Global extraction of quark TMDs

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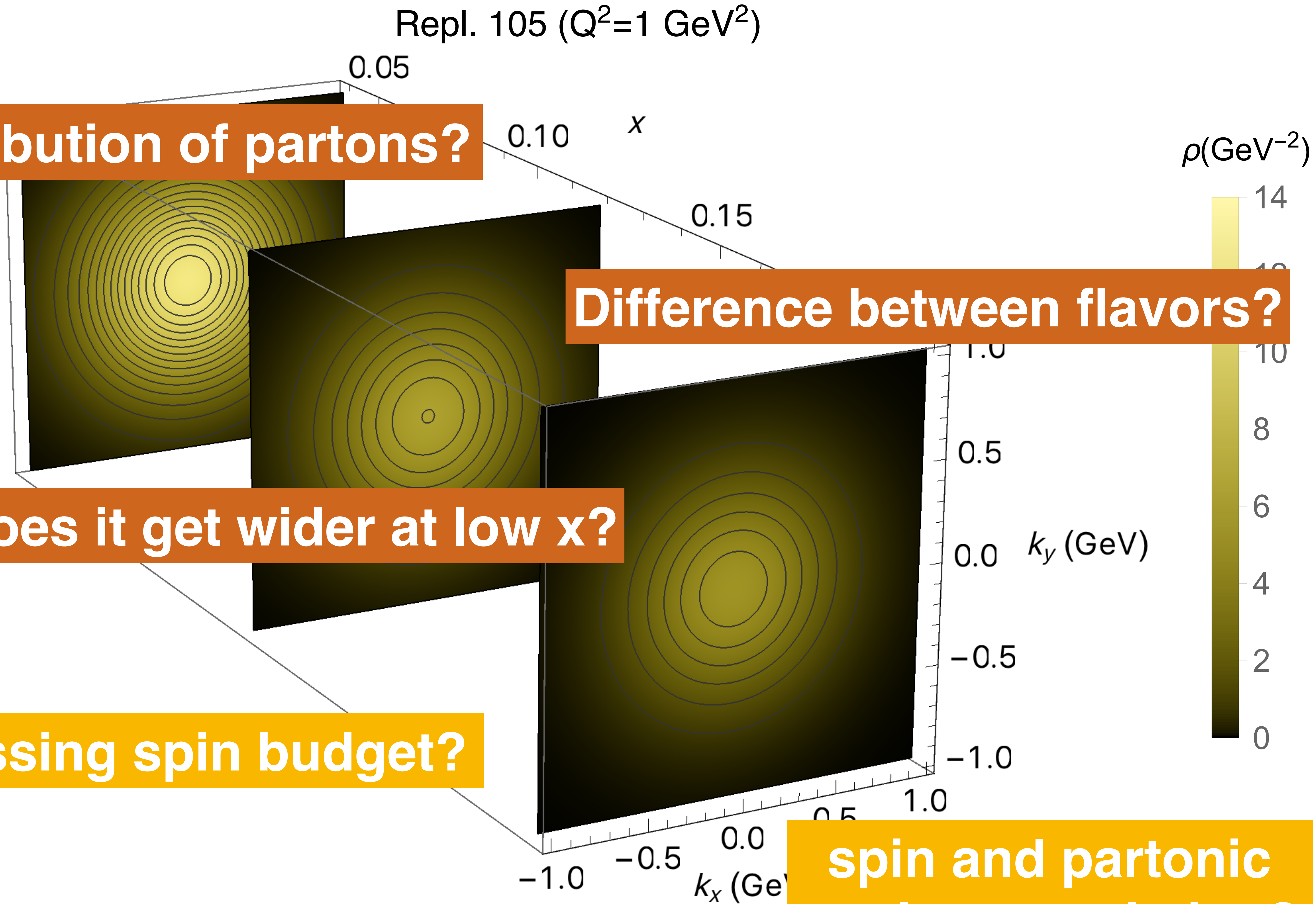
3DSPIN
MAPPING THE PROTON IN 3D

European Research Council

INFN
Istituto Nazionale di Fisica Nucleare

UNIVERSITÀ DI PAVIA

3DSPIN: structure of the nucleon



distribution of partons?

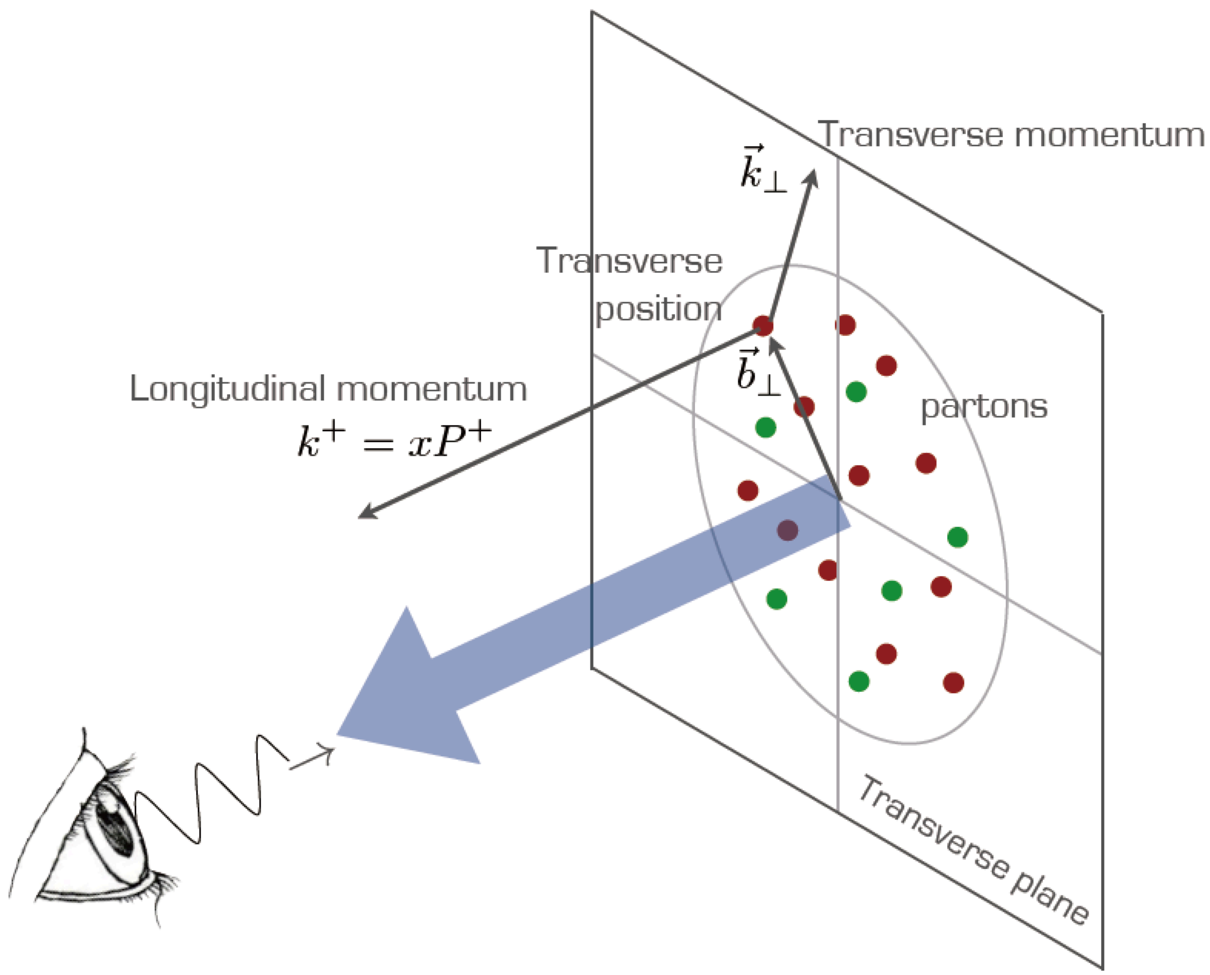
Difference between flavors?

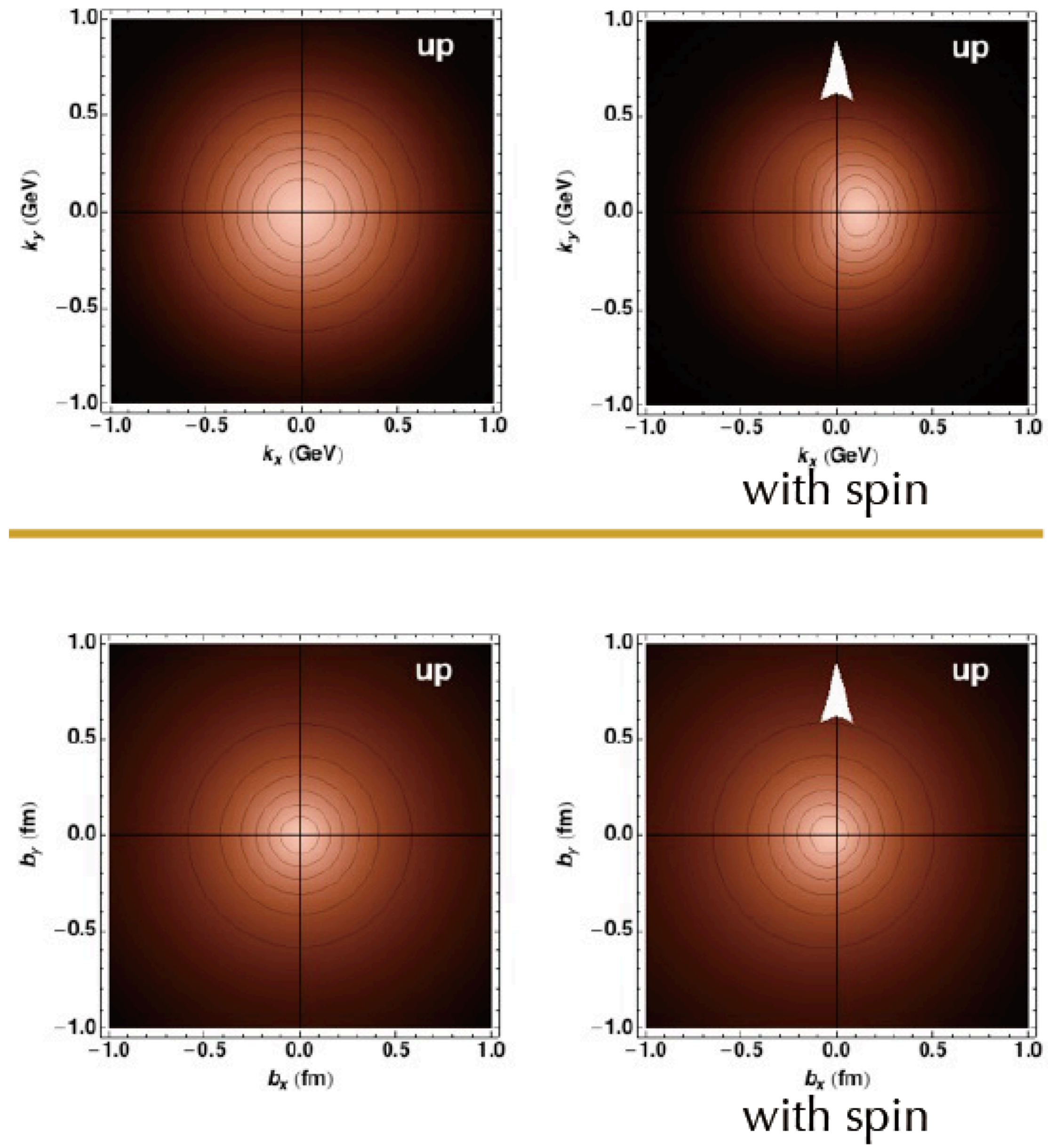
Does it get wider at low x?

missing spin budget?

spin and partonic motion correlation?

Orbital motion - Nucleon Structure from 1D to 3D





Generalized parton distribution (GPD)
Transverse momentum dependent parton distribution (TMD)

[Bacchetta's talk (2016)]

Transverse Momentum Distributions: TMD PDF

Unpolarized

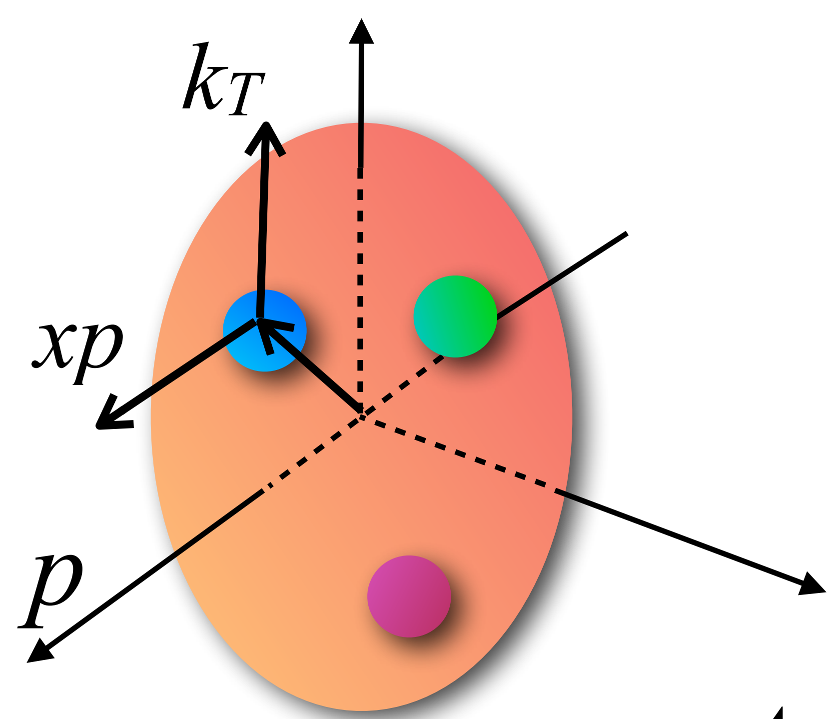
nucleon pol. \ quark pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

dependence on:

longitudinal momentum fraction x

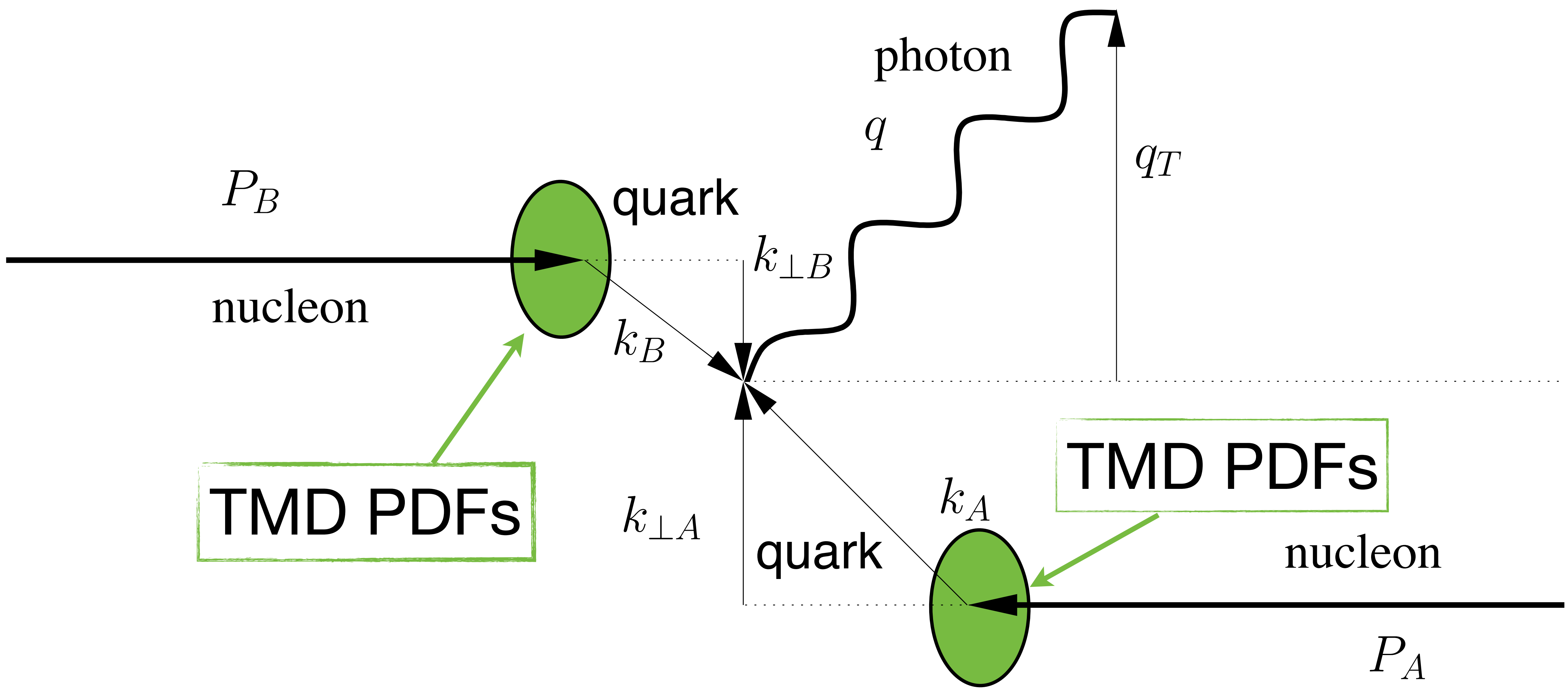
transverse momentum $\boldsymbol{k}_\perp$

energy scale



Extraction from SIDIS & Drell-Yan

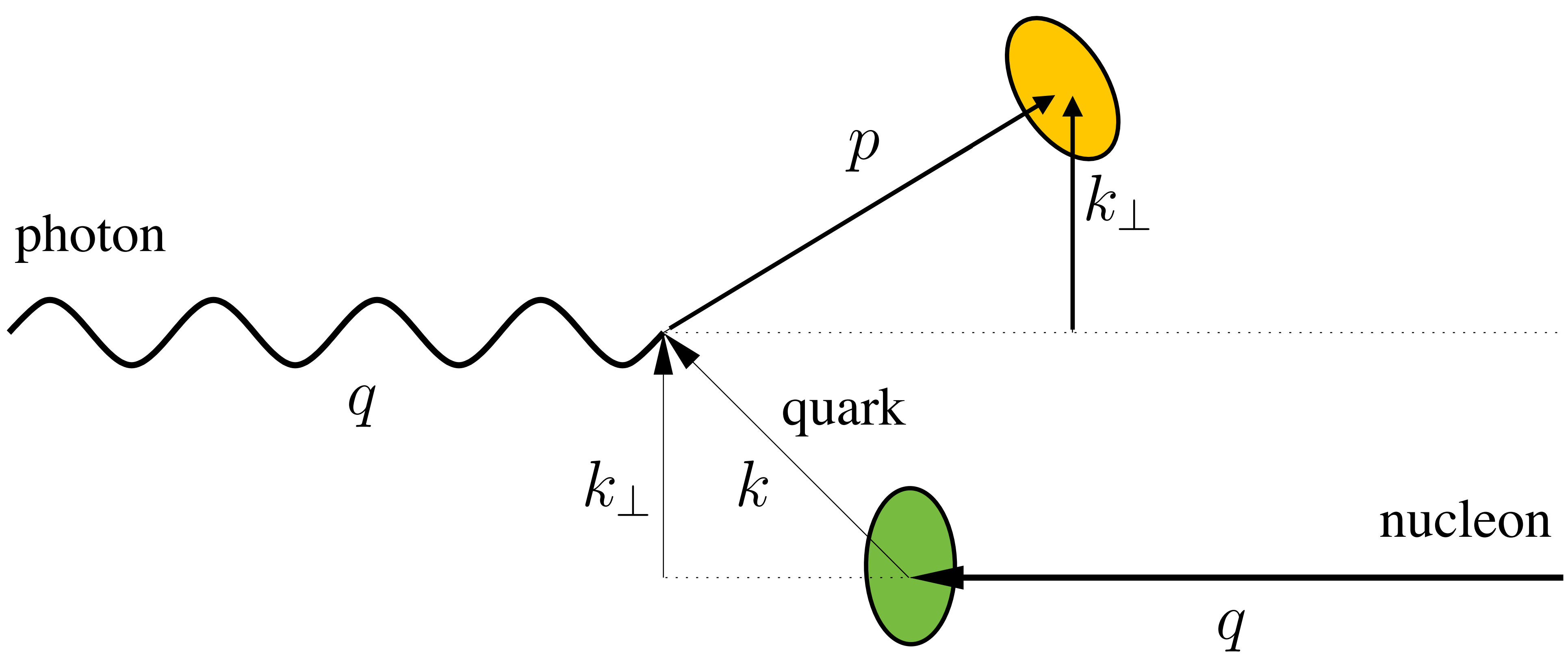
Drell-Yan \ Z production



$$A + B \to \gamma^* \to l^+ l^- \qquad A + B \to Z \to l^+ l^-$$

Extraction from SIDIS & Drell-Yan

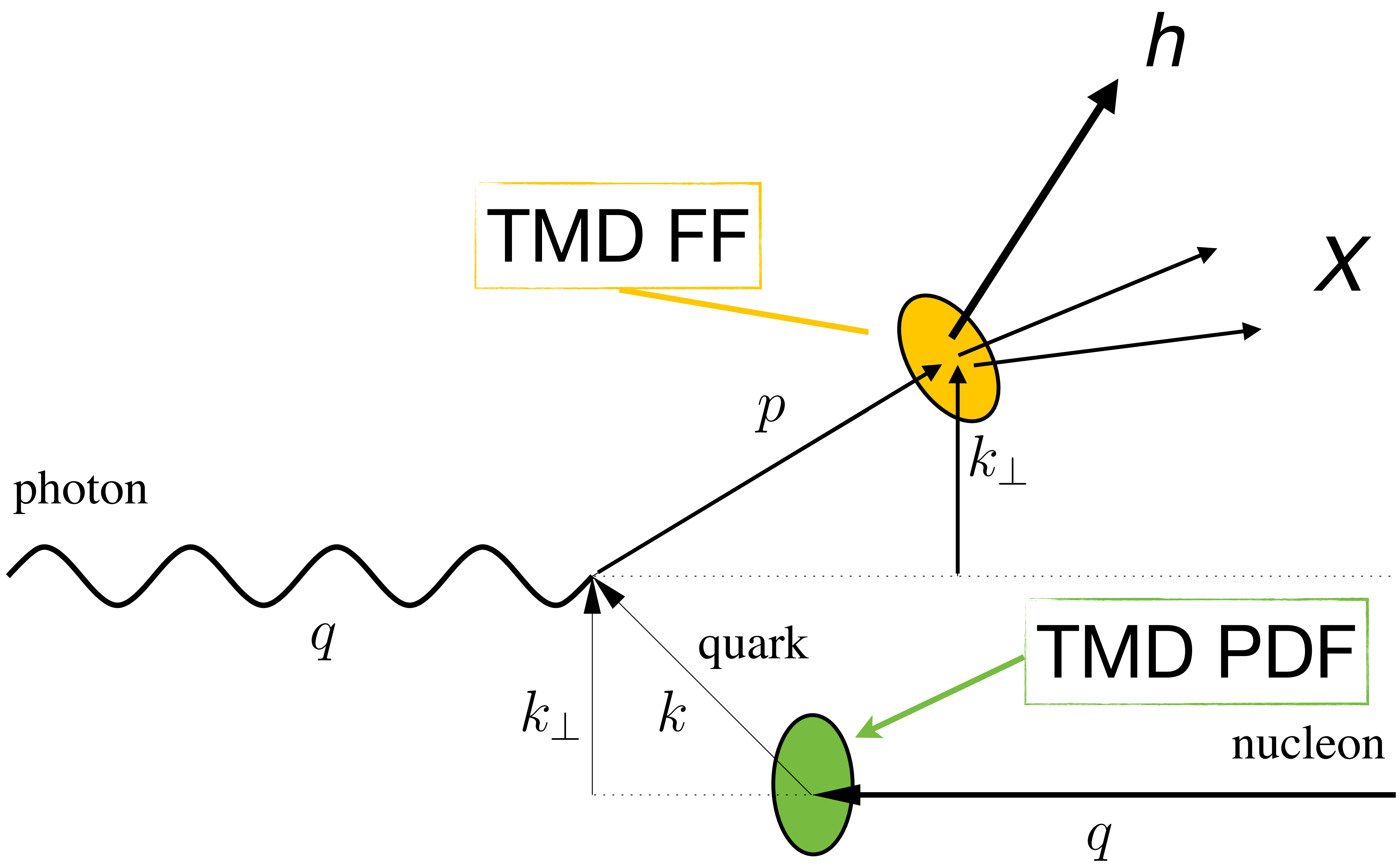
Semi-inclusive Deep Inelastic Scattering



$$l(\ell) + N(\mathcal{P}) \rightarrow l(\ell') + h(\mathcal{P}_h) + X$$

Extraction from SIDIS & Drell-Yan

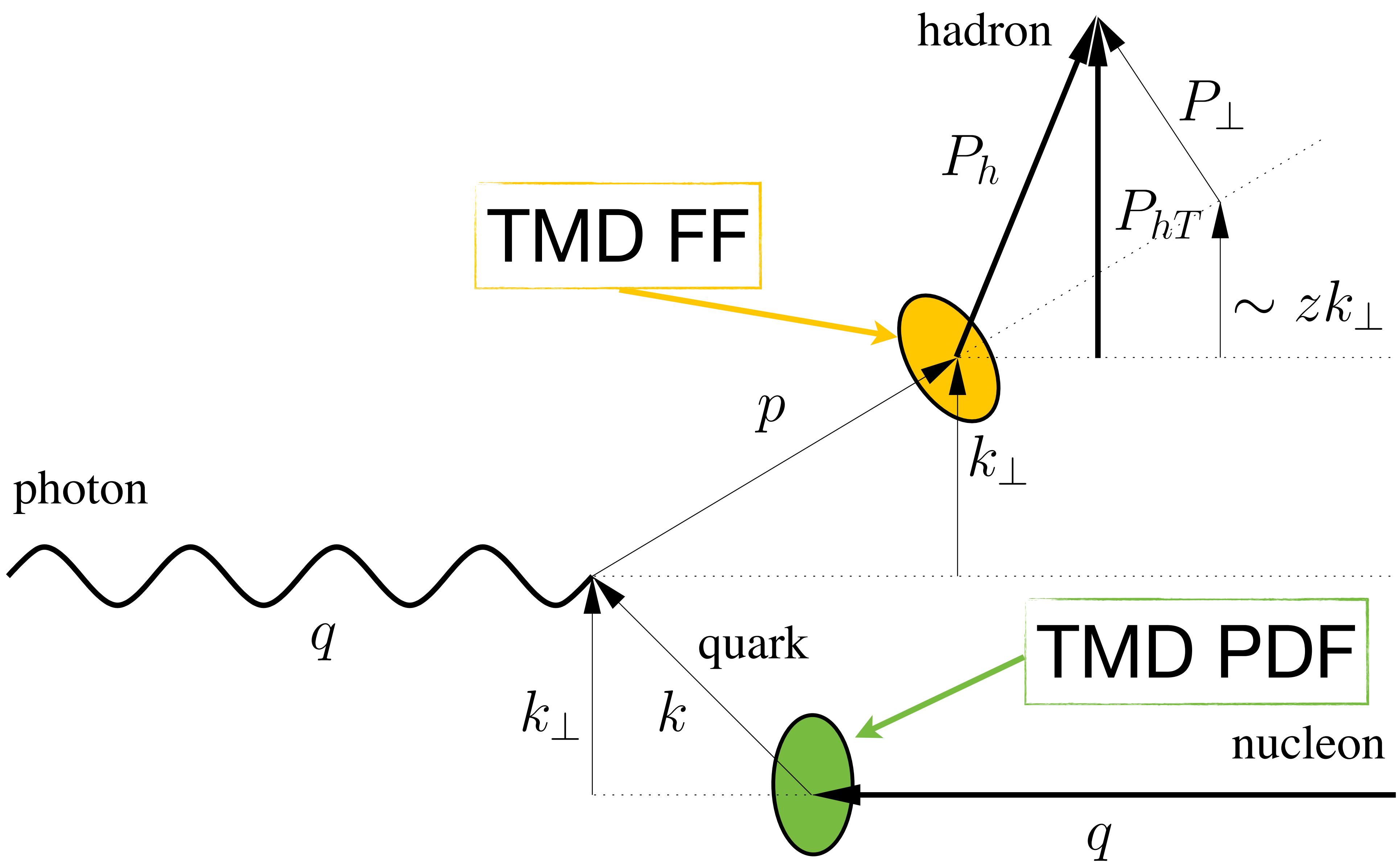
Semi-inclusive Deep Inelastic Scattering



