Pion polarizabilities in ChPT

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Nucl. Phys. B 728 (2005) 31-54 (neutral pions)

Nucl. Phys. B 745 (2006) 84-108 (charged pions)

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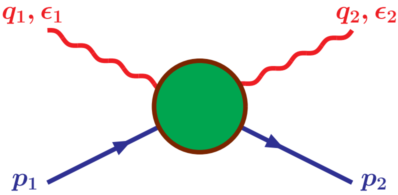
Pion polarizabilities: definition



Transverse gauge $\epsilon_i = (0, \vec{\epsilon}_i)$

LAB frame $p_1 = (M_\pi, \vec{0})$

Expansion in $q_i = (\omega_i, \vec{q}_i)$:

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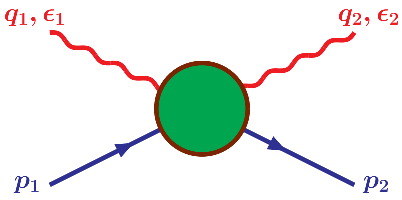
Pion polarizabilities: definition



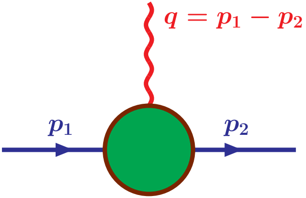
Transverse gauge $\epsilon_i = (0, \vec{\epsilon}_i)$

LAB frame $p_1 = (M_\pi, \vec{0})$

Expansion in $q_i = (\omega_i, \vec{q}_i)$:

$$T_{\gamma\pi^+\to\gamma\pi^+} = \underbrace{-2\,e^2\,\vec{\epsilon}_1\cdot\vec{\epsilon}_2}_{\text{Born term}} + 8\,\pi\,M_\pi\,\underbrace{\left\{\alpha_\pi\,\omega_1\omega_2\,\vec{\epsilon}_1\cdot\vec{\epsilon}_2 + \beta_\pi\,(\vec{\epsilon}_1\times\vec{q}_1)\cdot(\vec{\epsilon}_2\times\vec{q}_2)\right\}}_{\text{el-mag polarizabilities}} + \dots$$

- The electric, α_π, and magnetic, β_π, polarizabilities characterize the response of hadrons to their two-photon interactions at low energies
- These quantities are analogous to electromagnetic radii and magnetic moments which characterize the response of hadrons to their single-photon interactions at low energies



$$< \pi(p_2)|j^\mu|\pi(p_1) >= e(p_1 + p_2)^\mu F_\pi(q^2)$$

$$F_\pi(q^2) = 1 + \frac{1}{6} r_\pi^2 q^2 + \ldots$$

- The concept of the polarizability of molecules, atoms and nuclei was applied for the first time to hadrons in

  A. Klein, Phys. Rev. 99 (1955) 998,
  A.M. Baldin, Nucl. Phys. 18 (1960) 310,
  V.A. Petrun'kin, JETP 13 (1961) 804

- Many theoretical papers afterwards
- Only a few experiments

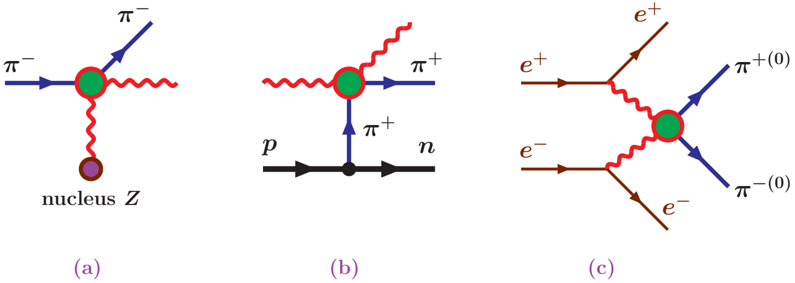
The units of measurement

As follows from the definition, the dipole pion polarizabilities are proportional to

$$\alpha_\pi(\beta_\pi) \sim \frac{\alpha}{M_\pi}\frac{1}{\Lambda^2} \approx 4 \times 10^{-4}\,\mathrm{fm}^3$$

where the hadronic scale $\Lambda \sim 4\pi F_\pi \sim 1\,\mathrm{GeV}$.
Then a natural choice of units for the dipole polarizabilities is $10^{-4}\,\mathrm{fm}^3$

Experiment

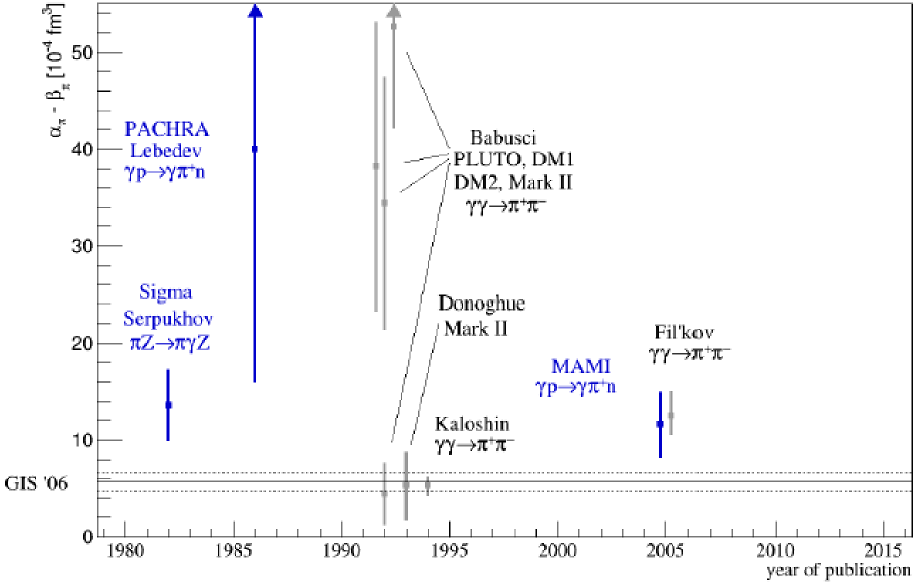


(a) The scattering of high energy pions off the Coulomb field of heavy nucleus.

