Transverse Momentum Dependent Parton Distributions

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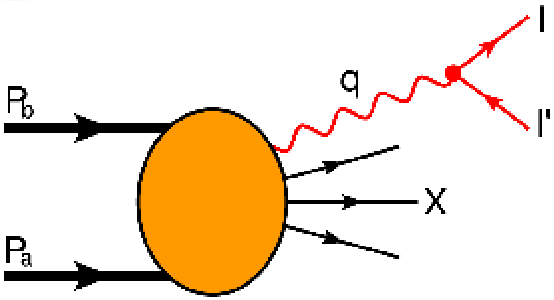
The Drell-Yan process

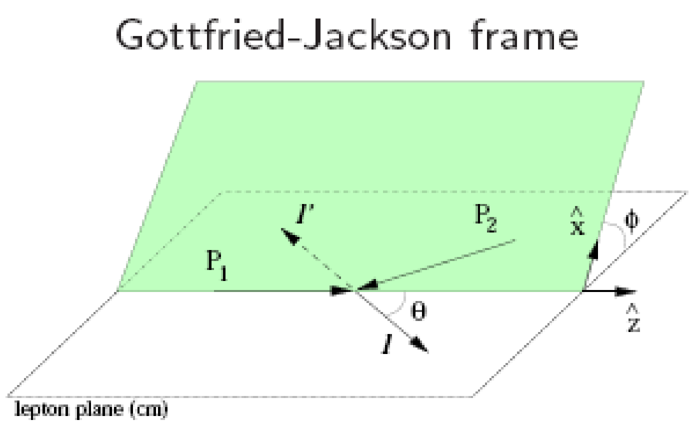
Kinematics (lepton pair produced by one decaying gauge boson):

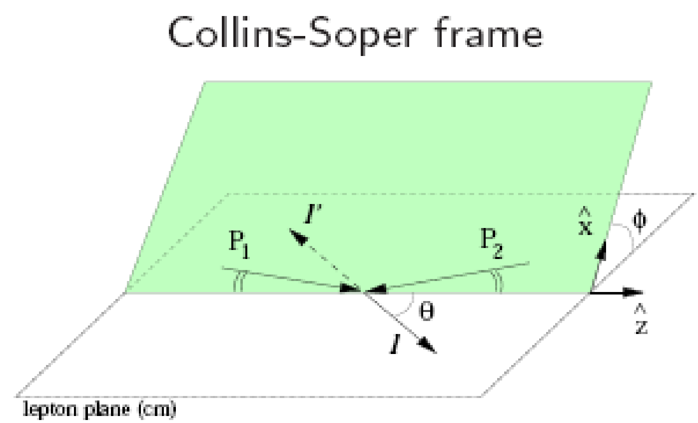


$$\frac{d\sigma}{d^4l\, d^4l'} = \frac{d\sigma}{d^4q\, d^4l} \propto \frac{\delta^+(l^2)\delta^+((q-l)^2)}{4F} \sum_X |M|^2 \delta^{(4)}(P_a + P_b - q - P_X)$$

Disentangling onshell conditions → Dilepton rest frame

Gottfried-Jackson frame and Collins-Soper frame:





Lepton angles:

$$d^4l \rightarrow d\Omega = d\phi\, d\cos\theta$$

Diff. CS including angular dependences:

$$\frac{d^6\sigma}{d^4q\, d\Omega} = 2\frac{d^6\sigma}{dy\, dQ^2\, d^2\vec{q}_T d\Omega}$$

Angular structure functions

Separation of the leptonic part (generated by one photon):

$$\frac{d\sigma}{d^4q\,d\Omega} \propto L_{\mu\nu}W^{\mu\nu}$$

with: $L_{\mu\nu} = 4\Big(l_\mu l'_\nu + l_\nu l'_\mu - \frac{Q^2}{2} g_{\mu\nu}\Big)$ → **Limited number of structure function**

Hadronic Tensor:

$$W^{\mu\nu}(P_a, S_a; P_b, S_b; q) = \int \frac{d^4x}{(2\pi)^4}\, \mathrm{e}^{iq\cdot x} \langle a,b|J^\mu(0)J^\nu(x)|a,b\rangle$$

Parameterization constraint by current conservation, hermiticity and parity

Decomposition into 4 + 8 +8 + 28 = 48 structure functions $F(x_a, x_b, q_T^2, Q^2)$

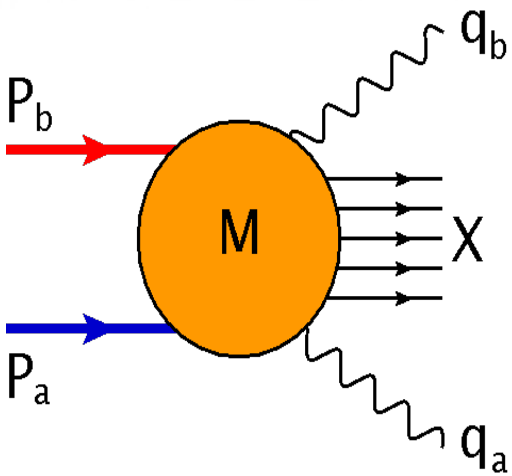
[Arnold, Metz, M.S., PRD 79, 034005] **e.g. unpolarized Drell-Yan**

$$\frac{d\sigma_{UU}}{dx_a\,dx_b\,d^2q_T\,d\Omega} = \frac{\alpha^2 s}{2Q^2 F}\Big((1+\cos^2\Theta)F^1_{UU} + (1-\cos^2\Theta)F^2_{UU} + \sin 2\Theta\cos\phi F^{\cos\phi}_{UU} + \sin^2\Theta\cos 2\phi F^{\cos 2\phi}_{UU}\Big)$$

