Réunion COMPASS, CERN,
13 octobre 2016

New look at the string model of quark fragmentation

Xavier ARTRU (IPN-Lyon)

Collaborateurs :

- Algérie : Zouina Belghobsi,
  Laboratoire de Physique Théorique, Univerisité de Jijel

  Essma Redouane-Salah, Université de M'sila

- Italie : Albi Kerbizi, Université de Trieste

Purpose of this study

Understand the $\mathbf{p}_T$ correlations which come from kinematics, to disentangle them from those which depend on quark spin, like the single and di-hadron Collins asymmetries (for transversity) and Jet handedness (for helicity) and also from the Bose-Einstein correlations.

Outlines

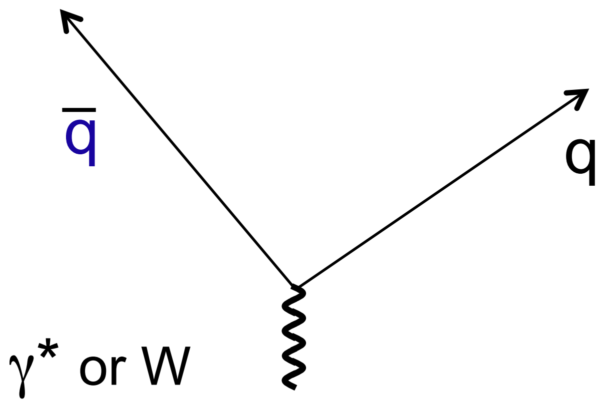
- jets and confinement
- string and multiperipheral approaches
- multiperipheral dynamics
- recursive method
- quark line reversal (or ‘left-right symmetry’)
- correspondance String model / multiperipheral models
- PYTHIA and ‘‘Lyon’’ splitting functions
- correlations between quark transverse momenta
- jet axis, primordial $\mathbf{k}_T$
- non-existence of the jet axis
- lower $\mathbf{p}_T$ of the first-rank hadron

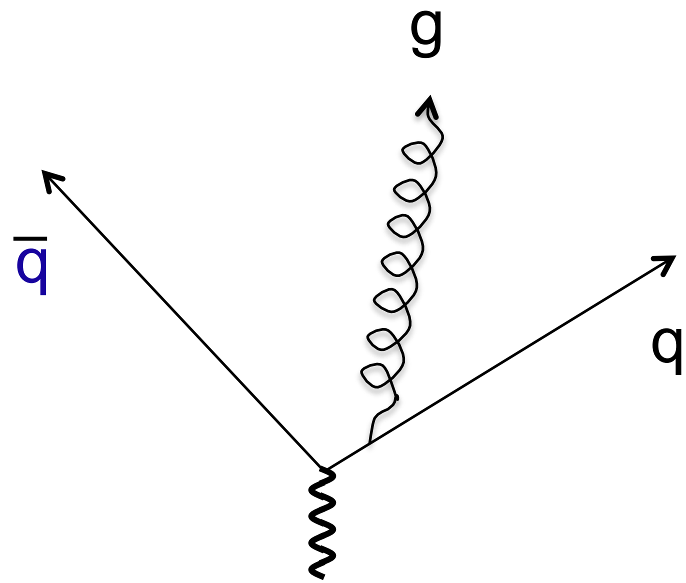
all this *WITHOUT SPIN* (for the moment)

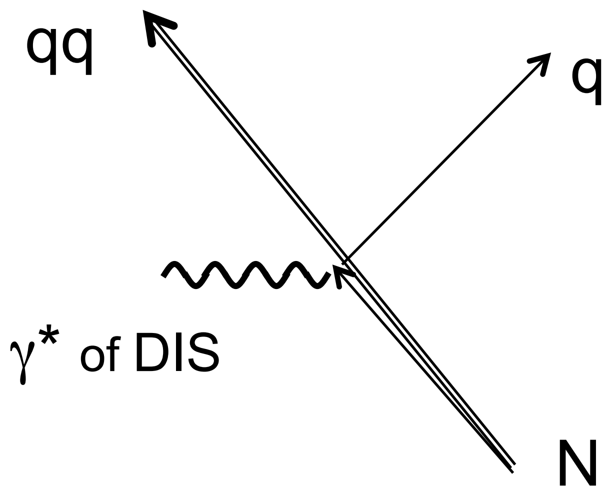
References :

- X.A., Z. Belghobsi, E. Redouane-Salah, Phys. Rev. **D**94 (Aug. 2016)
- Master thesis of A. Kerbizi (Trieste, Sept. 19, 2015)

