Radiative Corrections for COMPASS

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JG|U

SFB 1044
THE LOW-ENERGY FRONTIER
OF THE STANDARD MODEL

PRISMA

Motivation

Lepton-nucleon scattering – the goal:

High-precision measurements of the 'nucleon structure'

➜ measure form factors, structure functions, (generalized) parton distribution functions, ...

- at low Q^2 elastic and quasi-elastic scattering
  ➜ form factors, polarizabilities, ...
- at high Q^2 deep inelastic scattering
  ➜ parton distribution functions, GPDs, GDAs, ...

The interesting physics is encoded in FFs, PDFs, ...
test the dynamics of the strong interaction
QCD precision physics

Lepton scattering: only via electromagnetic and weak interaction

➜ well-controlled and separable perturbative treatment

Overview

- Properties of radiative corrections
  - folding and unfolding
  - leptonic radiation
  - kinematical effects
  - a few examples
  - radiation from quarks

➜ Implementation in DJANGOH

Radiative corrections

Measure FFs, PDFs, etc by comparing data with theoretical predictions:

$$\sigma_{\rm exp} = \sigma_{\rm theory}[F_n(x, Q^2, \ldots)]$$

High precision requires knowledge of higher-order corrections

$$\sigma_{\rm theory} = \sigma^{(0)}[F_n] + \alpha_{\rm em}\sigma^{(1)}[F_n] + \ldots$$

- Emission of real photons
  experimentally often not distinguished from non-radiative processes:
  soft photons, collinear photons
  ➜ ”radiative corrections”
- Virtual corrections: loop diagrams
  needed to cancel infrared divergences (Bloch-Nordsieck)
- Electroweak effects
  Z-, W-boson exchange ($O(G_F)$)
  and higher-order electroweak corrections ($O(\alpha G_F)$)

Classification of $O(\alpha)$ corrections

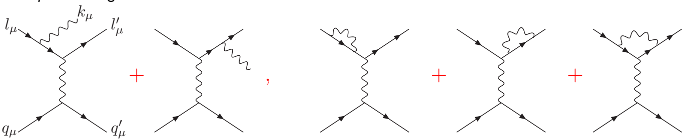
- Radiation from the lepton
  model independent (universal)
- Radiation from the hadronic initial/final state
  parton model: radiation from quarks
  to be considered as a part of the nucleon structure
- Interference of leptonic and hadronic radiation
  2γ exchange
  new structure
- vacuum polarization (γ and Z-boson self energy)
  universal
- purely weak corrections

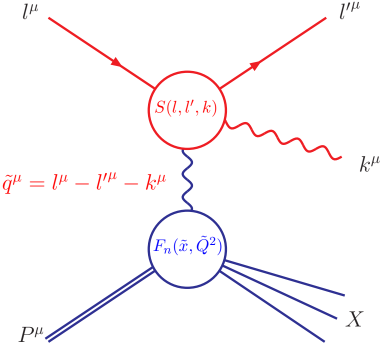
Note: for NC-scattering straightforward separation
Rule: respect gauge invariance
IR divergences: need to combine real and virtual radiation

Leptonic radiation

Feynman diagrams for leptonic radiation at $O(\alpha)$ (NC)

for *eq* scattering:





radiative leptonic tensor $S_{\mu\nu}(l, l', k)$ is

- gauge invariant
- infrared finite
- universal

(includes Born + loops: $\delta^{(4)}(k^\mu)$)

Observed cross section:
convolution of true cross section $\otimes$ radiator function

$$\mathrm{d}\sigma^{\mathrm{obs}}(p, q) = \int \frac{d^3 k}{2k^0} R(l, l', k)\, \mathrm{d}\sigma^{\mathrm{true}}(p, q-k)$$

or, for the structure functions:

$$F_n^{\mathrm{obs}}(x, Q^2) = \int \mathrm{d}\tilde{x}\mathrm{d}\tilde{Q}^2\, R_n(x, Q^2; \tilde{x}, \tilde{Q}^2)\, F_n^{\mathrm{true}}(\tilde{x}, \tilde{Q}^2)$$