Classification of structure functions helpful for data analysis

→ *Parton model*: 24 leading twist structure functions

Diphoton production



Two highly energetic real photons produced with

$$q \equiv q_a + q_b$$

$$\frac{d\sigma}{d^4q_a\, d^4q_b} = \frac{d\sigma}{d^4q\, d^4q_a} \propto \frac{\delta^+(q_a^2)\delta^+((q-q_a)^2)}{2\times 4F} \sum_X |M|^2 \delta^{(4)}(P_a + P_b - q - P_X)$$

Convenient choice: Diphoton rest frame $\rightarrow$ **Collins-Soper frame**

$$\frac{d^6\sigma}{dy\, dQ^2\, d^2\vec{q}_T\, d\Omega_a}$$

Unfortunately: No separation into ***hadronic – photonic*** parts possible!
$\rightarrow$ *all* angular modulations are allowed, in principle.

$$\frac{d^6\sigma}{dy\, dQ^2\, d^2\vec{q}_T\, d\Omega_a} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} C_{lm}(y, Q^2, q_T^2)\, Y_{lm}(\Omega_a)$$

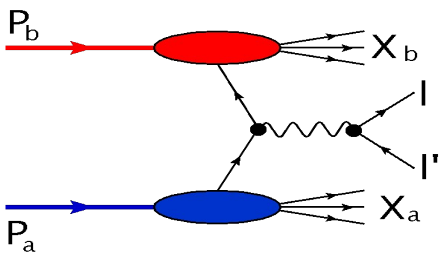
$$C_{00} = \frac{d^4\sigma}{dy\, dQ^2\, d^2q_T}, \ldots$$

However, we can calculate the cross section in the parton model.

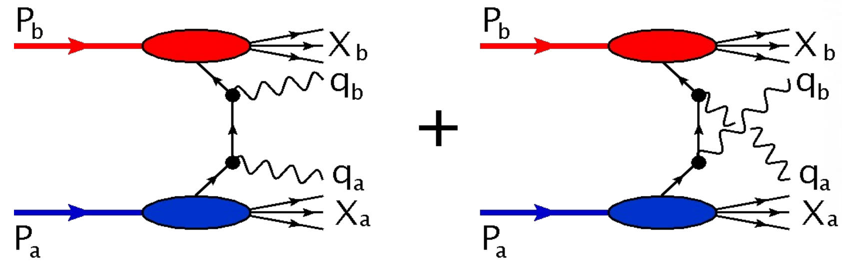
TMD tree-level formalism

Parton model tree-level at O(α_s^0):

Drell-Yan dilepton production:



Diphoton production:



Only relevant at very small q_T: $\Lambda_{QCD} \sim q_T \ll Q$

$$\left(\frac{d\sigma}{d^4q\, d\Omega}\right) \propto \int d^2k_{aT} \int d^2k_{bT}\, \delta^{(2)}(\vec{k}_{aT} + \vec{k}_{bT} - \vec{q}_T)\, \mathrm{Tr}\left[\Phi(x_a, \vec{k}_{aT})\, H(x_a, x_b, q_a, q_b)\, \bar{\Phi}(x_b, \vec{k}_{bT})\, H^\dagger\right] + \mathcal{O}(\tfrac{M}{Q})$$

k_T - correlator: $$\Phi_{ij}(x, \vec{k}_T) = \int \frac{dz^- d^2z_T}{(2\pi)^2}\, \mathrm{e}^{ik\cdot z} \langle P, S|\, \bar{q}_j(0)\, \mathcal{W}^{?/DY}[0\,;\, z]\, q_j(z)\, |P, S\rangle\Big|_{z^+=0}$$

→ can be parameterized in terms of TMDs according to quark / nucleon spin

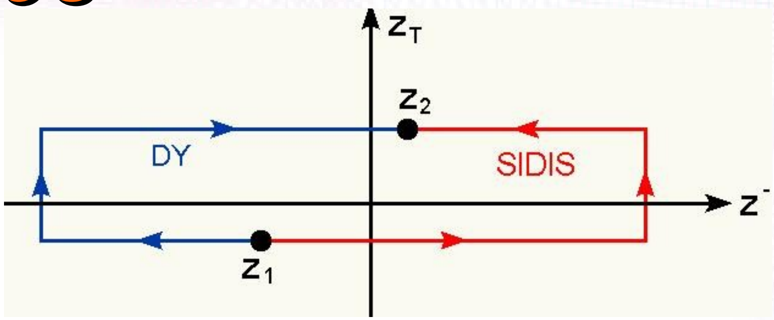
Main result of the TMD tree-level formalism:

$$\left(\frac{d^6\sigma^{hh\to\gamma\gamma X}}{dy\, dQ^2\, d^2q_T\, d\Omega}\right)(\Lambda \sim q_T \ll Q) = \frac{2}{\sin^2\theta}\left(\frac{d\sigma^{hh\to l^+l^- X}}{dy\, dQ^2\, d^2q_T\, d\Omega}\right)(\Lambda \sim q_T \ll Q \,|\, e_q \to e_q^2)$$

