Extraction of GPDs from DVCS off unpolarized target

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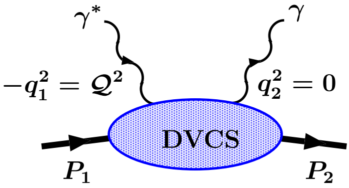
Outline

Probing the proton with two photons

- Deeply virtual Compton scattering (DVCS) [Müller '92, et al. '94]



$$t = (P_2 - P_1)^2\,, \quad q = (q_1 + q_2)/2$$

Generalized Bjorken limit:

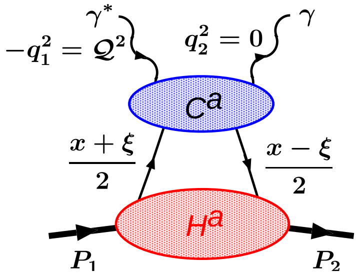
$$-q^2 \simeq \mathcal{Q}^2/2 \to \infty$$

$$\xi = \frac{-q^2}{2P\cdot q} \to \text{const}$$

- To twist-two accuracy (and neglecting gluon transversity) cross-section can be expressed in terms of four Compton form factors (CFF)

$$\mathcal{H}(\xi, t, \mathcal{Q}^2),\ \mathcal{E}(\xi, t, \mathcal{Q}^2),\ \tilde{\mathcal{H}}(\xi, t, \mathcal{Q}^2),\ \tilde{\mathcal{E}}(\xi, t, \mathcal{Q}^2).$$

Factorization of CFFs $\longrightarrow$ GPDs



- Compton form factor is a convolution:

$$^a\mathcal{H}(\xi, t, \mathcal{Q}^2) = \int \mathrm{d}x \; C^a(x, \xi, \mathcal{Q}^2/\mu^2) \; H^a(x, \eta = \xi, t, \mu^2)$$

a=NS,S(Σ,G)

- $H^a(x, \eta, t, \mu^2)$ — Generalized parton distribution (GPD)

Model-dependent extraction of GPDs

$$ {}^{a}\mathcal{H}(\xi, t, \mathcal{Q}^2) = \int \mathrm{d}x \; C^a(x, \xi, \mathcal{Q}^2/\mu^2) \; H^a(x, \eta = \xi, t, \mu^2) $$

- convolution is not generally invertible so we can extract GPDs only by modelling them and comparing to experiment

Our present model (details on the next few slides):

- We model sea quark and gluon GPDs in conformal moment space, we expand them in t-channel SO(3) partial waves, and take into account LO QCD evolution.
- We model valence quark GPDs by parametrizing them on $\eta = x$ trajectory, which at LO gives $\Im\mathrm{m}\mathcal{H}$ directly, and $\Re\mathrm{e}\mathcal{H}$ via dispersion relations. Here we ignore evolution which is probably negligible in kinematic region where valence quarks are not.

Modelling GPDs in moment space

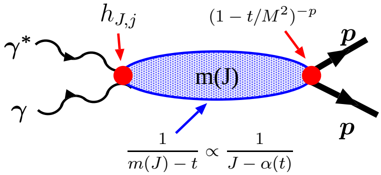
- Instead of considering momentum fraction dependence $H(x, \ldots)$
- …it is convenient to make a transform into complementary space of conformal moments j:

$$H_j^q(\eta, \ldots) \equiv \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} \mathrm{d}x \, \eta^j \, C_j^{3/2}(x/\eta) \, H^q(x, \eta, \ldots)$$

- H_j do not mix under evolution at LO
- Measurable on the lattice [M. Goeckeler's talk]
- For integer j, they are even polynomials in η
- In our case, they are continued to complex j to allow OPE series summation

Modelling conformal moments of GPDs

- $H_j(t)$ are modelled by SO(3) partial wave decomposition of t-channel $\gamma^*\gamma$ scattering



$$H_j(\eta, t) = \sum_J^{j+1} h_{J,j} \frac{1}{J-\alpha(t)} \frac{1}{\left(1-\frac{t}{M^2(J)}\right)^p} \eta^{j+1-J} d^J_{0,\nu}(\eta)$$

- $d^J_{0,\nu}(\eta)$ — Wigner SO(3) functions (Legendre, Gegenbauer,...)

