Gluonic Poles in "G"-GPM & Universality

11 October 2010 ECT*

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1)arXiv:1009.1936 [hep-ph] w/ Z. Kang
2) A. Mukherjee and P. Mulders in prep

Outline

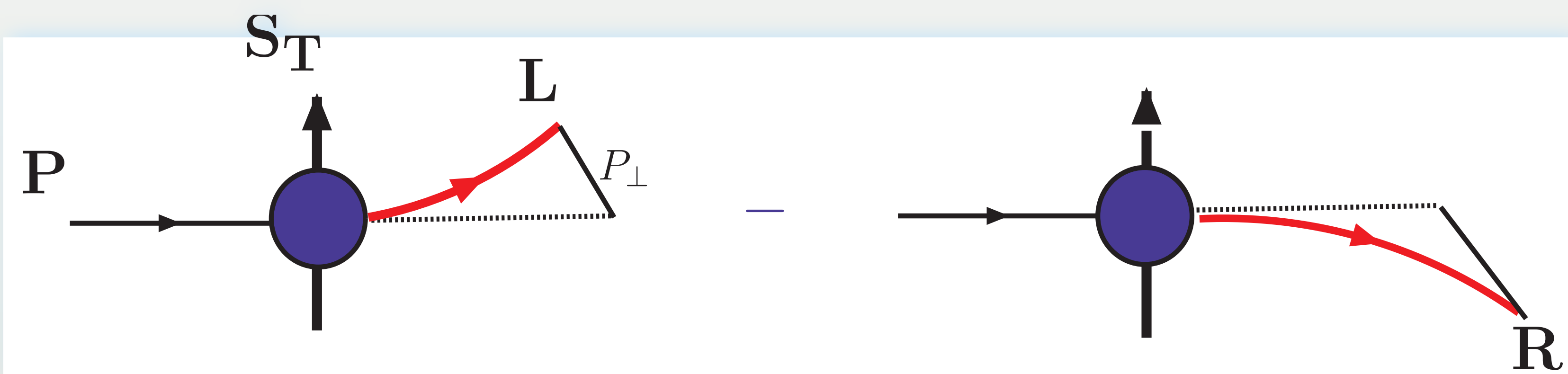
- **Transverse structure spin Effects in TSSAs**
- **Gauge links-Color Gauge Inv.-“T-odd” TMDs**
- **T-odd PDFs via FSIs ... Summing gauge link**

“QCD calc “ FSIs Gauge Links-Color Gauge Inv. “T-odd” TMDs

- Generalizing the Generalized Parton Model (GPM)--effects of FSI and ISI on color structure
- Connection to twist three & Gluonic Poles
- Universality and gluonic poles in fragmentation

Transverse SPIN Observables SSA (TSSA) $P^{\uparrow}P \to \pi X$

- Single Spin Asymmetry

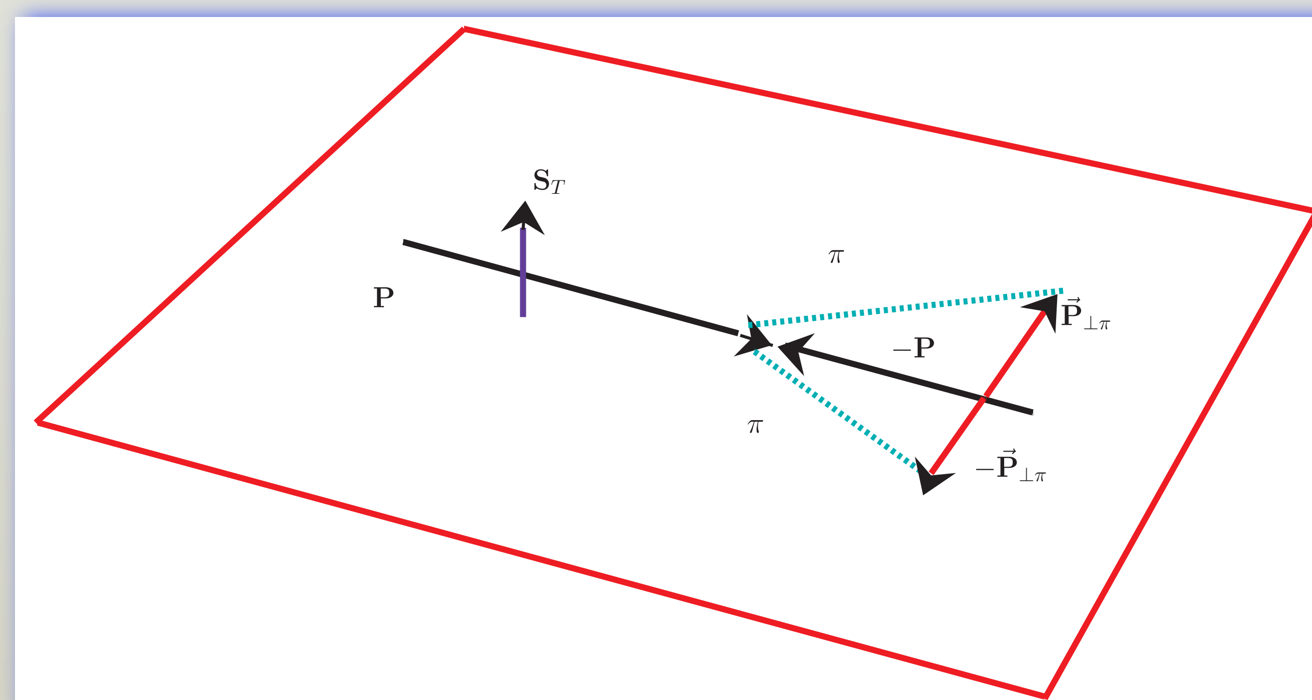


Parity Conserving interactions: SSAs Transverse Scattering plane

$$\Delta\sigma \sim iS_T \cdot (\mathbf{P} \times P_\perp^{\pi})$$

- Rotational invariance $\sigma^{\downarrow}(x_F, \boldsymbol{p}_\perp) = \sigma^{\uparrow}(x_F, -\boldsymbol{p}_\perp)$
  $\Rightarrow$ ***Left-Right Asymmetry***

$$\boldsymbol{A}_N = \frac{\sigma^{\uparrow}(x_F,\boldsymbol{p}_\perp)-\sigma^{\uparrow}(x_F,-\boldsymbol{p}_\perp)}{\sigma^{\uparrow}(x_F,\boldsymbol{p}_\perp)+\sigma^{\uparrow}(x_F,-\boldsymbol{p}_\perp)} \equiv \Delta\sigma$$

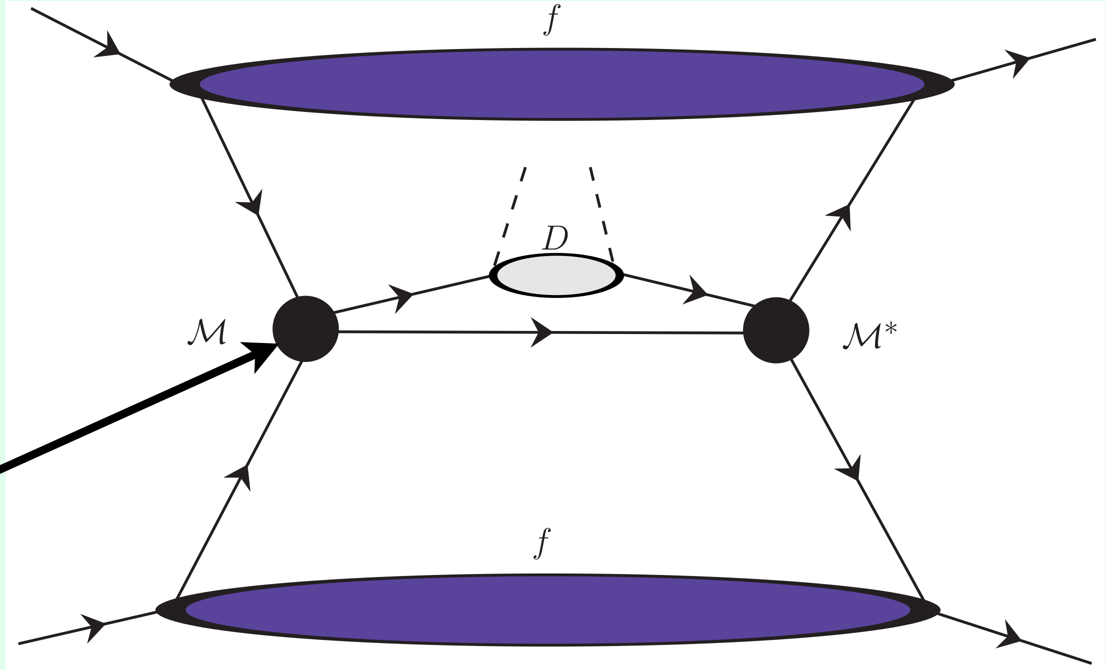


Transverse Polarization in Inclusive Reactions $P^{\uparrow} P \to \pi X$

e.g. Goldstein & Owens NPB 76

Transv. polarization cross section “interference” of helicity flip and non-flip amps.

quark-quark scattering



Elastic scattering of 2 quarks of different flavor

6 independent helicity Amps $M_{\lambda'_{q_1},\lambda'_{q_2};\lambda_{q_1},\lambda_{q_2}}$

$$M_{++,++} \equiv \Phi_1 \quad M_{--,++} \equiv \Phi_2 \quad M_{+-,+-} \equiv \Phi_3$$

$$M_{-+,+-} \equiv \Phi_4 \quad M_{-+,++} \equiv \Phi_5 \quad M_{++,+-} \equiv \Phi_6$$

$$A_N = \frac{\hat{\sigma}^{\uparrow} - \hat{\sigma}^{\downarrow}}{\hat{\sigma}^{\uparrow} + \hat{\sigma}^{\downarrow}} \sim \mathrm{Im}\left[\Phi_6(\Phi_1 + \Phi_3)^* - \Phi_5(\Phi_2 - \Phi_4)^*\right]$$

Interference of helicity flip and non-flip amps

1) requires breaking of chiral symmetry m_q/E

2) phases require higher order corrections

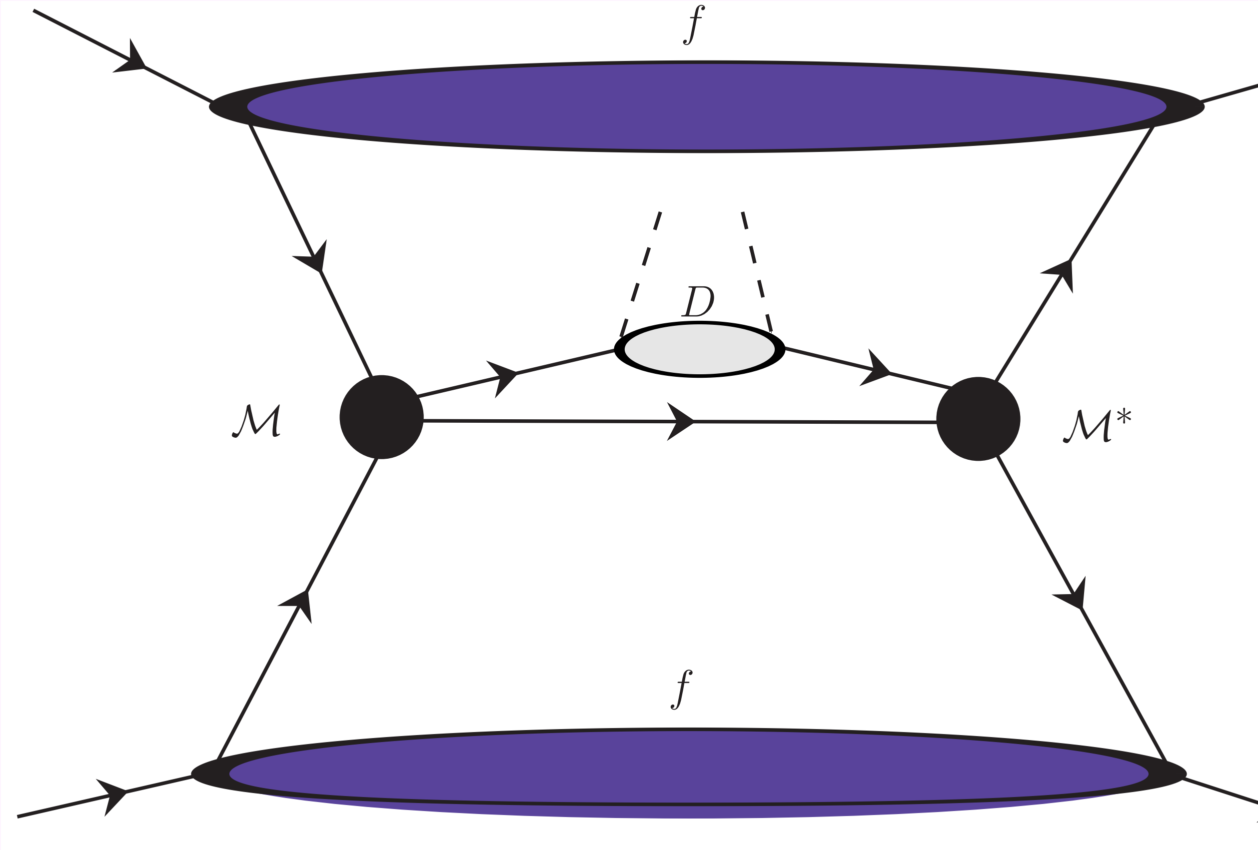
Collinear factorized QCD parton dynamics

$$\Delta\sigma^{pp^\uparrow \to \pi X} \sim f_a \otimes f_b \otimes \Delta\hat{\sigma} \otimes D^{q\to\pi}$$

$$\Delta\hat{\sigma} \equiv \hat{\sigma}^\uparrow - \hat{\sigma}^\downarrow$$

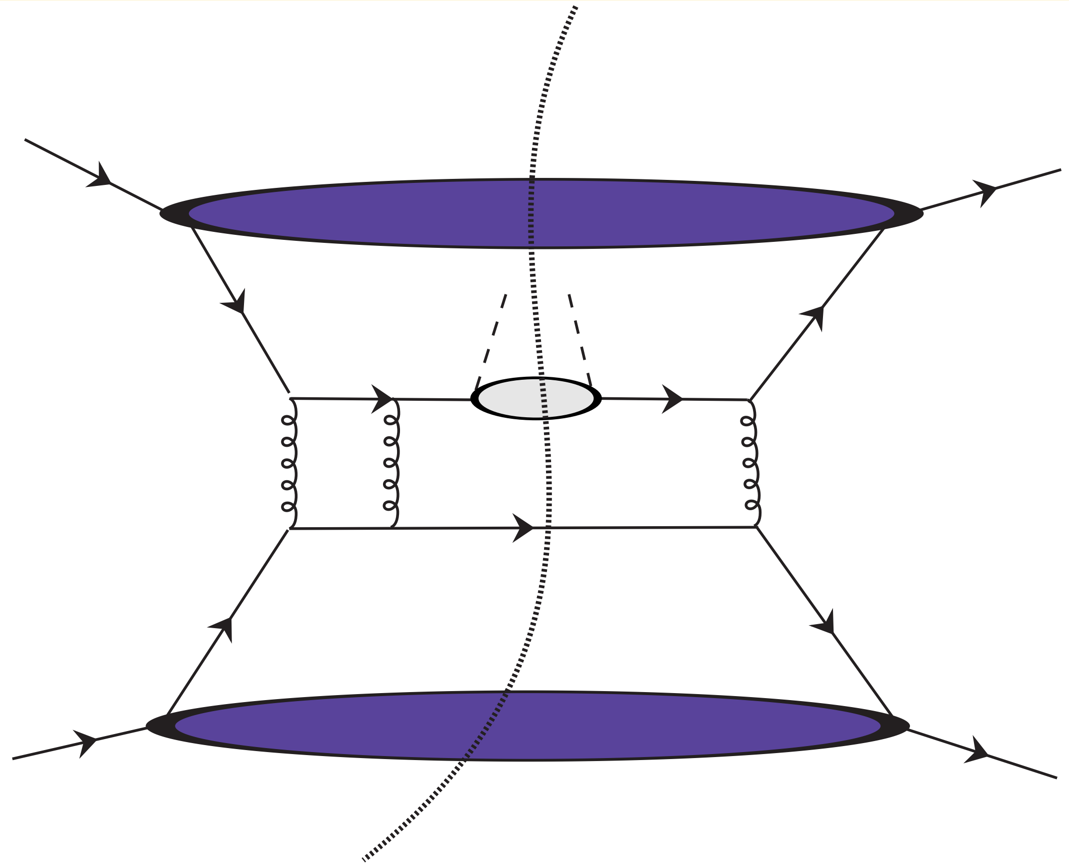
$$|\uparrow / \downarrow\rangle = (|+\rangle \pm i|-\rangle)$$

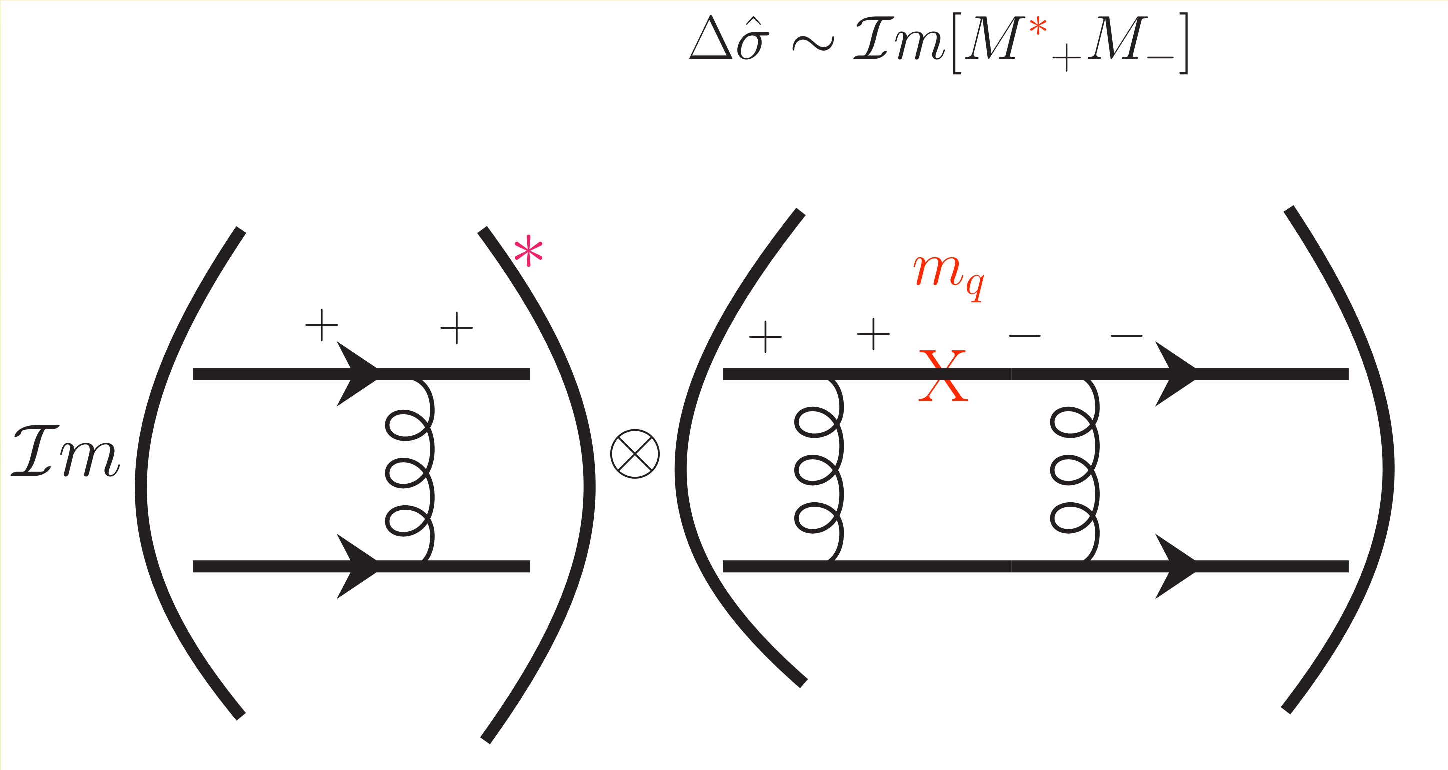
$$\hat{a}_N = \frac{\hat{\sigma}^\uparrow - \hat{\sigma}^\downarrow}{\hat{\sigma}^\uparrow + \hat{\sigma}^\downarrow} \sim \frac{\mathrm{Im}\left(\mathcal{M}^{+*}\mathcal{M}^-\right)}{|\mathcal{M}^+|^2 + |\mathcal{M}^-|^2}$$



⋆ **TSSA requires relative phase btwn** ***different*** **helicity amps**

Factorization Theorem & SSAs at Partonic level





- **Born amps are real -- need "loops"----> phases**
- **QCD interactions conserve helicity up to corrections** $\mathcal{O}\left(\frac{m_q}{E_q}\right)$

Twist three and trivial in chiral limit

$A_N \propto \frac{m_q}{E}\alpha_s$ at the partonic level

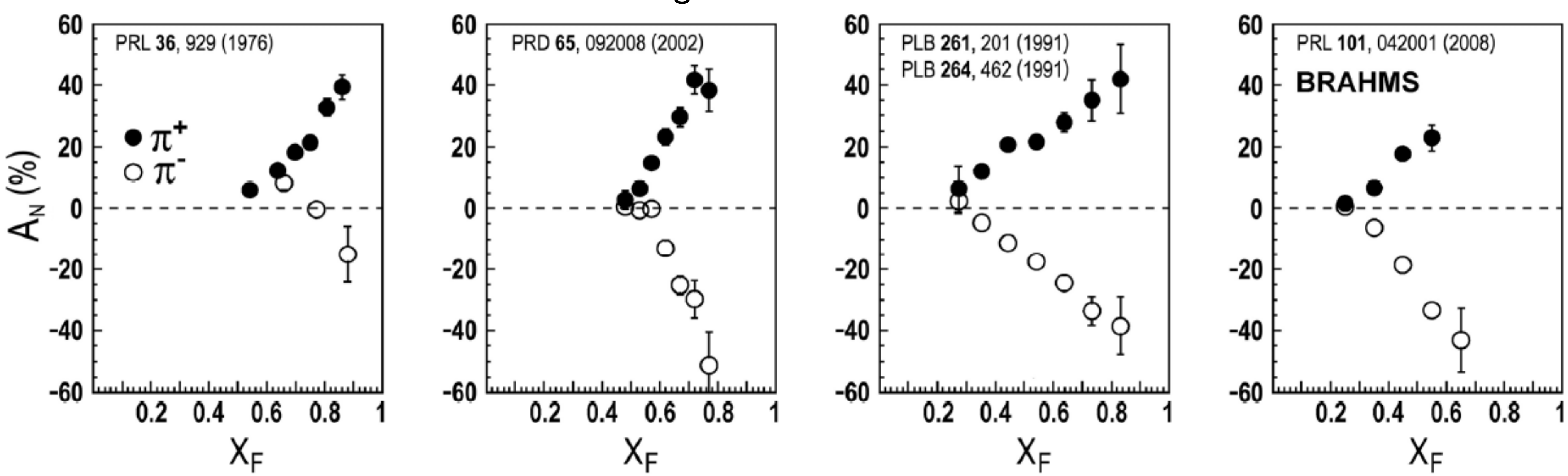
Kane & Repko, PRL: 1978

Large Transverse Polarization in Inclusive Reactions

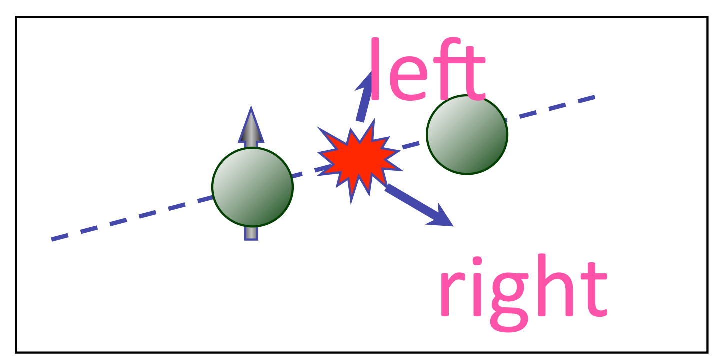
Transverse Single-Spin Asymmetries: From Low to High Energies!