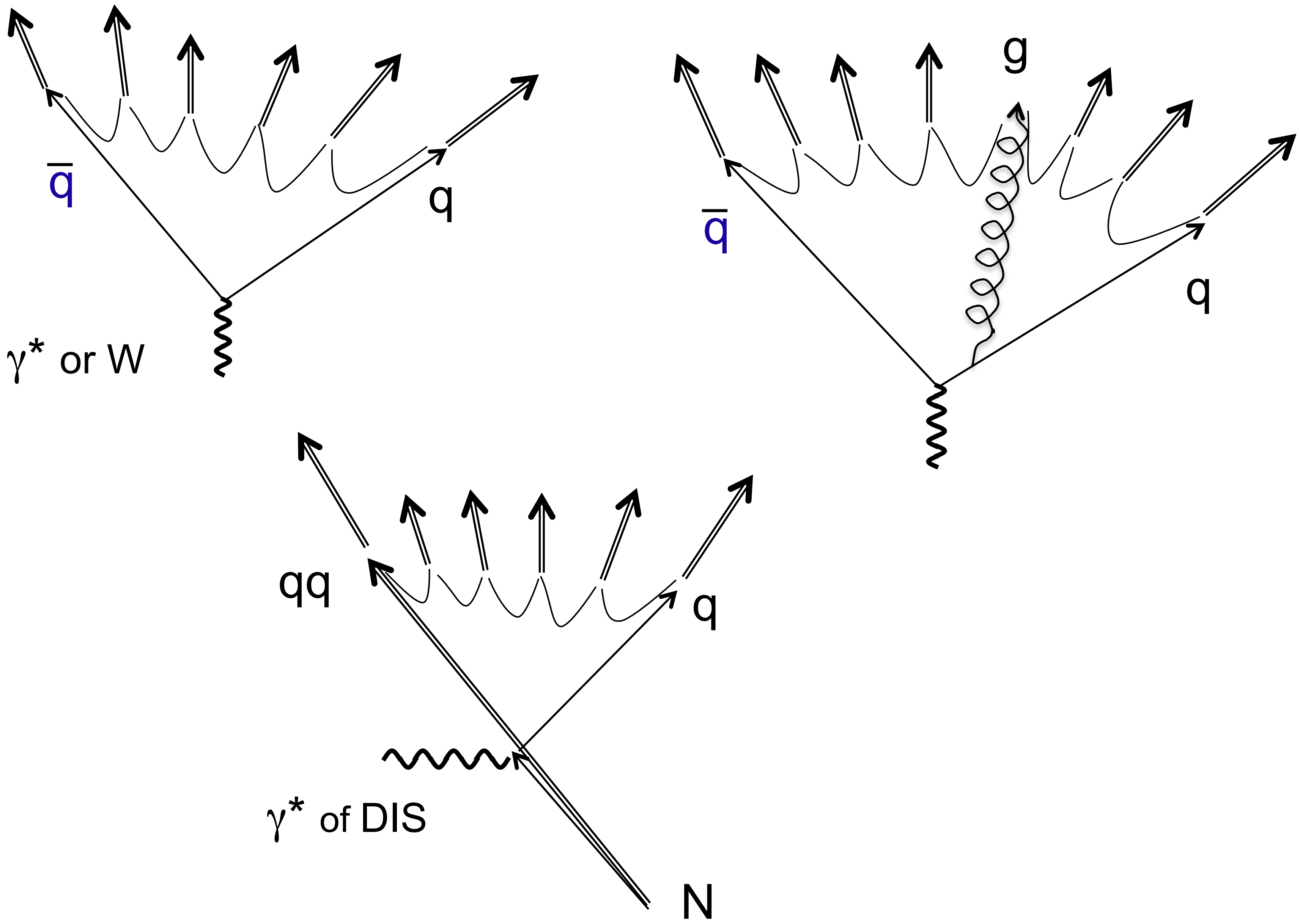
partons





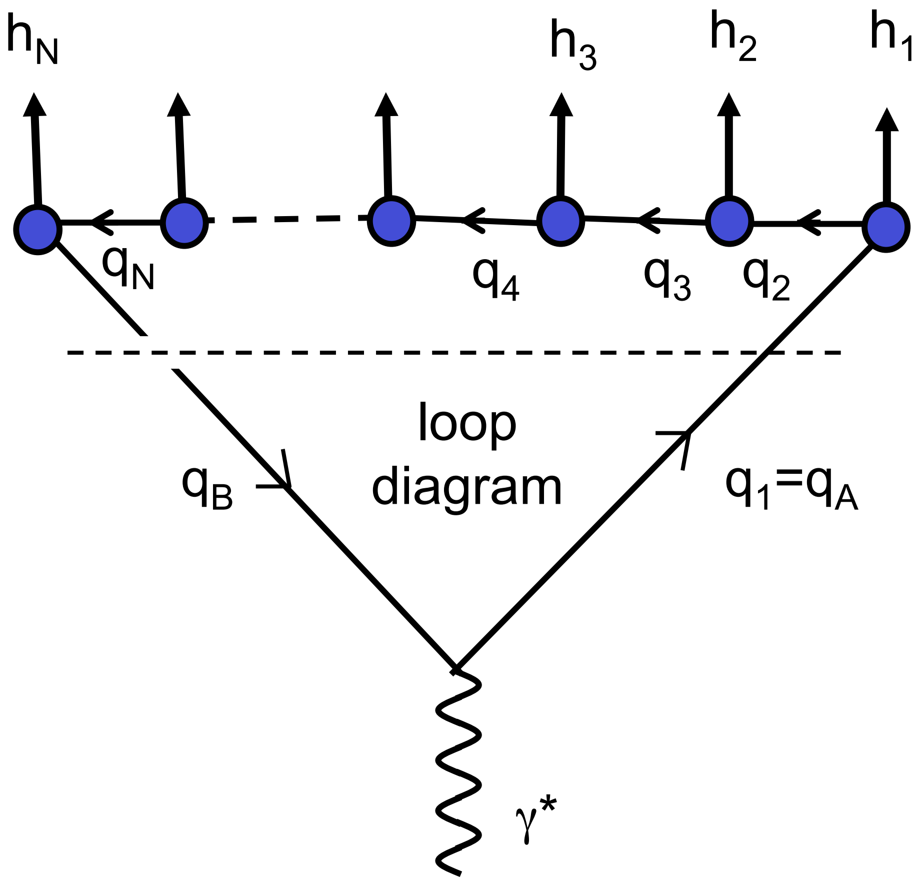


jets

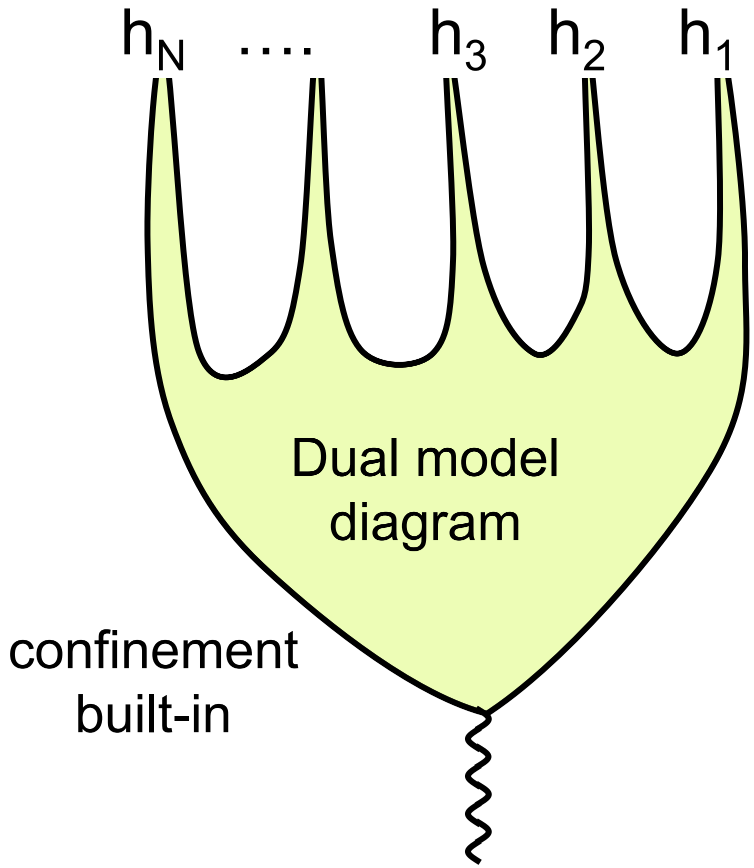


Two pictures of $e^+e^- \rightarrow q + qbar \rightarrow$ hadrons

Quark **M**ulti**P**eripheral (QMPM)

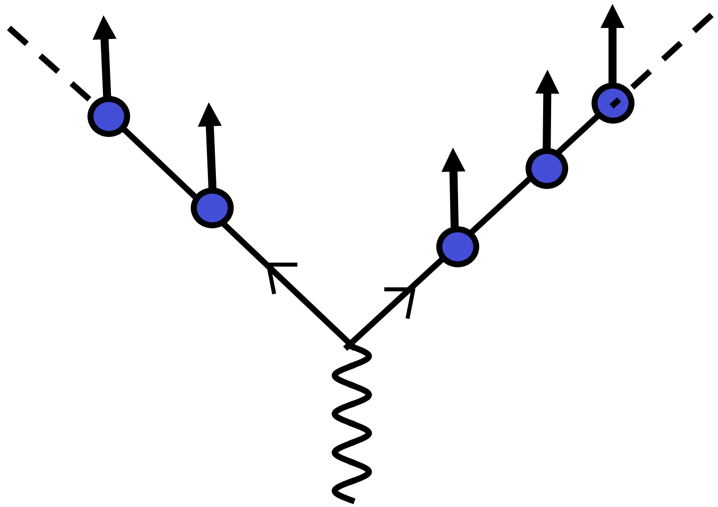


String **F**ragmentation (SFM)



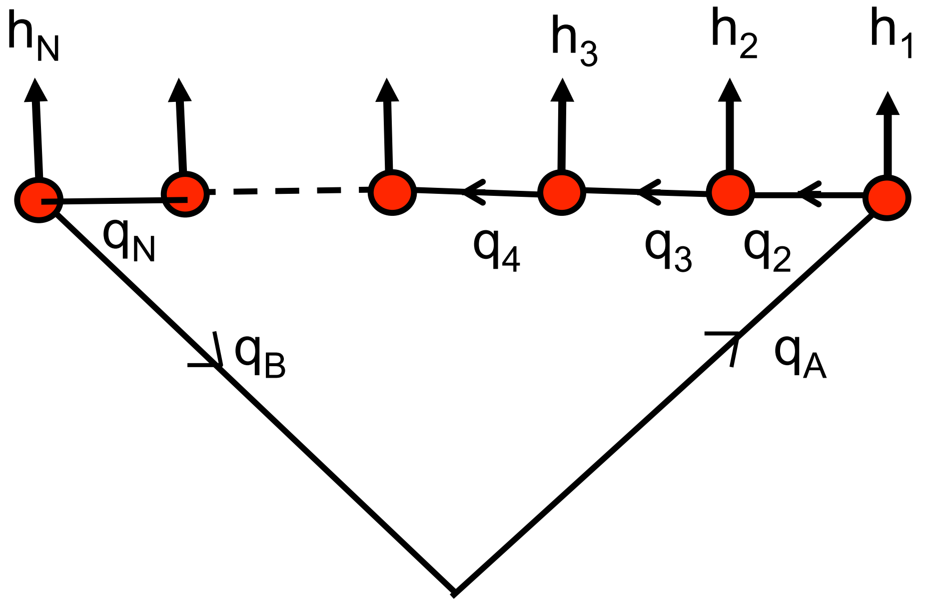
A bad picture :

