POLARIZED PDFs and HIGHER TWIST from NLO ANALYSIS of DIS and SIDIS

Elliot Leader

Imperial College London

in collaboration with

A. V. Sidorov (Dubna) and D. B. Stamenov (Sofia)
CONTENTS

1) From experiment to $g_1(x, Q^2)$ in DIS—kinematics.

2) DIS theoretical expression for $g_1(x, Q^2)$.

3) Extension to SIDIS.

4) Data sample.

5) Results and comparison with DSSV.

6) Controversy about Higher Twist.

7) Spin sum rule.
From experiment to $g_1(x, Q^2)$ in DIS

Measured asymmetries:

$$A_{\parallel} = \frac{d\sigma \leftrightarrow - d\sigma \Rightarrow}{2d\sigma_{unpold}}$$

$$A_{\perp} = \frac{d\sigma \rightarrow \uparrow - d\sigma \rightarrow \downarrow}{2d\sigma_{unpold}}$$

$$(A_{\parallel}, A_{\perp}) \Rightarrow (A_1, A_2) \Rightarrow (g_1, g_2)$$
From experiment to $g_1(x, Q^2)$ in DIS

Measured asymmetries:

$$A_\parallel = \frac{\leftarrow \sigma - \rightarrow \sigma}{2\sigma_{unpold}} \quad A_\perp \equiv \frac{\uparrow \sigma - \downarrow \sigma}{2\sigma_{unpold}}$$

$$(A_\parallel, A_\perp) \Rightarrow (A_1, A_2) \Rightarrow (g_1, g_2)$$

If both $A_\parallel$ and $A_\perp$ measured: $\Rightarrow \frac{g_1}{F_1}$
From experiment to $g_1(x, Q^2)$ in DIS

Measured asymmetries:

$$A_{\parallel} = \frac{d\sigma \leftarrow - d\sigma \Rightarrow}{2d\sigma_{\text{unpold}}} \quad A_{\perp} \equiv \frac{d\sigma \uparrow \rightarrow d\sigma \Rightarrow}{2d\sigma_{\text{unpold}}}$$

$$(A_{\parallel}, A_{\perp}) \Rightarrow (A_1, A_2) \Rightarrow (g_1, g_2)$$

If both $A_{\parallel}$ and $A_{\perp}$ measured: $\Rightarrow \frac{g_1}{F_1}$

If only $A_{\parallel}$ measured:

$$\frac{A_{\parallel}}{D} = (1 + \gamma^2) \left[ \frac{g_1}{F_1} \right] + (\eta - \gamma) A_2$$

$$\frac{A_{\parallel}}{D} \approx (1 + \gamma^2) \left[ \frac{g_1}{F_1} \right] \quad \gamma^2 = \frac{4M^2x^2}{Q^2}$$
NB $\gamma$ cannot be ignored in the SLAC, HERMES and JLab kinematic regions.
 NB $\gamma$ cannot be ignored in the SLAC, HERMES and JLab kinematic regions.

It is ignored in the DSSV analysis
NB $\gamma$ cannot be ignored in the SLAC, HERMES and JLab kinematic regions.

It is ignored in the DSSV analysis

Taking $F_1$ from experiment $\Rightarrow g_1(x, Q^2)_{exp}$
We utilize (in $\overline{MS}$ scheme)

\[
g_{1}(x, Q^2)_{exp} = g_{1}(x, Q^2)_{LT} + g_{1}(x, Q^2)_{TMC} + g_{1}(x, Q^2)_{HT} \\
= g_{1}(x, Q^2)_{LT} + g_{1}(x, Q^2)_{TMC} + \frac{h(x)}{Q^2}
\]
We utilize (in $\overline{MS}$ scheme)

\[
g_1(x, Q^2)_{\text{exp}} = g_1(x, Q^2)_{LT} + g_1(x, Q^2)_{TMC} + g_1(x, Q^2)_{HT}
= g_1(x, Q^2)_{LT} + g_1(x, Q^2)_{TMC} + \frac{h(x)}{Q^2}
\]