p_{beam}=12 Gev/c	p_{beam}=22 Gev/c	p_{beam}=200 Gev/c	
ANL	BNL	FNAL	RHIC
$\sqrt{s}$=4.9 GeV	$\sqrt{s}$=6.6 GeV	$\sqrt{s}$=19.4 GeV	$\sqrt{s}$=62.4 GeV

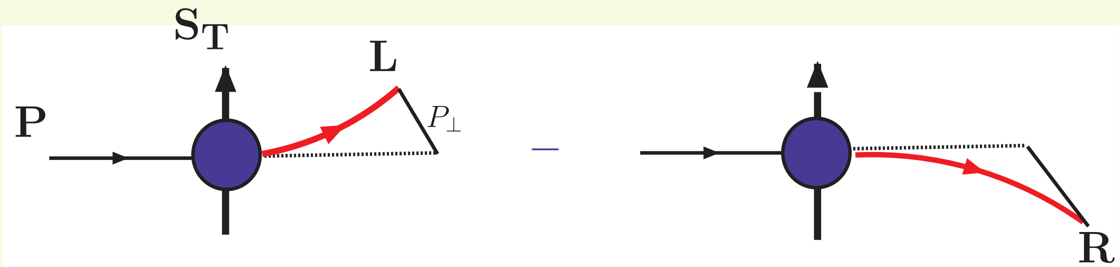
Fixed target | Collider

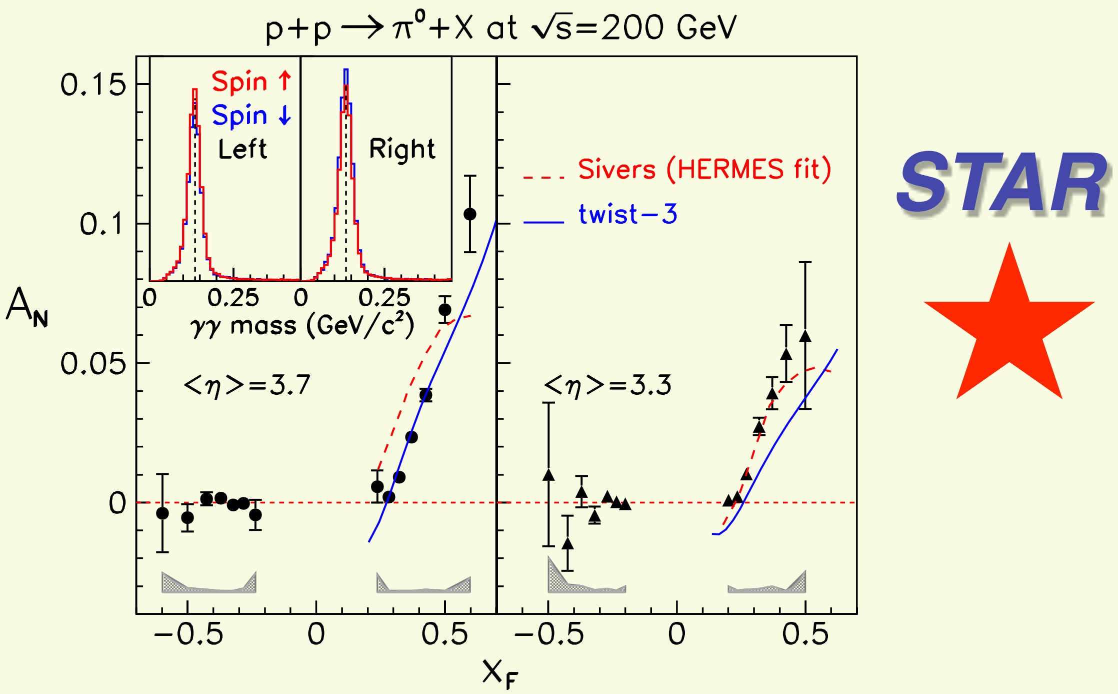


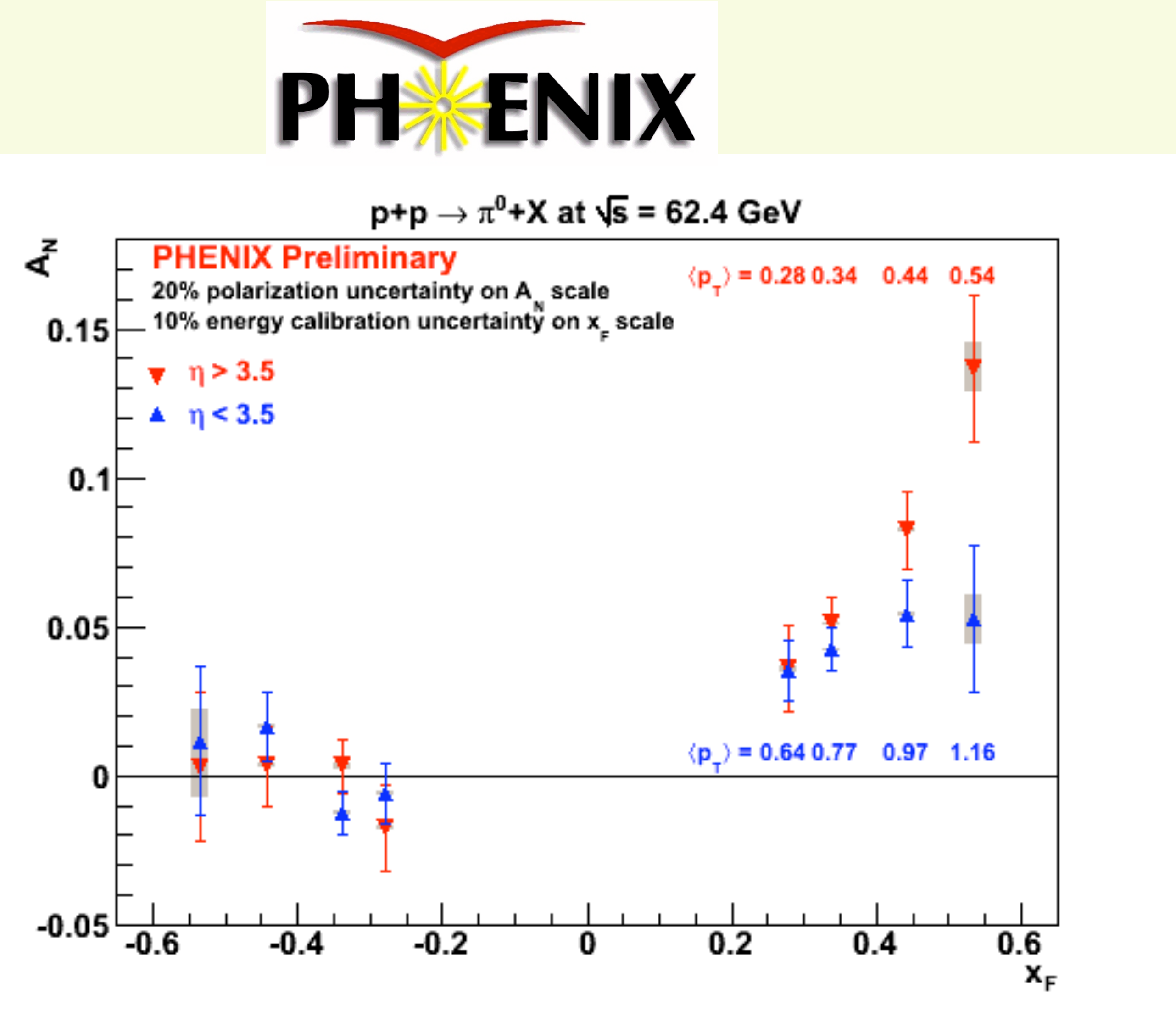
$$x_F = 2p_{long}/\sqrt{s}$$



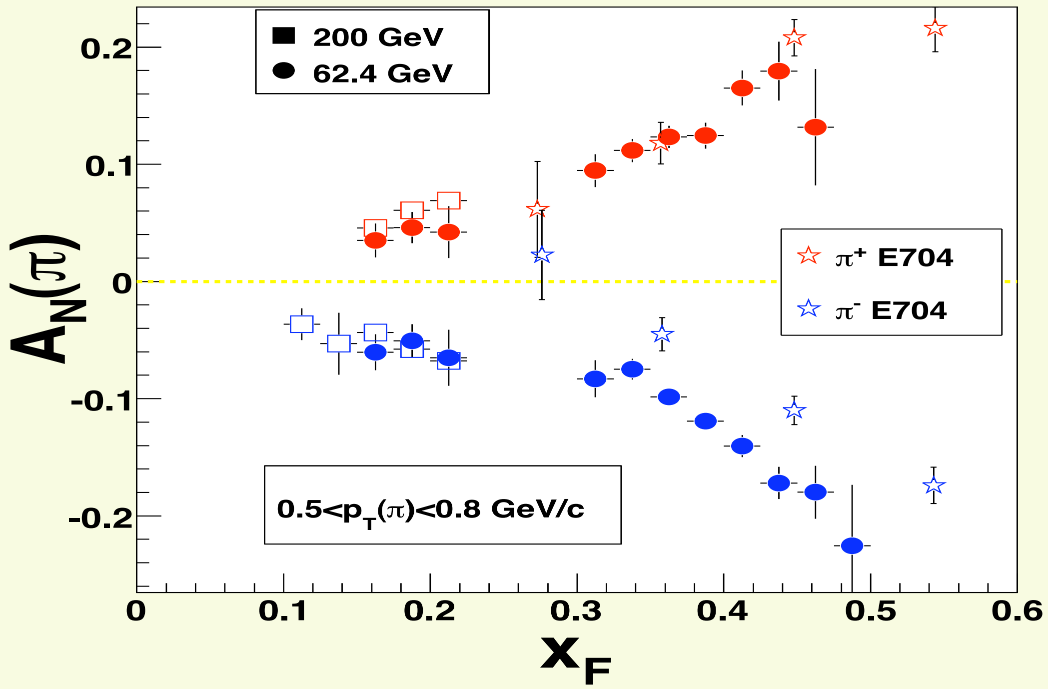
Modern Era Transverse SSA's at $\sqrt{s}$ = 62.4 & 200 GeV at RHIC







PRL101, 042001 (2008)



Polarization in inclusive Λ and $\overline{\Lambda}$ production at large p_T

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Physics Department, University of Wisconsin, Madison, Wisconsin 53706

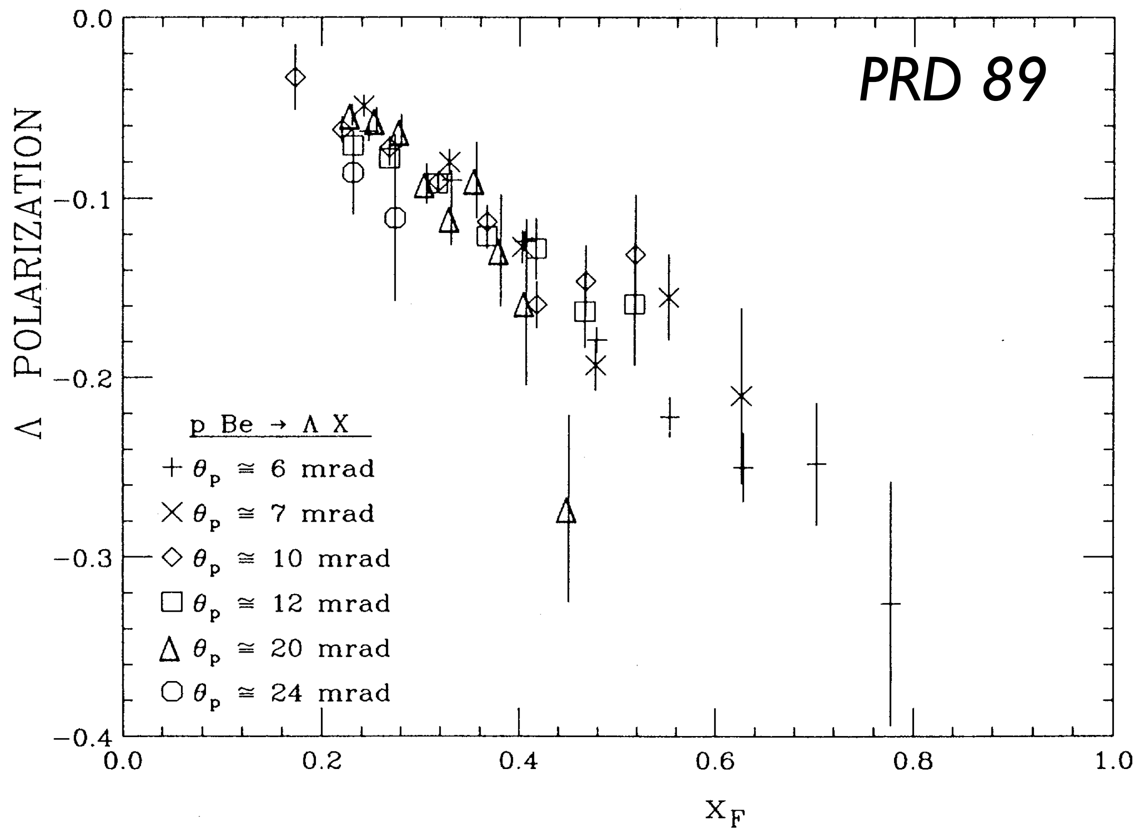


FIG. 4. The Λ polarization is shown as a function of x_F for all production angles. Over this range of production angles and within experimental uncertainties, the polarization is angle (or p_T) independent.

$$P_\Lambda = \frac{\sigma^{pp\to\Lambda^\uparrow X} - \sigma^{pp\to\Lambda^\downarrow X}}{\sigma^{pp\to\Lambda^\uparrow X} + \sigma^{pp\to\Lambda^\downarrow X}}$$

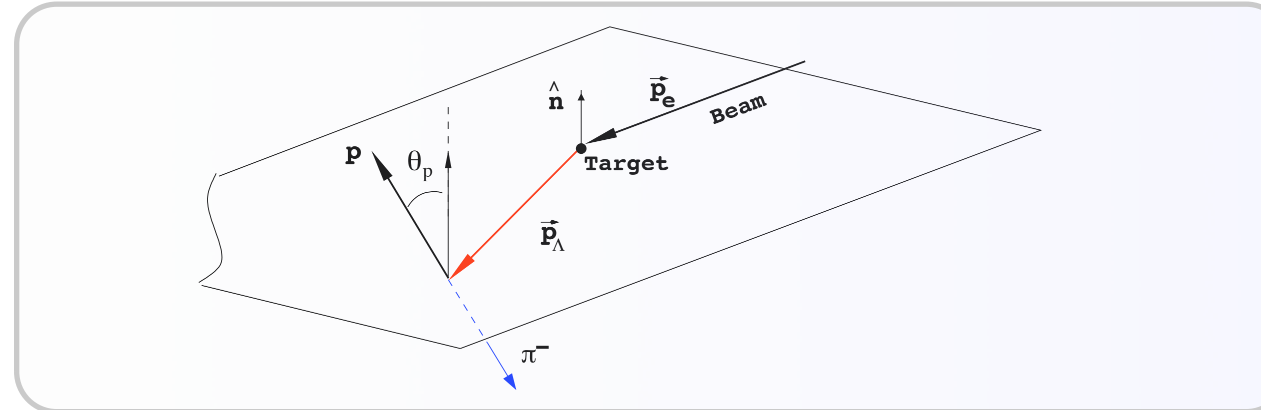


FIG. 1: Schematic diagram of inclusive Λ production and decay. The angle θ_p of the decay proton with respect to the normal $\hat{n}$ to the production plane is defined in the Λ rest frame.

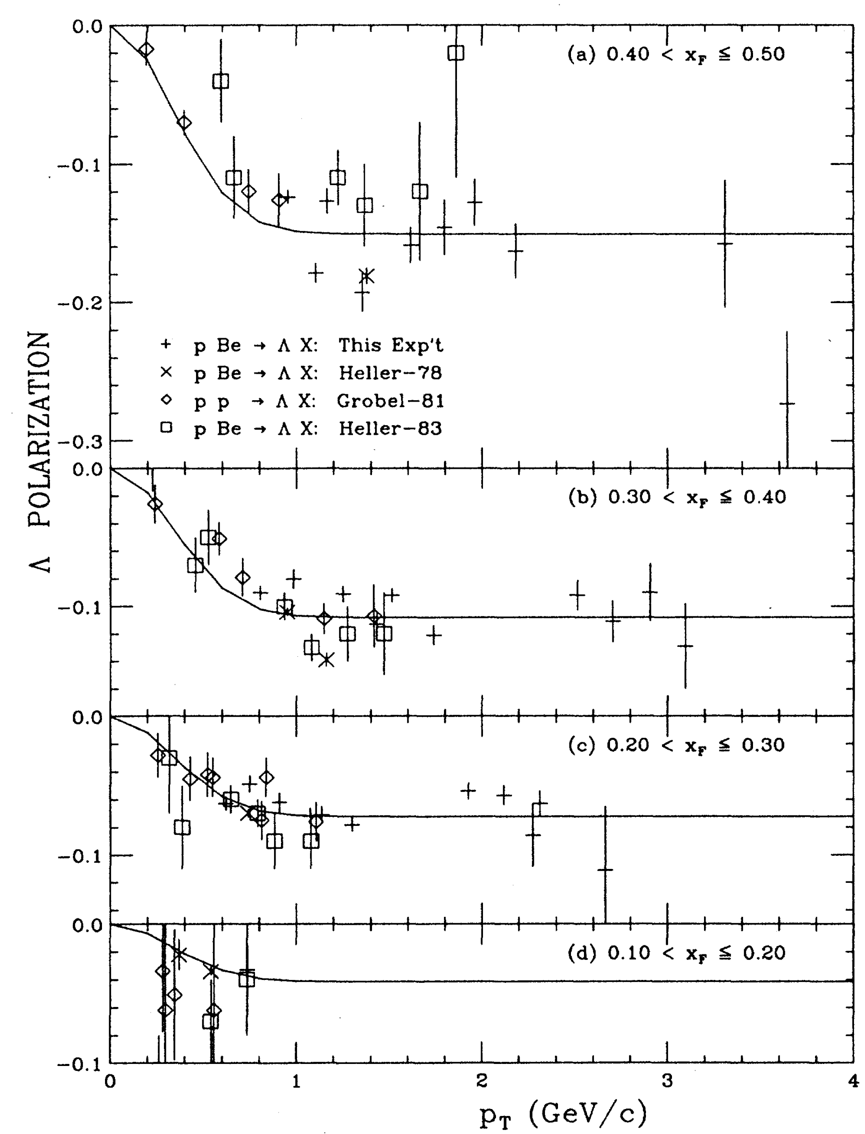


FIG. 5. Inclusive Λ polarization as a function of p_T with x_F restricted to each of the four ranges indicated in (a)–(d). The data plotted are from this experiment and Refs. 3, 23, and 24. All four experiments used the same spectrometer and measurement techniques. Errors when not shown are smaller than the points. The lines are a fit to the $p+$Be data using Eq. (9). Note

Comment

- Largest TSSA least understood

QCD test-Λ Production $pp \longrightarrow \Lambda^{\uparrow}\ X$

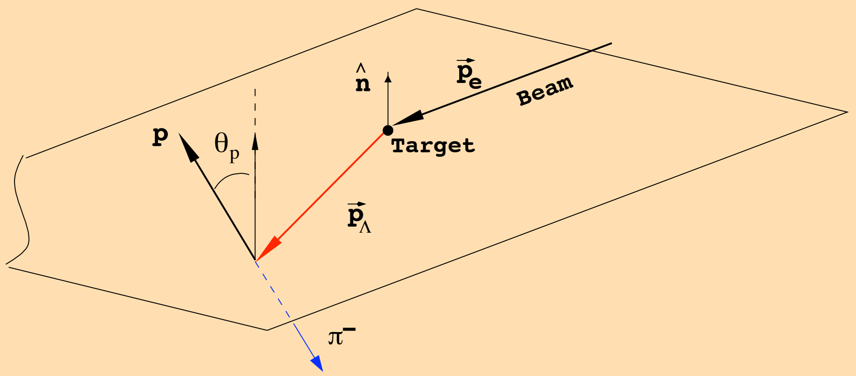


FIG. 1: Schematic diagram of inclusive Λ production and decay. The angle θ_p of the decay proton with respect to the normal $\hat{n}$ to the production plane is defined in the Λ rest frame.

Partonic Model Test.....

$$P_\Lambda = \frac{\sigma^{pp\to\Lambda^{\uparrow}X} - \sigma^{pp\to\Lambda^{\downarrow}X}}{\sigma^{pp\to\Lambda^{\uparrow}X} + \sigma^{pp\to\Lambda^{\downarrow}X}}$$

Interference of loops and tree level Phases in *hard part* $\Delta\hat{\sigma}$

- **Need strange quark to polarize a Λ**

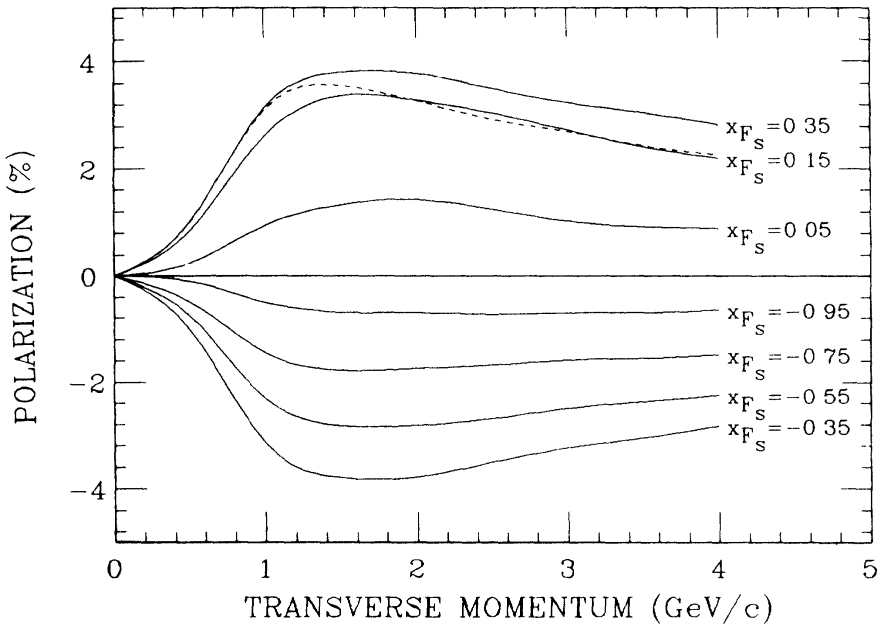


FIG. 4. Strange-quark polarization in the proton c.m. frame, $P_{\text{c.m.}} = 14$ GeV/c(400-GeV beam), after the convolution for the initial state gluons. x_{F_s} is the Feynman x for the strange quark. Dashed curve corresponds to $P_{\text{c.m.}} = 30.6$ GeV/c.