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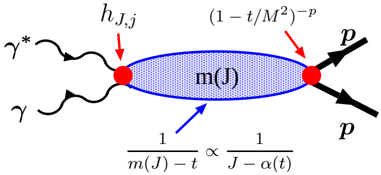

$$H_j(\eta, t) = \sum_J^{j+1} h_{J,j} \frac{1}{J - \alpha(t)} \frac{1}{\left(1 - \frac{t}{M^2(J)}\right)^p} \eta^{j+1-J} d^J_{0,\nu}(\eta)$$

- $d^J_{0,\nu}(\eta)$ — Wigner SO(3) functions (Legendre, Gegenbauer,...)
- Similar to "dual" parametrization [Polyakov, Shuvaev '02; K. Semenov's talk]

Leading partial wave (PW) model

- Taking just a leading partial wave $J = j + 1$ gives ansatz:

$$\mathbf{H}_j(\eta, t, \mu_0^2) = \begin{pmatrix} N'_\Sigma F_\Sigma(t)\, \mathrm{B}\big(1 + j - \alpha_\Sigma(0), 8\big) \\ N'_G F_G(t)\, \mathrm{B}\big(1 + j - \alpha_G(0), 6\big) \end{pmatrix}$$

$$\alpha_a(t) = \alpha_a(0) + 0.15t \qquad F_a(t) = \frac{j + 1 - \alpha(0)}{j + 1 - \alpha(t)} \left(1 - \frac{t}{M_0^{a2}}\right)^{-p_a}$$

...corresponding in forward case to PDFs of form

$$\Sigma(x) = N'_\Sigma\, x^{-\alpha_\Sigma(0)}\, (1 - x)^7 \; ; \qquad G(x) = N'_G\, x^{-\alpha_G(0)}\, (1 - x)^5$$

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- $M_0^G = \sqrt{0.7}\,\mathrm{GeV}$ is fixed by the J/ψ production data
- Single free parameter: M_0^Σ (after DIS F_2 fit fixes the rest)

For small ξ (small x_{Bj}) valence quarks are less important $\Rightarrow \Sigma \approx$ sea

Subleading PW — flexible models

$$\mathbf{H}_j(\eta,t) = \underbrace{\begin{pmatrix} N'_{\rm sea}\,F_{\rm sea}(t)\,\mathrm{B}\big(1+j-\alpha_{\rm sea}(0),8\big) \\ N'_{\mathrm{G}}\,F_{\mathrm{G}}(t)\,\mathrm{B}\big(1+j-\alpha_{\mathrm{G}}(0),6\big) \end{pmatrix}}_{\text{skewness } r\approx 1.6 \text{ (too large)}} + \underbrace{\begin{pmatrix} \text{subleading partial waves, } \eta\text{-dependence!} \end{pmatrix}}_{\substack{<0 \\ \text{negative skewness}}}$$

- Leading PW model: works only at (N)NLO [K.K., D. Müller and K. Passek-Kumerički '07]
- Leading and second PW: LO fits are fine, but gluon $H^G(x,x,t)$ tends to be negative for small x [K.K. and D. Müller '08]
- Adding third PW: everything fine [K.K. and D. Müller '10]
- Strengths of second and third quark and gluon partial wave are 4 additional model parameters.

Modelling valence GPDs

- Neglecting evolution allows a simple model of GPDs on $\eta = x$ trajectory:

$$H^{\mathrm{val}}(x,x,t) = 1.35\, r \left(\frac{2x}{1+x}\right)^{-\alpha(t)} \left(\frac{1-x}{1+x}\right)^{b} \frac{1}{\left(1 - \frac{1-x}{1+x}\frac{t}{M^2}\right)},$$

$$\tilde{H}^{\mathrm{val}}(x,x,t) = 0.6\, \tilde{r} \left(\frac{2x}{1+x}\right)^{-\alpha(t)} \left(\frac{1-x}{1+x}\right)^{\tilde{b}} \frac{1}{\left(1 - \frac{1-x}{1+x}\frac{t}{\tilde{M}^2}\right)}.$$

- $\alpha(t) = 0.43 + 0.85\, t/\mathrm{GeV}^2$ from $(\rho,\, \omega)$ Regge trajectories; non-Regge t-dependence taken from spectator model [Hwang, Müller '07]
- Six free parameters: $r,\ b,\ M,\ \tilde{r},\ \tilde{b},\ \tilde{M}$
- GPD E kinematically suppressed and neglected
- GPD $\tilde{E}$ modelled by pion pole contribution (2 parameters)

Usage of dispersion relations

[Teryaev '05; K.K., Müller and Passek-K. '07, '08; Diehl and Ivanov '07]

- LO perturbative prediction is "handbag" amplitude

$$\mathcal{H}(\xi, t, \mathcal{Q}^2) \stackrel{\text{LO}}{=} \int_{-1}^{1} dx \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H(x, \xi, t, \mathcal{Q}^2)$$

- giving access to GPD on the "cross-over" line $\eta = x$

$$\frac{1}{\pi} \Im\mathrm{m}\mathcal{H}(\xi = x, t, \mathcal{Q}^2) \stackrel{\text{LO}}{=} H(x, x, t, \mathcal{Q}^2) - H(-x, x, t, \mathcal{Q}^2)$$