\[
g_1(x, Q^2)_{LT} = \frac{1}{2} \sum_{\text{flavors}} e_q^2 \left\{ [\Delta q(x, Q^2) + \Delta \bar{q}(x, Q^2)] + \frac{\alpha_s(Q^2)}{2\pi} \int_1^1 \frac{dy}{y} \left\{ \Delta C_q(x/y) [\Delta q(y, Q^2) + \Delta \bar{q}(y, Q^2)] + \Delta C_G(x/y) \Delta G(y, Q^2) \right\} \right\}
\]
We utilize (in $\overline{MS}$ scheme)

\[
g_1(x, Q^2)_{\text{exp}} = g_1(x, Q^2)_{\text{LT}} + g_1(x, Q^2)_{\text{TMC}} + g_1(x, Q^2)_{\text{HT}}
\]

\[
= g_1(x, Q^2)_{\text{LT}} + g_1(x, Q^2)_{\text{TMC}} + \frac{h(x)}{Q^2}
\]

\[
g_1(x, Q^2)_{\text{LT}} = \frac{1}{2} \sum_{\text{flavors}} e_q^2 \left\{ \Delta q(x, Q^2) + \Delta \bar{q}(x, Q^2) \right\}
\]

\[
+ \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \left\{ \Delta C_q(x/y) \left[ \Delta q(y, Q^2) + \Delta \bar{q}(y, Q^2) \right] + \Delta C_G(x/y) \Delta G(y, Q^2) \right\}
\]

Inclusive DIS determines ONLY the sum of quark and antiquark densities
Important difference between UNPOLARIZED and POLARIZED DIS:

About half of data are at MODERATE $Q^2$ and $W^2$ i.e.

$$1 \lesssim Q^2 \lesssim 4 GeV^2 \quad 4 \lesssim W^2 \lesssim 10 GeV^2$$
Important difference between UNPOLARIZED and POLARIZED DIS:

About half of data are at MODERATE $Q^2$ and $W^2$ i.e.

$$1 \lesssim Q^2 \lesssim 4\,\text{GeV}^2 \quad 4 \lesssim W^2 \lesssim 10\,\text{GeV}^2$$

We believe Higher Twist corrections are important. $\gamma^2$ term should not be neglected!
Extension to SIDIS

Aside from a kinematic factor, the SIDIS polarized cross-section, in NLO is

\[
\Delta \sigma^h_p|_{NLO} = \sum_i e_i^2 \Delta q_i \left[ 1 + \frac{\alpha_s}{2\pi} \Delta C_{qq} \otimes \right] D^h_{qi}
\]

\[
+ \left( \sum_i e_i^2 \Delta q_i \right) \otimes \frac{\alpha_s}{2\pi} \Delta C_{qg} \otimes D^h_G
\]

\[
+ \Delta G \otimes \frac{\alpha_s}{2\pi} \Delta C_{gq} \otimes \left( \sum_i e_i^2 D^h_{qi} \right)
\]

This involves a double convolution and thus a double Mellin Transform.
The measured asymmetry is

\[ A^h_{\parallel}(x, Q^2)_{exp} = \frac{\Delta \sigma^h_p|_{exp}}{\sigma^h_p|_{exp}} \]
The measured asymmetry is

\[ A^h_{\parallel}(x, Q^2)_{exp} = \frac{\Delta \sigma^h_p|_{exp}}{\sigma^h_p|_{exp}} \]

TMC and HT corrections not known for SIDIS.....should be less important for kinematic range of present data. Thus use:

\[ A^h_{\parallel}(x, Q^2)_{exp} = \frac{\Delta \sigma^h_p|_{NLO}}{\sigma^h_p|_{exp}} \]
The measured asymmetry is

\[ A^h_{\parallel}(x, Q^2)_{exp} = \frac{\Delta \sigma^h_p|_{exp}}{\sigma^h_p|_{exp}} \]

TMC and HT corrections not known for SIDIS.....should be less important for kinematic range of present data. Thus use:

\[ A^h_{\parallel}(x, Q^2)_{exp} = \frac{\Delta \sigma^h_p|_{NLO}}{\sigma^h_p|_{exp}} \]