Dharmartna & Goldstein PRD 1990

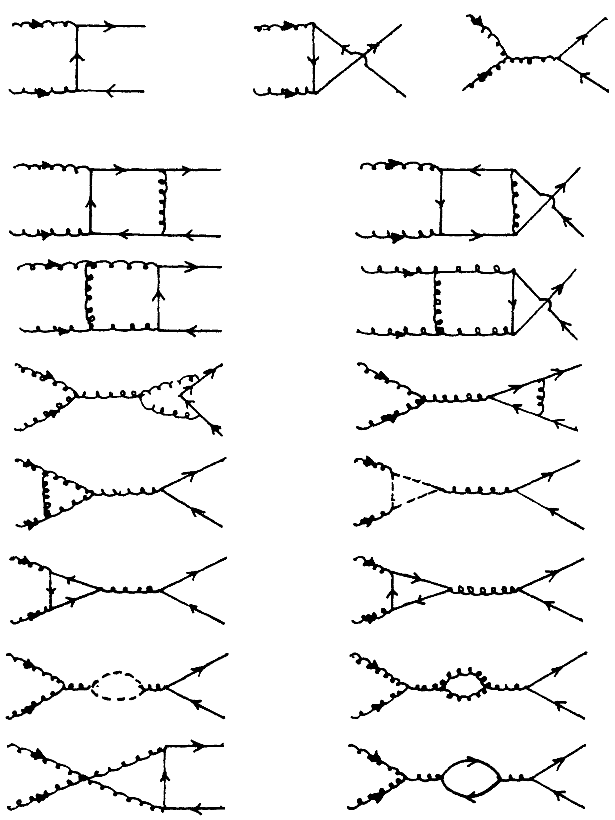
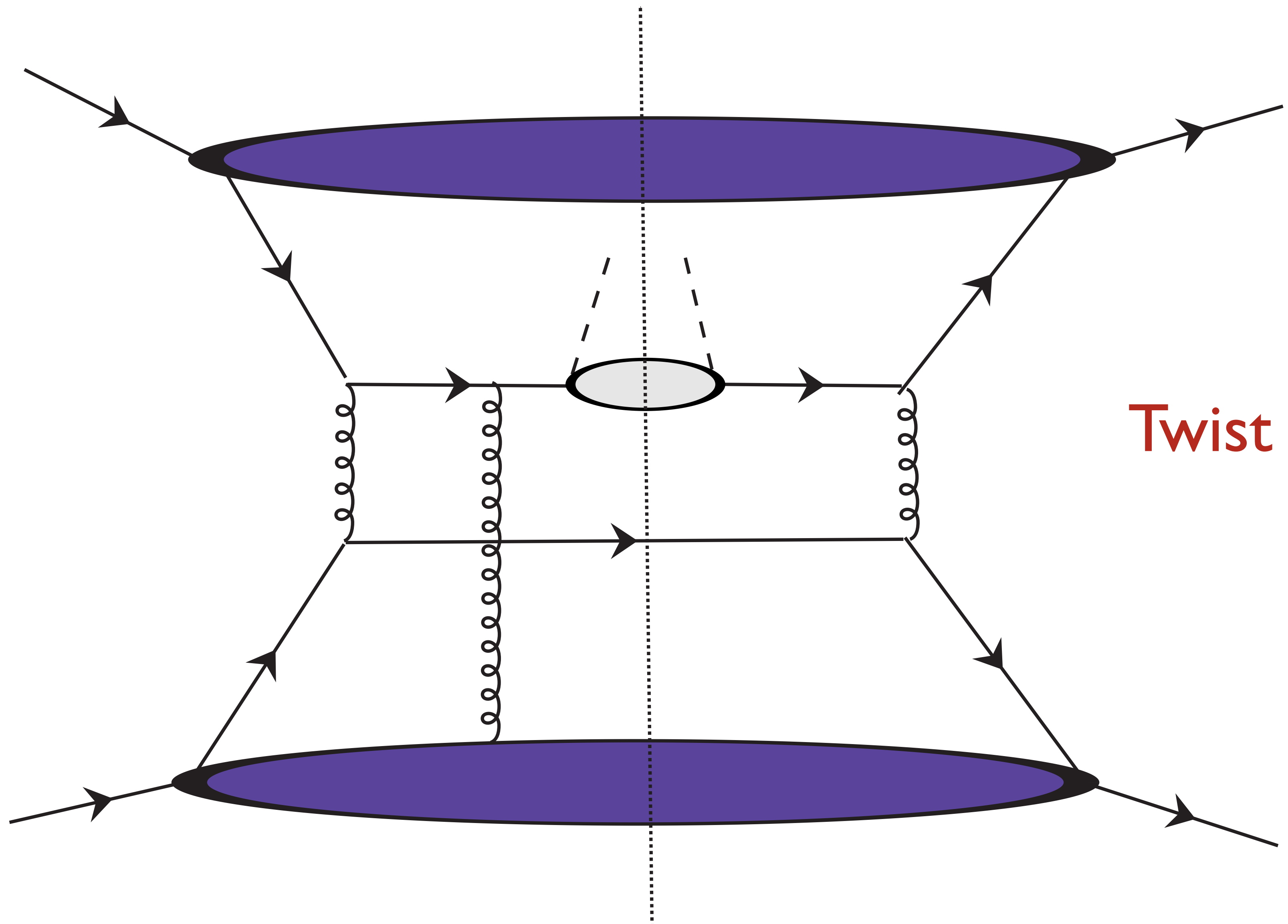


FIG. 1. Feynman diagrams for gluon fusion, $g+g\to s+\bar{s}$. In the second order, only the diagrams which contribute to the imaginary amplitude are shown.

Not the full story @ Twist 3 approach ETQS approach
Gluonic Poles

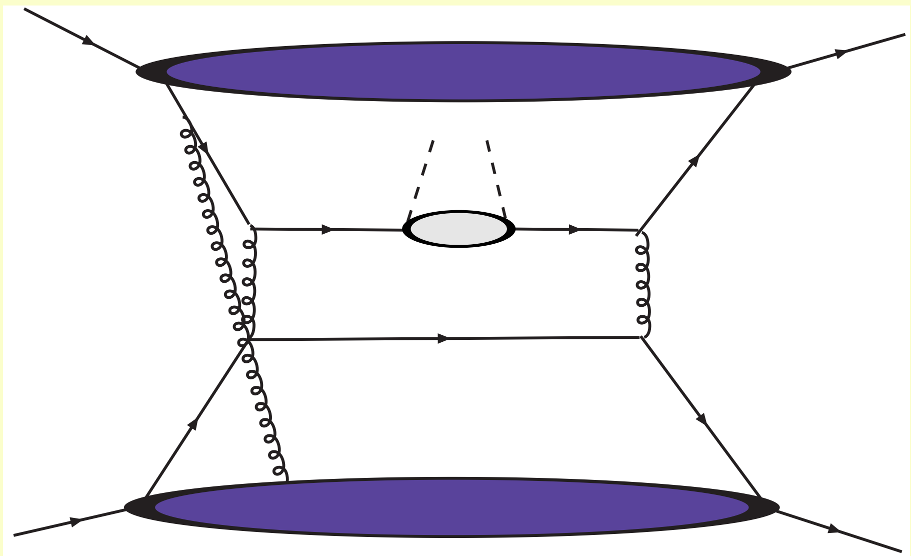


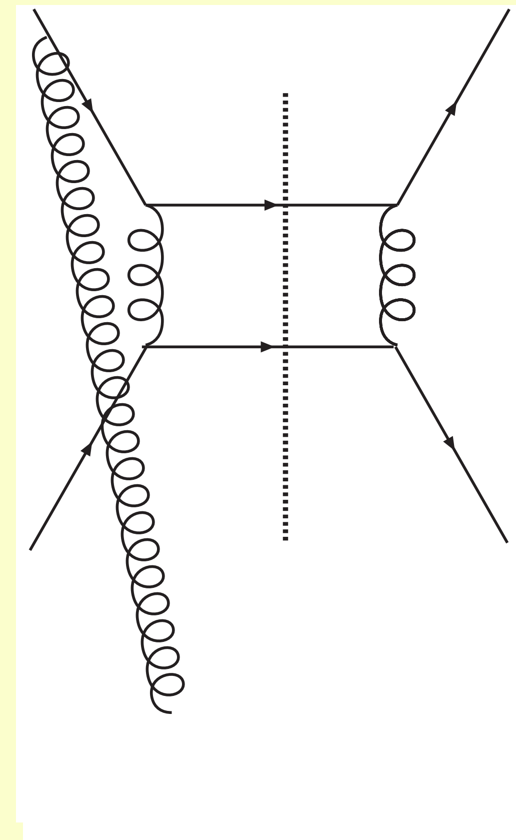
Twist three and non-trival?!

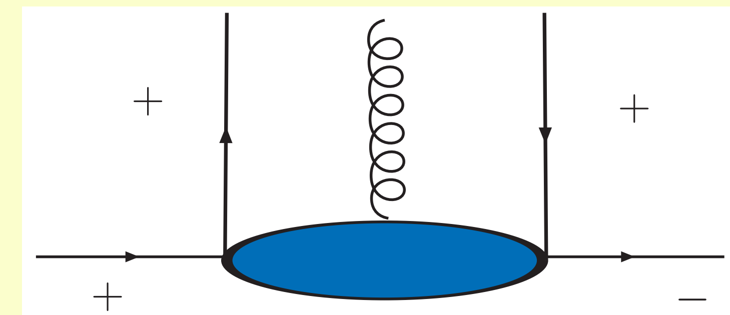
Phases in soft poles propagators-hard subprocesses Efremov & Teryaev Yad. Fiz & PLB 1984-1985
Factorization and Pheno: Qiu, Sterman 1991,1999..., Koike et al, 2000, ... 2010, Ji, Qiu, Vogelsang, Yuan, 2005 ... 2008 ..???, Yuan, Zhou 2008, 2009, Kang, Qiu, 2008, 2009 ...
Kouvaris Ji, Qiu, Vogelsang! 2006, Vogelsang and Yuan 2007, Bacchetta et al. 2007

$Q \sim P_T >> \Lambda_{\rm qcd}$ Co-linear Twist 3 Mechanism

Phases in $soft$ poles of propagator in hard subprocess Efremov & Teryaev :PLB 1982







T_F

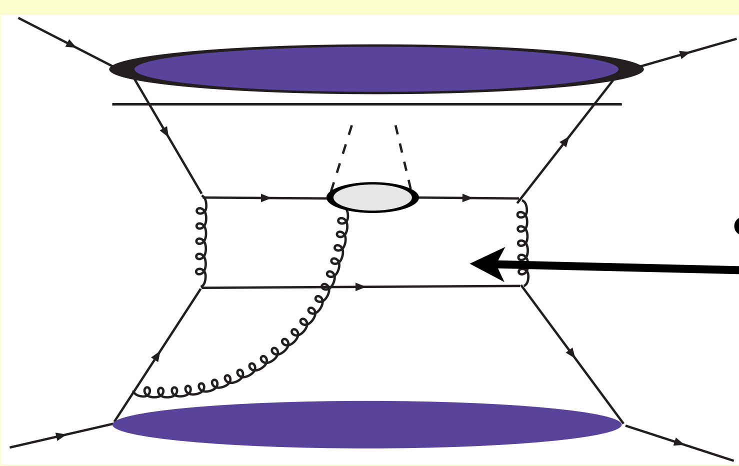
⋆ **Get helicity flips and phases** $m_q \rightarrow\sim M_H$

- $\Delta\sigma \sim f_a \otimes T_F \otimes H_{ETQS} \otimes D^{q\rightarrow\pi}$

$$\frac{1}{xs + i\epsilon} = \mathcal{P}\left(\frac{1}{xs}\right) \pm i\pi\delta(xs)$$

quark-gluon-quark correlator

Transversity in pp Koike 2002



...

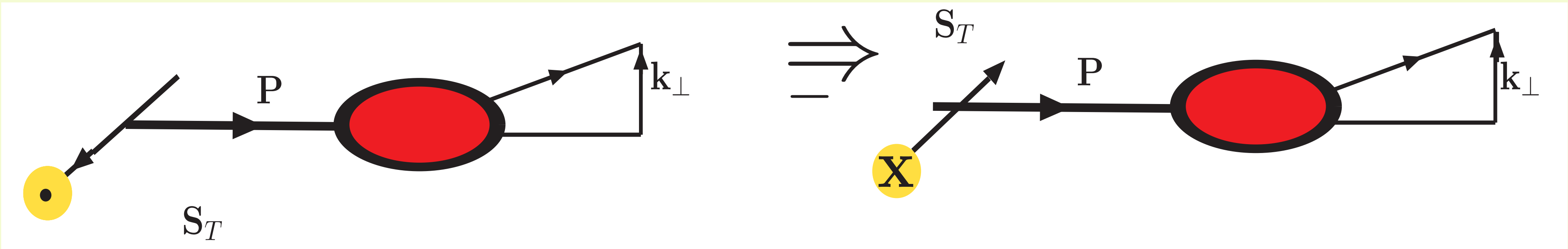
quark-gluon-quark correlator-frag

$$\Delta\sigma \sim \delta q(x) f(x') \otimes \hat{H}(z_1, z_2) \otimes \hat{\sigma}$$

TSSAs thru "T-odd" non-pertb. **spin-orbit** *correlations....*

Sensitivity to $p_T \sim \mathbf{k}_T << \sqrt{Q^2}$

- Sivers PRD: 1990 **TSSA is associated w/ correlation** *transverse* **spin and momenta in initial state hadron**

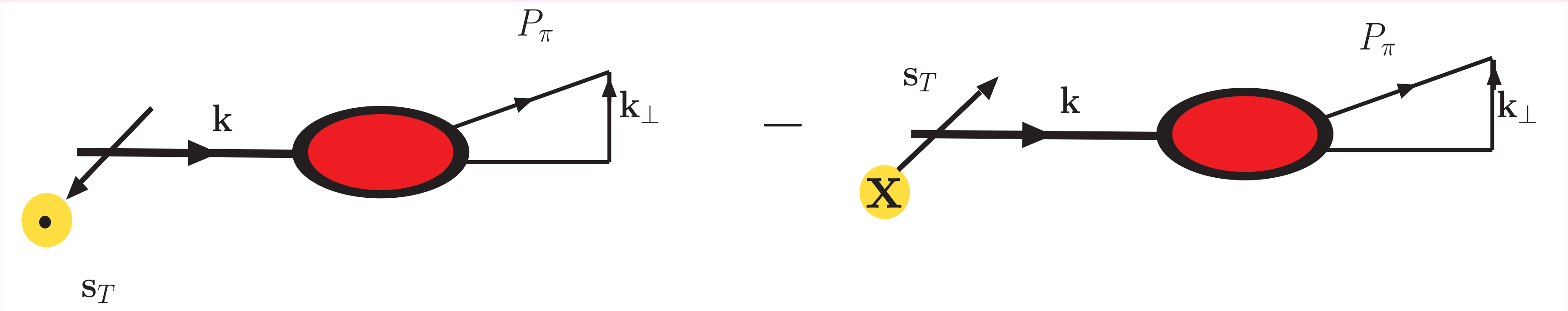


$$\Delta\sigma^{pp^{\uparrow}\to\pi X} \sim D \otimes f \otimes \Delta f^{\perp} \otimes \hat{\sigma}_{Born}$$

$$\boldsymbol{\Delta f^{\perp}(x, k_\perp) = i S_T \cdot (P \times k_\perp)}\, f_{1T}^{\perp}(x, \boldsymbol{k}_\perp)$$

.... Fragmentation

- **Collins NPB: 1993** **TSSA is associated with** $\mathit{transverse}$ **spin of fragmenting quark and transverse momentum of final state hadron**

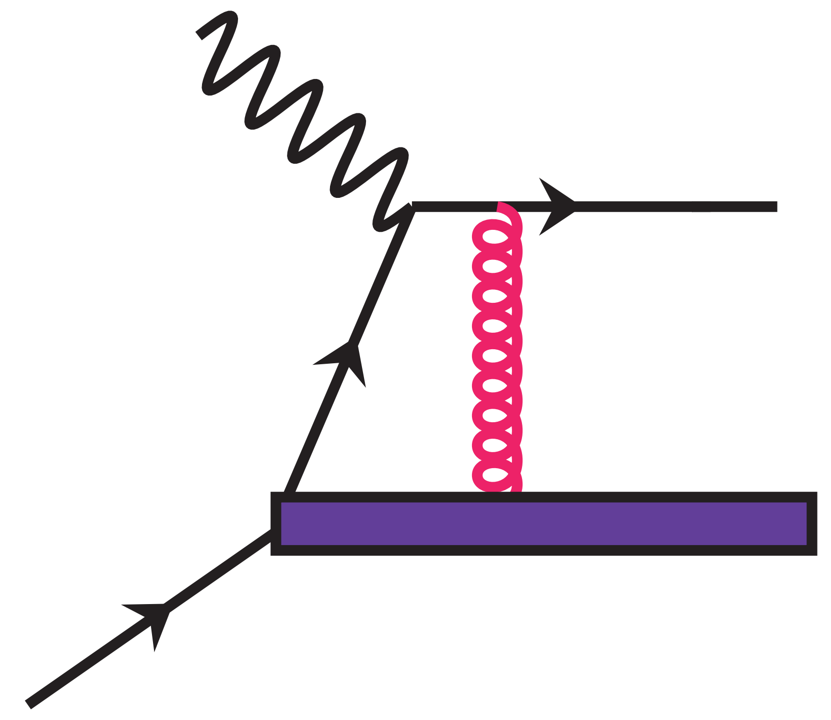


$$\Delta\sigma^{ep^\uparrow \to e\pi X} \sim \Delta D^\perp \otimes f \otimes \hat{\sigma}_{Born}$$

$$\boldsymbol{\Delta D^\perp(x, p_\perp) = i s_T \cdot (P \times p_\perp)}\, H_1^\perp(x, \boldsymbol{p}_\perp)$$

Reaction Mechanism-FSI phases in TSSAs
unsupressed Leading TWIST

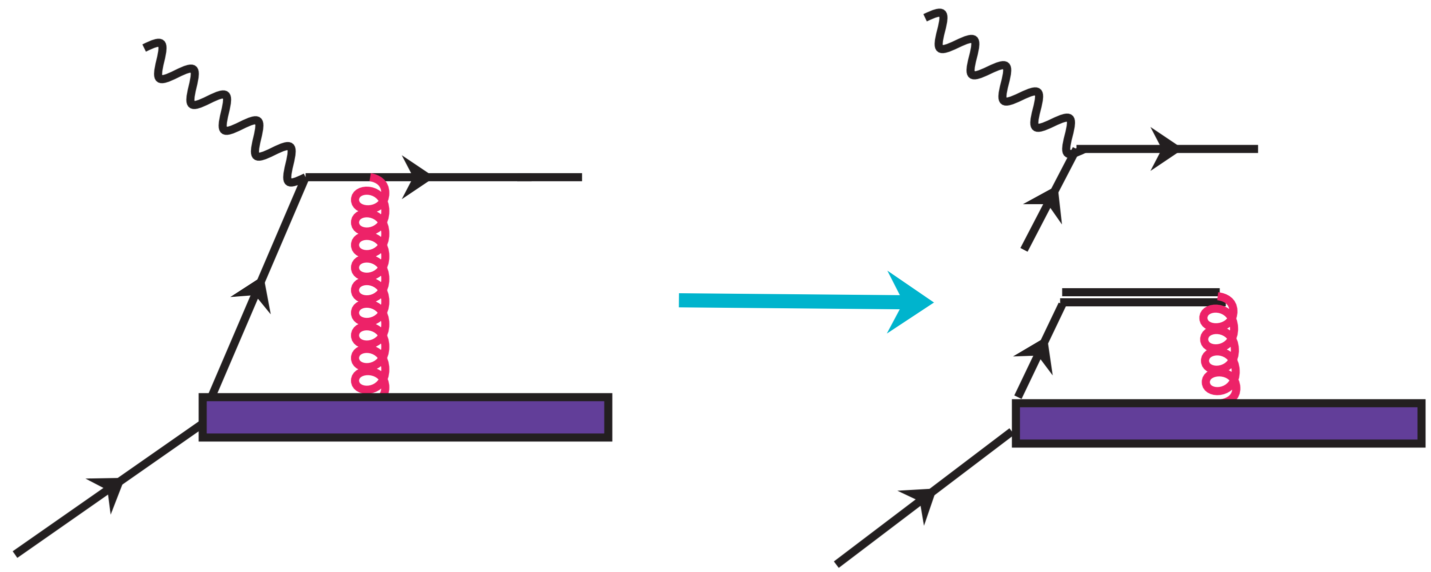
$$e\ p^{\uparrow} \to e \pi X$$



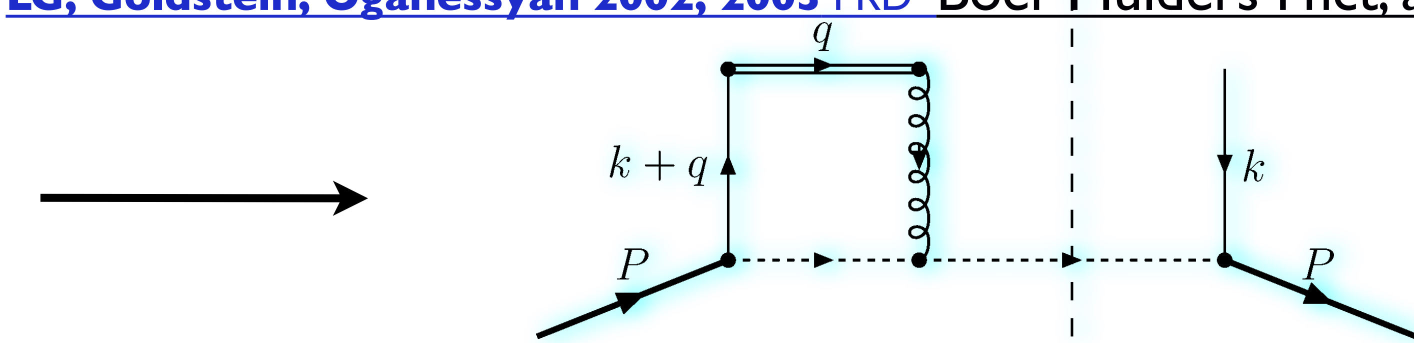
$$\boldsymbol{\Delta f^{\perp}(x, k_{\perp}) = i S_T \cdot (P \times k_{\perp})}\ f_{1T}^{\perp}(x, \boldsymbol{k}_{\perp})$$

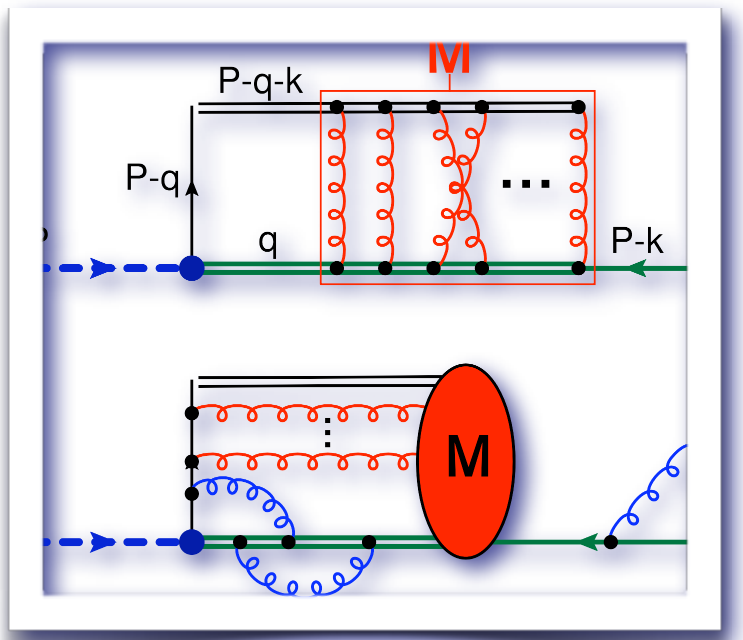
- Brodsky, Hwang, Schmidt PLB: 2002
  SIDIS w/ transverse polarized nucleon target
- **Unsurpressed reaction mech. Boer PRD 1999 context of DY process RHIC**

FSI phases in TSSAs unsupressed



- Ji, Yuan PLB: 2002 -Sivers fnct. FSI emerge from Color Gauge-links
- ***Collins PLB 2002- Gauge link Sivers function doesn't vanish***
- **LG, Goldstein, Oganessyan 2002, 2003** PRD Boer-Mulders Fnct, and Sivers -spectator model





- **LG, M. Schlegel, PLB 2010** Boer-Mulders Fnct, and Sivers beyond summing the FSIs through the gauge link

Factorization & Sensitivity to $P_T \sim k_\perp$ $\longrightarrow$ TMDs

John Collins

Nuclear Physics B396 (1993) 161–182

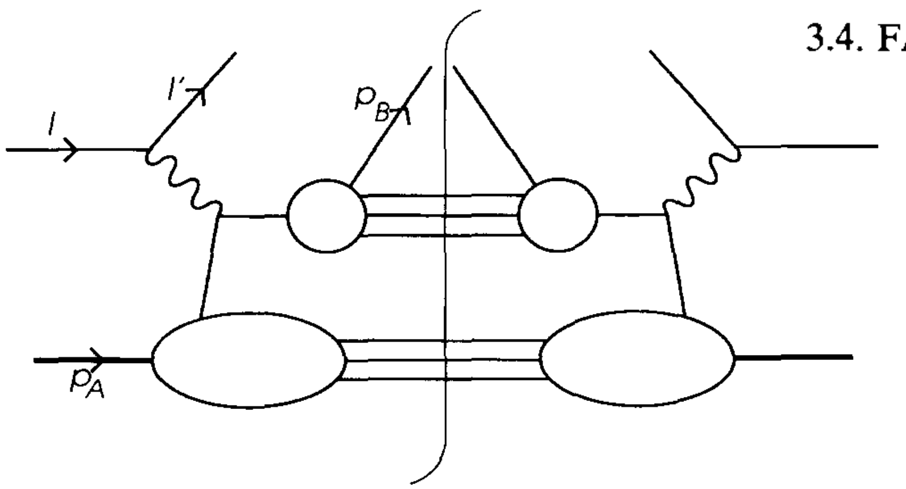


Fig. 2. Parton model for semi-inclusive deeply inelastic scattering.

