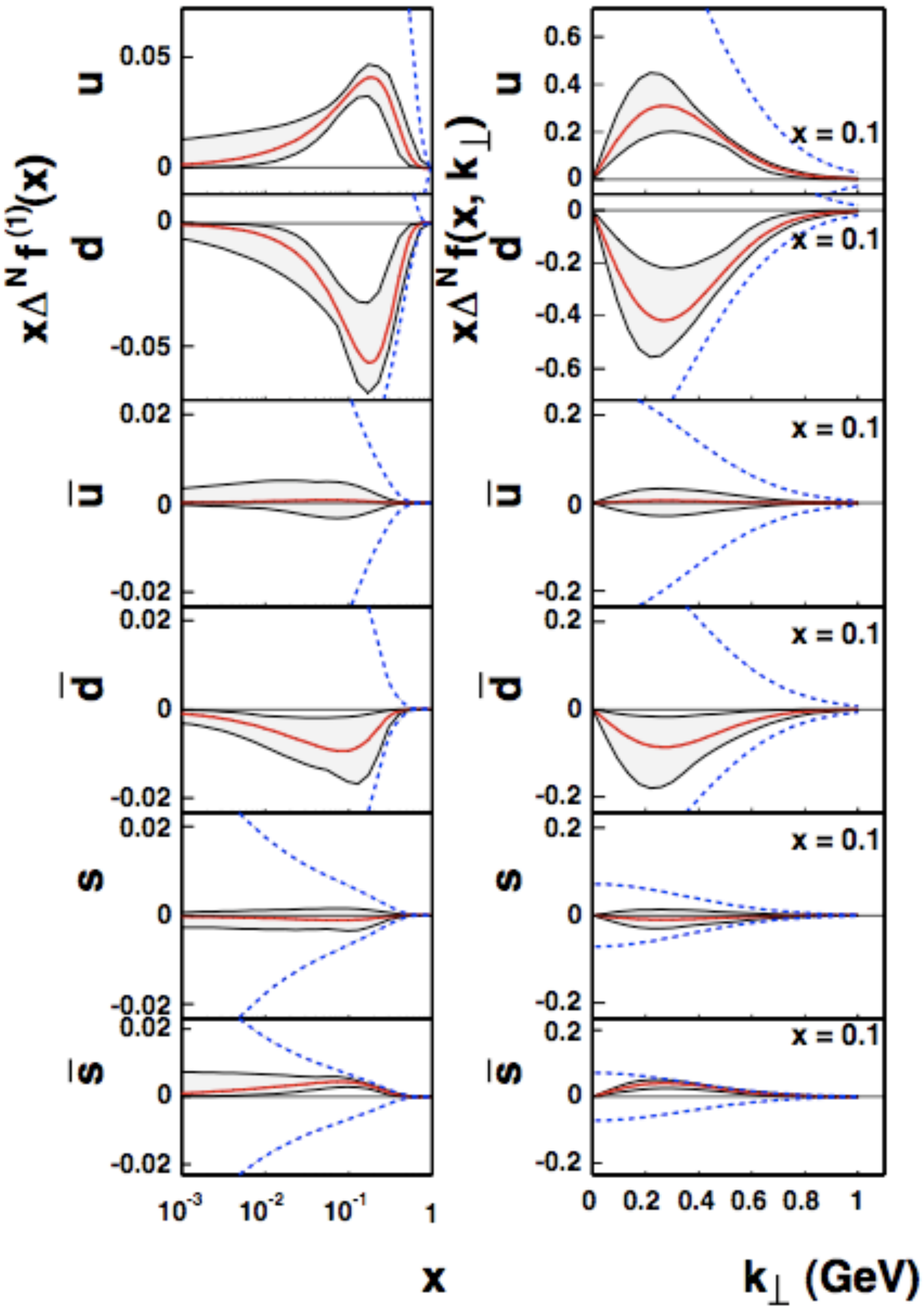
(from fitting $\cos\phi$ data in unpolarized cross section)

Sivers functions:
old fit - old data





M.A., M. Boglione, U. D'Alesio, A. Kotzinian, S. Melis, F. Murgia, A. Prokudin, C. Türk
EPJA 39, 89 (2009)



extracted Sivers functions, old fit

(from HERMES old proton and COMPASS deuteron data)

$$\Delta^N f_{u/p^\uparrow} > 0$$

$$\Delta^N f_{d/p^\uparrow} < 0$$

$$\Delta^N f_{\bar{s}/p^\uparrow} > 0$$

11 parameters

N_u	N_d	N_s
$N_{\bar{u}}$	$N_{\bar{d}}$	$N_{\bar{s}}$
α_u	α_d	α_{sea}
β	$M_1(\mathrm{GeV/c})$	

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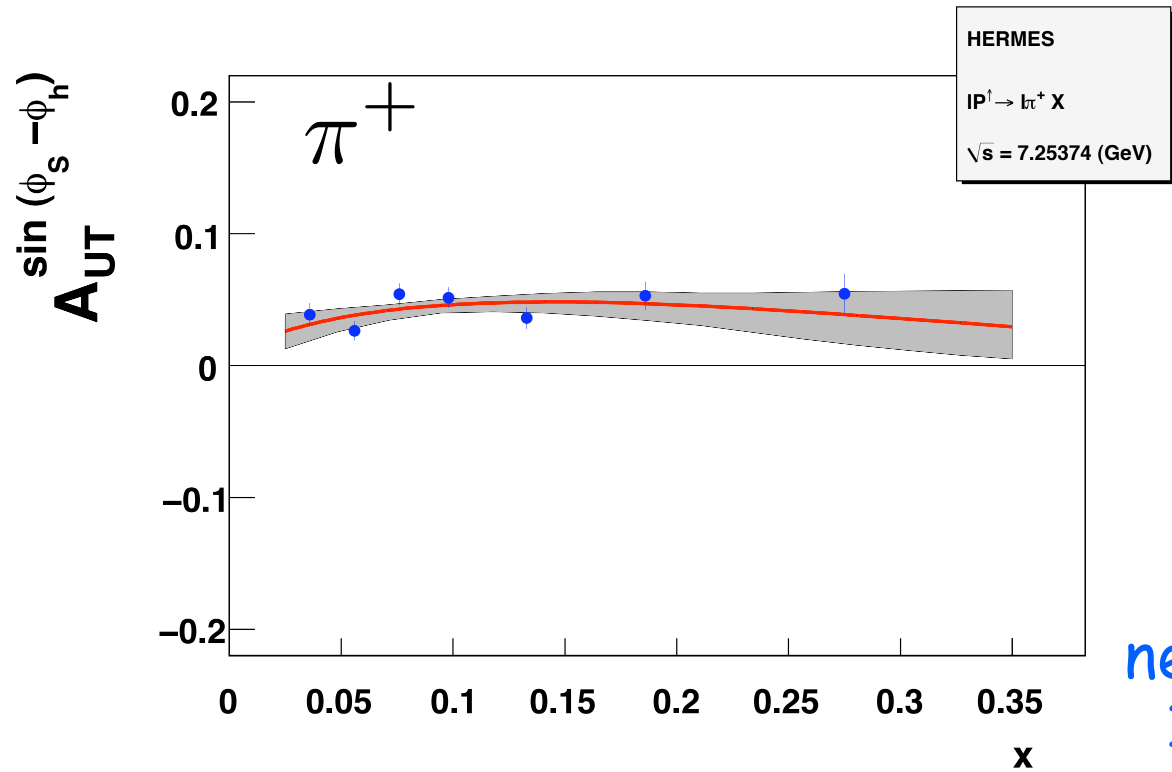
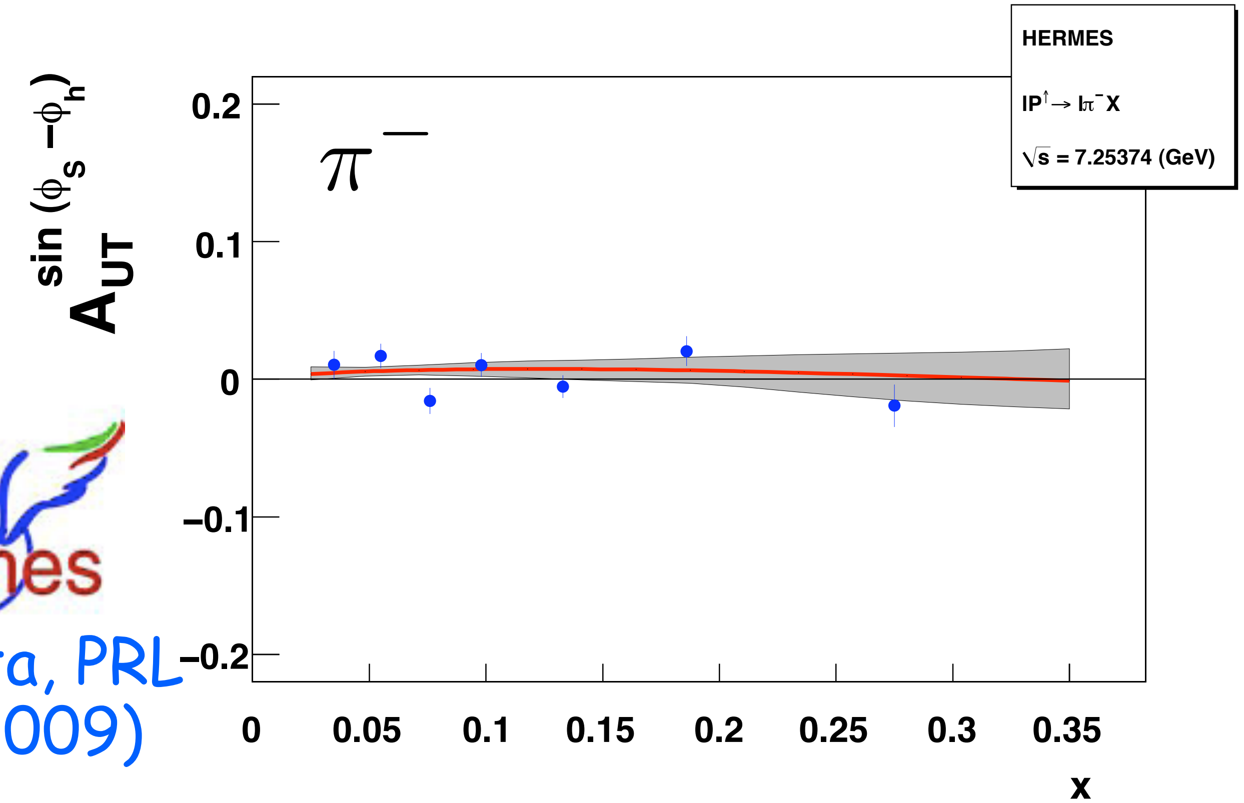
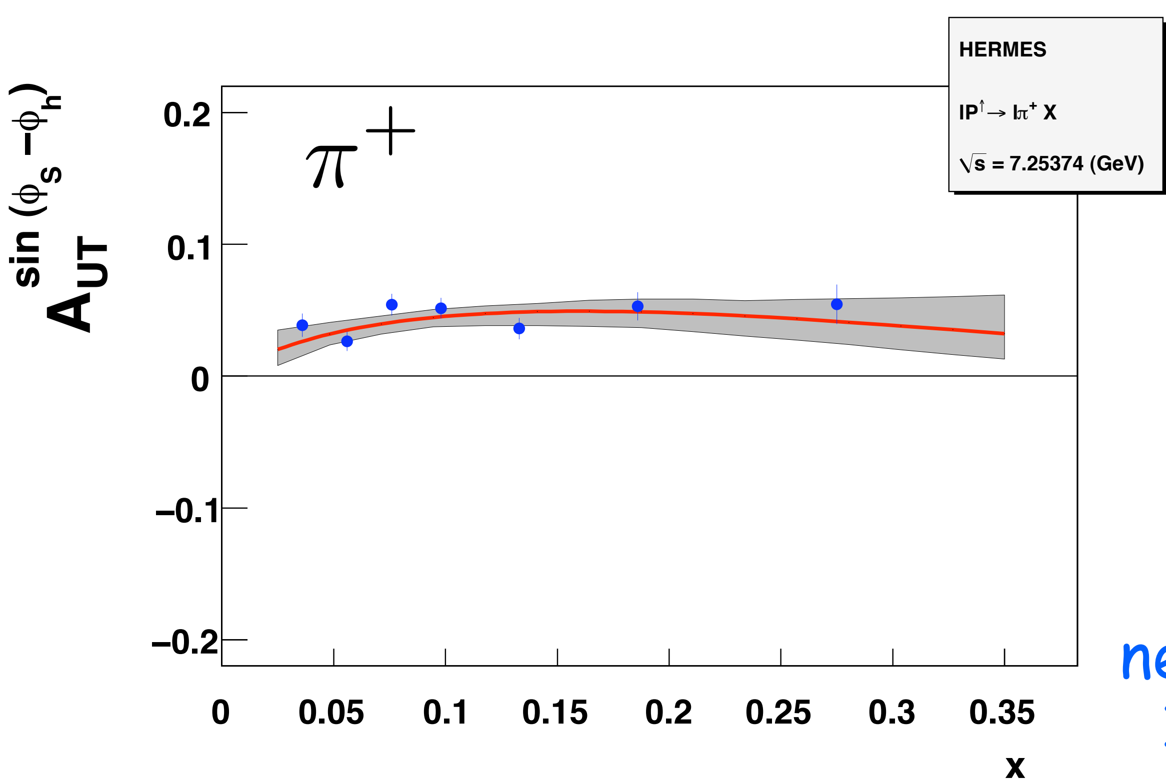
Sivers asymmetry, new fit, valence quarks only, no sea



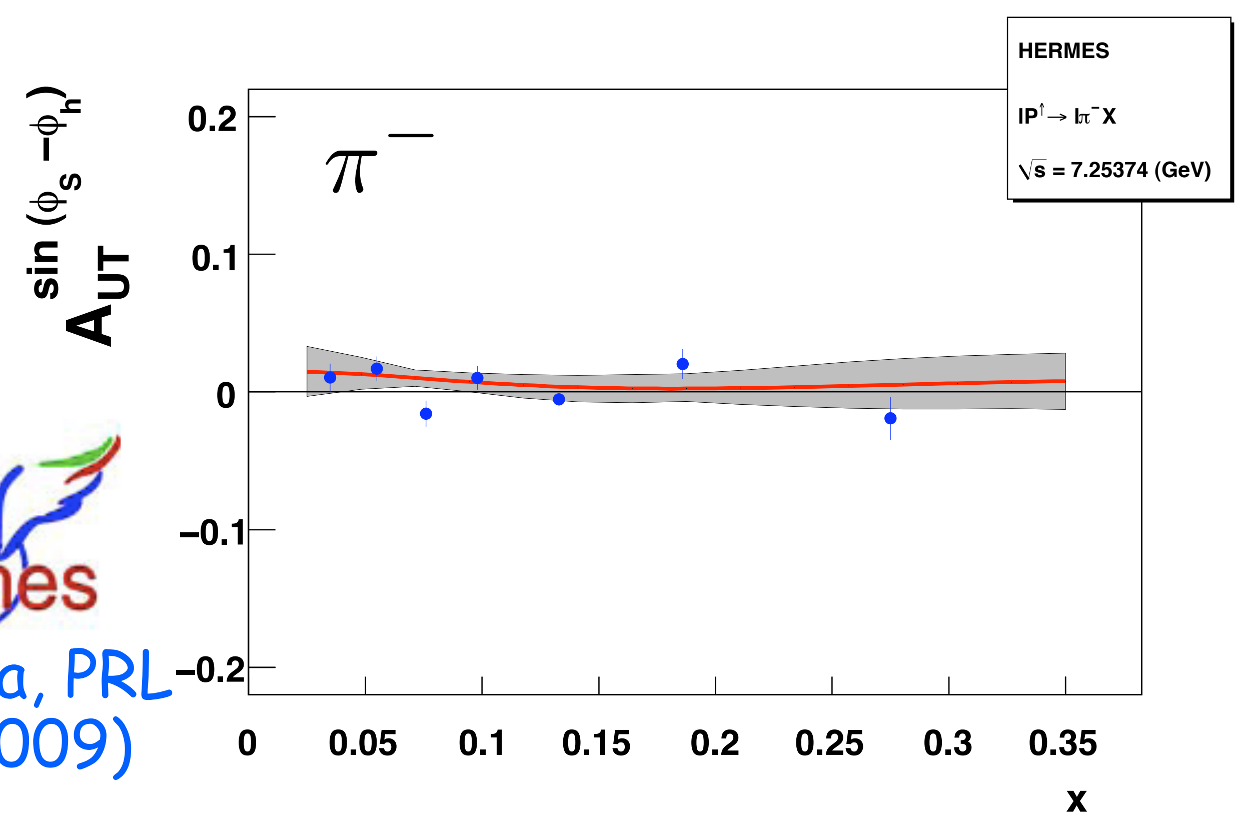
new data, PRL 103 (2009)



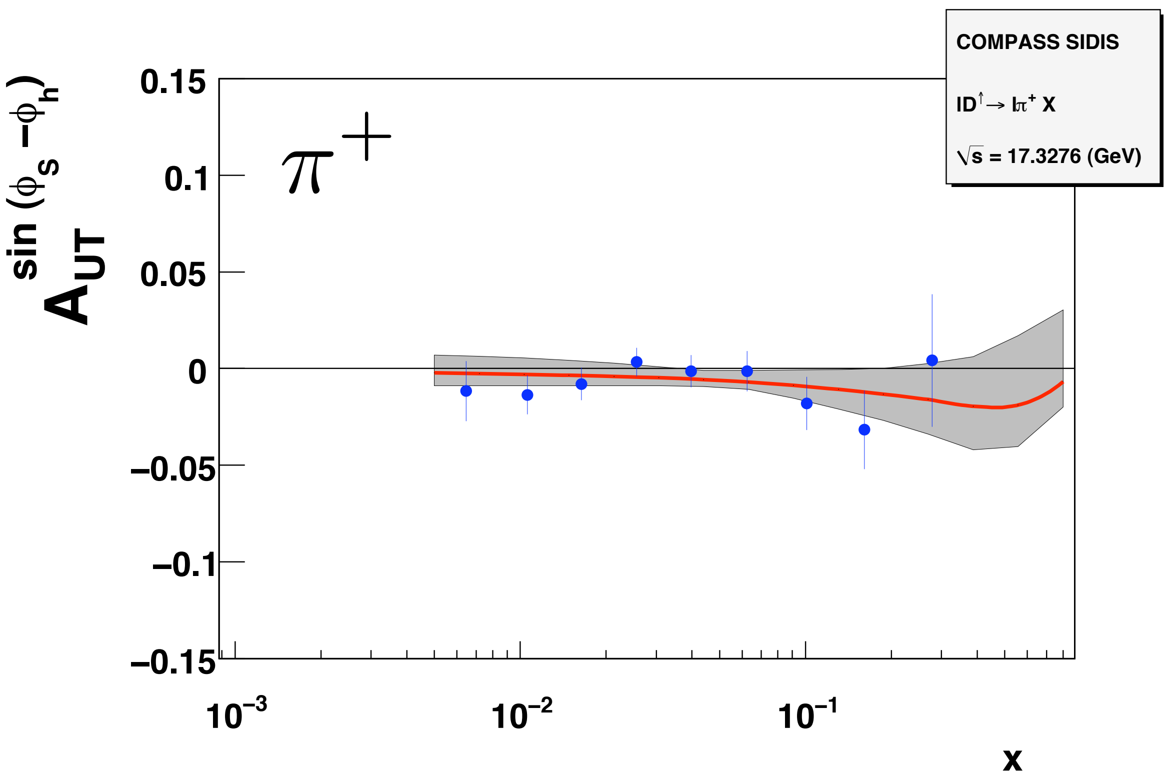
Sivers asymmetry, new fit, valence quarks + sea



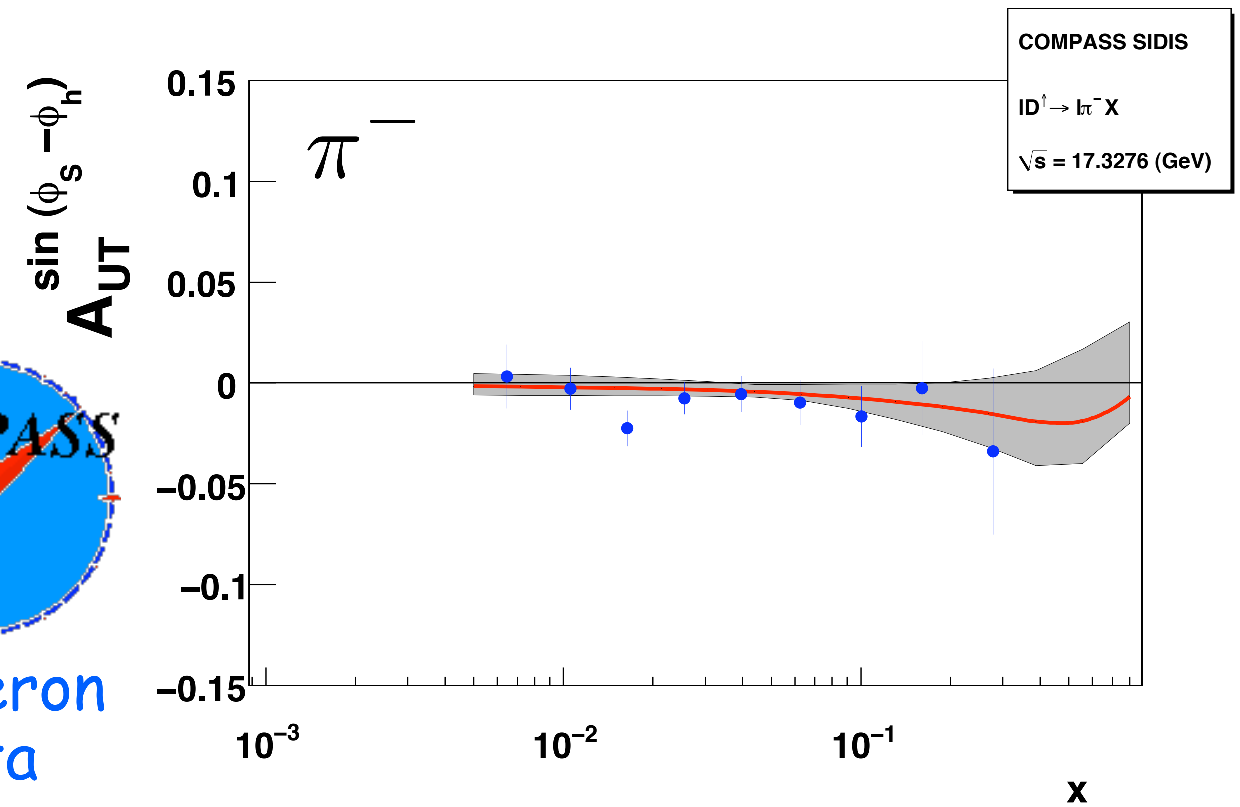
new data, PRL 103 (2009)