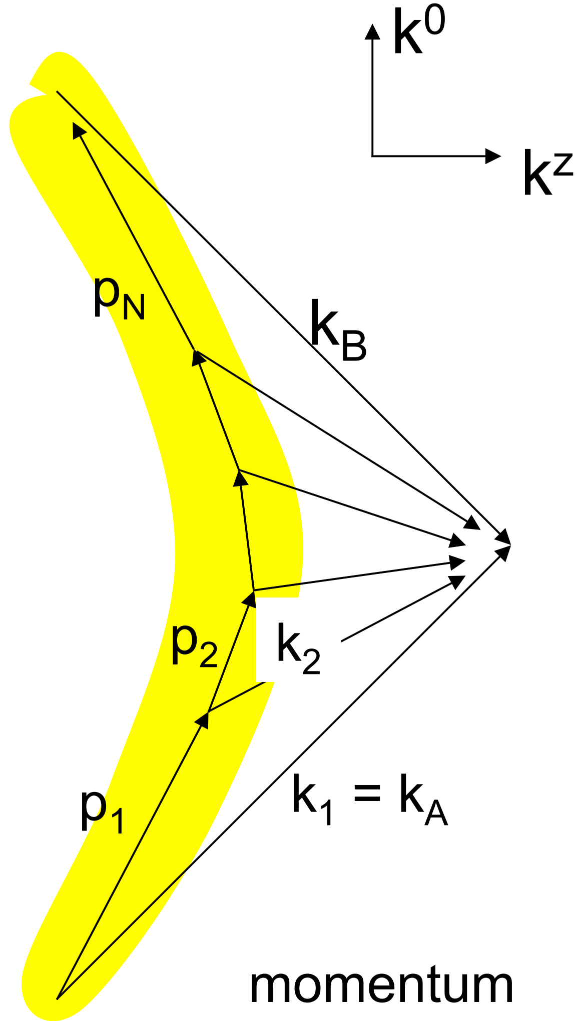
two **independent** cascades of "quark decays"



- No confinement
- Would produce two isotropic distributions in c.m. frame of each jet

Multiperipheral dynamics





quark virtuality : $-k^2 = |k^+k^-| + \mathbf{k}^2_T$

momentum diagram

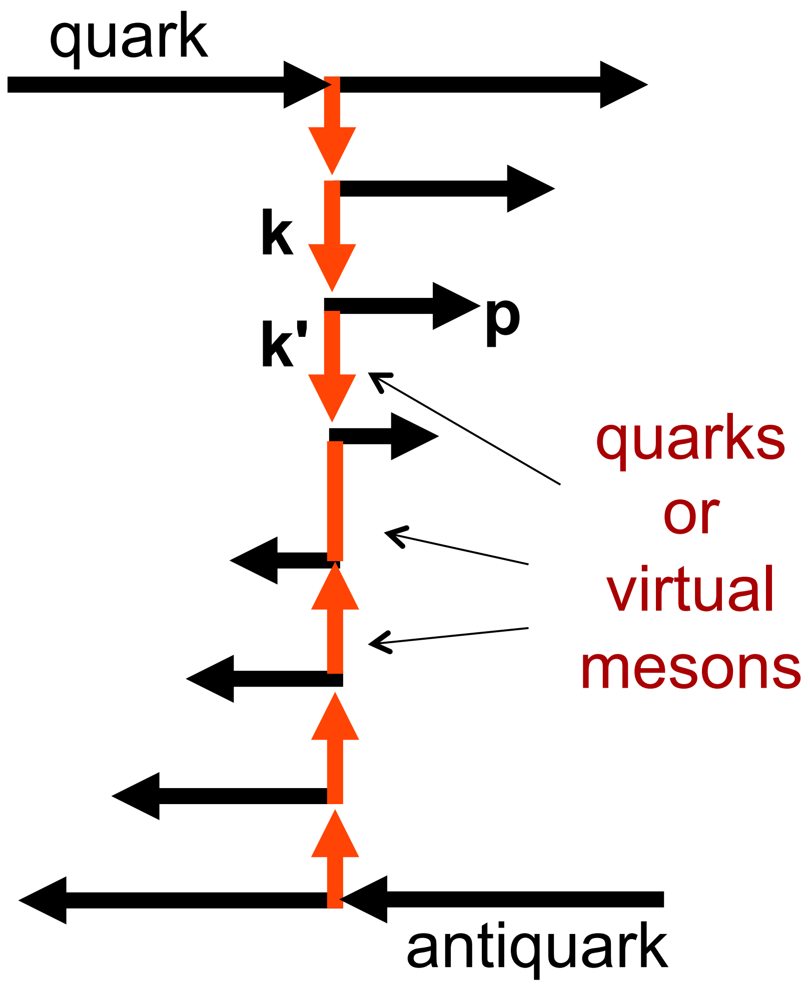
Cutoff in $\mathbf{k}_T$ ➔ Cutoff in $\mathbf{p}_T$

Cutoff in $|k^+k^-|$ ➔ h_1, h_2, h_3,… are nearly ordered in rapidity

Local **C**ompensation of **C**harges and **T**ransverse **M**omenta

Recursive fragmentation model

- with virtual mesons : Krzywicky & Petersson ; Finkel & Peccei (1972)
- with quarks : Feynman & Field



Splitting distribution for $\boldsymbol{k \rightarrow p + k'}$:

$f(Z)\, dZ$ (Krzywicky…)

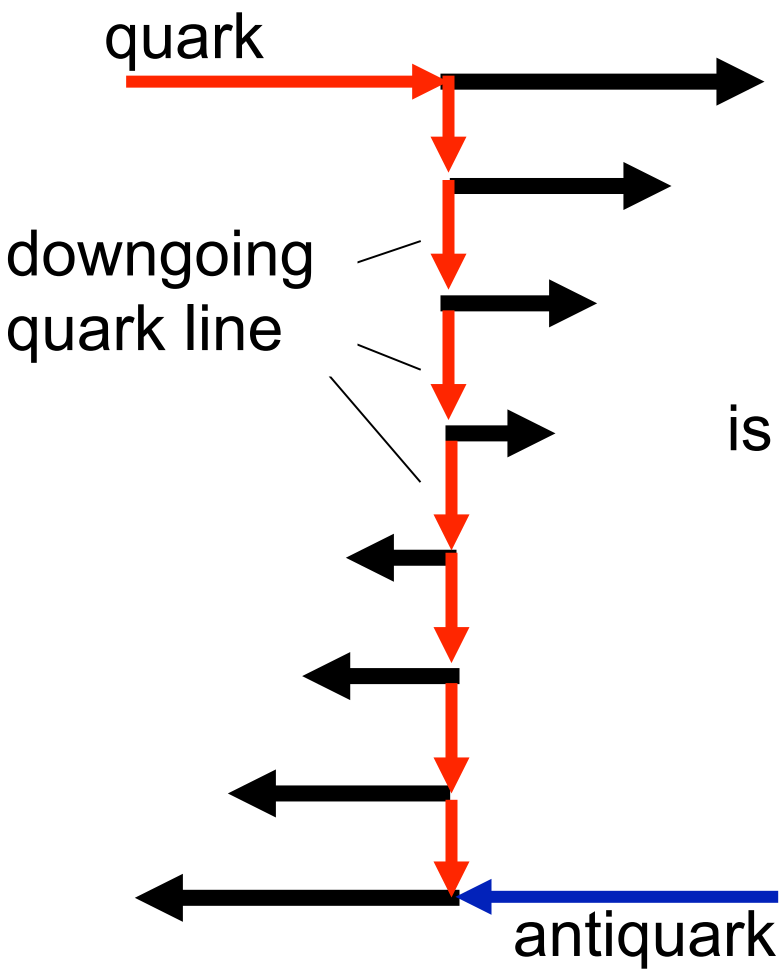
$f(Z, \mathbf{k'}_T)\, dZ\, d^2\mathbf{p}_T$ (Feynman-Field…)

$f\,(Z, \mathbf{p}_T\,, \mathbf{k'}_T)\, dZ\, d^2\mathbf{p}_T$ (Lund-symmetric)