- while dispersion relation connects it to $\Re\mathrm{e}\mathcal{H}$ and at the most one subtraction constant $\mathcal{C}_{\mathcal{H}} = -\mathcal{C}_{\mathcal{E}}$; $\mathcal{C}_{\tilde{\mathcal{H}}} = \mathcal{C}_{\tilde{\mathcal{E}}} = 0$

$$\Re\mathrm{e}\mathcal{H}(\xi, t, \mathcal{Q}^2) =$$

$$\frac{1}{\pi} \mathrm{PV} \int_{0}^{1} d\xi' \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right) \Im\mathrm{m}\mathcal{H}(\xi', t, \mathcal{Q}^2) + \underbrace{\frac{C}{(1 - t/M_C{}^2)^2}}_{\mathcal{C}_{\mathcal{H}}(t, \mathcal{Q}^2)}$$

Example fit result (preliminary)

- experimental data organized in standardized computer- and human-readable files
- Expressions for observables from [Belitsky, Müller and Kirchner '01, Belitsky and Müller '10]
- MINUIT minimizing routine [James and Roos '75]
- 15 parameter fit to 175 experimental points: $\chi^2/d.o.f = 132/160$

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M02S = 0.51 +- 0.02
SECS = 0.28 +- 0.02
SECG = -2.79 +- 0.12
THIS = -0.13 +- 0.01
THIG = 0.90 +- 0.05
  Mv = 4.00 +- 3.33 (edge)
  rv = 0.62 +- 0.06
  bv = 0.40 +- 0.67
   C = 8.78 +- 0.98
  MC = 0.97 +- 0.11
 tMv = 0.88 +- 0.24
 trv = 7.76 +- 1.39
 tbv = 2.05 +- 0.40
 rpi = 3.54 +- 1.77
 Mpi = 0.73 +- 0.37
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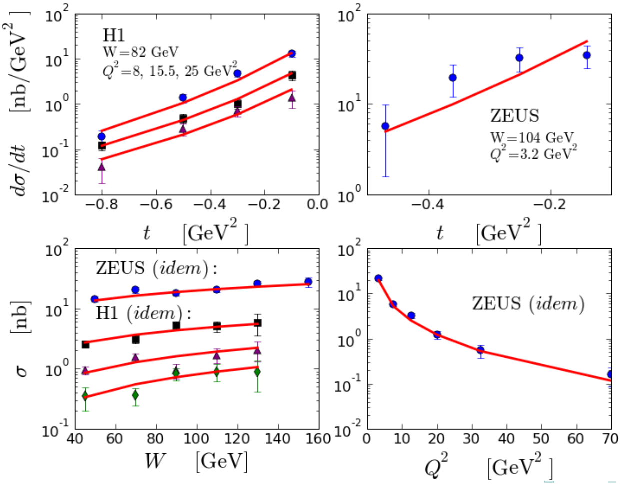
H1 (2007), ZEUS (2008)

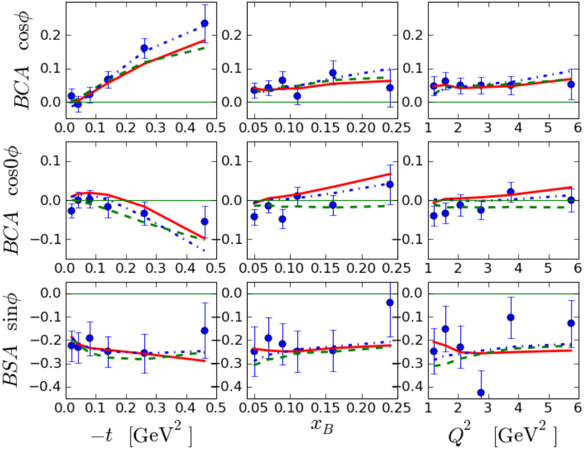
- 107 measurements of $\sigma^{\rm DVCS}$ and $d\sigma^{\rm DVCS}/dt \sim |\mathcal{H}|^2$



HERMES (2008)

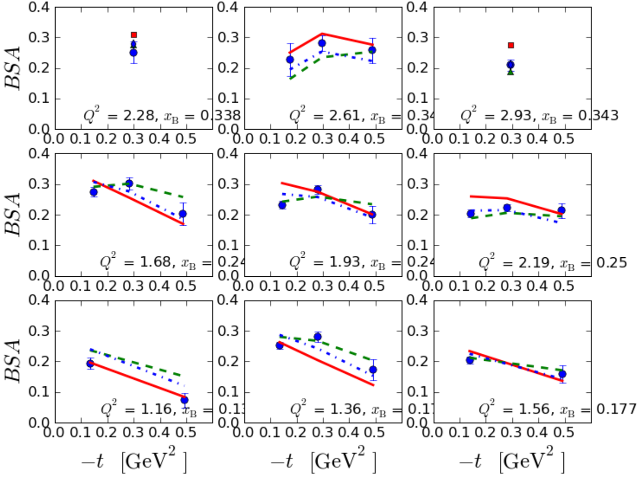
(18 points) $BCA \equiv \frac{\mathrm{d}\sigma_{e^+} - \mathrm{d}\sigma_{e^-}}{\mathrm{d}\sigma_{e^+} + \mathrm{d}\sigma_{e^-}} \sim A_C^{\cos 0\phi} + A_C^{\cos 1\phi} \cos\phi \quad \sim \Re\mathrm{e}\mathcal{H}$