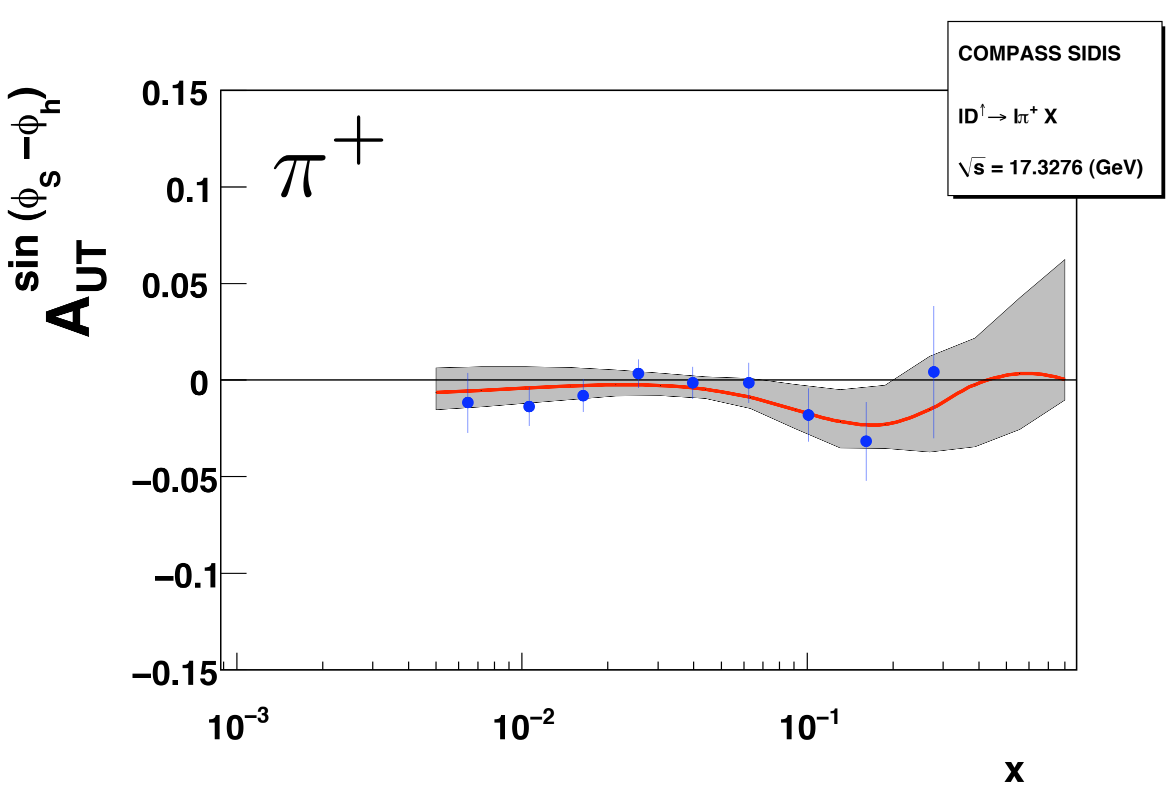
Sivers asymmetry, new fit, valence quarks only, no sea



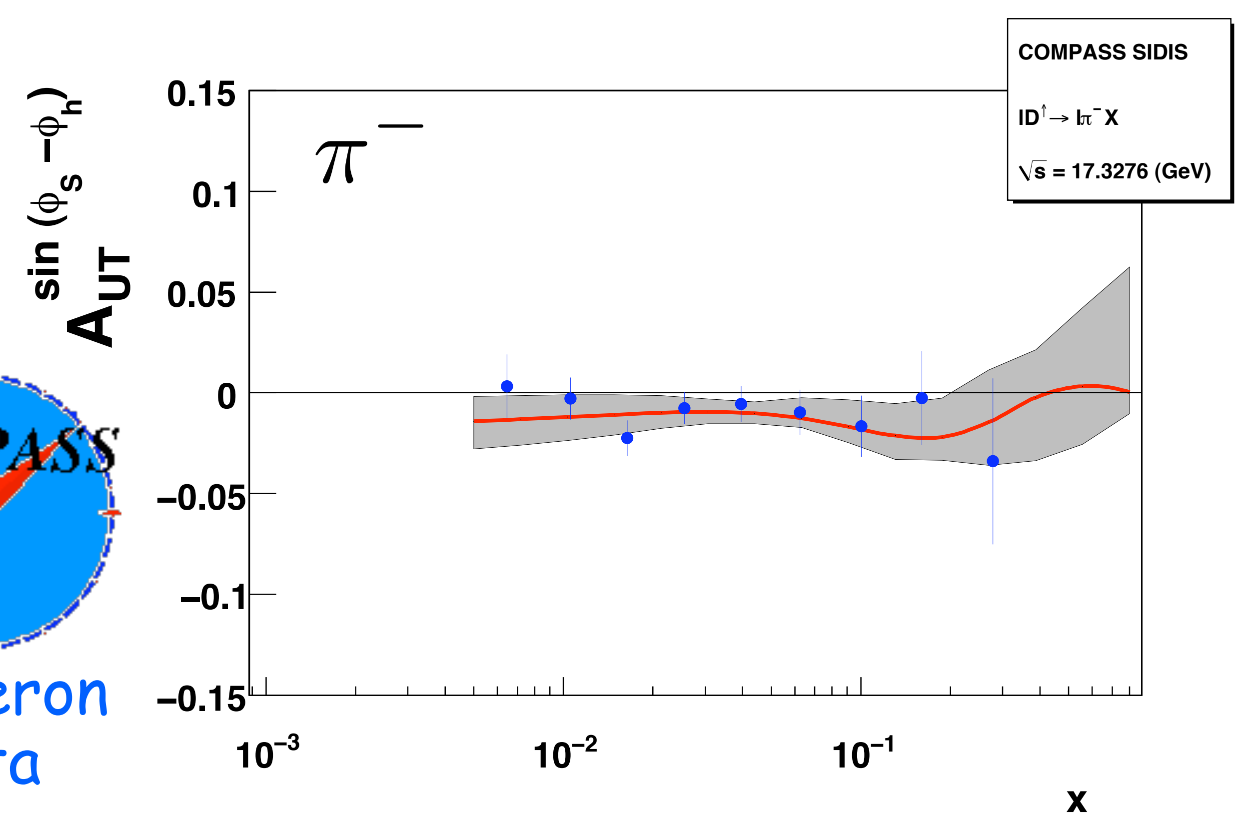
deuteron data



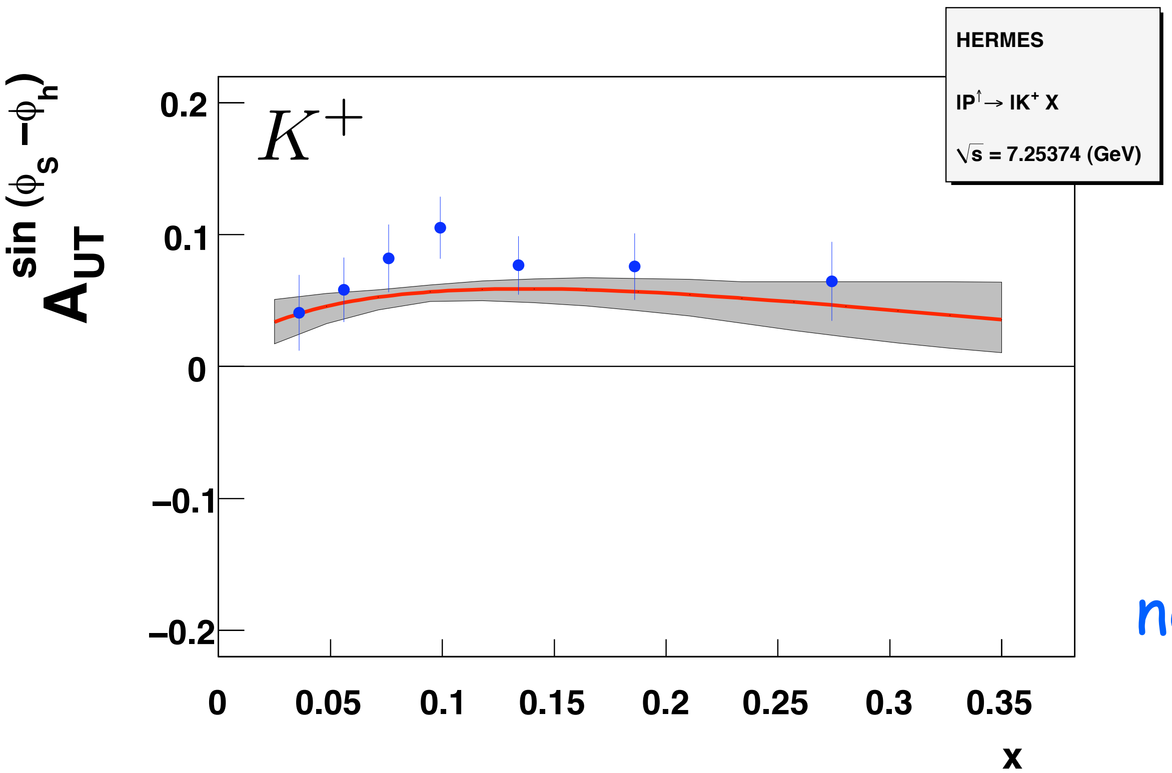
Sivers asymmetry, new fit, valence quarks + sea



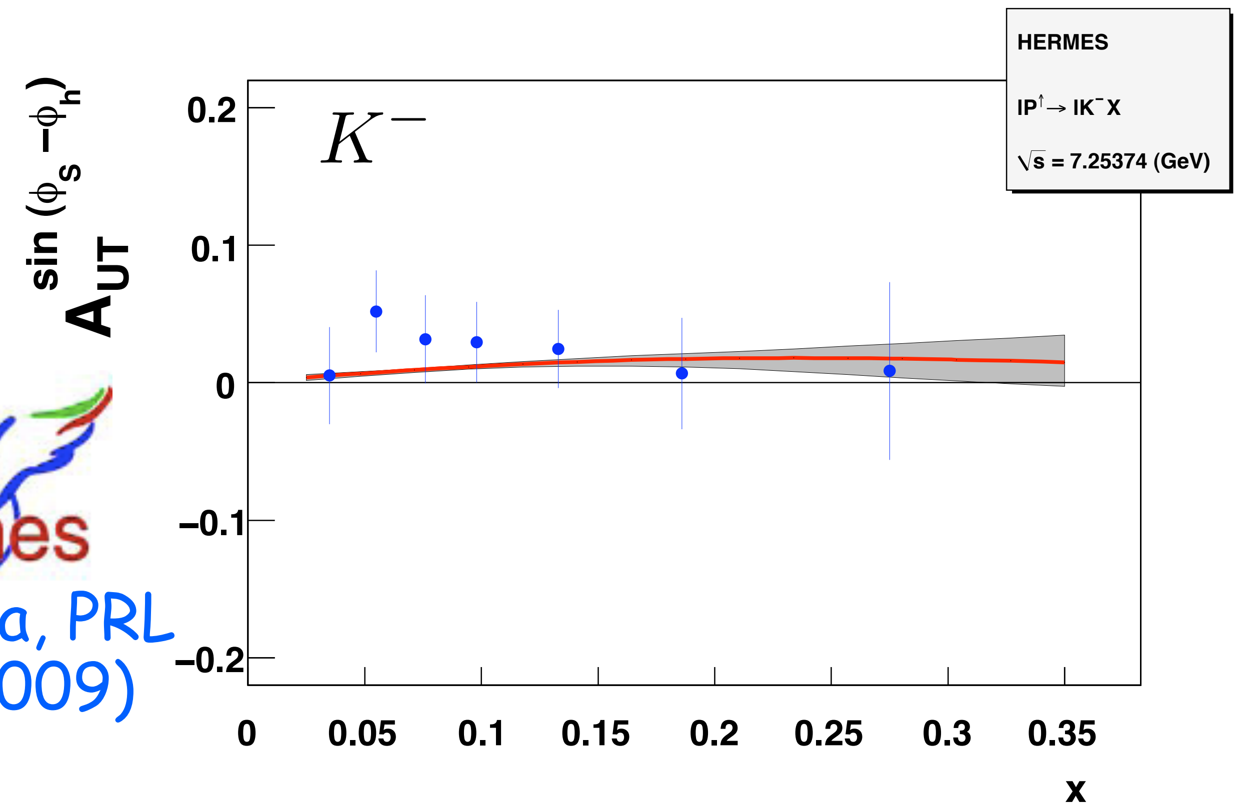
deuteron data



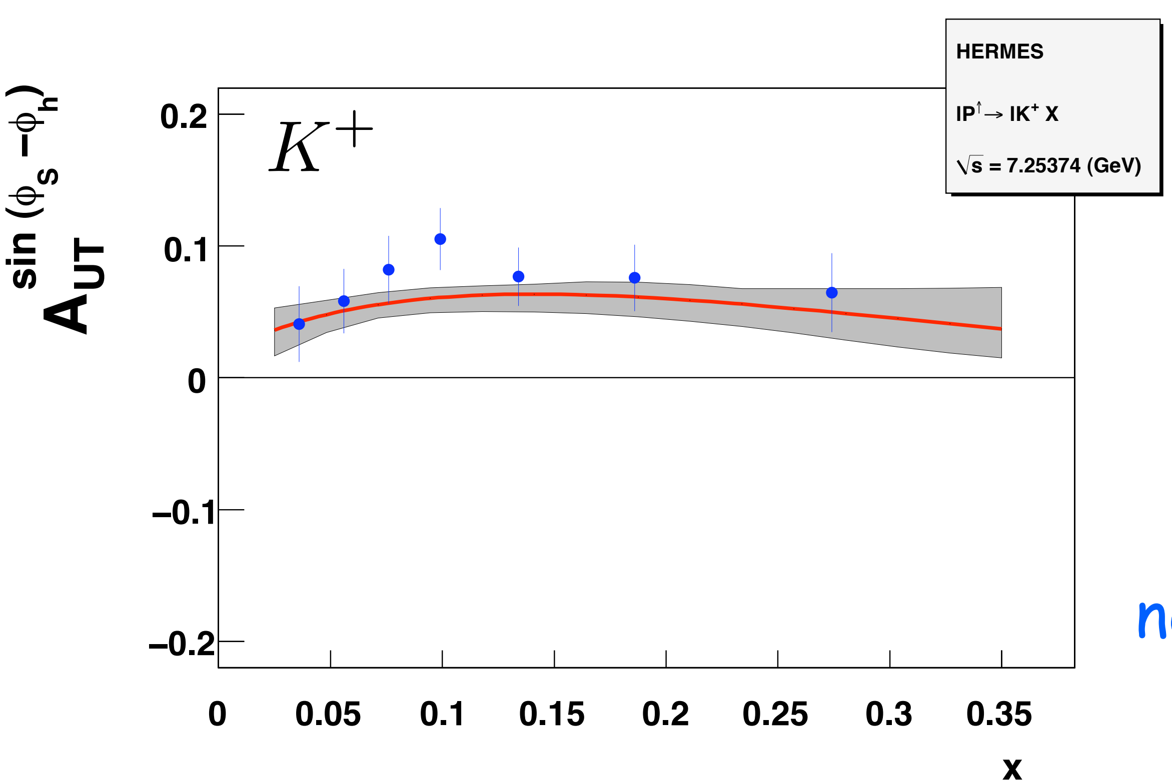
Sivers asymmetry, new fit, valence quarks only, no sea



new data, PRL 103 (2009)



Sivers asymmetry, new fit, valence quarks + sea



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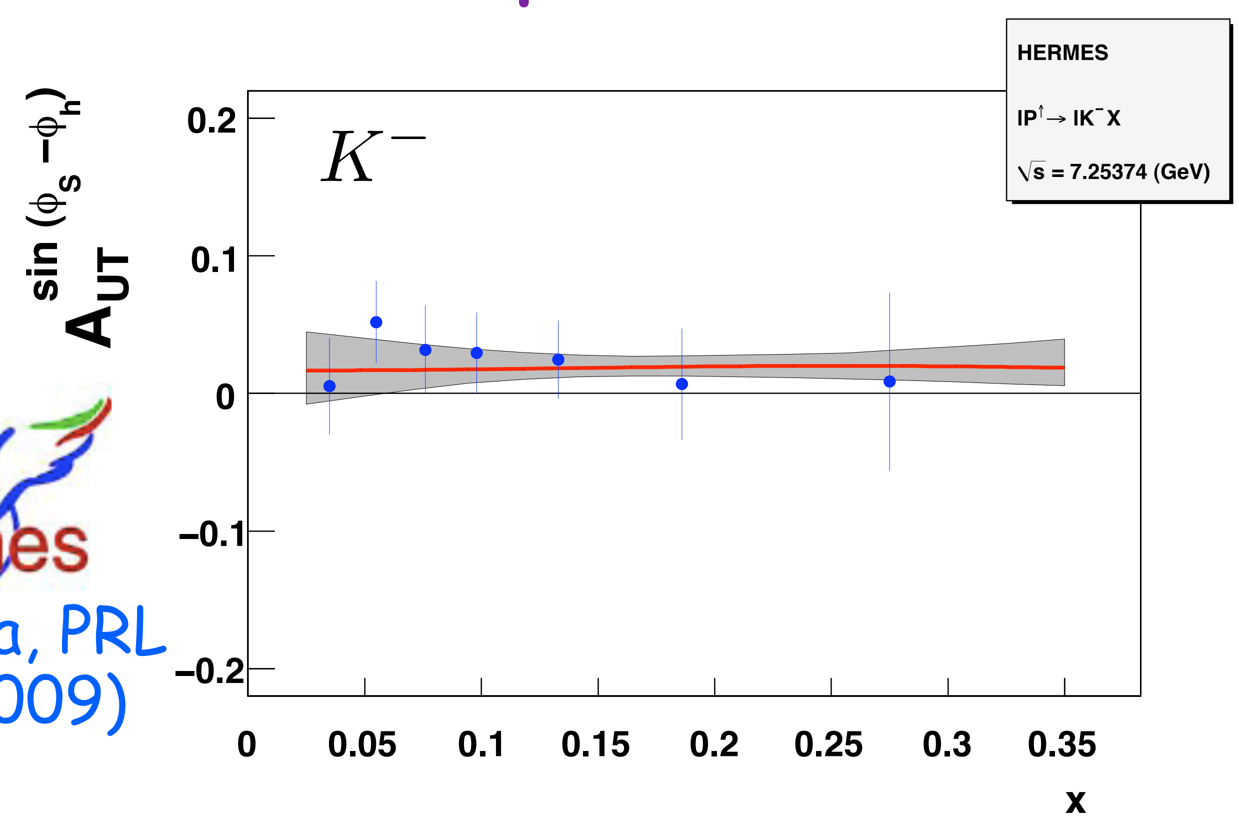
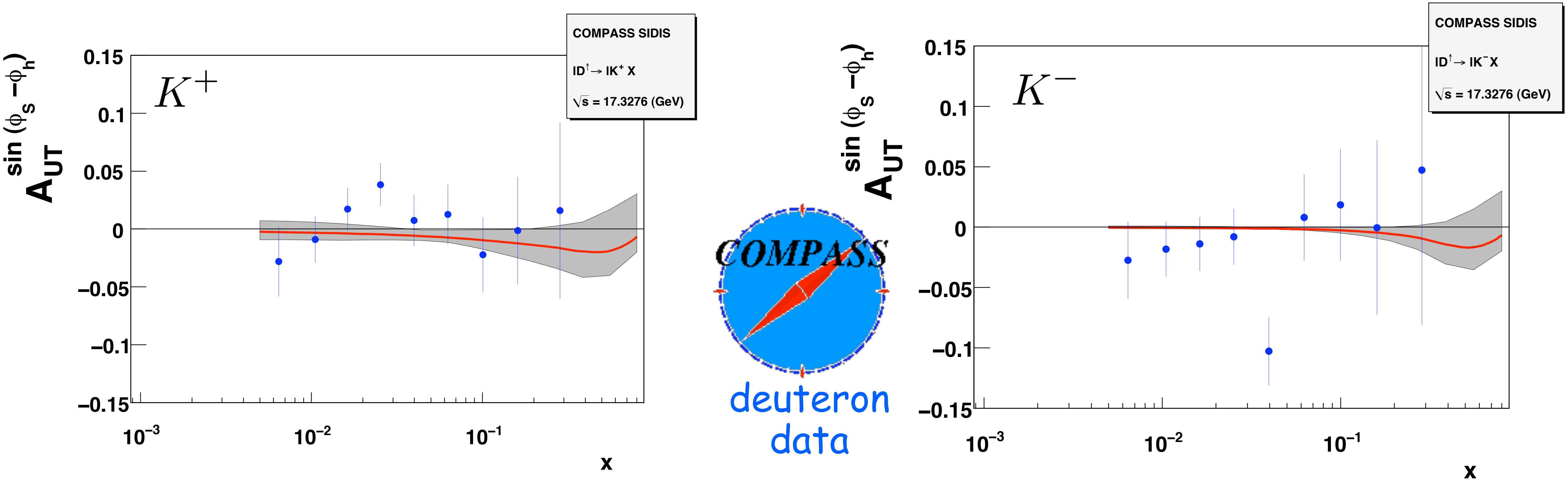
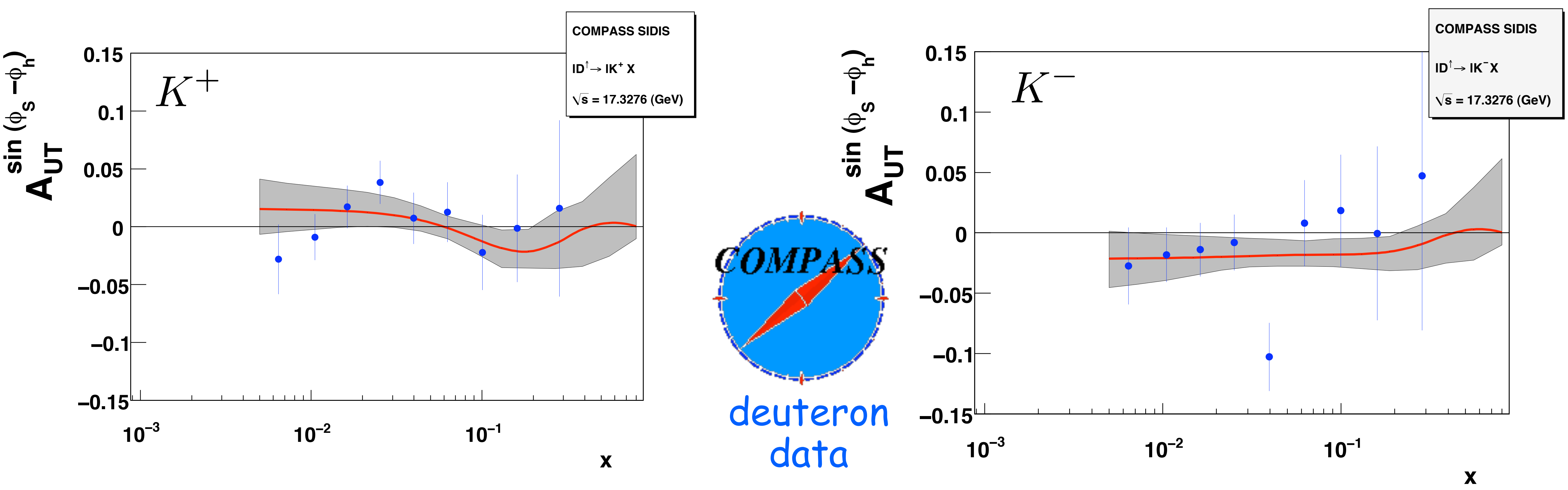