$$l(\ell) + N(\mathcal{P}) \to l(\ell') + h(\mathcal{P}_h) + X$$

Extraction from SIDIS & Drell-Yan

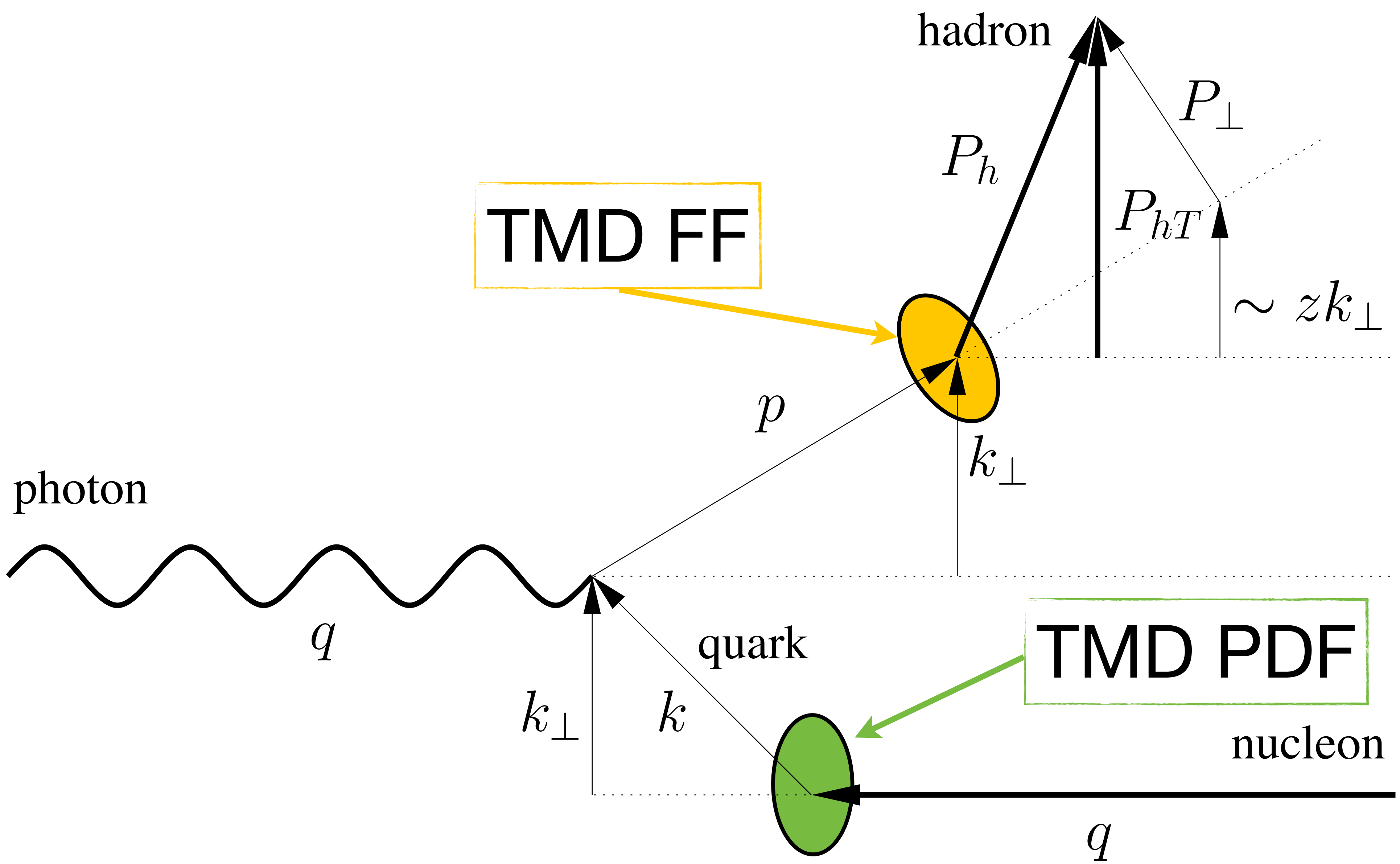
Semi-inclusive Deep Inelastic Scattering



$$l(\ell) + N(\mathcal{P}) \to l(\ell') + h(\mathcal{P}_h) + X$$

Extraction from SIDIS & Drell-Yan

Semi-inclusive Deep Inelastic Scattering



universality

$$l(\ell) + N(\mathcal{P}) \to l(\ell') + h(\mathcal{P}_h) + X$$

TMDs: Fragmentation Function

quark pol.

U	L	T
D_1		$H_1^\perp$

Unpolarized

TMD Fragmentation Functions
(TMD FFs)

dependence on:

longitudinal momentum fraction **z**

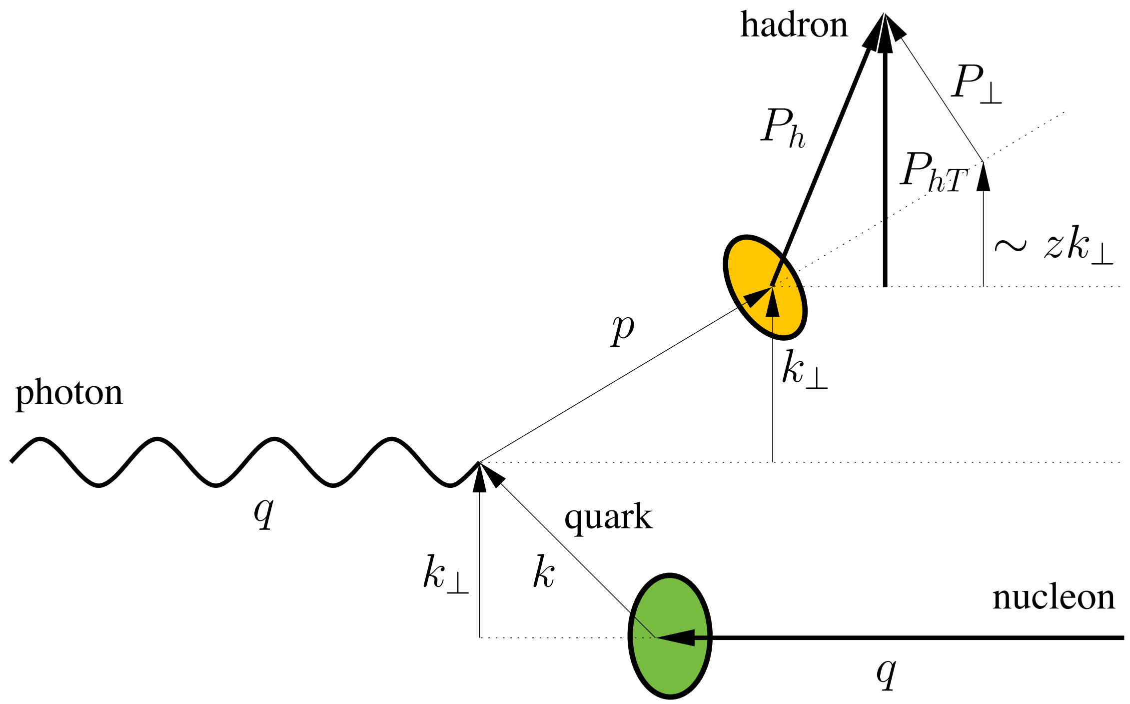
transverse momentum $\boldsymbol{P}_\perp$

energy scale

Structure functions and TMDs

multiplicities

$$m_N^h\left(x,z,\boldsymbol{P}_{hT}^2,Q^2\right)=\frac{d\sigma_N^h/\left(dx\,dz\,d\boldsymbol{P}_{hT}^2\,dQ^2\right)}{d\sigma_{DIS}/\left(dx\,dQ^2\right)}\approx\frac{\pi F_{UU,T}\left(x,z,\boldsymbol{P}_{hT}^2,Q^2\right)}{F_T(x,Q^2)}$$



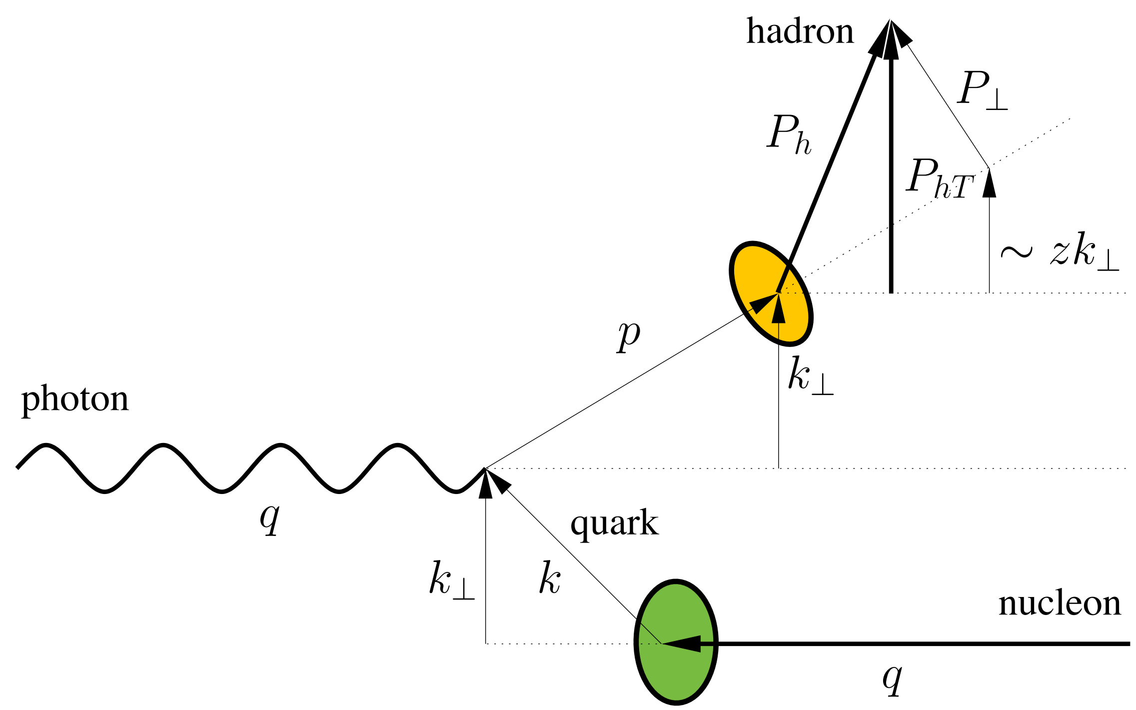
$$F_{UU,T}\left(x,z,P_{hT}^2,Q^2\right)=\sum_a \mathcal{H}_{UU,T}^a\left(Q^2;\mu^2\right)\int d^2k_T\,d^2P_T\,f_1^a\left(x,k_T^2;\mu^2\right)D_1^{h/a}\left(z,P_T^2;\mu^2\right)$$

$$\cdot\delta^2(zk_T-P_{hT}+P_T)+Y_{UU,T}(Q^2,P_{hT}^2)+\mathcal{O}\left(M^2/Q^2\right)$$

Structure functions and TMDs

multiplicities

$$m_N^h\left(x,z,\boldsymbol{P}_{hT}^2,Q^2\right)=\frac{d\sigma_N^h/\left(dxdzd\boldsymbol{P}_{hT}^2dQ^2\right)}{d\sigma_{DIS}/\left(dxdQ^2\right)}\approx\frac{\pi F_{UU,T}\left(x,z,\boldsymbol{P}_{hT}^2,Q^2\right)}{F_T(x,Q^2)}$$



$$F_{UU,T}\left(x,z,P_{hT}^2,Q^2\right)=\sum_a\mathcal{H}_{UU,T}^a\left(Q^2;\mu^2\right)\int d^2k_T\,d^2P_T\,f_1^a\left(x,k_T^2;\mu^2\right)D_1^{h/a}\left(z,P_T^2;\mu^2\right)$$

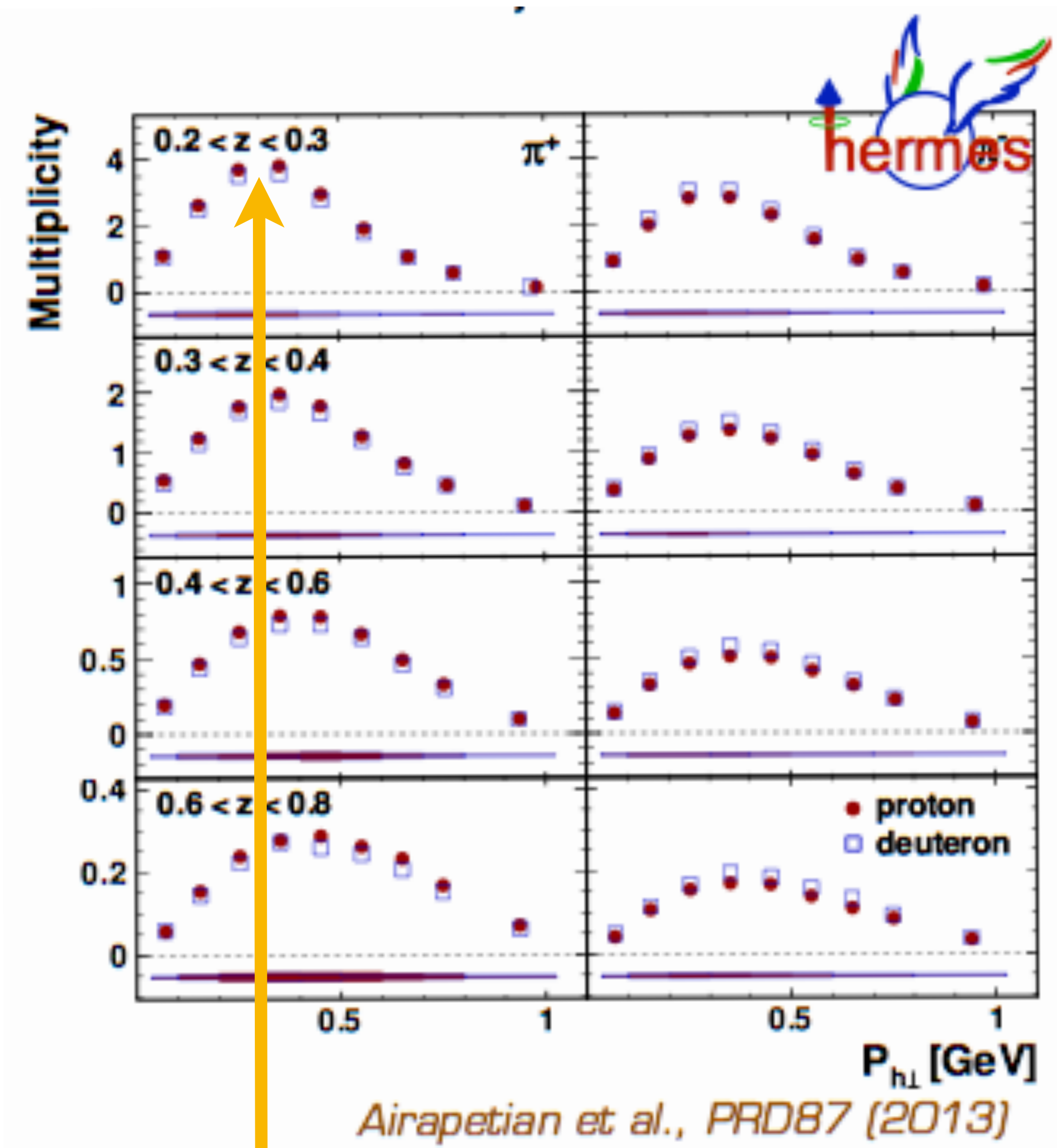
$$\cdot\delta^2(zk_T-P_{hT}+P_T)+Y_{UU,T}(Q^2,P_{hT}^2)+\mathcal{O}\left(M^2/Q^2\right)$$

not implemented in present fits

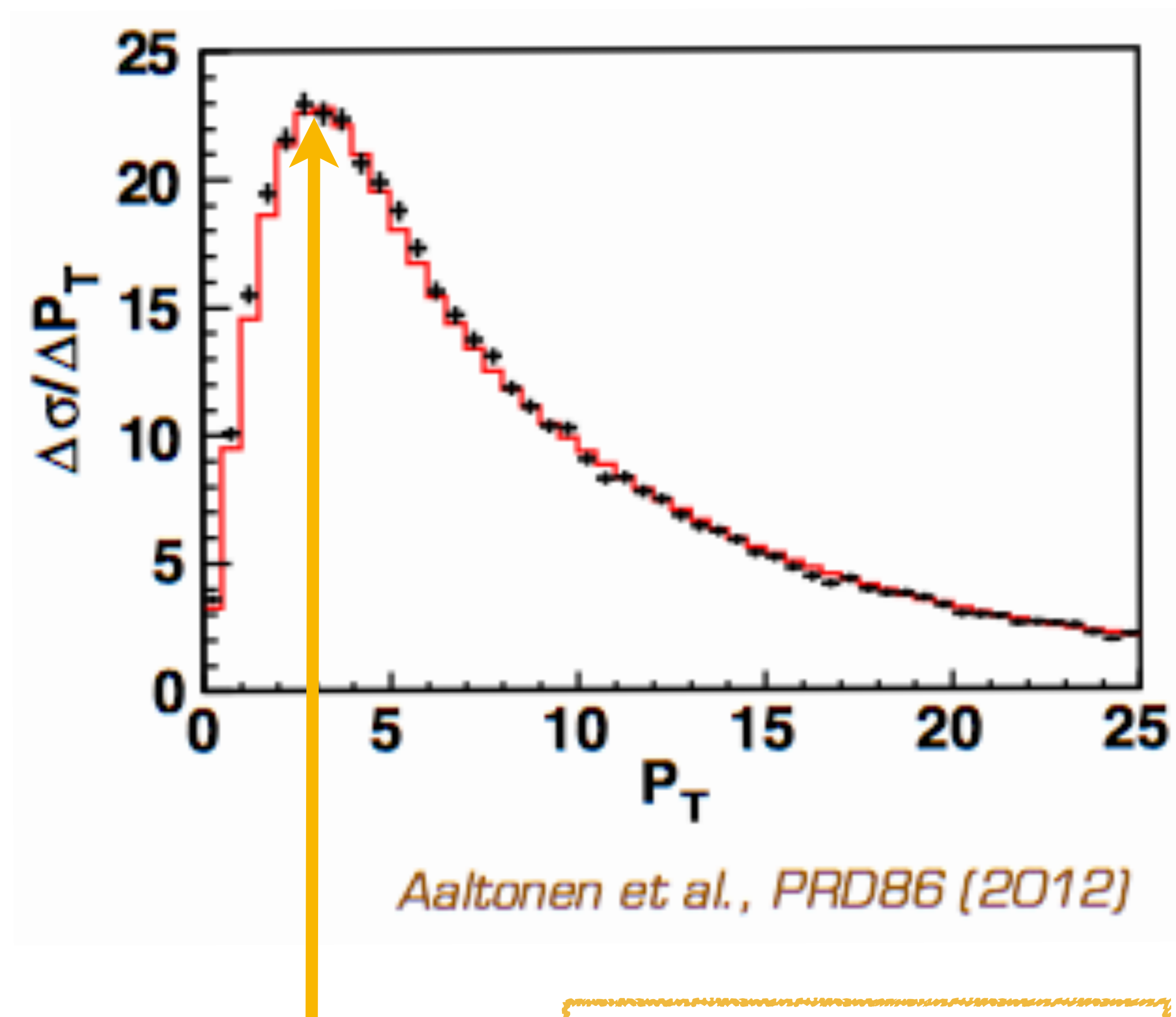
TMD Evolution

to reproduce shift of
TMD peak with energy scale

HERMES, $Q \approx 1.5$ GeV



CDF, $Q \approx 91$ GeV



Width of TMDs changes of one order of magnitude → EVOLUTION

Evolved TMDs

Fourier transform: b_T space

$$f_1^a(x, k_\perp; \mu^2) = \frac{1}{2\pi} \int d^2 b_\perp e^{-i b_\perp \cdot k_\perp} \widetilde{f}_1^a(x, b_\perp; \mu^2)$$

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

nonperturbative part of evolution

(Wilson Coefficient)

(Sudakov form factor)

collinear PDF

pQCD

non-perturbative part of TMD

Perturbative ingredients

PAVIA 2017

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T)\ln\frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

$A_1(\mathcal{O}(\alpha_S^1))$	$A_2(\mathcal{O}(\alpha_S^2))$	$A_3(\mathcal{O}(\alpha_S^3))$	$\dots$
	$B_1(\mathcal{O}(\alpha_S^1))$	$B_2(\mathcal{O}(\alpha_S^2))$	$\dots$
$C_0(\mathcal{O}(\alpha_S^0))$	$C_1(\mathcal{O}(\alpha_S^1))$	$C_2(\mathcal{O}(\alpha_S^2))$	$\dots$
$H_0(\mathcal{O}(\alpha_S^0))$	$H_1(\mathcal{O}(\alpha_S^1))$	$H_2(\mathcal{O}(\alpha_S^2))$	$\dots$
	$Y_1(\mathcal{O}(\alpha_S^1))$	$Y_2(\mathcal{O}(\alpha_S^2))$	$\dots$

Perturbative ingredients

PAVIA 2017

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \left(\tilde{C}_{a/i} \otimes f_1^i\right)(x, b_*; \mu_b)\, e^{\tilde{S}(b_*;\mu_b,\mu)}\, e^{g_K(b_T)\ln\frac{\mu}{\mu_0}}\, \hat{f}_{\mathrm{NP}}^a(x, b_T)$$

NLL

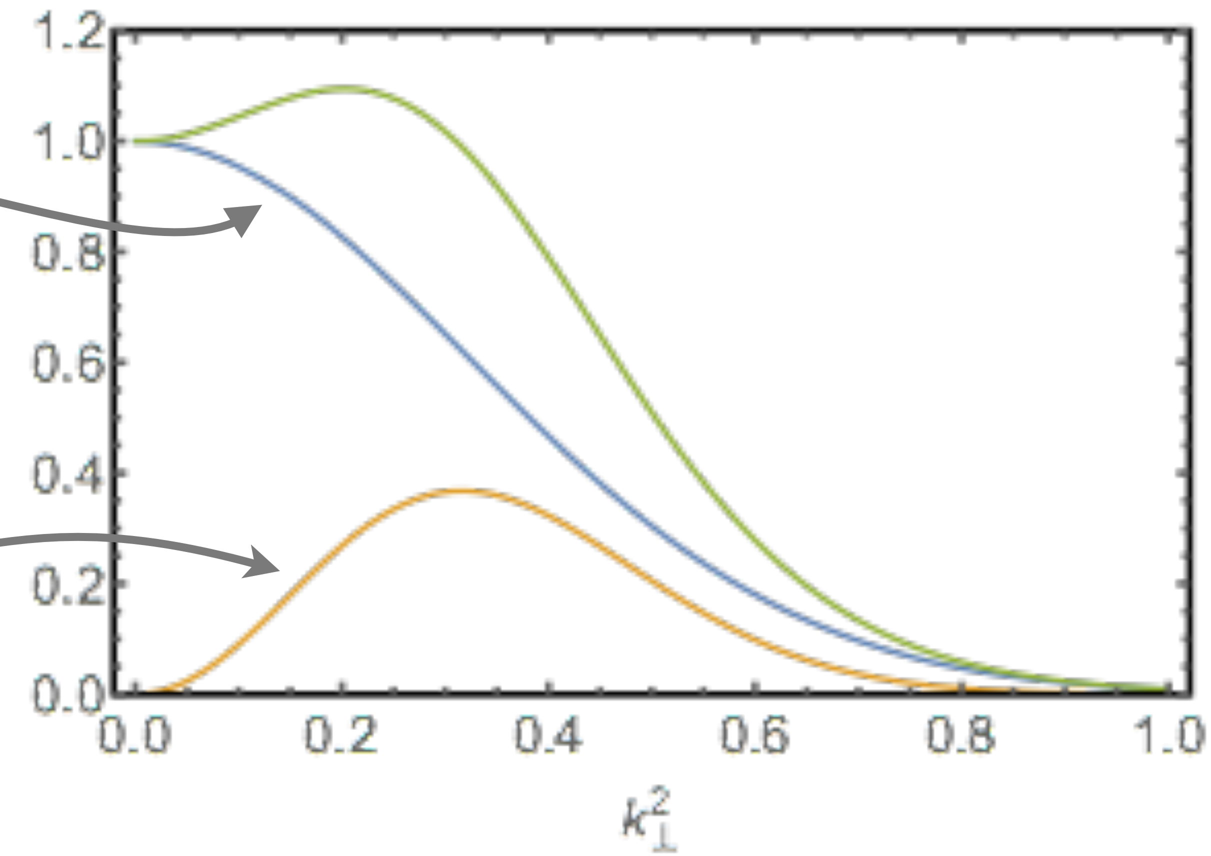
$A_1(\mathcal{O}(\alpha_S^1))$	$A_2(\mathcal{O}(\alpha_S^2))$	$A_3(\mathcal{O}(\alpha_S^3))$	$\ldots$
	$B_1(\mathcal{O}(\alpha_S^1))$	$B_2(\mathcal{O}(\alpha_S^2))$	$\ldots$
$C_0(\mathcal{O}(\alpha_S^0))$	$C_1(\mathcal{O}(\alpha_S^1))$	$C_2(\mathcal{O}(\alpha_S^2))$	$\ldots$
$H_0(\mathcal{O}(\alpha_S^0))$	$H_1(\mathcal{O}(\alpha_S^1))$	$H_2(\mathcal{O}(\alpha_S^2))$	$\ldots$
	$Y_1(\mathcal{O}(\alpha_S^1))$	$Y_2(\mathcal{O}(\alpha_S^2))$	$\ldots$