Can be extended to include higher-order effects: multi-photon emission, soft-photon exponentiation, e^+e^--pair creation,
R_n known analytically to second order, $O(\alpha^2)$

(p^μ: proton, $q^\mu = l^\mu - l'^\mu$, shifted momentum transfer $\tilde{Q}^2 = -(q-k)^2$)

In turn: Unfolding
determination of the true structure functions F_n from the measured ones, i.e. invert

$$F_n^{\rm obs}(x, Q^2) = \int \mathrm{d}\tilde{x}\mathrm{d}\tilde{Q}^2 R_n(x, Q^2; \tilde{x}, \tilde{Q}^2) F_n^{\rm true}(\tilde{x}, \tilde{Q}^2)$$

A typical approach in practice: iterative solution

But, the problem is "ill-defined",
i.e., no unique solution, large uncertainties, numerically unstable

Difficult to treat radiative and detector effects separately (acceptance cuts, efficiencies, ...)

A comparison of unfolding algorithms (H1, ZEUS, NMC):
Badelek et al., J. Phys. G. Nucl. Part. Phys. 19 (1993) 1671

with partial fractioning, write: $R(l, l', k) = \frac{J}{k \cdot l} + \frac{F}{k \cdot l'} + \frac{C}{\tilde{Q}^2} + \ldots$

- initial state radiation, $k \cdot l$ small for $\sphericalangle(e_{in}, \gamma) \to 0$
- final state radiation, $k \cdot l'$ small for $\sphericalangle(e_{out}, \gamma) \to 0$
- Compton peak, $\tilde{Q}^2$ small for $p_T(e_{out}) \simeq p_T(\gamma)$

ISR, FSR: narrow peaks, width $\simeq \sqrt{m_l/E_l}$: collinear or mass singularities

upon angular integration: large logarithm $\propto \frac{\alpha}{\pi} \log \frac{Q^2}{m_e^2} \simeq 10\,\%$

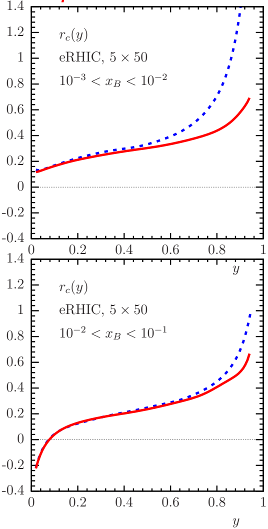
Note: $E^2_{\gamma,\mathrm{max}} \propto Q^2 \frac{1-x}{x}$

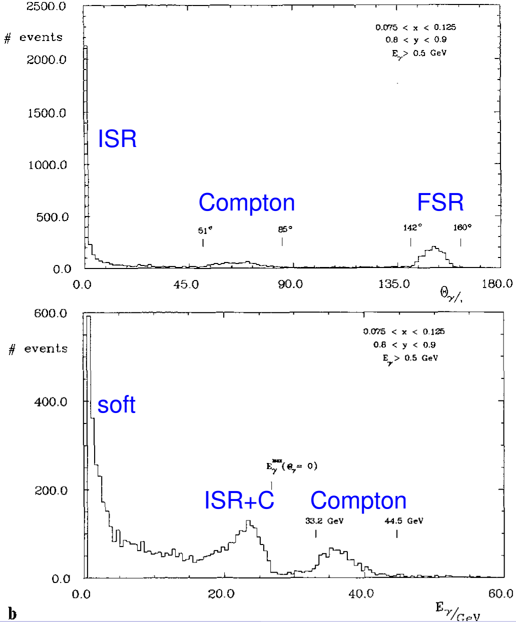
➜ large corrections at large Q^2 and at small x

➜ Radiation suppressed at small Q^2 and at large x,
large negative corrections from uncancelled virtual contributions