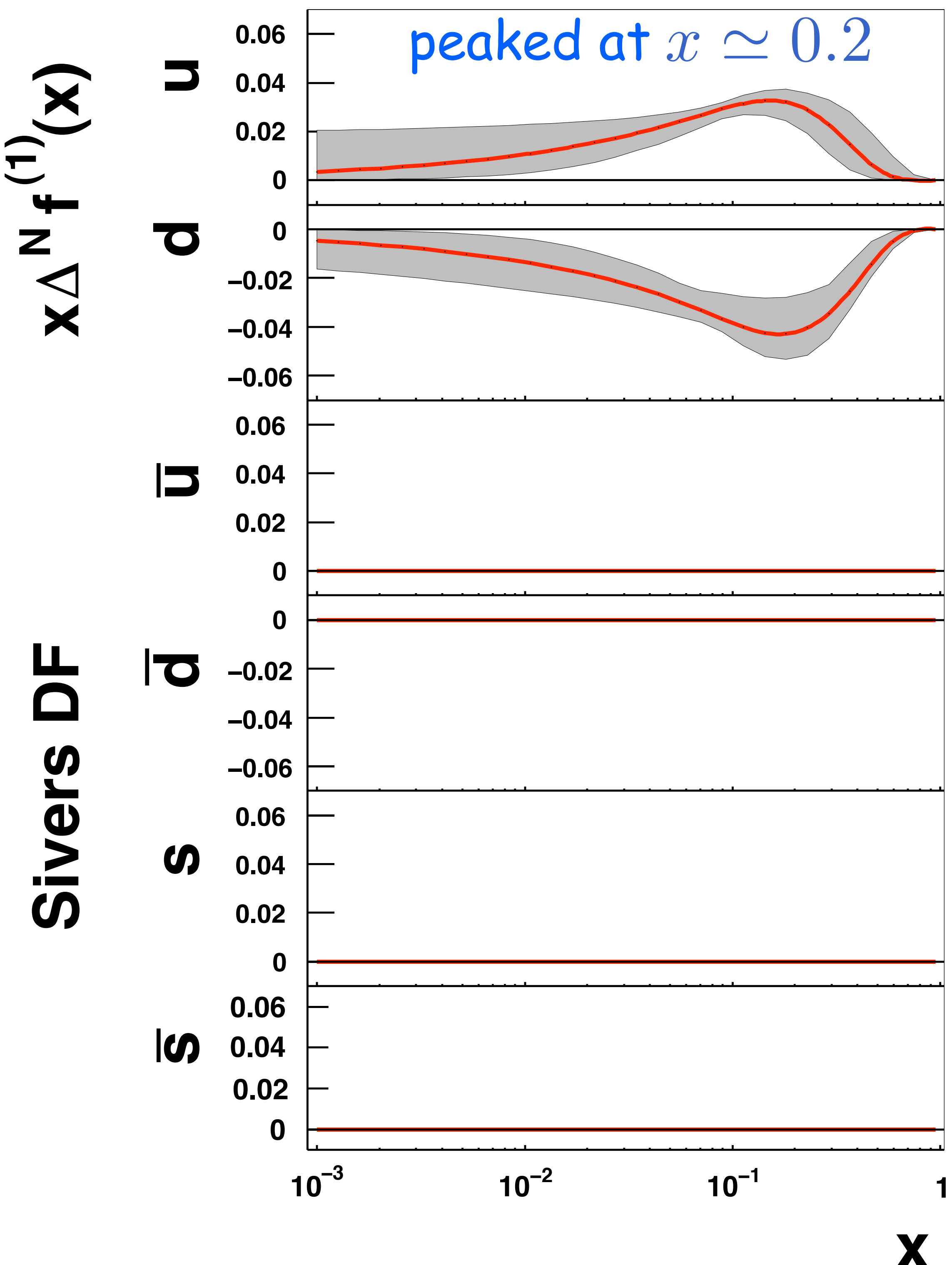
Sivers asymmetry, new fit, valence quarks only, no sea



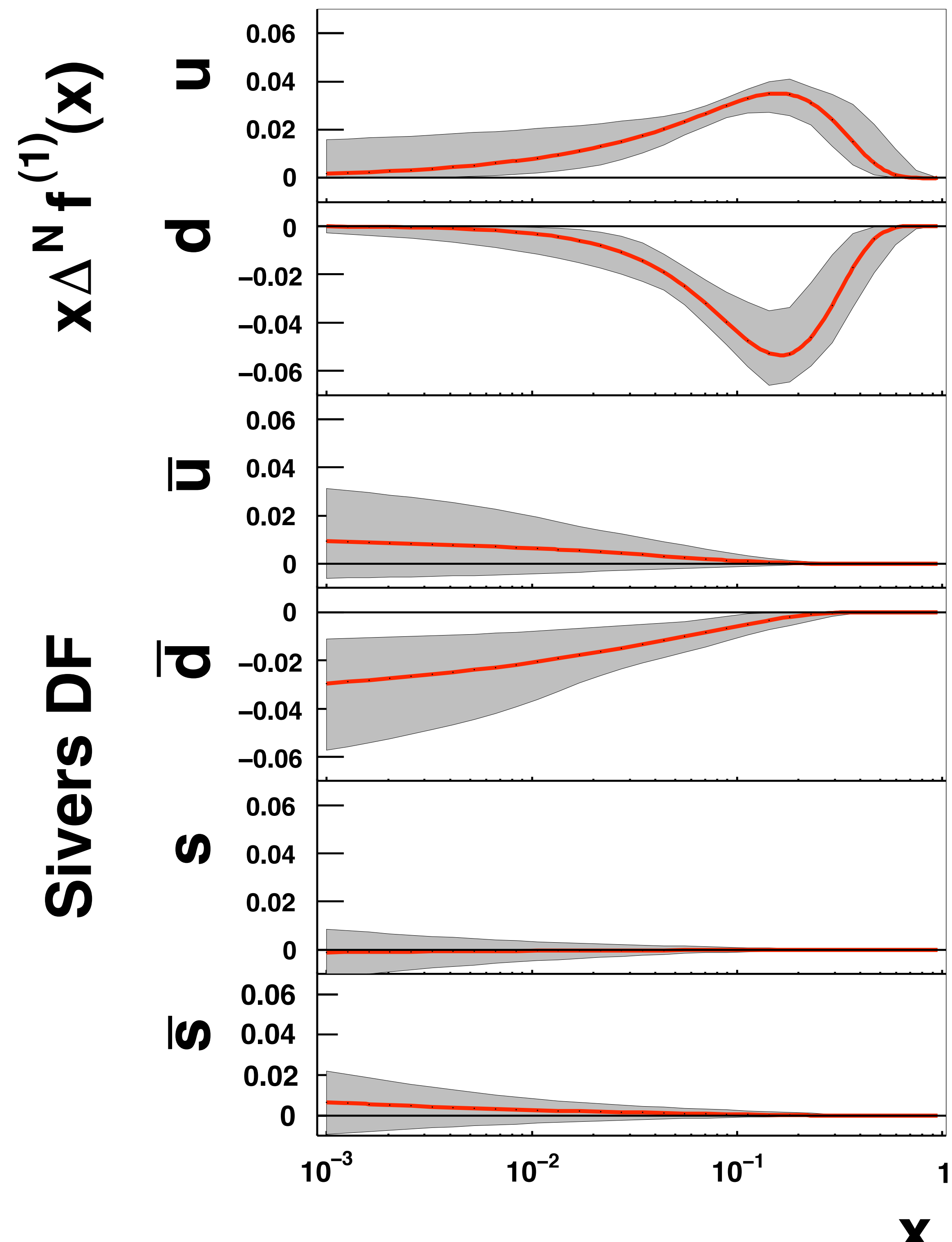
Sivers asymmetry, new fit, valence quarks + sea



extracted Sivers functions (work in progress)

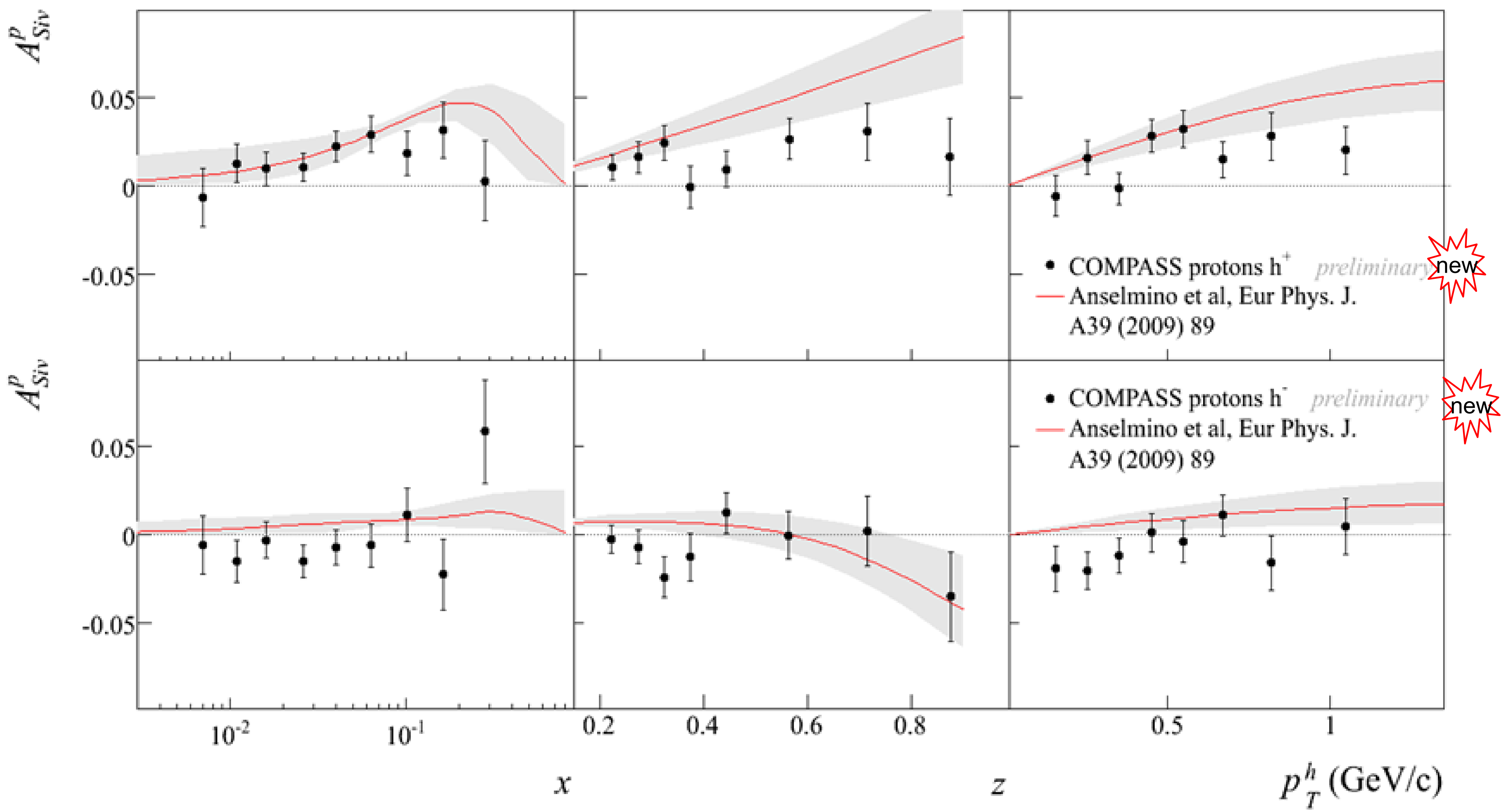


only valence



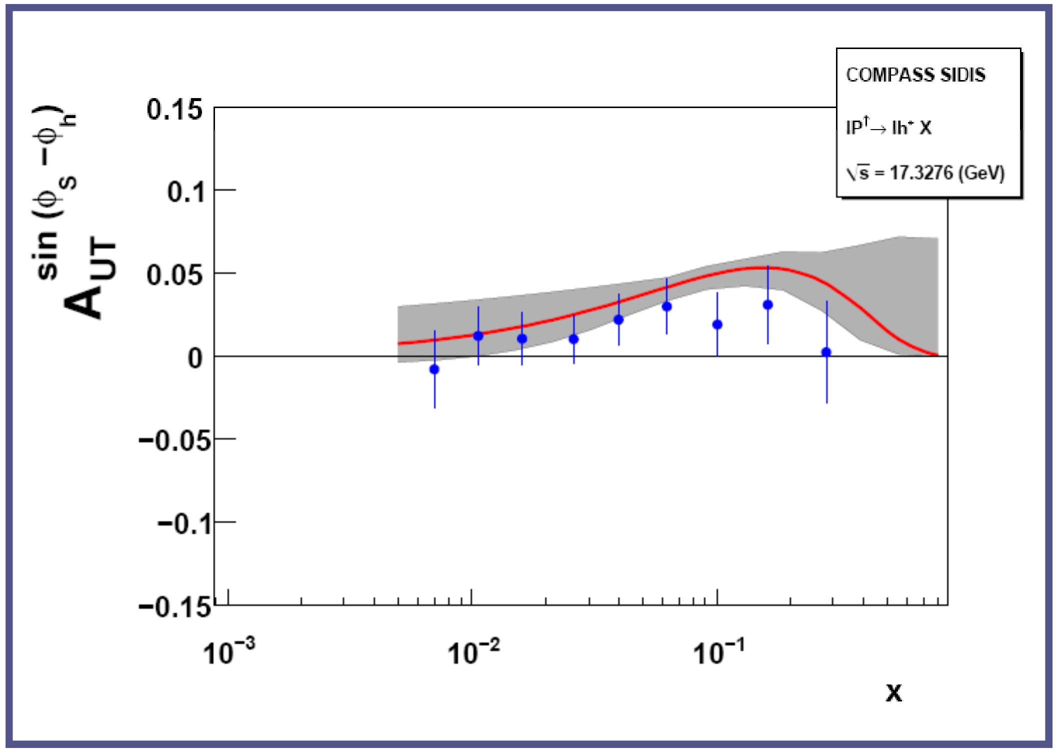
valence + sea

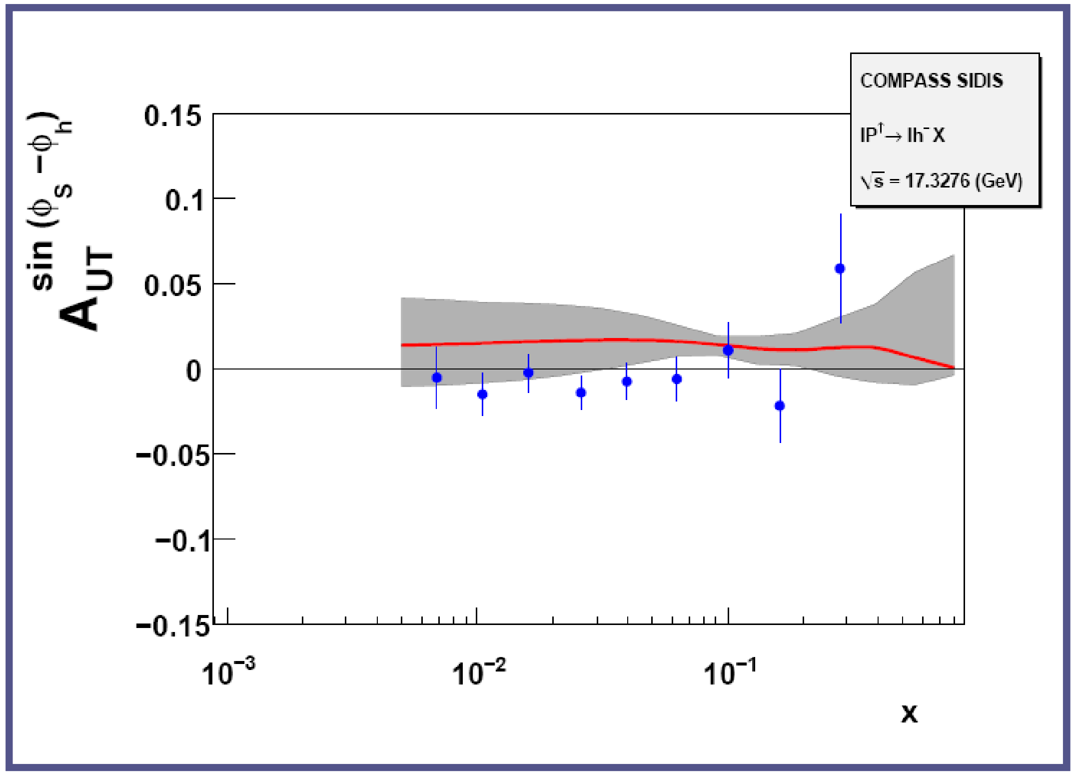
predictions for Sivers asymmetry at COMPASS proton target, old fit - comparison with new data