Use DSS Fragmentation Functions.....will use others as well
Note that DSS FFs are significantly different from others:

\[ D_{\pi^+}^g \gg \text{Krezer (KRE) or Albino, Kniehl and Kramer (AKK) at large } x. \]

\[ D_{\pi^+}^{s \bar{s}} \gg \text{AKK for } x \leq 0.7 \]

\[ D_{s + \bar{s}}^{K^+} \gg \text{KRE, } \ll \text{AKK} \]

\[ D_{g}^{K^+} \ll \text{KRE and AKK} \]

This needs study!
\[ \Delta u + \Delta \bar{u} = A_U x^{\alpha_U} (1 - x)^{\beta_U} (1 + \epsilon_U \sqrt{x} + \gamma_U x) \]

\[ \Delta \bar{u} = A_{\bar{u}} x^{\alpha_U} (1 - x)^{\beta} (1 + \gamma_{\bar{u}} x) \]

\[ \Delta d + \Delta \bar{d} = A_D x^{\alpha_D} (1 - x)^{\beta_D} (1 + \gamma_D x) \]

\[ \Delta \bar{d} = A_{\bar{d}} x^{\alpha_D} (1 - x)^{\beta} \]

\[ \Delta s = \Delta \bar{s} = A_s x^{\alpha_s} (1 - x)^{\beta} (1 + \gamma_s x) \]

\[ \Delta G = A_G x^{\alpha_G} (1 - x)^{\beta} (1 + \gamma_G x) \]

16 free parameters
The Data Sample

Inclusive DIS: 841 experimental points
Semi-inclusive DIS: 202 experimental points
The Data Sample

Inclusive DIS: 841 experimental points
Semi-inclusive DIS: 202 experimental points

Compared to DSSV, we use new COMPASS data on inclusive $A_1$(proton) and on semi-inclusive asymmetries for $\pi^\pm$ and $K^\pm$. 
The Data Sample

Inclusive DIS: 841 experimental points
Semi-inclusive DIS: 202 experimental points

Compared to DSSV, we use new COMPASS data on inclusive $A_1$(proton) and on semi-inclusive asymmetries for $\pi^\pm$ and $K^\pm$.

DIS: $\chi^2_{NExpP} = 0.85$    SIDIS: $\chi^2_{NExpP} = 0.90$

Overall $\chi^2_{DOF} = 0.88$
Fits to SIDIS data
Fits to SIDIS data
Predictions for COMPASS proton SIDIS data
Results and comparison with DSSV

$Q^2 = 2.5 \text{ GeV}^2$

$x(\Delta u + \Delta \bar{u})$

$x(\Delta d + \Delta \bar{d})$
Results and comparison with DSSV

\[ x\Delta u \quad Q^2 = 2.5 \text{ GeV}^2 \]

\[ x\Delta d \quad Q^2 = 2.5 \text{ GeV}^2 \]
Results and comparison with DSSV

Note: DSSV use $\alpha_{\bar{s}} = \alpha_{\bar{d}}$ and find $= 0.16$
LSS find: $\alpha_{\bar{s}} = 0.05 \pm 0.02$  $\alpha_{\bar{d}} = 0.55 \pm 0.12$
Results and comparison with DSSV

Redo DIS including term \((1 + \gamma x)\) to permit sign change.

\[ \Delta s \text{ is controversial} \]
Results and comparison with DSSV

\[ x\Delta G \]

\[ Q^2 = 2.5 \text{ GeV}^2 \]
We also find an acceptable solution with positive $\Delta G$

\[ x\Delta G \]

\[ Q^2 = 2.5 \text{ GeV}^2 \]

\[ \text{LSS'10} \]
\[ \text{LSS'10 (pos)} \]

NB: has very little effect on $\Delta \bar{u}, \Delta \bar{d}, \Delta \bar{s}$.