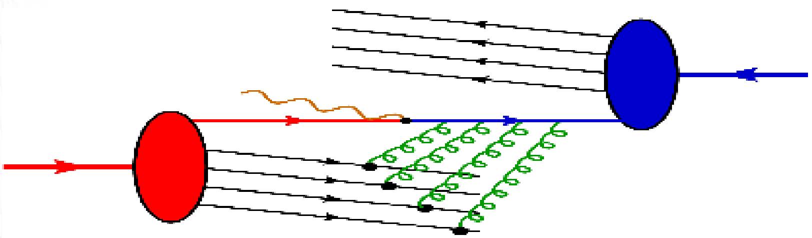
Wilson lines

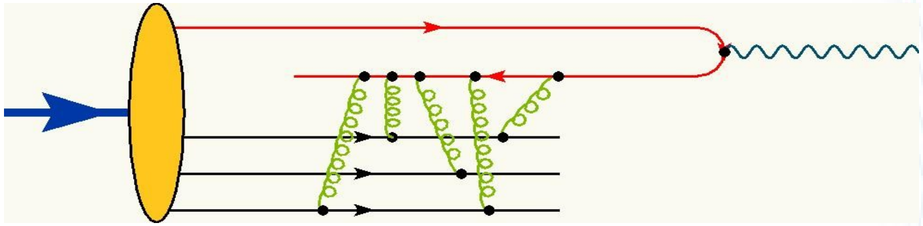
Wilson line process-dependent in DY/SIDIS:

$$\mathcal{W}[z_1; z_2] = \mathcal{P}e^{-ig\int_{z_1}^{z_2} ds\cdot A(s)}$$

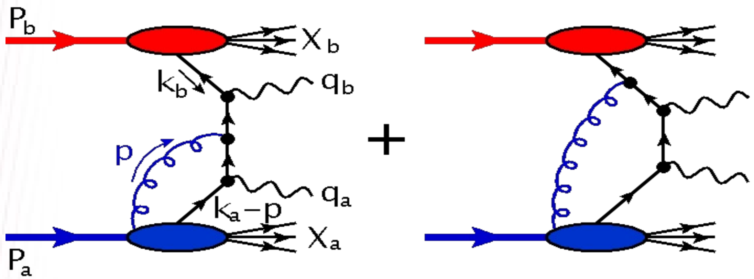


Physics: Initial / Final state interactions





Wilson line in diphoton production:



+crossed

Check for A^+, $A_T^i(z^- = -\infty)$

Diagrams topologically different to DY, *but* cancellations between diagrams

$$\mathcal{W}^{\gamma\gamma}[0;z]\Big|_{z^+=0} = 1 - ig\int_0^{-\infty} d\lambda\, A^+(\lambda n) - ig\int_{0_T}^{z_T} d\vec{y}_T\cdot\vec{A}_T(-\infty,0,\vec{y}_T) - ig\int_{-\infty}^{0} d\lambda\, A^+(\lambda n + z_T) + \mathcal{O}(g^2) = \mathcal{W}^{DY}[0;z]\Big|_{z^+=0}$$

Example: Sivers effect

k_T – correlator for unpolarized quarks:

$$\frac{1}{2}\mathrm{Tr}[\Phi(x,\vec{k}_T)\,\gamma^+] = f_1(x,\vec{k}_T^2) - \frac{\epsilon_T^{ij}\,k_T^i\,S_T^j}{M} f_{1T}^{\perp}(x,\vec{k}_T^2)$$

Sivers function → time-reversal odd → sign switch:

$$f_{1T}^{\perp}\Big|_{DIS} = -f_{1T}^{\perp}\Big|_{DY}$$

Can be determined from SIDIS data of a transverse target SSA (HERMES, COMPASS):

$$A_{UT}^{Siv} \sim \frac{f_{1T}^{\perp}\otimes D_1}{f_1\otimes D_1}$$

k_T – deconvolution through Gaussian ansatz

$$f(x,\vec{k}_T^2)\;=\;f(x)\exp\left[-\vec{k}_T^2/\langle k_T^2\rangle\right]$$

<u>Fit of the Sivers function to data:</u>

[Anselmino et al., EPJA 39, 89],
[Schweitzer, M.S., 0805.2137 and in prep.]