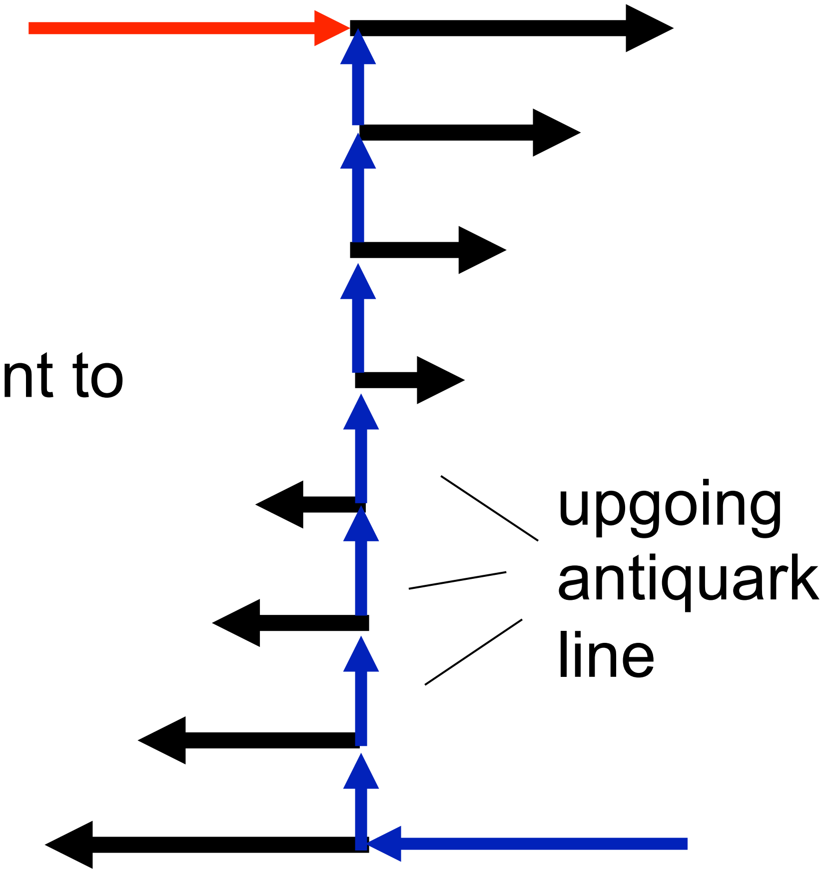
$Z = p^+/k^+ \;\; ; \;\; k^+ = k^0 + k^z$

$\color{red}{z_4} = p_4^{\,+}/k_A^{\,+} = \color{blue}{Z_4}\,(1 - z_3)(1 - z_2)(1 - z_1)$

An important constraint :
Symmetry of ***quark line reversal***



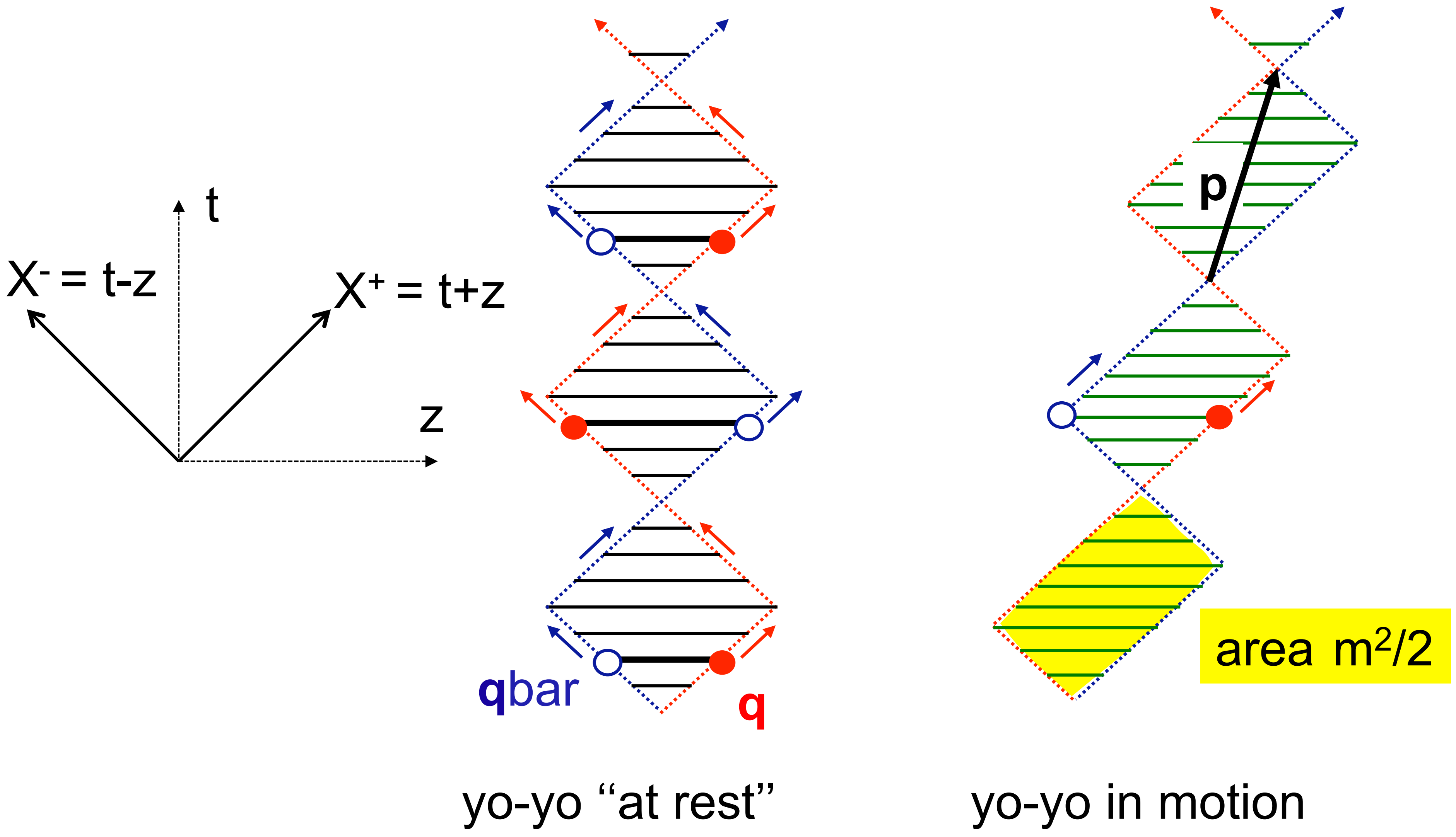
is equivalent to



➔ Lund-***symmetric*** model

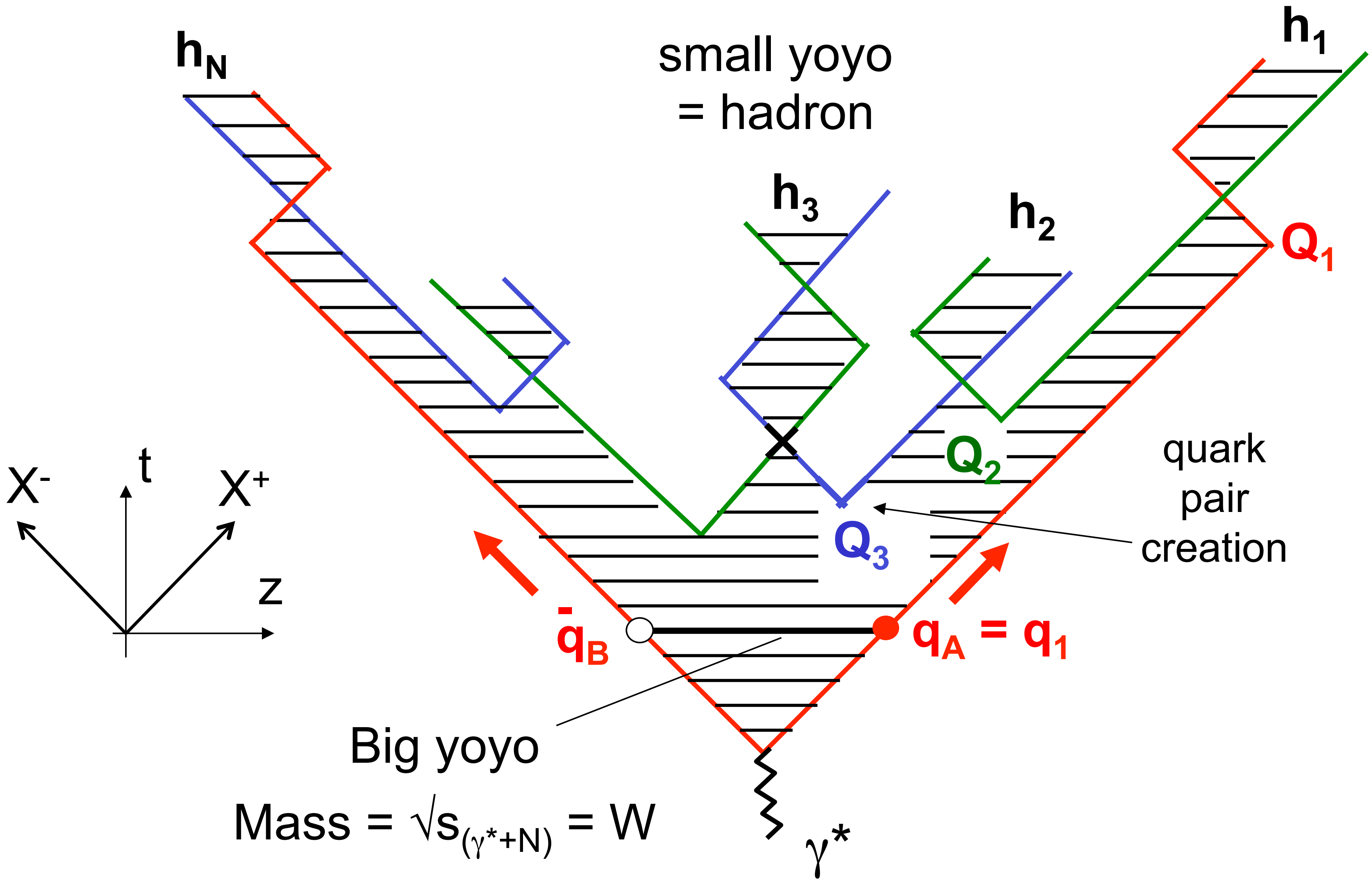
The string fragmentation model or ‘‘yoyo’’ model