(b) The radiative pion photoproduction from the proton.

(c) The pion pair production in photon-photon collisions.

Plot: T.Nagel, PhD TUM, 2012



GIS'06 = Gasser, Ivanov, Sainio, Nucl. Phys. B745 (2006) 84

General properties of pion polarizabilities

- **Classical sum rule (Petrun'kin'64):**

$$\alpha_\pi = \frac{\alpha}{3m} < r_\pi^2 > +2\alpha \sum_{n\neq 0} \frac{|<n|\mathcal{D}|0>|^2}{E_n - E_0}$$

where $\mathcal{D}$ is the electric dipole operator.

$$\alpha_{\pi^\pm} \mapsto (3.5 - 6.8)$$

- **The optical theorem relates the sum of polarizabilities to an unsubtracted forward dispersion relation**

$$(\alpha + \beta)_\pi = \frac{M_\pi^2}{\pi^2} \int_{4M_\pi^2}^{\infty} \frac{ds'}{(s' - M_\pi^2)^2} \sigma_{tot}^{\gamma\pi}(s') > 0$$

Low Energy Theorem

- **Using current algebra/PCAC gives the relation of $\alpha_\pi (\beta_\pi)$ with the vector F_V and axial F_A structure constants for radiative pion decays $\pi^- \to e\nu\gamma$ (Terent'ev'73):**

$$\alpha_{\pi^\pm} = -\beta_{\pi^\pm} = \frac{\alpha}{8\pi^2 F_\pi^2 M_\pi} \frac{F_A}{F_V} = 2.7 \pm 0.4$$

Update: PIBETA Coll.: PRL 103, 051802 (2009): = 2.78 (10)

Models

- Sum rule for axial constant (Das,Mathur,Okubo'67)

$$F_A = \frac{< r^2_{\pi^\pm} >}{3} F_\pi - \frac{1}{F_\pi} \int ds \, \frac{\rho^V(s) - \rho^A(s)}{s^2}$$

- Quark loop diagrams (Gerasimov'79)

$$\alpha_{\pi^\pm} \approx 6$$

- Assuming $\rho(s) = (M^4/g^2)\delta(s - M^2)$ and using VDM and KSFR relations give (Holstein'90):

$$\alpha_{\pi^\pm} \sim \frac{\alpha}{M_\pi M_A^2} \approx 2.6$$

Models

- **Nonlinear σ-model, chiral quark loops, superconducting type σ-model (Volkov,Pervushin'75, ...)**

$$\alpha_{\pi^\pm} \mapsto (5.0 - 5.8)$$

- **Quark confinement model (Ivanov,Mizutani'92)**

$$\alpha_{\pi^\pm} \approx 3.6$$

- **Nonlocal chiral quark model (Dorokhov,Broniowski'03)**

$$\alpha_{\pi^\pm} \approx 2.9$$

- Chiral expansion to one-loop
  (Bijnens & Cornet'88; Donoghue & Holstein'89):

$$(\alpha+\beta)_\pi = 0\,,$$
$$\alpha_{\pi^\pm} = \frac{\alpha}{8\,\pi^2\,F_\pi^2\,M_\pi}\cdot\frac{1}{6}\,(\bar\ell_6-\bar\ell_5)$$
$$= \frac{\alpha}{8\pi^2 M_\pi F_\pi^2}\,\frac{F_A}{F_V} = 2.7 \pm 0.4$$

This reproduces the Terent'ev low energy theorem.

- A fit of $\gamma\gamma \to \pi^+\pi^-$ data by using dispersion relations.
  Fil'kov and Kashevarov'2006

$$(\alpha+\beta)_{\pi^+} = 0.18^{+0.11}_{-0.02}$$
$$(\alpha-\beta)_{\pi^+} = 13.0^{+2.6}_{-1.9}$$

Effective Lagrangians: $\mathcal{L}_{\rm QCD} \longrightarrow \mathcal{L}_{\rm eff}$ for $E << M_\rho$

Weinberg'1979; Gasser, Leutwyler 1984,1985

$$\mathcal{L}_{\rm eff} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \ldots$$

$\mathcal{L}_{\rm eff}$ expressed in observed hadron fields, has the same symmetry as QCD.

