The Polarized Bjorken Sum Rule: Differential and Integral

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Contents

Introduction

- One goal in analyzing polarized deep-inelastic data is the measurement of the valence quark distributions and of the strong coupling $\alpha_s(M_Z^2)$.
- $\alpha_s(M_Z^2)$ can be accessed also using the Bjorken sum rule. This would be the ‘integral’ method.
- A more advantageous approach consists in the ‘differential’ method, i.e. measuring the flavor non-singlet contributions to $g_1(x, Q^2)$ and extracting $\alpha_s(M_Z^2)$ using non-singlet scale evolution.
- In the following we discuss the theoretical background for this possibility covering the different contributions.
- Recently the complete $O(\alpha_s^2)$ heavy flavor corrections have been calculated and the massless corrections are available effectively at $\mathrm{N}^3\mathrm{LO}$.
- A comprehensive world data analysis reaching this level has not been performed yet and would be rather timely.
- We expect an experimental error for $\alpha_s(M_Z^2) \sim \pm 0.0050$ or better with a remaining very small theory error.

The Data and their Scaling Violations

- Non-singlet combinations can be formed experimentally.

$$\frac{g_1^d(x,Q^2)}{\frac{1}{2}(1-\frac{3}{2}\omega_D)} = g_1^p(x,Q^2) + g_1^n(x,Q^2)$$

consider:

$$\Delta g_1(x,Q^2) = g_1^p(x,Q^2) - g_1^n(x,Q^2)$$

- At LO:

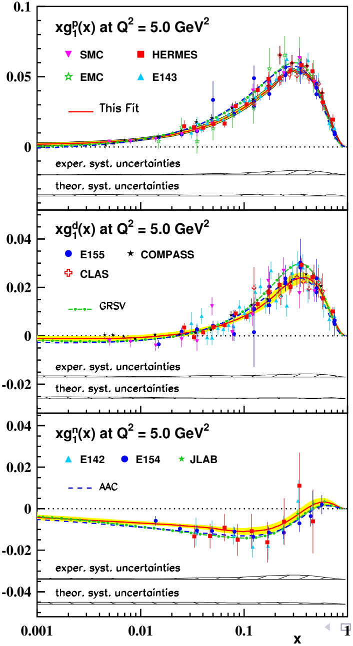
$$\Delta g_1^{\mathrm{NS,LO}}(x,Q^2) = \frac{1}{3}\left[\Delta u_v - \Delta d_v\right] + \frac{2}{3}\left[\Delta \bar{u} - \Delta \bar{d}\right]$$

The difference of the sea-quark densities does not necessarily vanish.

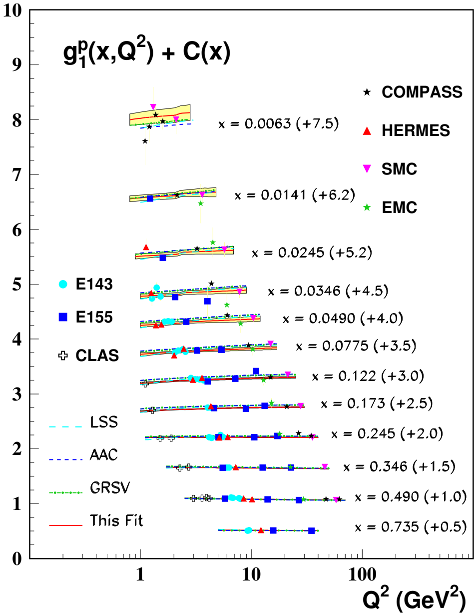
- At HO [in Mellin space]:

$$\Delta g_1^{\mathrm{NS}}(N,Q^2) = \left[1 + \sum_{l=1}^{3} a_s^l C_{g_1}^{\mathrm{NS},(l)}(N)\right] \mathrm{E}^{\mathrm{NS}}(N,Q^2,Q_0^2)\Delta g_1^{\mathrm{NS,LO}}(x,Q_0^2)$$