(18 points) $BSA \equiv \frac{\mathrm{d}\sigma_{e\uparrow} - \mathrm{d}\sigma_{e\downarrow}}{\mathrm{d}\sigma_{e\uparrow} + \mathrm{d}\sigma_{e\downarrow}} \sim A_{LU}^{\sin 1\phi} \sin\phi \quad \sim \Im\mathrm{m}\mathcal{H}$



CLAS (2007)

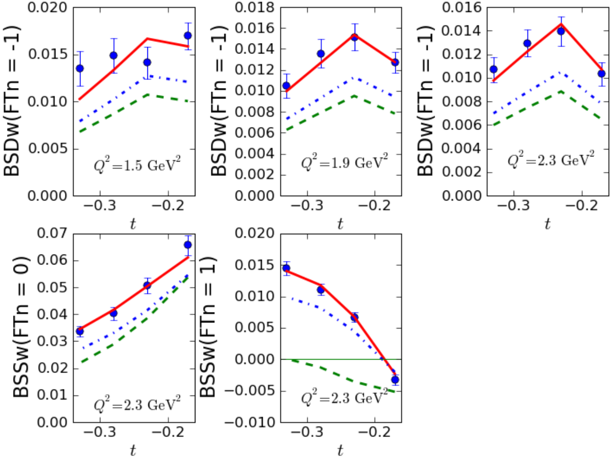
- (12 points, $|t| \le 0.3\,\mathrm{GeV}^2$) $\sin\phi$ harmonics of BSA



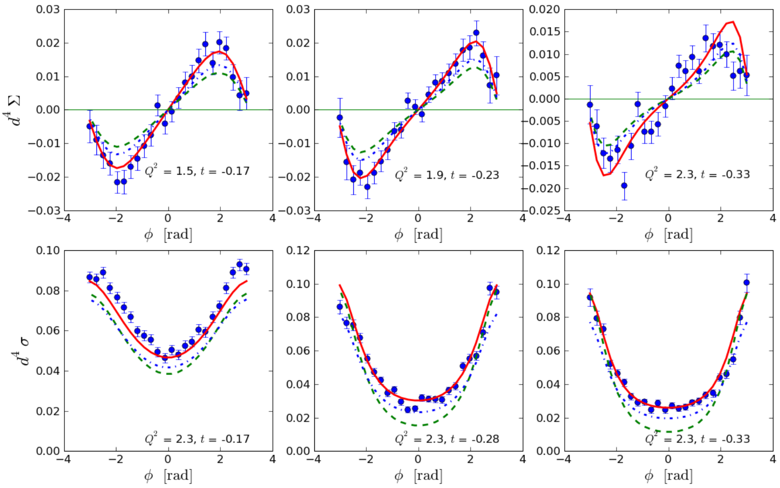
- Some outliers here?!

HALL A (2006) — harmonics

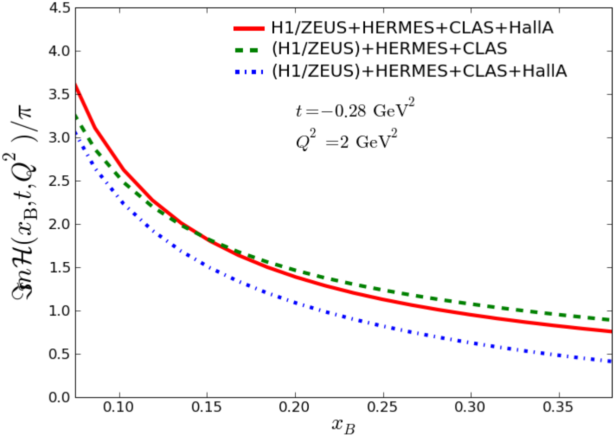
- (12 pts) BSD=$\mathrm{d}\sigma_{e\uparrow} - \mathrm{d}\sigma_{e\downarrow}$ sinϕ harmonic
- (8 pts) BSS=$\mathrm{d}\sigma_{e\uparrow} + \mathrm{d}\sigma_{e\downarrow}$ cos0ϕ and cosϕ harmonics
- Fit is OK only with unusually large $\Re\mathrm{e}\mathcal{T}_{\mathrm{DVCS}}$ ($\rightarrow \tilde{\mathcal{H}}, \tilde{\mathcal{E}}$)