$$f_{1T}^{\perp,q}(x)\;=\;A^q\,f_1^q(x)$$

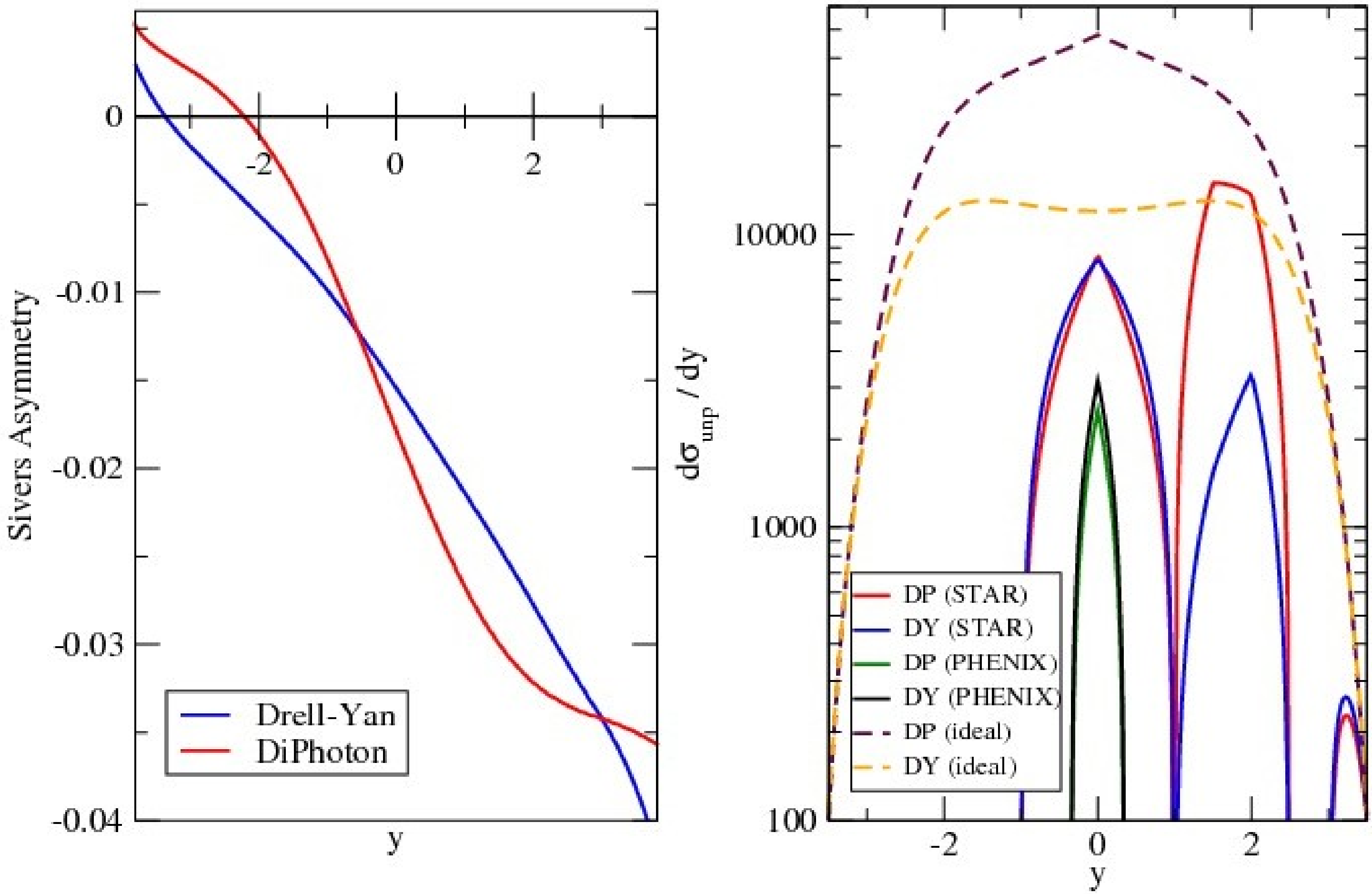
- non-zero sea-quark Sivers functions.
- Take into account finalized HERMES proton data and COMPASS deuteron data
- small statistical errors, χ^2/ d.o.f. ~ 1.07

<u>Sivers effect in Diphoton/DY process:</u>

$$A_{TU}^{Siv,DP/DY} \sim \frac{2/\sin^2\theta\; f_{1T}^{\perp}\otimes\bar{f}_1}{2/\sin^2\theta\; f_1\otimes\bar{f}_1}$$

Sivers Asymmetry vs. pair rapidity $y = (\eta_a + \eta_b)/2$

Bins[GeV]: $4 < Q < 10$, $0 < q_T < 1$, $|\eta| < 1$ and $3 < \eta < 4$ (STAR), $|\eta| < 0.35$ (PHENIX)



Sivers Asymmetry roughly equal in DY and DP

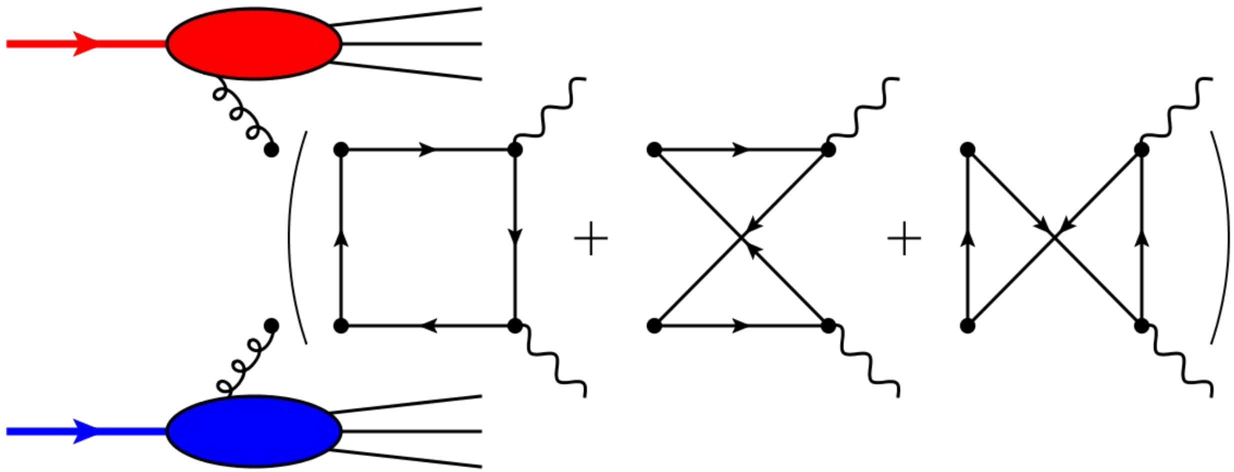
$e_q^2 \to e_q^4$ u-quark dominance

Rapidity enhancement seems to cancel in SSA

Photon pair production rate
up to 5 - 10 times larger
for large rapidity differences