A. Martin, DIS2010

Sivers Function
New fit predictions
for new COMPASS proton data

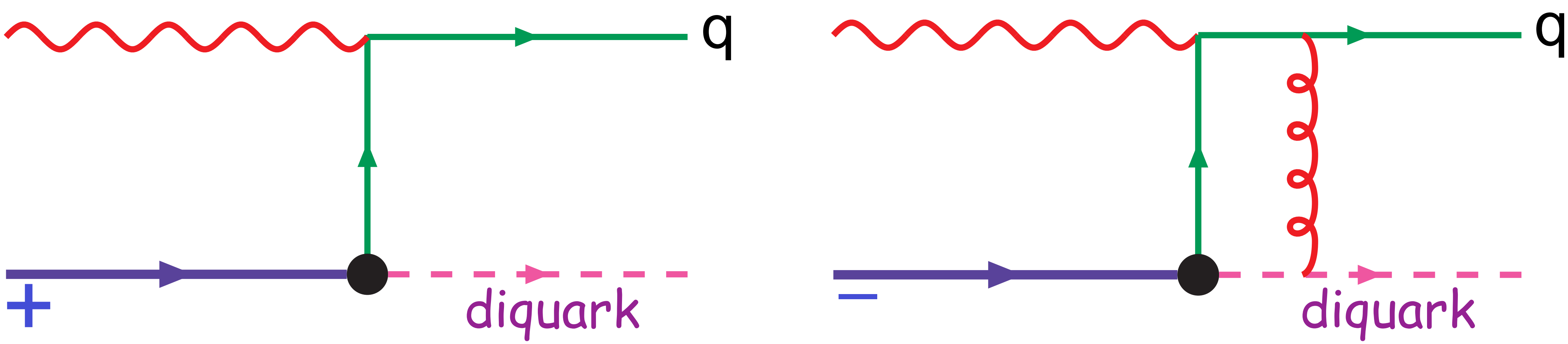




M. Boglione, SPIN2010

Quark models for Sivers function

Brodsky, Hwang, Schmidt: final state interactions

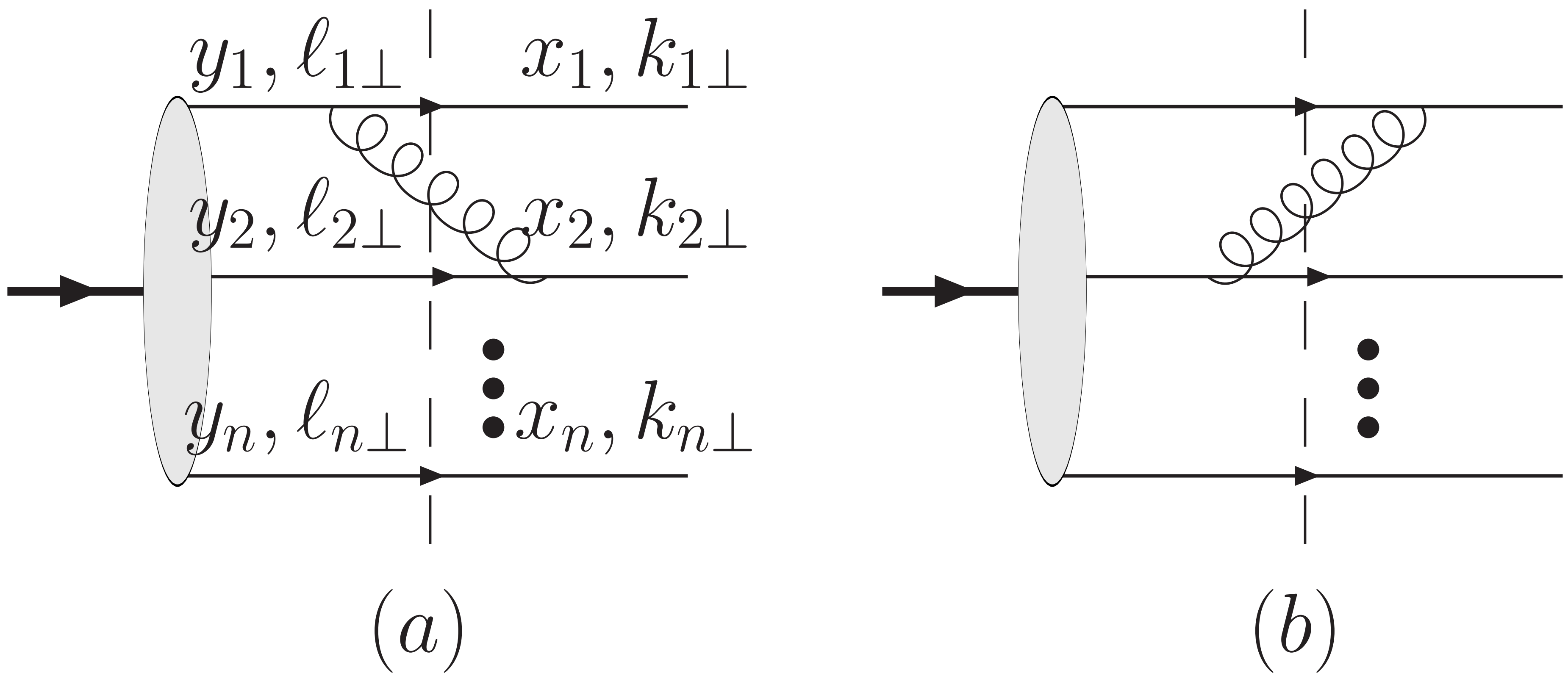


recent quark-diquark model of all twist-2 TMDs: Bacchetta, Conti, Radici, arXiv:0807.0323 (PRD 78, 074010, 2008); Bacchetta, Radici, Conti, Guagnelli, arXiv:1003.1328

very recent quark bag model of all twist-2 and twist-3 TMDs: Avakian, Efremov, Schweitzer, Yuan, arXiv:1001.5467 (supports Gaussian $k_\perp$ dependence of TMDs in valence x-region)

Sivers function from light-front wave function

Brodsky, Pasquini, Xiao, Yuan, arXiv:1001.1163
Pasquini, Yuan, arXiv:1001.5398



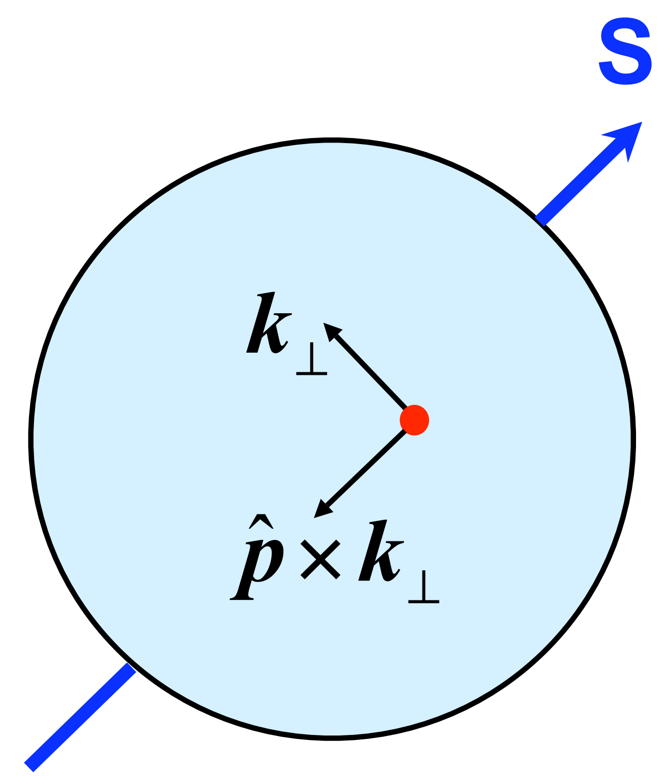
in all models one has:

$$[f_{1T}^{q\perp}]_{\mathrm{SIDIS}} = -[f_{1T}^{q\perp}]_{\mathrm{DY}}$$

see also Hwang, arXiv:1003.0867 - incorporation of final state interactions into the light-cone wave function

What could we learn from the Sivers distribution?

number density of partons with longitudinal momentum fraction x and transverse momentum $\boldsymbol{k}_\perp$, inside a proton with spin $\mathbf{S}$



$$\boxed{\sum_a \int dx\, d^2\boldsymbol{k}_\perp\, \boldsymbol{k}_\perp\, f_{a/p^\uparrow}(x, \boldsymbol{k}_\perp) \equiv \sum_a \langle \boldsymbol{k}_\perp^a \rangle = 0}$$

M. Burkardt, PR **D69**, 091501 (2004)

same naive sum rule as expected for free partons (no final state interactions)

Total amount of intrinsic momentum carried by partons of flavour a

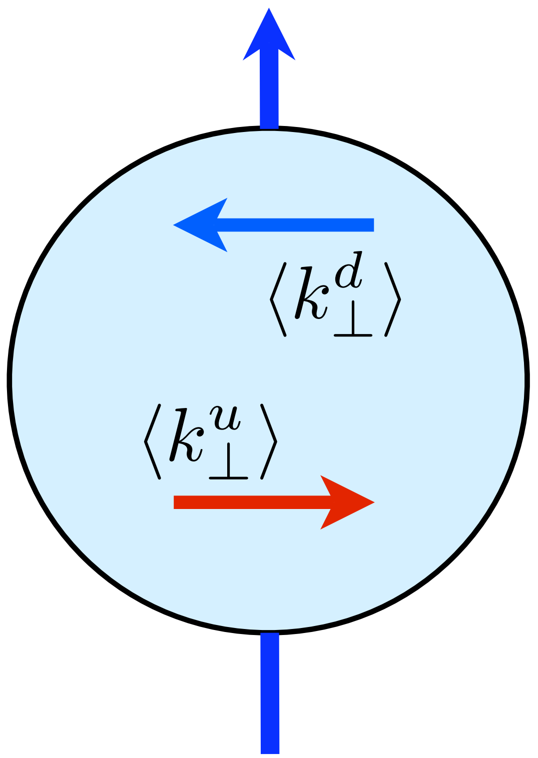
$$\langle \boldsymbol{k}_\perp^a \rangle = \left[\frac{\pi}{2} \int_0^1 dx \int_0^\infty dk_\perp \, k_\perp^2 \, \Delta^N f_{a/p^\uparrow}(x, k_\perp) \right] (\boldsymbol{S} \times \hat{\boldsymbol{P}})$$

$$= m_p \int_0^1 dx \, \Delta^N f^{(1)}_{q/p^\uparrow}(x) \, (\boldsymbol{S} \times \hat{\boldsymbol{P}}) \equiv \langle k_\perp^a \rangle \, (\boldsymbol{S} \times \hat{\boldsymbol{P}})$$

$$\langle k_\perp^u \rangle + \langle k_\perp^d \rangle = -17^{+37}_{-55} \ (\mathrm{MeV}/c)$$

$$[\, \langle k_\perp^u \rangle = 96^{+60}_{-28} \qquad \langle k_\perp^d \rangle = -113^{+45}_{-51} \,]$$

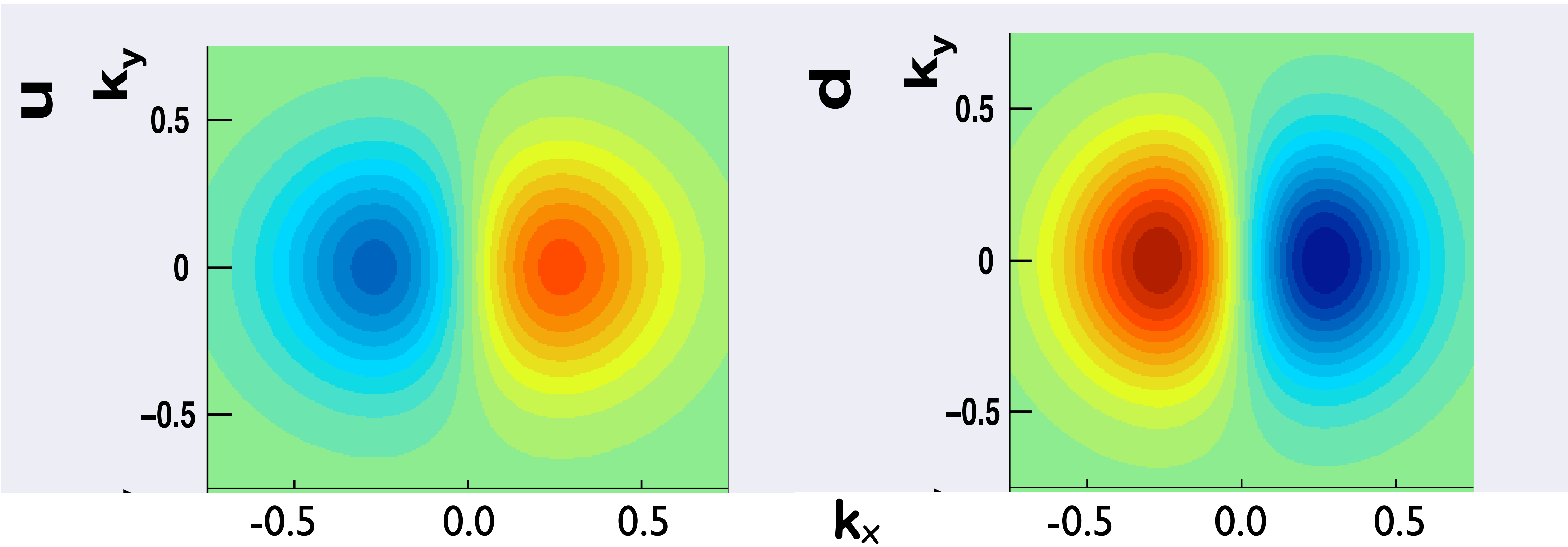
$$\langle k_\perp^{\bar{u}} \rangle + \langle k_\perp^{\bar{d}} \rangle + \langle k_\perp^s \rangle + \langle k_\perp^{\bar{s}} \rangle = -14^{+43}_{-66} \ (\mathrm{MeV}/c)$$



Burkardt sum rule almost saturated by u and d quarks alone; little residual contribution from gluons

$$-10 \le \langle k_\perp^g \rangle \le 48 \ (\mathrm{MeV}/c)$$

Sivers u and d quark densities in transverse momentum space



proton moving into the screen, polarization along y-axis

blue: less quarks red: more quarks x = 0.2 k in GeV/c

$$\langle k_\perp^u \rangle = 96^{+60}_{-28} \qquad \langle k_\perp^d \rangle = -113^{+45}_{-51} \ (\mathrm{MeV}/c)$$

courtesy of A. Prokudin

Sivers function and orbital angular momentum

D. Sivers

Sivers mechanism originates from $\boldsymbol{S} \cdot \boldsymbol{L}_q$ then it is related to the quark orbital angular momentum

Sivers function and proton anomalous magnetic moment

M. Burkardt, S. Brodsky, Z. Lu, I. Schmidt

Both the Sivers function and the proton anomalous magnetic moment are related to correlations of proton wave functions with opposite helicities

$$\int_0^1 \mathrm{d}x \, \mathrm{d}^2\boldsymbol{k}_\perp \, \Delta^N f_{q/p^\uparrow}(x, k_\perp) = C \, \kappa_q$$

in qualitative agreement with large z data:

$$\frac{A_{UT}^{\sin(\phi_{\pi^+} - \phi_S)}}{A_{UT}^{\sin(\phi_{\pi^-} - \phi_S)}} \sim \frac{\kappa_u}{\kappa_d}$$

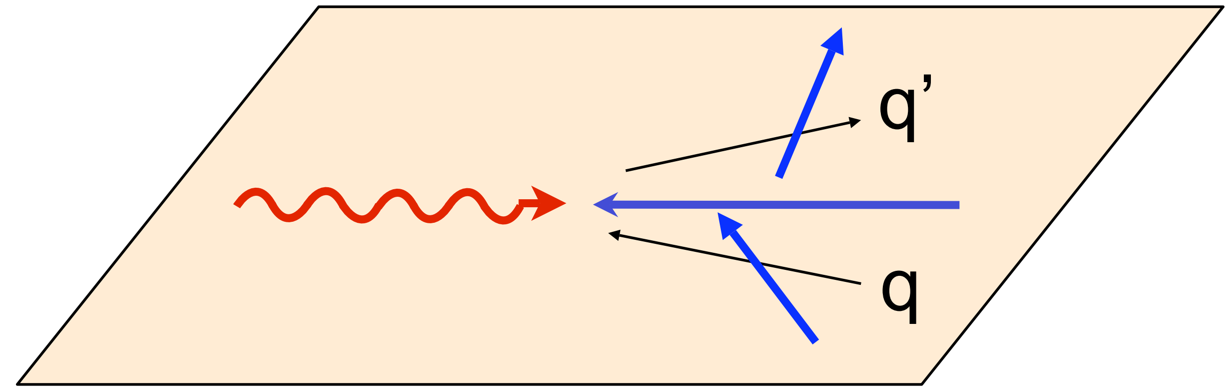
Sivers effect now observed by two experiments (+ hints from Jlab-HallA), but needs further measurements (small and large x regions need exploration, measure Sivers asymmetry for jet production)

and if $(\text{Sivers})_{SIDIS} \neq -\,(\text{Sivers})_{D\text{-}Y}$?

A_N in $AB \to CX$, which Sivers function? other mechanisms? Collins effect?

Collins effect in SIDIS - $F_{UT}^{\sin(\phi+\phi_S)}$

$$D_{h/q,\boldsymbol{s}_q}(z,\boldsymbol{p}_\perp) = D_{h/p}(z,p_\perp) + \frac{1}{2}\Delta^N D_{h/q^\uparrow}(z,p_\perp)\,\boldsymbol{s}_q\cdot(\hat{\boldsymbol{p}}_q\times\hat{\boldsymbol{p}}_\perp)$$



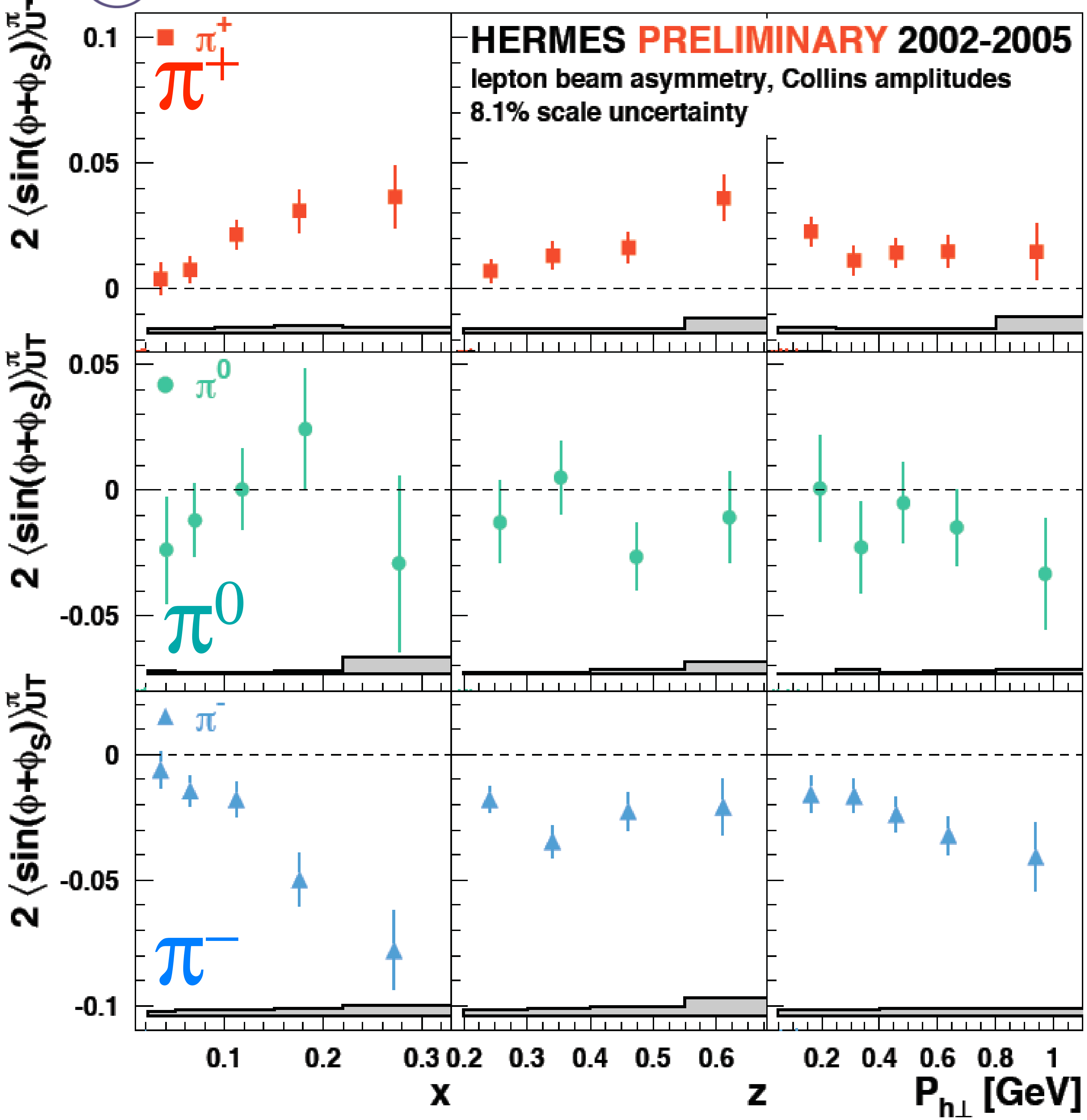
$$\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow = \sum_q h_{1q}(x,k_\perp)\otimes \mathrm{d}\Delta\hat{\sigma}(y,\boldsymbol{k}_\perp)\otimes \Delta^N D_{h/q^\uparrow}(z,\boldsymbol{p}_\perp)$$