HALL A (2006) — ϕ-dependence

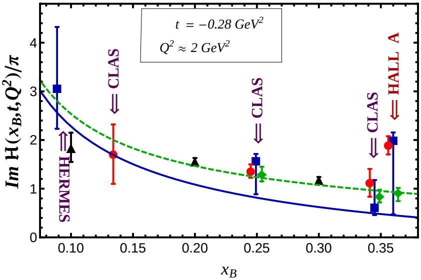


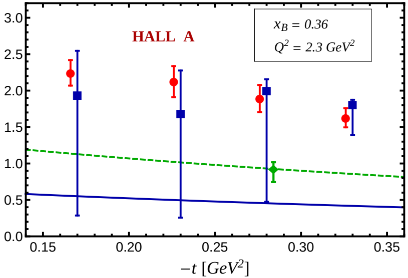
Resulting $\Im m\mathcal{H}(x_{\rm B}, t, Q^2)$



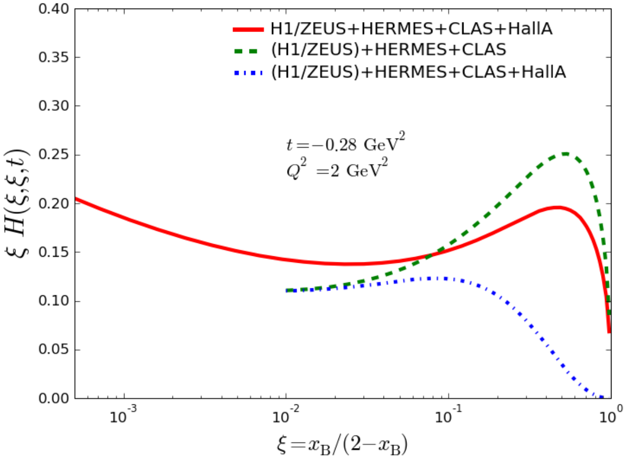
Comparison to other fitting approaches

- We have rough agreement with other fitting approaches (dual-model fit [Moutarde '09], model-independent fits: [Guidal '08], [Guidal and Moutarde '09]), although there are some discrepancies for Hall A data





Resulting $H(x, x, t)$



- Shape of GPD at very large-x still very unknown, but predictions for COMPASS/EIC/JLAB@12 are possible

H1 beam charge asymmetry

$$BCA \equiv \frac{d\sigma_{e^+} - d\sigma_{e^-}}{d\sigma_{e^+} + d\sigma_{e^-}} = \frac{\mathcal{A}_{\text{Interference}}}{|\mathcal{A}_{\text{DVCS}}|^2 + |\mathcal{A}_{\text{BH}}|^2} \overset{\text{LO}}{\propto} F_1 \Re\mathrm{e}\mathcal{H} + \frac{|t|}{4M^2} F_2 \Re\mathrm{e}\mathcal{E}$$

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- Model E_{sea} as $(\mathcal{B}_{\mathrm{sea}}/N_{\mathrm{sea}})H_{\mathrm{sea}}$ and take $\mathcal{B}_{\mathrm{sea}} \equiv \int \mathrm{d}x\, x\, E_{\mathrm{sea}}$ as a parameter

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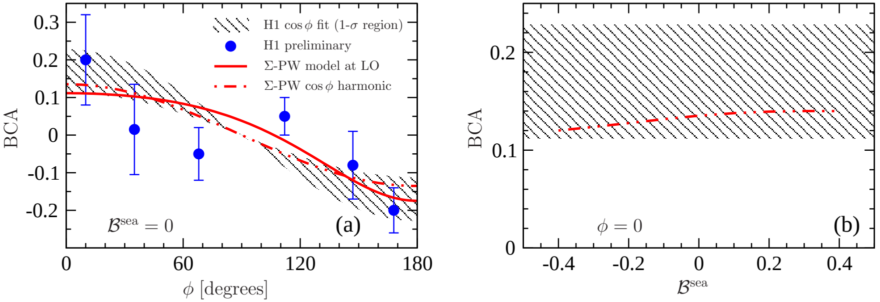
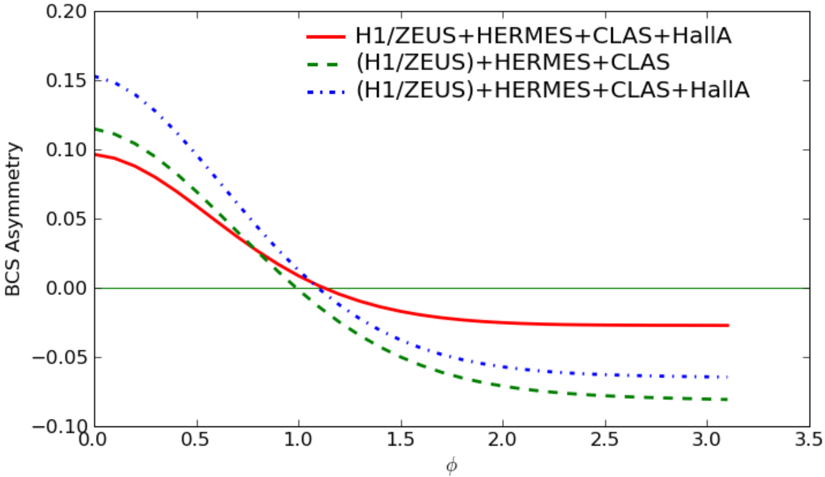