$r_c(y) = d\sigma_{O(\alpha)}(y)/d\sigma_{\mathrm{Born}}(y) - 1$, influence of a cut on $W_{had} > 1.4$ GeV

$e^- p$ at 5×50 GeV^2 $\qquad$ $e^- p$ at 30×325 GeV^2





ep at HERA:

$E_e = 30$ GeV

$0.075 \leq x \leq 0.125$

$0.8 \leq y \leq 0.9$

$E_\gamma > 0.5$ GeV

HERACLES/DJANGOH generates events in 4 NC channels:

- non-radiative, including soft-photon radiation below a (unphysical) cut-off and virtual (loop) corrections
- radiative with peak $\frac{J}{k \cdot l}$, called ISR
- radiative with peak $\frac{F}{k \cdot l'}$, called FSR
- radiative with peak $\frac{C}{\tilde{Q}^2}$, called Compton

(+ channels for CC, elastic radiative tail)

Note:
- the separation based on partial fractioning, $R(l, l', k) = \frac{I}{k \cdot l} + \frac{F}{k \cdot l'} + \frac{C}{\tilde{Q}^2}$ (formal, not physical)
- non-radiative is not equal to Born

$$F_n^{\rm obs}(x, Q^2) = \int \mathrm{d}\tilde{x}\mathrm{d}\tilde{Q}^2 R_n(x, Q^2; \tilde{x}, \tilde{Q}^2) F_n^{\rm true}(\tilde{x}, \tilde{Q}^2)$$

Note: shifted kinematics, e.g.,

$$Q^2 = -(l - l')^2 \to \tilde{Q}^2 = -(l - l' - k)^2$$

➜ expect strong dependence on experimental prescriptions for measuring kinematic variables

- leptonic variables: measure E and θ of scattered lepton ➜ x and Q^2
- hadronic variables: E, θ from hadronic final state ➜ $\tilde{x}$ and $\tilde{Q}^2$
- mixed variables: combine information from leptonic and hadronic final state

➜ need full Monte-Carlo modelling

Radiation of (hard) photons ➜ shifted kinematic variables:

$$Q^2 = -(l - l')^2 \to \tilde{Q}^2 = -(l - l' - k)^2$$

and

$$x = \frac{Q^2}{2P \cdot (l - l')} \to \tilde{x} = \frac{\tilde{Q}^2}{2P \cdot (l - l' - k)}$$

Radiator function is folded with

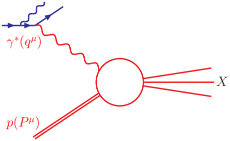
$$d\sigma(\tilde{x}, \tilde{Q}^2) \propto \frac{1}{\tilde{Q}^2}$$

➜ correction factor $d\sigma_{O(\alpha)}(x, Q^2)/d\sigma_{\rm Born}(x, Q^2)$ enhanced by $Q^2/\tilde{Q}^2$
Note: $\tilde{Q}^2 \ll Q^2$ possible: $\tilde{Q}^2_{\rm min} = \frac{x^2}{1-x} M_N^2$

➜ radiative tail, Compton peak

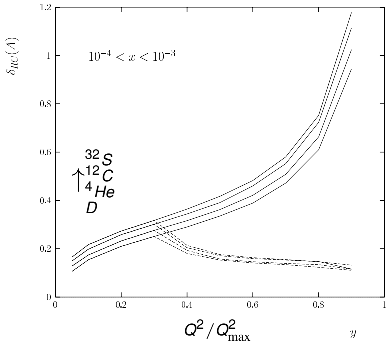
back to photoproduction

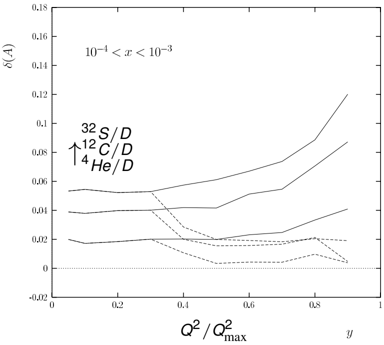
➜ γ-PDF



$\delta_{RC}(A) = d\sigma_{O(\alpha)}(A)/d\sigma_{\rm Born}(A) - 1$ for nuclei with $A = 2Z$ and
$\delta(A)$: corrections to the ratio $d\sigma_{O(\alpha)}(A)/d\sigma_{O(\alpha)}(D)$