Simplest string motion : the relativistic yo-yo

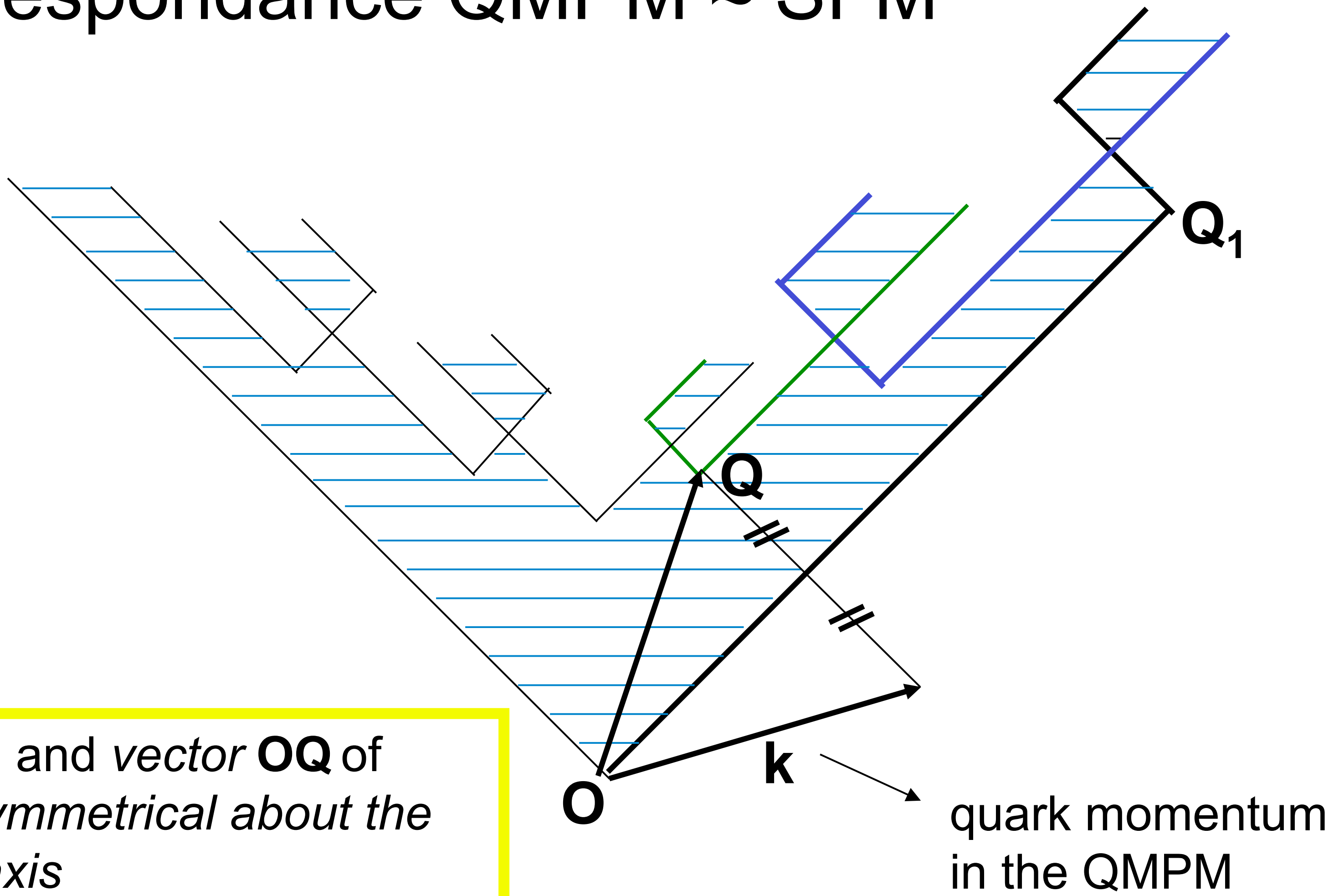


(the string tension $\kappa \approx 1$ GeV/fermi is taken as unity)

Space-time history of string fragmentation



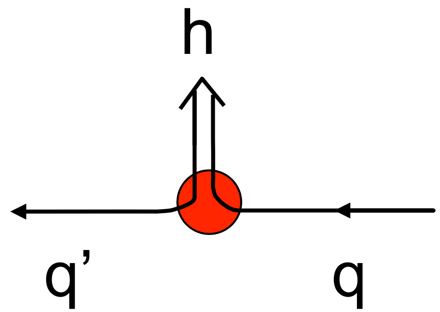
Correspondance QMPM ≈ SFM



k of QMPM and *vector* **OQ** of SFM are *symmetrical about the X^+ (= t+z) axis*

String **F**ragmentation **M**odel and **Q**uark **M**ulti**P**eripheral **M**odel are two complementary pictures.

The PYTHIA splitting function



$$f(q \to h+q') \propto \exp(-b_T \mathbf{k}'^2_T) \times Z^{-1} \times (1-Z)^a \times \exp\{-Z^{-1} b_L (m_h^2 + \boldsymbol{p}_T^2)\} \times N^{-1}(m_h^2+\boldsymbol{p}_T^2)$$

where

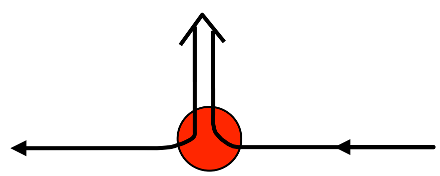
$$N(m_h^2+\boldsymbol{p}_T^2) = \int dZ\, Z^{-1} \times (1-Z)^a \times \exp\{-Z^{-1} b_L (m_h^2+\boldsymbol{p}_T^2)\}$$

The **PYTHIA algorithm :**

- draw $\mathbf{k}'_T$ first, with the $\exp(-b_T \mathbf{k}'^2_T)$ distribution
- draw Z with the distribution on the 3rd line

➔ no ($\mathbf{k}_T$, $\mathbf{k}'_T$) correlation, in spite of the factor $\exp\{-b_L(m_h^2+\boldsymbol{p}_T^2)/Z\}$ which penalizes large $|\mathbf{k}_T - \mathbf{k}'_T|$.
The factor $N^{-1}(m_h^2+\boldsymbol{p}_T^2)$ cancels this correlation.