Dashed lines: error bands — NB: Warning: error bands do not reflect functional uncertainty!!!
The controversy about Higher Twist

Following Operator Product Expansion (OPE), LSS use

\[ g_1(x, Q^2)_{\text{exp}} = g_1(x, Q^2)_{LT} + g_1(x, Q^2)_{TMC} + g_1(x, Q^2)_{HT} \]

\[ = g_1(x, Q^2)_{LT} + g_1(x, Q^2)_{TMC} + \frac{h(x)}{Q^2} \]

Higher twist corrections: the exactly known kinematical target mass corrections (TMC) and genuine dynamical higher twist terms (HT).
The controversy about Higher Twist

Following Operator Product Expansion (OPE), LSS use

\[ g_1(x, Q^2)_{\text{exp}} = g_1(x, Q^2)_{LT} + g_1(x, Q^2)_{TMC} + g_1(x, Q^2)_{HT} \]

\[ = g_1(x, Q^2)_{LT} + g_1(x, Q^2)_{TMC} + \frac{h(x)}{Q^2} \]

Higher twist corrections: the exactly known kinematical target mass corrections (TMC) and genuine dynamical higher twist terms (HT).
Possible slow scale i.e. \( Q^2 \) dependence in \( h(x) \), the precise form of which is unknown, neglected compared to \( 1/Q^2 \) variation.
We find significant HT contribution

Very important for CLAS data.
Blümlein and Böttcher (BB) (arXiv:1005.3113 v1) disagree

They use

\[ g_1(x, Q^2)_{\text{exp}} = g_1(x, Q^2)_{LT} \left[ 1 + \frac{C(x)}{Q^2} \right] \]

where any \( Q^2 \) dependence in \( C(x) \) is neglected.
Blümlein and Böttcher (BB) (arXiv:1005.3113 v1) disagree

They use

\[ g_1(x, Q^2)_{\text{exp}} = g_1(x, Q^2)_{\text{LT}} \left[ 1 + \frac{C(x)}{Q^2} \right] \]

where any \( Q^2 \) dependence in \( C(x) \) is neglected.

BB find no evidence for HT i.e. their \( C(x) \) for protons and neutrons is compatible with zero.
Thus

\[ C(x) = \frac{h(x)}{g_1(x, Q^2)_{LT}} \]

If legitimate to neglect the scale dependence in \( h(x) \) then \( C(x) \) must vary considerably with \( Q^2 \), contradicting the use of \( C(x) \) as \( Q^2 \)-independent.
Thus

\[ C(x) = \frac{h(x)}{g_1(x, Q^2)_{LT}} \]

If legitimate to neglect the scale dependence in \( h(x) \) then \( C(x) \) must vary considerably with \( Q^2 \), contradicting the use of \( C(x) \) as \( Q^2 \)-independent.

If legitimate to neglect the \( Q^2 \) dependence in \( C(x) \), then \( h(x) \) must vary considerably with \( Q^2 \).
Thus

\[ C(x) = \frac{h(x)}{g_1(x, Q^2)_{LT}} \]

If legitimate to neglect the scale dependence in \( h(x) \) then \( C(x) \) must vary considerably with \( Q^2 \), contradicting the use of \( C(x) \) as \( Q^2 \)-independent.

If legitimate to neglect the \( Q^2 \) dependence in \( C(x) \), then \( h(x) \) must vary considerably with \( Q^2 \).

Two approaches incompatible and their results incommensurate. One of the two methods (or perhaps both) has to be incorrect.
Thus

\[ C(x) = \frac{h(x)}{g_1(x, Q^2)_{LT}} \]

If legitimate to neglect the scale dependence in \( h(x) \) then \( C(x) \) must vary considerably with \( Q^2 \), contradicting the use of \( C(x) \) as \( Q^2 \)-independent.

If legitimate to neglect the \( Q^2 \) dependence in \( C(x) \), then \( h(x) \) must vary considerably with \( Q^2 \).

Two approaches incompatible and their results incommensurate. One of the two methods (or perhaps both) has to be incorrect.