LO

Model: non perturbative elements

input TMD PDF (Q^2=1GeV^2)

$$\hat{f}^a_{NP} = \mathcal{F}.\mathcal{T}. \text{ of } \left(e^{-\frac{k_T^2}{g1a}} + \lambda k_T^2 e^{-\frac{k_T^2}{g1a}} \right)$$



inspired by models (p-wave contribution)

x-dependence of TMD width

$$g_1(x) = N_1 \frac{(1-x)^\alpha \, x^\sigma}{(1-\hat{x})^\alpha \, \hat{x}^\sigma}$$

where

$$N_1 \equiv g_1(\hat{x})$$
$$\hat{x} = 0.1$$

Model: non perturbative elements

11 parameters:

$N_1, \alpha, \sigma, \lambda$ 4 for TMD PDF

$N_3, N_4, \beta, \delta, \gamma, \lambda_F$ 6 for TMD FF

$$g_K = -g_2 \frac{b_T^2}{2}$$

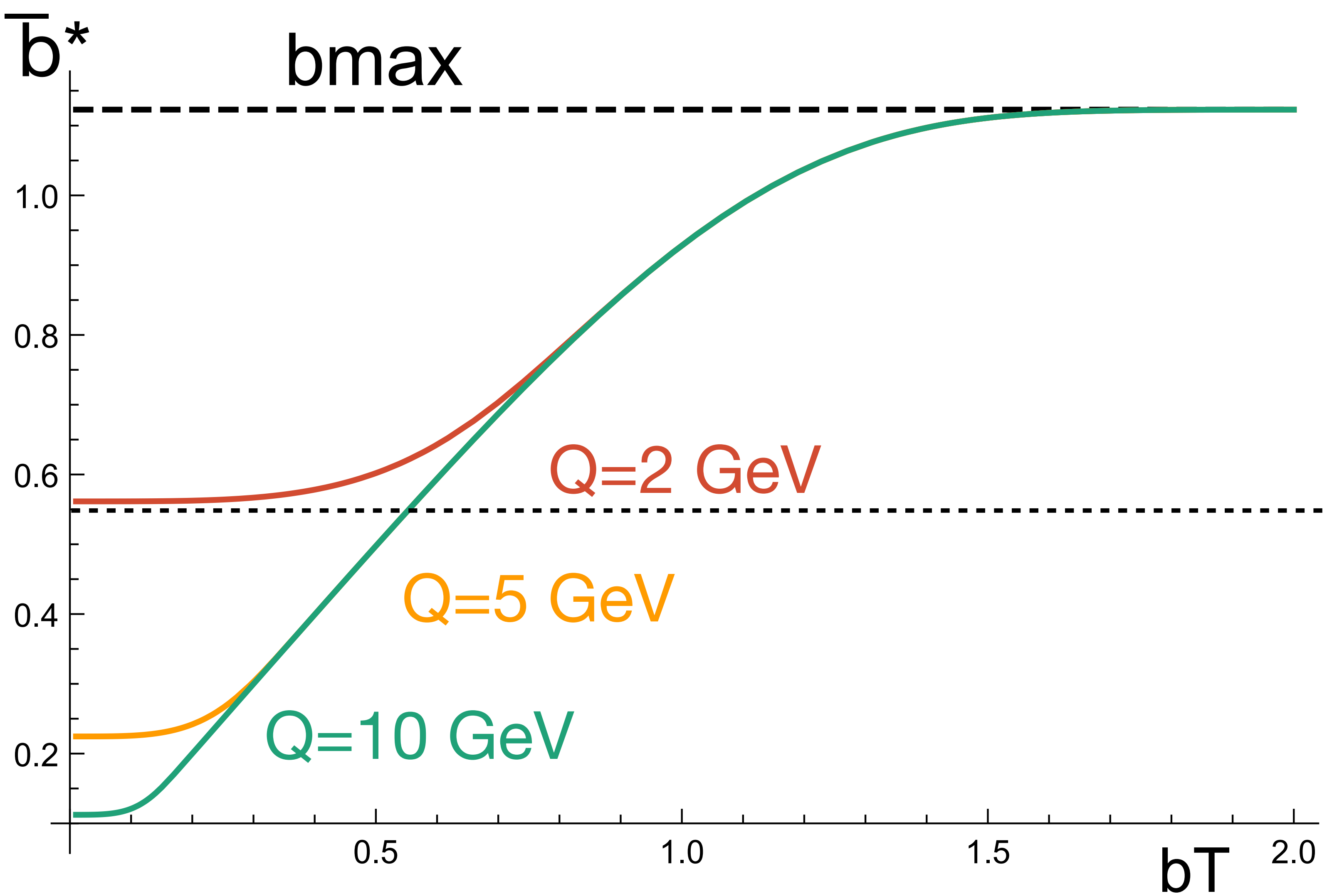
1 for NP contribution to TMD evolution

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

These are choices!

$$\mu_b = 2e^{-\gamma_E}/b_*$$

$$\bar{b}_\star(b_T; b_{\rm min}, b_{\rm max}) = b_{\rm max} \left(\frac{1 - e^{-b_T^4/b_{\rm max}^4}}{1 - e^{-b_T^4/b_{\rm min}^4}} \right)^{1/4}$$



$$b_{\rm max} = 2e^{-\gamma_E}$$

$$b_{\rm min} = 2e^{-\gamma_E}/Q$$

The importance of bmin is a signal that we are exiting the proper TMD region and approaching the region of collinear factorization, especially in SIDIS data at low Q

Experimental data

COMPASS

SIDIS μN

6252
data points

hermes

SIDIS eN

1514
data points

E288

E605

Drell-Yan

203
data points

Z Production

90
data points

Data selection and analysis

$Q2 > 1.4\ \mathrm{GeV}^2$

$0.2 < z < 0.7$

$P_{hT}\ ,\ q_T < \mathrm{Min}[0.2Q\ ,\ 0.7Qz] + 0.5\ \mathrm{GeV}$

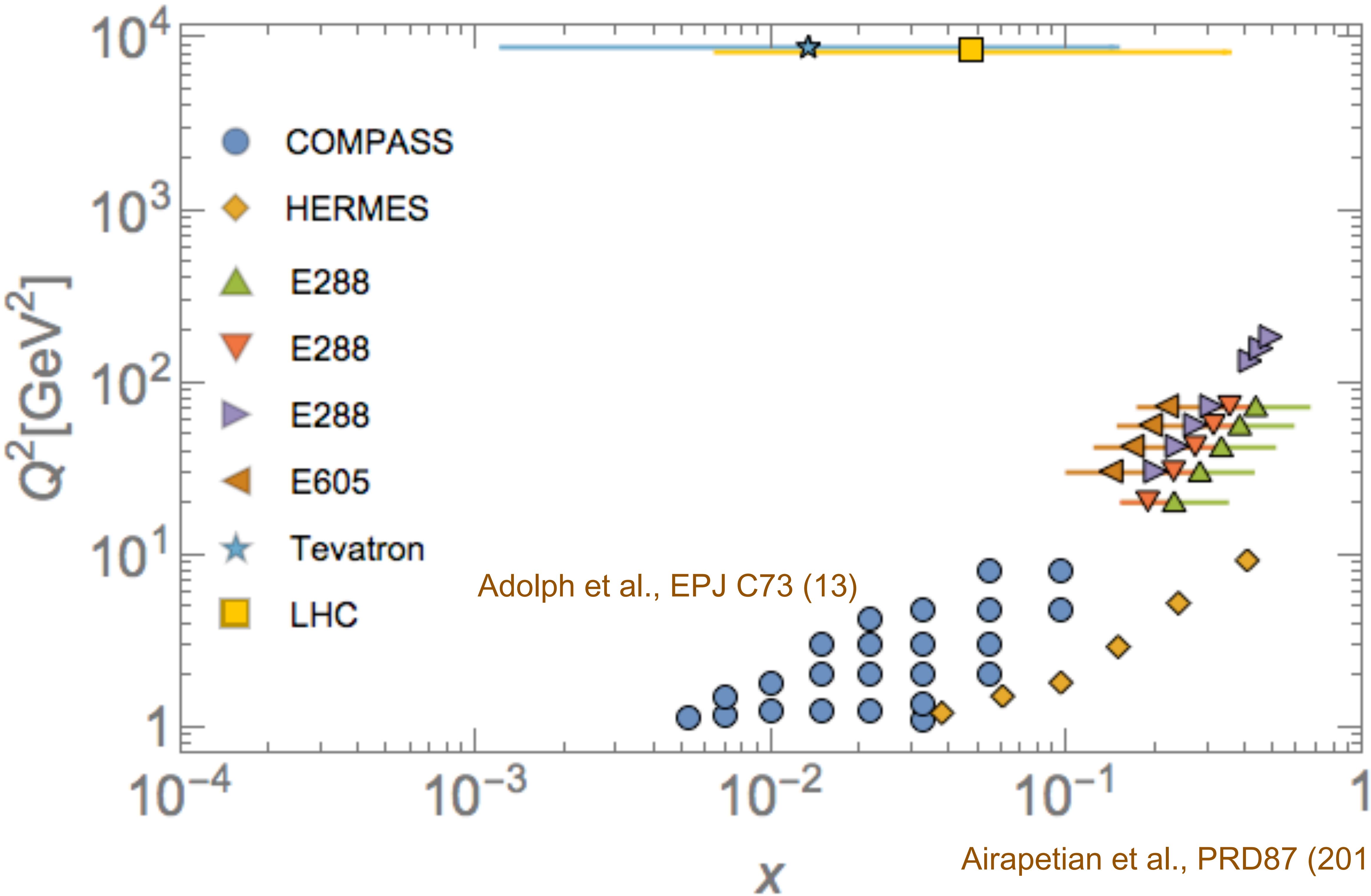
Motivations behind kinematical cuts

TMD factorization ($Ph_T/z \ll Q^2$)

Avoid target fragmentation (low z)

and exclusive contributions (high z)

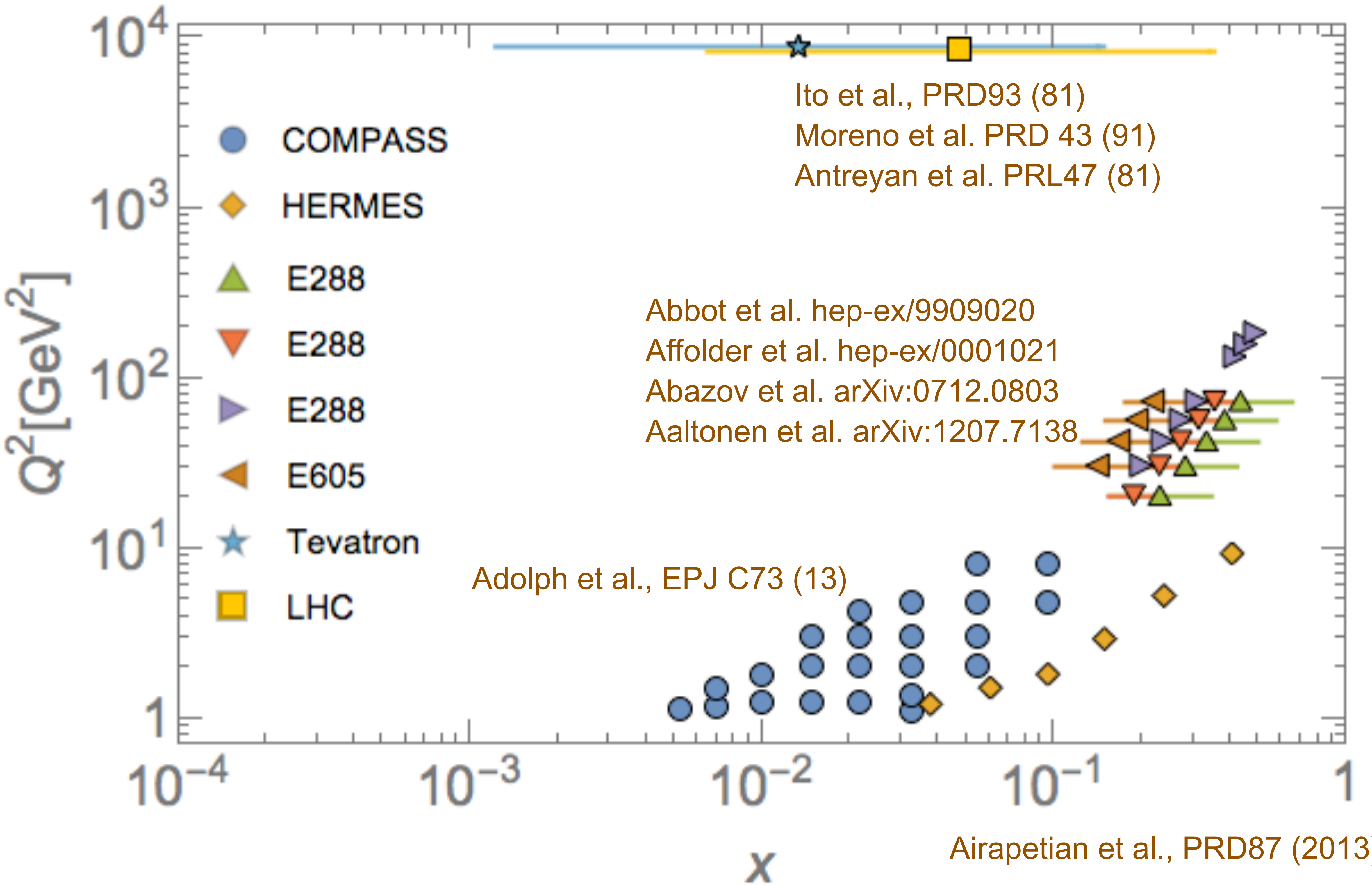
Data region



Adolph et al., EPJ C73 (13)

Airapetian et al., PRD87 (2013)

Data region



10^4
10^3
10^2
10^1
1
Q^2[GeV^2]
COMPASS
HERMES
E288
E288
E288
E605
Tevatron
LHC
Ito et al., PRD93 (81)
Moreno et al. PRD 43 (91)
Antreyan et al. PRL47 (81)
Abbot et al. hep-ex/9909020
Affolder et al. hep-ex/0001021
Abazov et al. arXiv:0712.0803
Aaltonen et al. arXiv:1207.7138
Adolph et al., EPJ C73 (13)
10^-4
10^-3
10^-2
10^-1
1
x
Airapetian et al., PRD87 (2013)

	Framework	**SIDIS** HERMES	**SIDIS** COMPASS	**DY**	**Z production**	**# points**
KN 2006	NLL/NLO	✗	✗	✓	✓	98
Pavia 2013	No Evo	✓	✗	✗	✗	1539
Torino 2014	No Evo	✓ (separately)	✓ (separately)	✗	✗	576 (H) 6284 (C)
DEMS 2014	NNLL/NLO	✗	✗	✓	✓	223
Pavia 2017	NLL/LO	✓	✓	✓	✓	8059
SV 2017	NNLL/NNLO	✗	✗	✓	✓	309

An almost global fit

	Framework	HERMES	COMPASS	DY	Z production	N of points
Pavia 2017 (+ JLab)	LO-NLL	✔	✔	✔	✔	8059

[JHEP06(2017)081]

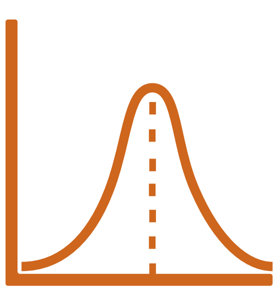
Summary of results

Total number of data points: 8059

Total number of free parameters: 11

➛ 4 for TMD PDFs ➛ 6 for TMD FFs

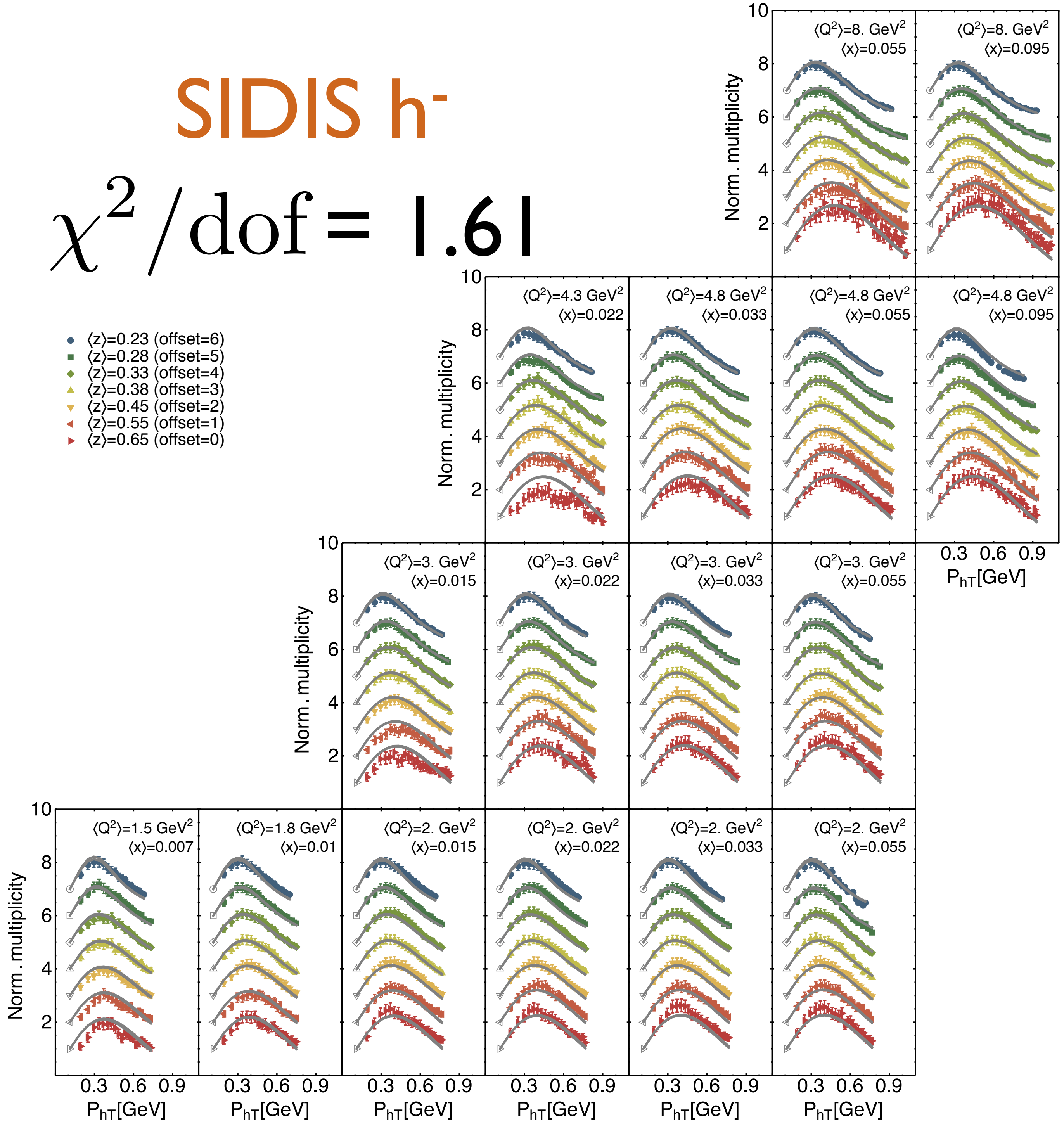
➛ 1 for TMD evolution



$$\chi^2/d.o.f. = 1.55 \pm 0.05$$

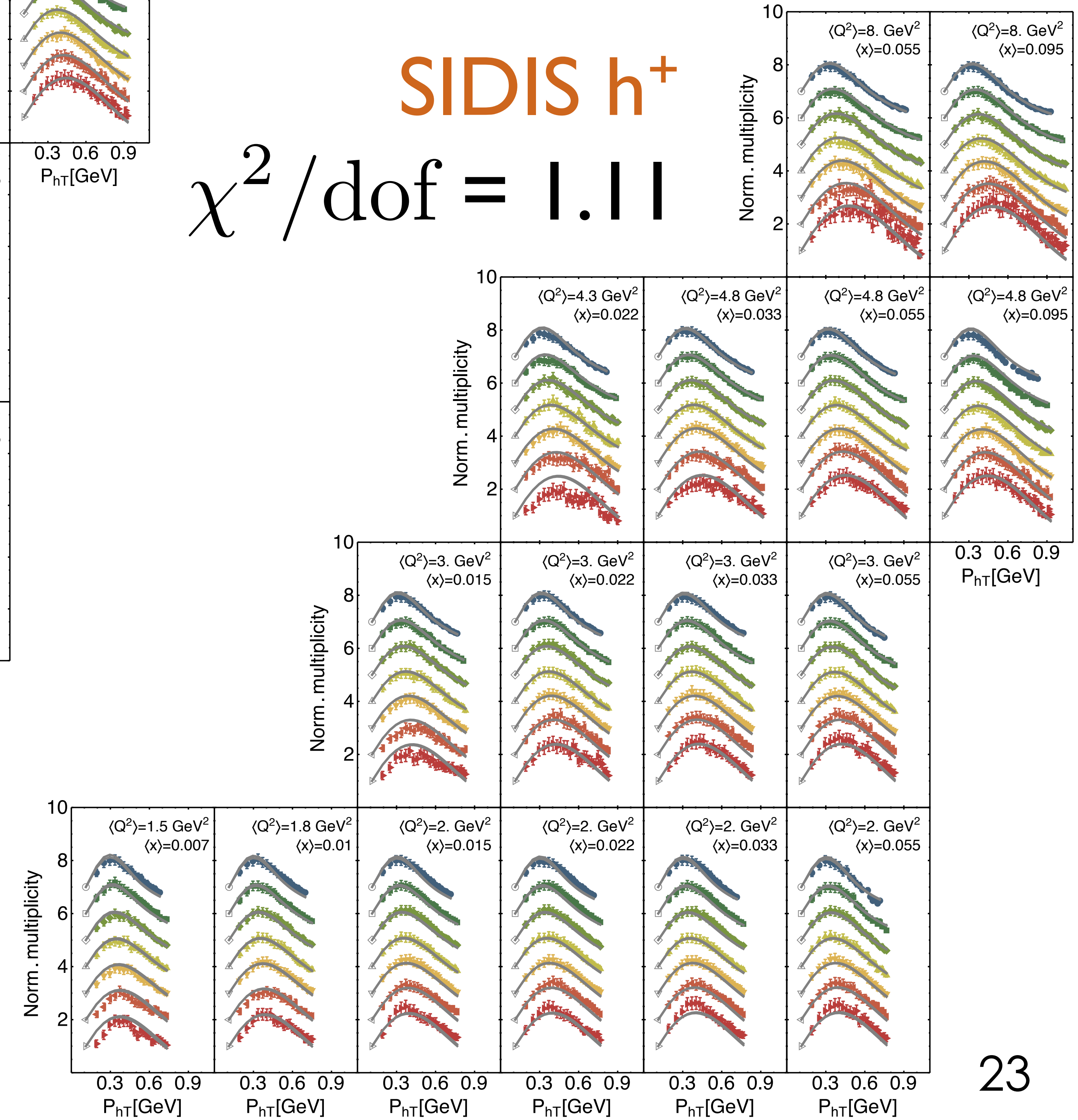
SIDIS h^-

$$\chi^2/\mathrm{dof} = 1.61$$

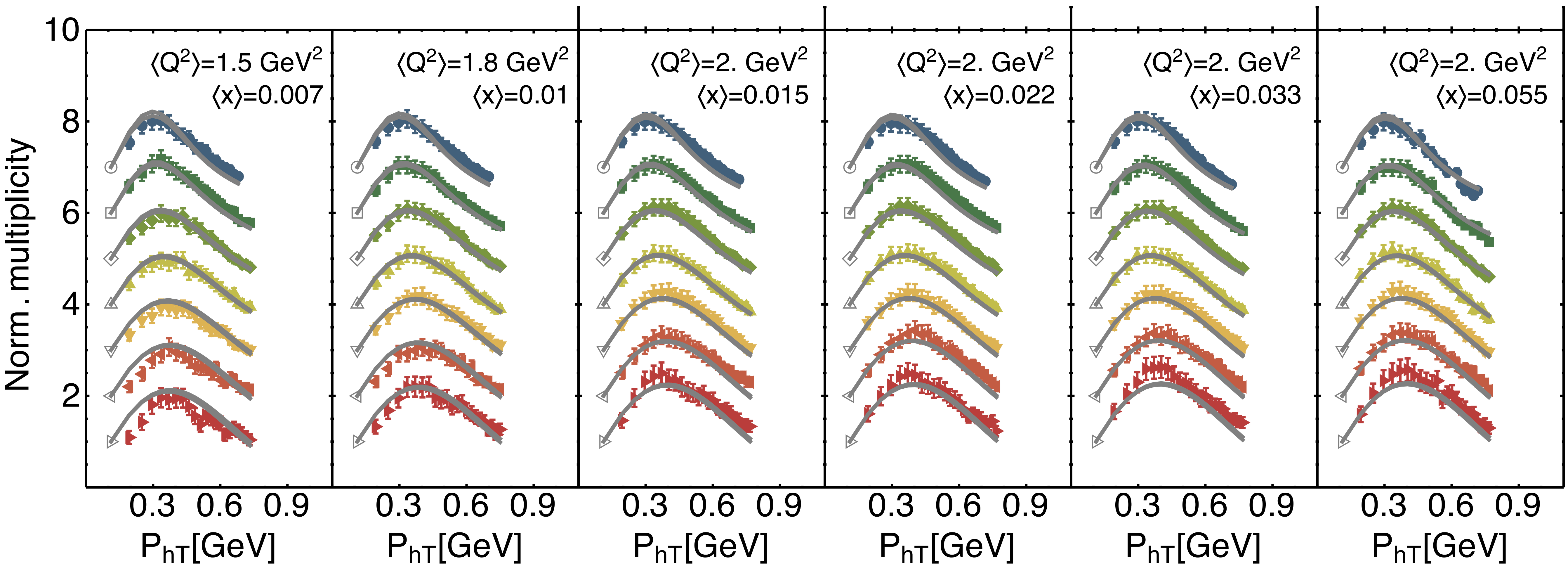


SIDIS h^+

$$\chi^2/\mathrm{dof} = 1.11$$



COMPASS data
SIDIS h+

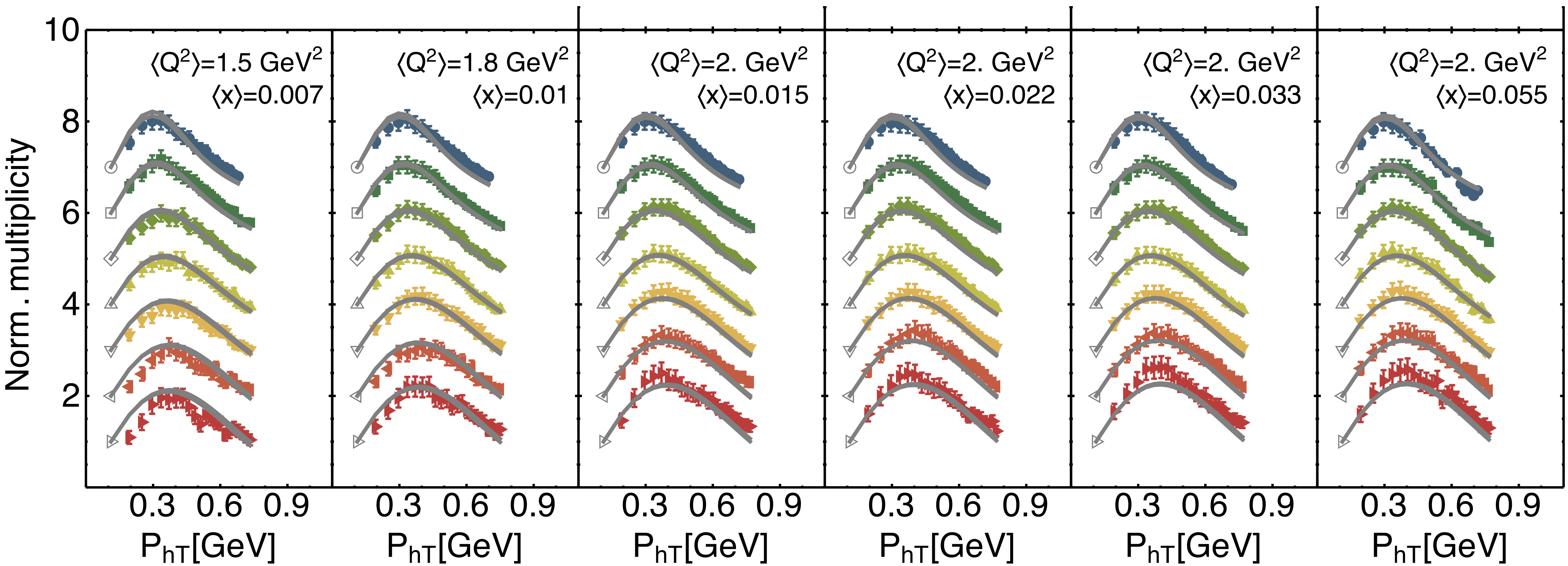


to avoid known problems
with Compass data normalization:

Observable

$$\frac{m_N^h\left(x, z, \boldsymbol{P}_{hT}^2, Q^2\right)}{m_N^h\left(x, z, \min[\boldsymbol{P}_{hT}^2], Q^2\right)}$$

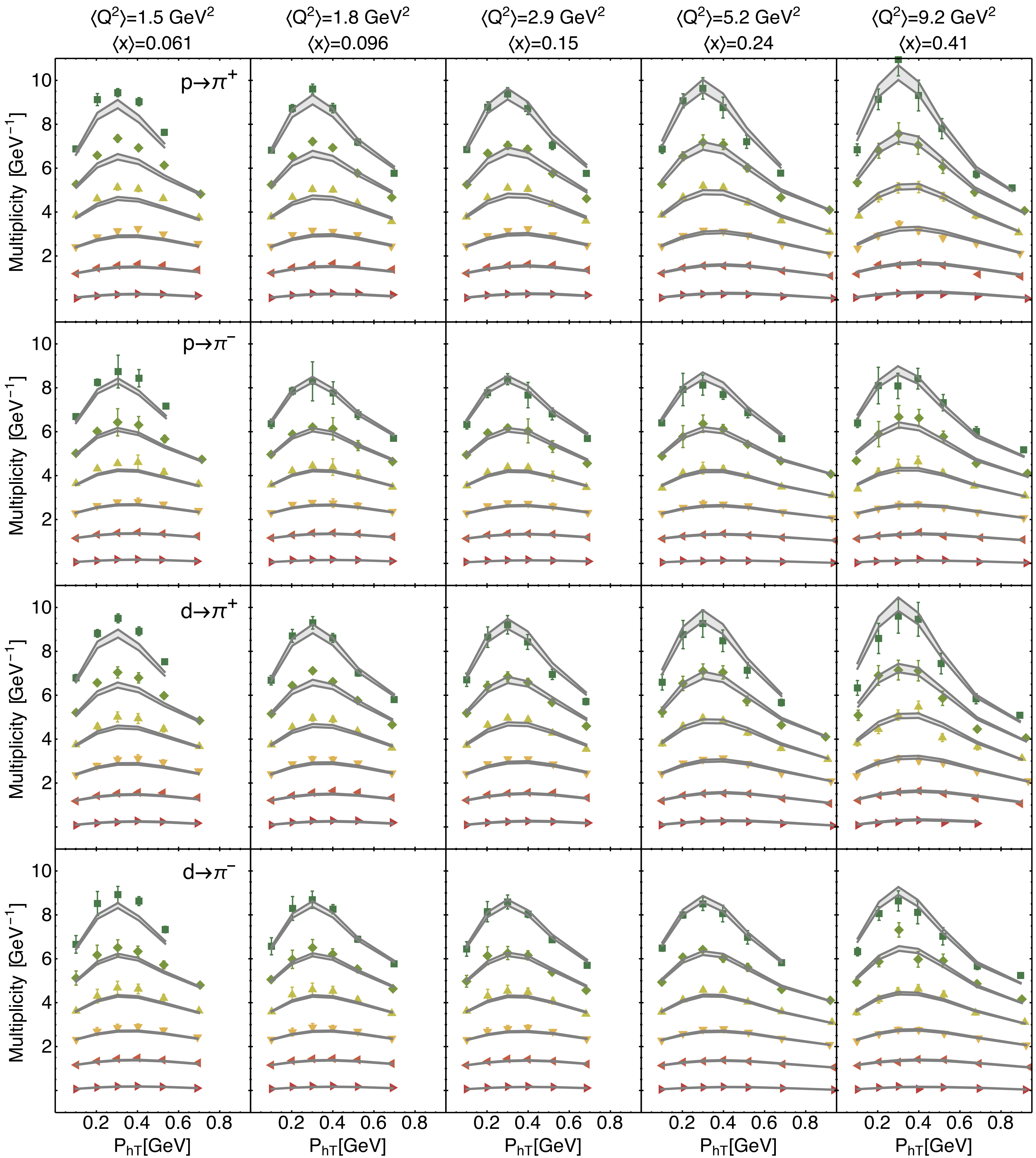
COMPASS data
SIDIS h^+



Observable:

$$\frac{m_N^h\left(x, z, \boldsymbol{P}_{hT}^2, Q^2\right)}{m_N^h\left(x, z, \min[\boldsymbol{P}_{hT}^2], Q^2\right)}$$

Hermes data pion production



π

χ^2/dof

4.83

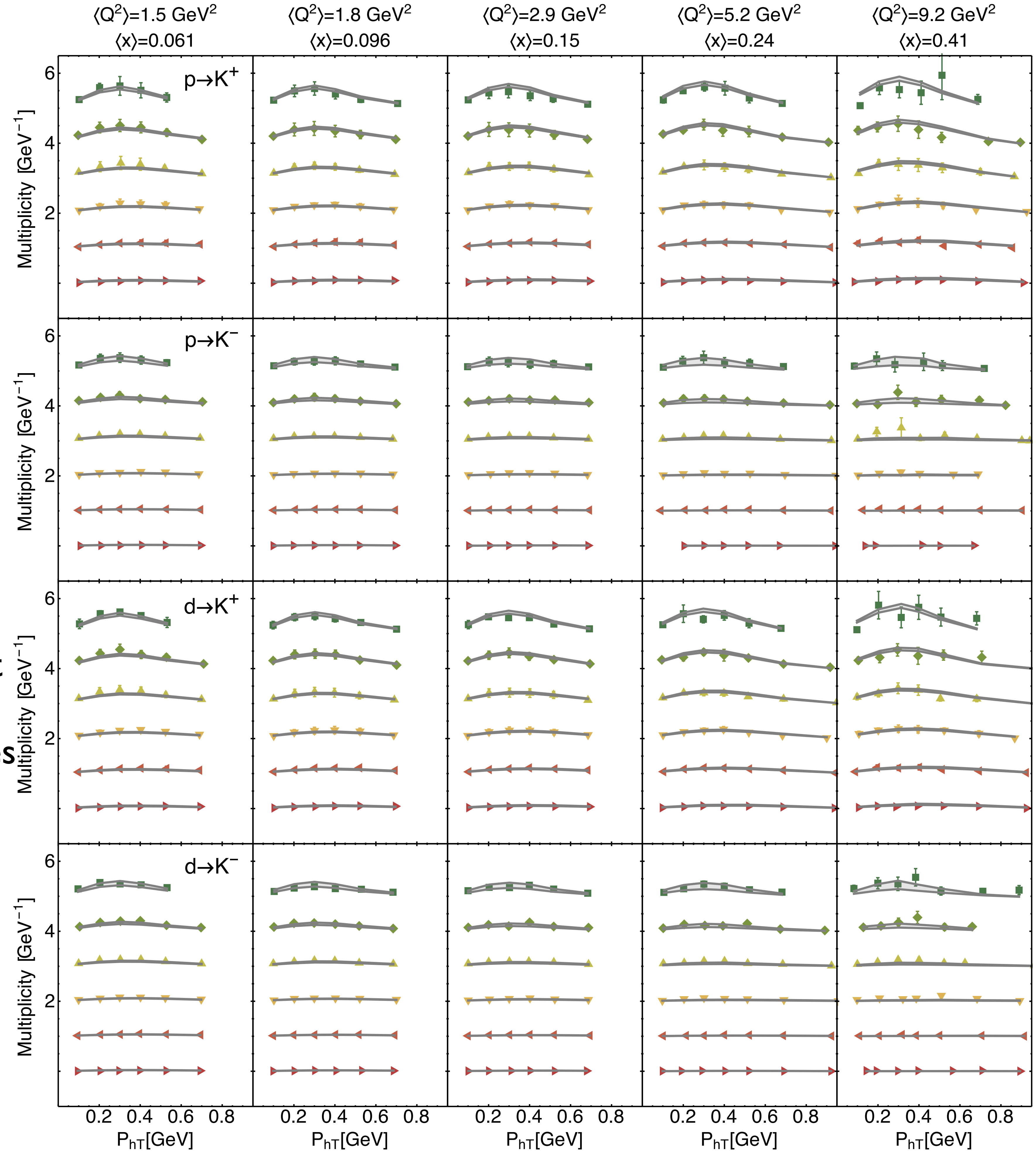
2.47

3.46

2.00

Hermes data kaon production

χ^2/dof



0.91

0.82

1.31

2.54

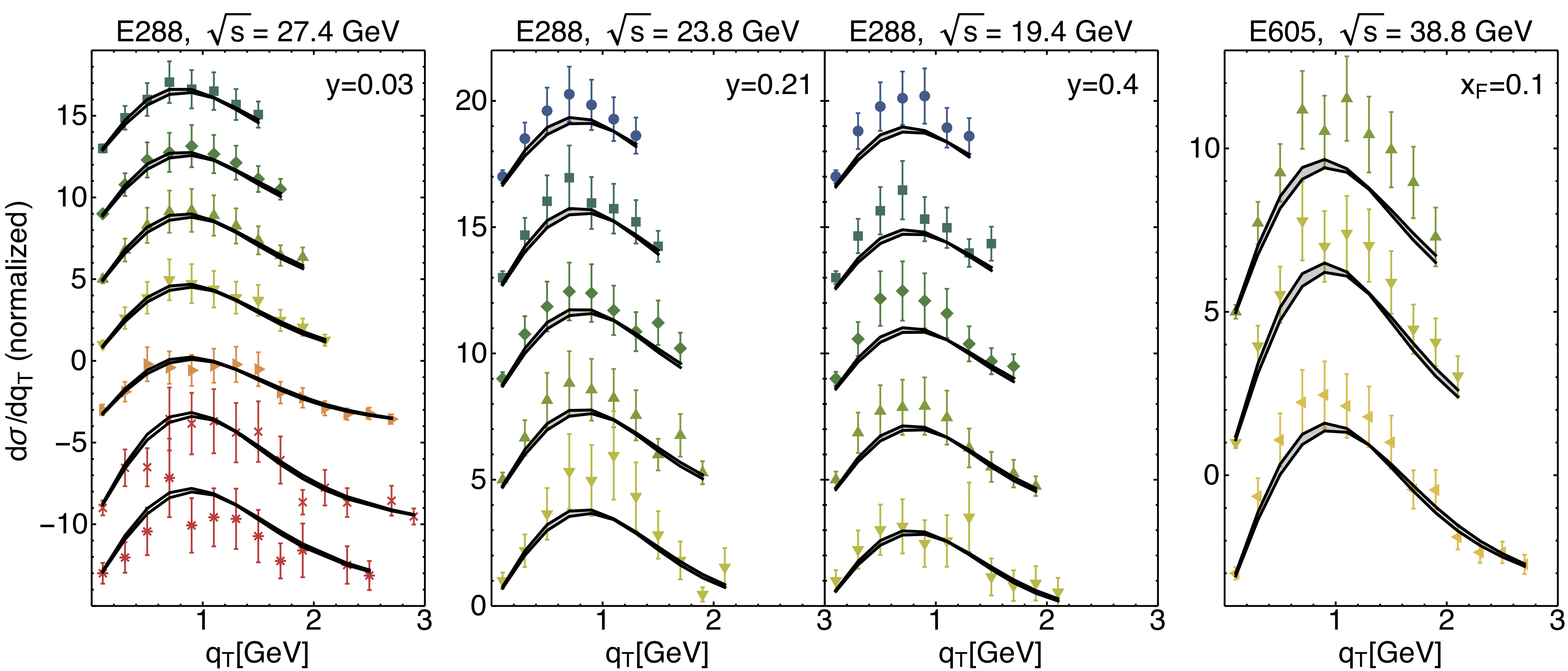
hermes

K

better agreement than pions: larger uncertainties form FFs

Drell-Yan data

χ^2/dof **0.32** **0.84** **0.99** **1.12**



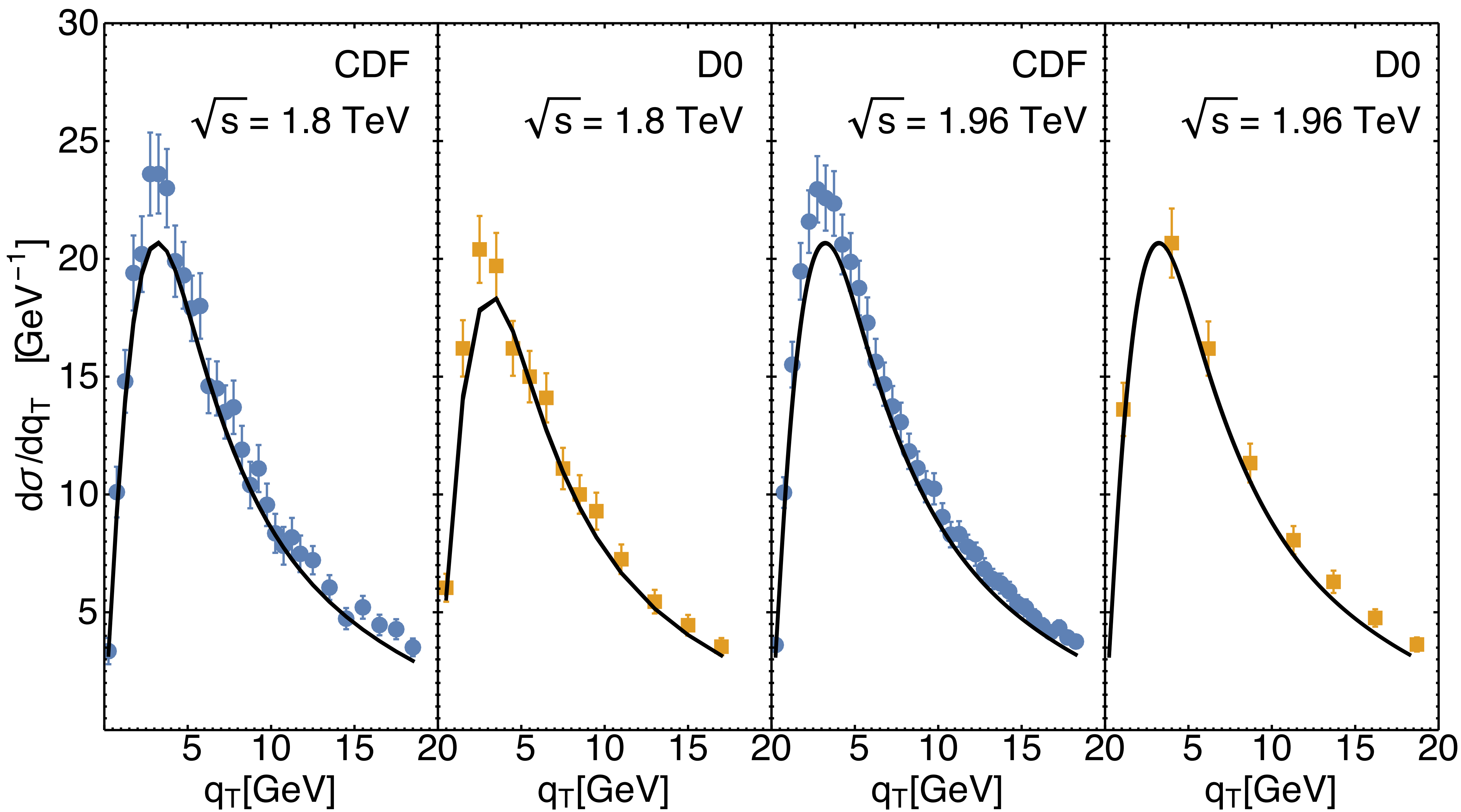
Fermilab

Q^2 Evolution: The peak is now at about 1 GeV, it was at 0.4 GeV for SIDIS

Z-boson production data

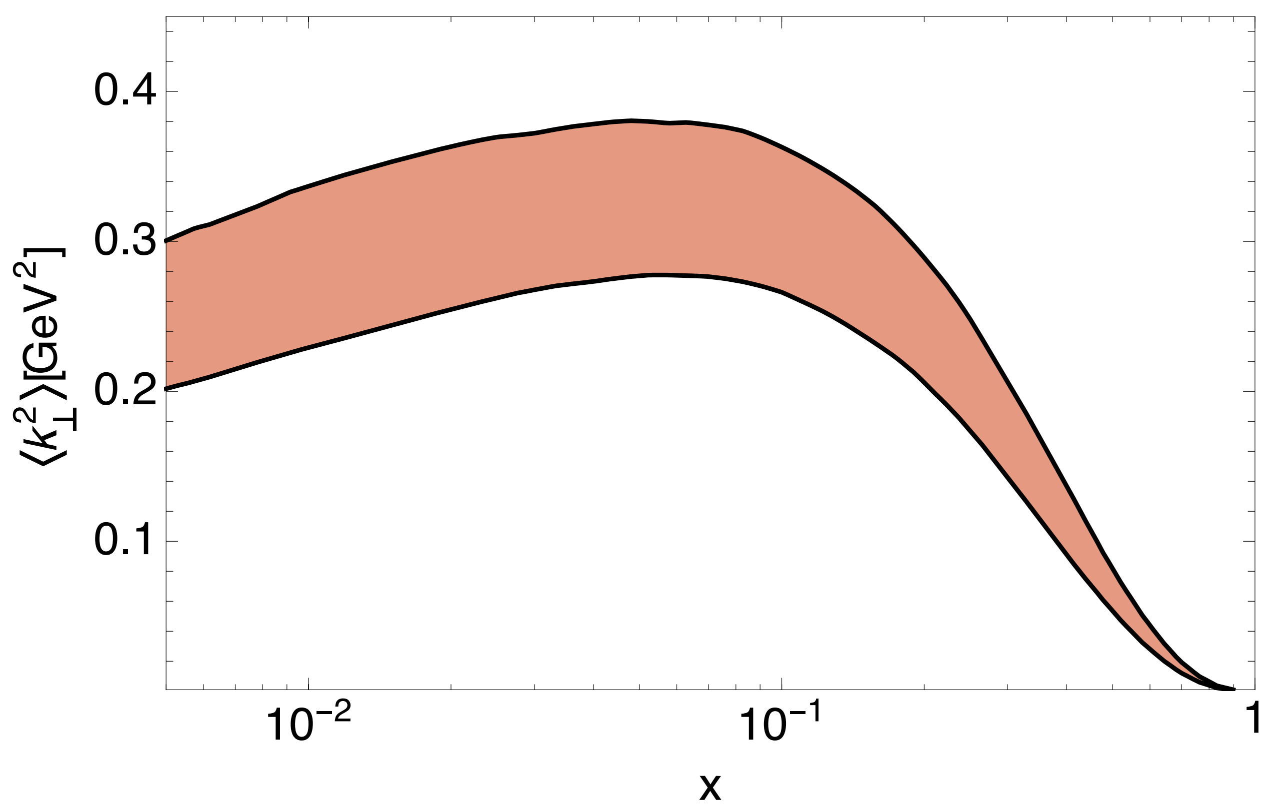
normalization : fixed from DEMS fit, different from exp.
(not really relevant for TMD parametrizations)