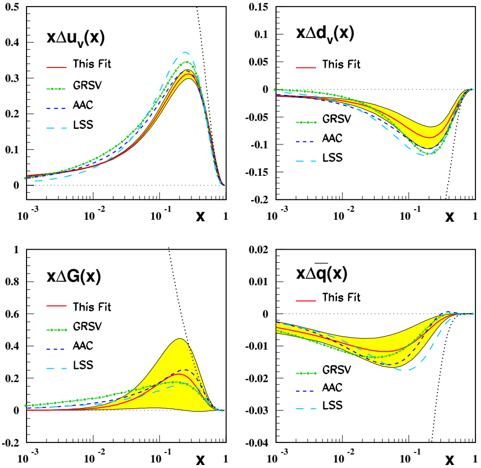
$a_s = \alpha_s/(4\pi)$, $C_{g_1}^{\mathrm{NS},(l)}$ - Wilson coefficients



0.1
0.05
0
$xg_1^p(x)$ at Q^2 = 5.0 GeV^2
SMC
HERMES
EMC
E143
This Fit
exper. syst. uncertainties
theor. syst. uncertainties
0.04
0.02
0
-0.02
$xg_1^d(x)$ at Q^2 = 5.0 GeV^2
E155
COMPASS
CLAS
GRSV
exper. syst. uncertainties
theor. syst. uncertainties
0.04
0.02
0
-0.02
-0.04
$xg_1^n(x)$ at Q^2 = 5.0 GeV^2
E142
E154
JLAB
AAC
exper. syst. uncertainties
theor. syst. uncertainties
0.001
0.01
0.1
1
x


$g_1^p(x,Q^2)$ + C(x)
COMPASS
HERMES
SMC
EMC
E143
E155
CLAS
LSS
AAC
GRSV
This Fit
x = 0.0063 (+7.5)
x = 0.0141 (+6.2)
x = 0.0245 (+5.2)
x = 0.0346 (+4.5)
x = 0.0490 (+4.0)
x = 0.0775 (+3.5)
x = 0.122 (+3.0)
x = 0.173 (+2.5)
x = 0.245 (+2.0)
x = 0.346 (+1.5)
x = 0.490 (+1.0)
x = 0.735 (+0.5)
0
1
2
3
4
5
6
7
8
9
10
1
10
100
Q^2 (GeV^2)

NLO pdfs



BB 2010: Nucl.Phys. B841 (2010) 205 at $Q_0^2 = 4$ GeV^2.

NS Parton Evolution

$$
\begin{aligned}
f^{\rm NS}(N,Q^2) \quad = \quad & f^{\rm NS}(N,Q_0^2)\left(\frac{a}{a_0}\right)^{-\hat{P}_0(N)/\beta_0}\Biggl\{1-\frac{1}{\beta_0}(a-a_0)\left[\hat{P}_1^-(N)-\frac{\beta_1}{\beta_0}\hat{P}_0(N)\right] \\
& -\frac{1}{2\beta_0}\left(a^2-a_0^2\right)\left[\hat{P}_2^-(N)-\frac{\beta_1}{\beta_0}\hat{P}_1^-(N)+\left(\frac{\beta_1^2}{\beta_0^2}-\frac{\beta_2}{\beta_0}\right)\hat{P}_0(N)\right] \\
& +\frac{1}{2\beta_0^2}(a-a_0)^2\left(\hat{P}_1^-(N)-\frac{\beta_1}{\beta_0}\hat{P}_0(N)\right)^2 \\
& -\frac{1}{3\beta_0}\left(a^3-a_0^3\right)\Biggl[\hat{P}_3^-(N)-\frac{\beta_1}{\beta_0}\hat{P}_2^-(N)+\left(\frac{\beta_1^2}{\beta_0^2}-\frac{\beta_2}{\beta_0}\right)\hat{P}_1^-(N) \\
& +\left(\frac{\beta_1^3}{\beta_0^3}-2\frac{\beta_1\beta_2}{\beta_0^2}+\frac{\beta_3}{\beta_0}\right)\hat{P}_0(N)\Biggr]+\frac{1}{2\beta_0^2}(a-a_0)\left(a_0^2-a^2\right) \\
& \times\left(\hat{P}_1^-(N)-\frac{\beta_1}{\beta_0}\hat{P}_0(N)\right)\left[\hat{P}_2^-(N)-\frac{\beta_1}{\beta_0}\hat{P}_1^-(N)-\left(\frac{\beta_1^2}{\beta_0^2}-\frac{\beta_2}{\beta_0}\right)\hat{P}_0(N)\right] \\
& -\frac{1}{6\beta_0^3}(a-a_0)^3\left(\hat{P}_1^-(N)-\frac{\beta_1}{\beta_0}\hat{P}_0(N)\right)^3\Biggr\}.
\end{aligned}
$$

$$f^{\rm NS}(1,Q^2) \quad = \quad f^{\rm NS}(1,Q_0^2),\ \hat{P}_k(1)=0, \forall k,$$ No evolution of the first moment!