- The leading p^2-order in chiral SU(2) (pions and photons only):

$$\mathcal{L}_2 = \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger + M^2(U + U^\dagger)\rangle \quad U \in SU(2)\,, \text{contains pions}$$

$$U = \sigma + i\,\pi/F\,, \quad \sigma^2 + \frac{\pi^2}{F^2} = 1_{2\times 2}\,, \quad \pi = \begin{pmatrix} \pi^0 & \sqrt{2}\,\pi^+ \\ \sqrt{2}\,\pi^- & -\pi^0 \end{pmatrix}$$

$$D_\mu U = \partial_\mu U - i(QU - UQ)A_\mu\,, \qquad Q = \frac{e}{2}\mathrm{diag}(1,-1)\,,$$

- $M^2 = (m_u + m_d)B$
- F, B are low-energy constants (LECs) not fixed by chiral symmetry
- $\mathcal{L}_2$ - nonrenormalizable quantum field theory

Higher order Lagrangians

$$p^4 \quad \to \quad \mathcal{L}_4 = \sum_{i=1}^{10} \ell_i K_i = \frac{\ell_1}{4} \langle D_\mu U \, D^\mu U^\dagger \rangle^2 + \cdots ,$$

$$p^6 \quad \to \quad \mathcal{L}_6 = \sum_{i=1}^{57} c_i P_i ,$$

($57 \to 56$) Haefeli, Ivanov, Schmid, Ecker 2007

- Local monomials K_i, P_i are known

Gasser, Leutwyler 1984,1985; Bijnens, Colangelo,Ecker 1999

- LECs ℓ_i, c_i absorb the divergences at order p^4 and p^6
- Notation later on: $\ell_i, c_i \to \ell_i^r, c_i^r$ UV finite parts of ℓ_i, c_i;
  $\ell_i^r, c_i^r \to \bar{\ell}_i, \bar{c}_i$ scale independent parts of ℓ_i, c_i.

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  $\ell_i^r, c_i^r \to \bar{\ell}_i, \bar{c}_i$ scale independent parts of ℓ_i, c_i.
- Calculations with $\mathcal{L}_{\text{eff}}$ give an expansion in quark masses and external momenta.

Chiral perturbation theory (ChPT)

Gasser, Leutwyler 1984,1985

$\gamma\gamma \to \pi\pi$ and pion polarizabilities in ChPT

- **Pion polarizabilities in ChPT to one-loop**

  Bijnens,Cornet 1988
  Donoghue,Holstein 1989

- **Chiral expansion to two-loops**

  $\gamma\gamma \to \pi^0\pi^0$ — **Bellucci,Gasser,Sainio 1994**

  $\gamma\gamma \to \pi^+\pi^-$ — **Burgi 1996**

- **Recalculation of the amplitudes** — **Gasser, Ivanov, Sainio 2005,2006**

$\gamma\gamma \to \pi\pi$ amplitude

Reaction $\gamma(q_1, \mu) + \gamma(q_2, \nu) \to \pi(p_1) + \pi(p_2)$

Gauge invariance and transversality of photons $\Longrightarrow$ two Lorentz structures:

$$V_{\mu\nu} = A(s,t,u)\, T_{1\,\mu\nu} + B(s,t,u)\, T_{2\,\mu\nu}$$

$$T_{1\,\mu\nu} = \frac{1}{2} s\, g_{\mu\nu} - q_{1\nu} q_{2\mu}\,,$$

$$T_{2\,\mu\nu} = 2\, s\, \Delta_\mu \Delta_\nu - \nu^2 g_{\mu\nu} - 2\,\nu\,(q_{1\,\nu}\Delta_\mu - q_{2\,\mu}\Delta_\nu)\,,$$

$$\Delta_\mu = (p_1 - p_2)_\mu\,, \qquad \nu = t - u.$$

Helicity amplitudes:

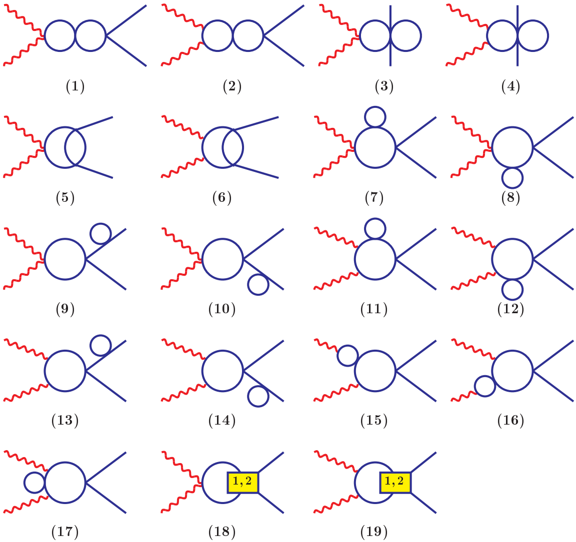
$$H_{++} = A + 2\,(4\,M_\pi^2 - s)\,B\,, \qquad \text{helicity non-flip}\,,$$

$$H_{+-} = \frac{8\,(M_\pi^4 - t\,u)}{s}\,B\,, \qquad \text{helicity flip}\,.$$

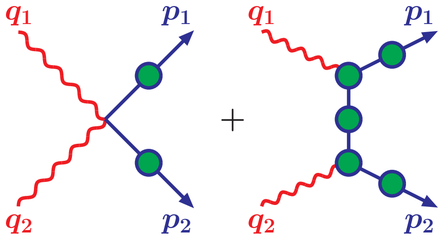
Polarizabilities (Born term is removed):

$$\frac{\alpha}{M_\pi} H_{+\mp}(s, t = M_\pi^2) = \underbrace{(\alpha_1 \pm \beta_1)}_{\text{dipole}} + \frac{s}{12}\underbrace{(\alpha_2 \pm \beta_2)}_{\text{quadrupole}} + \mathcal{O}(s^2)\,.$$