Contribution from inelastic tail:



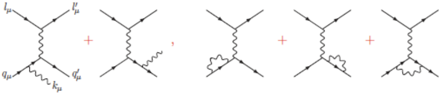


Full lines: leptonic variables without cuts (inelastic tail only)

dashed lines: with cuts on $E - p_z$ and $p_{T,\rm had}$ HERA workshop 1991

Hadronic radiation

at large Q^2: DIS, parton model
emission of photons
like emission of gluons



infrared divergences (soft photons / gluons) cancel with loops, collinear emission gives rise to corrections $\frac{\alpha}{2\pi} \log m_q^2$, but quark masses are ill-defined

➜ factorization: absorb collinear divergences into parton distribution functions

$$d\sigma = \sum_f d\hat{\sigma}_f (1 + \delta_f(Q^2; m_q^2)) q_f(x)$$

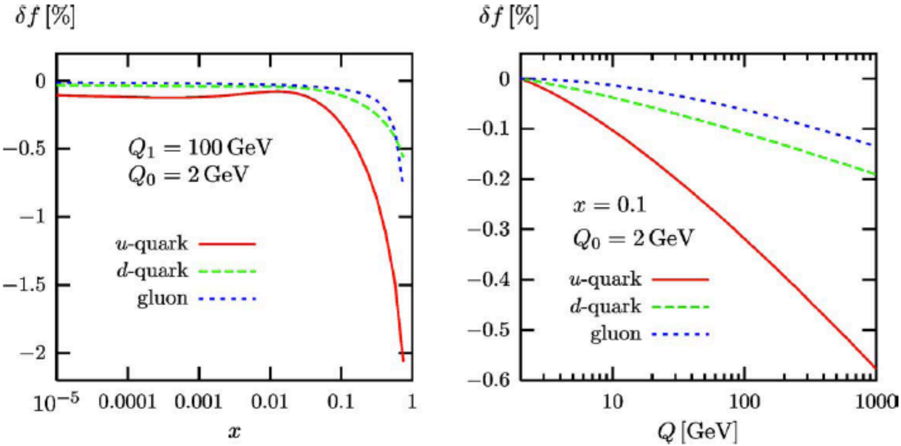
$$d\sigma = \sum_f d\hat{\sigma}_f (1 + \delta_f(Q^2; m_q^2)) q_f(x) = \sum_f d\hat{\sigma}_f \hat{q}_f(x, Q^2)$$

renormalized parton distribution functions

$$\hat{q}_f(x, Q^2) = (1 + \delta_f(Q^2; m_q^2)) q_f(x)$$

➜ modified scaling violations

well-known in QCD, $\overline{\text{MS}}$ factorization different charges of *u*- and *d*-quarks ➜ isospin-violating effect



implemented in MRST2004, NNPDF — HS'95; Roth, Weinzierl, PLB590

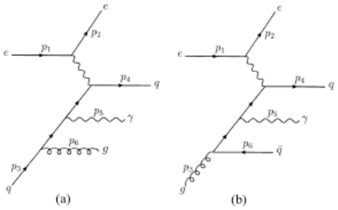
relevant for precision predictions, e.g. *W* production at the LHC

In practice: Do not include corrections due to radiation from quarks
Quarkonic QED corrections considered as contained in the PDFs

Direct photons

Acces to u/d PDF ratio

Need higher-order QCD corrections, $O(\alpha_s)$
including $q \to q + \gamma + g$, $g \to q + \bar{q} + \gamma$
and non-perturbative input for
$q, g \to \gamma$ fragmentation functions