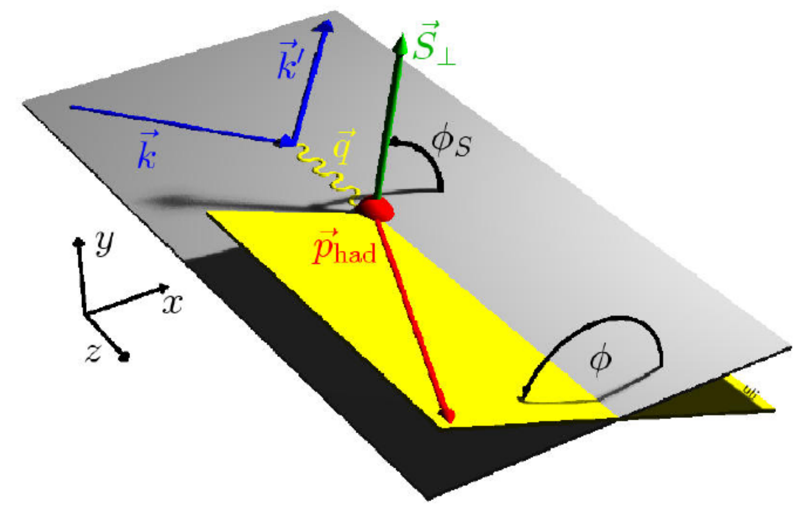
$$A_{UT}^{\sin(\phi+\phi_S)} \equiv 2\,\frac{\int \mathrm{d}\phi\,\mathrm{d}\phi_S\,[\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow]\,\sin(\phi+\phi_S)}{\int \mathrm{d}\phi\,\mathrm{d}\phi_S\,[\mathrm{d}\sigma^\uparrow + \mathrm{d}\sigma^\downarrow]}$$

$$\mathrm{d}\Delta\hat{\sigma} = \mathrm{d}\hat{\sigma}^{\ell q^\uparrow\to\ell q^\uparrow} - \mathrm{d}\hat{\sigma}^{\ell q^\uparrow\to\ell q^\downarrow}$$

Collins effect in SIDIS couples to transversity

$ep^{\uparrow} \rightarrow \pi X$ E_b=27GeV, $\sqrt{s}$~7 GeV

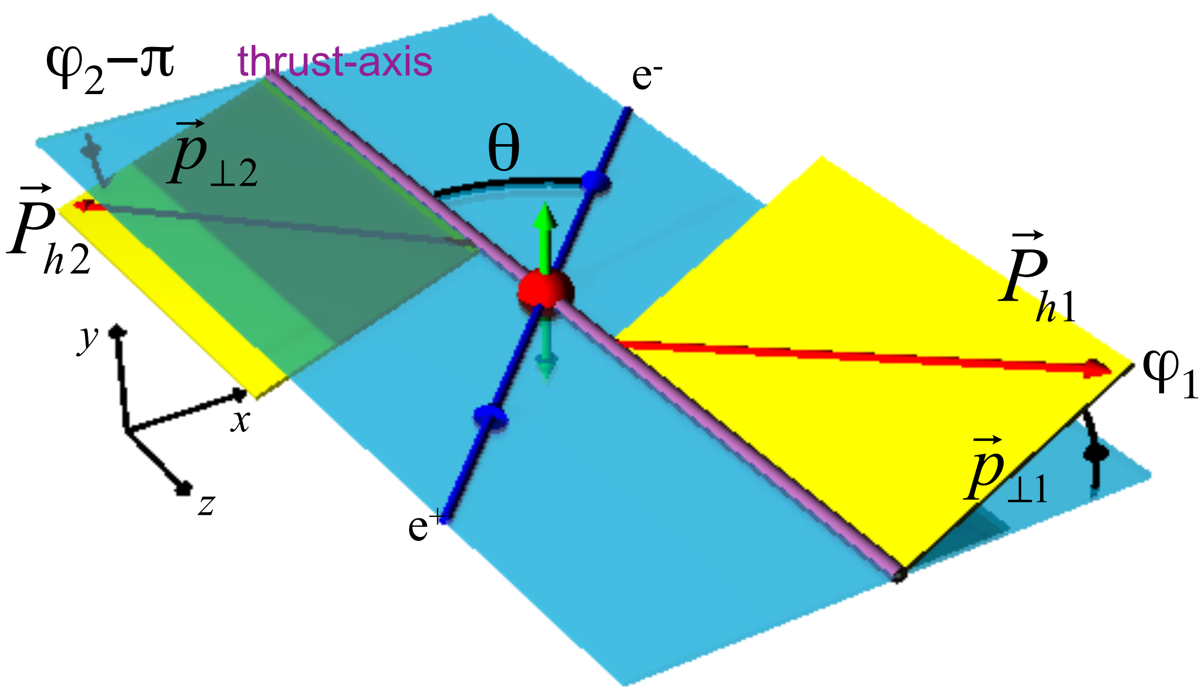


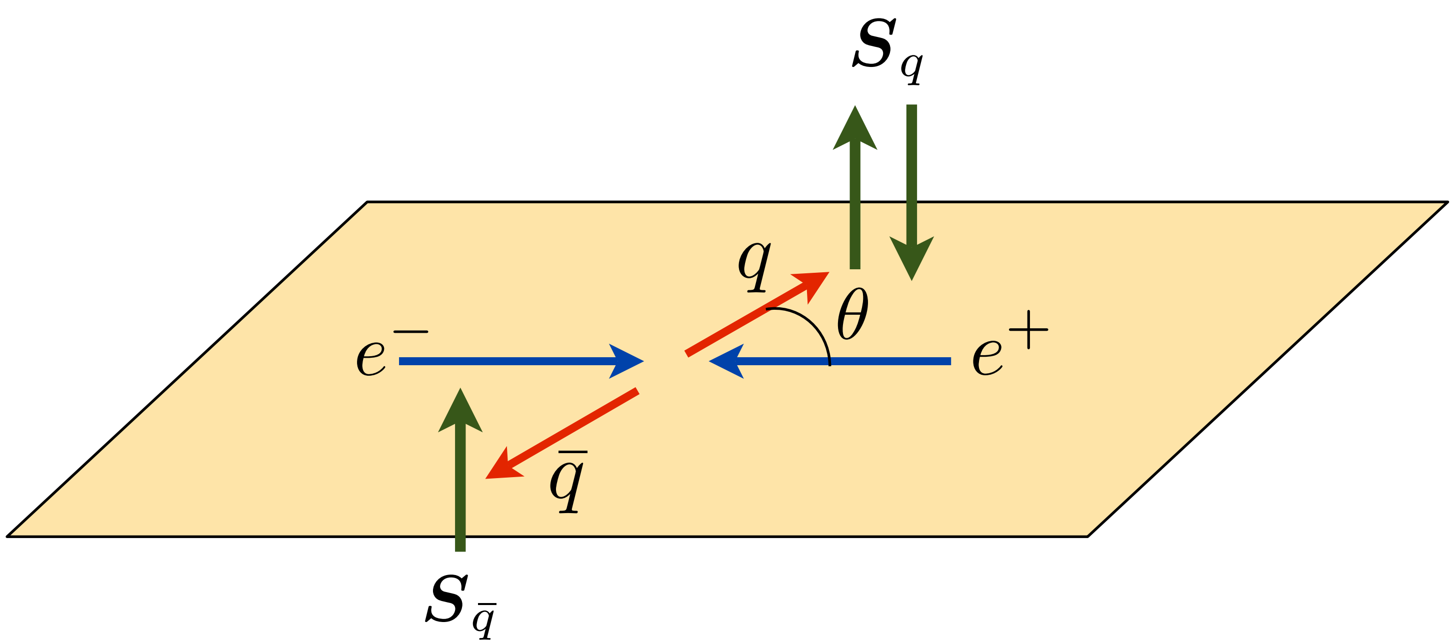


HERMES Collins asymmetry

Collins function from e^+e^- processes

BELLE @ KEK





$$\frac{d\sigma^{e^+e^-\to q^\uparrow \bar{q}^\uparrow}}{d\cos\theta} = \frac{3\pi\alpha^2}{4s}\, e_q^2\, \cos^2\theta \qquad \frac{d\sigma^{e^+e^-\to q^\downarrow \bar{q}^\uparrow}}{d\cos\theta} = \frac{3\pi\alpha^2}{4s}\, e_q^2$$

$$A_{12}(z_1, z_2, \theta, \varphi_1+\varphi_2) \equiv \frac{1}{\langle d\sigma\rangle} \frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{dz_1\, dz_2\, d\cos\theta\, d(\varphi_1+\varphi_2)}$$

$$= 1 + \frac{1}{4}\,\frac{\sin^2\theta}{1+\cos^2\theta}\,\cos(\varphi_1+\varphi_2) \times \frac{\sum_q e_q^2\, \Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$

Transversity and Collins parameterization

$$\Delta_T q(x, k_\perp) = \frac{1}{2} \mathcal{N}_q^T(x) \, \left[f_{q/p}(x) + \Delta q(x) \right] \, \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle_T}}{\pi \langle k_\perp^2 \rangle_T}$$

$$\Delta^N D_{h/q^\uparrow}(z, p_\perp) = 2 \, \mathcal{N}_q^C(z) \, D_{h/q}(z) \, h(p_\perp) \, \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle}$$

$$\mathcal{N}_q^T(x) = N_q \, x^\alpha (1-x)^\beta \, \frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha \beta^\beta}$$

$$\mathcal{N}_q^C(x) = N_q \, x^\gamma (1-x)^\delta \, \frac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^\delta \gamma^\delta} \qquad \langle k_\perp^2 \rangle_T = \langle k_\perp^2 \rangle$$

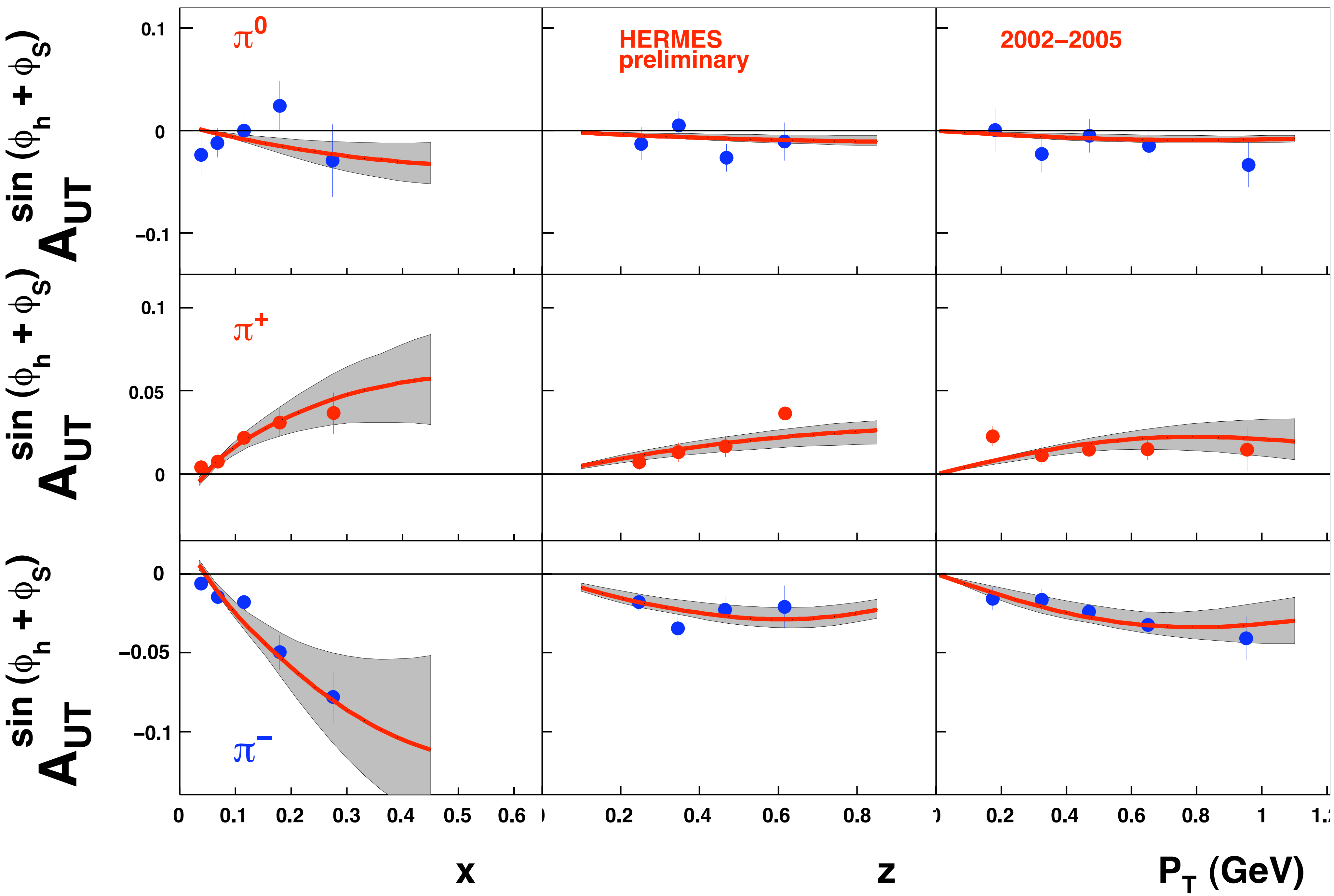
$$h(p_\perp) = \sqrt{2e} \, \frac{p_\perp}{M_1} \, e^{-p_\perp^2/M_1^2}$$

$$|N| \leq 1 \quad \Longrightarrow$$

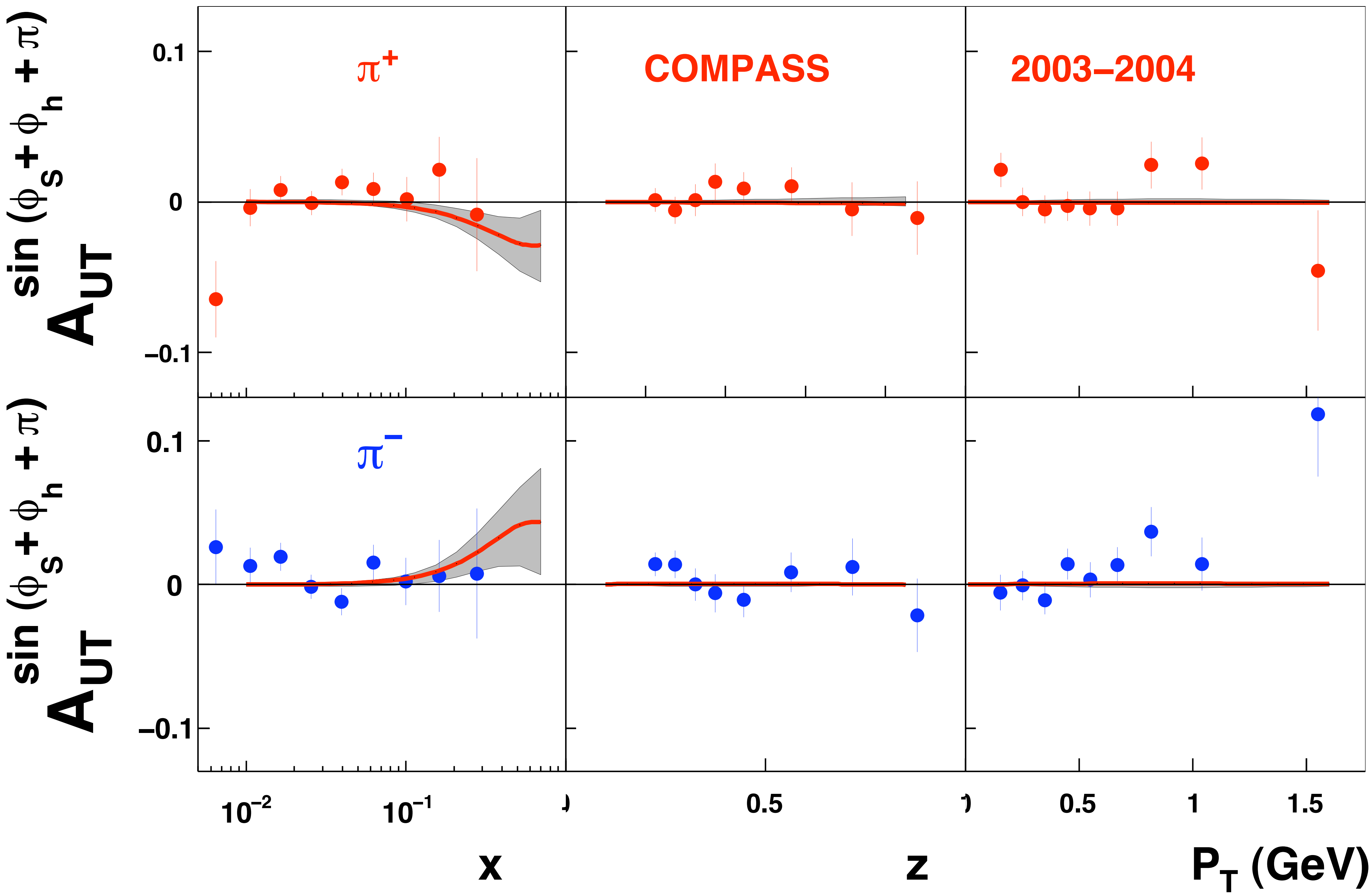
$$|\Delta_T q(x)| \leq \frac{1}{2} \left[f_{q/p}(x) + \Delta q(x) \right]$$

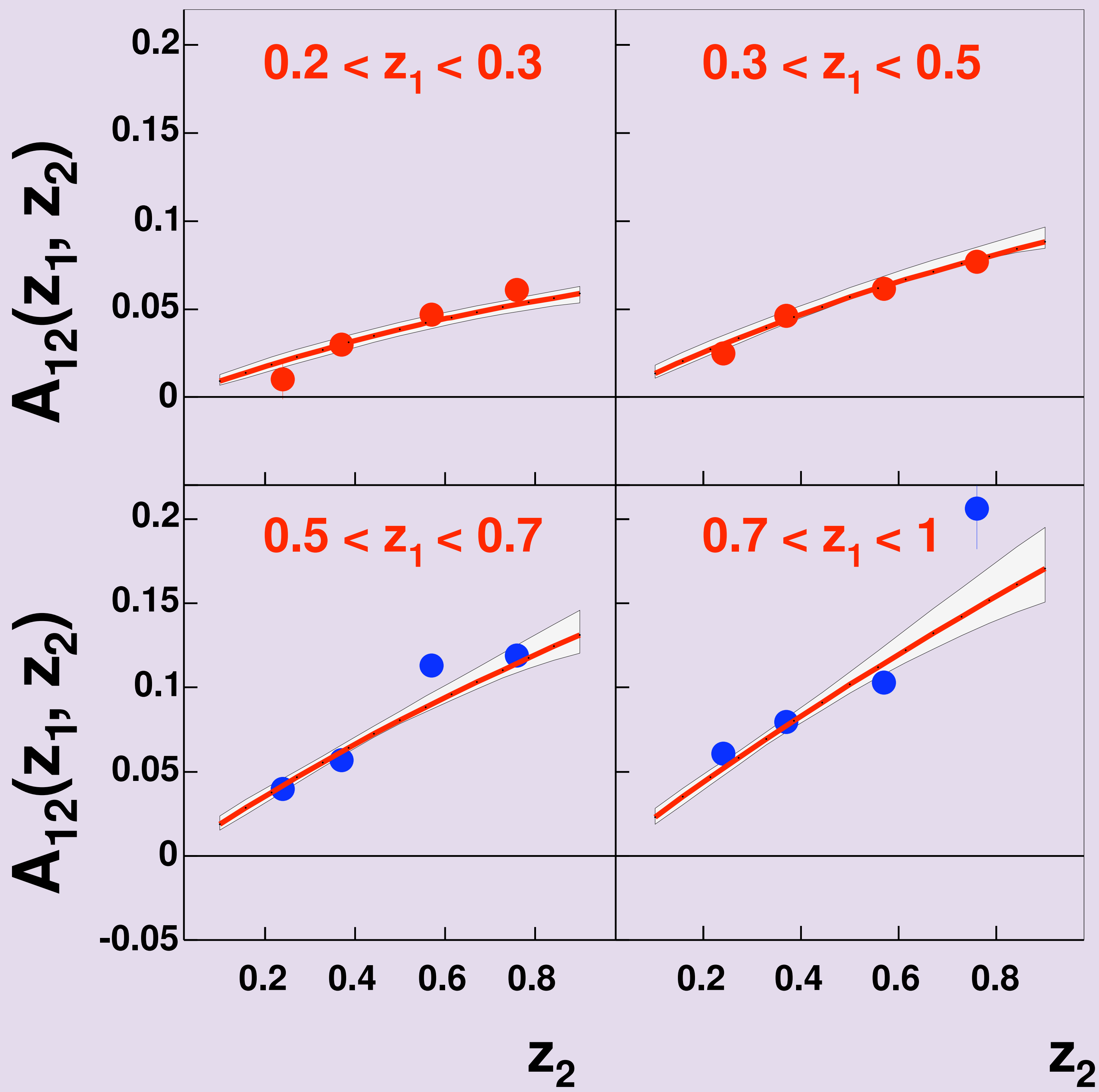
$$|\Delta^N D_{h/q^\uparrow}(z, p_\perp)| \leq 2 D_{h/q}(z, p_\perp)$$