Gluon TMDs in diphoton production

Unique feature of diphoton production → direct sensitivity to gluon TMDs at $O(\alpha_s^2)$



- Current conservation → "boxes" are IR – and UV-finite → effectively "tree-level"
- Large gluon distribution at smaller x compensates α_s^2 suppression → competing process to quark – antiquark generated diphotons
- Polarized gluon TMDs at smaller x → possible contributions feasible at RHIC
- Interaction of two gluons generates new azimuthal asymmetries that are absent for quark – antiquark scattering → e.g., $\cos(4\phi)$ asymmetry in unpol. scattering

Gluon TMD Correlator:

$$\Gamma_{\mu\nu;\lambda\eta}(x,\vec{k}_T) = \frac{1}{xP^+}\int\frac{dz^- d^2z_T}{(2\pi)^2}\,\mathrm{e}^{ik\cdot z}\langle P,S|\,F^{\alpha}_{\mu\nu}(0)\,\mathcal{W}^{\alpha\beta}[0;\,z]\,F^{\beta}_{\lambda\eta}(z)\,|P,S\rangle\Big|_{z^+=0}$$

Gluon TMDs:

unpolarized hadron:

$$\Gamma_U^{+i;+j}(x,\vec{k}_T) = \frac{\delta^{ij}}{2}\,f_1^g(x,\vec{k}_T^2) + \frac{k_T^i k_T^j - \frac{1}{2}\vec{k}_T^2\delta^{ij}}{2M^2}\,h_1^{\perp g}(x,\vec{k}_T^2)$$

long. pol. hadron:

$$\Gamma_L^{+i;+j}(x,\vec{k}_T) = S_L\frac{i\epsilon_T^{ij}}{2}\,g_1^g(x,\vec{k}_T^2) + S_L\frac{k_T^i\epsilon_T^{jk}k_T^k + (i\leftrightarrow j)}{4M^2}\,h_{1L}^{\perp g}(x,\vec{k}_T^2)$$

transv. pol. hadron:

$$\Gamma_T^{+i;+j}(x,\vec{k}_T) = -\frac{\delta^{ij}}{2}\frac{k_T\times S_T}{M}\,f_{1T}^{\perp g}(x,\vec{k}_T^2) + \frac{i\epsilon_T^{ij}}{2}\frac{\vec{k}_T\cdot\vec{S}_T}{M}\,g_{1T}^{\perp g}(x,\vec{k}_T^2)$$

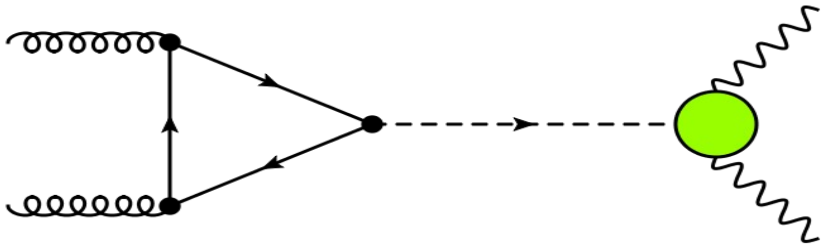
$$+\frac{\epsilon_T^{ik}\left(S_T^j k_T^k + k_T^j S_T^k\right) + (i\to j)}{8M}\,h_{1T}^g(x,\vec{k}_T^2) + \frac{k_T^i\epsilon_T^{jk}k_T^k + (i\leftrightarrow j)}{4M^2}\frac{\vec{k}_T\cdot\vec{S}_T}{M}\,h_{1T}^{\perp g}(x,\vec{k}_T^2)$$

	$\Phi^{[even]}(x,p_T)$		$\Phi^{[odd]}(x,p_T)$	
	even	odd	even	odd
U	f_1			$h_1^\perp$
L	g_{1L}	$h_{1L}^\perp$		
T	g_{1T}	h_{1T} $h_{1T}^\perp$	$f_{1T}^\perp$	

	$\Phi^{g[even]}(x,p_T)$		$\Phi^{g[odd]}(x,p_T)$	
		flip		flip
U	f_1^g	$h_1^{\perp g}$		
L	g_{1L}^g			$h_{1L}^{\perp g}$
T	g_{1T}^g		$f_{1T}^{\perp g}$	h_{1T}^g $h_{1T}^{\perp g}$

Gluon TMDs at the LHC:

Diphoton production → important process for Higgs production at LHC



→ Background process: diphoton production via quark-box → gluon TMDs feasible

Unpolarized gluon-gluon cross section ($q_T << Q$):

$$\frac{d\sigma_{UU}}{d^4q\, d\Omega} \sim \left(\frac{\alpha_s}{2\pi}\right)^2 \left(\mathcal{F}_1(\theta)[f_1^g \otimes f_1^g] + \cos(2\phi)\mathcal{F}_2(\theta)[h_1^{\perp g} \otimes f_1^g + f_1^g \otimes h_1^{\perp g}] + \cos(4\phi)\mathcal{F}_3(\theta)[h_1^{\perp g} \otimes h_1^{\perp g}]\right)$$

$\mathcal{F}_i$: non-trivial functions of sin(θ) and cos(θ) (Logarithms)

Factor α_s^2 compensated by (possibly) large unpol. and Boer-Mulders gluon TMDs

cos(4ϕ) induced by gluon Bour-Mulders functions only, no corresponding DY term.