3.4. FACTORIZATION WITH INTRINSIC TRANSVERSE MOMENTUM AND POLARIZATION

Collins Soper NPB 1981, & Sterman NPB 1985

Ralston Soper NPB 1979, Collins NPB 1993

Factorization parton model when P_T of the hadron small

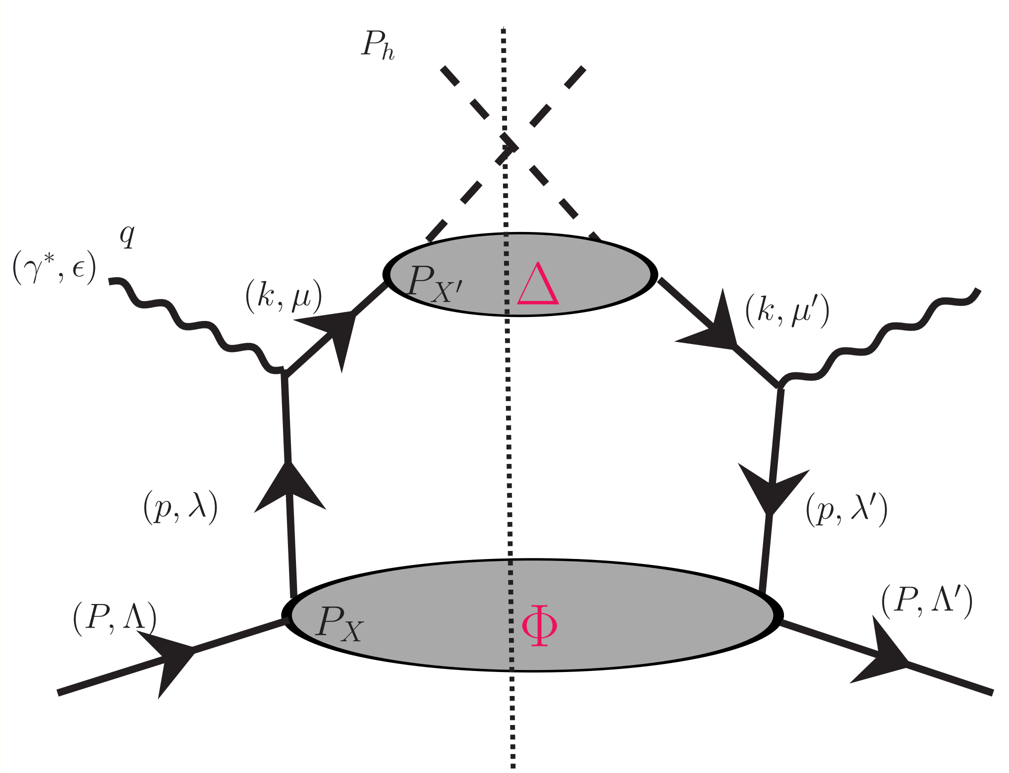
$$W^{\mu\nu}(q,P,S,P_h) \approx \sum_a e^2 \int \frac{d^2\mathbf{p}_T dp^- dp^+}{(2\pi)^4} \int \frac{d^2\mathbf{k}_T dk^- dk^+}{(2\pi)^4} \delta(p^+ - x_B P^+)\delta(k^- - \frac{P_h^-}{z})\delta^2(\mathbf{p}_T + \mathbf{q}_T - \mathbf{k}_T)$$

$$\times \mathrm{Tr}\left[\Phi(p,P,S)\gamma^\mu \Delta(k,P_h)\gamma^\nu\right]$$

$$W^{\mu\nu}(q,P,S,P_h) = \int \frac{d^2\mathbf{p}_T}{(2\pi)^4} \int \frac{d^2\mathbf{k}_T}{(2\pi)^4} \delta^2(\mathbf{p}_T - \frac{\mathbf{P}_{h\perp}}{z} - \mathbf{k}_T) \mathrm{Tr}\left[\left(\int dp^- \Phi\right)\gamma^\mu \left(\int dk^+ \Delta\right)\gamma^\nu\right]$$

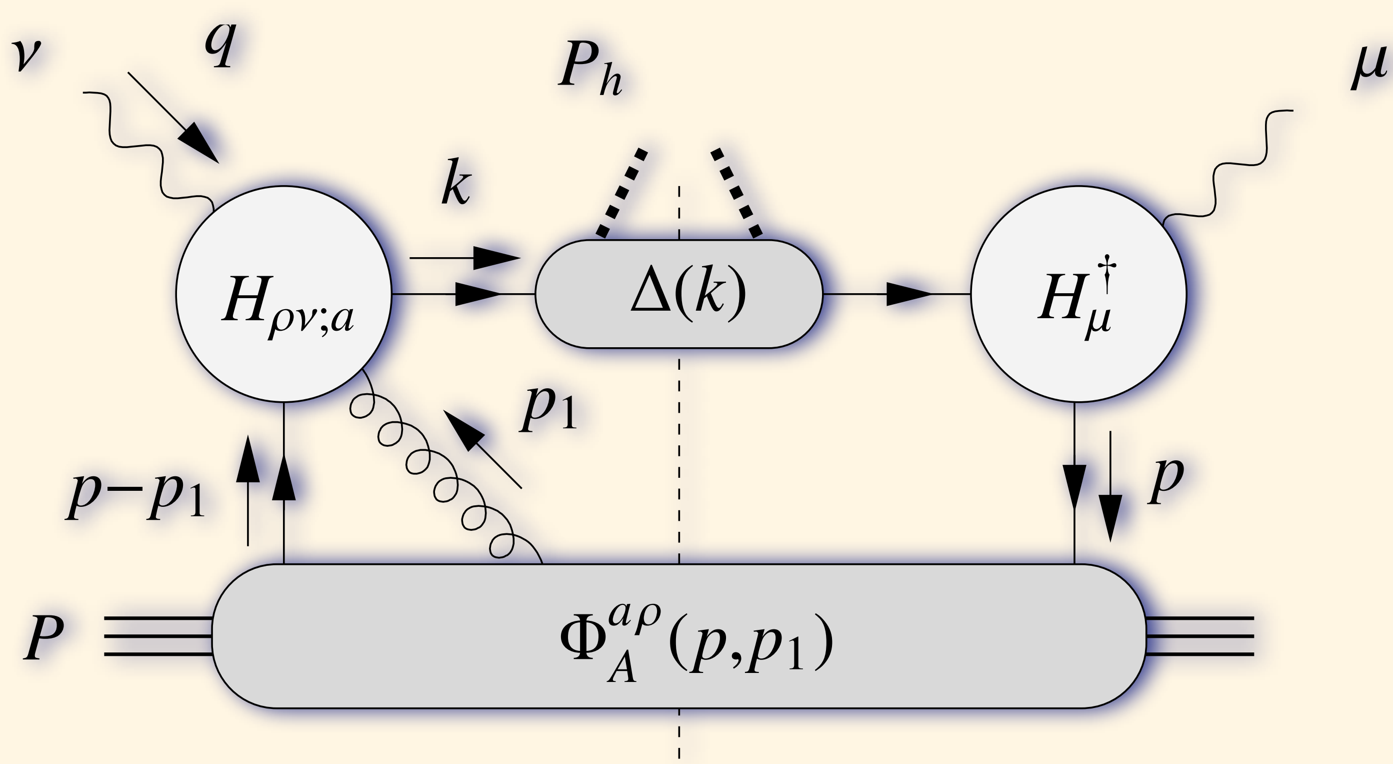
Small transverse momentum !!!

$$\Phi(x,\mathbf{p}_T,S) \equiv \int dp^- \Phi(p,P,S)\Big|_{p^+ = x_B P^+}, \qquad \Delta(z,\mathbf{k}_T) \equiv \int dk^+ \Delta(k,P_h)\Big|_{k^- = \frac{P^-}{z_h}}$$



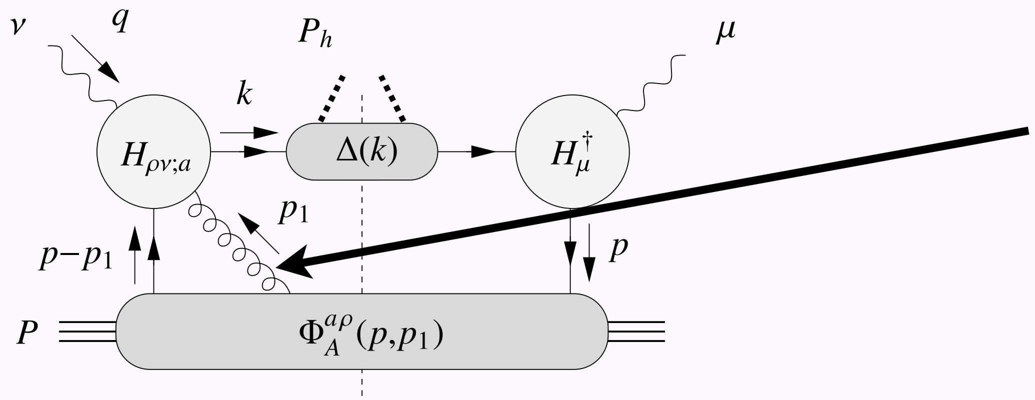
What about FSIs and TSSAs?
Extend Parton Model result-Gauge Links

- What are the "leading order" gluons that implement color gauge invariance?
- How is the correlator modified?



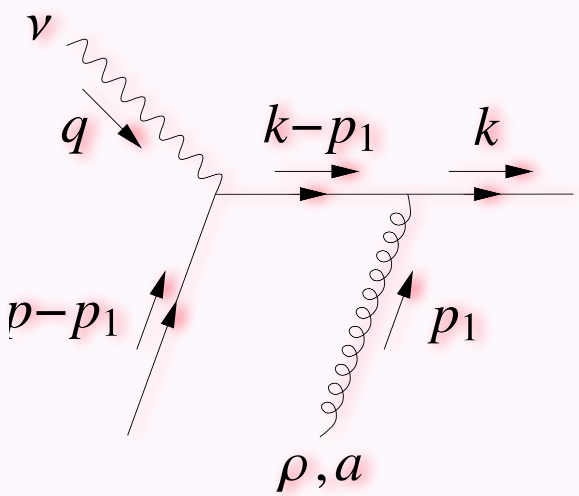
$$H_{\rho,\nu} = \gamma^{\nu}$$

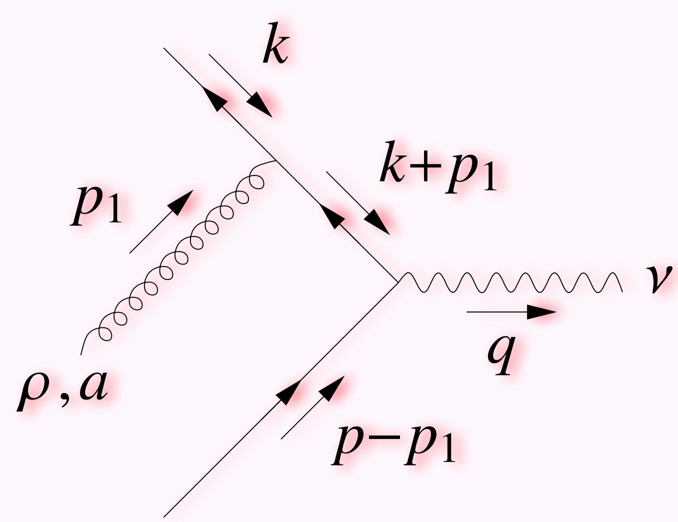
Summation of gluons from soft to hard at leading twist involves gluons collinear to hadron's momentum



Amsterdam grp Boer, Bomhof ,Mulders, Pijlman, et al. 2003 - 2008-Gauge link determined by re-summing all gluon interactions btwn soft and hard

$$\frac{1}{2M}\int d^4p\, d^4k\, d^4p_1\; \delta^4(p+q-k)\, \mathrm{Tr}\left[\, \Phi_A^{a\rho}(p,p_1)\, H_\mu^\dagger(p,k)\, \Delta(k)\, H_{\rho\nu;a}(p,k;p_1)\,\right]$$





The hard tree amplitudes in SIDIS and DY dressed with leading co-linear gluon insertions "eikonalize". Convoluting this hard amplitude with soft factors determines "[C]" factors

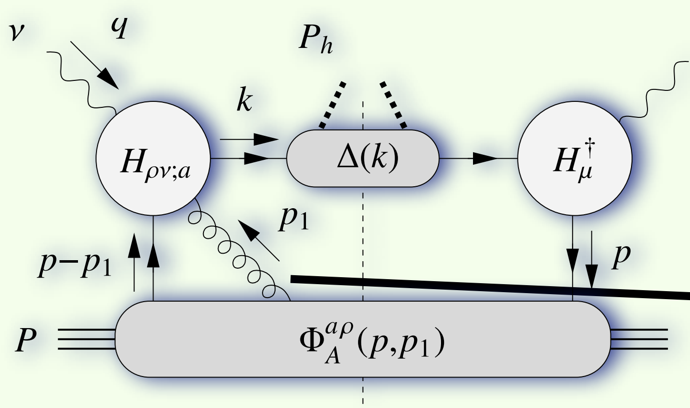
$$\longrightarrow \quad \int d^4p\, d^4k\, \delta^4(p+q-k)\mathrm{Tr}\left[\Phi^{[U^{C}_{[\infty;\xi]}]}(p) H_\mu^\dagger(p,k)\Delta(k)H_\nu(p,k)\right]$$

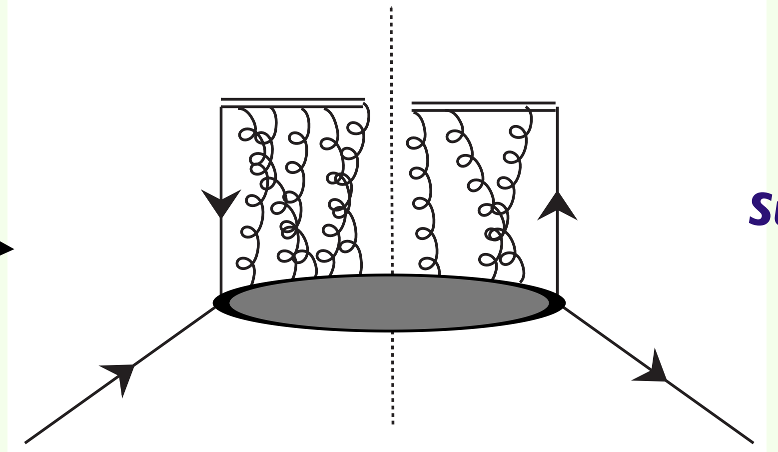
- **The path $[C]$ is fixed by hard subprocess within hadronic process.**

“T-Odd” Effects From Color Gauge Inv. Via Gauge links

Gauge link determined re-summing gluon interactions btwn soft and hard
Efremov,Radyushkin *Theor. Math. Phys. 1981*
Belitsky, Ji, Yuan NPB 2003,
Boer, Bomhof, Mulders Pijlman, et al. 2003 - 2008- NPB, PLB, PRD

$$\Phi^{[\mathcal{U}[C]]}(x,p_T) = \int \frac{d\xi^- d^2\xi_T}{2(2\pi)^3} e^{ip\cdot\xi} \langle P|\overline{\psi}(0)\mathcal{U}^{[C]}_{[0,\xi]}\psi(\xi^-,\xi_T)|P\rangle|_{\xi^+=0}$$

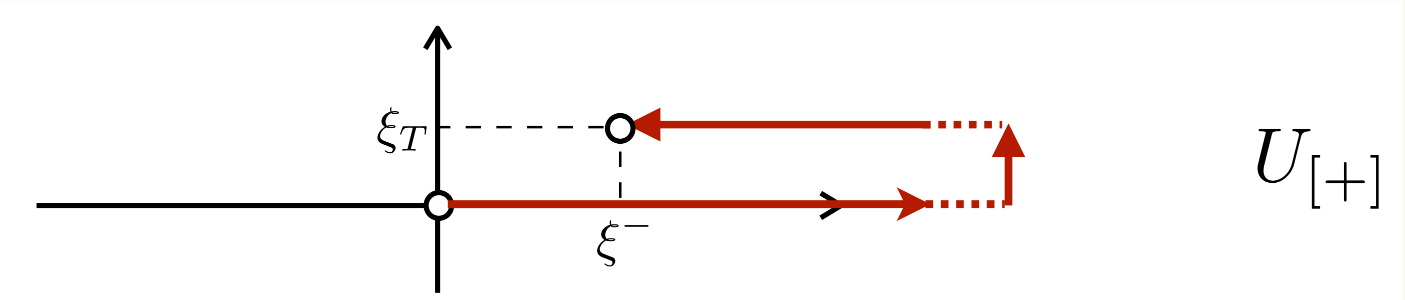




Summing gauge link with color
LG, M. Schlegel PLB 2010

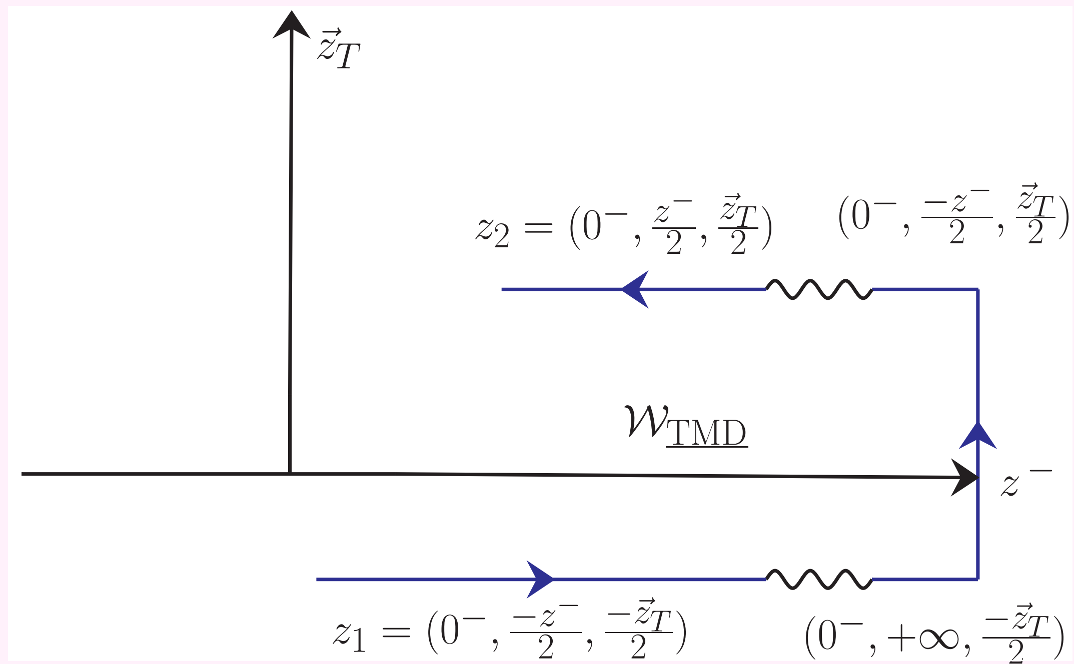
- **The path $[C]$ is fixed by hard subprocess within hadronic process.**

$$\int d^4p d^4k \delta^4(p+q-k)\mathrm{Tr}\left[\Phi^{[U^{C}_{[\infty;\xi]}}(p) H^\dagger_\mu(p,k)\Delta(k)H_\nu(p,k)\right]$$

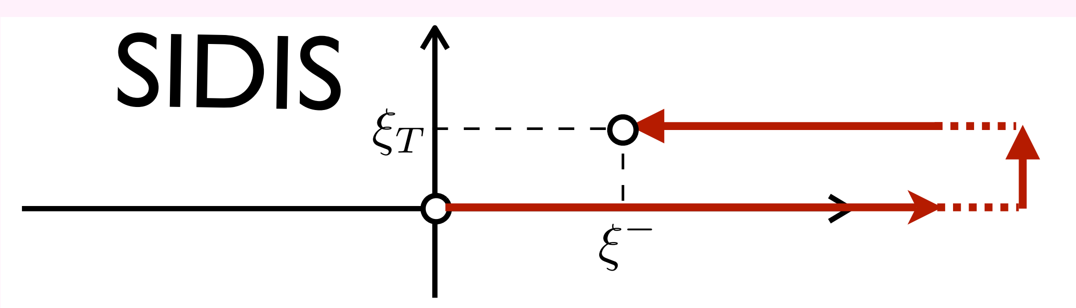


Wilson Line = Gauge links ... Path ordered Eikonal

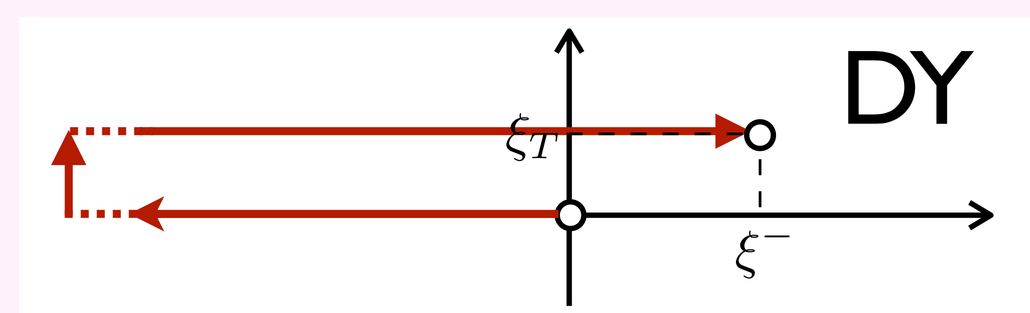
$$U^s_{[z_1,z_2]} = \mathcal{W}[z_1; z_2] = [z_1; z_2] = \mathcal{P}\mathrm{e}^{-ig\int_{z_1}^{z_2} ds\cdot A(s)}$$



Process Dependence, Collins plb 02, Brodsky et al. NPB 02, Boer Mulders Pijlman Bomhoff 03, 04 ...