χ^2/dof 1.36 1.11 2.00 1.73



Q² Evolution: The peak is now at about 4 GeV

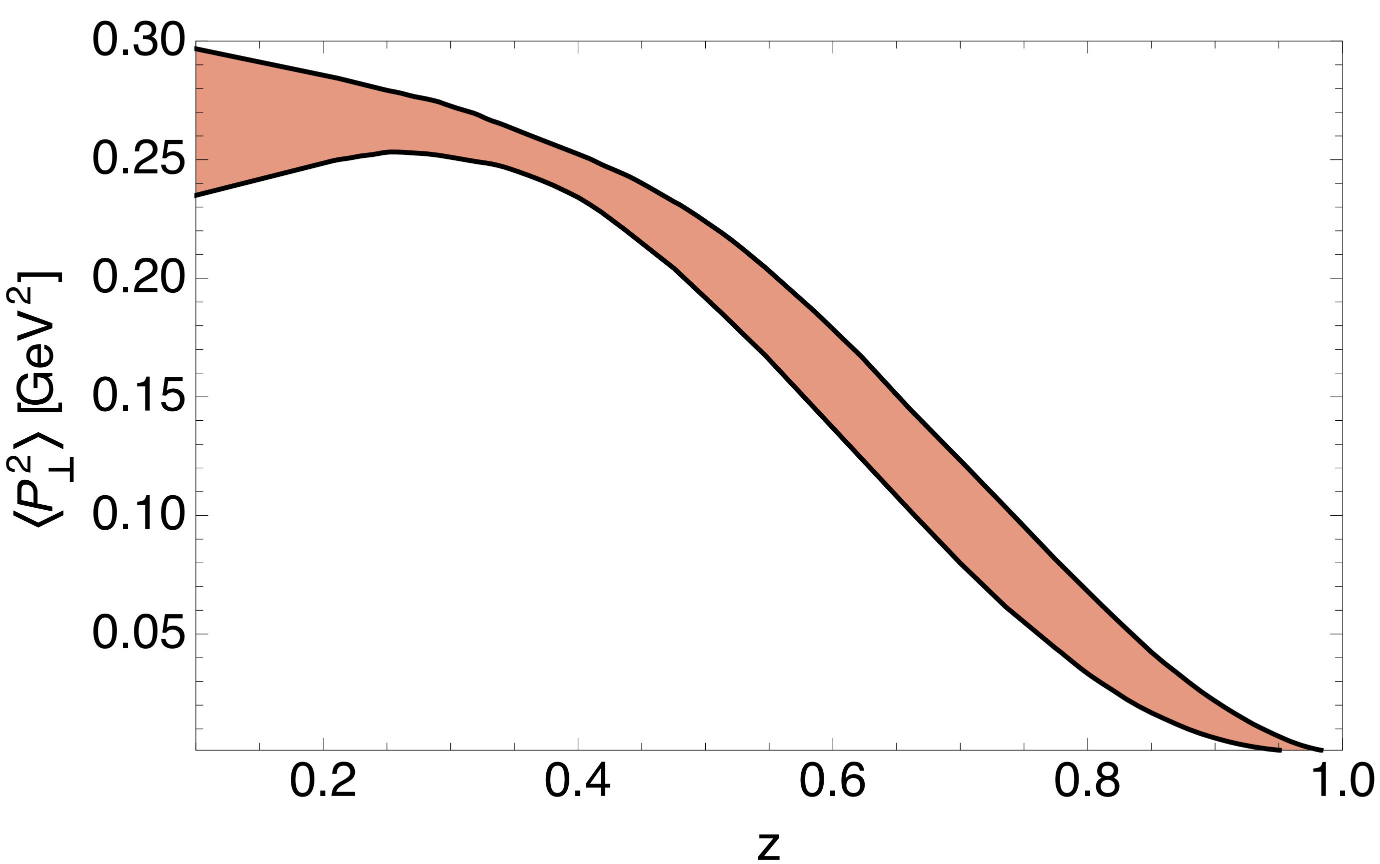
Mean transverse momentum



Change in TMD width x-dependence

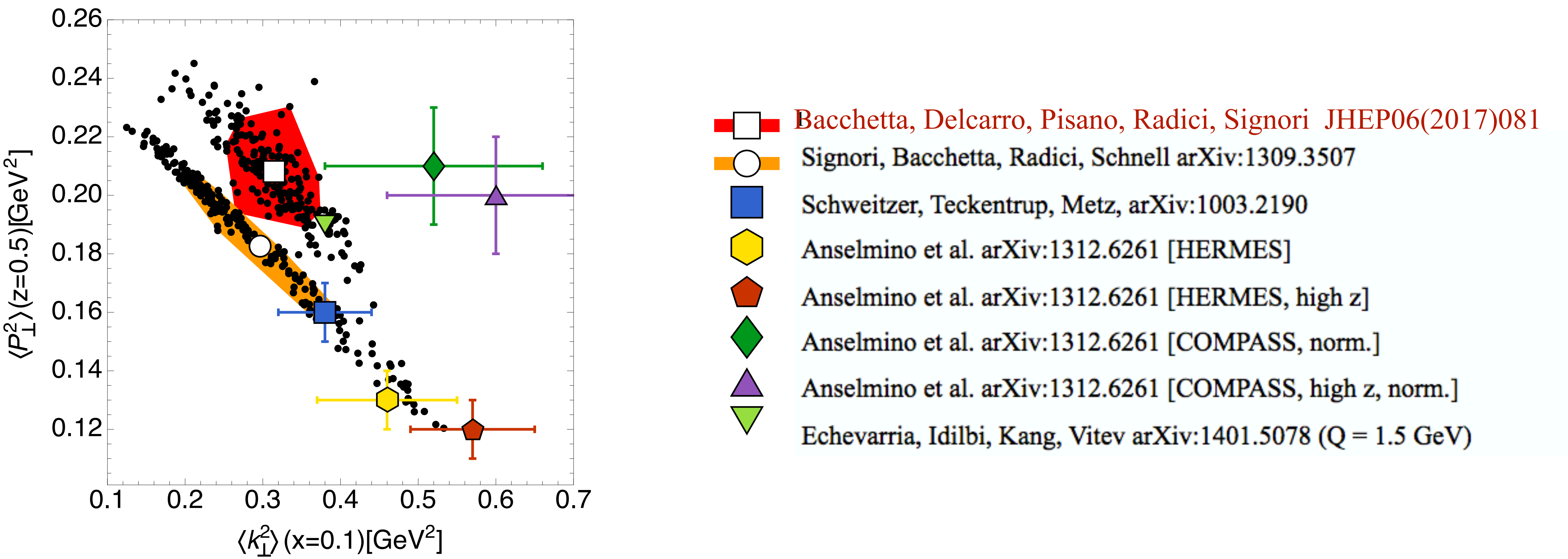
in TMD PDF

Q^2=1GeV^2



in TMD FF

Best fit value: transverse momenta



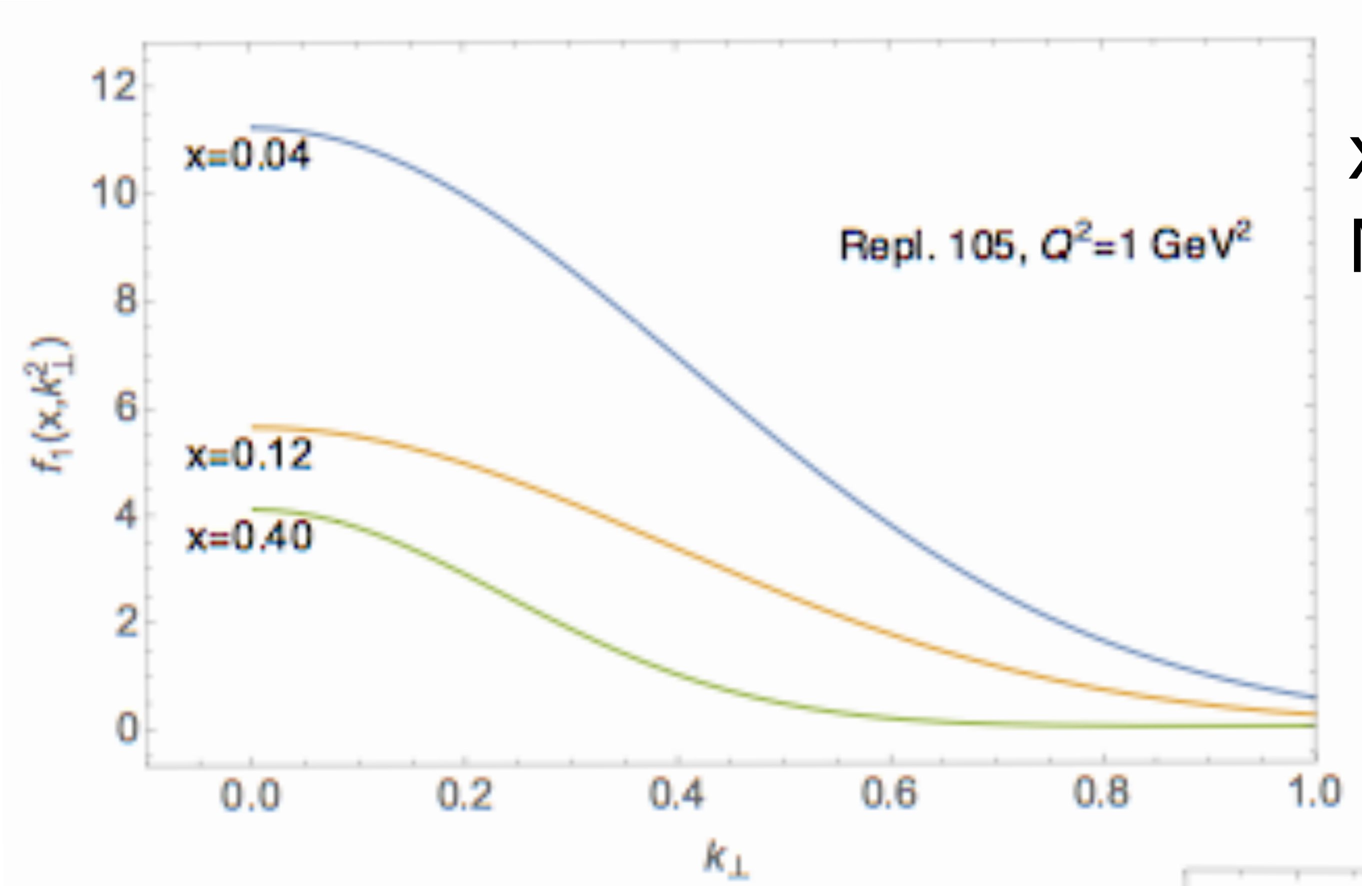
Red/orange regions: **68% CL** from replica method

Inclusion of DY/Z diminishes the correlation

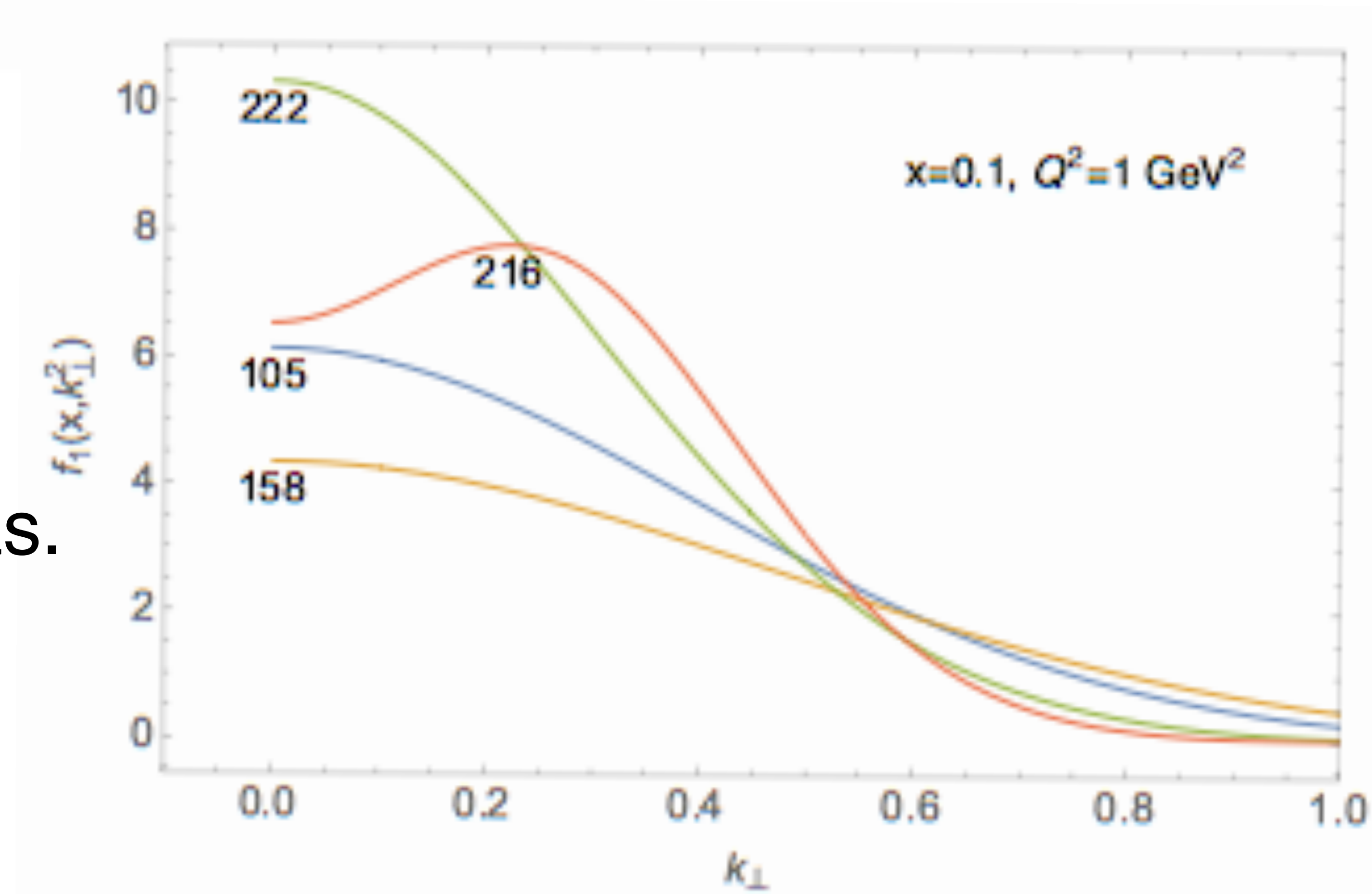
Inclusion of Compass increases the $\langle P_\perp^2 \rangle$ and reduces its spread

e+e- would further reduce the correlation

Shape uncertainties in replicas



x-dependence of a single replica.
Most of them are similar.



Shape of four selected replicas.
Still huge uncertainties.

Stability of results

Test of our default choices

How does the χ^2 of a single replica change if we modify them?

Original χ^2/dof = **1.51**

Normalization of HERMES data as done for COMPASS:
χ^2/dof = 1.27

Parametrizations for collinear PDFs
(NLO GJR 2008 default choice):
NLO MSTW 2008 (1.84), NLO CJ12 (1.85)

More stringent cuts
(TMD factorization better under control) χ^2/dof $\rightarrow$ 1
Ex: Q2 > 1.5 GeV^2; $0.25 < z < 0.6$; PhT < 0.2Qz $\Rightarrow$ χ^2/dof = 1.02 (477 bins)

What’s next

FIFA WORLD CUP

RUSSIA 2018

Perturbative ingredients

PAVIA 2018

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \left(\tilde{C}_{a/i} \otimes f_1^i\right)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T)\ln\frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

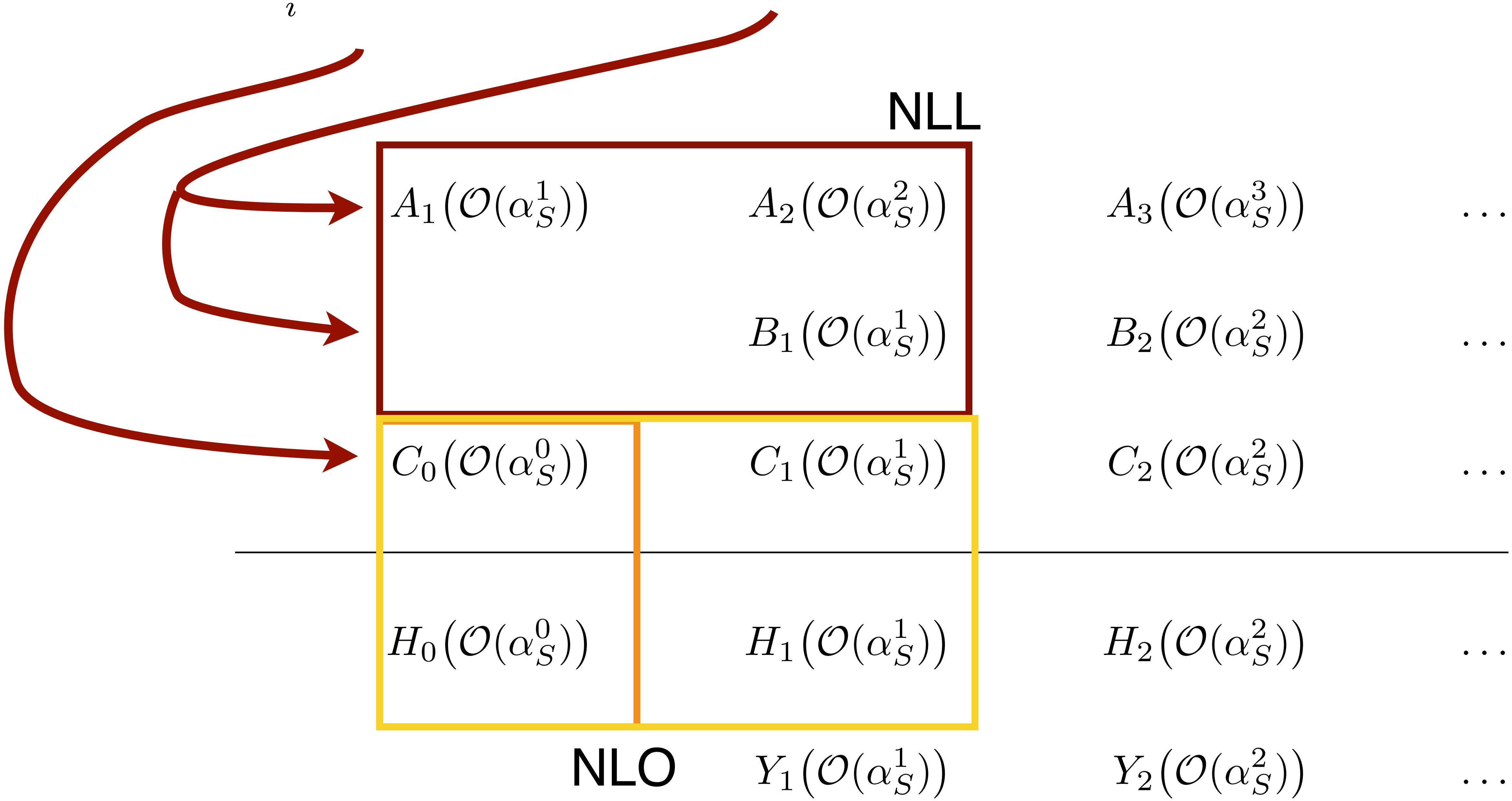
$A_1(\mathcal{O}(\alpha_S^1))$	$A_2(\mathcal{O}(\alpha_S^2))$	$A_3(\mathcal{O}(\alpha_S^3))$	$\ldots$
	$B_1(\mathcal{O}(\alpha_S^1))$	$B_2(\mathcal{O}(\alpha_S^2))$	$\ldots$
$C_0(\mathcal{O}(\alpha_S^0))$	$C_1(\mathcal{O}(\alpha_S^1))$	$C_2(\mathcal{O}(\alpha_S^2))$	$\ldots$

$H_0(\mathcal{O}(\alpha_S^0))$	$H_1(\mathcal{O}(\alpha_S^1))$	$H_2(\mathcal{O}(\alpha_S^2))$	$\ldots$
	$Y_1(\mathcal{O}(\alpha_S^1))$	$Y_2(\mathcal{O}(\alpha_S^2))$	$\ldots$

Perturbative ingredients

PAVIA 2018

$$\widetilde{f}_1^a(x,b_T;\mu^2) = \sum_i \big(\tilde{C}_{a/i}\otimes f_1^i\big)(x,b_*;\mu_b)\, e^{\tilde{S}(b_*;\mu_b,\mu)}\, e^{g_K(b_T)\ln\frac{\mu}{\mu_0}}\, \hat{f}_{\rm NP}^a(x,b_T)$$



Perturbative ingredients

PAVIA 2018

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \left(\tilde{C}_{a/i} \otimes f_1^i\right)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

$A_1(\mathcal{O}(\alpha_S^1))$ $A_2(\mathcal{O}(\alpha_S^2))$

$B_1(\mathcal{O}(\alpha_S^1))$

$C_0(\mathcal{O}(\alpha_S^0))$ $C_1(\mathcal{O}(\alpha_S^1))$

$H_0(\mathcal{O}(\alpha_S^0))$ $H_1(\mathcal{O}(\alpha_S^1))$

$Y_1(\mathcal{O}(\alpha_S^1))$

few % correction

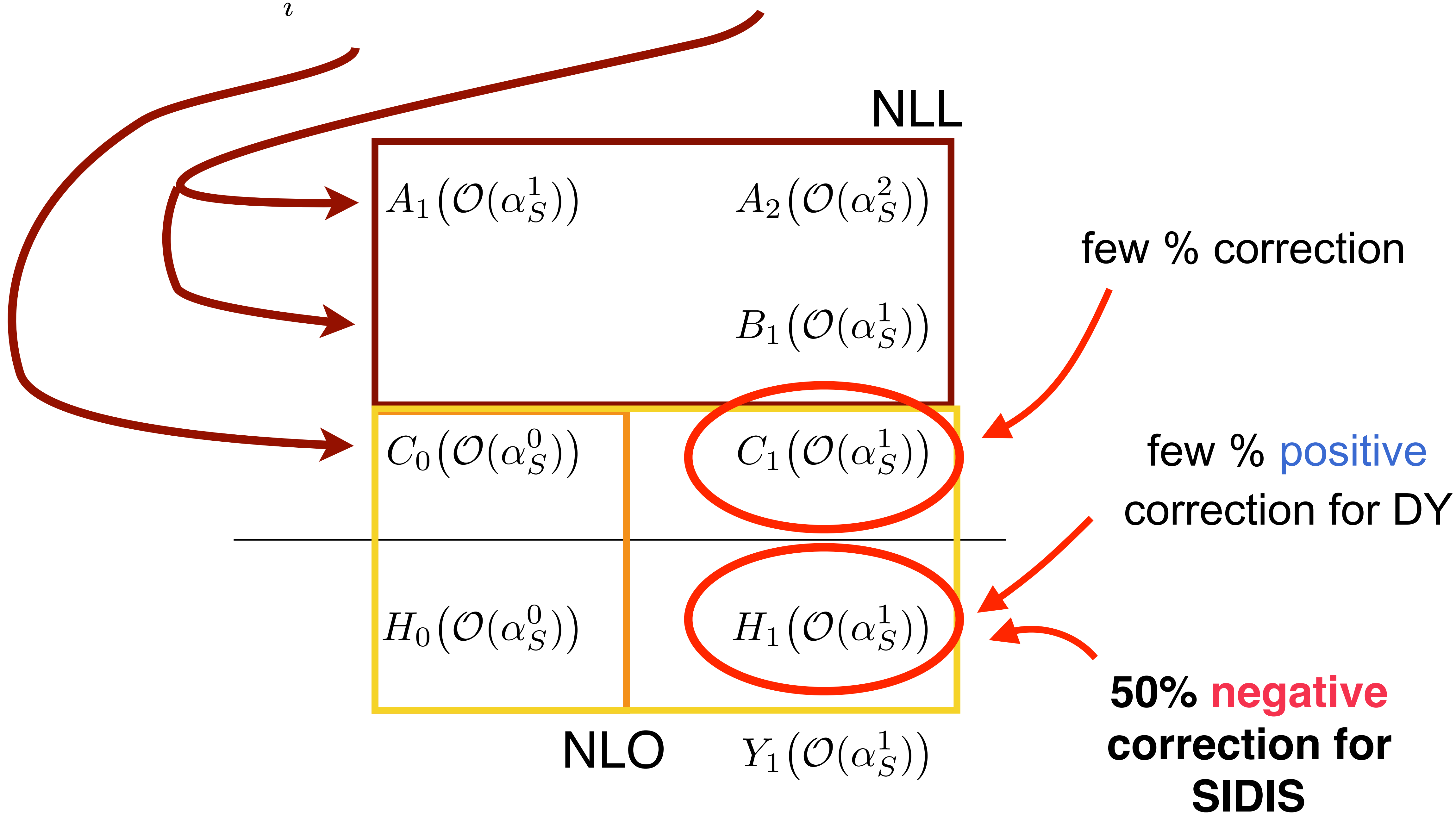
few % positive correction for DY

50% negative correction for SIDIS

Perturbative ingredients

PAVIA 2018

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\mathrm{NP}}^a(x, b_T)$$



Perturbative ingredients

PAVIA 2018

$$m_N^h\left(x,z,P_{hT}^2,Q^2\right)=\frac{d\sigma_N^h/\left(dx\,dz\,dP_{hT}^2\,dQ^2\right)}{d\sigma_{DIS}/\left(dx\,dQ^2\right)}$$

$$\approx\frac{F_{UU,T}\left(x,z,P_{hT}^2,Q^2\right)}{F_T\left(x,Q^2\right)+F_L\left(x,Q^2\right)\frac{1-y}{1-y+y^2/2}}$$

NLO

positive ~10% correction to the multiplicity

Data to be included

7 TeV
8 TeV

$$q\bar{q} \rightarrow Z_0/\gamma^* + X$$

7 TeV
8 TeV

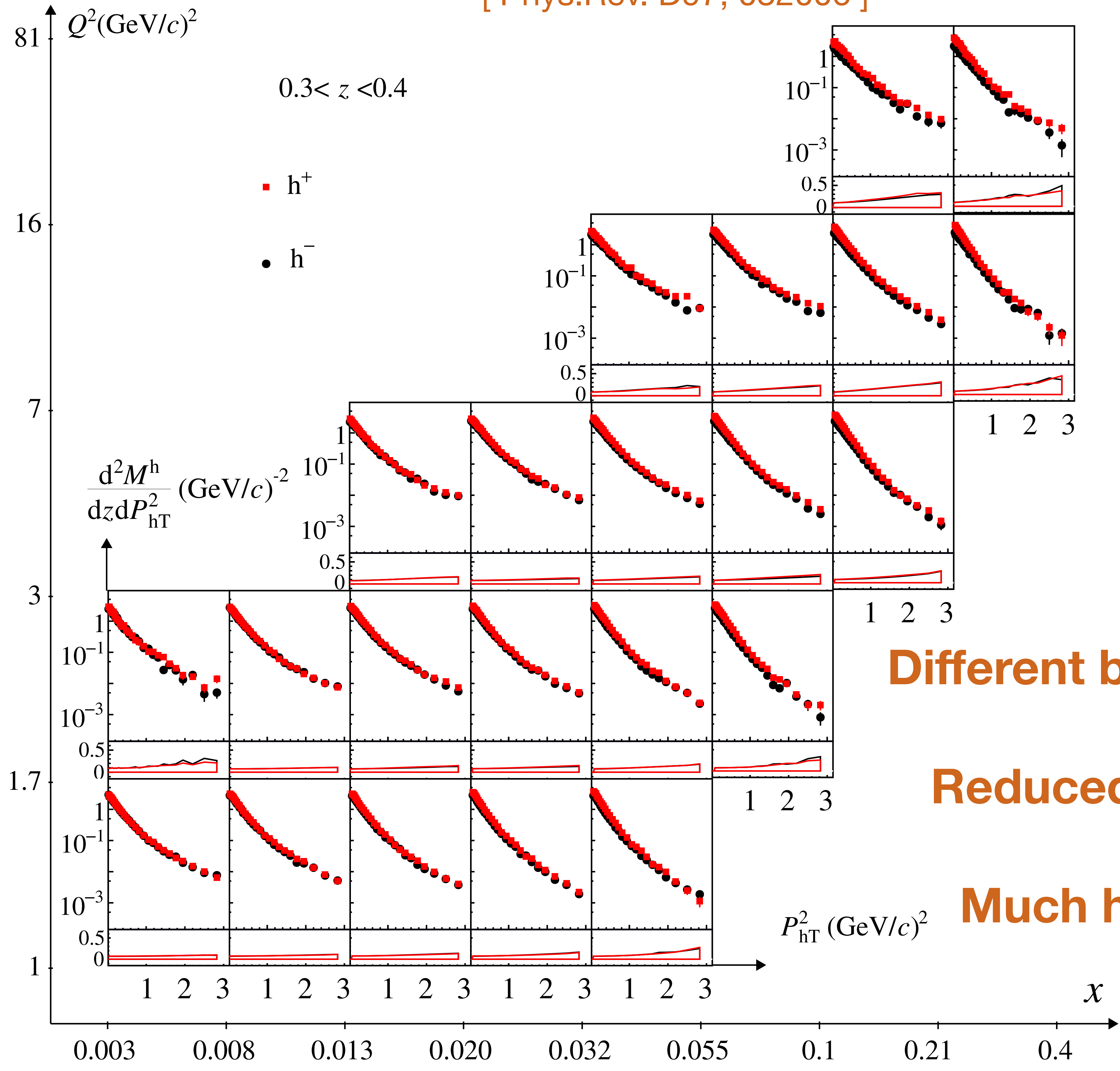
$$pp \rightarrow Z_0/\gamma^* \rightarrow (\mu^+ + \mu^- / e^+ + e^-)$$

7 TeV
8 TeV
13 TeV

$$pp \rightarrow Z_0 \rightarrow \mu^+ + \mu^-$$

Data to be included

[Phys.Rev. D97, 032006]



Different binning in Z (larger)