Collins asymmetry best fits



COMPASS data on deuteron





best fit of Belle data

9 parameters in fit of Collins + transversity

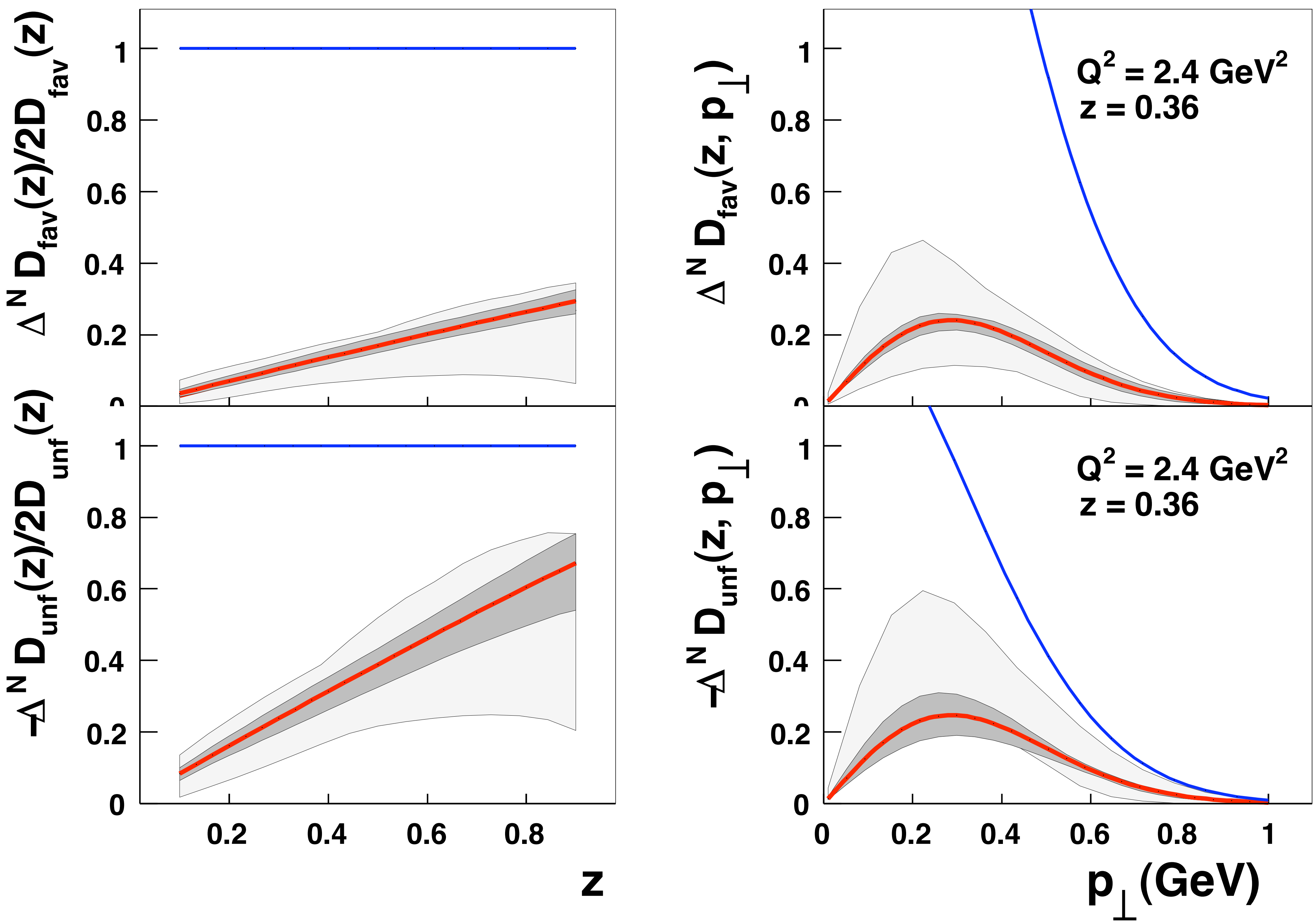
N_u^T N_d^T

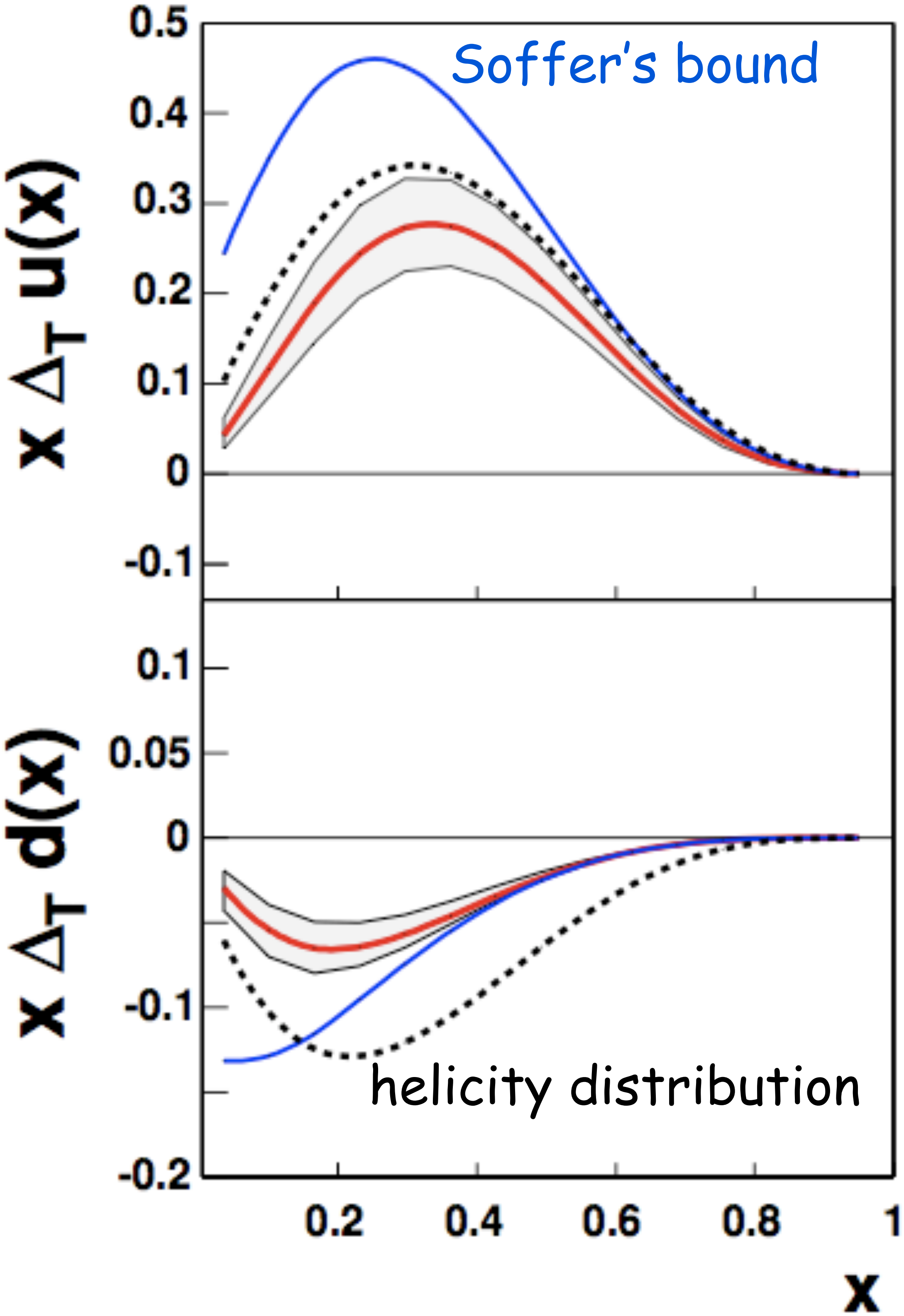
N_{fav}^C N_{unf}^C

α β

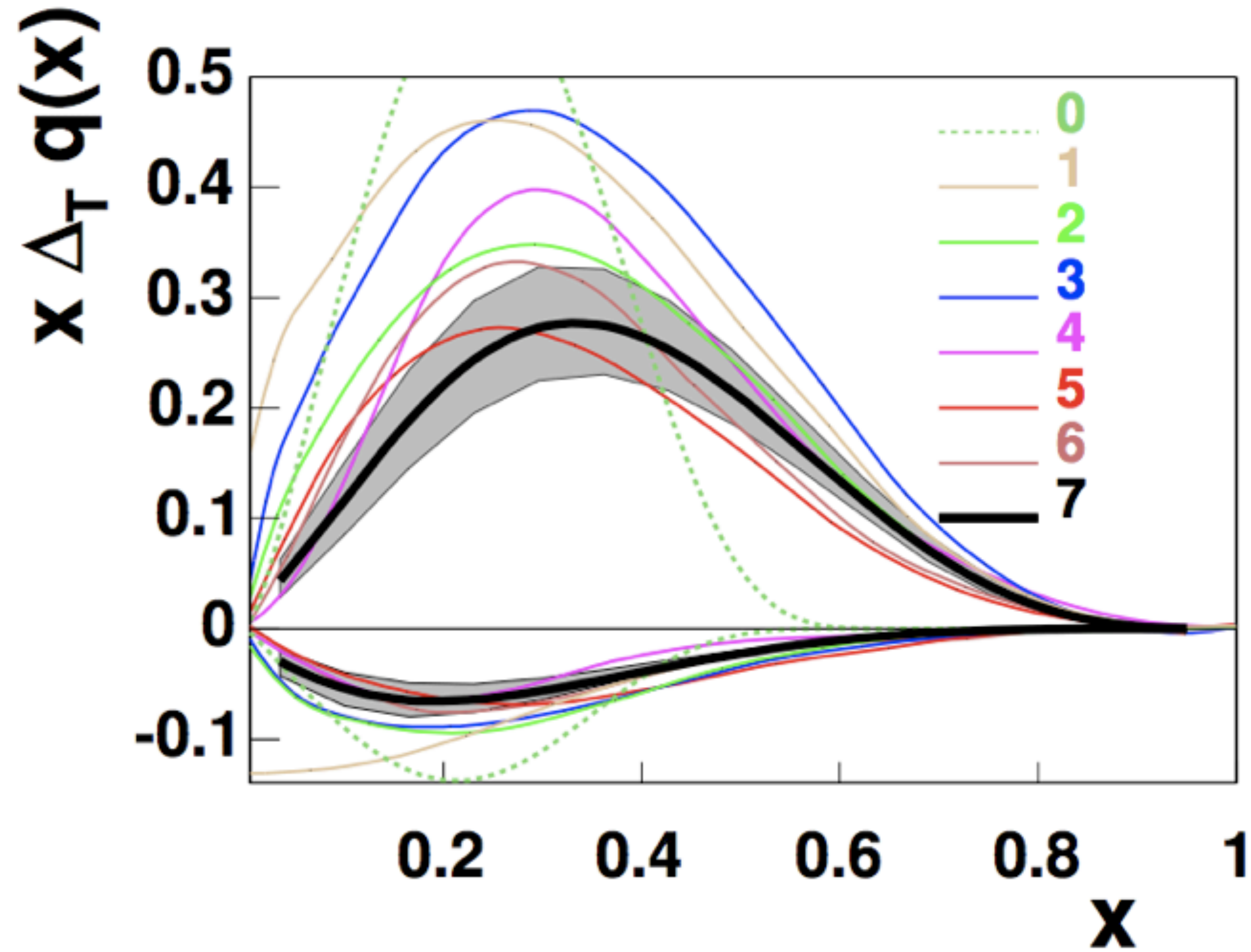
γ δ M_1

extracted Collins functions



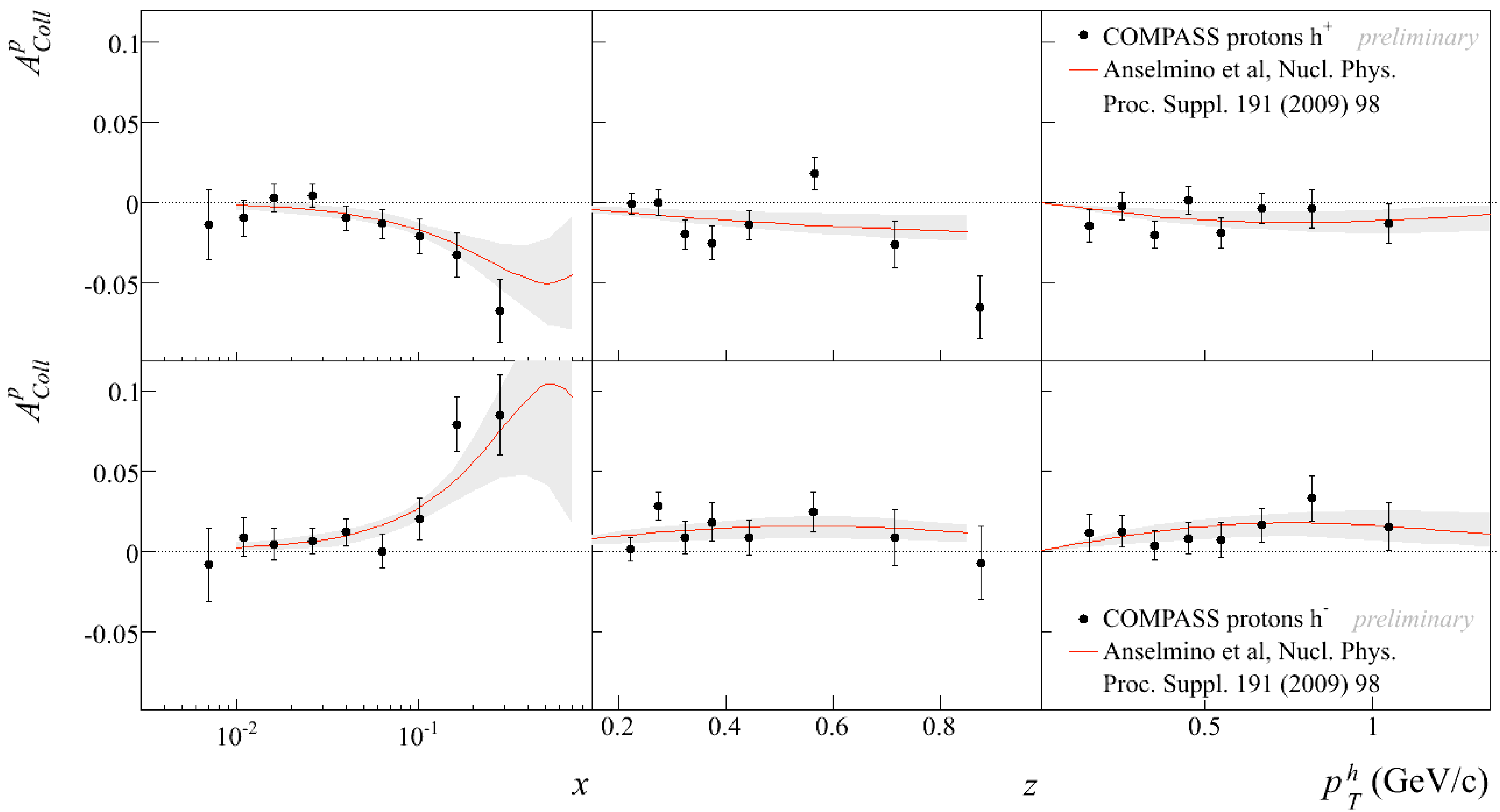


extracted transversity and comparison with models



M.A., M. Boglione, U. D'Alesio, A. Kotzinian, S. Melis, F. Murgia, A. Prokudin, C. Türk

Predictions for COMPASS, with a proton target, and comparison with data



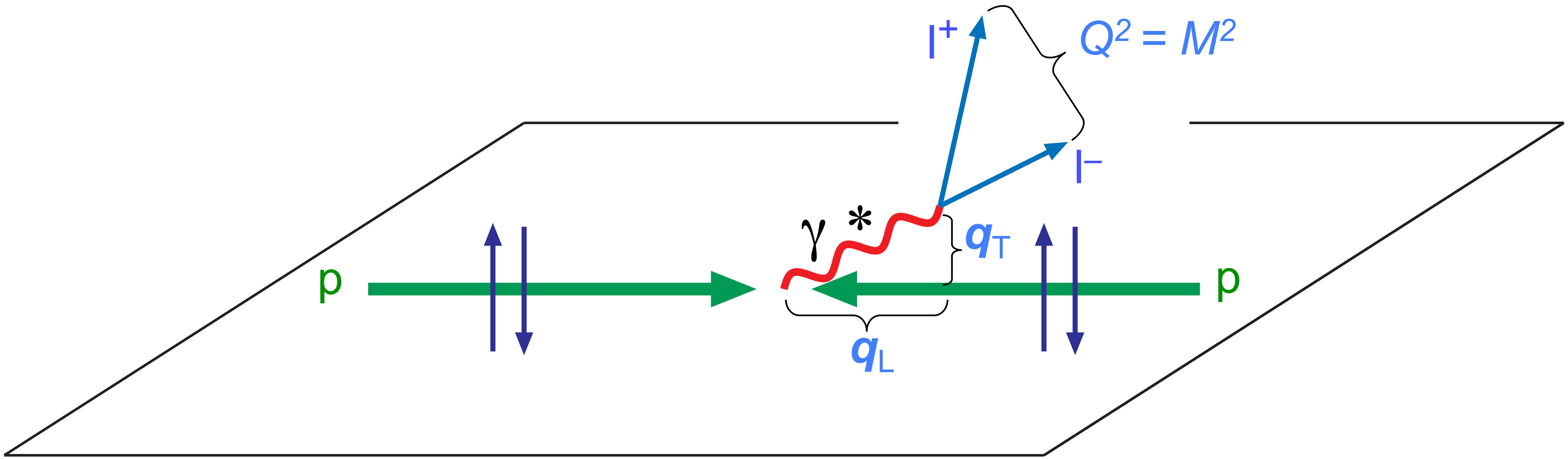
A. Martin, DIS2010

Collins effect observed by three independent experiments: HERMES, BELLE and COMPASS

Collins function expected to be universal

Collins function couples to Boer-Mulders function in unpolarized SIDIS to give a $\cos(2\Phi)$ asymmetry

Drell-Yan processes - TMDs



factorization holds, two scales, M^2, and $q_T \ll M$

$$d\sigma^{D-Y} = \sum_a f_q(x_1, \boldsymbol{k}_{\perp 1}; Q^2) \otimes f_{\bar{q}}(x_2, \boldsymbol{k}_{\perp 2}; Q^2)\, d\hat{\sigma}^{q\bar{q}\to\ell^+\ell^-}$$