Since LSS formulation is closer in structure to the OPE we believe it to be the correct way to implement HT corrections.
Another problem: BB utilize above for proton and deuteron data and extract the neutron value of $C(x)$ via

$$C_n(x) = \frac{2}{1 - 1.5\omega_D} C_d - C_p$$
Another problem: BB utilize above for proton and deuteron data and extract the neutron value of $C(x)$ via

$$C_n(x) = \frac{2}{1 - 1.5\omega_D}C_d - C_p$$

This is incorrect. The correct relation should be

$$C_n(x) = \frac{1}{g_{1n}(x, Q^2)_{LT}} \left[ \frac{2}{1 - 1.5\omega_D}g_{1d}(x, Q^2)_{LT}C_d(x) - g_{1p}(x, Q^2)_{LT}C_p(x) \right]$$
Another problem: BB utilize above for proton and deuteron data and extract the neutron value of $C(x)$ via

$$C_n(x) = \frac{2}{1 - 1.5\omega_D} C_d - C_p$$

This is incorrect. The correct relation should be

$$C_n(x) = \frac{1}{g_{1n}(x, Q^2)_{LT}} \left[ \frac{2}{1 - 1.5\omega_D} g_{1d}(x, Q^2)_{LT} C_d(x) - g_{1p}(x, Q^2)_{LT} C_p(x) \right]$$

Dangerous, since $g_{1n}(x, Q^2)_{LT}$ has a zero!
LSS Letter to BB—no response—so

followed by Version 2 of BB, abandoning factorized form for HT

“We prefer the additive case, since the twist-2 scaling violations of $g_1(X,Q^2)$ do not influence $C_{p,d,n}(x)$.”

No reference to LSS

Claim no evidence for HT, but central values essentially same as LSS. BB use only statistical errors, but, more important, define error bars by $\Delta \chi^2 = 9.3$. 
LSS method agrees with approach to HT of moments.

\[
\overline{h}^N \equiv \int_{0.0045}^{0.75} dx \: h^N(x) \quad N = p, n
\]

\[
\overline{h}^p = (-0.028 \pm 0.005) GeV^2 \quad \overline{h}^n = (0.018 \pm 0.008) GeV^2
\]
LSS method agrees with approach to HT of moments.

\[ \bar{h}^N \equiv \int_{0.0045}^{0.75} dx \, h^N(x) \quad N = p, n \]

\[ \bar{h}^p = (-0.028 \pm 0.005) GeV^2 \quad \bar{h}^n = (0.015 \pm 0.007) GeV^2 \]

\[ \bar{h}^p - \bar{h}^n = (-0.043 \pm 0.009) GeV^2 \]

Agrees first moment analysis of \( g_{1}^{(p-n)} \) of Duer et al. Also instanton model.
LSS method agrees with approach to HT of moments.

\[
\bar{h}^N \equiv \int_{0.0045}^{0.75} dx \, h^N(x) \quad N = p, n
\]

\[
\bar{h}^p = (-0.028 \pm 0.005) GeV^2 \quad \bar{h}^n = (0.015 \pm 0.007) GeV^2
\]

\[
\bar{h}^p - \bar{h}^n = (-0.043 \pm 0.009) GeV^2
\]

Agrees first moment analysis of \( g_1^{(p-n)} \) of Duer et al. Also instanton model.

\[
\bar{h}^p + \bar{h}^n = (-0.013 \pm 0.009) GeV^2
\]

\[|\bar{h}^p + \bar{h}^n| < |\bar{h}^p - \bar{h}^n|\]

Agrees \( 1/N_C \) expansion.
The spin sum rule: \( \overline{MS} : Q^2 = 4GeV^2 \)

\[
\frac{1}{2} = \frac{1}{2} \Delta \Sigma(Q^2) + \Delta G(Q^2) + \text{OAM}
\]

Positive \( \Delta G \)

\[
\Delta G = 0.316 \pm 0.190 \quad \Delta \Sigma = 0.207 \pm 0.034
\]

\[
J_{z} = (0.42 \pm 0.19) + \text{OAM}
\]
The spin sum rule: $\overline{MS}: Q^2 = 4 GeV^2$

\[
\frac{1}{2} = \frac{1}{2} \Delta \Sigma(Q^2) + \Delta G(Q^2) + \text{OAM}
\]