Reduced number of data

Much higher statistics

Preliminary results

Number of experimental data

Same kinematical cuts in x, Q^2, z, P_{hT}

Same data for DY	203
Z	90
SIDIS eN	1514

Total: 3931 data

COMPASS

SIDIS μN

2124

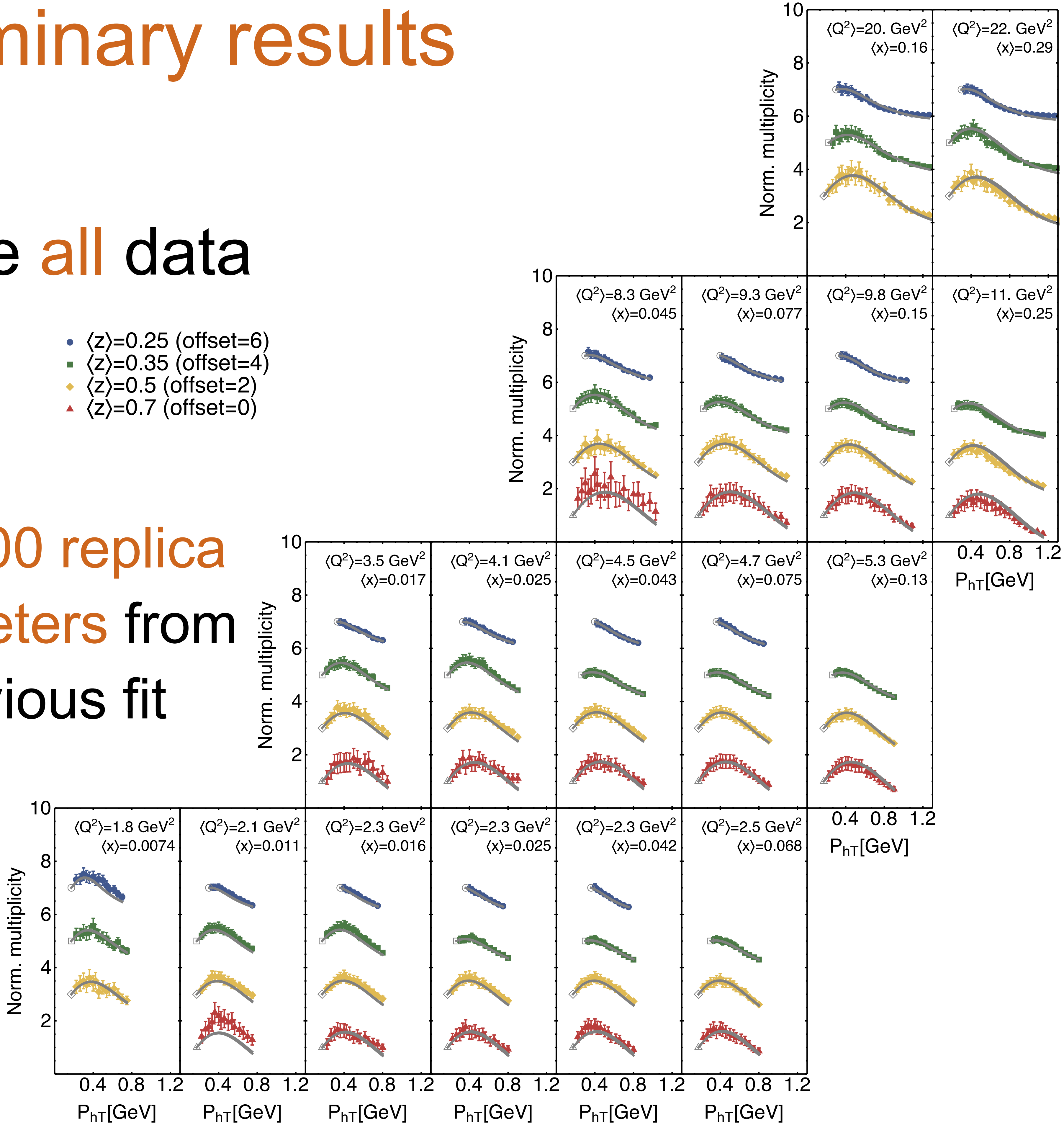
data points

Preliminary results

SIDIS h^+

Include all data

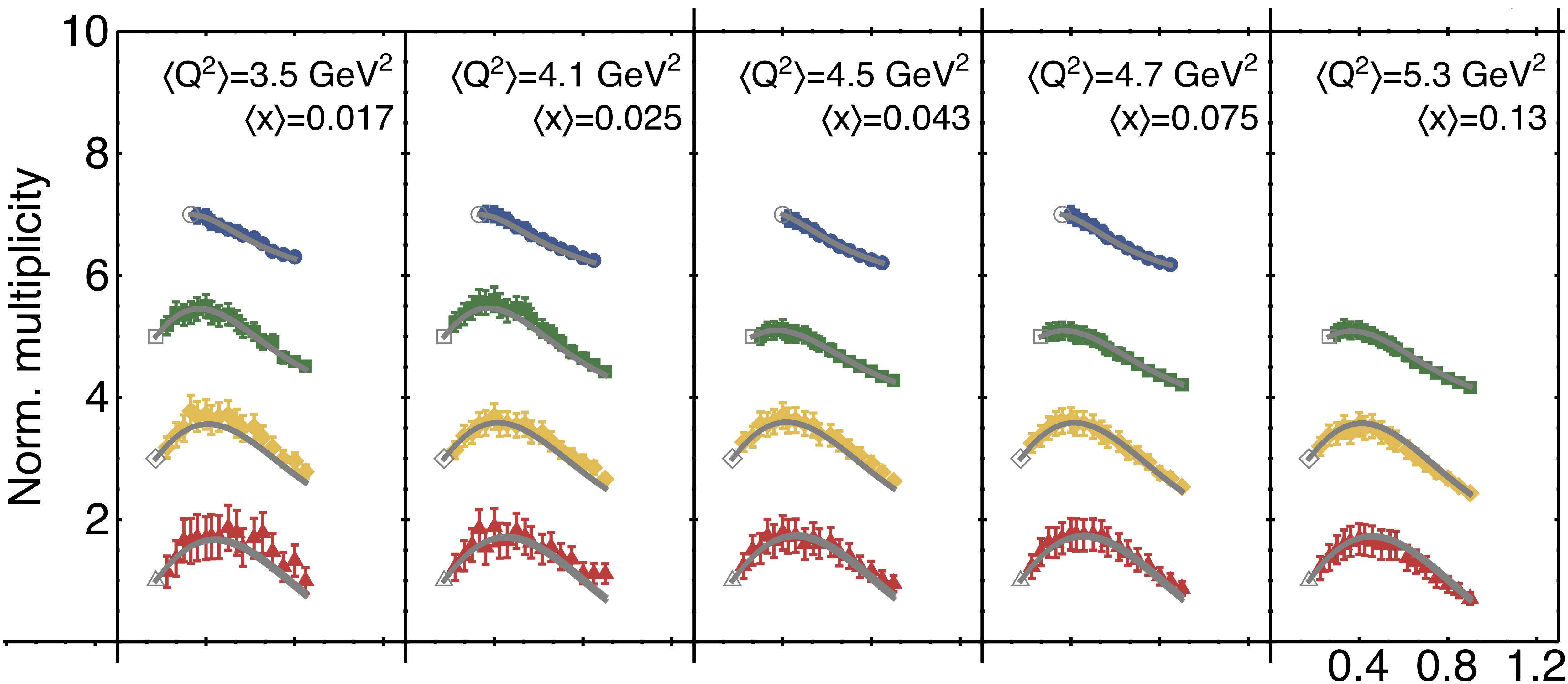
Use 200 replica parameters from previous fit



Normalized at 1st data point of bin

Include all data

SIDIS h^+

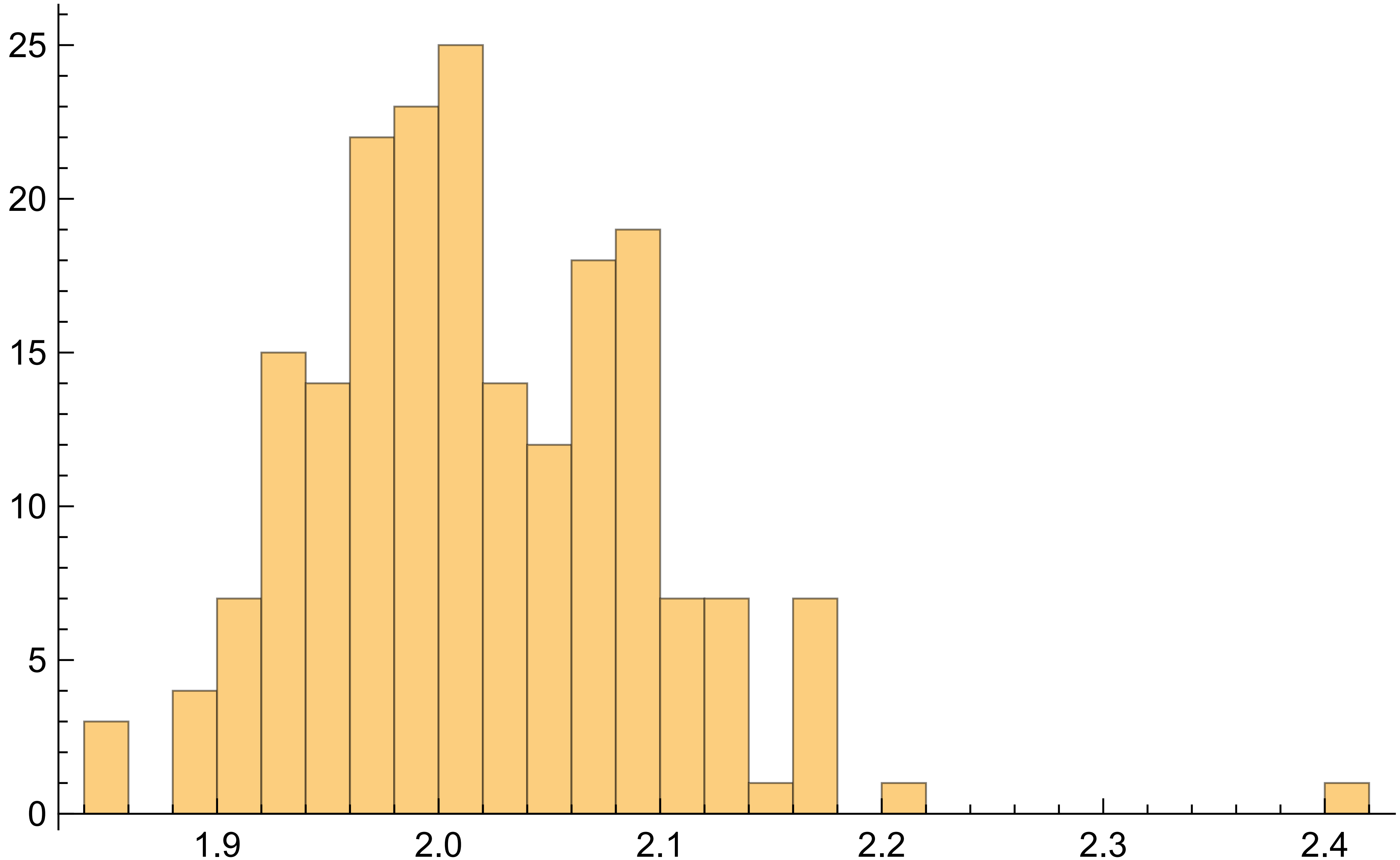


Use 200 replica parameters from previous fit

Normalized at 1st data point of bin

Include all data

SIDIS h^+



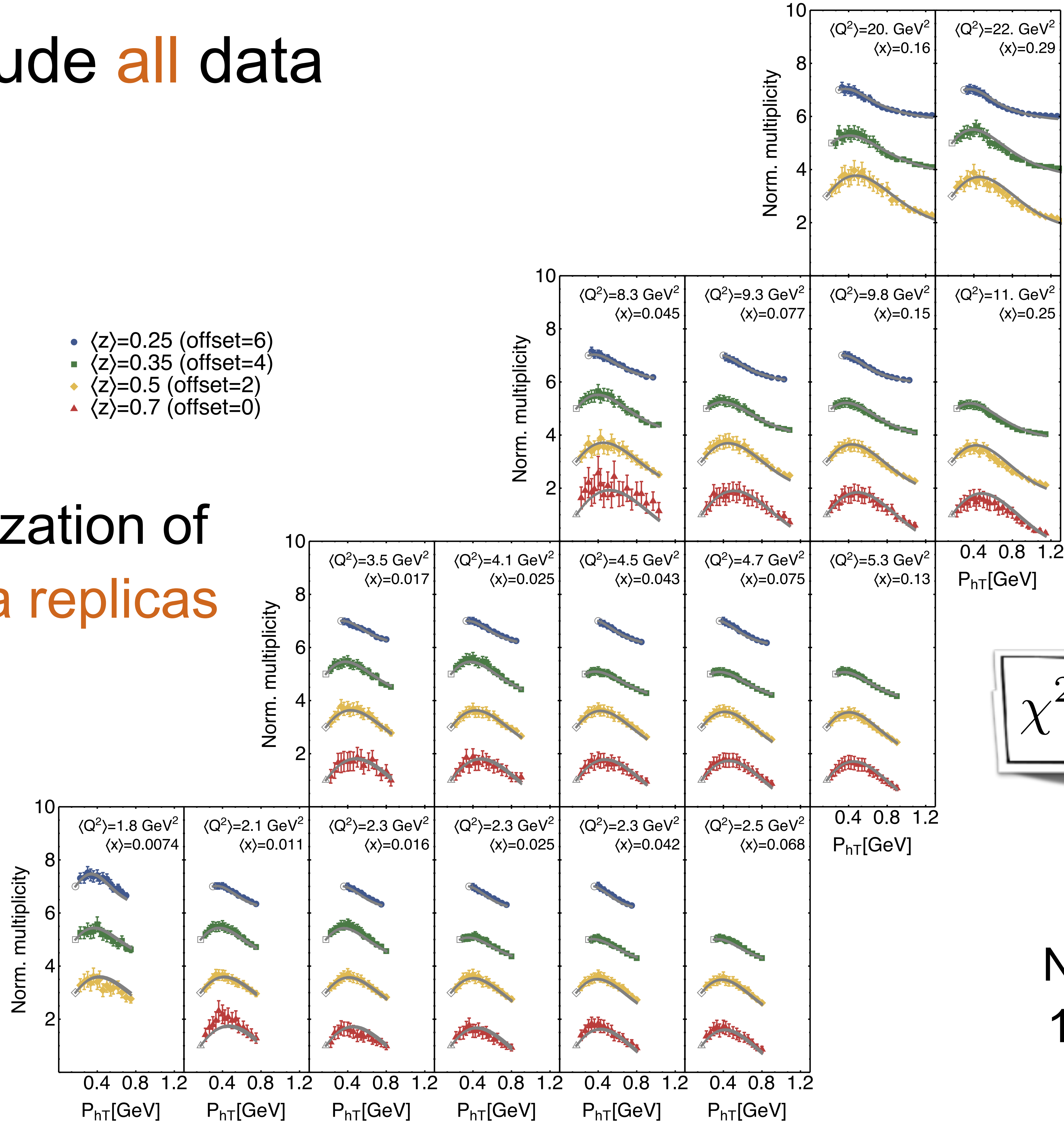
Use 200 replica parameters from previous fit

$\chi^2/\mathrm{dof} = 2.01$

Normalized at 1st data point of bin

Include all data

SIDIS h^+



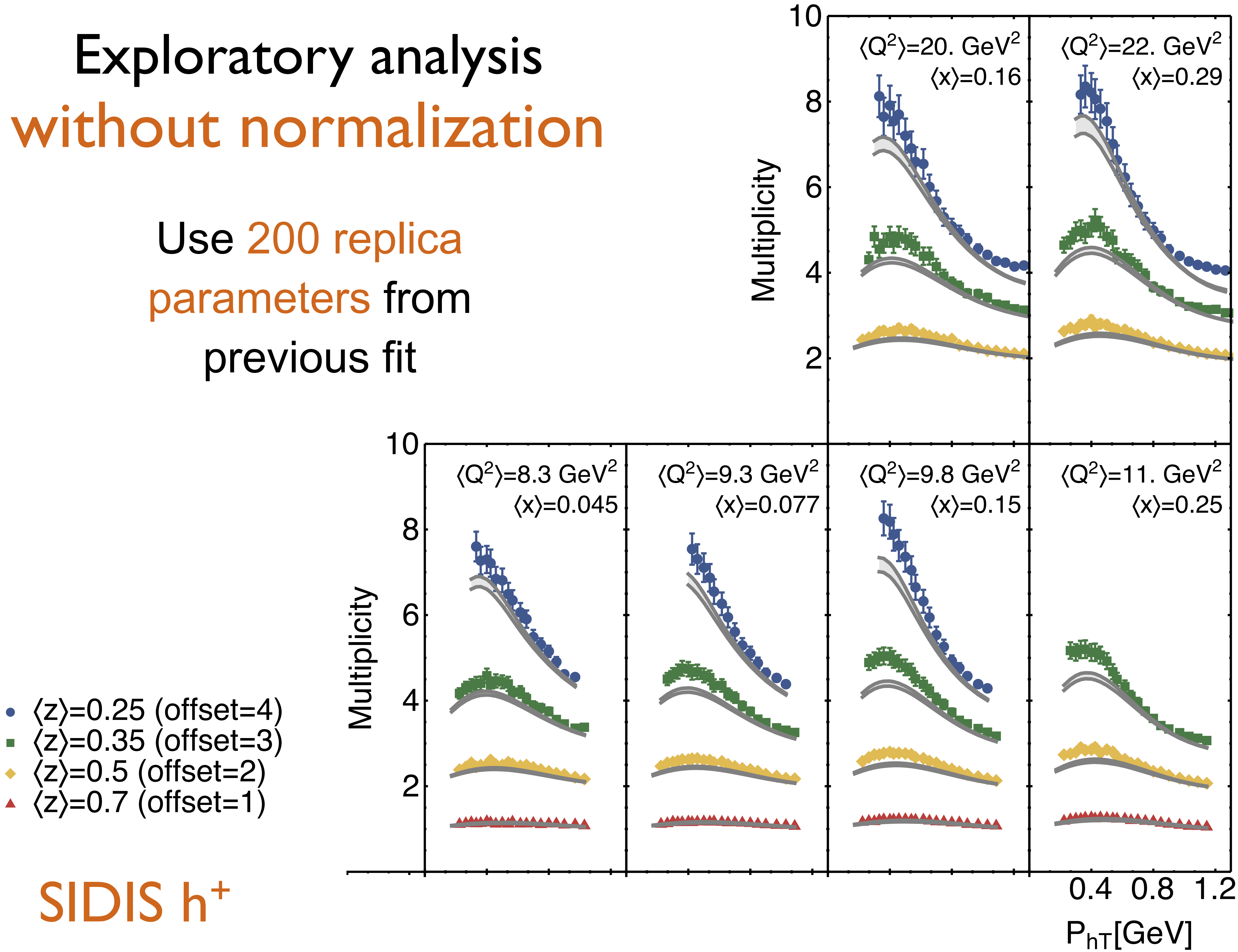
Minimization of 50 data replicas

$$\chi^2/\mathrm{dof} = 1.71$$

Normalized at 1st data point of bin

Exploratory analysis without normalization

Use 200 replica parameters from previous fit

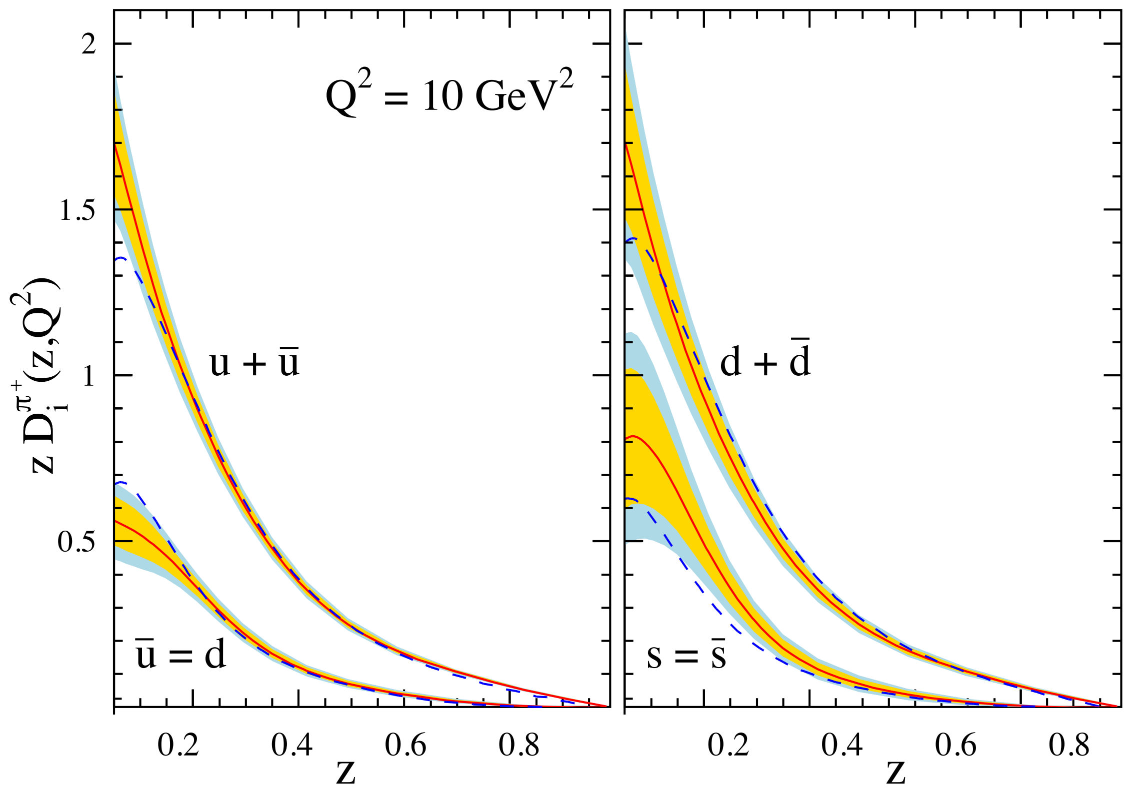


SIDIS h^+

Exploratory analysis
without normalization

Use 200 replica parameters from previous fit $\longrightarrow$ $\chi^2/\text{dof} > 4$

Sensitive to z value



FF DSS

$$\chi^2 = \sum \frac{(N_i \, th_j - exp_j)}{\delta_j^2}$$

if one keeps the **same parameters** and lets the normalization of each bin being fitted

$$\chi^2/\text{dof} = 1.68$$

N_i between 0.92 and 1.52 for 90% of bins (COMPASS 17)
(total range $0.55 < N_i < 1.66$)

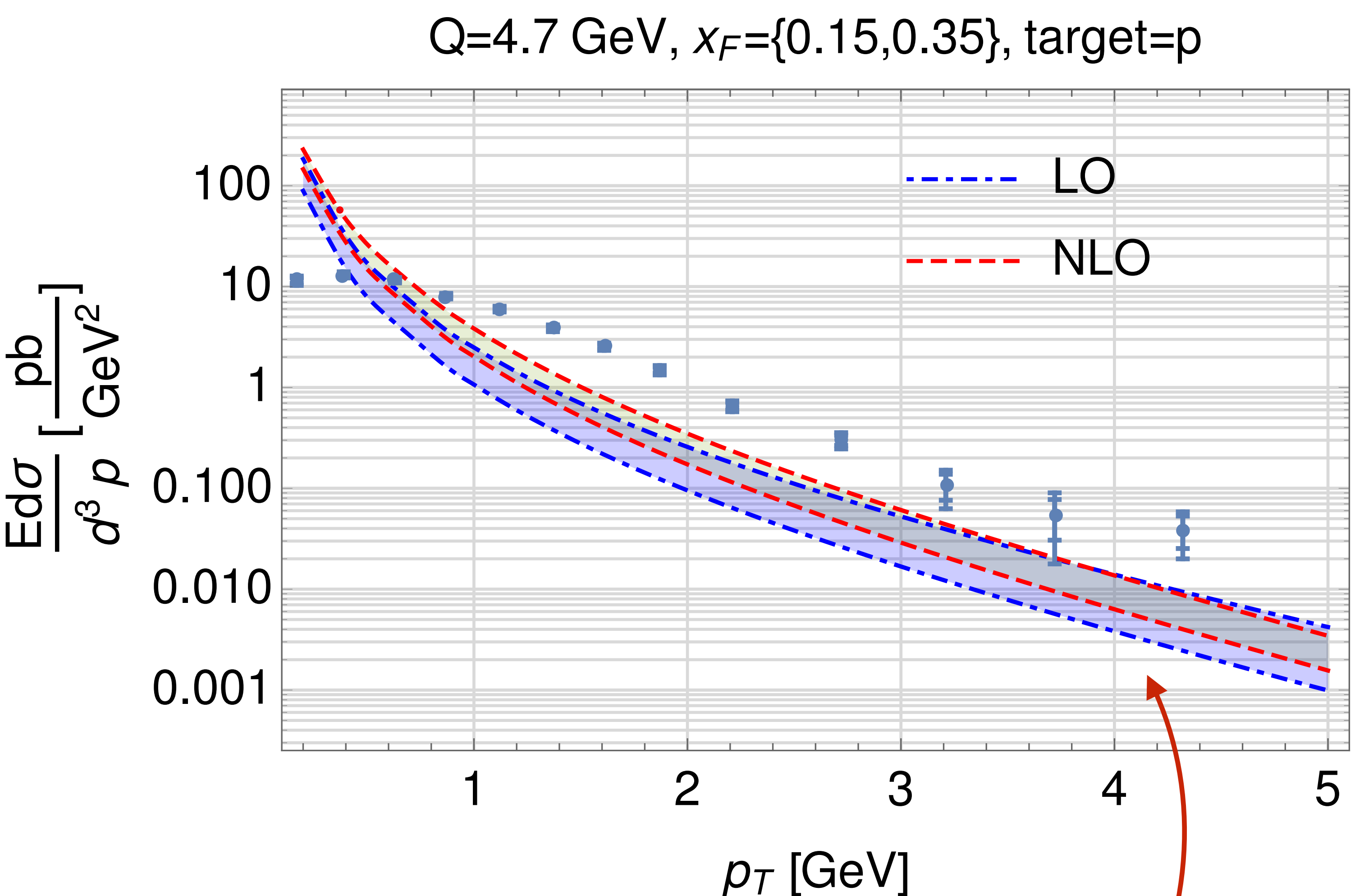
Issues at high p_T

Drell-Yan

E866/NuSea

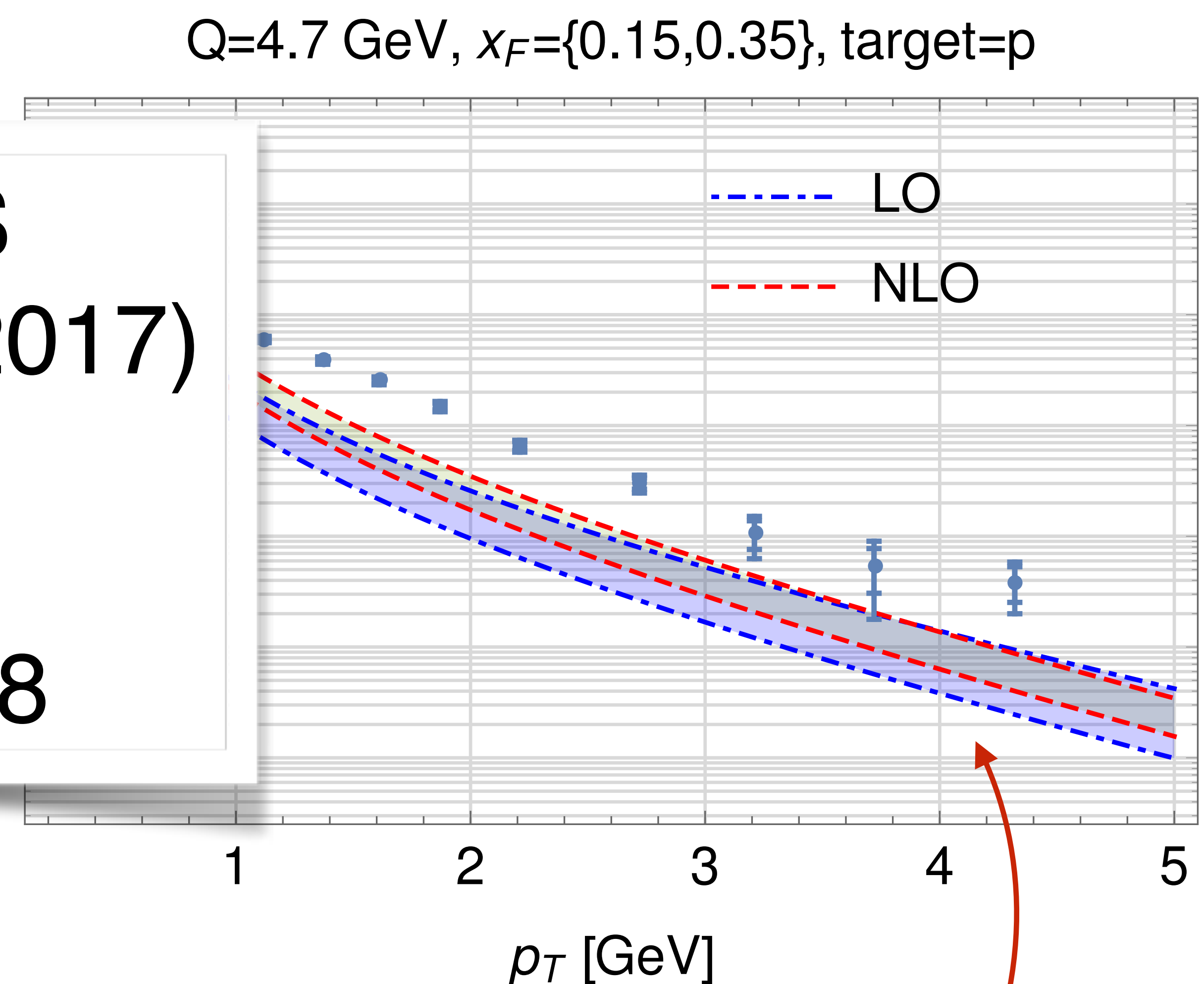
$$p\,p \to \mu^+\mu^- X$$

$$\sqrt{s} = 38.8\,GeV$$



Here collinear factorization should work! $q_T \simeq Q$

Drell-Yan



Similar issue in SIDIS
at high PhT (COMPASS 2017)