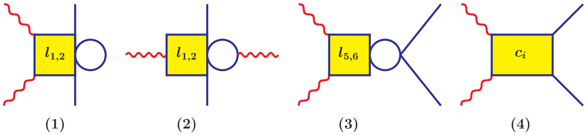
A set of two-loop diagrams generated by $\mathcal{L}_2$ and one-loop generated by $\mathcal{L}_4$



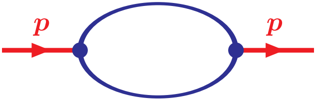
A class of one-particle reducible diagrams



One-loop graphs generated by $\mathcal{L}_4$ and counterterm from $\mathcal{L}_6$



Two-point subdiagram

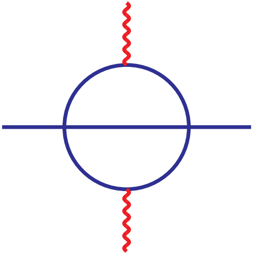


$$J(p^2) = \int \frac{d^d\ell}{(2\pi)^d i} \frac{1}{[1-\ell^2][1-(\ell+p)^2]}$$

$$= \frac{\Gamma(-w)}{(4\pi)^{2+w}} \int_0^1 dx\,[1-p^2\,x\,(1-x)]^w = \int_4^\infty \frac{[d\sigma]}{\sigma - p^2}$$

$w = d/2 - 2$

Acnode diagram



Invoke a dispersion relation for the function

$$\mathbf{I}(\mu, n; s) = \int_0^1 dx\, [x\,(1-x)]^n\, [1 - s\,x\,(1-x)]^\mu$$

$$= \int_4^\infty \frac{d\sigma\, \rho\,(\mu, n; \sigma)}{\sigma - s} \qquad (-1 < \mu < 0)$$

General structure of the amplitudes to two-loops

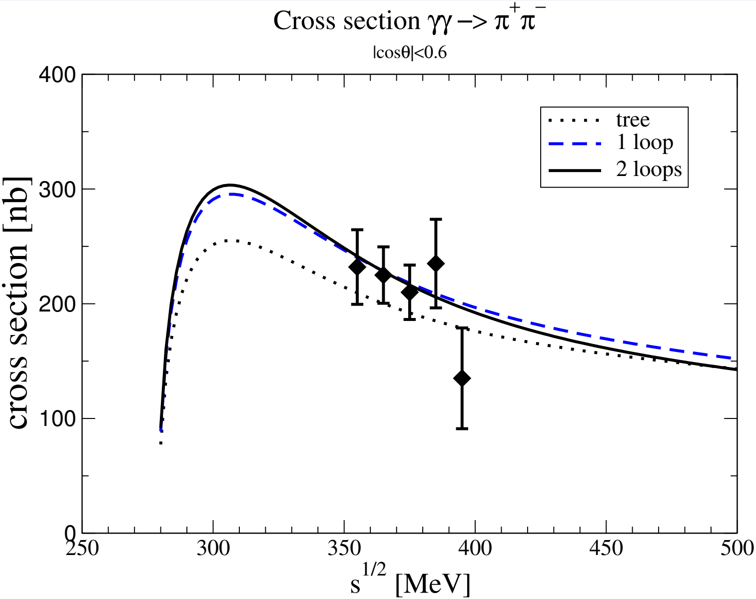
$$A = A_{\rm Born} + A_{1\,\rm loop} + A_{2\,\rm loop}\,; \qquad A_{2\,\rm loop} = U_A + P_A\,,$$
$$B = B_{\rm Born} + B_{2\,\rm loop}\,; \qquad B_{2\,\rm loop} = U_B + P_B\,.$$

$$P = M_\pi^2\, P_1 + s\, P_2\,, \qquad P_i = P_i\left(\bar{\ell}_i, \ell, c_i^r\right)\,, \qquad \text{Polynomial part}\,,$$

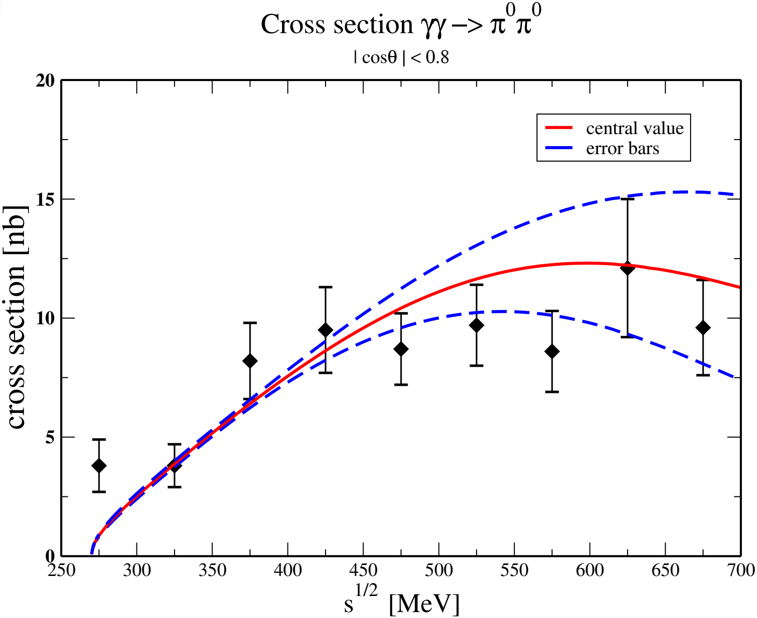
$$U = \bar{U} + \Delta\,, \qquad \bar{U}\left\{\bar{J}(s), \bar{G}(s);\ \bar{\ell}_i;\ s, t, u\right\}\,, \qquad \text{Unitarity part}\,,$$

$$\Delta = \Delta(s,t,u) \qquad \text{are genuine two-loop integrals}\,.$$

The quantities $\Delta_{A,B}$ were given in numerical form only in previous calculations.