The "Lyon" splitting function

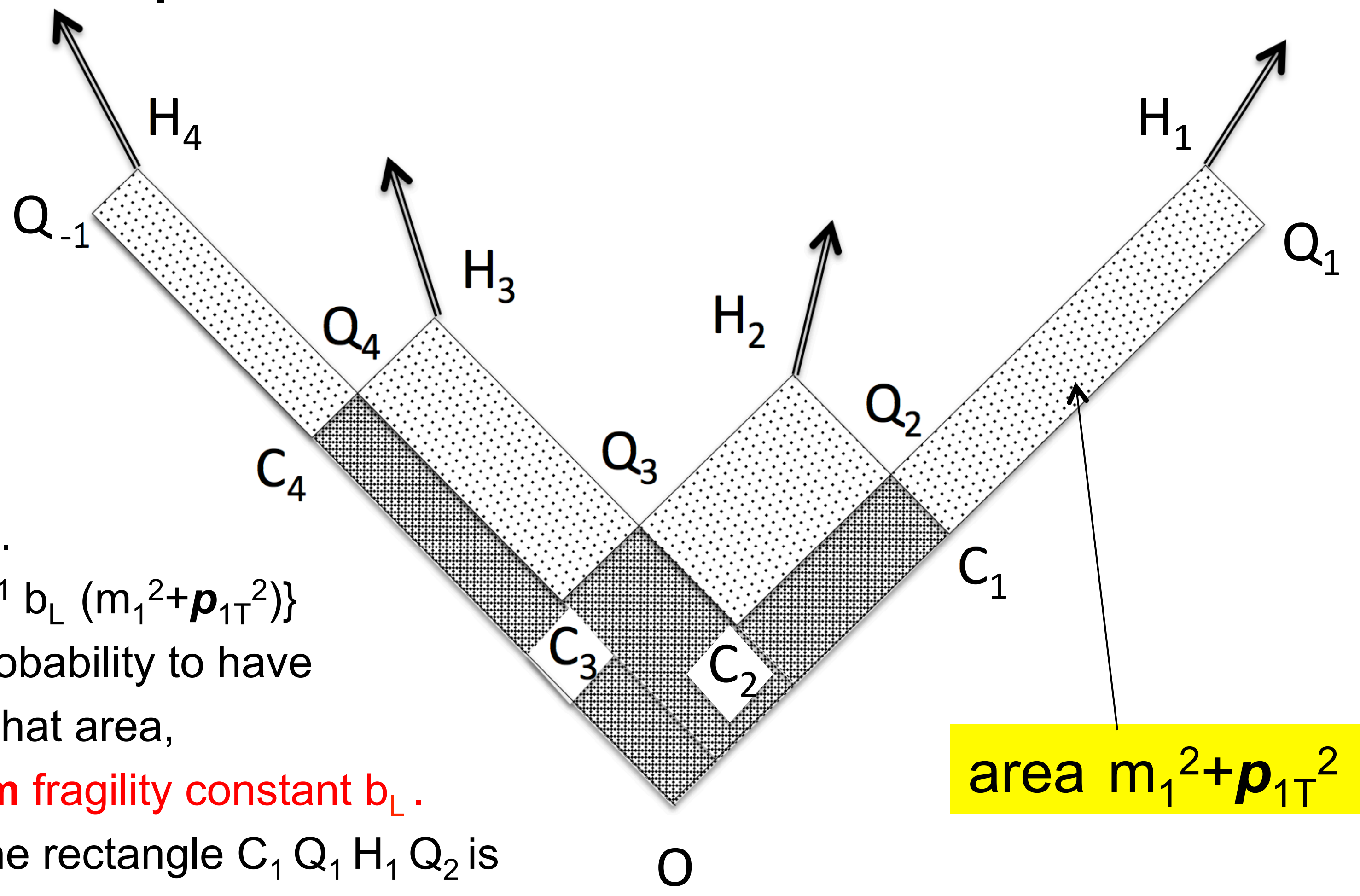


$$f(q \rightarrow h+q') \propto \exp(-b_T \mathbf{k'}_T^2)$$

$$\times Z^{-1} \times (1-Z)^a \times \exp\left\{-(Z^{-1} - c) \,.\, b_L \,.\, (m_h^2 + \boldsymbol{p}_T^2)\right\}$$

- The factor $N^{-1}(m_h^2+\boldsymbol{p}_T^2)$ has been thrown away.
- $\mathbf{k}_T$ and $\mathbf{k'}_T$ are correlated : $\langle \mathbf{k}_T \,.\, \mathbf{k'}_T \rangle$ is positive
- We added a new parameter c.
- Monte Carlo drawing of $\mathbf{k'}_T$ and Z is no more complicated than in PYTHIA

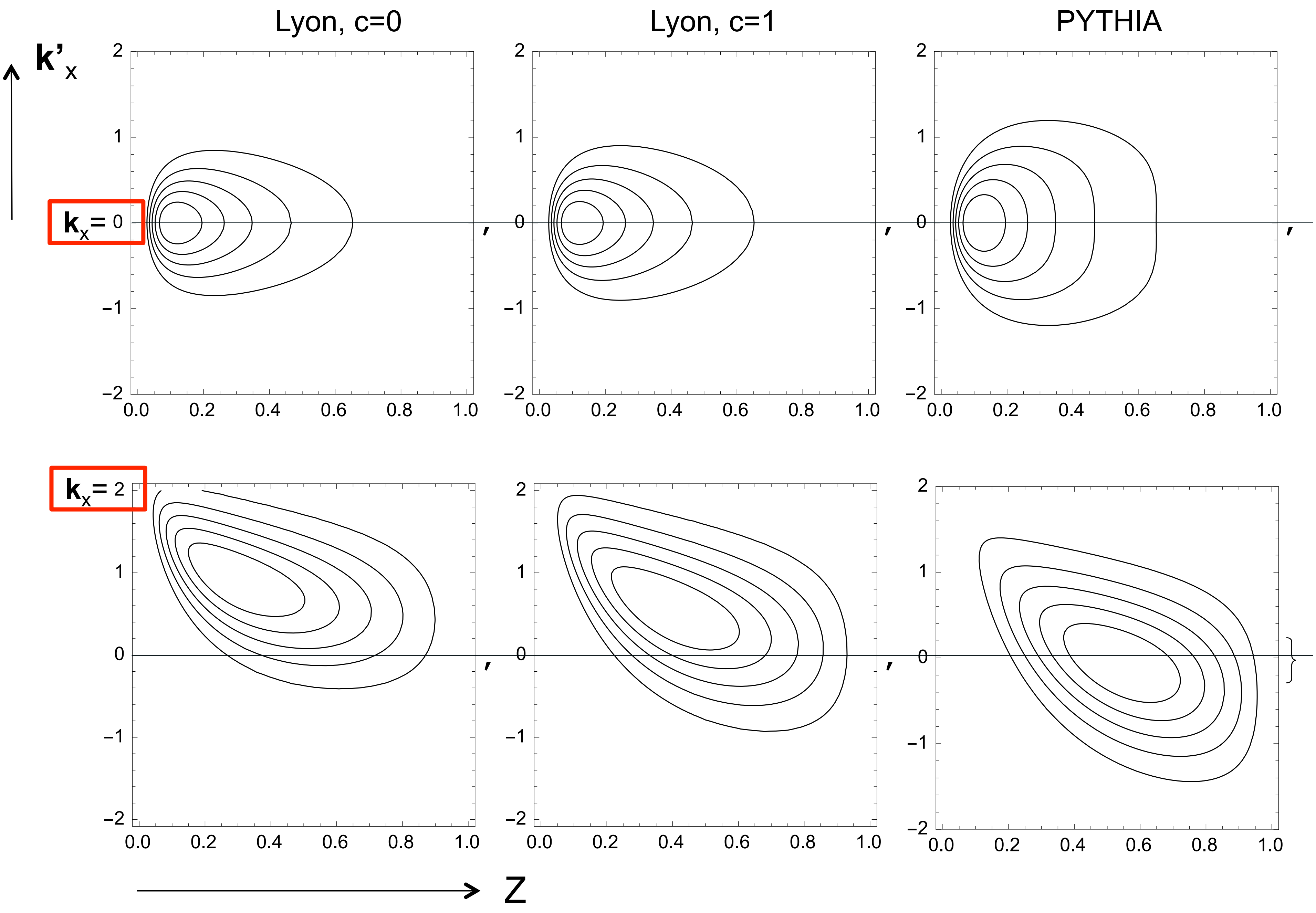
Meaning of the parameter c



In the exponential,
$Z_1^{-1}\, b_L\, (m_1^2+\boldsymbol{p}_{1T}^2)\}$
is the area in the
past lightcone of H_1.
The factor $\exp\{- Z_1^{-1}\, b_L\, (m_1^2+\boldsymbol{p}_{1T}^2)\}$
of PYTHIA is the probability to have
no string cutting in that area,
assuming a **uniform** fragility constant b_L .
However, a cut in the rectangle $C_1\, Q_1\, H_1\, Q_2$ is
kinematically forbidden. The meson h_1 is "born" in C_1, not H_1 . Thus we require

no string cutting in the past light cone of Q_2.