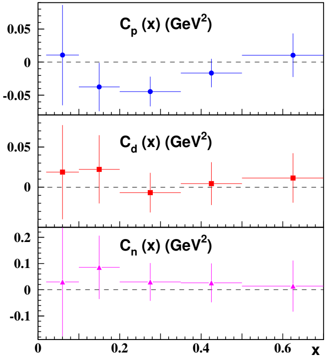
NS Parton Evolution

$$f^{\mathrm{NS}}(N, Q^2) = \mathrm{E}^{\mathrm{NS}}(N, Q^2, Q_0^2) f^{\mathrm{NS}}(N, Q_0^2)$$

- The NS evolution can be performed to 4-loop order.
- Although only a few moments are available for the splitting function, a Padè model works well, and one may associate a $\pm 100\%$ error to it. It's impact is far below possible foreseeable accuracies for $\alpha_s(M_Z^2)$.
- The essential corrections come form the Wilson coefficients.
- Massless case: 3 Loop Order Vermaseren et al. 2005
- Massive case: 2 Loop Order Blümlein et al. 2015

Higher Twist

Important to determine within a correlated fit.

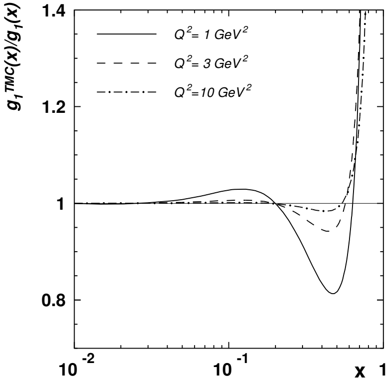


$$g_1(x, Q^2) = g_1^{\mathrm{LT}}(x, Q^2) + \frac{C_i(x, \langle Q^2 \rangle)}{Q^2}$$

No possibility to remove these effects by cuts, given the present World data.

The Target Mass Corrections

- Mandatory corrections to be carried out. A. Piccione and G. Ridolfi, Nucl.Phys. B513 (1998) 301; J. Blümlein and A. Tkabladze, Nucl. Phys. B443 (1999) 427.



Available in *x* and *N* space.

The Advantage of the Differential Method

- Data points have not to be moved, but are fitted in situ.
- No real extrapolation assumptions, in particular not at small x.
- The analysis can be carried out to N^{3}LO for the massless corrections.
- In case of the massive corrections the exact $O(\alpha_s^2)$ corrections are available.

The Bjorken Sum Rule: the Integral Method

J.D. Bjorken, Phys. Rev. D **1** (1970) 1376

$$\int_0^1 dx \left[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)\right] = \frac{1}{6}\left|\frac{g_A}{g_V}\right| C_{\mathrm{pBJ}}(\hat{a}_s),$$

with $g_{A,V}$ the neutron decay constants, $g_A/g_V \approx -1.2767 \pm 0.0016$ and $\hat{a}_s = \alpha_s/\pi$.

Massless case:

$$C_{\mathrm{pBJ}}(\hat{a}_s) = 1 + \sum_{k=1}^{4} \hat{a}_s^k C_k(N_F)\ .$$

1-loop J. Kodaira, S. Matsuda, T. Muta, K. Sasaki and T. Uematsu, Phys. Rev. D **20** (1979) 627
2-loop S.G. Gorishnii and S.A. Larin, Phys. Lett. B **172** (1986) 109
3-loop S.A. Larin and J.A.M. Vermaseren, Phys. Lett. B **259** (1991) 345
4-loop NS P.A. Baikov, K.G. Chetyrkin and J.H. Kühn, Phys. Rev. Lett. **104** (2010) 132004
4-loop SI P.A. Baikov, K.G. Chetyrkin and J.H. Kühn, Nucl. Part. Phys. Proc. **261-262** (2015) 3; S.A. Larin, Phys. Lett. B **723** (2013) 348

The Bjorken Sum Rule

SU(3) non-singlet contributions:

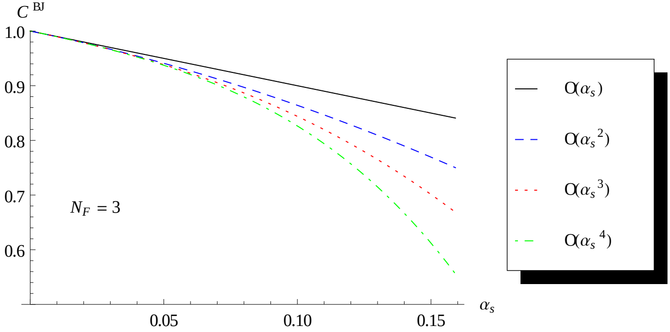
$$\begin{aligned}
C^{\mathrm{NS}}_{\mathrm{pBJ}} &= 1 - \hat{a}_s + \hat{a}_s^2\left[-\frac{55}{12} + \frac{N_F}{3}\right] \\
&+ \hat{a}_s^3\left[\frac{55\zeta_5}{2} - \frac{44\zeta_3}{9} - \frac{13841}{216} + N_F\left(\frac{61\zeta(3)}{54} - \frac{5\zeta_5}{3} + \frac{10339}{1296}\right)\right. \\
&\left. - N_F^2\frac{115}{648}\right] + \hat{a}_s^4\left[-\frac{2695\zeta_7}{16} + \frac{343175\zeta_5}{864} - \frac{363\zeta_3^2}{8} + \frac{8213\zeta_3}{48}\right. \\
&- \frac{17865665}{20736} + N_F\left(-\frac{32743\zeta_3}{2592} + \frac{11\zeta_3^2}{2} - \frac{53215\zeta_5}{1296} + \frac{245\zeta_7}{24}\right. \\
&\left.\left. + \frac{10134475}{62208}\right) + N_F^2\left(\frac{103\zeta_3}{432} - \frac{\zeta_3^2}{6} + \frac{5\zeta_5}{12} - \frac{169523}{20736}\right) + N_F^3\frac{605}{5832}\right] \\
&= 1 - \hat{a}_s + \hat{a}_s^2(-4.5833 + 0.3333N_F) \\
&+ \hat{a}_s^3(-41.4399 + 7.6073N_F - 0.1775N_F^2) \\
&+ \hat{a}_s^4(-479.4475 + 123.3914N_F - 7.6975N_F^2 + 0.1037N_F^3)
\end{aligned}$$

The Bjorken Sum Rule

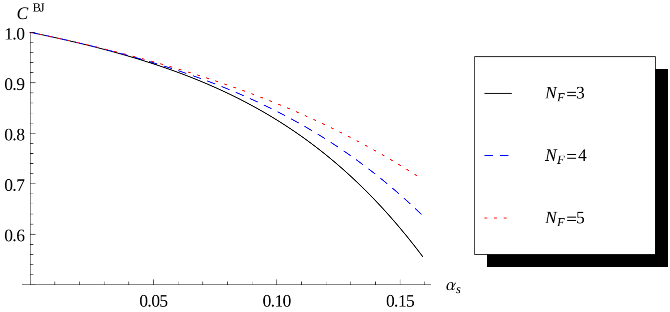
SU(3) singlet contributions:

$$
\begin{aligned}
C^{\mathrm{S}}_{\mathrm{pBJ}} &= \hat{a}_s^4 \frac{10}{9}\left(11 - \frac{2}{3}N_F\right)\sum_{q=1}^{N_F} e_q \\
&= 0 \quad N_F = 3 \\
&= a_s^4 \frac{500}{81} = a_s^4 6.173, \quad N_F = 4 \\
&= a_s^4 \frac{230}{81} = a_s^4 2.938, \quad N_F = 5
\end{aligned}
$$

The Bjorken Sum Rule



The Bjorken Sum Rule



The Bjorken Sum Rule

Massive Contributions

Switching on heavy flavors: from threshold to asymptotia

1) There are no logarithmic contributions $\propto \ln^k(Q^2/m^2)$, due to fermion-number conservation in the inclusive non-singlet case.
2) Only power corrections $\propto (m^2/Q^2)^l$ will contribute. These corrections start with $O(\alpha_s^2)$.
3) Down to which scale are hard corrections are perturbatively reliable? $\Longrightarrow \quad Q^2 \gtrsim 4\ \mathrm{GeV}^2$.

$$\int_0^1 dx \left[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)\right] = \frac{1}{6}\left|\frac{g_A}{g_V}\right| C_{\mathrm{pBJ}}(\hat{a}_s),$$