Experimental data from MARK II (SLAC) 1990


Cross section γγ –> π0π0
| cosθ | < 0.8
central value
error bars
cross section [nb]
s1/2 [MeV]
0
5
10
15
20
250
300
350
400
450
500
550
600
650
700

Analytic results for the dipole polarizabilities

$$(\alpha_1 \pm \beta_1)_{\pi^+} = \frac{\alpha}{16\,\pi^2\,F_\pi^2\,M_\pi}\left\{c_{1\pm} + \frac{M_\pi^2\,d_{1\pm}}{16\,\pi^2\,F_\pi^2} + O(M_\pi^4)\right\},$$

$$c_{1+} = 0\,, \qquad c_{1-} = \frac{2}{3}\,\bar{\ell}_\Delta\,,$$

$$d_{1+} = 8\,b^r - \frac{4}{9}\left\{\ell\left(\ell + \frac{1}{2}\,\bar{\ell}_1 + \frac{3}{2}\,\bar{\ell}_2\right) - \frac{53}{24}\,\ell + \frac{1}{2}\,\bar{\ell}_1 + \frac{3}{2}\,\bar{\ell}_2 + \frac{91}{72} + \Delta_+\right\}$$

$$d_{1-} = a_1^r + 8\,b^r - \frac{4}{3}\left\{\ell\left(\bar{\ell}_1 - \bar{\ell}_2 + \bar{\ell}_\Delta - \frac{65}{12}\right) - \frac{1}{3}\,\bar{\ell}_1 - \frac{1}{3}\,\bar{\ell}_2 + \frac{1}{4}\,\bar{\ell}_3 - \bar{\ell}_\Delta\bar{\ell}_4 + \frac{187}{108} + \Delta_-\right\}$$

$$\Delta_+ = \frac{8105}{576} - \frac{135}{64}\pi^2 = \underbrace{-6.75,}_{\text{Burgi:}-8.69} \qquad \Delta_- = \frac{41}{432} - \frac{53}{64}\pi^2 = \underbrace{-8.08.}_{\text{Burgi:}-8.73}$$

Analytic results for the quadrupole polarizabilities

$$(\alpha_2 \pm \beta_2)_{\pi^+} = \frac{\alpha}{16\,\pi^2\,F_\pi^2\,M_\pi^3}\left\{c_{2\pm} + \frac{M_\pi^2\,d_{2\pm}}{16\,\pi^2\,F_\pi^2} + O(M_\pi^4)\right\},$$

$$c_{2+} = 0\,, \qquad c_{2-} = 2\,,$$

$$d_{2+} = -\frac{2062}{27} + \frac{10817}{1440}\pi^2 + \frac{8}{45}\,\bar{\ell}_1 + \frac{8}{15}\,\bar{\ell}_2$$

$$d_{2-} = 12\,a_2^r - 24\,b^r - \ell(10 + 4\bar{\ell}_\Delta) - \frac{8}{15}\,\bar{\ell}_3 + 4\,\bar{\ell}_4 - 4\bar{\ell}_\Delta - \frac{218}{45}\,\bar{\ell}_1 - \frac{238}{45}\,\bar{\ell}_2 - \frac{56}{45} - \frac{1199}{1920}\pi^2$$

Numerics

Numerical values of LECs

$$\bar{\ell}_1 = -0.4 \pm 0.6\,,\;\; \bar{\ell}_2 = 4.3 \pm 0.1\,,\;\; \bar{\ell}_3 = 2.9 \pm 2.4\,,\;\; \bar{\ell}_4 = 4.4 \pm 0.2$$

Colangelo, Gasser, Leutwyler 2001

$$\bar{\ell}_\Delta \doteq \bar{\ell}_6 - \bar{\ell}_5 = 3.0 \pm 0.3\,.$$

Bijnens, Talavera 1997

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$$\bar{\ell}_1 = -0.4 \pm 0.6\,,\ \bar{\ell}_2 = 4.3 \pm 0.1\,,\ \bar{\ell}_3 = 2.9 \pm 2.4\,,\ \bar{\ell}_4 = 4.4 \pm 0.2$$

Colangelo, Gasser, Leutwyler 2001

$$\bar{\ell}_\Delta \doteq \bar{\ell}_6 - \bar{\ell}_5 = 3.0 \pm 0.3\,.$$

Bijnens, Talavera 1997

$$\begin{aligned} a_1^r &= -4096\pi^4\,(6\,c_6^r + c_{29}^r - c_{30}^r - 3\,c_{34}^r + c_{35}^r + 2\,c_{46}^r - 4\,c_{47}^r + c_{50}^r) \\ a_2^r &= 256\pi^4\,(8\,c_{29}^r - 8\,c_{30}^r + c_{31}^r + c_{32}^r - 2\,c_{33}^r + 4\,c_{44}^r + 8\,c_{50}^r - 4\,c_{51}^r) \\ b^r &= -128\pi^4\,(c_{31}^r + c_{32}^r - 2\,c_{33}^r - 4\,c_{44}^r) \end{aligned}$$

Resonance ρ, a_1, b_1 exchange at $\mu = M_\rho$

$$(a_1^r, a_2^r, b^r) = (-3.2, 0.7, 0.4)$$

ENJL model with large N_c (Bijnens & Prades)

$$(a_1^r, a_2^r, b^r) = (-8.7, 5.9, 0.38)$$

We use $b^r = 0.4 \pm 0.4$ and vary a_1^r from -10 to 0.