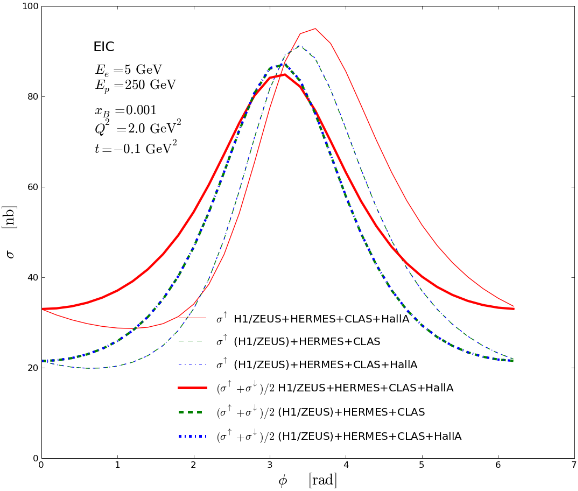
- We cannot extract $\mathcal{B}_{\mathrm{sea}}$ from H1 data

Prediction for COMPASS beam charge-spin asymmetry

$$A_{\mathrm{BCSA}}(\phi) = \frac{d\sigma^{\uparrow+} - d\sigma^{\downarrow-}}{d^{\uparrow+}\sigma + d^{\downarrow-}\sigma}$$



Prediction for EIC cross section



Problems with standard fitting approaches

1. Choice of fitting function introduces theoretical bias leading to systematic error which cannot be estimated
2. Propagation of uncertainties from experiment to fitted function is difficult. (Correlations are usually lost.)

Fitting with neural networkds

1. Neural networks make bias-free interpolation of data
2. Training networks on Monte Carlo replicated data preserves experimental uncertainties and their correlations [Giele et al. '01]

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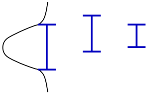

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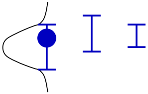

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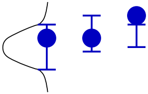

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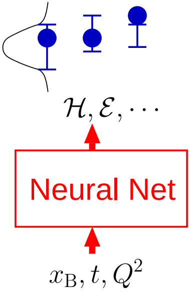

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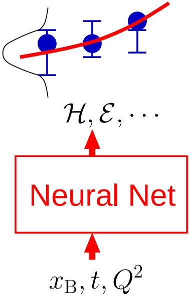

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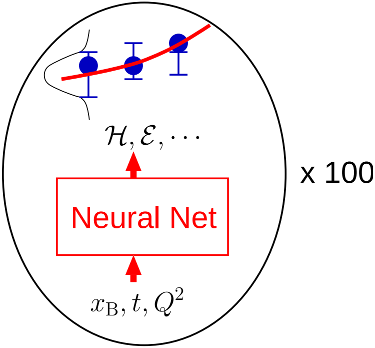

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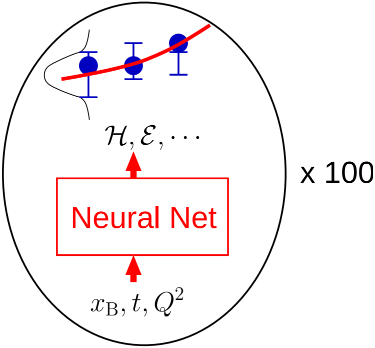
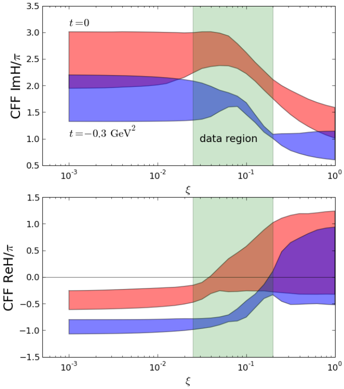


- Already successfully applied to PDF fitting by [NNPDF] group. Has maybe even larger potential in GPD fitting with GPD being less-known function of more variables.

Sample result (preliminary)

- 78 neural nets with neuron architecture 2-16-12-2 trained on CLAS BSA and HERMES BCA



Error bands unreliable at the moment!

- There are other interesting machine-learning approaches to parton structure [S. Liuti et al.]