N.Sato @QCDevo2018

Here collinear factorization should work! $q_T \simeq Q$

Conclusions

First global extraction of TMDs from SIDIS, DY and Z boson

Test of the universality and evolution formalism of partonic TMDs

New Data

- compatible with parameters obtained from previous analysis
- requires further considerations on normalisation

What we need

- flavor separation (proton target, identified hadrons)
- average z-value of bins
- new unpolarised Drell-Yan data (possible p-p, p-pbar?) especially at high q_T

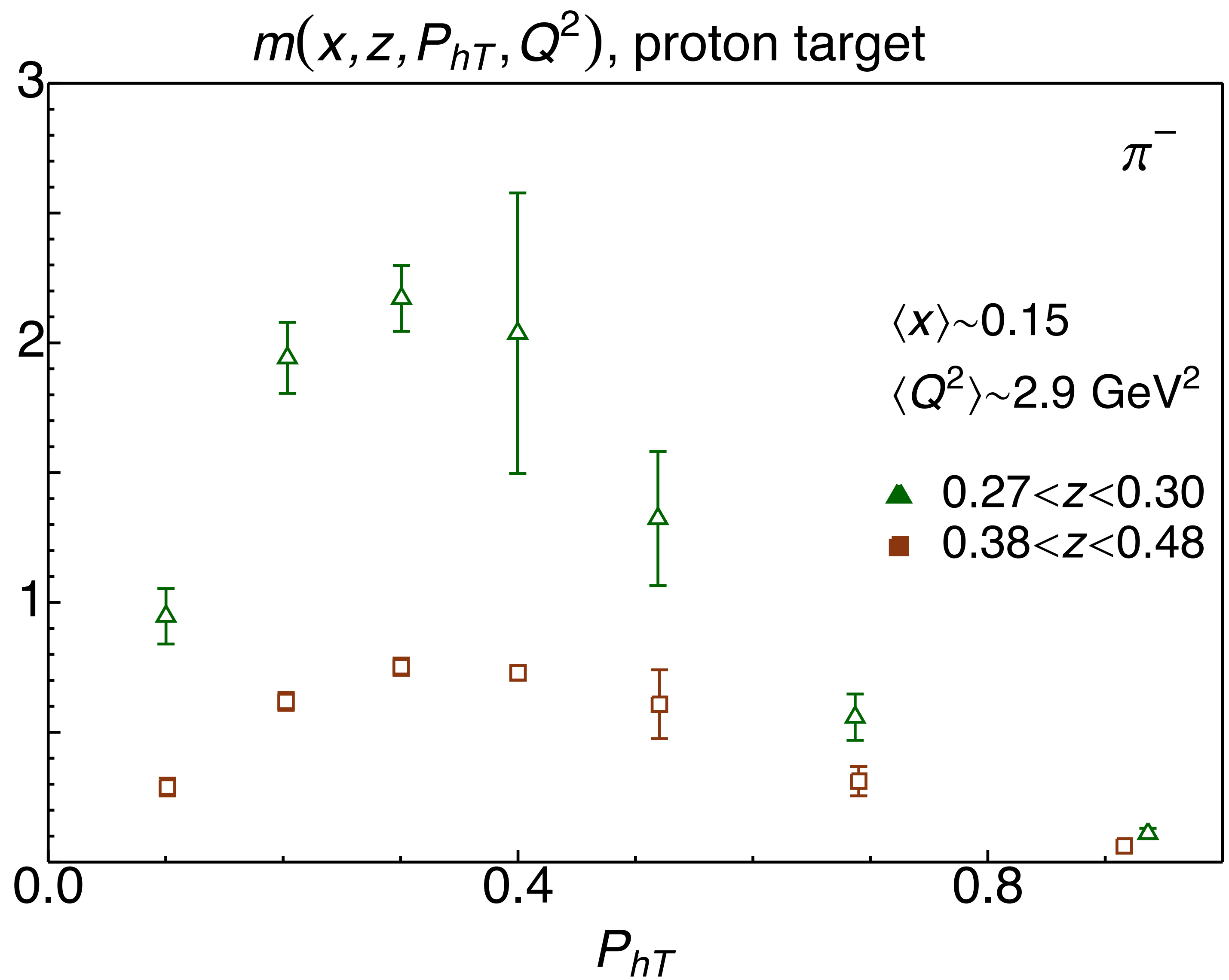
low energy Drell-Yan data

Experiment	Reaction	Year	TMD fits	PDF fits	high-qT tail
R209	p-p	1981	✔	✘	✔
E288	p-Cu, p-Pt	1981	✔	✘	✘
E605	p-Cu	1991	✔	✔	✘
E866	p-p, p-d	2003	✘	✔	✔

$$20\,GeV \lesssim \sqrt{s} \lesssim 60\,GeV$$

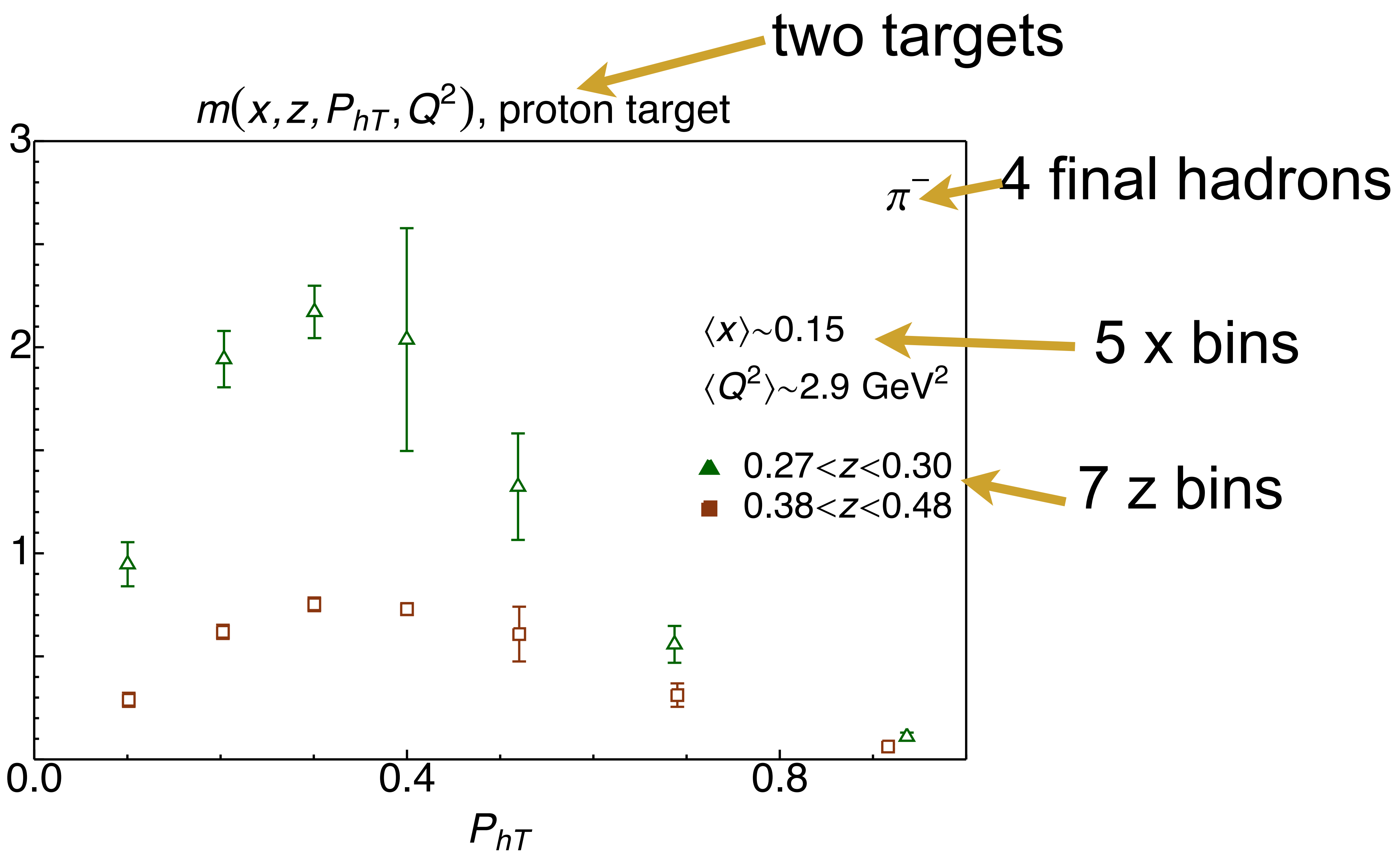
BACKUP

The replica method



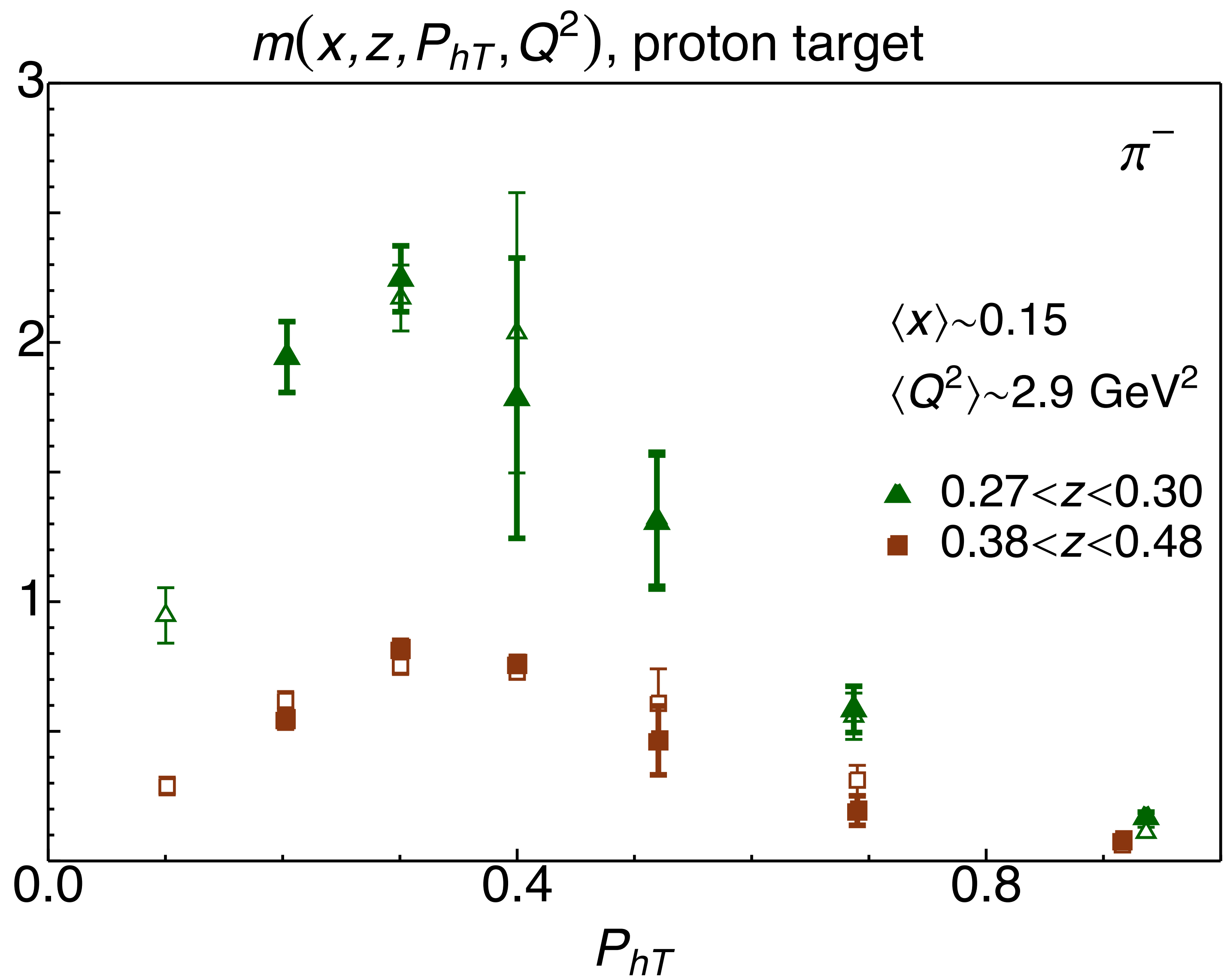
Example of original data

The replica method



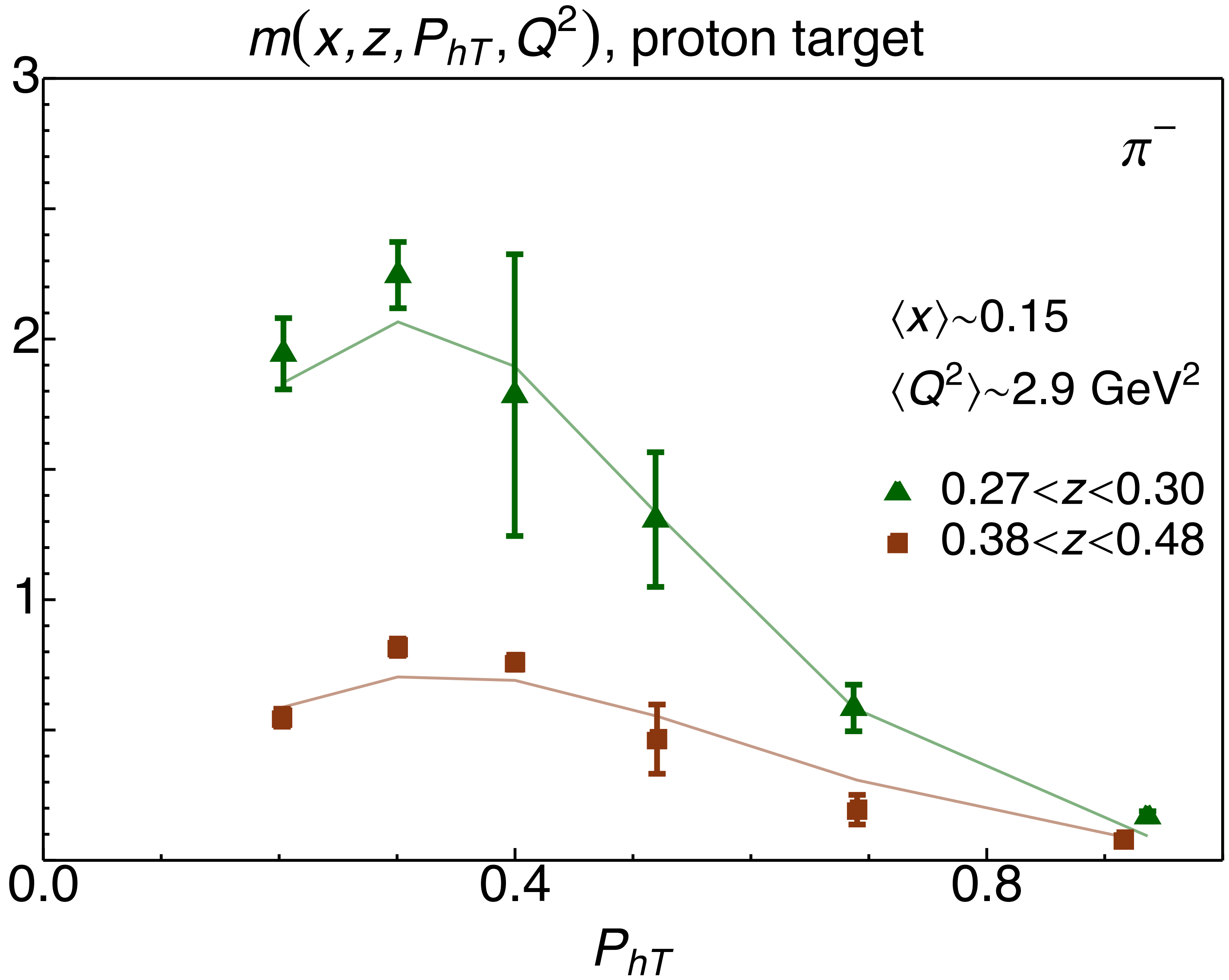
Example of original data

The replica method



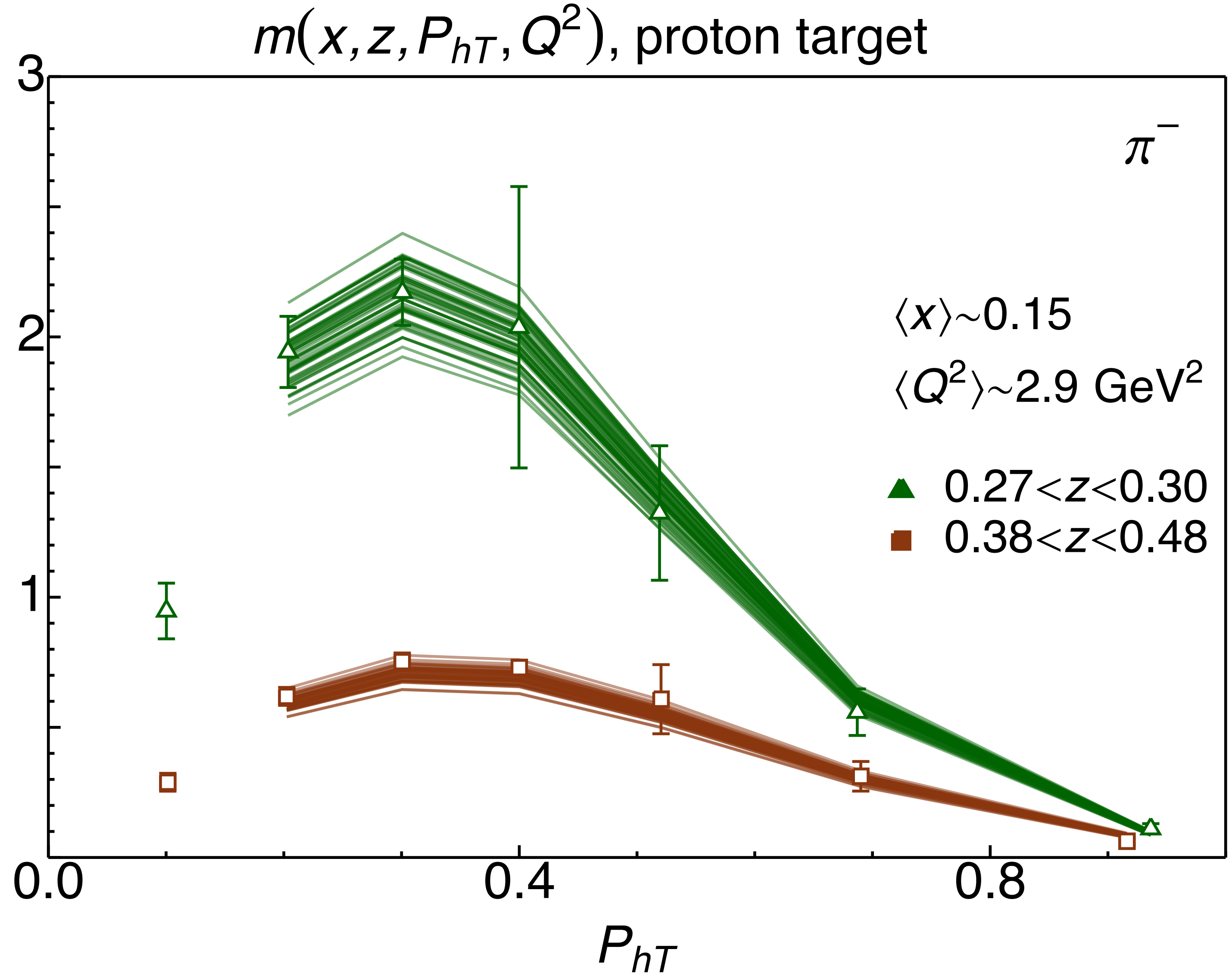
Data are replicated (with Gaussian distribution)

The replica method



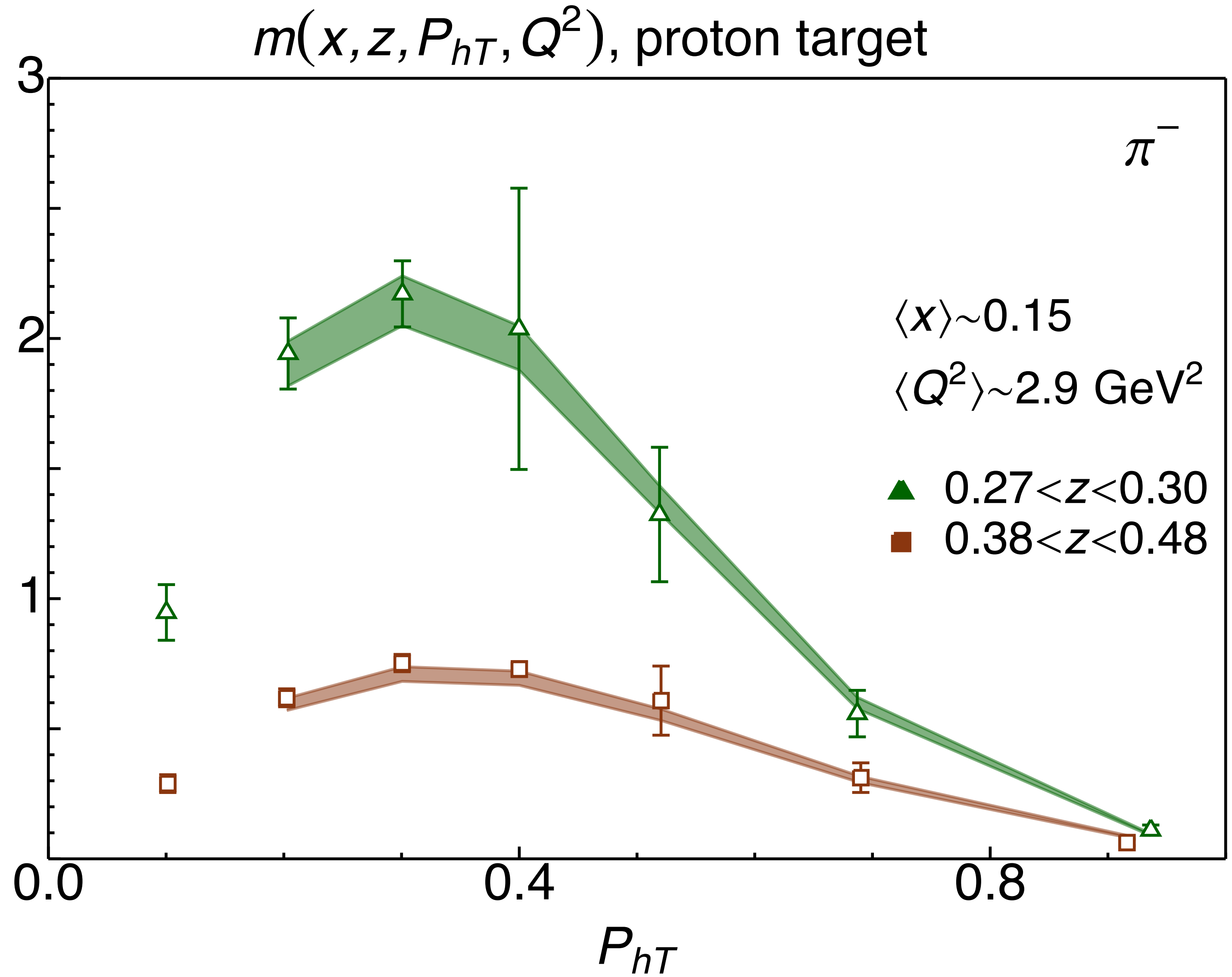
The fit is performed on the replicated data

The replica method



The procedure is repeated 200 times

The replica method



For each point, a central 68% confidence interval is identified

Previous fit studies

	Framework	HERMES	COMPASS	DY	Z production	N of points
KN 2006 hep-ph/0506225	LO-NLL	✘	✘	✔	✔	98
Pavia 2013 (+Amsterdam, Bilbao) arXiv:1309.3507	No evo (QPM)	✔	✘	✘	✘	1538
Torino 2014 (+JLab) arXiv:1312.6261	No evo (QPM)	✔ (separately)	✔ (separately)	✘	✘	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NLO-NNLL	✘	✘	✔	✔	223
EIKV 2014 arXiv:1401.5078	LO-NLL	1 (x,Q^2) bin	1 (x,Q^2) bin	✔	✔	500 (?)
Pavia 2017 (+ JLab)	LO-NLL	✔	✔	✔	✔	**8059**

μ and b_* prescriptions

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T)\ln\frac{\mu}{\mu_0}} \hat{f}_{\mathrm{NP}}^a(x, b_T)$$

μ and b$_*$ prescriptions

Choice **Choice**

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}^a_{\rm NP}(x, b_T)$$

μ and b* prescriptions

Choice **Choice**

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*; \mu_b, \mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

$$\mu_b = 2e^{-\gamma_E}/b_* \qquad b_* \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\rm max}^2}}$$

Collins, Soper, Sterman, NPB250 (85)

$$\mu_b = 2e^{-\gamma_E}/b_* \qquad b_* \equiv b_{\rm max}\left(1 - e^{-\frac{b_T^4}{b_{\rm max}^4}}\right)^{1/4}$$

Bacchetta, Echevarria, Mulders, Radici, Signori
arXiv:1508.00402

$$\mu_b = Q_0 + q_T \qquad b_* = b_T$$

DEMS 2014

μ and b* prescriptions

Choice **Choice**

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

$$\mu_b = 2e^{-\gamma_E}/b_* \qquad b_* \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\max}^2}}$$

Collins, Soper, Sterman, NPB250 (85)

$$\mu_b = 2e^{-\gamma_E}/b_* \qquad b_* \equiv b_{\max}\left(1 - e^{-\frac{b_T^4}{b_{\max}^4}}\right)^{1/4}$$

Bacchetta, Echevarria, Mulders, Radici, Signori
arXiv:1508.00402

$$\mu_b = Q_0 + q_T \qquad b_* = b_T$$

DEMS 2014

Complex-b prescription

Laenen, Sterman, Vogelsang, PRL 84 (00)

Pavia 2017 perturbative ingredients

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T)\ln\frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

$A_1(\mathcal{O}(\alpha_S^1))$ ✔	$A_2(\mathcal{O}(\alpha_S^2))$ ✔	$A_3(\mathcal{O}(\alpha_S^3))$	...
	$B_1(\mathcal{O}(\alpha_S^1))$ ✔	$B_2(\mathcal{O}(\alpha_S^2))$	...
$C_0(\mathcal{O}(\alpha_S^0))$ ✔	$C_1(\mathcal{O}(\alpha_S^1))$	$C_2(\mathcal{O}(\alpha_S^2))$	...