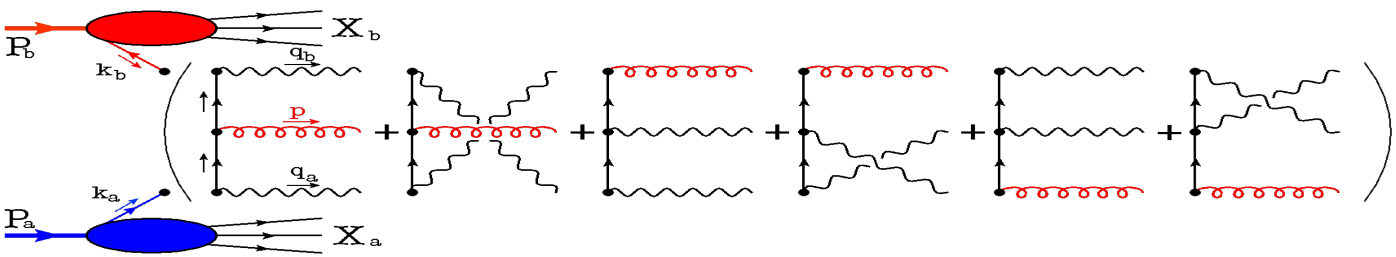
Polarized collisions (RHIC 500GeV): gluon Sivers function, work in progress...

High - q_T of diphoton production

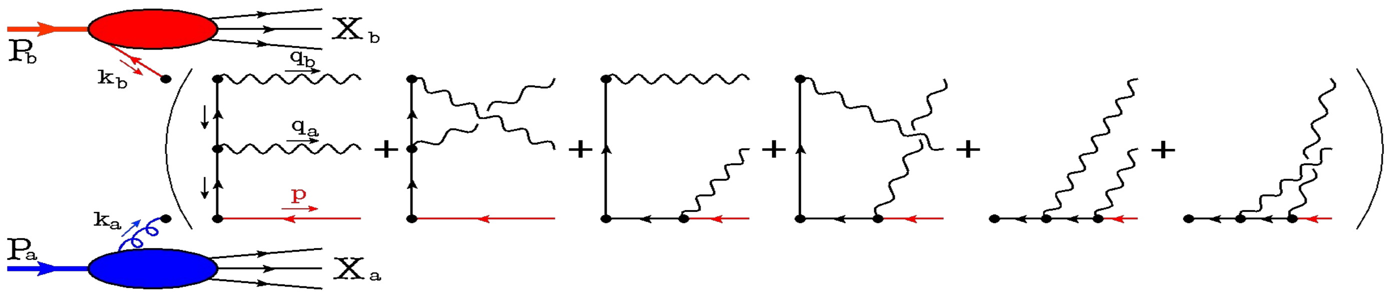
At large $q_T \sim Q$ → transverse momentum generated by gluon radiation

→ collinear parton model calculation

quark – antiquark scattering:



quark – gluon scattering:



However: No model-independent angular decomposition!

Diphoton angles enter the partonic cross section in numerator and denominator
→ All angular depedencies are allowed.

Situation simplifies for smaller q_T → Expansion in $1/q_T$

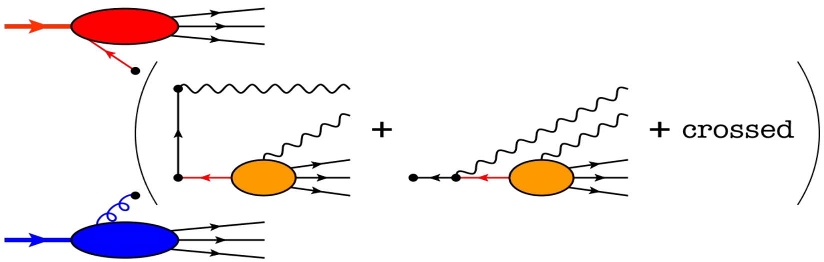
Leading order (Q^2/q_T^2) → "TMD-rule" still applies!

$$\sigma^{DP} = \frac{2}{\sin^2\theta}\sigma^{DY}(e_q \to e_q^2) + \mathcal{O}(1/q_T)$$

Higher orders → "TMD-rule" broken, collinear divergences

Isolation of direct photons

Hide collinear divergence in photon fragmentation function:



- Potentially endangers TMD-factorization
- Photon FF unknown

Circumvent the problem → Isolation [Frixione PLB 429,369; Frixione, Vogelsang NPB 568, 60]

Experimental necessity → diphotons from π^o-decays

Define "cone" in rapidity – azimuthal angle space:

$$\mathcal{C}_\gamma(R_0) \equiv \left\{ (\eta,\phi) \,|\, \sqrt{(\eta-\eta_\gamma)^2+(\phi-\phi_\gamma)^2} \leq R_0 \right\}$$

1. "Traditional" Criterium: allow certain percentage of hadronic energy inside the cone

$$E_T(R_0) \leq \epsilon q_{T\gamma}$$

- Boost-invariant criterium.
- Infra-red safe.
- Allows certain contribution from fragmentation photons.