The exponential factor becomes $\exp\{ - (Z^{-1}- 1)\, b_L\, (m_h^2+\boldsymbol{p}_T^2) \}$.
For flexibility, we put - c instead of – 1.



Lyon, c=0
Lyon, c=1
PYTHIA
k'x
kx= 0
kx= 2
0.0
0.2
0.4
0.6
0.8
1.0
Z

Tentative to define a *theoretical* jet axis, or *string axis*

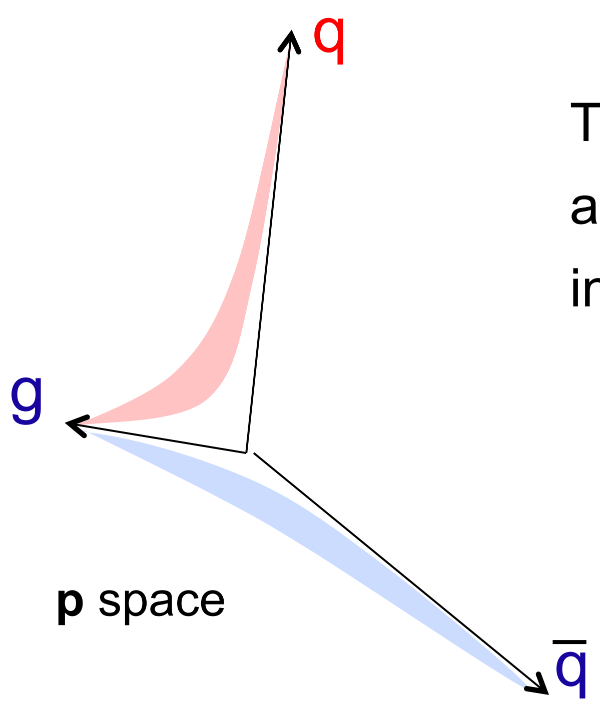
A string is spanned between **two** colored objects (quark, diquark, gluon).

The jet axis should be defined by the 4-momenta k_A and k_B of these objects.

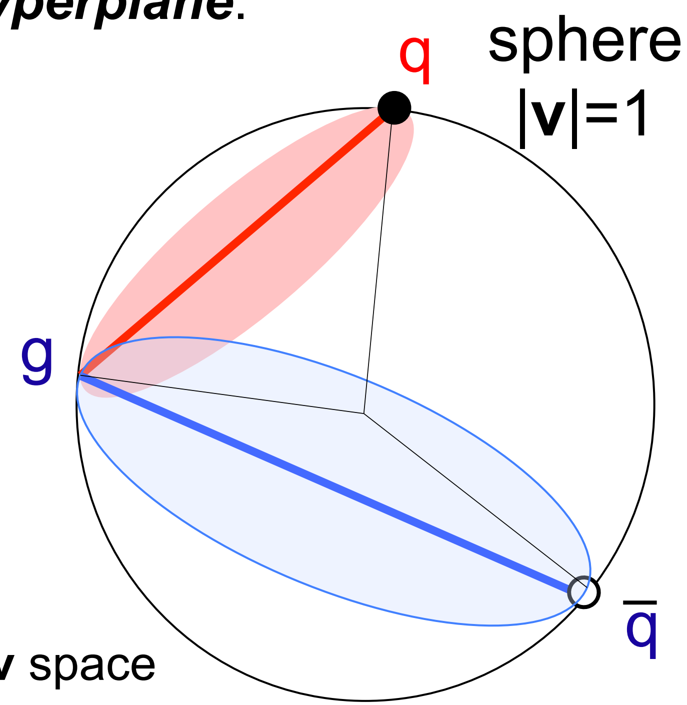
A hadron of this jet has 4-momentum $p = z^+ k_A + z^- k_B + \mathbf{p}_{T/\mathrm{string}}$.

$\mathbf{p}_T = 0$ defines a *2-D hyperplane* spanned by k_A and k_B .

One should speak of a ***jet hyperplane*** or ***string hyperplane***.



The particles of a jet are roughly aligned in *velocity space.*



$e^+e^- \rightarrow$ q + qbar + gluon

Practical jet axis, primordial $\mathbf{k}_T$

The string hyperplane is not accessible experimentally. In DIS, a *practical* jet axis is defined by the momentum **Q** of the virtual photon (γ^*).

Relative to **Q**, the struck quark (A) has a transverse momentum $\mathbf{k}_{T/\mathbf{Q}}$, called *primordial transverse momentum*.

The nucleon remnant (B) has the opposite $\mathbf{k}_{T/\mathbf{Q}}$.

From $p = z^+ k_A + z^- k_B + \mathbf{p}_{T/\text{string}}$:

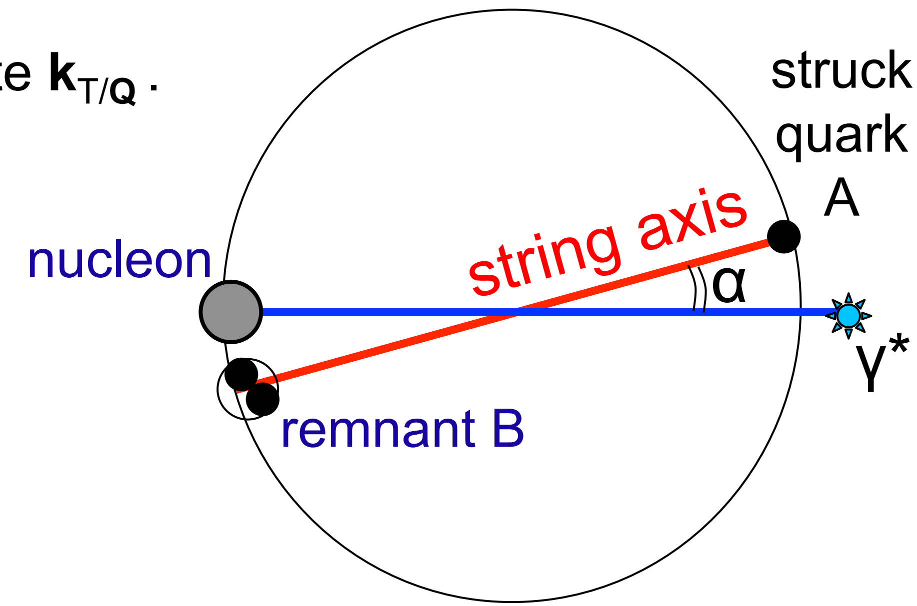
$$\mathbf{p}_{T/\mathbf{Q}} = \mathbf{p}_{T/\text{string}} + x_F\, \mathbf{k}_{T/\mathbf{Q}} ,$$

with $z^+ = z = p^+ / p_A^+$, $z^- = p^- / p_B^-$,

$x_F = z^+ - z^- = 2\, p_z(\text{c.m.}) / W$ **.**

Then, $\langle \mathbf{p}_{T/\mathbf{Q}} \rangle^2 = \langle \mathbf{p}_{T/\text{string}} \rangle^2 + x_F^2 \langle \mathbf{k}_{T/\mathbf{Q}} \rangle^2$.