$H_0(\mathcal{O}(\alpha_S^0))$ ✔	$H_1(\mathcal{O}(\alpha_S^1))$	$H_2(\mathcal{O}(\alpha_S^2))$	...
	$Y_1(\mathcal{O}(\alpha_S^1))$	$Y_2(\mathcal{O}(\alpha_S^2))$	...

Model: non perturbative elements

input TMD FF (Q^2=1GeV^2)

$$\hat{D}_{1NP}^{a\to h} = \text{ F.T. of } \quad \frac{1}{g_{3a\to h} + \left(\lambda_F/z^2\right) g_{4a\to h}^2} \left(e^{-\frac{\boldsymbol{P}_\perp^2}{g_{3a\to h}}} + \lambda_F \frac{\boldsymbol{P}_\perp^2}{z^2}\, e^{-\frac{\boldsymbol{P}_\perp^2}{g_{4a\to h}}} \right)$$

sum of two different gaussians
with different variance
with kinematic dependence on transverse momenta

width z-dependence

$$g_{3,4}(z) = N_{3,4} \frac{\left(z^\beta + \delta\right)\ (1-z)^\gamma}{\left(\hat{z}^\beta + \delta\right)\ (1-\hat{z})^\gamma}$$

where

$$N_{3,4} \equiv g_{3,4}(\hat{z})$$

$$\hat{z} = 0.5$$

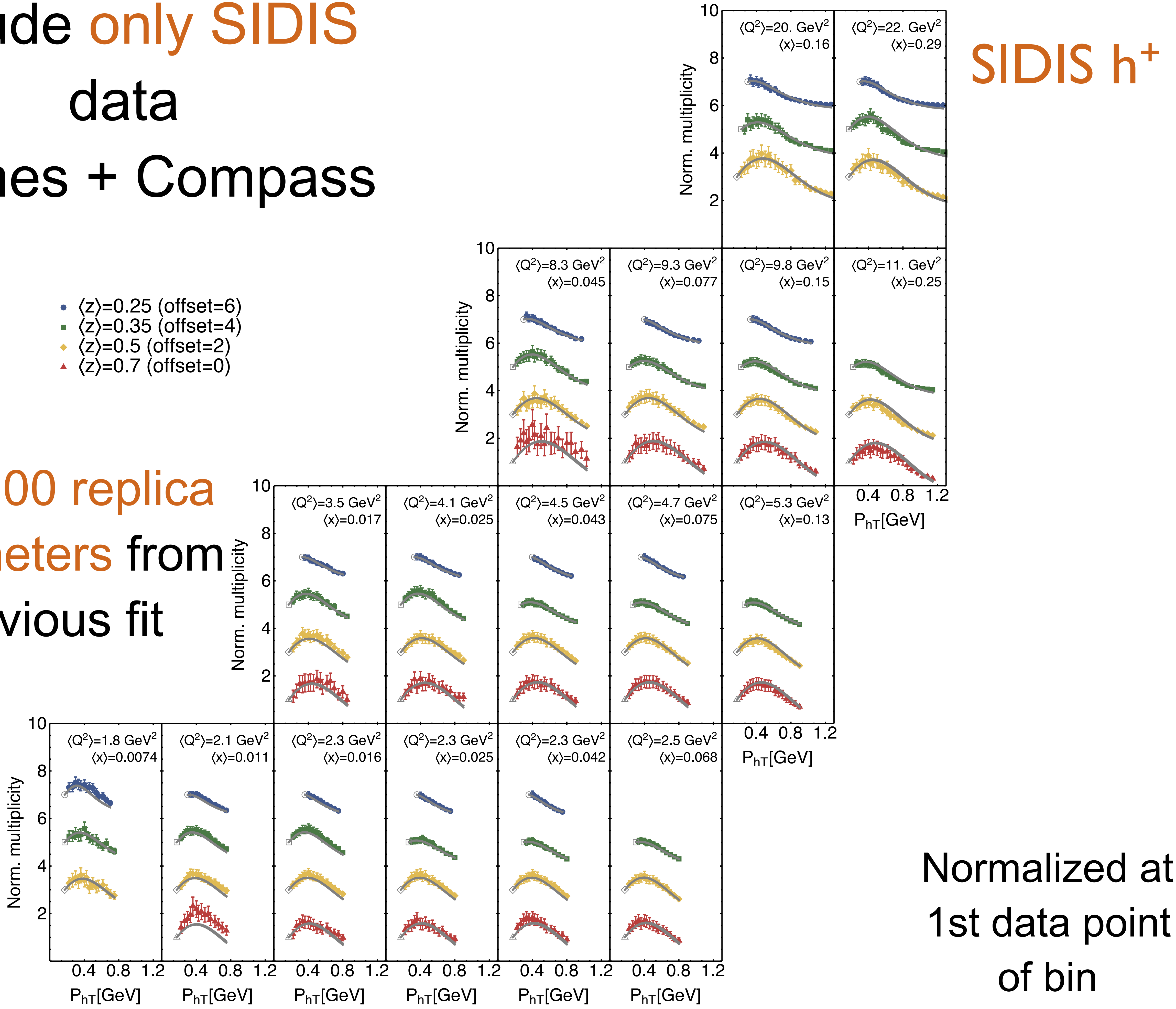
Average transverse momenta

$$\langle \boldsymbol{k}_\perp^2 \rangle(x) = \frac{g_1(x) + 2\lambda g_1^2(x)}{1 + \lambda g_1(x)}$$

$$\langle \boldsymbol{P}_\perp^2 \rangle(z) = \frac{g_3^2(z) + 2\lambda_F g_4^3(z)}{g_3(z) + \lambda_F g_4^2(z)}$$

Include only SIDIS data Hermes + Compass

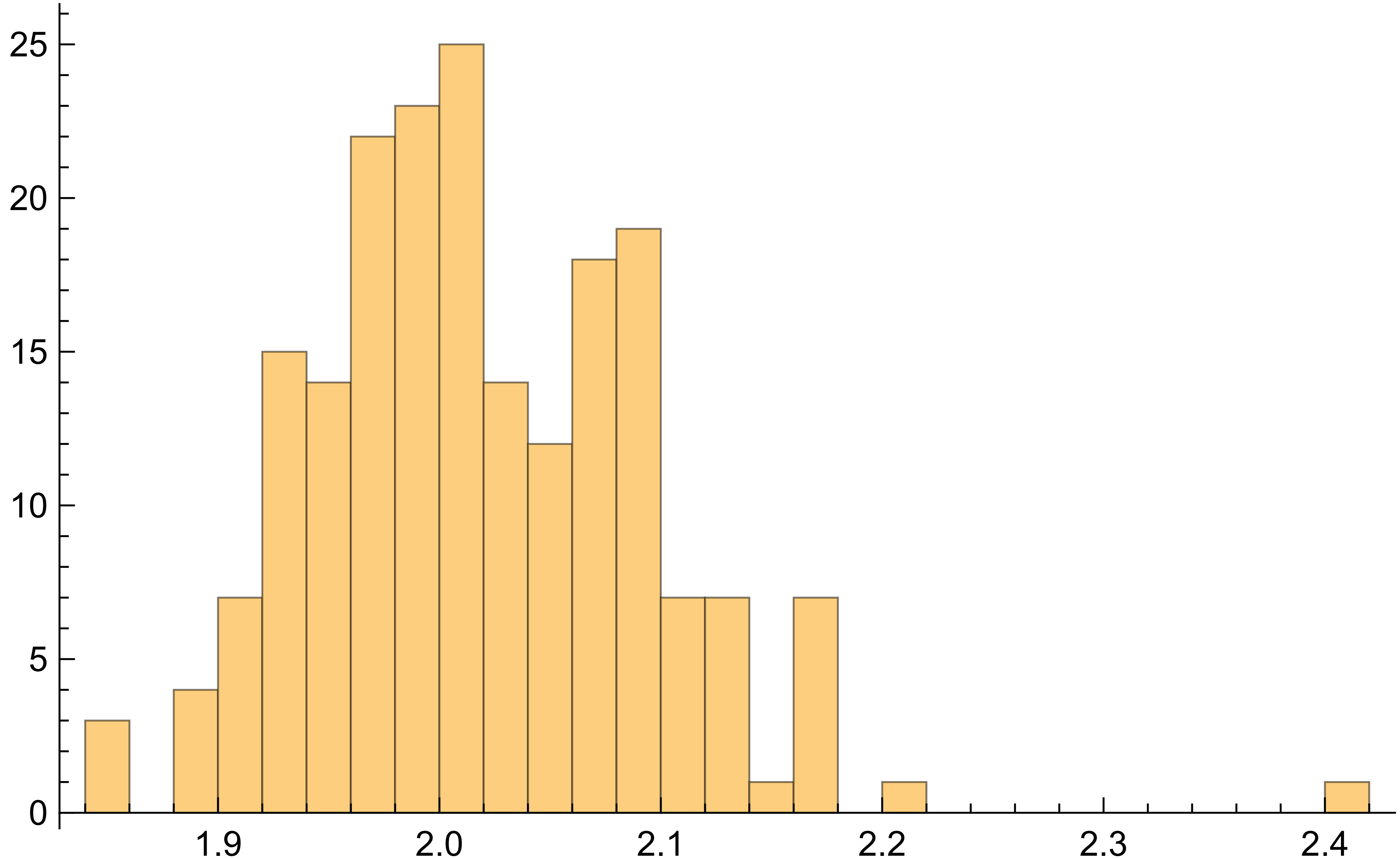
Use 200 replica parameters from previous fit



Normalized at 1st data point of bin

Include only SIDIS data

SIDIS h^+



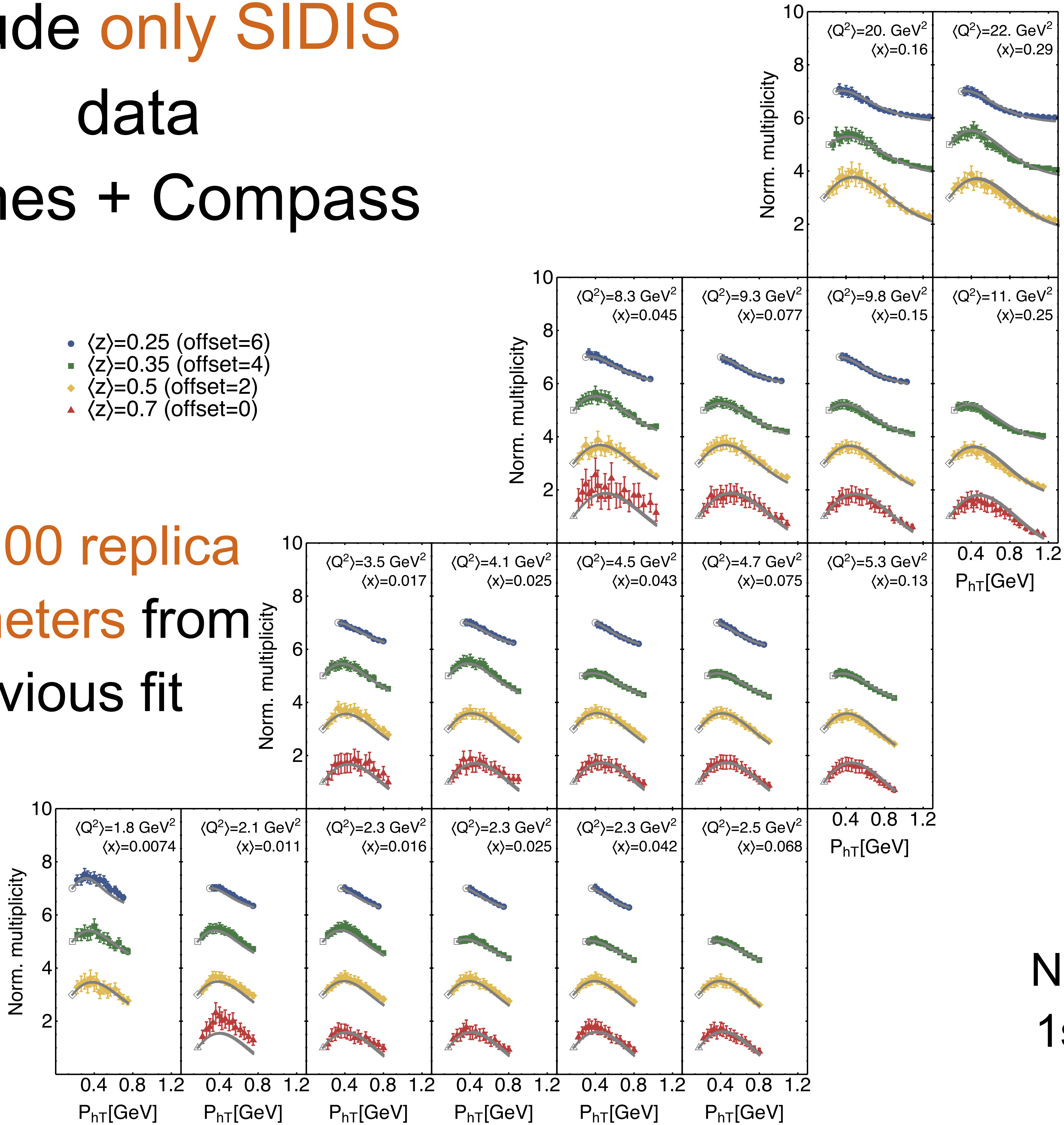
Use 200 replica parameters from previous fit

χ^2/dof = **2.07**

Normalized at 1st data point of bin

Include only SIDIS data
Hermes + Compass

SIDIS h^+

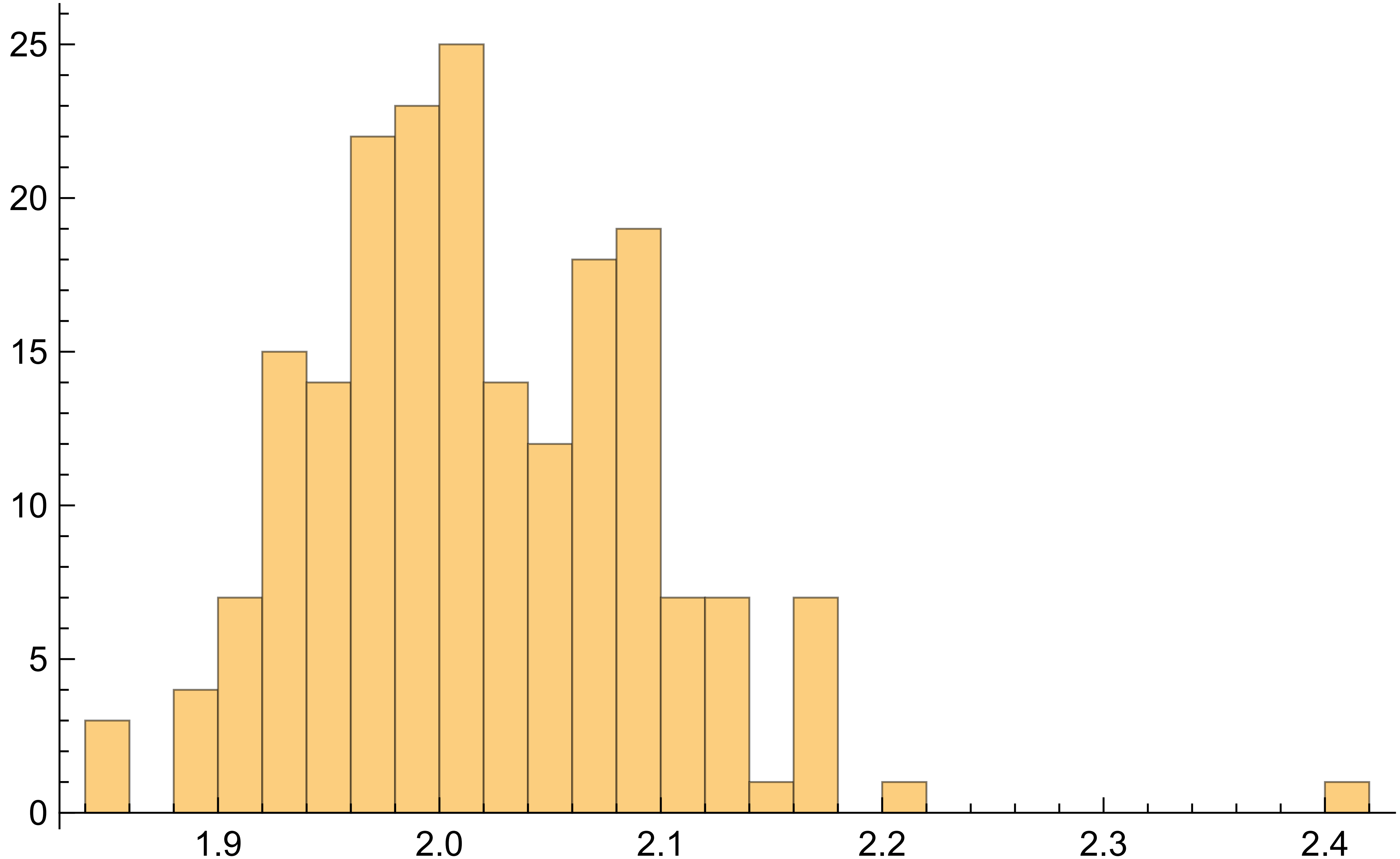


Use 200 replica parameters from previous fit

Normalized at 1st data point of bin

Include only SIDIS data

SIDIS h^+



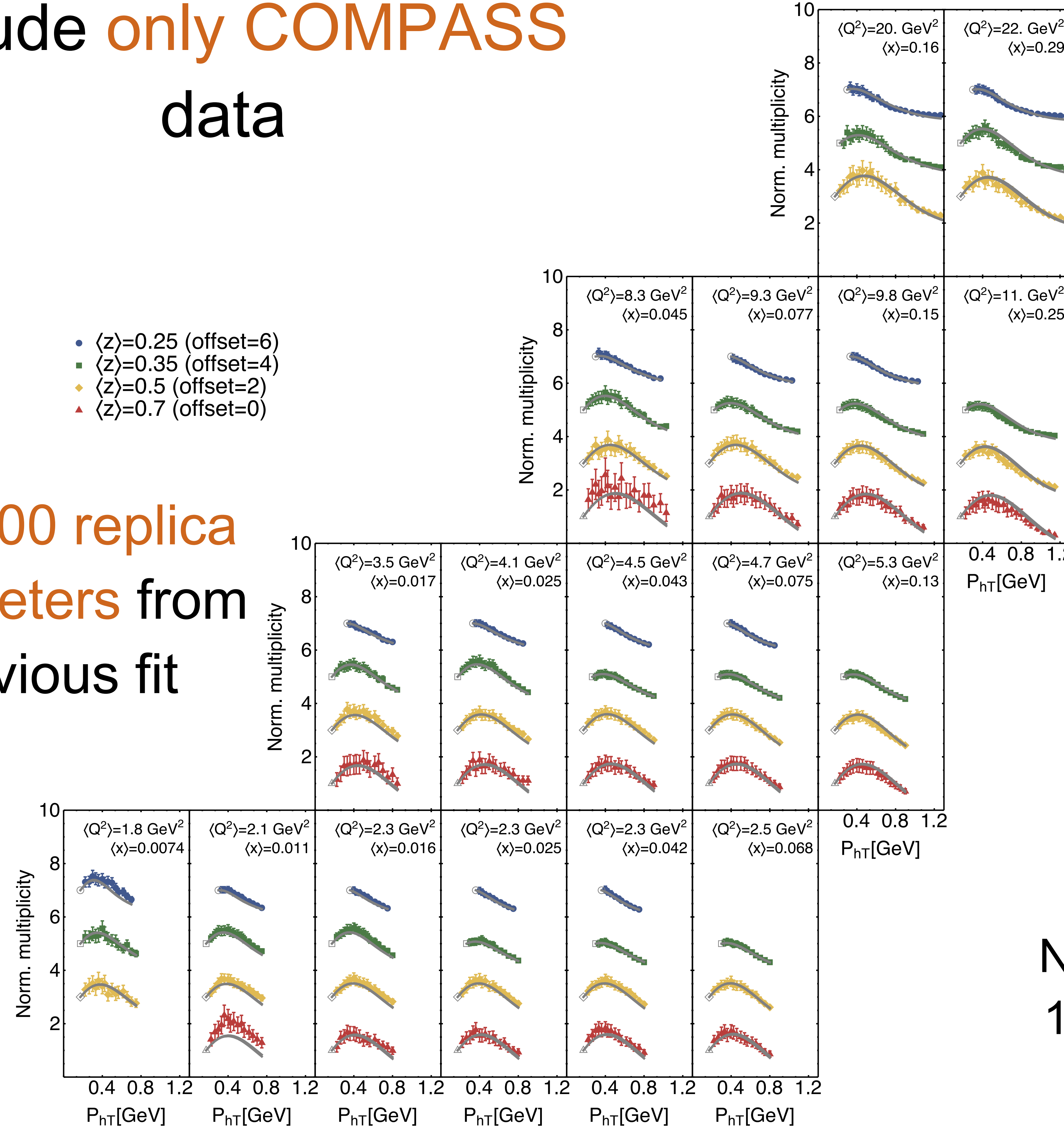
Use 200 replica parameters from previous fit

χ^2/dof = **2.07**

Normalized at 1st data point of bin

Include only COMPASS data

SIDIS h⁺

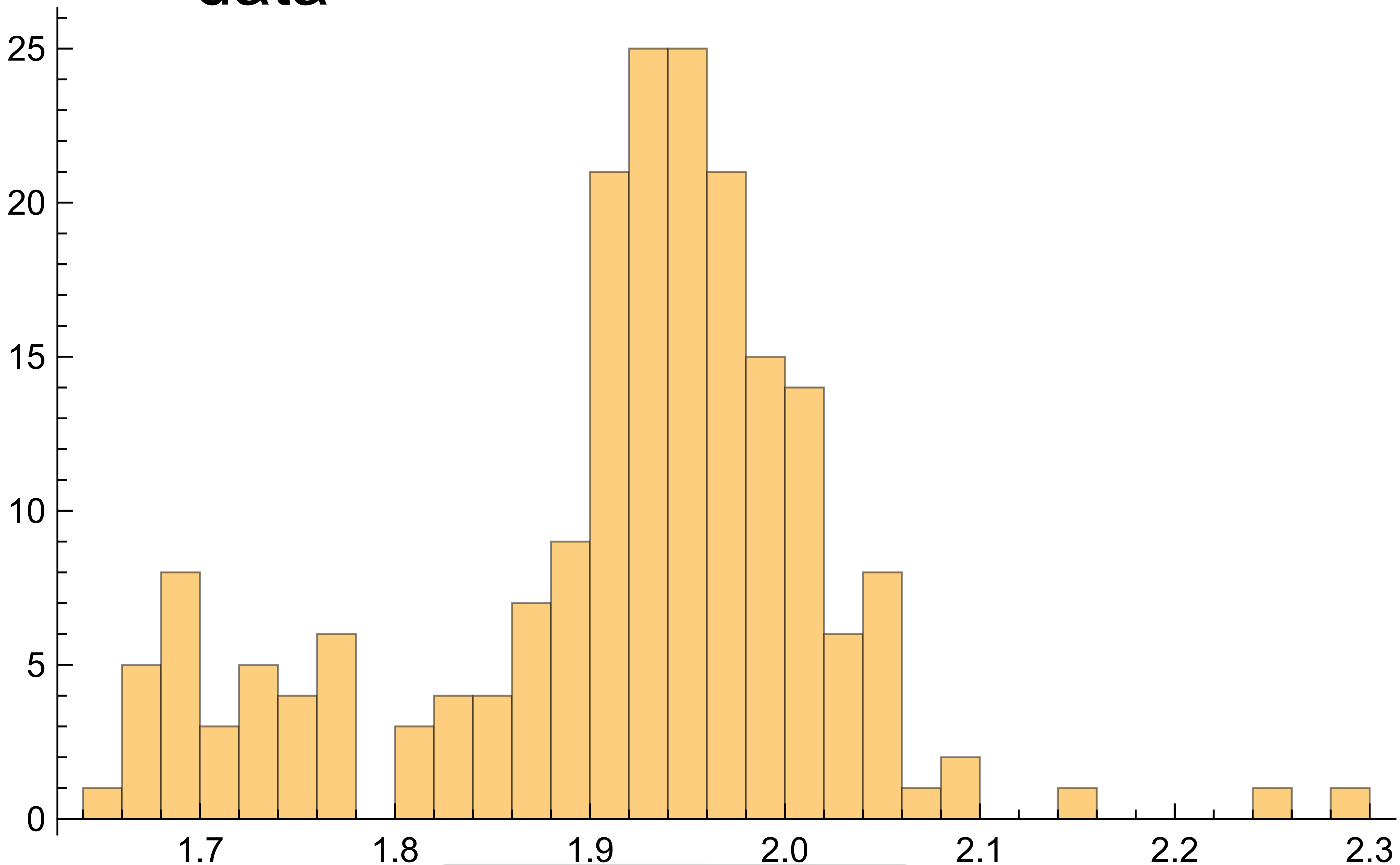


Use 200 replica parameters from previous fit

Normalized at 1st data point of bin

Include only COMPASS data

SIDIS h^+



Use 200 replica parameters from previous fit

$\chi^2/\mathrm{dof} = 1.91$

Normalized at 1st data point of bin