New look at the string model of quark fragmentation

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Purpose of this study

Understand the $p_T$ correlations which come from kinematics, to disentangle them from those which depend on quark spin, like the single and di-hadron Collins asymmetries (for transversity) and Jet handedness (for helicity) and also from the Bose-Einstein correlations.
Outlines

• jets and confinement
• string and multiperipheral approaches
• multiperipheral dynamics
• recursive method
• quark line reversal (or ‘left-right symmetry’)
• correspondance String model / multiperipheral models
• PYTHIA and “Lyon” splitting functions
• correlations between quark transverse momenta
• jet axis, primordial $k_T$
• non-existence of the jet axis
• lower $p_T$ of the first-rank hadron

all this WITHOUT SPIN (for the moment)

References :
partons

\[ \bar{q} \quad q \]

\( \gamma^* \) or \( W \)

\[ \bar{q} \quad q \]

\[ g \]

\[ \gamma^* \text{ of DIS} \]

\[ N \]
Two pictures of $e^+e^- \rightarrow q + \bar{q} \rightarrow \text{hadrons}$

**Quark Multi Peripheral (QMPM)**

**String Fragmentation (SFM)**

Dual model diagram

Confinement built-in

Quark $\rightarrow$ String

$\gamma^*$
A bad picture:

Two **independent** cascades of "quark decays"

- No confinement
- Would produce two isotropic distributions in c.m. frame of each jet
Multiperipheral dynamics

\[ -k^2 = |k^+k^-| + k_z^2 \]

Cutoff in \( k_T \) \( \Rightarrow \) Cutoff in \( p_T \)

Cutoff in \( |k^+k^-| \) \( \Rightarrow \) \( h_1, h_2, h_3, \ldots \) are nearly ordered in rapidity

Local Compensation of Charges and Transverse Momenta
Recursive fragmentation model

- with virtual mesons: Krzywicky & Petersson; Finkel & Peccei (1972)
- with quarks: Feynman & Field

Splitting distribution for $k \rightarrow p + k'$:

$$f(Z) \, dZ$$  
(Krzywicky…)

$$f(Z, k'_T) \, dZ \, d^2p_T$$  
(Feynman-Field…)

$$f(Z, p_T, k'_T) \, dZ \, d^2p_T$$  
(Lund-symmetric)

$$Z = p^+/k^+ \quad k^+ = k^0 + k^z$$

$$z_4 = p_4^+/k_A^+ = Z_4 \, (1- z_3) \,(1- z_2) \,(1- z_1)$$
An important constraint:
Symmetry of *quark line reversal*

\[ \text{Downgoing quark line} \quad \text{is equivalent to} \quad \text{Upgoing antiquark line} \]

\[ \text{Lund-symmetric model} \]
The string fragmentation model
or “yoyo” model
Simplest string motion: the relativistic yo-yo

\[ X^- = t - z \quad \text{and} \quad X^+ = t + z \]

\[ \text{yo-yo “at rest”} \quad \text{yo-yo in motion} \]

(\text{the string tension } \kappa \approx 1 \text{ GeV/fermi is taken as unity})
Space-time history of string fragmentation

Mass = \( \sqrt{s_{(\gamma^*+N)}} = W \)

Big yoyo

small yoyo

= hadron

quark pair creation

q_B

q_A = q_1

\[ h_N \]

\[ h_3 \]

\[ h_2 \]

\[ h_1 \]

\[ Q_3 \]

\[ Q_2 \]

\[ Q_1 \]
Correspondance $\text{QMPM} \approx \text{SFM}$

$k$ of QMPM and vector $\mathbf{OQ}$ of SFM are symmetrical about the $X^+ (=t+z)$ axis.

**String Fragmentation Model** and **Quark MultiPeripheral Model** are two complementary pictures.
The PYTHIA splitting function

\[
f(q \rightarrow h + q') \propto \exp(-b_T k_T'^2) \\
\times Z^{-1} \times (1-Z)^a \times \exp\{- Z^{-1} b_L (m_h^2 + p_T^2)\} \\
\times N^{-1}(m_h^2 + p_T^2)
\]

where

\[
N(m_h^2 + p_T^2) = \int dZ Z^{-1} \times (1-Z)^a \times \exp\{- Z^{-1} b_L (m_h^2 + p_T^2)\}
\]

The PYTHIA algorithm:

- draw \( k_T' \) first, with the \( \exp(-b_T k_T'^2) \) distribution
- draw \( Z \) with the distribution on the 3rd line

\( \Rightarrow \) no \((k_T, k_T')\) correlation, in spite of the factor \( \exp\{-b_L(m_h^2 + p_T^2)/Z\} \)
which penalizes large \( |k_T - k_T'| \).