Positive $\Delta G$

\[
\Delta G = 0.316 \pm 0.190 \quad \Delta \Sigma = 0.207 \pm 0.034
\]

\[
J_z = (0.42 \pm 0.19) + \text{OAM}
\]

Changing sign $\Delta G$

\[
\Delta G = -0.339 \pm 0.458 \quad \Delta \Sigma = 0.254 \pm 0.042
\]

\[
J_z = (-0.21 \pm 0.46) + \text{OAM}
\]
Summary

- LSS: NLO analysis of DIS and SIDIS (DSSV: also RHIC). **LSS includes TMC and Higher Twist terms**
Summary

• LSS: NLO analysis of DIS and SIDIS (DSSV: also RHIC). LSS includes TMC and Higher Twist terms
• $\Delta u + \Delta \bar{u}, \Delta \bar{u}, \Delta d + \Delta \bar{d}, \Delta \bar{d}$ reasonably well determined. Some disagreement with DSSV
Summary

• LSS: NLO analysis of DIS and SIDIS (DSSV: also RHIC). **LSS includes TMC and Higher Twist terms**
• $\Delta u + \Delta \bar{u}$, $\Delta \bar{u}$, $\Delta d + \Delta \bar{d}$, $\Delta \bar{d}$ reasonably well determined. Some disagreement with DSSV
• SIDIS imposes sign changing $\Delta \bar{s}$, as in DSSV, but LSS smaller in magnitude
Summary

• LSS: NLO analysis of DIS and SIDIS (DSSV: also RHIC). LSS includes TMC and Higher Twist terms
• $\Delta u + \Delta \bar{u}, \Delta \bar{u}, \Delta d + \Delta \bar{d}, \Delta \bar{d}$ reasonably well determined. Some disagreement with DSSV
• SIDIS imposes sign changing $\Delta \bar{s}$, as in DSSV, but LSS smaller in magnitude
• $\Delta \bar{s}|_{SIDIS}$ very different from $1/2[\Delta s + \Delta \bar{s}]_{DIS}$: Cause? $\Delta s \neq \Delta \bar{s}$? COMPASS says difference negligible. Fragmentation functions responsible??
Summary

• LSS: NLO analysis of DIS and SIDIS (DSSV: also RHIC). **LSS includes TMC and Higher Twist terms**

  - $\Delta u + \Delta \bar{u}, \Delta \bar{u}, \Delta d + \Delta \bar{d}, \Delta \bar{d}$ reasonably well determined. Some disagreement with DSSV

  - **SIDIS imposes sign changing $\Delta \bar{s}$**, as in DSSV, but LSS smaller in magnitude

  - $\Delta \bar{s}|_{SIDIS}$ very different from $1/2[\Delta s + \Delta \bar{s}]_{DIS}$: Cause? $\Delta s \neq \Delta \bar{s}$? COMPASS says difference negligible. Fragmentation functions responsible??

• Higher Twist: LSS disagrees with BB, but agrees with moment studies
Summary

• LSS: NLO analysis of DIS and SIDIS (DSSV: also RHIC). LSS includes TMC and Higher Twist terms
  • $\Delta u + \Delta \bar{u}, \Delta \bar{u}, \Delta d + \Delta \bar{d}, \Delta \bar{d}$ reasonably well determined. Some disagreement with DSSV
  • SIDIS imposes sign changing $\Delta \bar{s}$, as in DSSV, but LSS smaller in magnitude
  • $\Delta \bar{s}|_{SIDIS}$ very different from $1/2[\Delta s + \Delta \bar{s}]_{DIS}$. Cause? $\Delta s \neq \Delta \bar{s}$? COMPASS says difference negligible. Fragmentation functions responsible??
  • Higher Twist: LSS disagrees with BB, but agrees with moment studies
  • $\Delta G$ still ambiguous. EIC, large $Q^2$ and small $x$ could resolve.
SHORT BIBLIOGRAPHY

**LSS2010**: arXiv:1010.0574

DIS review: Kuhn, Chen, Leader, Prog. Part. Nucl. Phys., 63 (2009) 1