instead of z^2



Velocity space, c.m. frame

$\mathbf{k}_{T/\mathbf{Q}} = \alpha W/2$

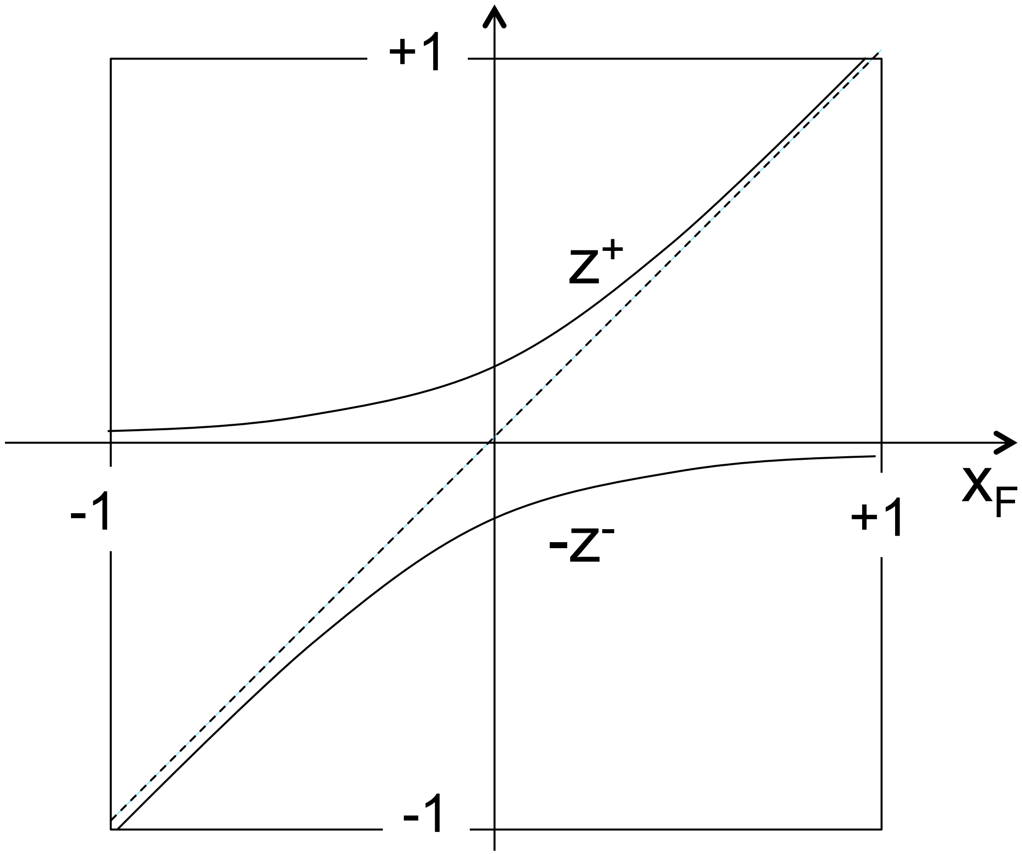
Relation between x_F, and z

In the hadronic c.m. frame

$z = z^+ = p^+ / W$,

$z^- = p^- / W$,

$x_F = 2\, p_z / W$



$$z^+ = \tfrac{1}{2}\,[\, x_F + \mathrm{sqrt}\{x_F^{\,2} + 4(m^2+\mathbf{p}_T^{\,2})/W^2\}\,]$$

Lower $\mathbf{p}_T$ of the first-rank hadron

$\mathbf{p}_{T/string} = (\mathbf{k}_T - \mathbf{k'}_T)_{/string}$

$\rightarrow \langle \mathbf{p}_T^2 \rangle_{/string} = 2 \langle \mathbf{k}_T^2 \rangle - 2 \langle \mathbf{k}_T . \mathbf{k'}_T \rangle \sim 2 \langle \mathbf{k}_T^2 \rangle_{/string}$

except fo the first rank, because $\mathbf{k}_{1T/string} = 0$.

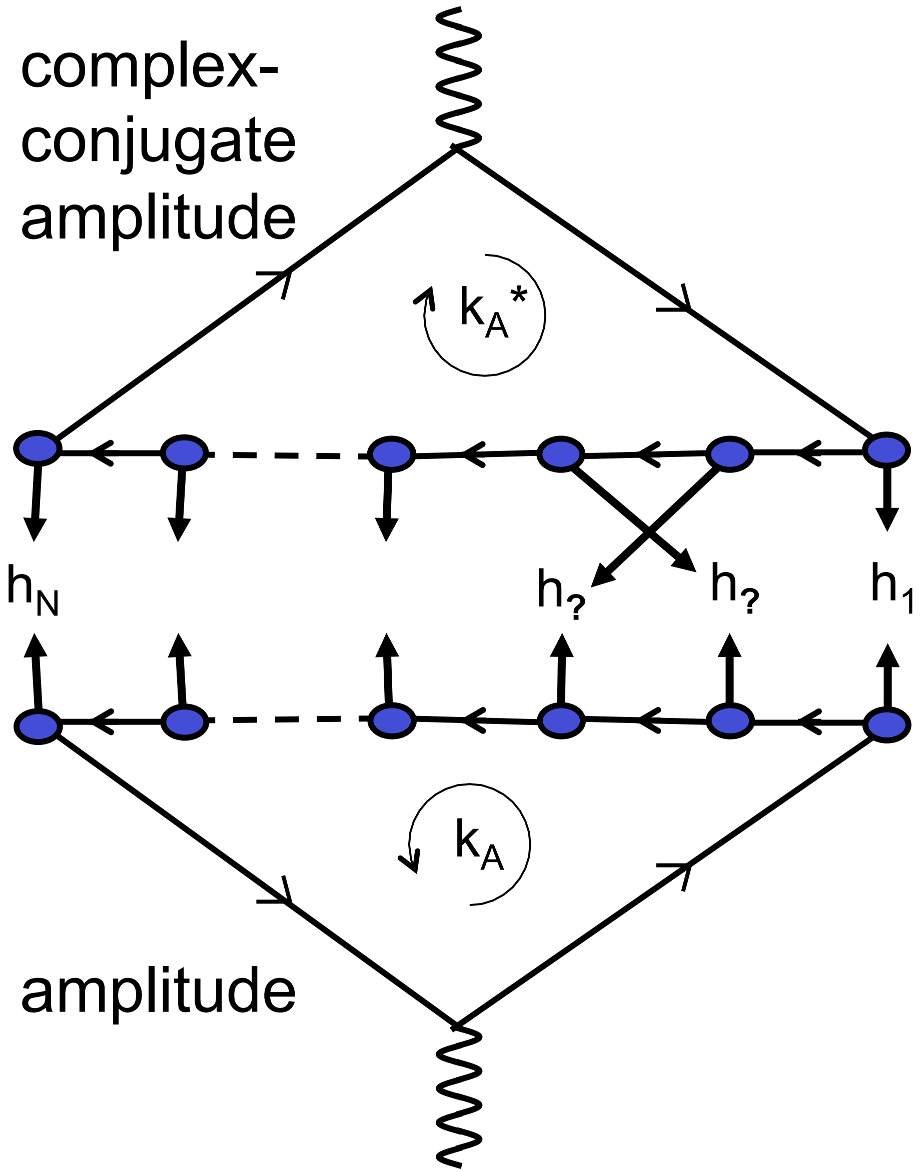
➔ the recursive model predicts, at large but equal z,

$$\langle \mathbf{p}_T^2 \rangle_{\text{favored}} < \langle \mathbf{p}_T^2 \rangle_{\text{unfavored}}$$

For instance, in DIS on protons : $\langle \mathbf{p}_T^2 \rangle(\pi^+) < \langle \mathbf{p}_T^2 \rangle(\pi^-)$

Is it the case experimentally ?

Non-existence of a theoretical jet axis



k_A and k_A^* are internal momenta of quark loops
= integration variables.

$k_A - k_A^*$ has no classical counter-part. When $k_A \neq k_A^*$ one can speak of a $k_A \times k_A^*$ interference.

Also the **ranks** of the hadrons are theoretically ambiguous, due to crossing between identical particles.

Main conclusions

- We have proposed a new splitting formula introducing the $\mathbf{k}_T$ correlations, which are missing in PYTHIA.

- Our splitting function also depends on a new parameter *c*. Kinematical constraints forbid string decay in some regions, suggesting c=1.

- In DIS, $\langle \mathbf{p}_{T/\mathbf{Q}} \rangle^2 = \langle \mathbf{p}_{T/string} \rangle^2 + x_F^2 \langle \mathbf{k}_{T/\mathbf{Q}} \rangle^2$.

- The recursive models generally predict a smaller $\langle \mathbf{p}_T^2 \rangle$ for a favored meson than for an unfavored one, at large but equal z. This has to be clarified on the experimental side.

- Jet axis and hadron rank cannot be defined withoput ambiguity, even theoretically.

Thank you for attention !