P&T



$$\Phi^{[+]*}(x, p_T) = i\gamma^1\gamma^3\Phi^{[-]}(x, p_T)i\gamma^1\gamma^3$$

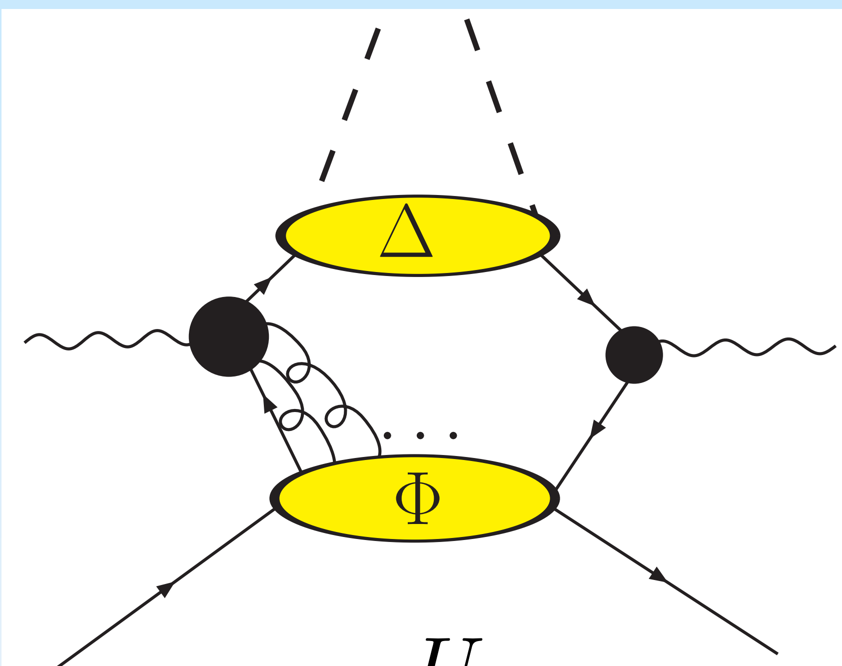
"Generalized Universality" Fund. Prediction of QCD Factorization

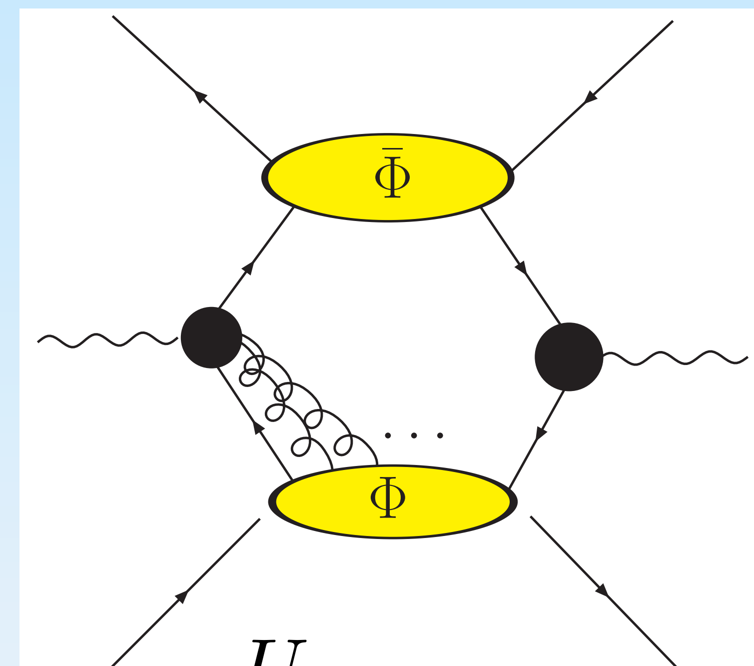
$$f_{1T\,sidis}^{\perp}(x,k_T) = -f_{1T\,DY}^{\perp}(x,k_T) \qquad p_T \sim \mathbf{k}_T << \sqrt{Q^2}$$

EIC conjunction with DY exp. E906-Fermi, RHIC II, Compass, JPARC

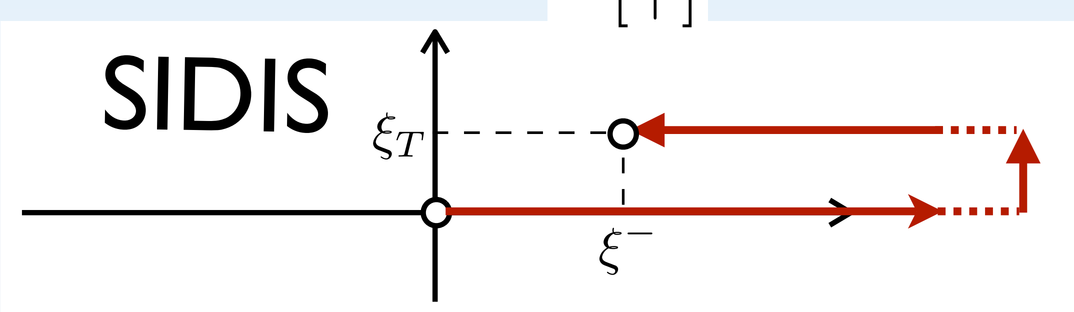
Process Dependence, Collins plb 02, Brodsky et al. NPB 02, Boer Mulders Pijlman Bomhoff 03, 04 ...

$$d\sigma = L_{\mu\nu}W^{\mu\nu} \Rightarrow$$

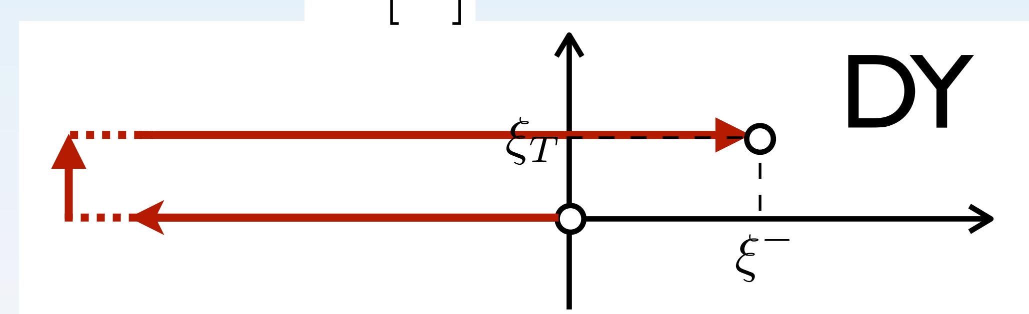




$U_{[+]}$ $U_{[-]}$

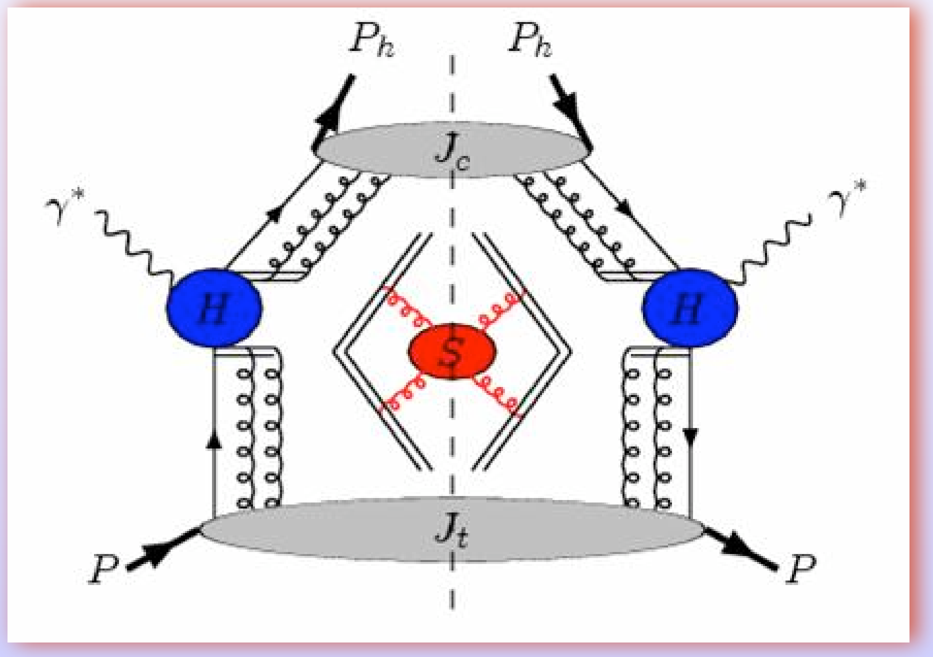


P&T



$$\Phi^{[+]*}(x,p_T) = i\gamma^1\gamma^3\Phi^{[-]}(x,p_T)i\gamma^1\gamma^3$$

Ji, Ma, Yuan: PLB, PRD 2004, 2005 Extend factorization of CS-NPB: 81



Also see Bacchetta Boer Diehl Mulders JHEP 08

$$F_{UU,T}(x,z,P_{h\perp}^2,Q^2) = \mathcal{C}\big[f_1 D_1\big]$$

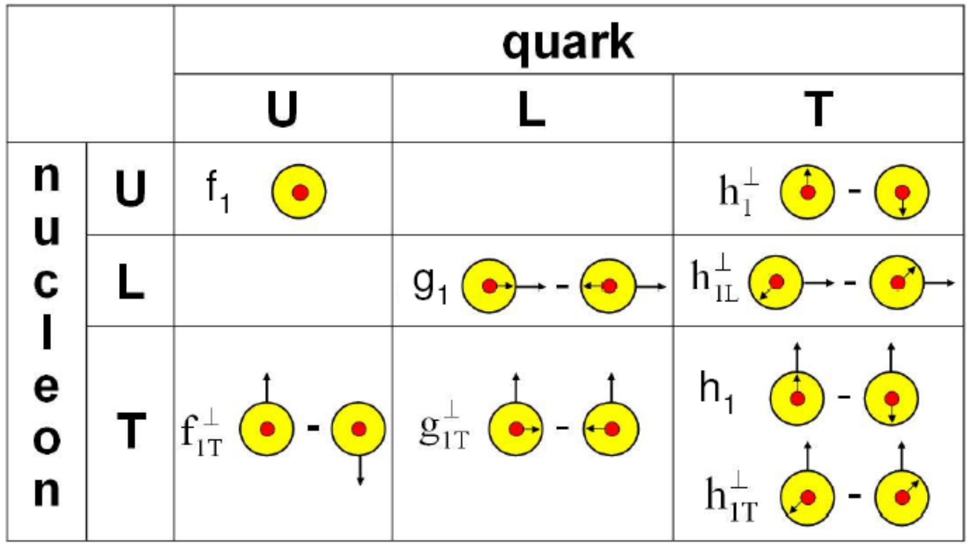
$$= \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, d^2\boldsymbol{l}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T - \boldsymbol{P}_{h\perp}/z\big)$$

$$x \sum_a e_a^2\, f_1^a(x,p_T^2,\mu^2)\, D_1^a(z,k_T^2,\mu^2)\, U(l_T^2,\mu^2) H(Q^2,\mu^2)$$

TMD PDF TMD FF Soft factor Hard part

Collins, Soper, NPB 193 (81)
Ji, Ma, Yuan, PRD 71 (05)

Leading Twist Contributions



$$\mathcal{F}_{AB} = \mathcal{C}[w \otimes f \otimes D]$$

UU	1	$f_1 =$	$\otimes$	$D_1 =$
	$\cos(2\phi_h^l)$	$h_1^\perp =$	$\otimes$	$H_1^\perp =$
UL	$\sin(2\phi_h^l)$	$h_{1L}^\perp =$	$\otimes$	$H_1^\perp =$
UT	$\sin(\phi_h^l + \phi_S^l)$	$h_1 =$	$\otimes$	$H_1^\perp =$
	$\sin(\phi_h^l - \phi_S^l)$	$f_{1T}^\perp =$	$\otimes$	$D_1 =$
	$\sin(3\phi_h^l - \phi_S^l)$	$h_{1T}^\perp =$	$\otimes$	$H_1^\perp =$
LL	1	$g_1 =$	$\otimes$	$D_1 =$
LT	$\cos(\phi_h^l - \phi_S^l)$	$g_{1T} =$	$\otimes$	$D_1 =$

Summary of Trans polz effects in QCD

- Realization that FSI and ISI btwn struck parton and target remnant provide necessary phases that lead to non-vanishing TSSAs

- Two scale factorization in terms of TMDs

$$p_T \sim \mathbf{k}_T << \sqrt{Q^2}$$

- One large scale factorization in terms twist 3 approach $Q \sim P_T >> \Lambda_{\rm qcd}$

- Connection btwn two approaches? Unified picture Ji, Qiu, Vogelsang, Yuan PRL 2006

$$\Lambda_{\rm QCD} << q_T << Q$$

Generalizing the Generalized Parton Model GPM

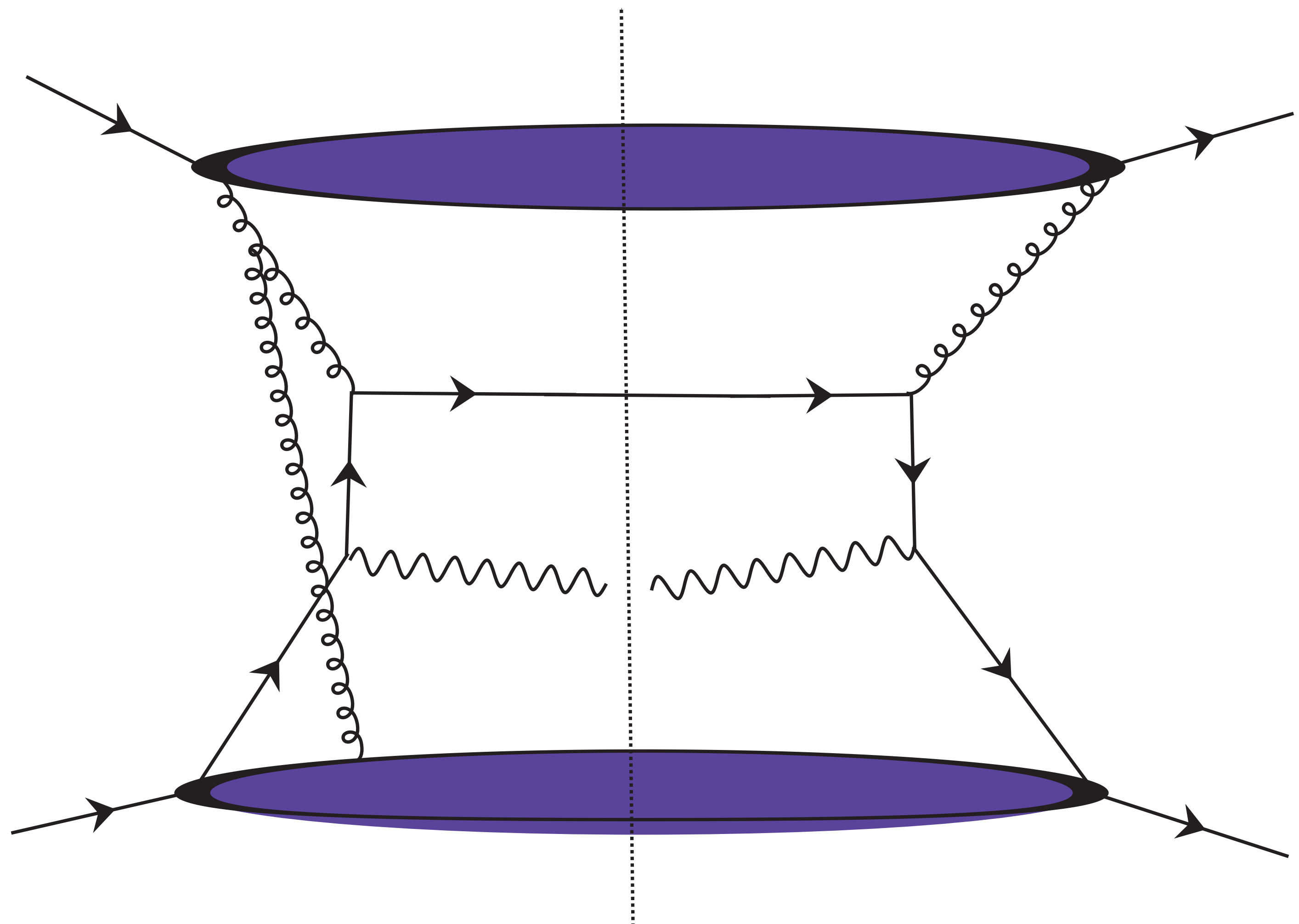
- Feynman and Field (& Fox PRD 77 & 78)-incorporate intrisic k_T
- Include Transverse spin pol. w/ intrisic k_t --Anselmino, Boglione, Murgia PLB 94--see talk of Mauro Anselmino this wkshp.
- Pheno.... Torino group and other 1994-2010 inclusive processes
- Weighted and unweighted asymmetries in dijet, photon & jet (safer reactions) Bacchetta Bomhoff Mulders Pijlman 2005 PRD & w/ D'Alseio and Murgia 2007 PRL, Qiu, Vogelsang, Yuan PRD 2007
- Inclusive processes studied by Kouvaris, Qiu, Vogelsang, Yuan PRD 2006, $pp \to \pi X$ $\quad pp \to \gamma X$
- What happens when you adopt ansazt of GPM including dynamical reaction mechanism of FSI/ISI in inclusive processes
- Now use process dependent Sivers function
- Since this approach is twist three is there connection w/ twist 3 ?

Direct Photon in GPM with

$$\Delta\sigma^{pp^{\uparrow}\to\gamma X} \sim \Delta f_a \otimes f_b \otimes \Delta\hat{\sigma}$$

(assuming factorization)

???



One should use the Sivers function that is determined from color structure of hard processes in calculating SSA for inclusive pp collisions in GPM

adpot approach of Qiu, Vogelsang, Yuan PRD 07, Kouvaris, Qiu, Vogelsang, Yuan PRD 06

Spin Dependent Cross Section in GPM

$$pp \to \gamma X$$

A_N is defined by the ratio $\quad A_N = E_\gamma \frac{d\Delta\sigma}{d^3P_\gamma} \Big/ E_\gamma \frac{d\sigma}{d^3P_\gamma}.$

$$E_\gamma \frac{d\Delta\sigma}{d^3P_\gamma} = \frac{\alpha_{em}\alpha_s}{S}\sum_{a,b}\int \frac{dx_a}{x_a} d^2k_{aT} \Delta^N f_{a/A}^{\rm DIS}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT}) \times \int \frac{dx_b}{x_b} d^2k_{bT} f_{b/B}(x_b, k_{bT}) H^U_{ab\to\gamma}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u}).$$

GPM Anselmino et al.

$$E_\gamma \frac{d\Delta\sigma}{d^3P_\gamma} = \frac{\alpha_{em}\alpha_s}{S}\sum_{a,b}\int \frac{dx_a}{x_a} d^2k_{aT} \Delta^N f_{a/A}^{ab\to\gamma}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT}) \times \int \frac{dx_b}{x_b} d^2k_{bT} f_{b/B}(x_b, k_{bT}) H_{ab\to\gamma}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u}),$$

GPM w/color
LG & Z. Kang
arXiv:1009.1936 [hep-ph]

process-dependent Sivers function denoted as $\Delta^N f_{a/A}^{ab\to c}(x_a, k_{aT})$

how to get it ?

Spin Dependent Cross Section in GPM

$$pp \longrightarrow \pi X$$

A_N is defined by the ratio $\quad A_N \equiv E_h \frac{d\Delta\sigma}{d^3P_h} \Big/ E_h \frac{d\sigma}{d^3P_h}.$

$$E_h \frac{d\Delta\sigma}{d^3P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dx_a}{x_a} d^2k_{aT} \Delta^N f_{a/A}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT}) \int \frac{dx_b}{x_b} d^2k_{bT} f_{b/B}(x_b, k_{bT}) \times \int \frac{dz_c}{z_c^2} D_{h/c}(z_c) H^U_{ab\to c}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}),$$

GPM Anselmino et al.

$$E_h \frac{d\Delta\sigma}{d^3P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dx_a}{x_a} d^2k_{aT} \Delta^N f^{ab\to c}_{a/A}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT}) \int \frac{dx_b}{x_b} d^2k_{bT} f_{b/B}(x_b, k_{bT}) \times \int \frac{dz_c}{z_c^2} D_{h/c}(z_c) H^U_{ab\to c}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}),$$

GPM w/color LG & Z. Kang

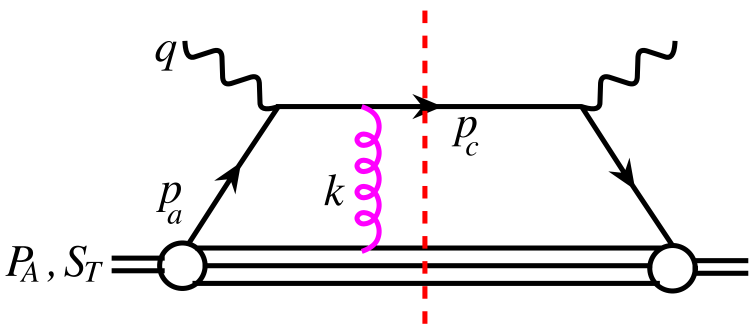
arXiv:1009.1936 [hep-ph]

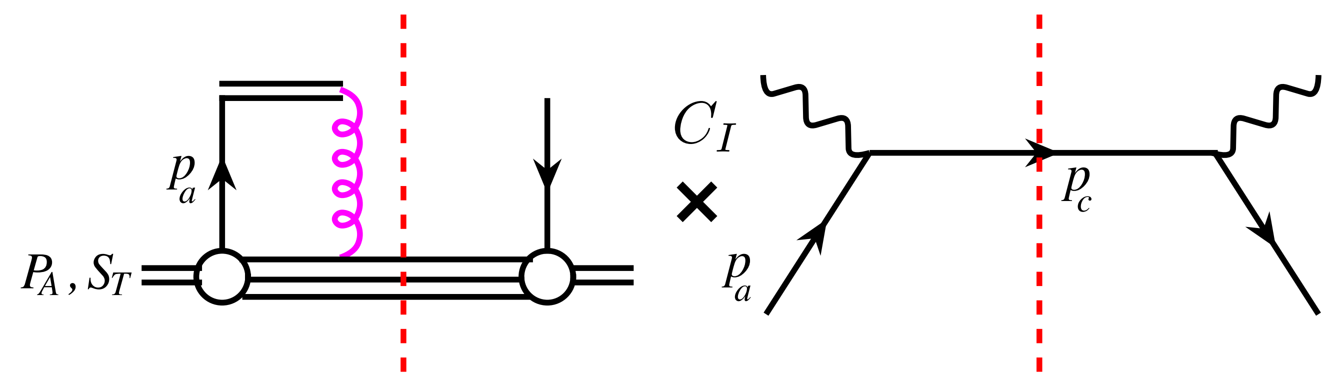
process-dependent Sivers function denoted as $\Delta^N f^{ab\to c}_{a/A}(x_a, k_{aT})$

how to get it ?