The factor \( N^{-1}(m_h^2 + p_T^2) \) cancels this correlation.
The “Lyon” splitting function

\[ f(q \rightarrow h + q') \propto \exp(-b_T k'_T^2) \]
\[ \times Z^{-1} \times (1-Z)^a \times \exp\{- (Z^{-1} - c) \cdot b_L \cdot (m_h^2 + p_T^2)\} \]

- The factor \( N^{-1}(m_h^2 + p_T^2) \) has been thrown away.
- \( k_T \) and \( k'_T \) are correlated: \( \langle k_T \cdot k'_T \rangle \) is positive
- We added a new parameter \( c \).
- Monte Carlo drawing of \( k'_T \) and \( Z \) is no more complicated than in PYTHIA
Meaning of the parameter $c$

In the exponential, $Z_1^{-1} b_L (m_1^2 + p_{1T}^2)$ is the area in the past lightcone of $H_1$. The factor $\exp\{- Z_1^{-1} b_L (m_1^2 + p_{1T}^2)\}$ of PYTHIA is the probability to have no string cutting in that area, assuming a uniform fragility constant $b_L$. However, a cut in the rectangle $C_1 Q_1 H_1 Q_2$ is kinematically forbidden. The meson $h_1$ is “born” in $C_1$, not $H_1$. Thus we require no string cutting in the past light cone of $Q_2$.

The exponential factor becomes $\exp\{- (Z^{-1} - 1) b_L (m_h^2 + p_T^2)\}$. For flexibility, we put $-c$ instead of $-1$. 
Tentative to define a *theoretical* jet axis, or *string axis*

A string is spanned between **two** colored objects (quark, diquark, gluon). The jet axis should be defined by the 4-momenta $k_A$ and $k_B$ of these objects. A hadron of this jet has 4-momentum $p = z^+ k_A + z^- k_B + p_{T/string}$.

$p_T=0$ defines a *2-D hyperplane* spanned by $k_A$ and $k_B$.

One should speak of a *jet hyperplane* or *string hyperplane*.

The particles of a jet are roughly aligned in *velocity space*.

\[ e^+ e^- \rightarrow q + \bar{q} + \text{gluon} \]
Practical jet axis, primordial $k_T$

The string hyperplane is not accessible experimentally. In DIS, a practical jet axis is defined by the momentum $Q$ of the virtual photon ($\gamma^*$). Relative to $Q$, the struck quark (A) has a transverse momentum $k_{T/Q}$, called *primordial transverse momentum*.

The nucleon remnant (B) has the opposite $k_{T/Q}$.

From $p = z^+ k_A + z^- k_B + p_{T/string}$:

$$p_{T/Q} = p_{T/string} + x_F k_{T/Q},$$

with $z^+ = z = p^+ / p_A^+$, $z^- = p^- / p_B^-$, $x_F = z^+ - z^- = 2 p_z (c.m.) / W$.

Then, $\langle p_{T/Q} \rangle^2 = \langle p_{T/string} \rangle^2 + x_F^2 \langle k_{T/Q} \rangle^2$.

Instead of $z^2$
Relation between $x_F$, and $z$

In the hadronic c.m. frame

$z = z^+ = p^+/W$,

$z^- = p^-/W$,

$x_F = 2\, p_z/W$

\[
z^+ = \frac{1}{2} \left[ x_F + \sqrt{x_F^2 + 4(m^2 + p_T^2)/W^2} \right]
\]
Lower $p_T$ of the first-rank hadron

$$p_{T/\text{string}} = (k_T - k'_T)_{/\text{string}}$$

$$\rightarrow \langle p_T^2 \rangle_{/\text{string}} = 2 \langle k_T^2 \rangle - 2 \langle k_T \cdot k'_T \rangle \sim 2 \langle k_T^2 \rangle_{/\text{string}}$$

*except for the first rank*, because $k_{1T/\text{string}} = 0$.

*the recursive model predicts, at large but equal $z$,*

$$\langle p_T^2 \rangle_{\text{favored}} < \langle p_T^2 \rangle_{\text{unfavored}}$$

For instance, in DIS on protons: $\langle p_T^2 \rangle (\pi^+) < \langle p_T^2 \rangle (\pi^-)$

Is it the case experimentally?
$k_A$ and $k_A^*$ are internal momenta of quark loops = integration variables.

$k_A - k_A^*$ has no classical counterpart. When $k_A \neq k_A^*$ one can speak of a $k_A \times k_A^*$ interference.

Also the ranks of the hadrons are theoretically ambiguous, due to crossing between identical particles.
Main conclusions

• We have proposed a new splitting formula introducing the $k_T$ correlations, which are missing in PYTHIA.

• Our splitting function also depends on a new parameter $c$. Kinematical constraints forbid string decay in some regions, suggesting $c=1$.

• In DIS, $\langle p_{T/Q}^2 \rangle = \langle p_{T/string}^2 \rangle + x_F^2 \langle k_{T/Q}^2 \rangle$.

• The recursive models generally predict a smaller $\langle p_T^2 \rangle$ for a favored meson than for an unfavored one, at large but equal $z$. This has to be clarified on the experimental side.

• Jet axis and hadron rank cannot be defined without ambiguity, even theoretically.

Thank you for attention!