The Bjorken Sum Rule: $O(\alpha_s^2)$ HQ contributions

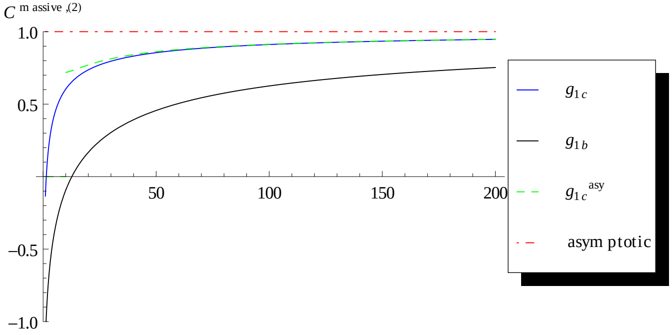
$$
\begin{aligned}
C_{\rm pBJ}^{\text{massive},(2)} = & \; 3C_F T_F \Bigg\{ \frac{6\xi^2 + 2735\xi + 11724}{5040\xi} - \frac{\sqrt{\xi+4}}{\xi^{3/2}} \frac{(3\xi^3 + 106\xi^2 + 1054\xi + 4812)}{5040} \\
& \times \ln\left[\frac{\sqrt{1+\frac{4}{\xi}}+1}{\sqrt{1+\frac{4}{\xi}}-1}\right] - \frac{1}{\xi^2}\frac{5}{12}\ln^2\left[\frac{\sqrt{1+\frac{4}{\xi}}+1}{\sqrt{1+\frac{4}{\xi}}-1}\right] \\
& + \frac{(3\xi^2 + 112\xi + 1260)}{5040} \ln(\xi) \Bigg\},
\end{aligned}
$$

with $\xi = Q^2/m^2$. In the asymptotic region $\xi \gg 1$, $C_{\rm pBJ}^{\text{massive},(2)}$ behaves like

$$
C_{\rm pBJ}^{\text{massive},(2)} \propto 3C_F T_F \left\{ \frac{1}{2} - \frac{5}{12\xi^2}\ln^2(\xi) - \frac{4}{3\xi}\ln(\xi) + \frac{17}{9\xi} + O\left(\frac{\ln(\xi)}{\xi^2}\right) \right\}.
$$

Valid to about $Q^2 \simeq m_c^2$.

The Bjorken Sum Rule: $O(\alpha_s^2)$ HQ contributions



JB, G. Falcioni, A. De Freitas, DESY 15-171

Charm and bottom contributions as a function of $\xi = Q^2/m_c^2$; the flavor excitation $N_F \to N_F + 1$ is shown. Note the negative! corrections at low scales.

The Bjorken Sum Rule

Higher Twist Corrections

- Only vaguely known; not very reliable theoretical predictions yet

$$c_{\mathrm{HT}} \approx -0.025... + 0.03\mathrm{GeV}^2$$

I.I. Balitsky, V.M. Braun, A.V. Kolesnichenko Phys. Lett B242 (1990) 245; E: B318 (1993) 648; X. Ji, P. Unrau, Phys. Lett. B333 (1994) 228; B. Lampe and E. Reya, Phys. Rep. 332 (2000) 1.

- Better determine it by fitting.

Target Mass Corrections

- To be applied, cf. J. Blümlein and A. Tkabladze, Nucl. Phys. B443 (1999) 427 for the moments.

The Status of $\alpha_s(M_Z^2)$: polarized case

- Till now only NLO analyses, however, accounting for charm at LO: JB and H. Böttcher Nucl. Phys. B841 (2010) 205
- $\alpha_s(M_Z^2) = 0.1132^{+0.0043}_{-0.0051}$ EXP $^{+0.0029}_{-0.0015}$ FS $^{+0.0032}_{-0.0075}$ RS
- The higher order corrections will remove a significant part of the factorization and renormalization scale uncertainty.
- Yet an experimental error of $\sim \pm 0.005$ will remain.
- This is still a very interesting measurement. The next real leap forward can be made at the EIC, if it will be built.

Conclusions

- The polarized DIS data on $g_1(x, Q^2)$ can be used to project a non-singlet combination.
- The measurement of $\alpha_s(M_Z^2)$ at leading twist is currently possible including the 3-loop massless and 2-loop massive Wilson coefficients, using the differential method.
- Higher twist and target mass effects have to be accounted for. The former need to be fitted from the data.
- Using the above method the theory errors can be widely reduced.
- The current experimental error is expected to be ± 0.0050. It will be interesting to see, which central value is going to be obtained.