Estimating the uncertainties

- **Uncertainties from the LECs. Use the central values of the p^4 LECs and two sets of the p^6 LECs:**

$$(a_1^r, a_2^r, b^r) = \begin{cases} (-3.2,\ 0.7,\ 0.4) \\ (-8.7,\ 5.9,\ 0.38) \end{cases}$$

- **Dipole $(10^{-4}\ \mathrm{fm}^3)$ and quadrupole $(10^{-4}\ \mathrm{fm}^5)$ charged pion polarizabilities.**

	to one loop	to two-loops
$(\alpha_1 - \beta_1)_{\pi^+}$	6.0	5.7 [5.5]
$(\alpha_1 + \beta_1)_{\pi^+}$	0	0.16 [0.16]
$(\alpha_2 - \beta_2)_{\pi^+}$	11.9	16.2 [21.6]
$(\alpha_2 + \beta_2)_{\pi^+}$	0	−0.001 [−0.001]

Estimating the uncertainties

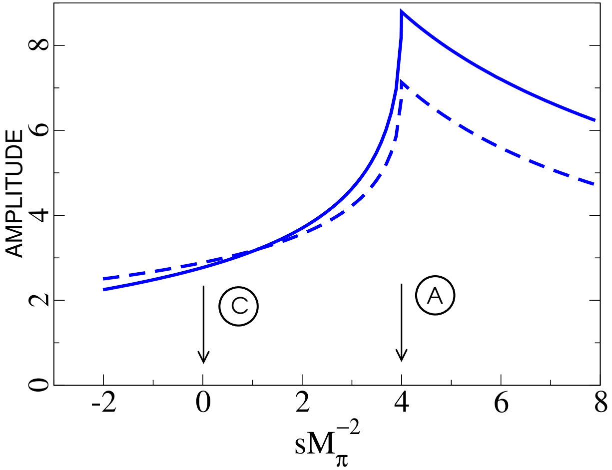
The contribution of the LECs at order p^6 to the polarizabilities.

	a_1^r	a_2^r	b^r	total
$(\alpha_1 - \beta_1)_{\pi^+}$	-0.14 [-0.37]	0	0.14 [0.13]	0 [-0.24]
$(\alpha_1 + \beta_1)_{\pi^+}$	0	0	0.14 [0.13]	0.14 [0.13]
$(\alpha_2 - \beta_2)_{\pi^+}$	0	0.72 [6.09]	-0.83 [-0.78]	-0.10 [5.31]
$(\alpha_2 + \beta_2)_{\pi^+}$	0	0	0	0

Resonance saturation generates additional uncertainty related to the choice of a scale μ. Using the choice $(a_1^r, a_2^r, b^r) = (-3.2,\ 0.7,\ 0.4)$ one gets

	$\mu = 500$ MeV	$\mu = 1$ GeV
$(\alpha_1 - \beta_1)_{\pi^+}$	6.1	5.5
$(\alpha_1 + \beta_1)_{\pi^+}$	0.20	0.13
$(\alpha_2 - \beta_2)_{\pi^+}$	14.6	17.2
$(\alpha_2 + \beta_2)_{\pi^+}$	-0.001	-0.001

Chiral expansion at the Compton threshold



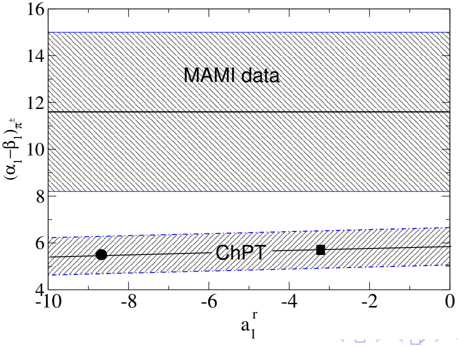
Example: spin non-flip amplitude. Solid line → two-loops, dashed line → one-loop

Value of the dipole polarizability $(\alpha_1 - \beta_1)_{\pi^\pm}$

- **Add the uncertainties from the couplings at order p^4, from b^r and from the scale dependence introduced by the resonance scheme in quadrature and obtain uncertainty**

$$\Delta = 0.80 \times 10^{-4}\mathrm{fm}^3$$

- **Plot the dependence of $(\alpha_1 - \beta_1)_{\pi^\pm}$ on the a_1^r.**



Value of the dipole polarizability $(\alpha_1 - \beta_1)_{\pi^\pm}$

- The final theoretical value for the dipole polarizability

$$(\alpha_1 - \beta_1)_{\pi^\pm} = (5.7 \pm 1.0) \times 10^{-4}\,\mathrm{fm}^3$$

- The central value is almost twice less than the value reported by Serpukhov and Mainz experiment and the value obtained by the dispersion relation analysis.

$$(\alpha_1 - \beta_1)_{\pi^+} = 13.0^{+2.6}_{-1.9} \times 10^{-4}\,\mathrm{fm}^3$$

* L. V. Fil'kov and V. L. Kashevarov, Phys.Rev. C73 (2006) 035210

Values of the polarizabilities (resonance saturation)

Uncertainties from the couplings at order p^4,

Uncertainties from the couplings at order p^6 (resonance saturation)

$$(a_1^r, a_2^r, b^r) = (-3.2 \pm 3.2,\ 0.7 \pm 0.7,\ 0.4 \pm 0.4),$$

Uncertainties from the scale dependence.