Classic example-same real pts opposite imaginary pts

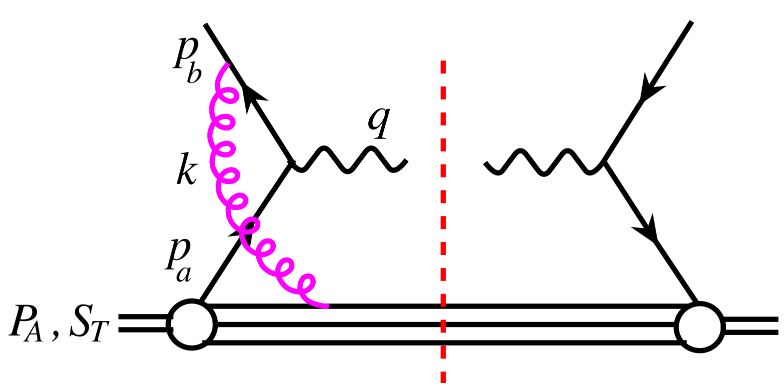
Final-state interaction in SIDIS

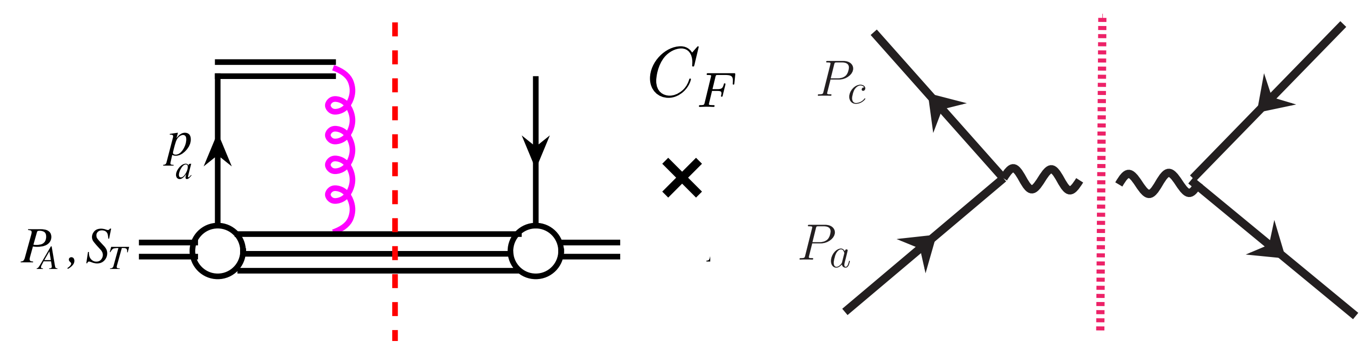




$$\bar{u}(p_c)(-ig)\gamma^- T^a \frac{i(\not{p}_c - \not{k})}{(p_c - k)^2 + i\epsilon} \approx \bar{u}(p_c)\left[\frac{g}{-k^+ + i\epsilon} T^a\right]$$

and initial-state interaction in DY

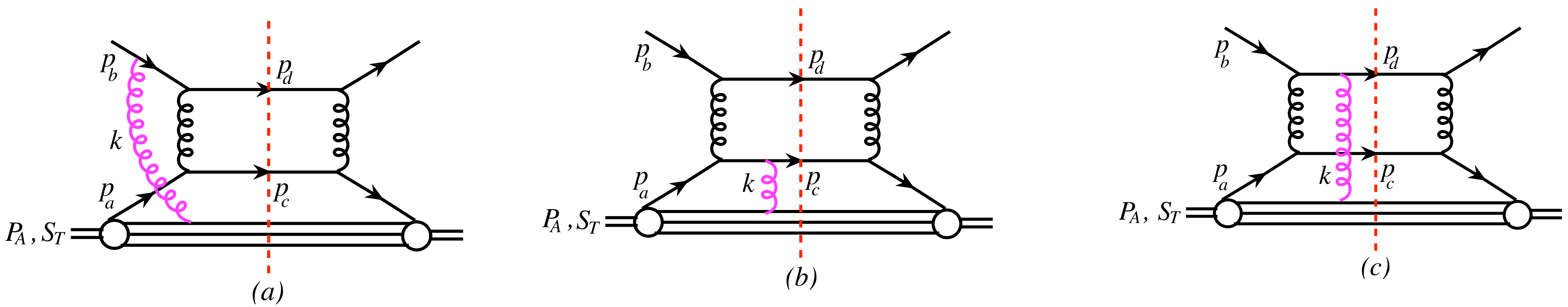




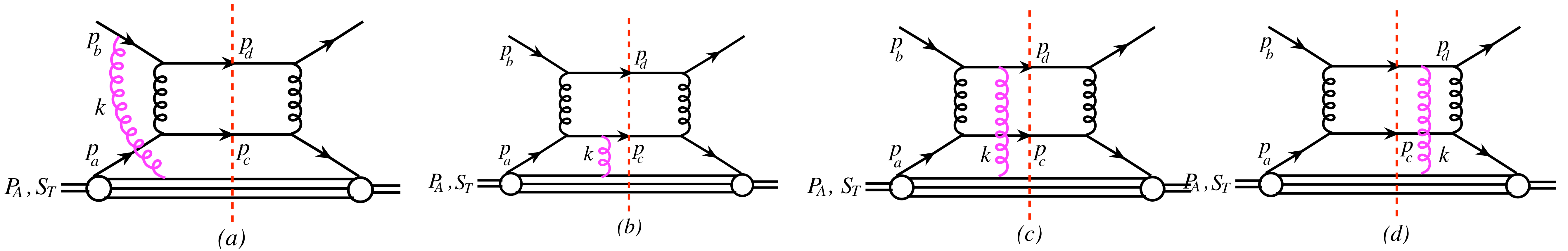
$$\bar{v}(p_b)(-ig)\gamma^- T^a \frac{-i(\not{p}_b + \not{k})}{(p_b + k)^2 + i\epsilon} \approx \bar{v}(p_b)\left[\frac{g}{-k^+ - i\epsilon} T^a\right],$$

Observation

- Crucial point: Sivers function in inclusive single particle production contains both ISI and FSI
- consider channel $qq' \to qq'$



One gluon exchange approx for ISI and FSI

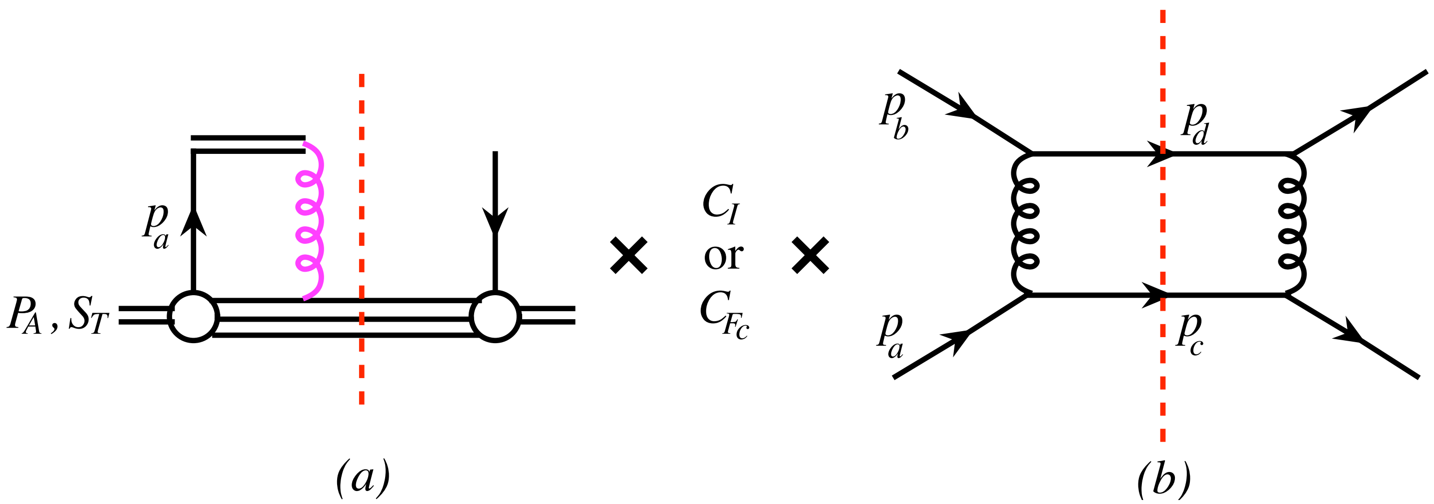


$$\left[\frac{-g}{-k^+ - i\epsilon}T^a\right]$$

$$C_I = -\frac{1}{2N_c^2},$$

$$\left[\frac{g}{-k^+ + i\epsilon}T^a\right]$$

$$C_{F_c} = -\frac{1}{4N_c^2},$$

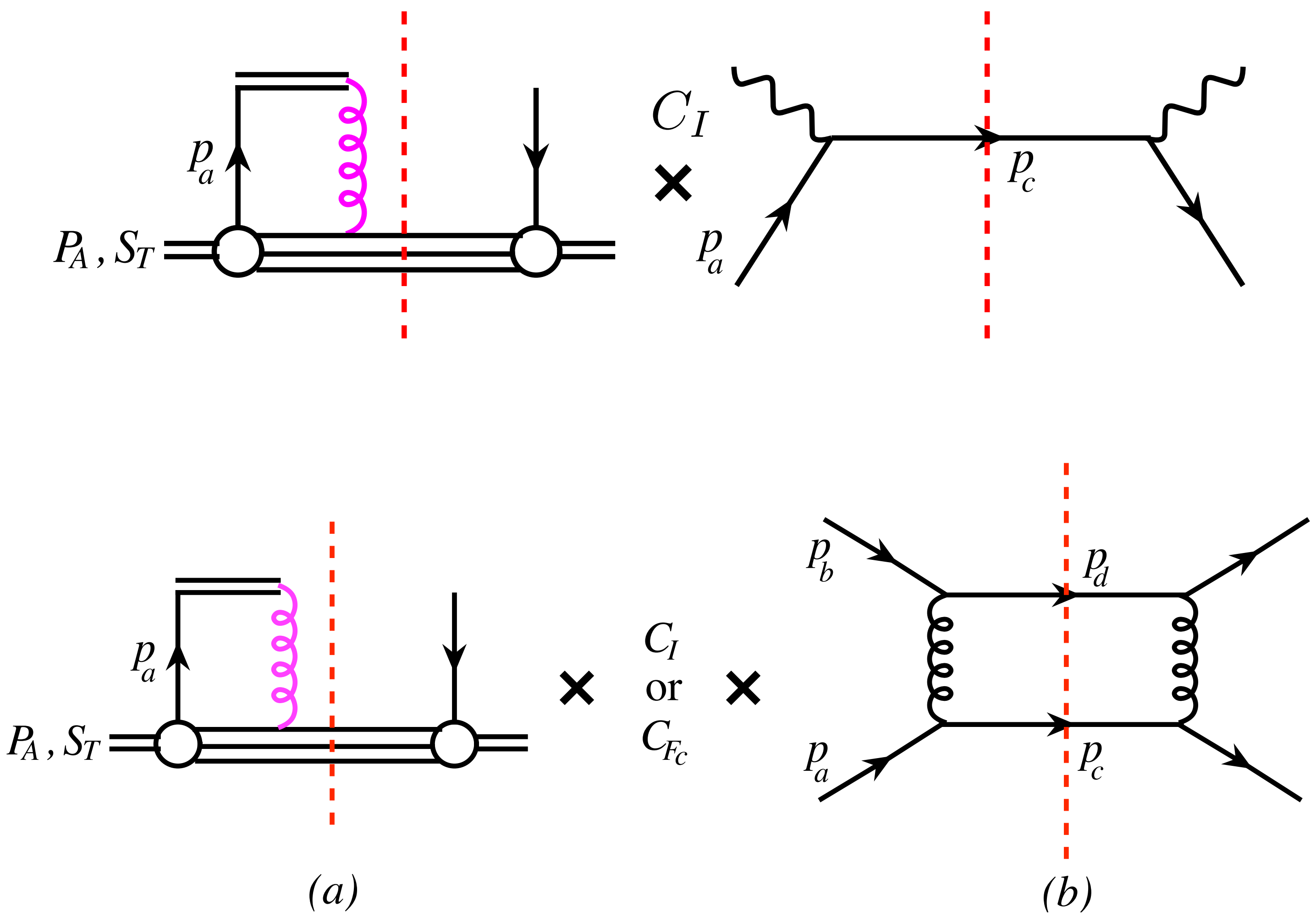


Note unpolarized color factor

$$C_u = \frac{N_c^2 - 1}{4N_c^2}$$

Comparing imag. pt of eikonal propagators for subprocess in SIDIS and inclusive single particle production

Sivers function probed in $qq' \to qq'$ process is related to those in SIDIS



$$\Delta^N f_{a/A}^{qq'\to qq'} = \frac{C_I + C_{F_c}}{C_u} \Delta^N f_{a/A}^{\mathrm{SIDIS}}.$$

Alternatively one can move color factors "process dependence" to hard parts

$$E_h \frac{d\Delta\sigma}{d^3P_h} = \frac{\alpha_s^2}{S}\sum_{a,b,c}\int \frac{dx_a}{x_a} d^2k_{aT}\Delta^N f_{a/A}^{ab\to c}(x_a,k_{aT})\frac{1}{2}S_A\cdot(\hat{P}_A\times\hat{k}_{aT})\int\frac{dx_b}{x_b}d^2k_{bT}f_{b/B}(x_b,k_{bT})$$
$$\times\int\frac{dz_c}{z_c^2}D_{h/c}(z_c)H^U_{ab\to c}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u}),$$

$$E_h \frac{d\Delta\sigma}{d^3P_h} = \frac{\alpha_s^2}{S}\sum_{a,b,c}\int \frac{dx_a}{x_a} d^2k_{aT}\Delta^N f_{a/A}^{\rm SIDIS}(x_a,k_{aT})\frac{1}{2}S_A\cdot(\hat{P}_A\times\hat{k}_{aT})\int\frac{dx_b}{x_b}d^2k_{bT}f_{b/B}(x_b,k_{bT})$$
$$\times\int\frac{dz_c}{z_c^2}D_{h/c}(z_c)H^{\rm Inc}_{ab\to c}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u}),$$

GPM

$$\Delta^N f_{a/A}^{\rm SIDIS} H^U_{qq'\to qq'} \equiv \Delta^N f_{a/A}^{\rm SIDIS} \left[C_u h_{qq'\to qq'}\right],$$

generalize GPM Replace with

$$\Delta^N f_{a/A}^{qq'\to qq'} H^U_{qq'\to qq'} = \frac{C_I + C_{F_c}}{C_u} \Delta^N f_{a/A}^{\rm SIDIS} H^U_{qq'\to qq'} = \Delta^N f_{a/A}^{\rm SIDIS} \left[C_I h_{qq'\to qq'} + C_{F_c} h_{qq'\to qq'}\right],$$

hard partonic c.s. w/o color factors

$$h_{qq'\to qq'} = 2\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}.$$

Then "modified" GPM is

$$E_h \frac{d\Delta\sigma}{d^3P_h} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dx_a}{x_a} d^2k_{aT} \Delta^N f_{a/A}^{\mathrm{SIDIS}}(x_a, k_{aT}) \frac{1}{2} S_A \cdot (\hat{P}_A \times \hat{k}_{aT}) \int \frac{dx_b}{x_b} d^2k_{bT} f_{b/B}(x_b, k_{bT})$$
$$\times \int \frac{dz_c}{z_c^2} D_{h/c}(z_c) H_{ab\to c}^{\mathrm{Inc}}(\hat{s}, \hat{t}, \hat{u}) \delta(\hat{s} + \hat{t} + \hat{u}),$$

$$H_{qq'\to qq'}^{\mathrm{Inc}} \equiv H_{qq'\to qq'}^{\mathrm{Inc-I}} + H_{qq'\to qq'}^{\mathrm{Inc-F}},$$

where,

$$H_{qq'\to qq'}^{\mathrm{Inc-I}} = C_I h_{qq'\to qq'}, \qquad H_{qq'\to qq'}^{\mathrm{Inc-F}} = C_{F_c} h_{qq'\to qq'}$$

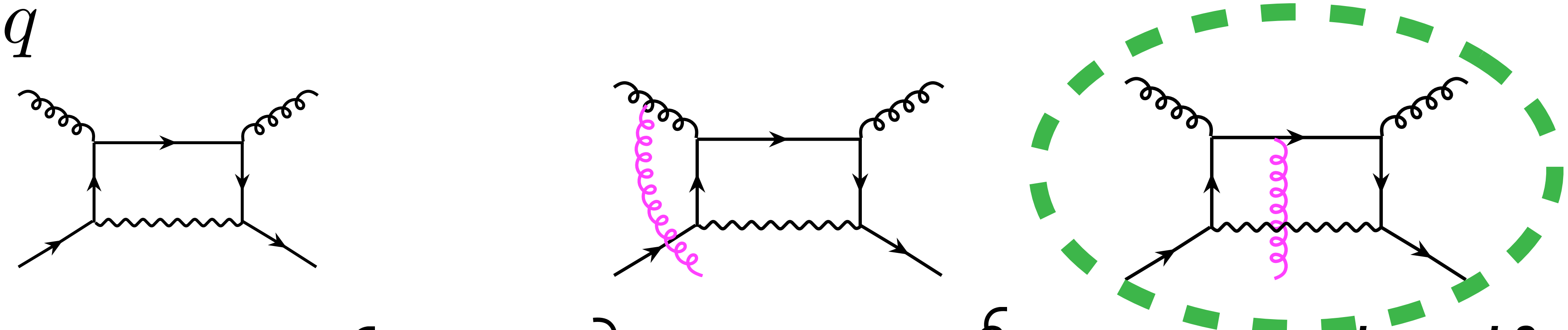
hard partonic c.s. w/o color factors

$$h_{qq'\to qq'} = 2\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}.$$

Color modification of hard cross sections due to Gauge Link

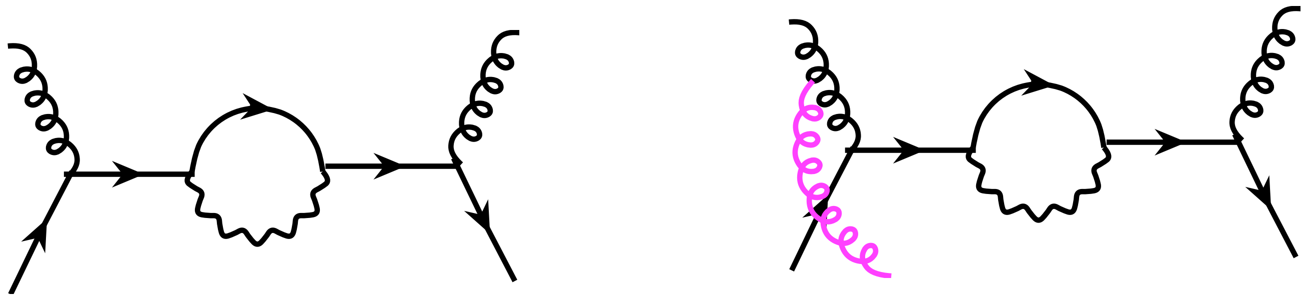
$qg \to \gamma q$

t-channel

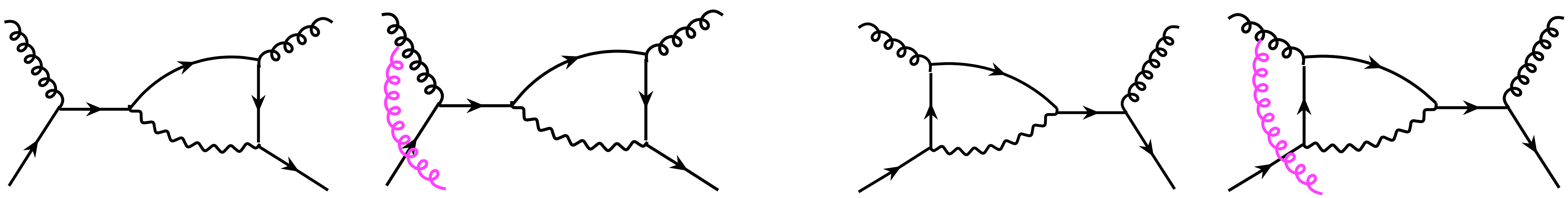


unobserved final state contribution vanishes

s-channel

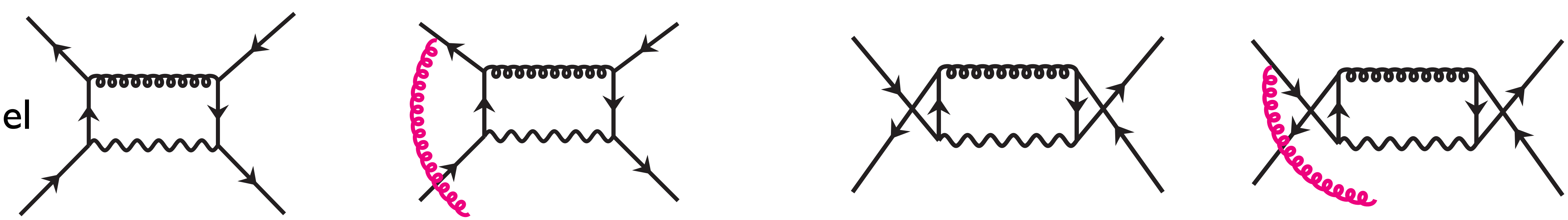


s-t interference



$\bar{q}q \to \gamma g$

t & u-channel



etc ....

t-u interference

The contributions for $pp \to \gamma X$

the various contributing partonic subprocesses are given by

$$\boxed{\begin{aligned} H^{\mathrm{Inc}}_{qg\to\gamma q} &= -H^{\mathrm{Inc}}_{\bar{q}g\to\gamma\bar{q}} = -\frac{N_c}{N_c^2-1}e_q^2\left[-\frac{\hat{t}}{\hat{s}}-\frac{\hat{s}}{\hat{t}}\right] \\ H^{\mathrm{Inc}}_{q\bar{q}\to\gamma g} &= -H^{\mathrm{Inc}}_{\bar{q}q\to\gamma g} = \frac{1}{N_c^2}e_q^2\left[\frac{\hat{t}}{\hat{u}}+\frac{\hat{u}}{\hat{t}}\right] \end{aligned}}$$

The contributions for $pp \to \pi X$

the various contributing partonic subprocesses are given by

$$H^{\mathrm{Inc-I}}_{qq'\to qq'} = -H^{\mathrm{Inc-I}}_{\bar{q}\bar{q}'\to \bar{q}\bar{q}'} = -\frac{1}{N_c^2}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]$$
$$H^{\mathrm{Inc-F}}_{qq'\to qq'} = -H^{\mathrm{Inc-F}}_{\bar{q}\bar{q}'\to \bar{q}\bar{q}'} = -\frac{1}{2N_c^2}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]$$

$$H^{\mathrm{Inc-I}}_{q\bar{q}'\to q\bar{q}'} = -H^{\mathrm{Inc-I}}_{\bar{q}q'\to \bar{q}q'} = -\frac{N_c^2-2}{2N_c^2}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]$$
$$H^{\mathrm{Inc-F}}_{q\bar{q}'\to q\bar{q}'} = -H^{\mathrm{Inc-F}}_{\bar{q}q'\to \bar{q}q'} = -\frac{1}{2N_c^2}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]$$

$$H^{\mathrm{Inc-I}}_{qq'\to q'q} = -H^{\mathrm{Inc-I}}_{\bar{q}\bar{q}'\to \bar{q}'\bar{q}} = -\frac{1}{N_c^2}\left[\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right]$$
$$H^{\mathrm{Inc-F}}_{qq'\to q'q} = -H^{\mathrm{Inc-F}}_{\bar{q}\bar{q}'\to \bar{q}'\bar{q}} = \frac{N_c^2-2}{2N_c^2}\left[\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right]$$

$$H^{\mathrm{Inc-I}}_{q\bar{q}'\to \bar{q}'q} = -H^{\mathrm{Inc-I}}_{\bar{q}q'\to q'\bar{q}} = -\frac{N_c^2-2}{2N_c^2}\left[\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right]$$
$$H^{\mathrm{Inc-F}}_{q\bar{q}'\to \bar{q}'q} = = -H^{\mathrm{Inc-F}}_{\bar{q}q'\to q'\bar{q}} = \frac{1}{N_c^2}\left[\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right]$$

$$H^{\mathrm{Inc-I}}_{qq\to qq} = -H^{\mathrm{Inc-I}}_{\bar{q}\bar{q}\to \bar{q}\bar{q}} = -\frac{1}{N_c^2}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right]+\frac{N_c^2+1}{N_c^3}\frac{\hat{s}^2}{\hat{t}\hat{u}}$$
$$H^{\mathrm{Inc-F}}_{qq\to qq} = -H^{\mathrm{Inc-F}}_{\bar{q}\bar{q}\to \bar{q}\bar{q}} = -\frac{1}{2N_c^2}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]+\frac{N_c^2-2}{2N_c^2}\left[\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right]+\frac{1}{N_c^3}\frac{\hat{s}^2}{\hat{t}\hat{u}}$$

$$H^{\mathrm{Inc-I}}_{q\bar{q}\to q'\bar{q}'} = -H^{\mathrm{Inc-I}}_{\bar{q}q\to \bar{q}'q'} = \frac{1}{2N_c^2}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]$$
$$H^{\mathrm{Inc-F}}_{q\bar{q}\to q'\bar{q}'} = -H^{\mathrm{Inc-F}}_{\bar{q}q\to \bar{q}'q'} = \frac{N_c^2-2}{2N_c^2}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]$$

$$H^{\mathrm{Inc-I}}_{q\bar{q}\to \bar{q}'q'} = -H^{\mathrm{Inc-I}}_{\bar{q}q\to q'\bar{q}'} = \frac{1}{2N_c^2}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]$$
$$H^{\mathrm{Inc-F}}_{q\bar{q}\to \bar{q}'q'} = -H^{\mathrm{Inc-F}}_{\bar{q}q\to q'\bar{q}'} = \frac{1}{N_c^2}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]$$

$$H^{\mathrm{Inc-I}}_{q\bar{q}\to q\bar{q}} = -H^{\mathrm{Inc-I}}_{\bar{q}q\to \bar{q}q} = -\frac{N_c^2-2}{2N_c^2}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]+\frac{1}{2N_c^2}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]-\frac{1}{N_c^3}\frac{\hat{u}^2}{\hat{s}\hat{t}}$$
$$H^{\mathrm{Inc-F}}_{q\bar{q}\to q\bar{q}} = -H^{\mathrm{Inc-F}}_{\bar{q}q\to \bar{q}q} = -\frac{1}{2N_c^2}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]+\frac{N_c^2-2}{2N_c^2}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]+\frac{1}{N_c^3}\frac{\hat{u}^2}{\hat{s}\hat{t}}$$

$$H^{\mathrm{Inc-I}}_{q\bar{q}\to \bar{q}q} = -H^{\mathrm{Inc-I}}_{\bar{q}q\to q\bar{q}} = -\frac{N_c^2-2}{2N_c^2}\left[\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right]+\frac{1}{2N_c^2}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]-\frac{1}{N_c^3}\frac{\hat{t}^2}{\hat{s}\hat{u}}$$
$$H^{\mathrm{Inc-F}}_{q\bar{q}\to \bar{q}q} = -H^{\mathrm{Inc-F}}_{\bar{q}q\to q\bar{q}} = \frac{1}{N_c^2}\left[\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}+\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]-\frac{N_c^2+1}{N_c^3}\frac{\hat{t}^2}{\hat{s}\hat{u}}$$

$$H^{\mathrm{Inc-I}}_{qg\to qg} = -H^{\mathrm{Inc-I}}_{\bar{q}g\to \bar{q}g} = \frac{1}{2(N_c^2-1)}\left[-\frac{\hat{s}}{\hat{u}}-\frac{\hat{u}}{\hat{s}}\right]+\frac{N_c^2}{2(N_c^2-1)}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\frac{\hat{u}}{\hat{s}}\right]$$
$$H^{\mathrm{Inc-F}}_{qg\to qg} = -H^{\mathrm{Inc-F}}_{\bar{q}g\to \bar{q}g} = \frac{1}{2N_c^2(N_c^2-1)}\left[-\frac{\hat{s}}{\hat{u}}-\frac{\hat{u}}{\hat{s}}\right]-\frac{1}{N_c^2-1}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]$$

$$H^{\mathrm{Inc-I}}_{qg\to gq} = -H^{\mathrm{Inc-I}}_{\bar{q}g\to g\bar{q}} = \frac{1}{2(N_c^2-1)}\left[-\frac{\hat{s}}{\hat{t}}-\frac{\hat{t}}{\hat{s}}\right]+\frac{N_c^2}{2(N_c^2-1)}\left[\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\frac{\hat{t}}{\hat{s}}\right]$$
$$H^{\mathrm{Inc-F}}_{qg\to gq} = -H^{\mathrm{Inc-F}}_{\bar{q}g\to g\bar{q}} = -\frac{1}{2(N_c^2-1)}\left[-\frac{\hat{s}}{\hat{t}}-\frac{\hat{t}}{\hat{s}}\right]-\frac{N_c^2}{2(N_c^2-1)}\left[\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\frac{\hat{s}}{\hat{t}}\right]$$