2. "Improved" Criterium: dynamically generated cone $R < R_0$

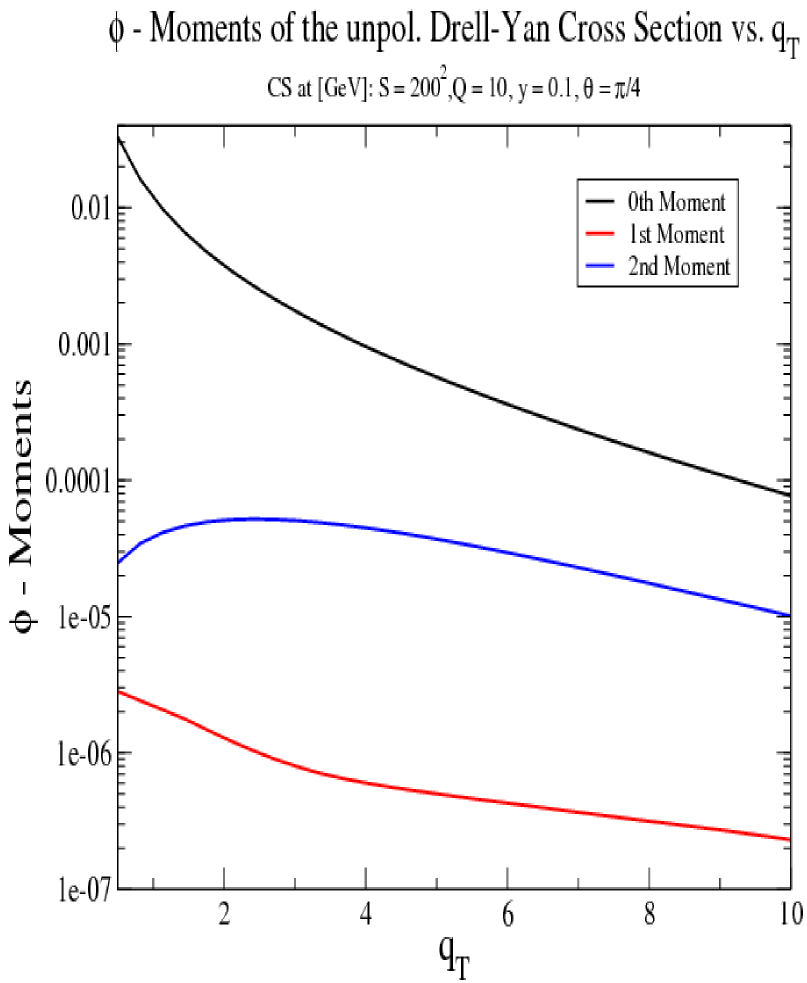
$$E_T(R) \leq \epsilon_\gamma \, q_{T\gamma} \, f(R)$$

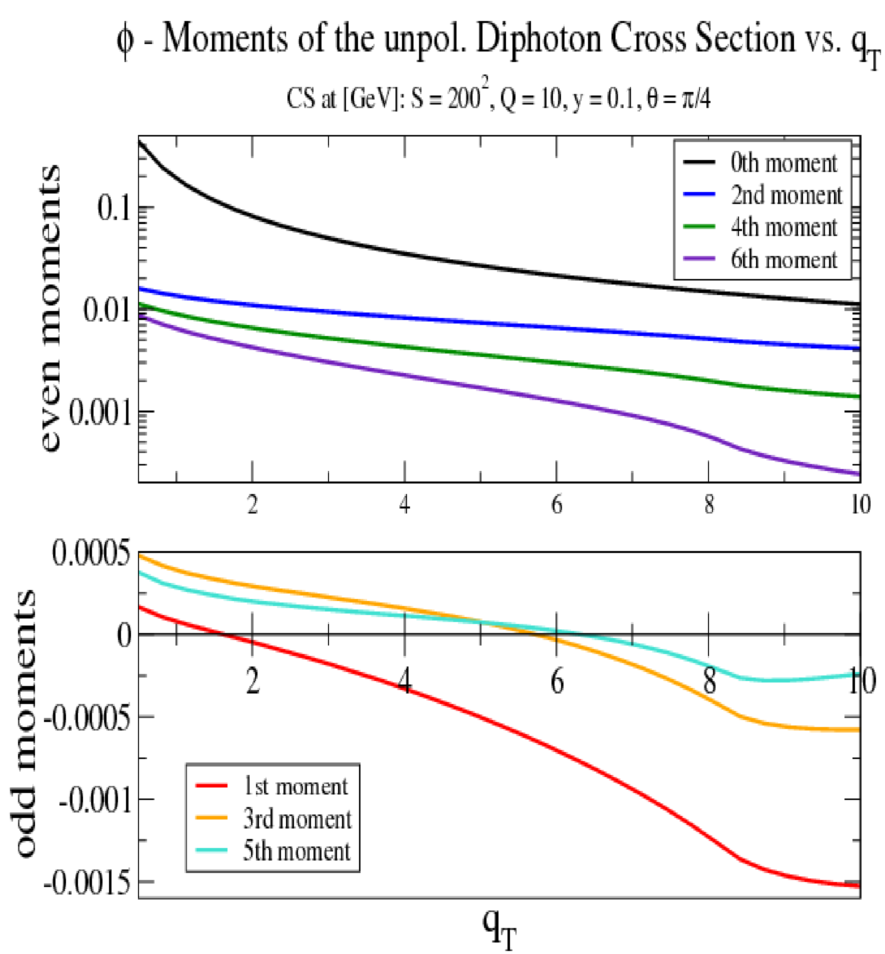
$$\lim_{R\to 0} f(R) = 0$$

- Boost-invariant criterium.
- Infra-red safe.
- Cuts out *all* fragmentation photons.
- Experimentally harder→ needs high resolution in η and ϕ.