	ChPT to one loop	ChPT to two-loops	dispersion relations *
$(\alpha_1 - \beta_1)_{\pi^+}$	6.0	5.7 ± 0.8	$13.0^{+2.6}_{-1.9}$
$(\alpha_1 + \beta_1)_{\pi^+}$	0	0.16 ± 0.14	$0.18^{+0.11}_{-0.02}$
$(\alpha_2 - \beta_2)_{\pi^+}$	12.0	16.0 ± 1.2	$25.0^{+0.8}_{-0.3}$
$(\alpha_2 + \beta_2)_{\pi^+}$	0	-0.001 ± 0.010	0.133 ± 0.015

* L. V. Fil'kov and V. L. Kashevarov, Phys.Rev. C73 (2006) 035210

Value of the quadrupole polarizability $(\alpha_2 - \beta_2)_{\pi^\pm}$

- Substantial two-loop correction to the one-loop result. (This can be seen from the behavior of H_{++}).
- Chiral logarithms at order p^6 generate now a rather large contribution.
- The LEC a_2^r enters with weight 12. As a result of this, it now matters which value of a_2^r is used, even though its contribution is suppressed by the factor $M_\pi^2(16\pi^2 F_\pi^2)^{-1} \simeq 0.014$.
- If we use the value of $a_2^r = 5.9$ from ENJL model it gives $(\alpha_2 - \beta_2)_{\pi^\pm} = 21.6$ which is close to the analysis of Fil'kov and Kashevarov $25.0^{+0.8}_{-0.3}$
- However, we cannot take the outcome as a reliable two-loop prediction of ChPT: compared to the one-loop result, the two-loop contribution generates nearly a 100 percent contribution in this case.

Values of the polarizabilities $(\alpha_{1,2} + \beta_{1,2})_{\pi^\pm}$

- These polarizabilities are related to the helicity flip amplitude which starts out at order p^6
- Thus we determined only its leading order term in the chiral expansion.
- We checked whether there are potentially large contribution to H_{+-} at order p^8 from the resonance exchange.
- In the case of $\gamma\gamma \to \pi^0\pi^0$ there is substantial contribution from ω-exchange.
- However, this resonance does not contribute here, and the contributions from ρ-exchange are very small, except for the contribution to the slope parametrized by $(\alpha_2 + \beta_2)_{\pi^\pm}$ which is affected by $-0.04 \times 10^{-4}\ \mathrm{fm}^5$.
- On the other hand, there are also contributions from one-loop graphs at order p^8, where each vertex coming from the Wess-Zumino-Witten Lagrangian at order p^4. No reason why these should be small compared to the leading term at order p^6.
- We, therefore, do not consider the chiral prediction for these quantities particularly reliable.

Pion polarizability via Primakoff reaction

A.G. Galperin, G.V. Mitselmakher, A.G. Olshevski and V.N. Pervushin, Yad. Fiz. 32 (1980) 1053

- The first observation of the Compton scattering off pion at SIGMA spectrometer (Serpukhov)
- The first measurement of pion polarizabilities

Main advantages of COMPASS

- One can use pion and muon beams of the same momentum with the same setup configuration.

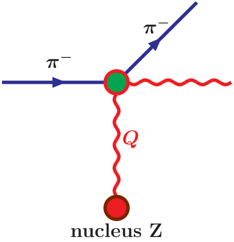
$$\pi^-(A,Z) \to \pi^-(A,Z)\gamma$$

$$\mu^-(A,Z) \to \mu^-(A,Z)\gamma$$

- Muon is the point-like particle and corresponding cross section for muon is known with high precision.
- So, muon data can be used as reference to control the systematics.

COMPASS

C. Adolph *et al.* Phys. Rev. Lett. 114 (2015) 062002



Primakoff reaction:

$\pi^- + Z \to \pi^- + Z + \gamma$

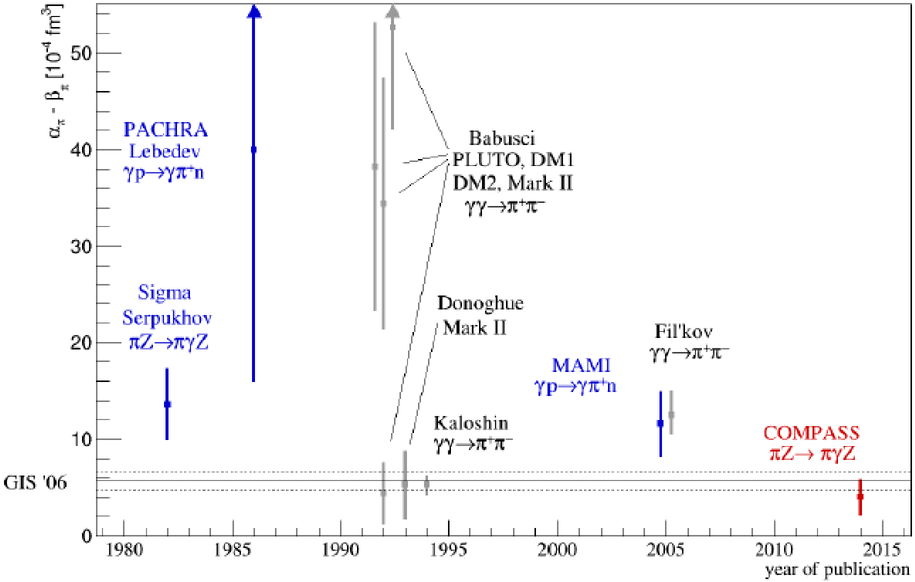
$E_\pi = 190$ GeV, $Q^2 < 0.0015$ GeV^2

$63 \cdot 10^3$ events

$$\alpha_\pi = 2.0 \pm 0.6\,(\text{stat}) \pm 0.7\,(\text{syst})$$

assumption: $\alpha_\pi + \beta_\pi = 0$

Plot: B. Badelek (COMPASS) 2015



COMPASS: C. Adolph et al. Phys. Rev. Lett. 114 (2015) 062002

Experimental information

Experiments		$(\alpha - \beta)_{\pi^\pm}$
$\gamma p \to \gamma \pi^+ n$	Mainz (2005)	$11.6 \pm 1.5_{\rm stat} \pm 3.0_{\rm syst} \pm 0.5_{\rm mod}$
L. Fil'kov, V. Kashevarov (2005)		$13.0^{+2.6}_{-1.9}$
$\gamma\gamma \to \pi^+\pi^-$	available data	
A. Kaloshin, V. Serebryakov (1994)		5.25 ± 0.95
$\gamma\gamma \to \pi^+\pi^-$	MARK II	
J.F. Donoghue, B. Holstein (1993)		5.4
$\gamma\gamma \to \pi^+\pi^-$	MARK II	
D. Babusci *et al.* (1992)		
$\gamma\gamma \to \pi^+\pi^-$	PLUTO	$38.2 \pm 9.6 \pm 11.4$
	DM 1	34.4 ± 9.2
	DM 2	52.6 ± 14.8
	MARK II	4.4 ± 3.2
$\gamma p \to \gamma \pi^+ n$	Lebedev Inst. (1986)	40 ± 24
$\pi^- Z \to \gamma \pi^- Z$	Serpukhov (1983)	$15.6 \pm 6.4_{\rm stat} \pm 4.4_{\rm syst}$
	COMPASS (2015)	$4.0 \pm 1.2_{\rm stat} \pm 1.4_{\rm syst}$

Kaon polarizabilities

- The first calculation was done by Donoghue and Holstein

  J. F. Donoghue and B. R. Holstein, Phys. Rev. D40, 3700 (1989)

- Recent review on hadron polarizabilities

  B. R. Holstein and S. Scherer, Ann. Rev. Nucl. Part. Sci. 64, 51 (2014) [arXiv:1401.0140 [hep-ph]].

- Recall the results for charged pion polarizabilities at order p^4:

$$
\begin{aligned}
(\alpha_1 - \beta_1)_{\pi^+} &= \frac{\alpha}{24\,\pi^2\,F_\pi^2\,M_\pi}\,\bar{\ell}_\Delta\,, \\
(\alpha_1 + \beta_1)_{\pi^+} &= 0\,.
\end{aligned}
$$

- Express SU(2) LECs ℓ_i^r via SU(3) LECs L_i^r:

$$
\begin{aligned}
\ell_5^r &= \frac{1}{12}\nu_K + L_{10}^r\,, \qquad \nu_K = \frac{1}{32\pi^2}\left(\ln\frac{M_K^2}{\mu^2} + 1\right), \\
\ell_6^r &= \frac{1}{6}\nu_K - 2L_9^r\,.
\end{aligned}
$$

Kaon polarizabilities

- Replace $F_\pi \to F_K$ and $M_\pi \to M_K$. One gets the expressions for the charged kaon polarizabilities at order p^4:

$$(\alpha_1 - \beta_1)_{K^+} = \frac{8\alpha}{F_K^2 M_K} (L_9^r + L_{10}^r) ,$$
$$(\alpha_1 + \beta_1)_{K^+} = 0 .$$

- The numerical values:

$$(\alpha_1)_{K^+} = 0.58 \times 10^{-4} \text{fm}^3 ,$$

J. F. Donoghue and B. R. Holstein, Phys. Rev. D 40, 3700 (1989).

$$(\alpha_1)_{K^+} = (0.64 \pm 0.10) \times 10^{-4} \text{fm}^3 .$$

F. Guerrero and J. Prades, Phys. Lett. B 405, 341 (1997)

- The quark confinement model predicts significantly higher value:

$$(\alpha_1)_{K^+} = 2.3 \times 10^{-4} \text{fm}^3 .$$

M. A. Ivanov and T. Mizutani, Phys. Rev. D 45, 1580 (1992).

Summary

- ChPT is successful tool to analyse low-energy physics
- Chiral expansion for the $\gamma\gamma \to \pi\pi$ amplitude at the Compton threshold converges quite rapidly
- Two-loop result for the dipole charged pion polarizability $(\alpha_1 - \beta_1)_{\pi^+}$ is in agreement with very well known low-energy theorem
- However, there is a clash almost a factor of 2 (!) between this result and several experiments
- The last precise measurement of α_{π^+} performed by COMPASS is found in agreement with ChPT.