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$$H^{\mathrm{Inc-F}}_{q\bar{q}\to gg} = -H^{\mathrm{Inc-F}}_{\bar{q}q\to gg} = -\frac{1}{2N_c}\left[\frac{\hat{u}}{\hat{t}}+\frac{\hat{t}}{\hat{u}}\right]+\frac{N_c}{2}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\frac{\hat{u}}{\hat{t}}\right]$$

Observations

- Hard amplitudes squared have same form in Mandelstam variables as twist-3 $\hat{s},\ \hat{t},\ \hat{u}$

Kouvaris,Qiu ,Vogelsang, and Yuan PRD 2006

- however $\hat{s},\ \hat{t},\ \hat{u}$ depend on k_T in GPM whereas in twist-3 approach there has been collinear expansion on hard and soft factors

- Indeed we have prelim. results that GPM expanded with respect to k_{aT} results in twist result

Collinear Expansion in GPM

- Implement delta function
- now "s" and "t" depend on k_{aT}
- expand k_{aT} and study contribution from Sivers function and hard cross section

$$E_h \frac{d\Delta\sigma}{d^2 P_h} = \frac{\alpha_s}{s} \sum_{abc} \int d^2 k_{aT} \frac{1}{M} \epsilon^{\alpha S_T n \hat{n}} k_{aT\alpha} \frac{1}{x_a} f_{1T}^{\perp\,\mathrm{sidis}}(x_a, k_{aT}^2) \Big|_{x_a = X + \frac{2P_{hT}\cdot k_{aT}/z}{x_b s + T/z}}$$
$$\times \int \frac{dx_b}{x_b} \int \frac{dz}{z^2} H_{ab\to c}^{\mathrm{Inc}}(\hat{s}, \hat{t}, \hat{u}) \frac{1}{x_b s + T/z}$$

that is.... in GPM

$$\hat{s} = (p_a + p_b)^2 = x_a x_b S + \quad \mathcal{O}(k_T^2)$$

$$\hat{t} = (x_a P_A + k_{aT} - \frac{P_h}{z})^2 = \frac{x_a}{z} T - \frac{2 P_{hT} \cdot k_{aT}}{z}$$

$$\hat{u} = (p_b - p_c)^2 = (x_b P_B - \frac{P_h}{z})^2 = \frac{x_b}{z} U$$

$$\delta(\hat{s} + \hat{t} + \hat{u}) = \frac{1}{x_b s + \frac{T}{z}} \delta(x_a - X - \frac{2 P_{hT} \cdot K_{aT}}{x_b s + \frac{T}{z}})$$

Collinear twist three

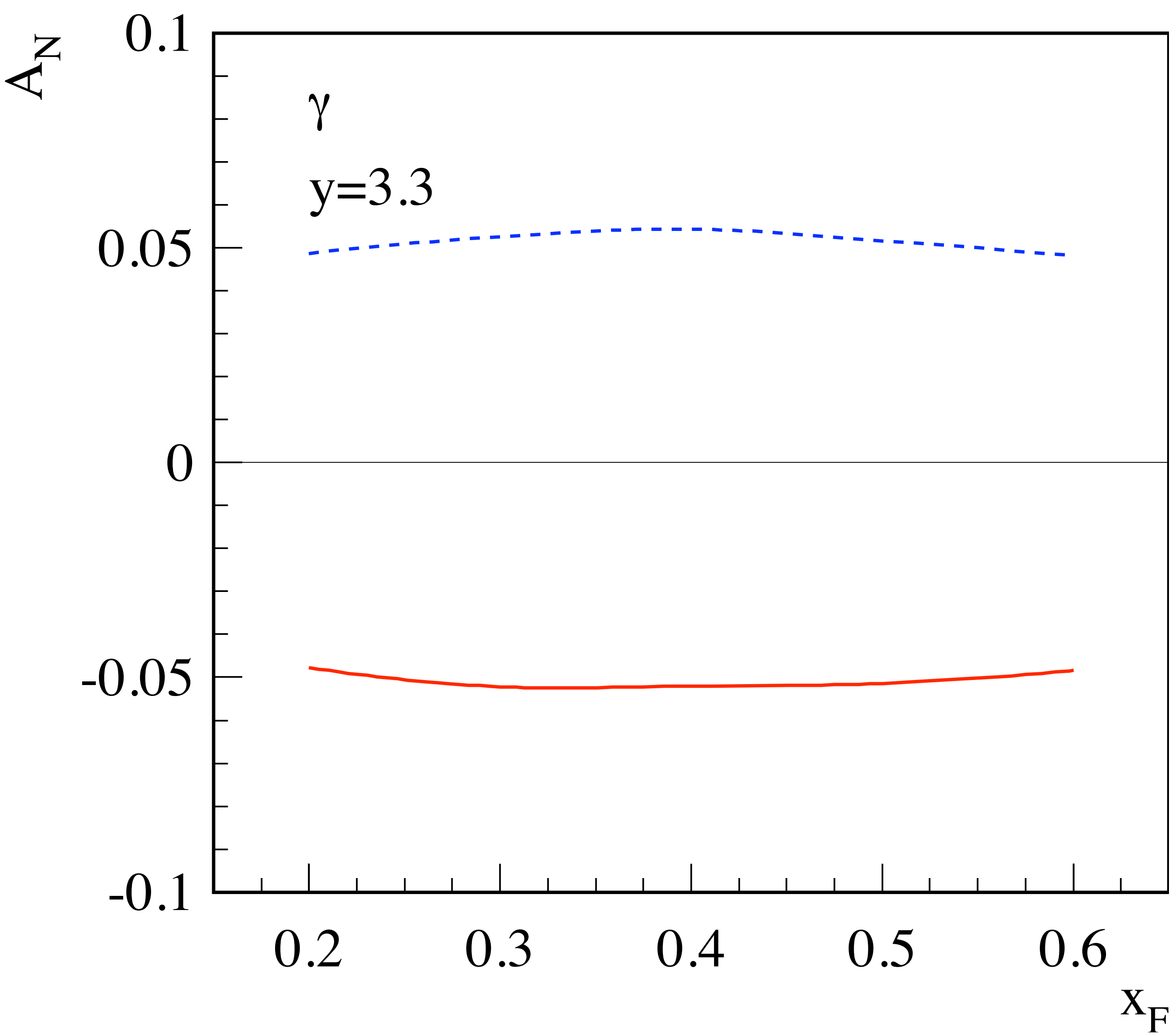
$$E_h \frac{d\Delta\sigma}{d^2 P_h} = \frac{\alpha_s}{s} \sum_{abc} \frac{1}{x}\left(T_F(x,x) - x\frac{d}{dx}T_F(X,X)\right)\frac{1}{-X}\frac{\epsilon^{P_h S_T n \bar{n}}/z}{x_b s + T/z}$$
$$\times \int \frac{dx_b}{x_b} \int \frac{dz}{z^2} H^{\mathrm{Inc}}_{ab\to c}(\hat{s},\hat{t},\hat{u}) \frac{1}{x_b s + T/z}$$

Same as Kouvaris, Qiu ,Vogelsang, and Yuan PRD 2006

- Twist 3 and twist 2 approach connection????

we have another term .... ~ T_F

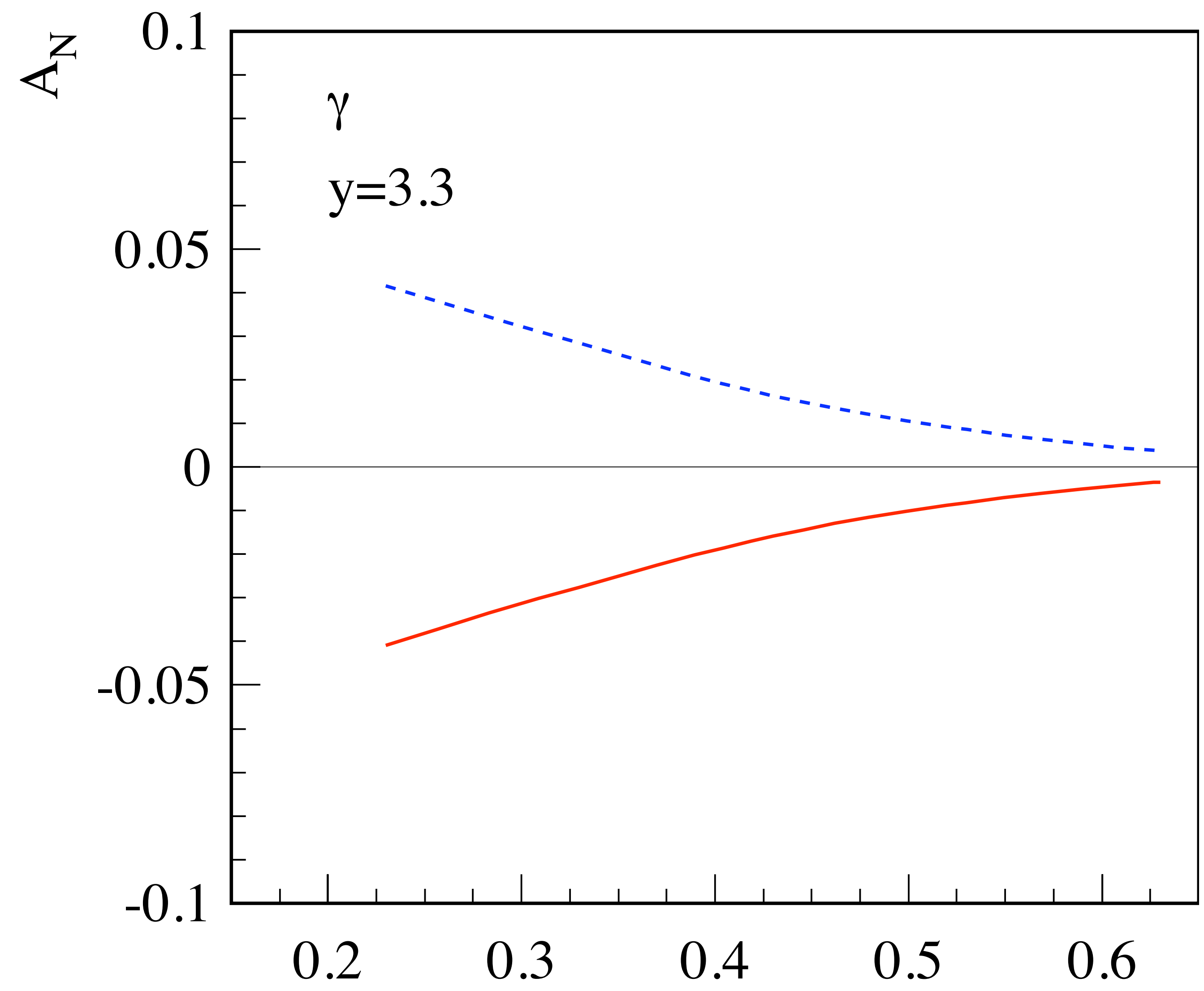
Based on old parameterization



We use GRV98 LO parton

the old Sivers function from [4], and Kretzer fragmentation function [5].

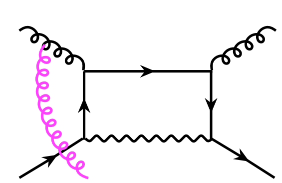
Based on new parameterization



We use GRV98 LO parton

, the latest Sivers function from [2], and DSS fragmentation function [3].

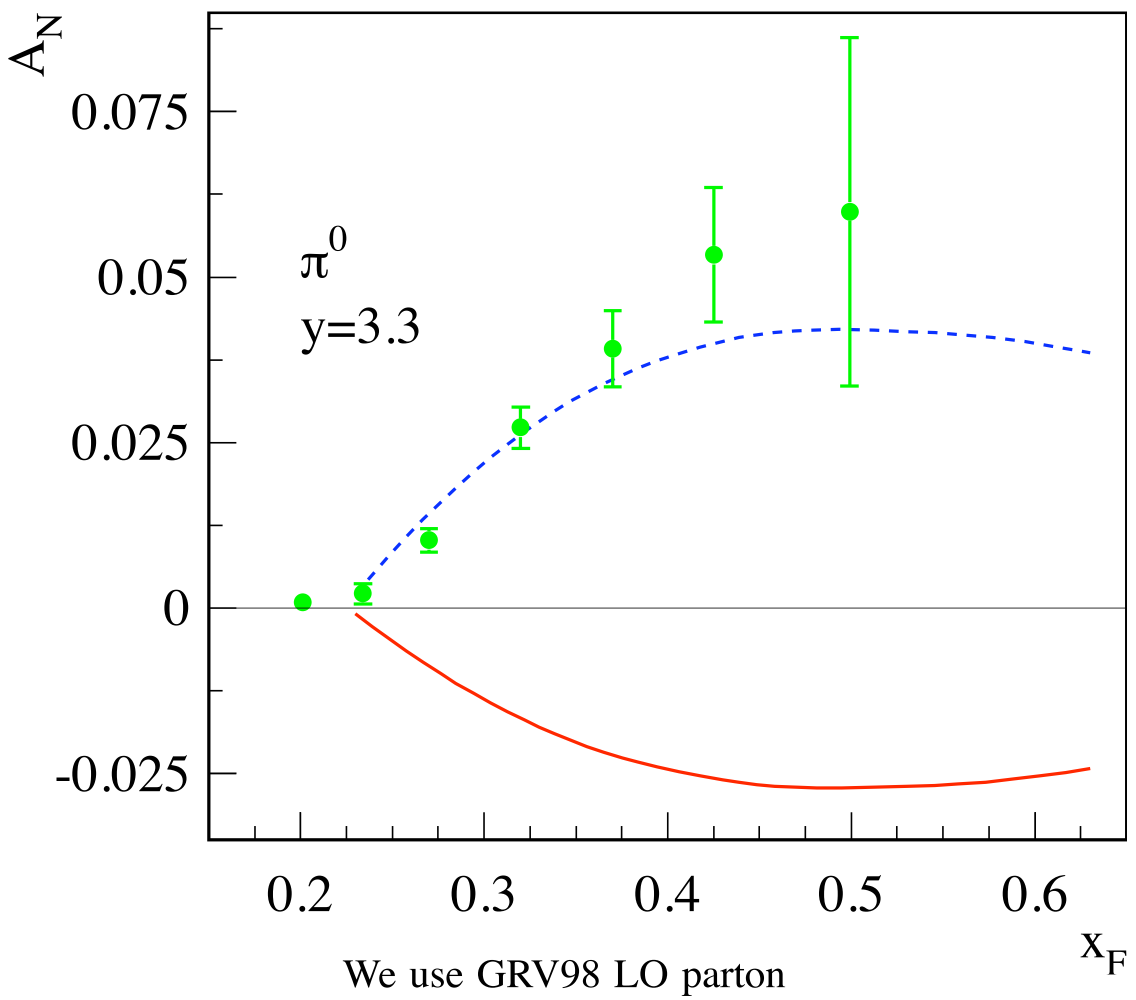
$$H^{\rm Inc}_{qg\to\gamma q} = -\frac{N_c}{N_c^2-1}e_q^2\left[-\frac{\hat t}{\hat s}-\frac{\hat s}{\hat t}\right]$$



ISI drives result

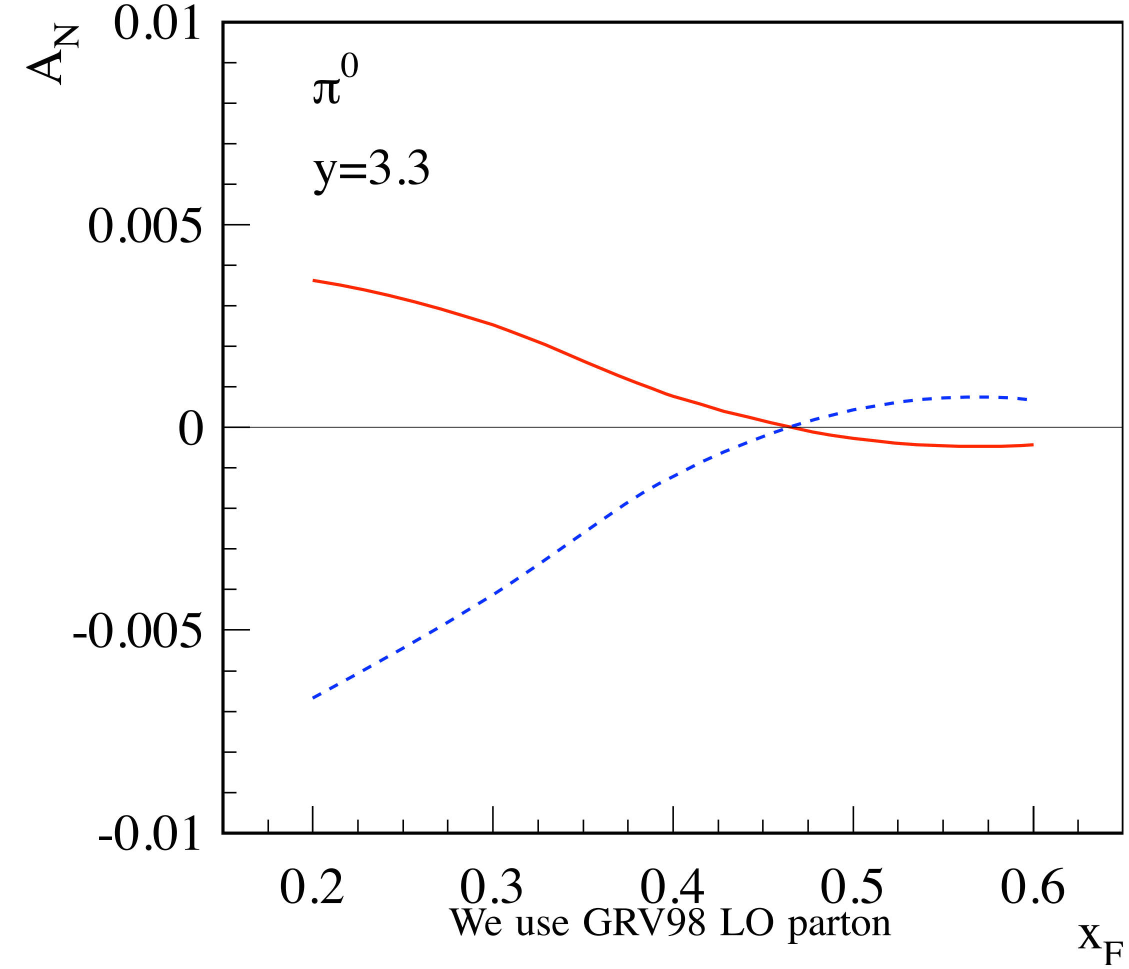
- In this connection see Kouvaris, Qiu, Vogelsang and Yuan PRD 2006

Based on old parameterization



the old Sivers function from [4], and Kretzer fragmentation function [5].

Based on new parameterization



, the latest Sivers function from [2], and DSS fragmentation function [3].

$$\Delta\sigma^{pp^{\uparrow}\to\pi X} \sim \Delta f_a \otimes f_b \otimes \Delta\hat{\sigma} \otimes D^{q\to\pi}$$

Must take into account....

Factorization Breaking for most other process

$$f_{1T}^{\perp},\, h_1^{\perp},\, D_{1T}^{\perp},\, H_1^{\perp}$$

- **Collins Qiu & Collins PRD 2007 & hep arXive**
- **Kang, Qiu, Zhang Gluonic Poles PRD 2010**
- **Mulders & Rogers Fact. breaking PRD 2010**

NO GENERALIZED TRANSVERSE MOMENTUM DEPENDENT . . . PHYSICAL REVIEW D **81,** 094006 (2010)

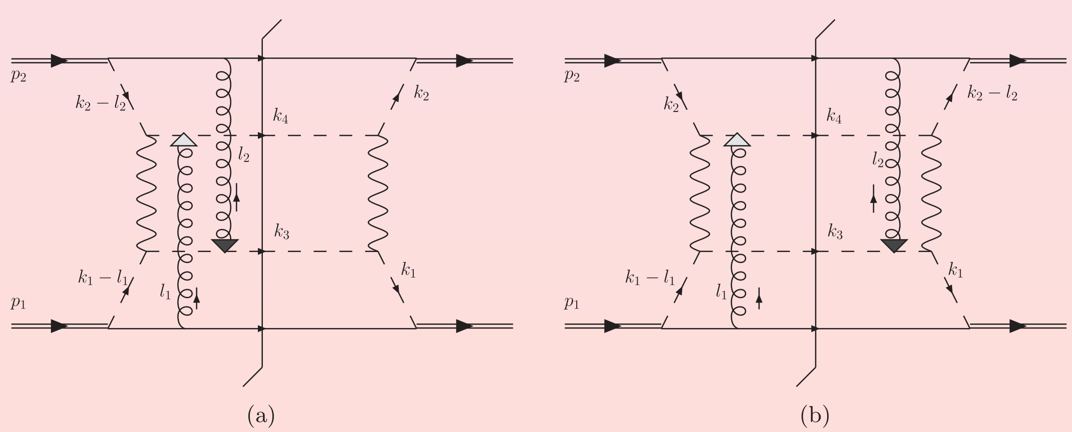


FIG. 8. Graphs that contribute to a violation of generalized TMD-factorization. Other graphs that should be included are those with all possible attachments of l_1 to the k_4 and k_2 lines, and all possible attachments of l_2 to the k_3 and k_1 lines, and all Hermitian conjugate graphs. In total there are 16 graphs of this type.