Need specialized calculations and programs:
Aurenche, Fontannaz, Guillet, Heinrich;
Kramer, HS for $ep \to \gamma$+jets

Monte-Carlo implementation

Monte-Carlo approach in HERACLES:

- inclusive lepton nucleon scattering
- complete QED and electroweak corrections at $O(\alpha)$
- NC and CC scattering, polarized lepton, polarized nucleon
- elastic tail

Full event generation in DJANGO:

- universal leptonic corrections at $O(\alpha)$
- interface to LEPTO, JETSET
- jets, parton showers, hadronic final state
- SOPHIA for low-mass hadronic final states

Originally for electron scattering at high energies: HERA

Resources

HERACLES: An Event Generator for *ep* Interactions at HERA Energies Including Radiative Processes: Version 1.0,
A. Kwiatkowski, H. Spiesberger, H.-J. Möhring,
Comput. Phys. Commun. 69 (1992) 155

DJANGO: Combined QED and QCD radiative effects in deep inelastic lepton-proton scattering: The Monte Carlo generator DJANGO6,
K. Charchula, G. A. Schuler, H. Spiesberger,
Comput. Phys. Commun. 81 (1994) 381

Proceedings of workshops:
"Physics at HERA", 1991 (with TERAD, Bardin et al.),
"Monte Carlo generators for HERA physics", 1998 (for CC, with HECTOR, Arbuzov et al.)

Used (and tested) extensively in data analyses of H1 and ZEUS

QCD-based event generation, valid at large Q^2: parton model
Need Parton Distribution Functions: interface to LHAPDF

HERACLES provides also models for low Q^2 structure functions

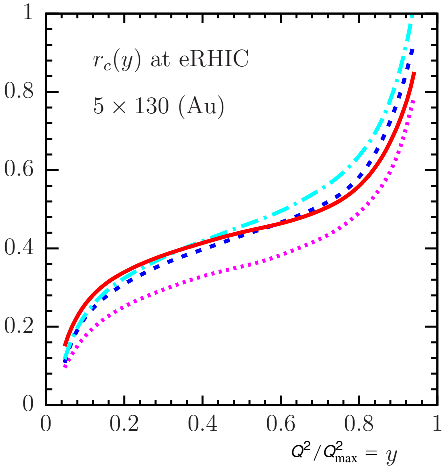
- exponentially low-Q^2 suppressed PDFs
- VDM or Regge inspired phenomenological parametrizations:
  ALLM: Abramowicz et al.,
  BK: Badelek, Kwiecinski,
  DL: Donnachie, Landshoff

Heavy nuclei:
Models for nuclear shadowing, nuclear parton distribution functions

Polarized target:
Models for polarized PDFs (see PRD88)

(Easy to extend)

$r_c(y) = d\sigma_{O(\alpha)}(y)/d\sigma_{\mathrm{Born}}(y) - 1$
for e^-Au at 5×130 GeV^2, $10^{-3} \leq x_{Bj} \leq 10^{-2}$,



different nuclear PDFs

- full red: CTEQ61M with EPS09
- dash-dotted light-blue: CTEQ61M with EPS08
- dashed blue: CTEQ61M with EKS98
- dotted magenta: HKN

There is substantial dependence on PDF input

... not a summary

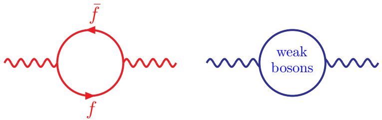
Not covered:

- How to correct data?
  Radiative corrections closely related to experimental conditions
- Vacuum polarization, electroweak loops, box graphs, parameter relations, Δr
- DJANGOH modified for μ scattering
  thanks to N. Pierre for testing for COMPASS

Appendix

Vacuum polarization

Self energy diagrams of the exchanged boson (γ and Z)