Summary

- Global fits to unpolarized target H1, ZEUS, HERMES and CLAS data are possible within assumption of GPD H dominance
- cross sections measured by Hall A require additional contributions ($\tilde{H}$ or $\tilde{E}$).
- **For the future**: Inclusion of data on DVCS with polarized target, and on meson production [T. Lautenschlager, K. Passek-Kumerički, et al. work in progress].

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The End

- Probability $P(\chi^2/\mathrm{d.o.f.})$ and parameter values

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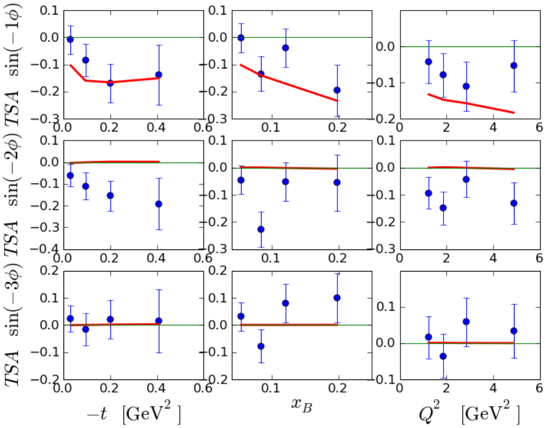
-------------------------   ------------------------   -----------------------------
 [unpolarized target]        [unp. + pol. target]        [unp. + pol. target]
 H1ZEUS+UNP5                 H1ZEUS+UNP5+TSA1            H1ZEUS+UNP5+TSA1 (Q2min=1.6)
 P(131.94, 160) = 0.95       P(196.41, 172) = 0.1        P(168.82, 156) = 0.23
-------------------------   ------------------------   -----------------------------
M02S = 0.51 +- 0.02          M02S = 0.54 +- 0.02          M02S = 0.52 +- 0.02
SECS = 0.28 +- 0.02          SECS = 1.49 +- 0.02          SECS = 0.57 +- 0.03
THIS = -0.13 +- 0.01         THIS = -0.50 +- 0.01         THIS = -0.22 +- 0.01
SECG = -2.79 +- 0.12         SECG = -3.34 +- 0.12         SECG = -3.30 +- 0.18
THIG = 0.90 +- 0.05          THIG = 0.94 +- 0.05          THIG = 1.09 +- 0.09
  Mv = 4.00 +- 3.33 (edge)     Mv = 4.00 +- 3.54  (e)       Mv = 4.00 +- 3.58 (e)
  rv = 0.62 +- 0.06            rv = 1.07 +- 0.04            rv = 1.03 +- 0.04
  bv = 0.40 +- 0.67            bv = 0.40 +- 0.02  (e)       bv = 0.40 +- 0.03 (e)
   C = 8.78 +- 0.98             C = 1.05 +- 0.30             C = 1.23 +- 0.33
  MC = 0.97 +- 0.11            MC = 4.00 +- 3.38  (e)       MC = 4.00 +- 3.36 (e)
 tMv = 0.88 +- 0.24           tMv = 1.32 +- 2.26           tMv = 1.03 +- 0.82
 trv = 7.76 +- 1.39           trv = 0.82 +- 0.19           trv = 0.92 +- 0.23
 tbv = 2.05 +- 0.40           tbv = 0.40 +- 0.16  (e)      tbv = 0.40 +- 0.33 (e)
 rpi = 3.54 +- 1.77           rpi = 3.38 +- 0.16           rpi = 3.38 +- 0.17
 Mpi = 0.73 +- 0.37           Mpi = 4.00 +- 2.33  (e)      Mpi = 4.00 +- 2.35 (e)
-------------------------------------------------------------------------------------
```

- Partial χ^2/npts