Conclusions

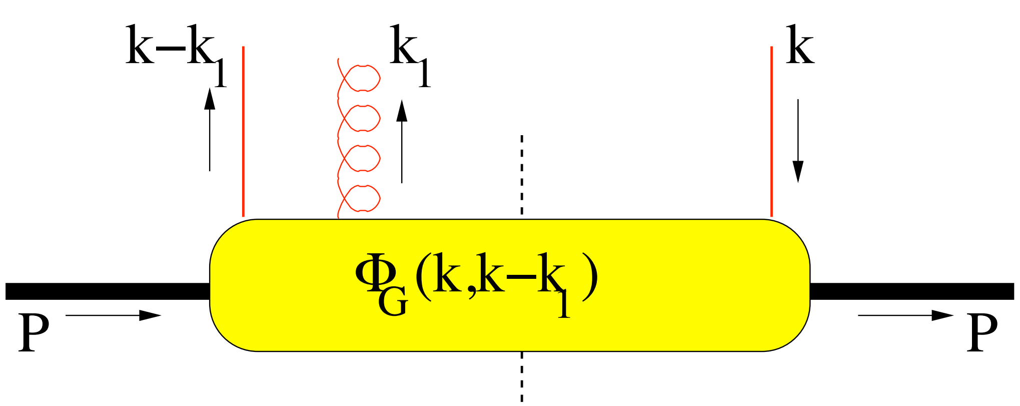
- Generalize GPM w/ color--should perform global analysis pheo test
- Elephant in the room is factorization and let alone universality
- Appears to be twist 3 and twist 2 approach connection-under investigation LG and Z. Kang

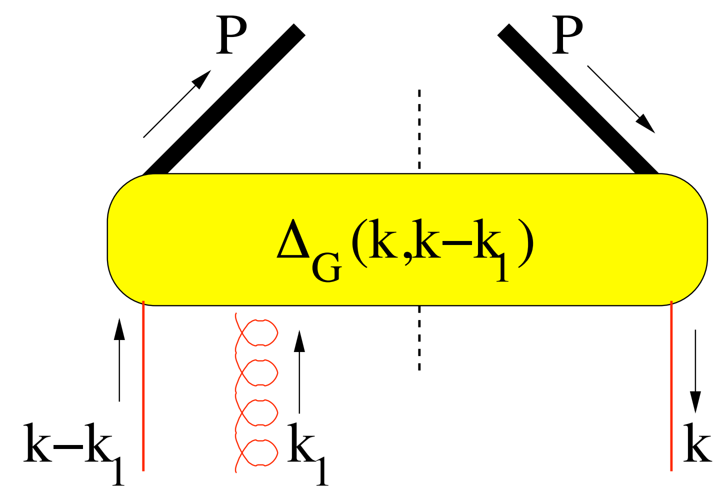
Universality & Gluonic Pole Matrix elements

Model independent generalization of spectator model L.G. A. Mukherjee & P. Mulders PRD 2008
in prep... this week?

Consider correlation functions as multi-particle scattering amplitudes

$$\Phi_G^{\alpha}(x, x-x_1) = \int \frac{d(\xi\cdot P)}{2\pi}\,\frac{d(\eta\cdot P)}{2\pi}\; e^{ix_1(\eta\cdot P)}e^{i(x-x_1)(\xi\cdot P)}$$
$$\times\, \langle P|\,\overline{\psi}(0)\,U^{n}_{[0;\eta]}\,gG^{n\alpha}(\eta)\,U^{n}_{[\eta;\xi]}\,\psi(\xi)\,|P\rangle\,\rfloor_{\rm LC}$$





The steps in these considerations:

1) The observation that the k^- and integrations in the quark-quark quark-quark-gluon correlators lead to light-front correlators, for which time-ordering is irrelevant

2)Therefore the matrix elements can be expressed as matrix elements of time-ordered products of operators then using LSZ formalism can study analytic structure poles and cuts

(Jaffe-1984) quark-quark and multi-parton correlators collinear correlators

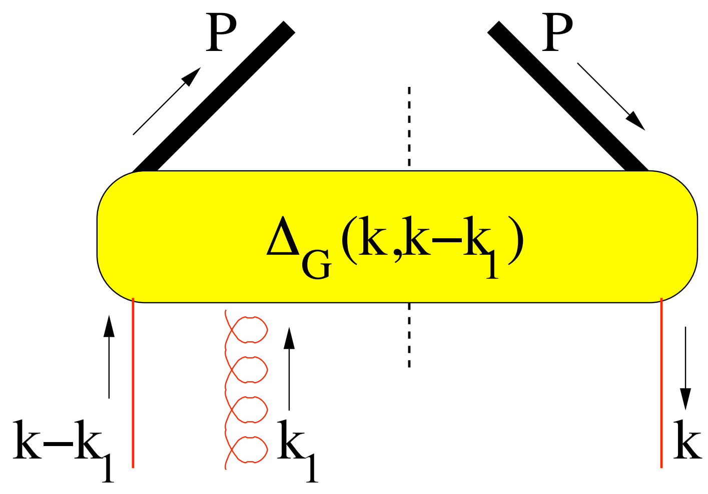
(Diehl-Gousset-1998) GPDs Radyushkin and Belitsky Phys. Rep. 2005

The steps in these considerations:

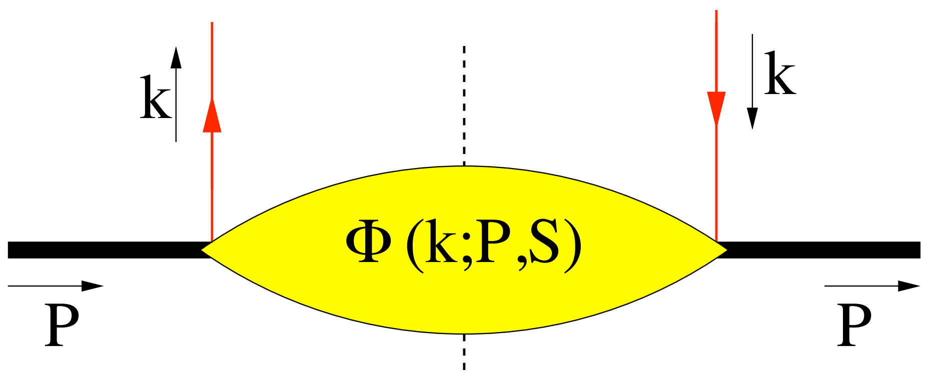
.....

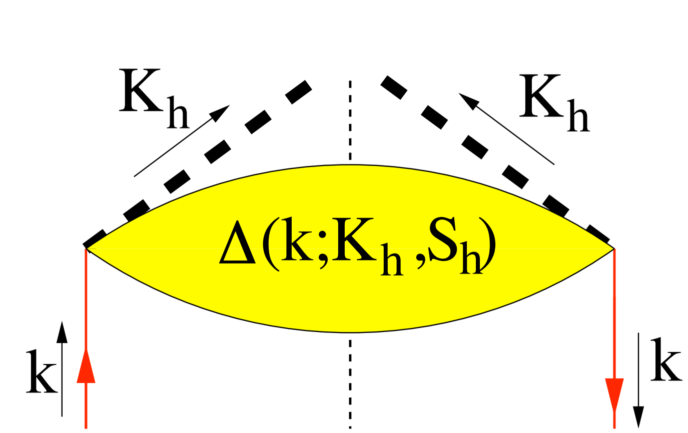
3) These pictures become just hadron-parton amplitudes, e.g. the quark-quark correlator is related to the forward antiquark-hadron scattering amplitude. Depending on the precise structure these are untruncated Greens functions-time ordered products. Can use LSZ formalism to study analytic/singluarity structure

Goal to study support properties in limit $x_1 \to 0$



Take simple example of quark-target amplitude Landshoff, Polkinghorne, and Short 1971 NPB applied to TMDs





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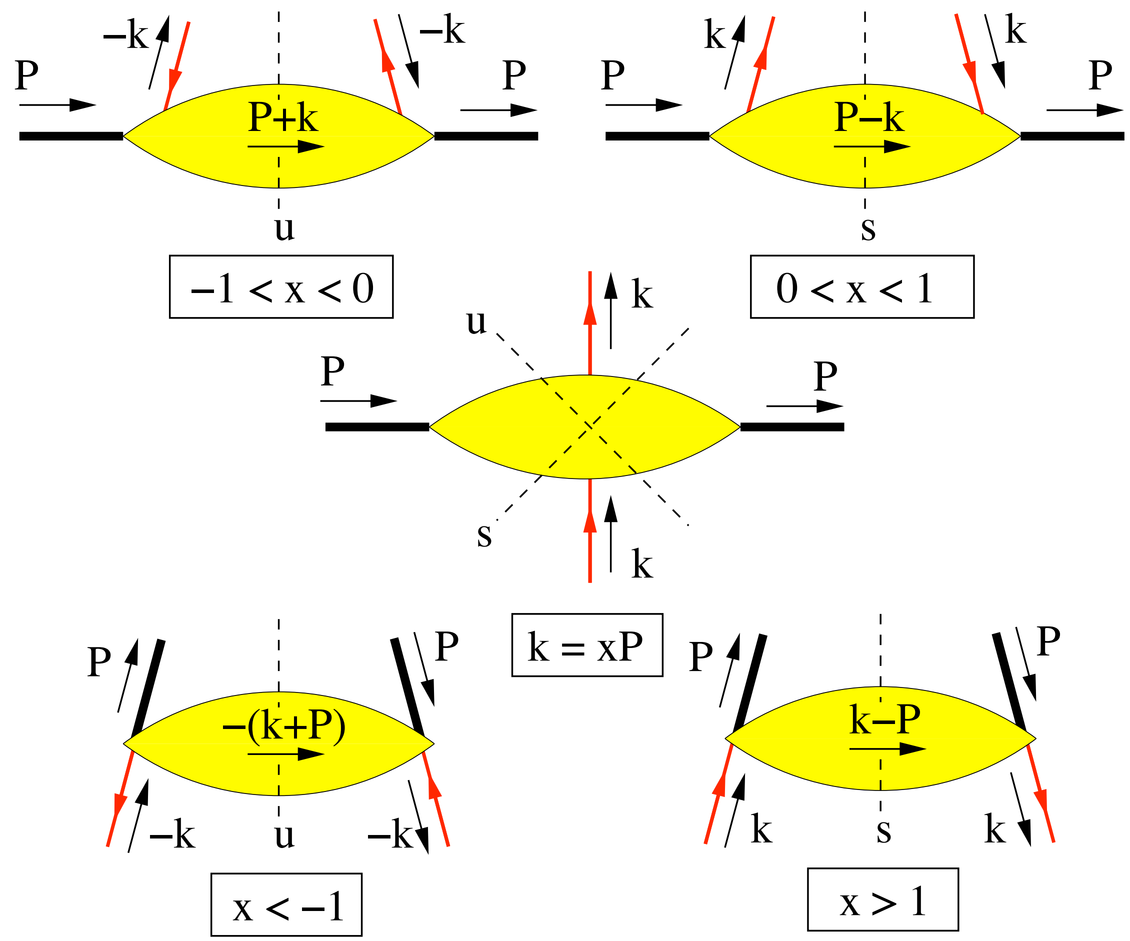
3) These pictures become just hadron-parton amplitudes, e.g. the quark-quark correlator is related to the forward antiquark-hadron scattering amplitude. Depending on the precise structure these are untruncated Greens functions-time ordered products. Can use LSZ formalism to study analytic/singluarity structure

Goal to study support properties

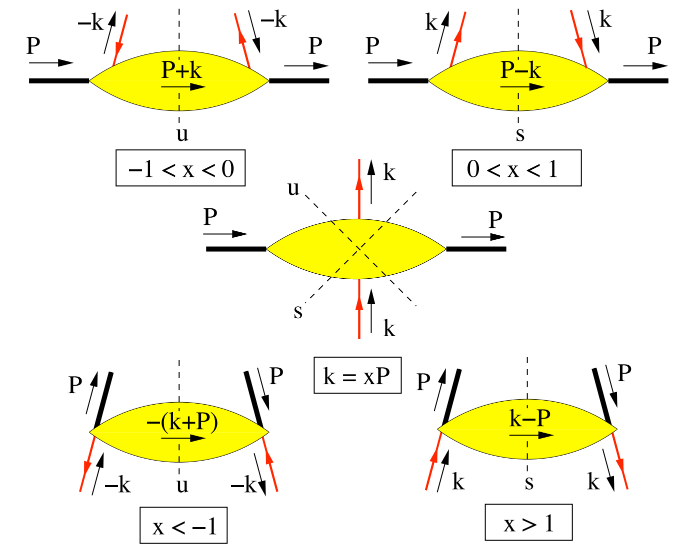
$$A(k^2; s, u)$$



Integrating parton correlators over k^- connects them to the anti-parton -hadron scattering four-point function.

Depending on the value of *x*,
the imaginary part of this amplitude $A(k^2; s, u)$ represents
the (anti)-parton distribution or fragmentation correlators.

TMDs



$$\Phi^\alpha(x, k_T^2) = \int dk^- \mathcal{A}^\alpha(s + i\epsilon, k^2 + i\epsilon, u + i\epsilon)\Big|_{LF}$$

$$= \int dk_1^- \mathcal{A}^\alpha(k_1^- + i\epsilon f_a(x))\Big|_{LF}$$

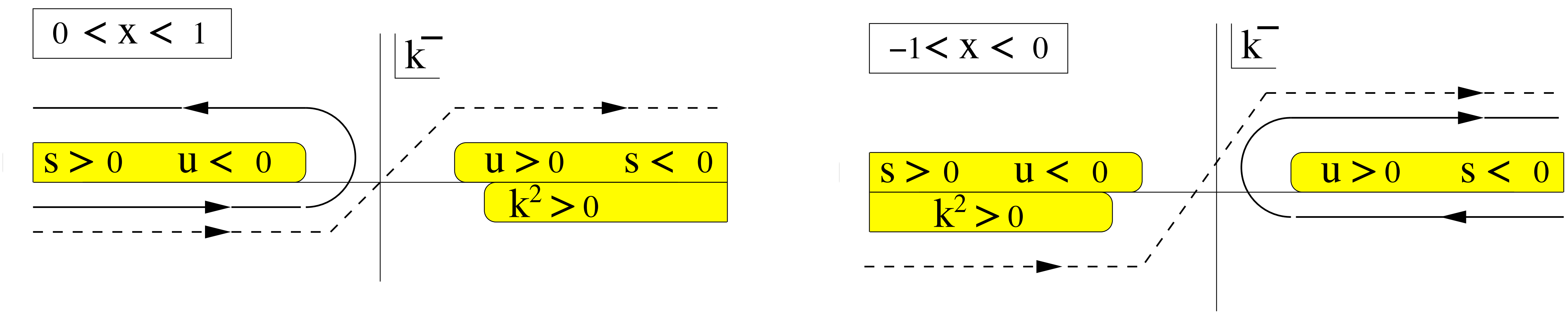
$$k^- = \frac{s + i\epsilon}{2(x-1)}$$

$$k^- = \frac{u + i\epsilon}{2(x+1)}$$

$$k^- = \frac{k^2 + i\epsilon}{2x}$$

Thus Support in x region PDFs

$$\Phi(x) = \theta(x)\,\theta(1-x)\,\mathrm{Disc}_{[s]}\mathcal{A} + \theta(-x)\,\theta(1+x)\,\mathrm{Disc}_{[u]}\mathcal{A}$$



The integration contours for k^- integration w/respect to the kinematic singularities in the (forward) anti-parton - hadron scattering amplitude for the case of (non-vanishing) distribution functions for quarks (a) and antiquarks (b).

Support in x region FFs

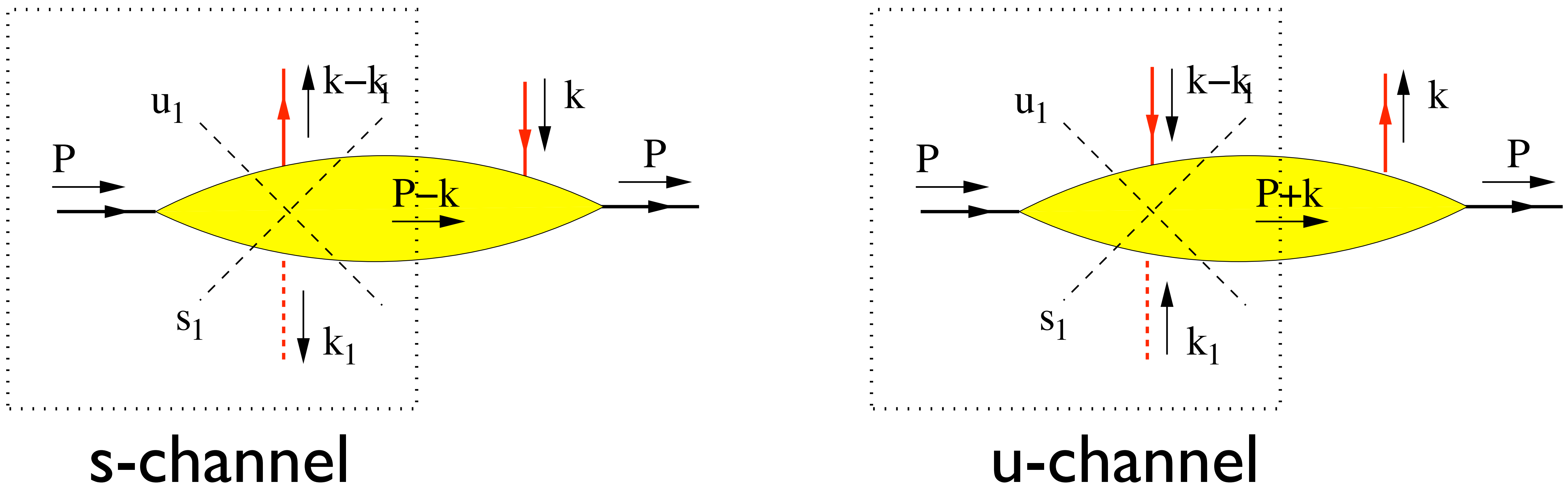
$$\Delta(x) = \theta(x-1)\,\mathrm{Disc}_{[s]}\mathcal{A} + \theta(-1-x)\,\mathrm{Disc}_{[u]}\mathcal{A}$$

$$= \theta(z)\,\theta(1-z)\,\mathrm{Disc}_{[s]}\mathcal{A} + \theta(-z)\,\theta(1+z)\,\mathrm{Disc}_{[u]}\mathcal{A}$$

The case for fragmentation is different since the parton propagator for positive k^2 contours in x and z not connected by analytic continuation Landshoff and Polkinghorn Phys. Rep. 1972

Extend analyticity study to multi-parton distribution and fragmentation function

$$\mathcal{A}(k^2; s, u; s_1, u_1; k_1^2, (k-k_1)^2)$$



The additional invariants for the amplitude

$$\mathcal{A}(k^2; s, u; s_1, u_1; k_1^2, (k-k_1)^2)$$

relevant for gluonic pole matrix elements, for the case s > 0 and for the case u > 0.

The additional invariants for the amplitude
$\mathcal{A}(k^2; s, u; s_1, u_1; k_1^2, (k-k_1)^2)$
relevant for gluonic pole matrix elements, for the case s > 0 and for the case u > 0.

$$s_1 = (P \mp k \pm k_1)^2 \quad \text{and} \quad u_1 = (P \mp k_1)^2$$

$$k_1^- = \frac{s_1 + i\epsilon}{2(x_1 - (x \mp 1))} + k^- \qquad k_1^- = \frac{u_1 + i\epsilon}{2(x_1 \mp 1)}$$

$$k_1^- = \frac{(k-k_1)^2 + i\epsilon}{2(x_1 - x)} + k^- \qquad k_1^- = \frac{k_1^2 + i\epsilon}{2x_1}$$

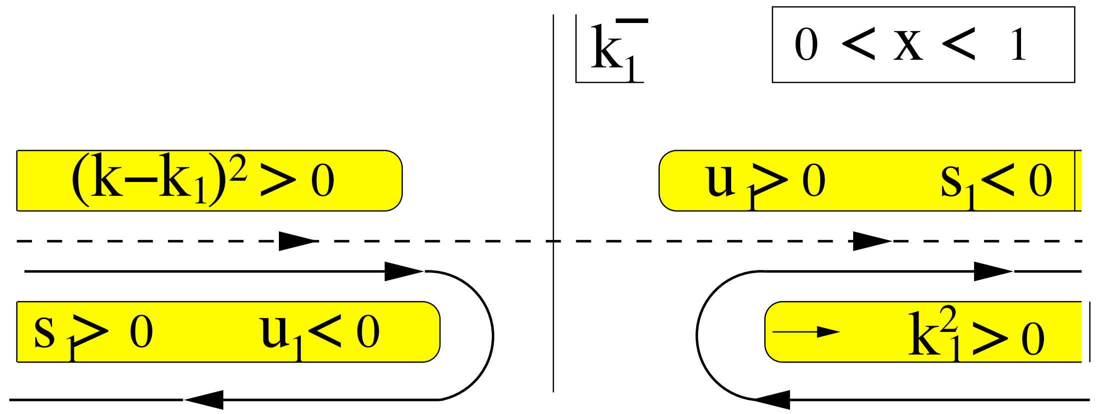
Comments

- Depending on the value of x_1 the integration contour in k_1^- bypasses the singularities encountered in the complex plane in a particular way, which dictates the support properties of the quark-gluon-quark correlation functions

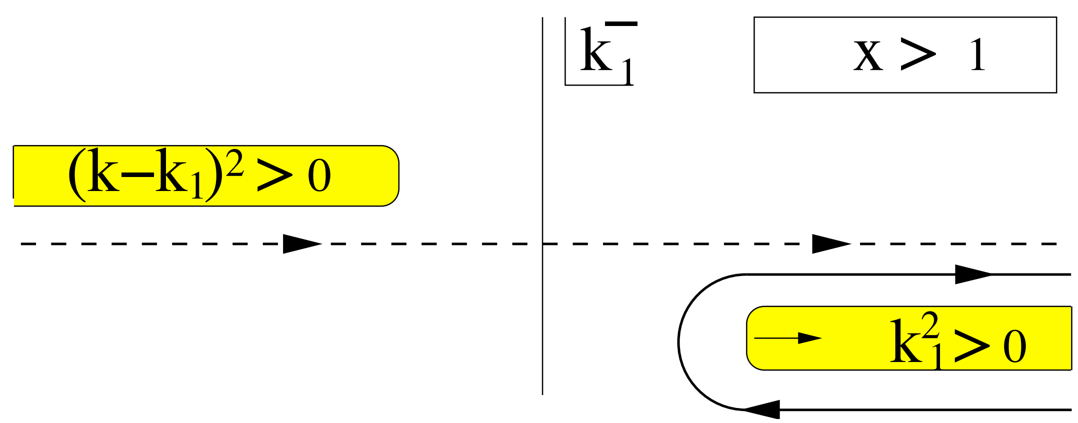
- The denominators in the expressions relating k_1^- to s_1 and u_1 tell us that only when $x_1 \in [x-1, 1]$ (for positive x) or $x_1 \in \; - \; , x$ (for negative x) the singularities in s_1 and u_1 are relevant

- study case of s-channel ($s > 0$) $0 < x < 1$
- Look at the gluonic poles $x_1 \to 0$
- $x_1 \to 0$ is in the interval $0 < x < 1$

$$\lim_{x_1 \to 0} \Delta_G(x, x - x_1) = \Delta_G(x, x) \to 0$$



$$\Phi_G(x, x - x_1)$$



$$\Delta_G(x, x - x_1)$$

For the case $x > 1$ the k_1^- integration can be wrapped around the cut k_1^2 which smoothly vanishes for $x_1 \to 0$ describes the by the arrow inside branch cut indicates that it harmlessly recedes to infinity

Agrees with earlier model analysis Collins, Metz PRL 2004
Agrees with earlier model analysis LG, A. Mukherjee, P. Mulders PRD 2008
Agrees with spectral analysis A. Metz, S. Meissner PRL 2009

Comments

- Text In wrapping the integration around the s- or u-cut
- Must assume convergence in the variable k^- or use subtracted relations

Conclusions

- Study support of multi-parton correlation functions through analytic structure of scattering amplitude
- Gluonic pole contribution to fragmentation function vanishes--model independent result
- Implies universality of Collins function
- Consistent with a number of past studies