direct product of TMDs
no fragmentation process

cross-section: most general pp leading-twist expression

S. Arnold, A. Metz and M. Schlegel, arXiv:0809.2262 [hep-ph]

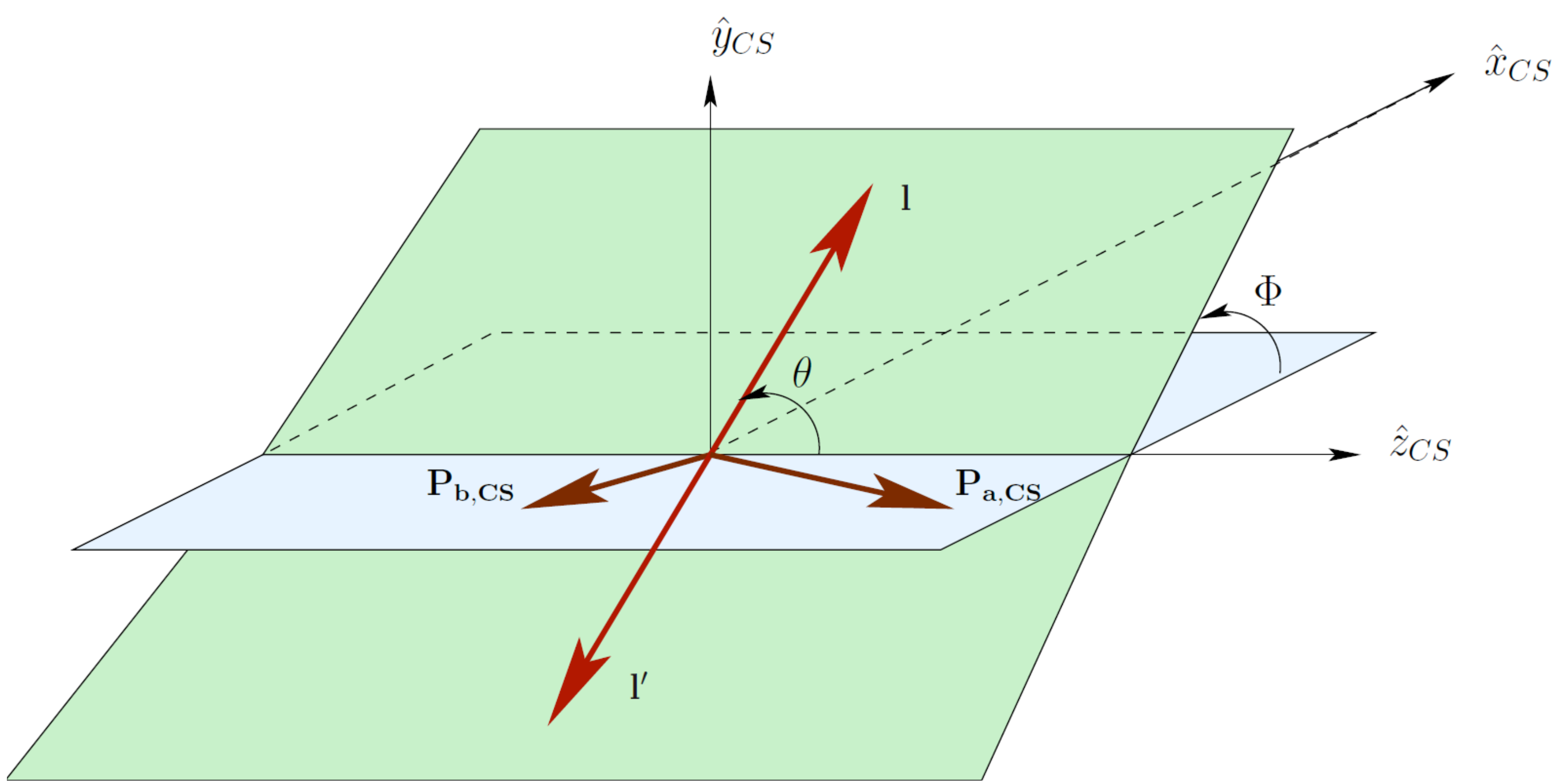
$$
\begin{aligned}
\frac{d\sigma}{d^4q\,d\Omega} = \frac{\alpha_{em}^2}{F\,q^2}\times
&\Big\{\Big((1+\cos^2\theta)\,F_{UU}^1 + (1-\cos^2\theta)\,F_{UU}^2 + \sin 2\theta\cos\phi\,F_{UU}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{UU}^{\cos 2\phi}\Big)\\
&+ S_{aL}\Big(\sin 2\theta\sin\phi\,F_{LU}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{LU}^{\sin 2\phi}\Big)\\
&+ S_{bL}\Big(\sin 2\theta\sin\phi\,F_{UL}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{UL}^{\sin 2\phi}\Big)\\
&+ |\vec{S}_{aT}|\Big[\sin\phi_a\Big((1+\cos^2\theta)\,F_{TU}^1 + (1-\cos^2\theta)\,F_{TU}^2 + \sin 2\theta\cos\phi\,F_{TU}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{TU}^{\cos 2\phi}\Big)\\
&\qquad + \cos\phi_a\Big(\sin 2\theta\sin\phi\,F_{TU}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{TU}^{\sin 2\phi}\Big)\Big]\\
&+ |\vec{S}_{bT}|\Big[\sin\phi_b\Big((1+\cos^2\theta)\,F_{UT}^1 + (1-\cos^2\theta)\,F_{UT}^2 + \sin 2\theta\cos\phi\,F_{UT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{UT}^{\cos 2\phi}\Big)\\
&\qquad + \cos\phi_b\Big(\sin 2\theta\sin\phi\,F_{UT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{UT}^{\sin 2\phi}\Big)\Big]\\
&+ S_{aL}\,S_{bL}\Big((1+\cos^2\theta)\,F_{LL}^1 + (1-\cos^2\theta)\,F_{LL}^2 + \sin 2\theta\cos\phi\,F_{LL}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{LL}^{\cos 2\phi}\Big)\\
&+ S_{aL}\,|\vec{S}_{bT}|\Big[\cos\phi_b\Big((1+\cos^2\theta)\,F_{LT}^1 + (1-\cos^2\theta)\,F_{LT}^2 + \sin 2\theta\cos\phi\,F_{LT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{LT}^{\cos 2\phi}\Big)\\
&\qquad\quad + \sin\phi_b\Big(\sin 2\theta\sin\phi\,F_{LT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{LT}^{\sin 2\phi}\Big)\Big]\\
&+ |\vec{S}_{aT}|\,S_{bL}\Big[\cos\phi_a\Big((1+\cos^2\theta)\,F_{TL}^1 + (1-\cos^2\theta)\,F_{TL}^2 + \sin 2\theta\cos\phi\,F_{TL}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{TL}^{\cos 2\phi}\Big)\\
&\qquad\quad + \sin\phi_a\Big(\sin 2\theta\sin\phi\,F_{TL}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{TL}^{\sin 2\phi}\Big)\Big]\\
&+ |\vec{S}_{aT}|\,|\vec{S}_{bT}|\Big[\cos(\phi_a+\phi_b)\Big((1+\cos^2\theta)\,F_{TT}^1 + (1-\cos^2\theta)\,F_{TT}^2 + \sin 2\theta\cos\phi\,F_{TT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{TT}^{\cos 2\phi}\Big)\\
&\qquad\quad + \cos(\phi_a-\phi_b)\Big((1+\cos^2\theta)\,\bar{F}_{TT}^1 + (1-\cos^2\theta)\,\bar{F}_{TT}^2 + \sin 2\theta\cos\phi\,\bar{F}_{TT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,\bar{F}_{TT}^{\cos 2\phi}\Big)\\
&\qquad\quad + \sin(\phi_a+\phi_b)\Big(\sin 2\theta\sin\phi\,F_{TT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{TT}^{\sin 2\phi}\Big)\\
&\qquad\quad + \sin(\phi_a-\phi_b)\Big(\sin 2\theta\sin\phi\,\bar{F}_{TT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,\bar{F}_{TT}^{\sin 2\phi}\Big)\Big]\Big\}
\end{aligned}
$$

Drell-Yan

48 structure functions ...

Case of one polarized nucleon only

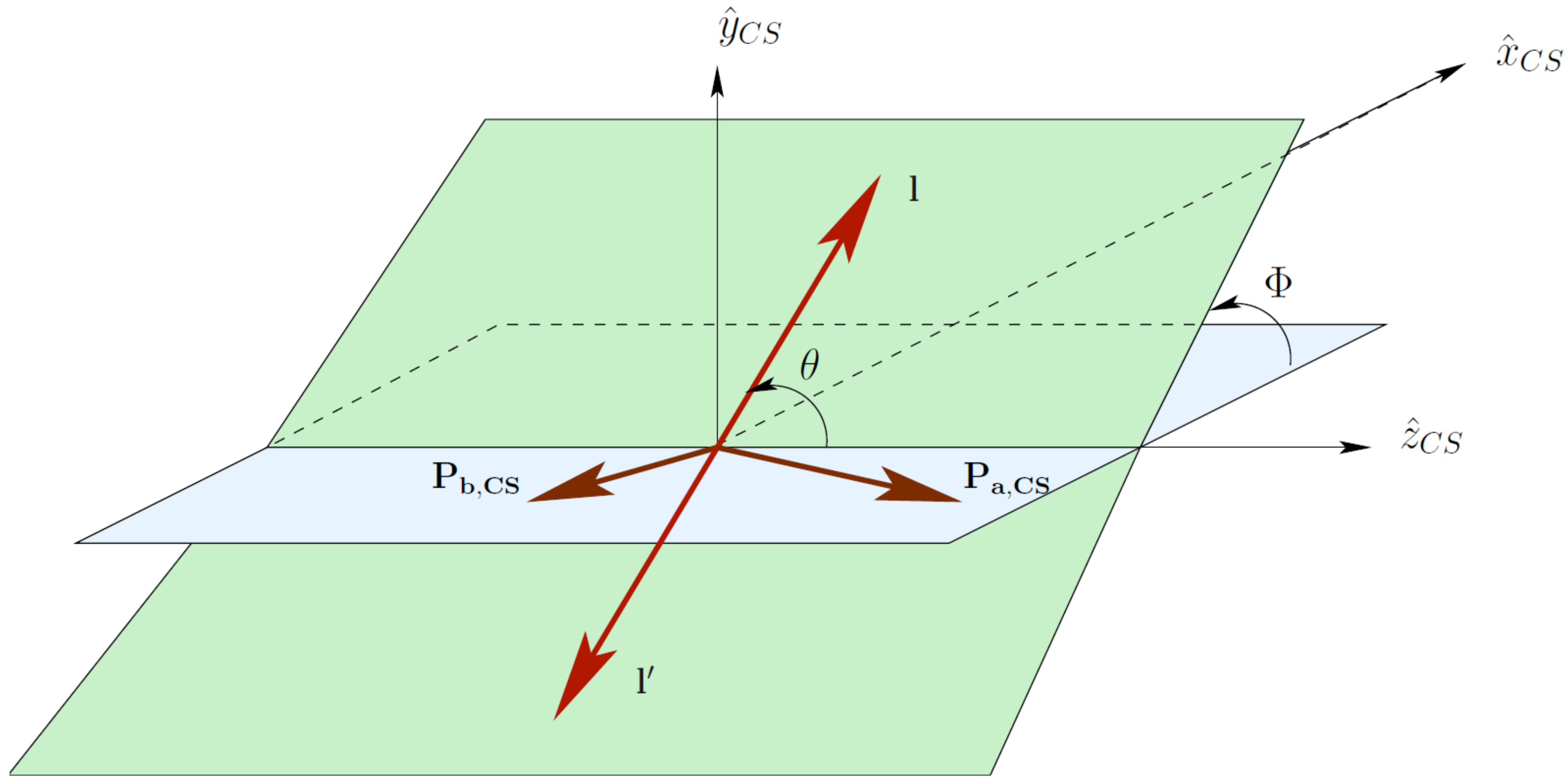
$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q\,\mathrm{d}\Omega} = \frac{\alpha^2}{\Phi\, q^2} \Big\{ & (1+\cos^2\theta)\, F_U^1 + (1-\cos^2\theta)\, F_U^2 + \sin 2\theta \cos\phi\, F_U^{\cos\phi} + \sin^2\theta \cos 2\phi\, F_U^{\cos 2\phi} \\
& + S_L \left(\sin 2\theta \sin\phi\, F_L^{\sin\phi} + \sin^2\theta \sin 2\phi\, F_L^{\sin 2\phi} \right) \\
& + S_T \Big[\left(F_T^{\sin\phi_S} + cos^2\theta\, \tilde{F}_T^{\sin\phi_S} \right) \sin\phi_S + \sin 2\theta \Big(\sin(\phi+\phi_S)\, F_T^{\sin(\phi+\phi_S)} \\
& \qquad + \sin(\phi-\phi_S)\, F_T^{\sin(\phi-\phi_S)} \Big) \\
& + \sin^2\theta \left(\sin(2\phi+\phi_S)\, F_T^{\sin(2\phi+\phi_S)} + \sin(2\phi-\phi_S)\, F_T^{\sin(2\phi-\phi_S)} \right) \Big] \Big\}
\end{aligned}
$$



Collins-Soper frame

Unpolarized cross section already very interesting

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\left(1+\lambda\cos^2\theta+\mu\,\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right)$$



Collins-Soper frame

naive collinear parton model: $\lambda = 1 \quad \mu = \nu = 0$

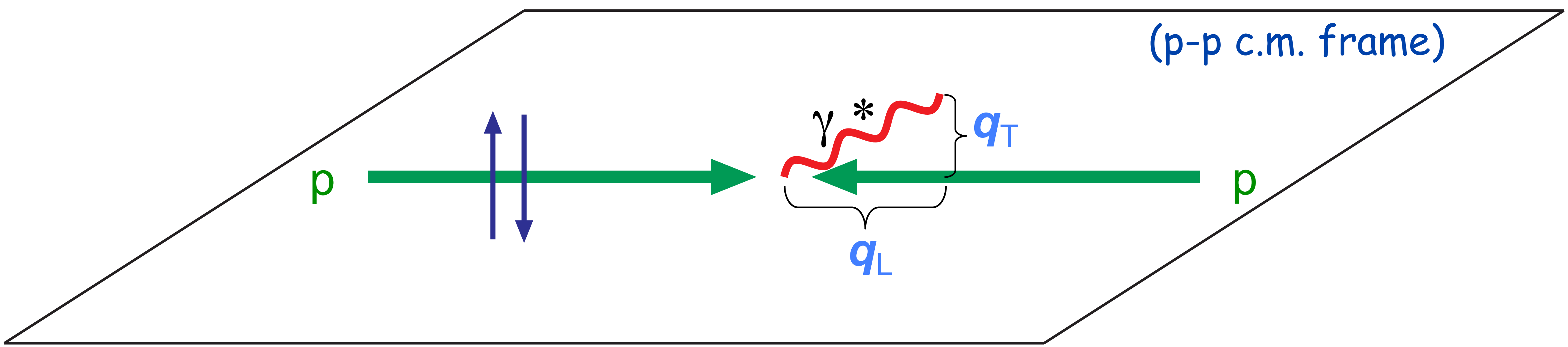
Sivers effect in D-Y processes

By looking at the $d^4\sigma/d^4q$ cross section one can single out the Sivers effect in D-Y processes

$$d\sigma^\uparrow - d\sigma^\downarrow \propto \sum_q \Delta^N f_{q/p^\uparrow}(x_1, \boldsymbol{k}_\perp) \otimes f_{\bar{q}/p}(x_2) \otimes d\hat{\sigma}$$

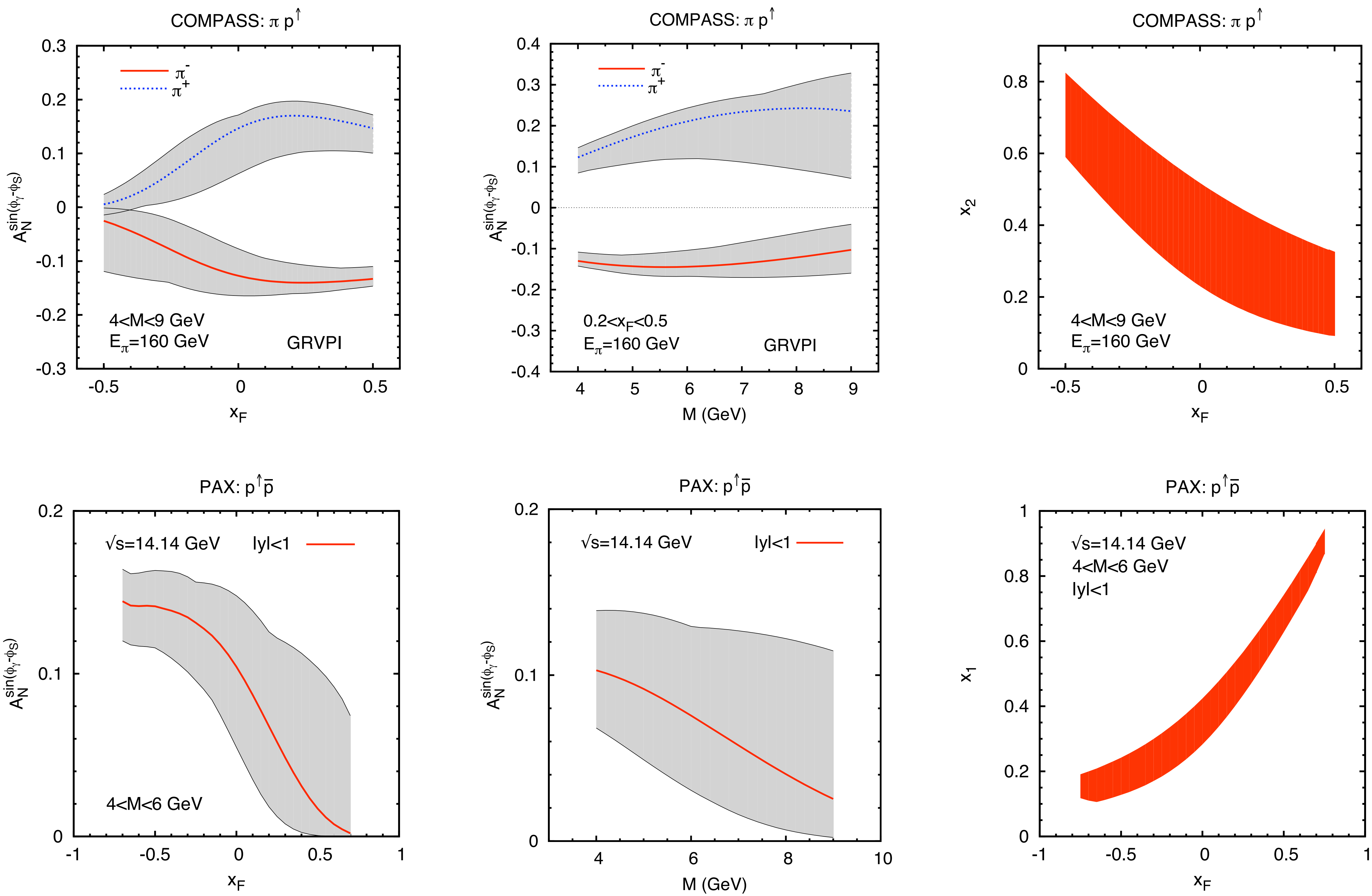
$q = u, \bar{u}, d, \bar{d}, s, \bar{s}$

$$A_N^{\sin(\phi_S - \phi_\gamma)} \equiv \frac{2\int_0^{2\pi} d\phi_\gamma \, [d\sigma^\uparrow - d\sigma^\downarrow] \, \sin(\phi_S - \phi_\gamma)}{\int_0^{2\pi} d\phi_\gamma \, [d\sigma^\uparrow + d\sigma^\downarrow]}$$

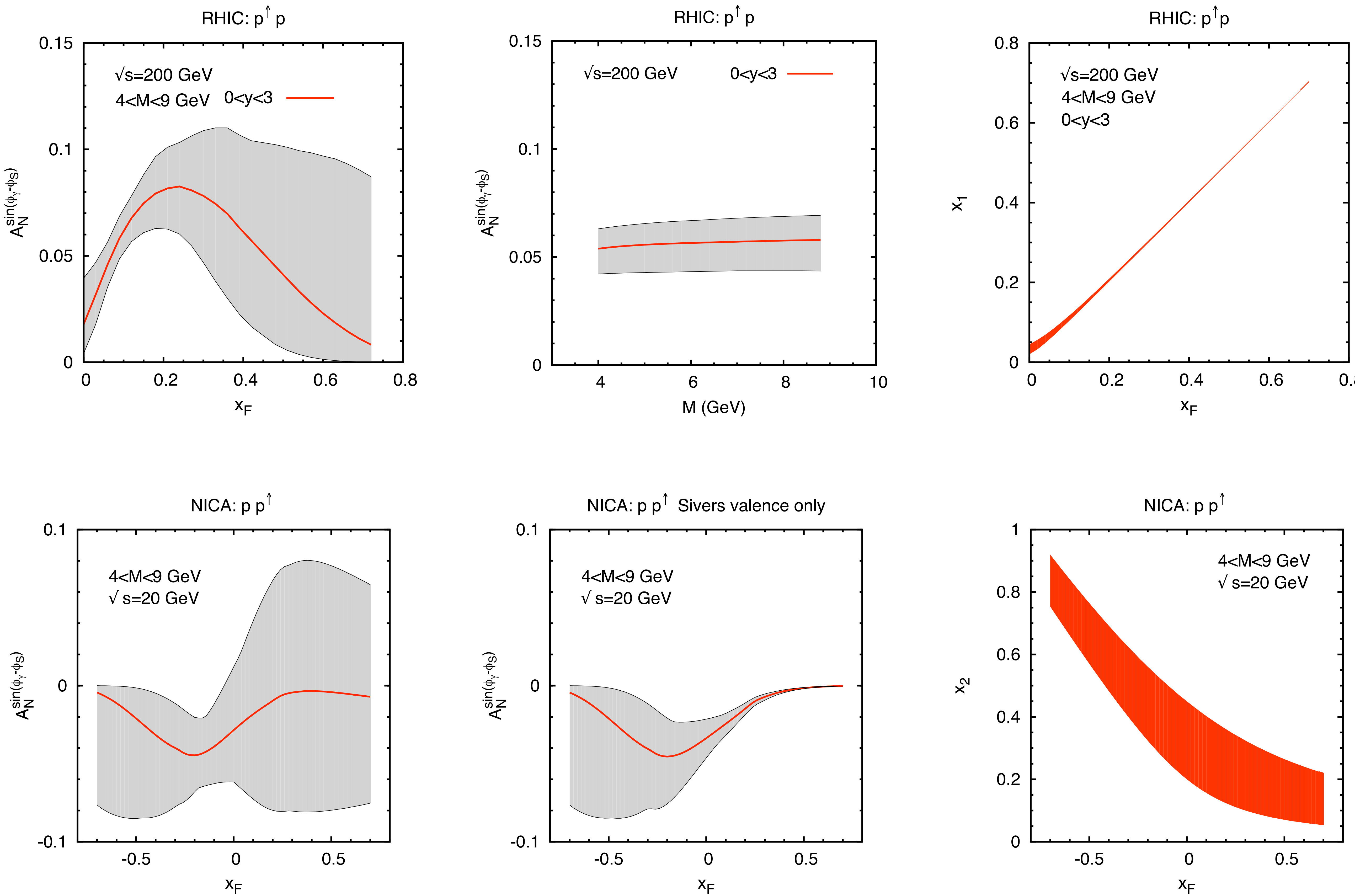


Predictions for A_N

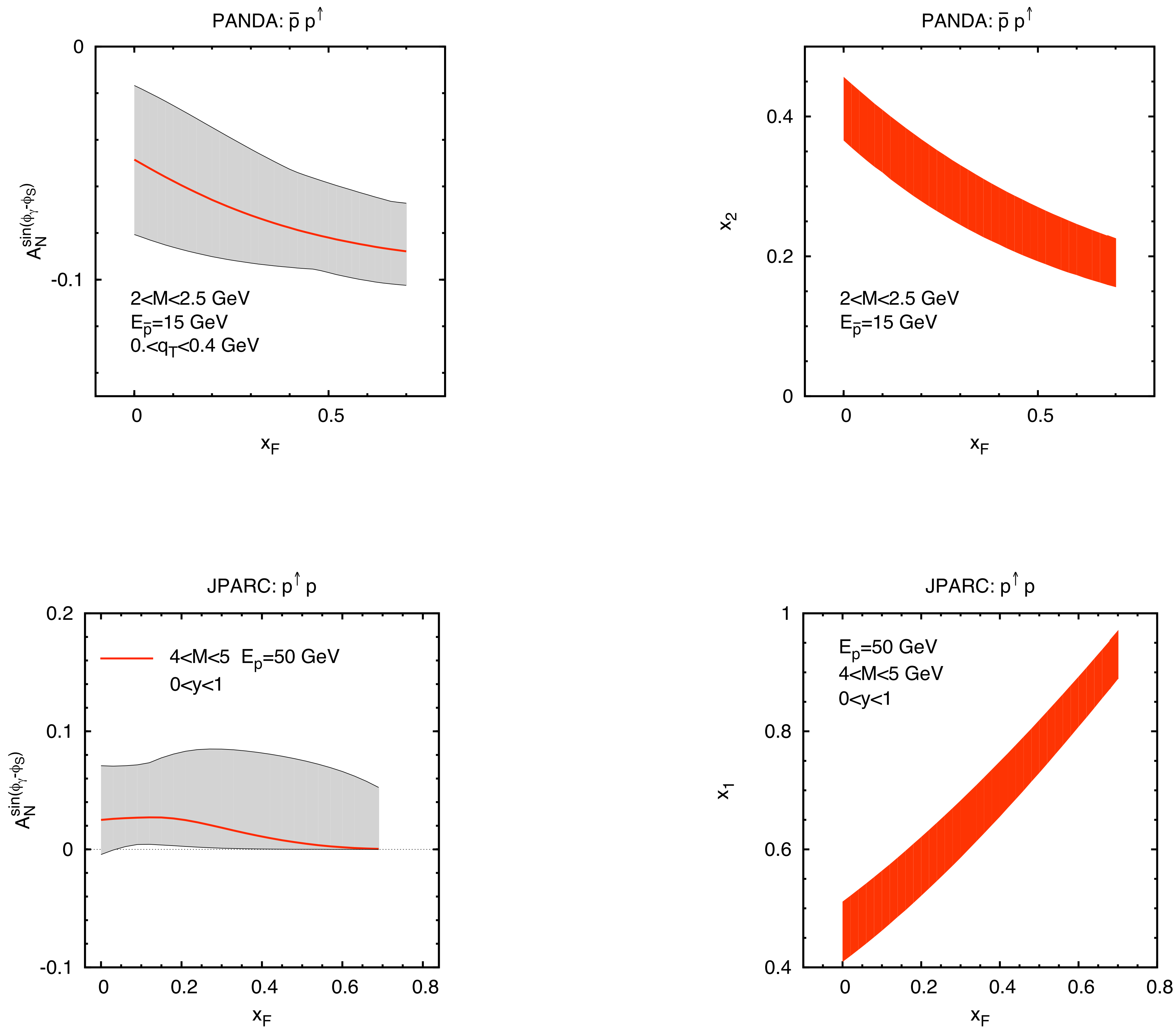
Sivers functions as extracted from SIDIS data, **with opposite sign**