```
H1ZEUS:    81.74/107          91.64/107          84.11/107
allUNP:    50.20/68           86.11/68           69.10/54 (cut)
  CLAS:    116.16/22          68.92/22           58.87/15 (cut)
CLASDM:    18.51/12           11.16/12            8.48/8 (cut)
  BSDw:     9.76/12           22.54/12           12.31/8 (cut)
  BSSw:     4.13/8            21.39/8            19.58/8
  TSA1:    608.33/12          18.66/12           15.62/10 (cut)
```

HERMES TSA


$TSA \quad \sin(-1\phi)$
$TSA \quad \sin(-2\phi)$
$TSA \quad \sin(-3\phi)$
$-t \quad [\mathrm{GeV}^2]$
x_B
$Q^2 \quad [\mathrm{GeV}^2]$

Mellin-Barnes representation of CFFs (I)

- Factorization formula for CFFs ...

$$^{S}\mathcal{H}(\xi, \Delta^2, \mathcal{Q}^2) = \int \mathrm{d}x \, \mathbf{C}(x, \xi, \mathcal{Q}^2/\mu^2) \, \mathbf{H}(x, \xi, \Delta^2, \mu^2)$$

- ... is in moment space written as conformal operator product expansion (COPE)

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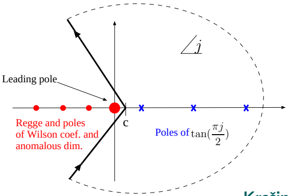
- However, this series converges only for unphysical $\xi > 1$
- To evaluate it for $\xi < 1$ we analytically continue in complex j plane and write the COPE sum as a Mellin-Barnes integral ...

Mellin-Barnes representation of CFFs (II)

- . . . using Sommerfeld-Watson transformation and dispersion relations:

$$^{\mathrm{S}}\mathcal{H}(\xi, \Delta^2, \mathcal{Q}^2) = 2\sum_{j=0}^{\infty} \xi^{-j-1} \mathbf{C}_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu))\, \mathbf{H}_j(\eta, \Delta^2, \mu^2)$$

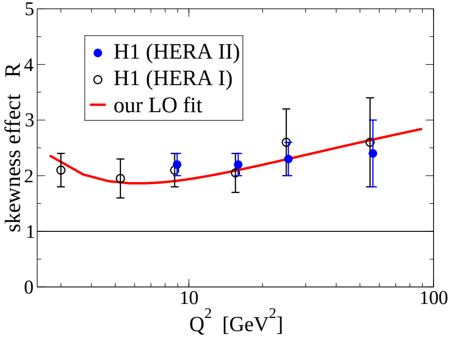
$$= \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj\, \xi^{-j-1}\left[i + \tan\left(\frac{\pi j}{2}\right)\right] \mathbf{C}_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu))\, \mathbf{H}_j(\xi, \Delta^2, \mu^2)$$



Skewness ratio — R

- . . . is discriminating feature of GPD models

$$R \equiv \frac{\mathcal{I}m\, A_{\mathrm{DVCS}}}{\mathcal{I}m\, A_{\mathrm{DIS}}}\bigg|_{t=0} \sim \frac{\sqrt{\sigma_{\mathrm{DVCS}}}}{\sigma_{\mathrm{DIS}}} \overset{LO}{\sim} \frac{H(x,x)}{q(\frac{2x}{1+x},0)}$$

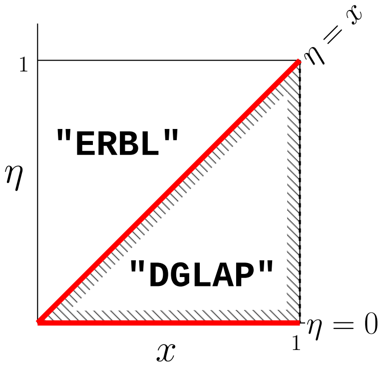


- measurement: $R \approx 2$

Skewness ratio (II) — r

- Skewness ratio is naturally defined by ratio of GPDs $H(x, \eta)$ at two physically relevant trajectories: $\eta = x$ and $\eta = 0$

$$r = \frac{H(x, x)}{H(x, 0)}$$



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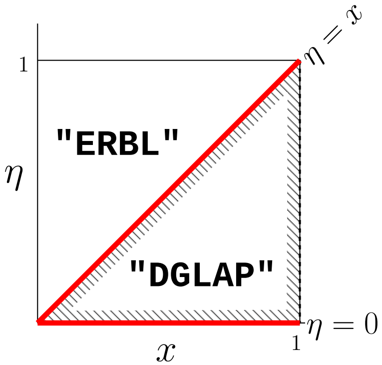

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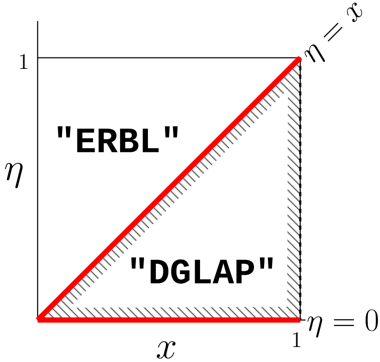

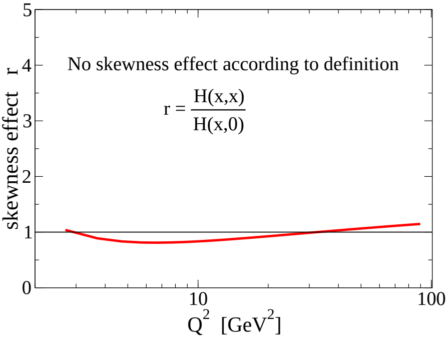

Skewness ratio (III)

- Simple GPD models are usually constrained by "natural" DVCS-to-DIS enhancement factor

  [Shuvaev et al. '99]

$$r = \left.\frac{2^{j+2}\Gamma(j+5/2)}{\sqrt{\pi}\Gamma(j+3)}\right|_{j=\alpha-1\approx 0.2} \approx 1.5$$

- ... and thus fail to reproduce data
- Having correct $r \approx 1$ skewness ratio is an important feature of models that aim to reproduce data at LO.