$\propto \log \frac{Q^2}{m_f^2} \to O(10\,\%)$ small, $O(1\,\%)$

Photon self energy = vacuum polarization, absorbed in the running fine structure constant:

$$\alpha \to \alpha(Q^2) = \frac{\alpha}{1 - \Pi_\gamma(Q^2)}$$

Z-boson self energy: a small correction if written in terms of:

$$\frac{\alpha}{s_W^2 c_W^2} \to \frac{M_Z^2 G_\mu \sqrt{2}}{\pi} \frac{1 - \Delta r}{1 - \Pi_Z(Q^2)}$$

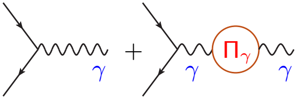
(with s_W^2, c_W^2: sin and cos of the weak mixing angle; G_μ the muon decay constant; Δr one-loop corrections to the muon decay: renormalization)

Effective couplings

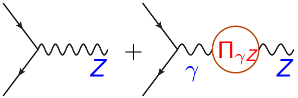
Including quantum loop corrections, universal contributions:

Self energy diagrams of the exchanged boson (γ, Z, W)

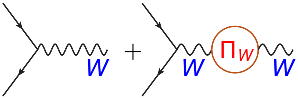
Schematically:



➜ $\alpha \to \alpha + \Delta\alpha = \alpha(Q^2)$

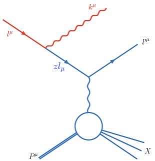


➜ $\sin^2\theta_W \to \sin^2\theta_W + \Delta\sin^2\theta_W = \sin^2\theta_{eff}(Q^2)$



➜ $G_F \to G_F(1 - \Delta r)$

e.g., for initial-state radiation: assume $k^\mu = (1-z)l^\mu$
➜ Radiator function



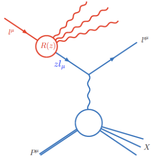
$$R_{\mathrm{ISR}} = \frac{\alpha}{2\pi}\,\frac{1+z^2}{1-z}\,\log\frac{Q^2}{m_e^2}$$

$(+\delta(1-z)$ from loops $\to$ $+$-distribution $1/(1-z)_+)$

$$\mathrm{d}\sigma_{\mathrm{ISR}} = \int \frac{\mathrm{d}z}{z} R_{\mathrm{ISR}}(z)\,\mathrm{d}\sigma_{\mathrm{Born}}(l^\mu \to z l^\mu)$$

(similar for final-state radiation)

Can be extended to include multi-photon emission:



$$R^{(2)}_{\mathrm{ISR}}(z) = \int_z^1 \frac{\mathrm{d}z'}{z'} R^{(1)}_{\mathrm{ISR}}(z')R^{(1)}_{\mathrm{ISR}}(z/z') + \ldots$$

Solution of evolution equations like DGLAP
Known at $O(\alpha^2)$ (complete) and partially at $O(\alpha^3)$

Corrections due to soft photons are universal

sum of real and virtual contributions: δ^{IR} (finite and gauge invariant)

$$1 + \delta^{\mathrm{tot}} = 1 + \delta^{\mathrm{IR}} + \delta^{\mathrm{fin}} \to \exp(\delta^{\mathrm{IR}})(1 + \delta^{\mathrm{fin}})$$

δ^{IR} contains $\log(E_\gamma^{\mathrm{max}})$ and $L_e = \log(m_e^2/Q^2)$:

$$1 + \frac{\alpha}{2\pi}(L_e - 1)\ln\frac{E_\gamma^{\mathrm{max}}}{E_e} + \ldots \to \left(\frac{E_\gamma^{\mathrm{max}}}{E_e}\right)^{\frac{\alpha}{2\pi}(L_e - 1)} (1 + \ldots)$$

(in the $\gamma^* p$ cms: $E_\gamma^{\mathrm{max}} = \frac{1}{2}\sqrt{y(1-x)S}$, i.e. important at low y and large x)

soft photon exponentiation Yennie, Frautschi, Suura, 1961