Define phi moments:

$$\langle \cos(n\phi) \rangle = \int_0^{2\pi} d\phi \cos(n\phi) \frac{d\sigma}{dy\, dQ^2\, d^2 q_T\, d\Omega}$$

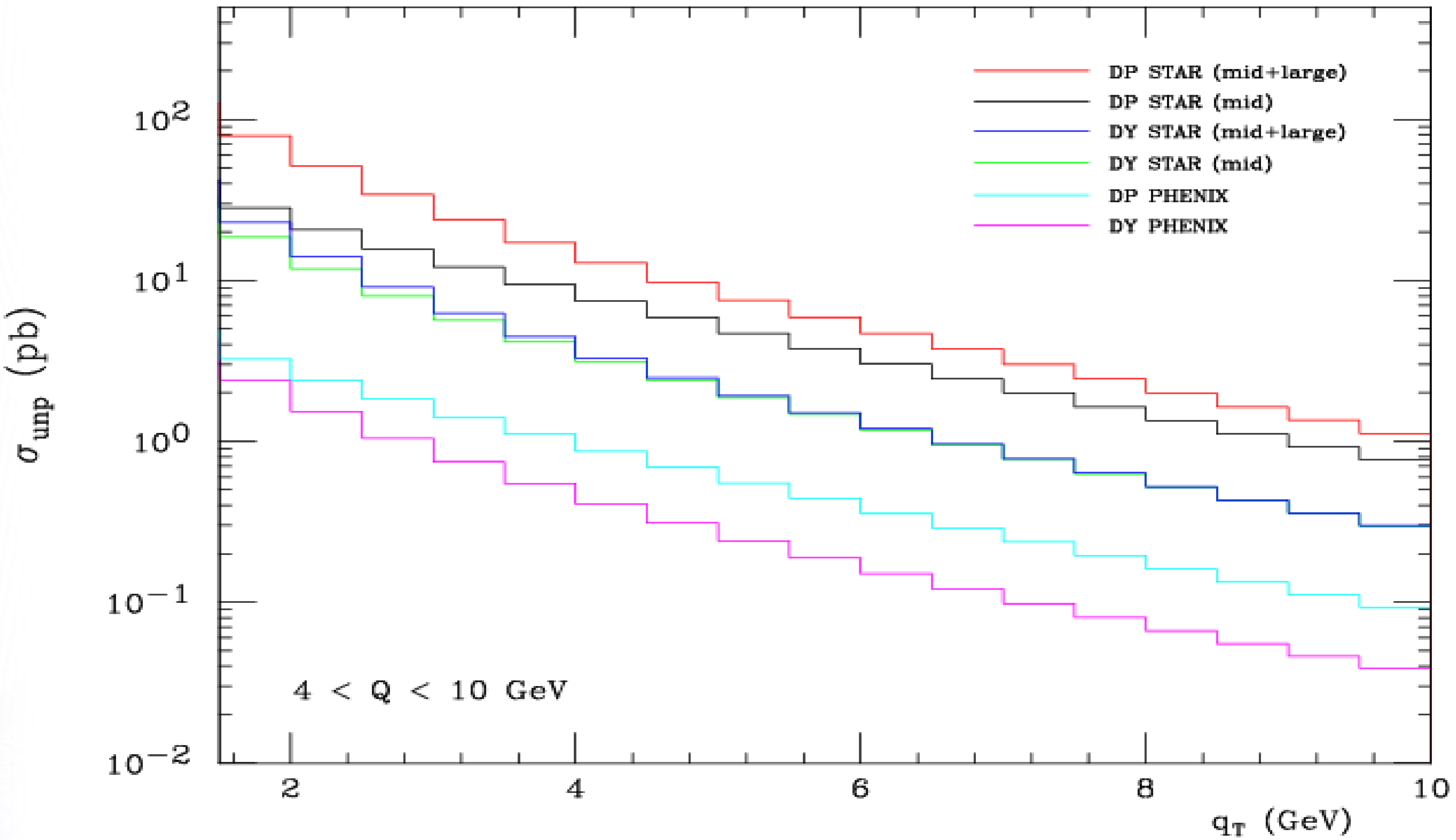




Numerical results

Predictions for the q_T-tail
for STAR and PHENIX including isolation:

$$\sigma_{\text{unp}} = \int_{\text{cuts}} dy\, dQ^2\, dq_T\, d\varphi_q\, d\Omega\, \frac{d\sigma}{dy\, dQ^2\, dq_T\, d\varphi_q\, d\Omega}$$



Also at larger qT → Diphoton production rate about 5 - 10 times larger than Drell-Yan

Summary:

- Drell-Yan cross section can be decomposed model-independently into angular structure function, not possible for photon pair production
- TMD-factorization at low q_T: Photon pair production similar to Drell-Yan
- Sivers effect similar in Photon pair production, but higher production rate → simultaneous measurement
- Photon pair production directly sensitive to Gluon TMDs via quark box → high energy experiments (LHC, RHIC)
- Collinear factorization at larger q_T: all azimuthal modulations possible for photon pair production in contrast to lepton pair production
- Expansion to smaller q_T: Azimuthal behaviour partly recovered → photon fragmentation or Isolation needed.