M.A., M. Boglione, U. D'Alesio, S. Melis, F. Murgia, A. Prokudin, e-Print: arXiv:0901.3078



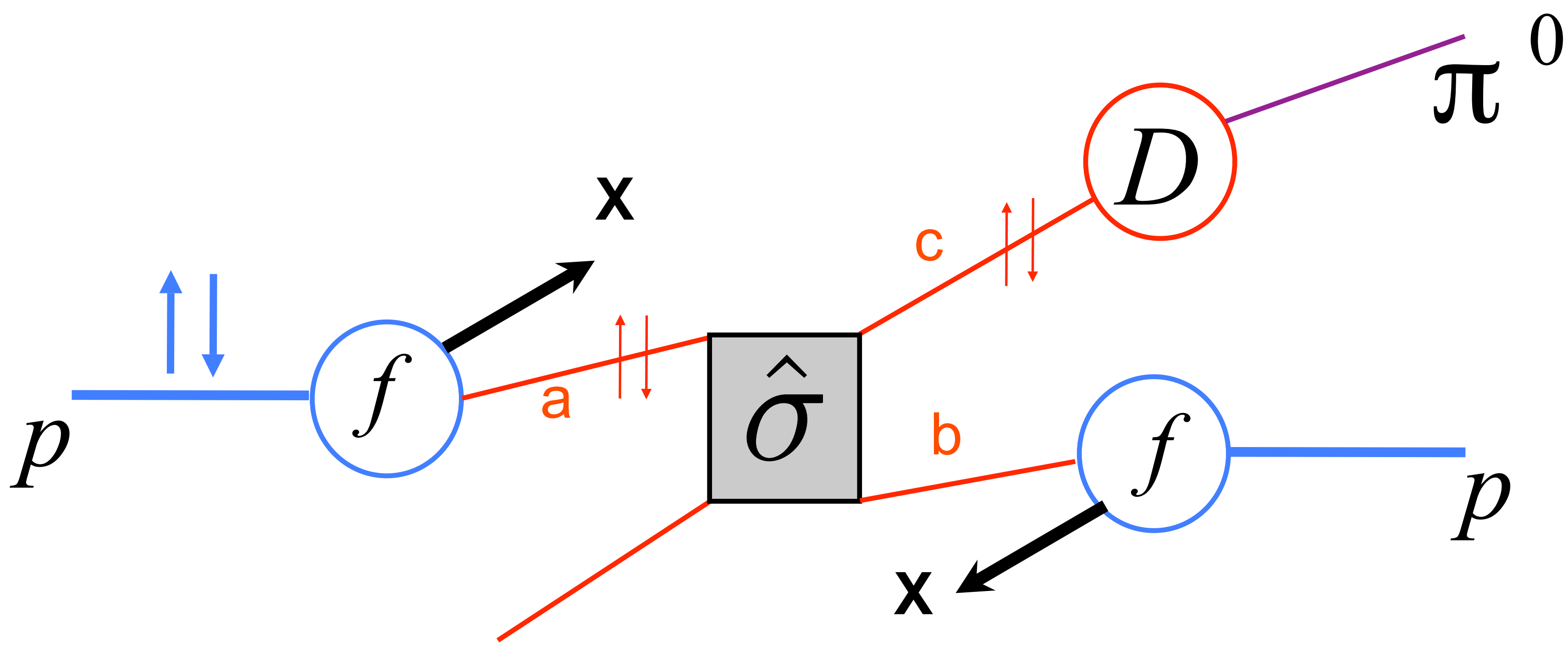
RHIC: p↑ p
√s=200 GeV
4<M<9 GeV
0<y<3
$A_N^{\sin(\phi_\gamma-\phi_S)}$
x_F
RHIC: p↑ p
√s=200 GeV
0<y<3
$A_N^{\sin(\phi_\gamma-\phi_S)}$
M (GeV)
RHIC: p↑p
√s=200 GeV
4<M<9 GeV
0<y<3
x_1
x_F
NICA: p p↑
4<M<9 GeV
√ s=20 GeV
$A_N^{\sin(\phi_\gamma-\phi_S)}$
x_F
NICA: p p↑ Sivers valence only
4<M<9 GeV
√ s=20 GeV
$A_N^{\sin(\phi_\gamma-\phi_S)}$
x_F
NICA: p p↑
4<M<9 GeV
√ s=20 GeV
x_2
x_F



PANDA: p̄ p↑
0
-0.1
A_N^{sin(φ_γ-φ_S)}
2<M<2.5 GeV
E_p̄=15 GeV
0.<q_T<0.4 GeV
0
0.5
x_F
PANDA: p̄ p↑
0.4
0.2
0
x_2
2<M<2.5 GeV
E_p̄=15 GeV
0
0.5
x_F
JPARC: p↑ p
0.2
0.1
0
-0.1
A_N^{sin(φ_γ-φ_S)}
4<M<5 E_p=50 GeV
0<y<1
0
0.2
0.4
0.6
0.8
x_F
JPARC: p↑ p
1
0.8
0.6
0.4
x_1
E_p=50 GeV
4<M<5 GeV
0<y<1
0
0.2
0.4
0.6
0.8
x_F

SSA in hadronic processes: TMDs, higher-twist correlations?

$$A_N \equiv \frac{d\sigma^{\uparrow} - d\sigma^{\uparrow}}{d\sigma^{\uparrow} + d\sigma^{\uparrow}}$$



Generalization of collinear scheme
(assuming factorization)

$$\mathrm{d}\sigma^{\uparrow} = \sum_{a,b,c=q,\bar{q},g} \underbrace{f_{a/p^{\uparrow}}(x_a, \boldsymbol{k}_{\perp a})} \otimes \underbrace{f_{b/p}(x_b, \boldsymbol{k}_{\perp b})} \otimes \mathrm{d}\hat{\sigma}^{ab\to cd}(\boldsymbol{k}_{\perp a}, \boldsymbol{k}_{\perp b}) \otimes \underbrace{D_{\pi/c}(z, \boldsymbol{p}_{\perp \pi})}$$

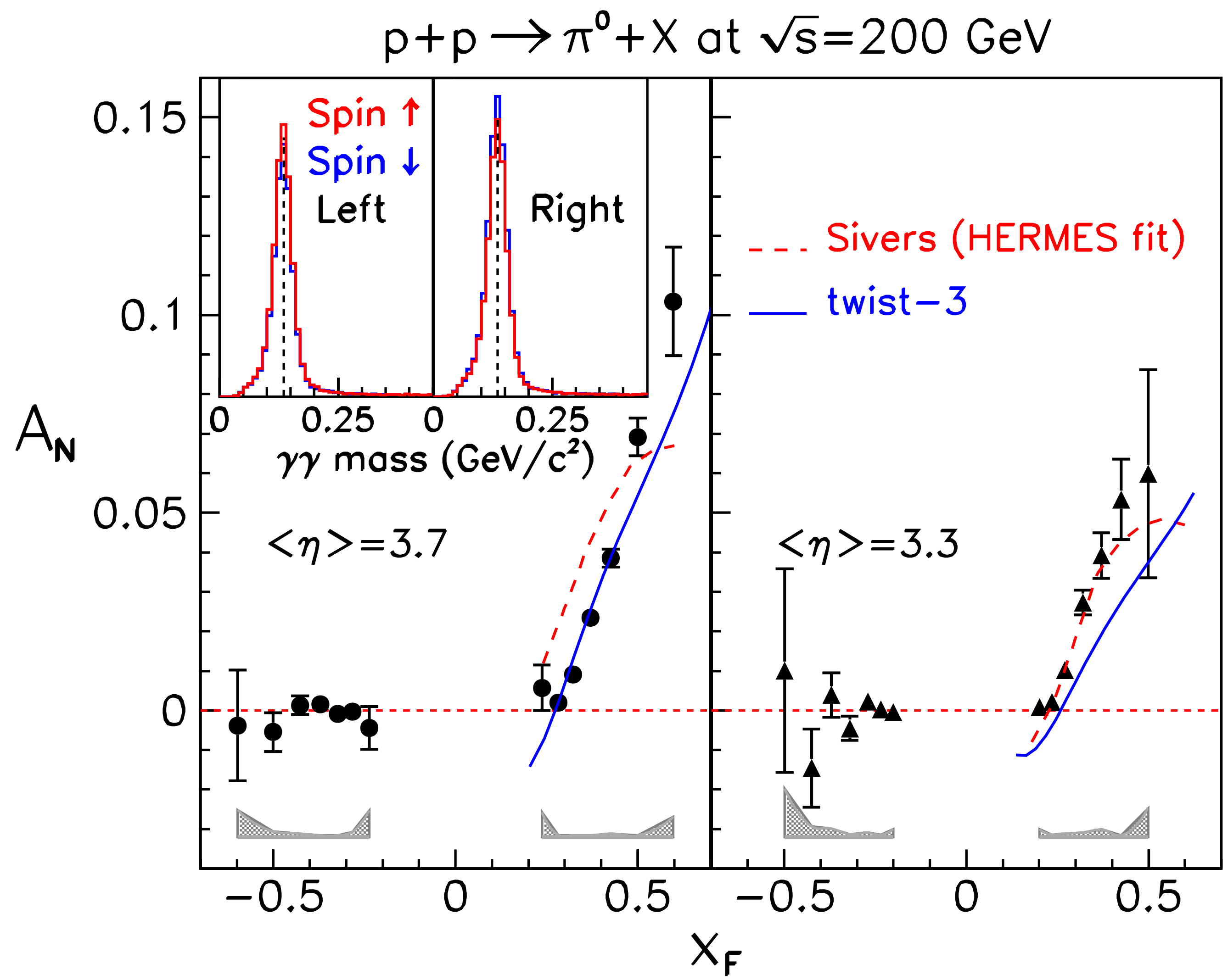
single spin effects in TMDs

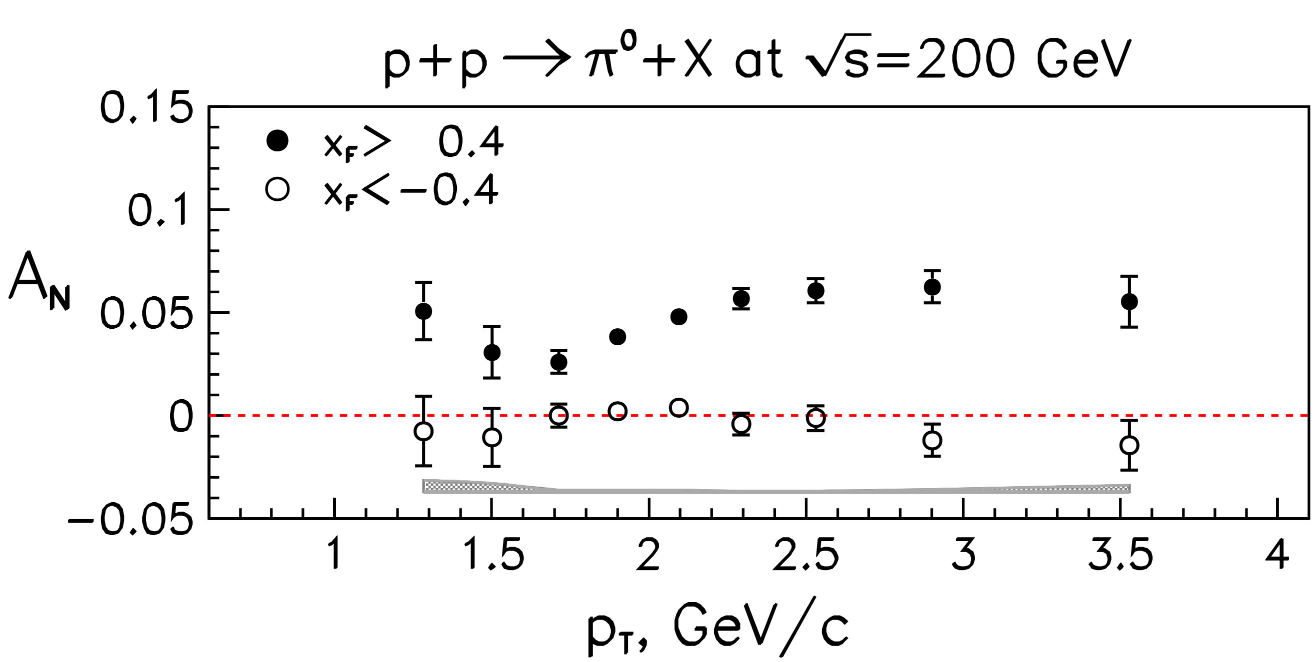
M.A., M. Boglione, U. D'Alesio, E. Leader, S. Melis, F. Murgia, A. Prokudin, ...

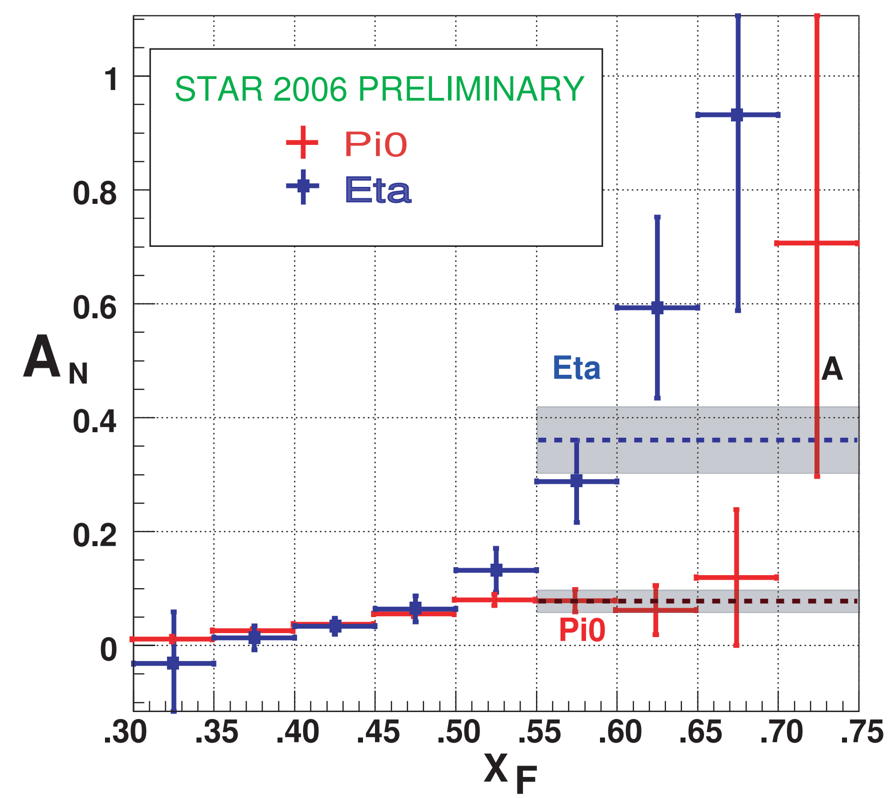
large SSAs observed

STAR data

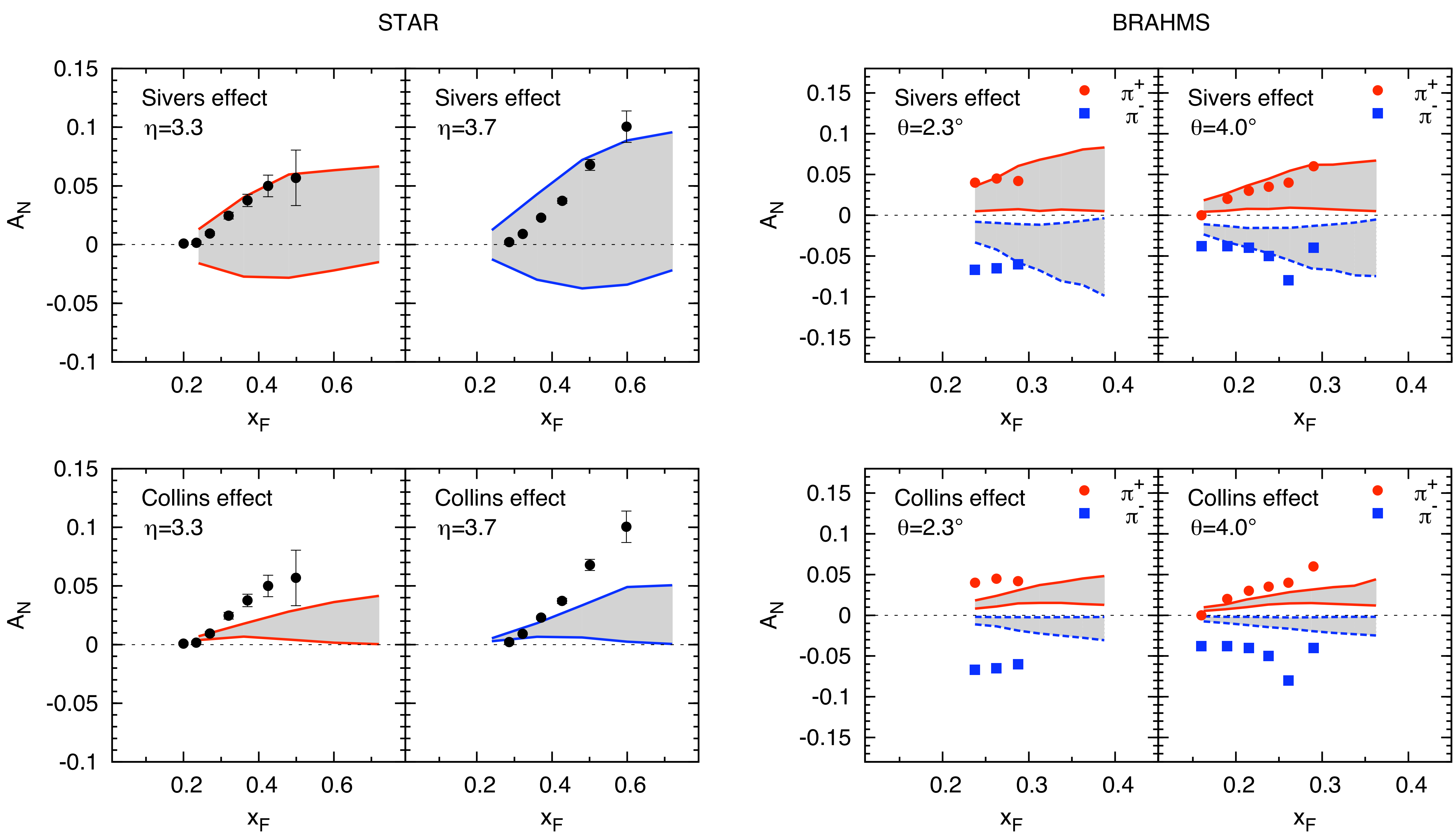
PRL 101, 222001 (2008)







contributions to A_N of SIDIS extracted Sivers, Collins and transversity distributions



a combination of Sivers and Collins effect might explain data

Conclusions

Both Collins and Sivers effects have been experimentally observed

First extractions of Sivers functions, from SIDIS data, mainly for u and d quarks; role of valence quarks not clear yet. Gluon Sivers function?

Role of Sivers function in other processes? Crucial test of sign change in Drell-Yan

First extractions of Collins functions (and transversity distributions) for u and d quarks. SIDIS and Belle data

Many open theoretical issues: Q^2 evolution of TMDs, factorization, universality, ...

The 3-dimensional exploration of the nucleon structure has just begun ....